Response of the Macroeconomy to Uncertainty Shocks: the Risk Premium Channel

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January 31, 2019

Abstract

Uncertainty shocks are also risk premium shocks. With countercyclical risk aversion (RA), a positive shock to uncertainty not only increases risk, but it also elevates RA as consumption growth falls. The combination of high RA and high uncertainty produces significant risk premia in bad times, which in turn exacerbate the decline of macroeconomic aggregates and equity prices. Empirically, we document that local projection coefficients capturing the data response to the interaction of risk aversion and uncertainty are statistically significant and economically large. Indeed, heightened levels of RA during the 2008 crisis amplified the drop in output and investment by 41% and 28%, respectively, at the recession trough. Theoretically, we show that a New-Keynesian model with endogenously time-varying risk aversion via Campbell and Cochrane (1999) can produce large falls in output and investment close to matching their data counterparts following positive uncertainty shocks.

JEL classification: C32, C63, E32, E44
Keywords: Risk Aversion, Uncertainty, Conditional IRF, Dynamic Economies.

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We thank Martin Andreasen, Nick Bloom, Brent Bundick, Sulkhan Chavleishvili (Discussant), Anthony Diercks (Discussant), Luca Gambetti (Discussant), Nicola Gennaioli, Ralph Luetticke, Ilaria Piatti (Discussant), Franck Portier, and Jesus Fernández-Villaverde for valuable comments. We also thank seminar participants at the 2017 SITE Volatility Conference (Stanford), the 25th Finance Forum Conference (Universitat Pompeu Fabra), the 49th Money, Macro, the Finance Conference (King's College London), University College London, the 2018 MFA Annual Meeting, the Frontiers of Finance (WBS), the 2018 SITE Asset Pricing Conference (Stanford), the 2018 Asset Pricing Conference at Collegio Carlo Alberto.

1 Introduction

Finance matters to the macroeconomy. In the postmortem of the 2008 Great Recession, numerous papers have highlighted the significance of the financial network connecting institutions, firms, and households in generating and prolonging the crisis. Examples include Schularick and Taylor (2012), Mian et al. (2017), and Mian and Sufi (2018). Of course, the idea of a debt or credit cycle is not new. Bernanke et al. (1999) establish the importance of external financing premium in affecting business activity.[1] Geanakoplos (2010) discusses the leverage cycle as a primary driver of macroeconomic fluctuations through collateralized borrowing. Brunnermeier and Sannikov (2014) explicitly incorporate debt financing in a general equilibrium model to show that endogenous risk faced by market participants produces large declines in macro aggregates when adverse shocks are realized. In this paper, we extend this line of research to show that, in the presence of time-varying risk aversion, risk premium of financial assets is a crucial determinant of the macroeconomic response to risk shocks.

The interaction between risk and risk aversion is well understood in the literature. In a simple setting, risk premium under the Consumption CAPM (Breeden, 1979) can be expressed as the product of risk aversion, risk in the form of consumption growth uncertainty, and the correlation between returns and consumption growth. With countercyclical risk aversion, uncertainty shocks not only increase risk but also increase the level of risk aversion as consumption growth declines. The combined effect leads to large risk premia in bad times, which causes investment and output to further deteriorate. This amplification channel is important because, as we show in the paper, the impact of uncertainty shocks is not fully realized unless its interaction with risk aversion is taken into account.

Empirically, we estimate the impulse response of the macroeconomy to uncertainty shocks in the presence of time-varying risk aversion employing the Smoothed

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[1] Also see Kiyotaki and Moore (1997).

Local Projection (SLP) method of Barnichon and Brownlees (2016). Practically speaking, this means performing separate predictive regressions of output, consumption, or investment on uncertainty and the interaction of uncertainty with risk aversion, controlling for standard predictors. The coefficient loading on the interaction term is called the state multiplier and can be interpreted as the intensifying reaction of the dependent variable to uncertainty shocks, due to the risk premium effect, in addition to the baseline response. In this setting, the impulse response of the variable of interest can then be calculated by combining the response to uncertainty alone and the response to the risk premium term, across forecast horizons. Since the estimated state multipliers are negative across forecast horizons, higher risk aversion results in an amplification of the decline of macroeconomic variables (output, consumption, or investment) following a positive shock to uncertainty.

Relying on the empirically estimated SLP regressions, one can perform counterfactual analysis in the data by comparing the fitted value of the forecast with the fitted value when the state multiplier is set to zero. As an application, we show that conditional on the fact that risk aversion was elevated during the 2008 crisis, the fall in output and investment driven by uncertainty was deepened by 41% and 28%, respectively.[2] This finding demonstrates that the interaction between risk aversion and uncertainty is a potentially important channel through which financial market conditions contributed to the deterioration of the macroeconomy during the crisis.

The notion of time-varying risk aversion has gained traction in the macroeconomics and finance literature in recent decades. Grounded in theoretical models with habit (Abel (1990), Constantinides (1990) and Campbell and Cochrane (1999)), aggregate risk aversion in the economy can exhibit counter-cyclical variation as evidenced by the countercyclical risk premium in stock returns.[3] Cohn, Engelmann, Fehr and Maréchal (2015) and Guiso, Sapienza and Zingales (2018) provide additional experimental evidence on countercyclical risk aversion. In the theory section

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[2] Depending on the risk aversion proxy and the forecasting horizon, the amplifying effect on investment can be higher.

[3] See Chan and Kogan (2002).

of this paper, we discuss the interaction between risk aversion and uncertainty within a DSGE model with stochastic volatility (SV) in productivity.

Specifically, we build a standard New-Keynesian model with Epstein and Zin (1989) – Weil (1990) preferences to allow for the separation of risk aversion from the elasticity of intertemporal substitution.[4] Moreover, we nest Campbell and Cochrane (1999) external habit inside the Epstein-Zin-Weil recursive utility to generate endogenous time variation in risk aversion, which is a function of the inverse of the surplus consumption ratio. We establish the following two main findings. First, risk aversion amplifies the magnitude of macroeconomic responses to uncertainty shocks through the risk premium channel. Higher uncertainty precipitates an economic decline that lowers surplus consumption and increases risk aversion. The interaction between risk aversion and uncertainty produces countercyclical risk premia that aggravates the negative impact of uncertainty further. Second, habit-induced time-varying risk aversion is a powerful mechanism that allows the model to reproduce the observed decline of output and investment in the data following a positive shock to uncertainty. In particular, the presence of time-varying risk aversion causes the drop in output to double and the fall in investment to deepen by about four times.

As validation of consistency, we reproduce the SLP analysis using simulated data from our New-Keynesian model featuring endogenous risk aversion and uncertainty in productivity. We show that the estimated state multipliers are negative across various specifications. Furthermore, the conditional impulse responses of simulated output, consumption, and investment are in line with those generated by actual data. In general, the dependent macroeconomic variable falls following a positive shock to uncertainty, and the decline is intensified when risk aversion increases.

High-order perturbation techniques have become one of the standard method for solving DSGE models.[5] It is also well know that risk premia are unaffected by first-

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[4] In Appendix F, we study the interaction of risk aversion and uncertainty in the open economy model of Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez and Uribe (2011). The amplification effect of high coefficient of risk aversion on uncertainty shocks holds also in this setup. We refer the readers to the appendix for details of the analysis.

[5] See Aruoba, Fernández-Villaverde and Rubio-Ramírez (2006) for a discussion about perturba-

order terms and completely determined by those second- and higher-order terms. A widespread macro-finance *separation* paradigm, first proposed by Tallarini (2000), suggests that the moments of macroeconomic quantities are not very sensitive to the addition of second- and higher-order terms. This result is important since it implies that by varying the risk aversion parameter while holding the other parameters of the model constant, one is able to fit the asset pricing facts without compromising the model's ability to fit the macroeconomic data.[6] Our paper suggests that risk aversion not only determines the level of asset returns, but it also matters in calibrating the model to match the standard deviations of macroeconomic variables to those in the data. The simultaneity of risk aversion and uncertainty in driving macroeconomic dynamics poses an additional challenge in our understanding of time-varying expected returns as risk cannot be filtered solely by observing macroeconomic volatilities. The macro-finance separation does not hold in DSGE models featuring stochastic volatility when the solution technique takes into account the non-linearity of the model.

Our paper is related to various streams of literature in economics. First, the use of time-varying uncertainty has a long tradition in the financial economics literature. E.g. Kandel and Stambaugh (1991) study the implications for asset returns of time-varying first and second moments of consumption growth in a model with a representative Epstein-Zin investor. In a similar spirit, Bansal and Yaron (2004) incorporate time-varying first and second moments of consumption growth and recursive preferences in an endowment economy, and show that stochastic volatility not only generates time-variation in risk premium but also significantly increases the mean of the equity risk premium. Our results add another dimension of complication in extending these types of macro-finance models that employ stochastic volatility from endowment economies to full general equilibrium as macroeconomic and asset pricing moments need to be calibrated simultaneously.

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tion and alternative solution methods.

[6] Risk aversion only appears in the perturbation solution in higher than first-order terms, see Koijen, van Binsbergen, Rubio-Ramírez and Fernández-Villaverde (2008).

Second, an increasing body of research studies how uncertainty fluctuations influence business cycle dynamics. Within the framework of irreversible investment (see Bernanke (1983), Dixit and Pindyck (1994), Abel and Eberly (1996), Hassler (1996)), Bloom (2009) studies the propagation of *firm-level* uncertainty shocks. Following an increase in uncertainty about future profitability, firms will slow down activities that cannot be easily reversed, i.e. they "wait and see". After the heightened uncertainty is resolved, pent-up demand for capital goods leads to an investment boom. Related, another growing literature stresses the interaction of risk and economic activity propagated through financial, rather than physical frictions. Using a model with financial frictions, Gilchrist, Sim and Zakrajsek (2014) argue that increases in firm risk lead to an increase in bond premia and the cost of capital which, in turn, triggers a decline in investment activity and measured aggregate productivity. Arellano, Bai and Kehoe (2016) show that firms downsize investment projects to avoid default when they face higher risk. Christiano, Motto and Rostagno (2014) analyze the macroeconomic implications of volatility shocks in the context of a financial accelerator model adapted from Bernanke, Gertler and Gilchrist (1999). Alfaro, Bloom and Lin (2018) show how firm's external financing cost can amplify the consequence of uncertainty shocks. In a closely related paper focusing on ambiguity aversion rather than risk aversion, Bianchi, Ilut and Schneider (2018) estimate a business cycle model with financial frictions and find that uncertainty about marginal product of capital and operating cost can induce comovements in investment, stock prices, leverage, and payout role over the business and medium-term cycles in a way consistent with the data. The analysis presented here shows that when risk aversion is elevated, uncertainty shocks have larger and prolonged impact. Our risk aversion channel is intimately tied to the financial market frictions channel such that volatility fluctuations drive macroeconomic outcomes: risk aversion can rise with tightening financial constraints or vice versa, and the macroeconomic reaction to uncertainty shocks may intensify as risk premium increase.

Third and more recently, the literature has also started investigating the impact of shocks to *aggregate* uncertainty. Justiniano and Primiceri (2008) and Fernández-Villaverde and Rubio-Ramírez (2007) estimate dynamic equilibrium models with heteroskedastic shocks and show that time-varying volatility helps to explain the Great Moderation between 1984 and 2007. Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014) find that risk shocks are an important factor in explaining business cycles in emerging market economies. Fernández-Villaverde, Guerrón-Quintana, Kuester and Rubio-Ramírez (2015) document the important role of fiscal volatility for output fluctuations. Basu and Bundick (2017) study the interaction of aggregate risk shocks with precautionary saving in an environment with nominal rigidities. We contribute to this literature by theoretically and empirically investigating the interaction of aggregate uncertainty with risk aversion.

Finally, Gourio (2012)[7] examines the joint implication of risk aversion and time-varying risk on macroeconomic dynamics in the context of time-varying probability of disaster risk. In contrast, our paper explores the interaction between stochastic volatility and risk aversion.[8] In a related paper, Chen, Cooper, Ehling and Xiouros (2018) study the impact of time-varying risk aversion on the real business cycle as risk aversion affects the optimal production decision of firms.

2 Risk Aversion and Uncertainty: Empirical Evidence

In this section we study the amplifying effect of risk aversion on uncertainty shocks by examining the dynamic impulse responses of macroeconomic aggregates in the data. Furthermore, we illustrate how elevated level of risk aversion during the 2008 financial crisis directly contributed to the sharpened decline in output and investment in the recession. Later, we repeat the empirical exercise on simulated data generated by the NK-EZ-Habit model to validate our finding.

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[7] Gourio (2013) extends the application to credit spreads.

[8] In Proposition 3, Gourio (2012) shows that uncertainty in the probability of disaster translates to a *level* shock to the time discount factor with constant volatility. Thus, our analysis of second moment shocks to productivity is different.

2.1 Macroeconomic Responses to Uncertainty Shocks and Time-Varying Risk Aversion

In this section we estimate the dynamic responses of macroeconomic quantities to uncertainty shocks when risk aversion is time-varying. The estimation of state-dependent impulse response functions has recently been the subject of expressed interest in macroeconomics, see e.g. Auerbach and Gorodnichenko (2012a), Auerbach and Gorodnichenko (2012b), and Ramey and Zubairy (2018) for investigations of the size of fiscal multipliers when the economy is in recession, or more broadly, during periods of economic slack. Tenreyro and Thwaites (2016) examine the response of the U.S. economy to monetary policy shocks predicated on the state of the business cycle. To the best of our knowledge, the role of risk aversion underlying the macroeconomic response to uncertainty shocks is unexplored so far.

To estimate the state-dependent IRFs, we rely on the smoothed version of Jorda (2005) local projections developed by Barnichon and Brownlees (2016).[9] The Smooth Local Projections (SLP) strikes a balance between the efficiency of Vector Autoregressions (VAR) and the robustness (to model misspecification) of the Local Projections (LP) approach. In practice, SLP consists in estimating LP under the assumption that the impulse response is a smooth function of the forecast horizon. Specifically, we estimate an h-step ahead predictive regressions,

$$y_{t+h} = \alpha_h + (\beta_{0,h} + \beta_{1,h} RA_t)\,\mathrm{UNC}_t + \sum_{i=0}^{p} \gamma_{i,h} w_{t-i} + u_{t+h} \qquad (1)$$

where h ranges from 0 to H and p is the number of lags used for the control variables, w_t. y_{t+h} is the h period ahead realization of the macroeconomic variable of interest. RA_t is the state variable. UNC_t is our measure of uncertainty. To capture state dependence, the response of y_{t+h} to uncertainty at time t is a linear function, $\beta_{0,h} + \beta_{1,h}\mathrm{RA}_t$, of risk aversion. In what follows, the $\beta_{1,h}$ coefficient capturing the amplification/contraction effect due to risk aversion is called the *state multiplier*.

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[9] We thank C. Brownless for clarifying various aspects about the SLP technique.

We are interested in knowing whether uncertainty shock has a larger effect on, e.g., output during high risk aversion states.

For our empirical application, we include gross domestic product (GDP), consumption, investment, hours worked, the GDP deflator, the Standard & Poor's 500 Stock Price Index, and a measure of the stance of monetary policy as control variables. Appendix A provides details on the data construction. We employ two alternative measures of uncertainty and of risk aversion, which gives a total of four possible specifications. Specifically, to proxy for uncertainty we use either the index of economic policy uncertainty (EPU) created by Baker, Bloom and Davis (2016), or the financial uncertainty series constructed in Ludvigson, Ma and Ng (2017) using the framework of Jurado, Ludvigson and Ng (2015). To proxy for risk aversion we use either the (inverse of) surplus consumption computed as the (negative) moving average of past consumption growth, $-\sum_{j=0}^{\infty} \phi^j \Delta c_{t+j}$, or the consumption-price ratio. With regard to the former, we follow Wachter (2006) and truncate the moving average to 40 quarters and employ a decay rate $\phi = 0.97$. Finally, both measures of risk aversion are standardized to have mean zero and unit variance, and we plot the response of the outcome variable y_{t+h} when RA_t is at its average value (i.e. $RA_t = 0$, dubbed average state), and when it is one standard deviation above ($RA_t = 1$) or below ($RA_t = -1$) its average value.

Our choice of risk aversion proxies is motivated by the Campbell and Cochrane (1999) habit model. In this model, risk aversion is time-varying and inversely related to the surplus consumption ratio. In turn, the surplus consumption is essentially consumption relative to a slow-moving average of past consumption (see also discussion in Wachter, 2006; Muir, 2017). Under the original calibration of the Campbell and Cochrane (1999) model, the surplus consumption ratio is nearly linear in the price-dividend ratio.[10] However, we prefer to work with the consumption-price ratio since Menzly, Santos and Veronesi (2004) show that - in an economy with multiple securities and predictable dividend growth - the consumption-price ratio is less

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[10] See Figure 3 in Campbell and Cochrane (1999) and Figure 3 in Wachter (2006).

sensitive than the dividend-price ratio to changes in expected dividend growth, and more sensitive to changes in the surplus ratio. As a consequence, the consumption-price ratio captures better variation in the aggregate discount rate induced by time-varying risk aversion.[11] Analogously, Lettau and Ludvigson (2001) argue that $\widehat{cay}$, the consumption-wealth ratio, is countercyclical in nature, and it is tied to time varying risk aversion in the Campbell and Cochrane (1999) framework, and Haddad and Muir (2018) employ $\widehat{cay}$ to capture aggregate or household risk aversion.[12] Finally, in Section 3 we show that the relationship between the slow moving average of past consumption and the price-consumption ratio with the surplus consumption and risk aversion continues to hold in our DSGE model with habit.

In our empirical exercise, we choose $p = 4$, and let all variables enter in log levels with the exception of the monetary policy measure. Also, to estimate the state dependent IRFs, we follow Barnichon and Brownlees (2016) and include the set of controls w_t and their interaction with the state variable, RA_t. We use a recursive identification scheme with the uncertainty measure ordered first and estimate our baseline empirical model using quarterly data over the 1967−2017 sample period.[13]

Figures 1 and 2 show the results when we employ EPU together with the surplus consumption ratio or the consumption-price ratio. Figures 3 and 4 report the same analysis when we replace EPU with the financial uncertainty index of Ludvigson, Ma and Ng (2017). In each figure, the left column plot the responses of GDP, consumption, and investment to an uncertainty shock that is realized (i) in a high risk aversion state ($RA_t = 1$), (ii) in an average state ($RA_t = 0$), and (iii) in a low risk aversion state ($RA_t = -1$).

[Insert Figures 1, 2, 3 and 4 about here.]

We start with EPU. Looking at the response when the risk premium channel is

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[11] Specifically, see Proposition 1 and 2 in Menzly et al. (2004).

[12] Haddad and Muir (2018) try to disentangle the effect of intermediary risk aversion from household risk aversion on asset prices.

[13] A similar identification strategy has been adopted in previous works (e.g. Bloom, 2009) and is standard in the literature.

shut off ($RA_t = 0$), we obtain the standard result in the literature (see, e.g., Baker et al., 2016; Basu and Bundick, 2017; Ludvigson et al., 2017) that higher uncertainty causes declines in output, consumption, and investment.[14] When the risk premium channel is active, Figures 1 and 2 present a novel and clear message about the state-dependent nature of uncertainty shocks: the response of the macroeconomy to a EPU shock is substantially larger when risk aversion is heightened. The peak decline in output, consumption, and investment when $RA_t = 1$ is roughly twice as large as the decline obtained in a low risk aversion environment when $RA_t = -1$. The gap between the responses in the high and low risk aversion state generally closes after two years.

The right column in Figures 1 and 2 plot the state multipliers obtained from SLP. Recall that the state multiplier, $\beta_{1,h}$, captures the extent to which time-varying risk aversion affects the IRF at each horizon. A negative value of the state multiplier implies that the IRF response to a positive uncertainty shock is more negative when the risk aversion is high in the economy ($RA_t = 1$). Independently from the risk aversion proxy, we find that the state multiplier is economically large and statistically significant for about eight quarters, and it generally becomes insignificant afterwards. Also, the state multiplier attains its minimum value after about one year, its value being slightly below -0.1% for output and consumption, and between -0.2% and -0.3% for investment. E.g., this implies that, after an EPU shock, investment is -0.5% below its steady state level when risk aversion is at its average value, and $-0.8\%(= -0.5 - 0.3)$ below steady state level when risk aversion is high (c.f. Panel (e) in Figures 1).

Turning to financial uncertainty we see results that are largely consistent with those discussed so far. In particular, the right columns in Figures 3 and 4 show that replacing EPU with the uncertainty index leaves the behavior of the state multiplier

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[14] Empirically, we find that the response obtained by estimating Eq. (1) and then setting $RA_t = 0$ is very close to the response obtained in absence of the interaction term between risk aversion and uncertainty (i.e. the standard unconditional impulse response). Therefore, we do not report the unconditional IRFs.

largely unaffected: the state multiplier still bottoms at about one quarter with values that are about -0.05% and -0.1% for output and consumption, and about -0.15% and -0.32% for investment. E.g., this implies that, after a financial uncertainty shock, investment is -0.85% below its steady state level when risk aversion is at its average value, and $-1.17\%(= -0.85 - 0.32)$ below steady state level when risk aversion is high (c.f. Panel (e) in Figures 4).

The Appendix shows additional robustness checks. First, motivated by the work of Santos and Veronesi (2016), we employ the financial intermediary leverage as measured by He et al. (2017) as a proxy for risk aversion.[15] Figure D.1 and D.2 show the results. In general, using leverage leads to more persistent differences between the response in the high and low risk aversion states. E.g., looking at Figure D.2 the response of output and consumption in the high risk aversion state converges to the response in the low risk aversion state only after four years. Figure D.3 shows the results when we use two alternative proxies for uncertainty and risk aversion recently proposed by Bekaert et al. (2017). Similar to Figures 1-4, the state multiplier significant decreases during the first six quarters. Thereafter, the state multiplier gradually converges to zero and become insignificant. The maximum value of the state multiplier is about -0.2 for output (at five quarters), -0.15 for consumption (at seven quarters) and -0.7 for investment (at six quarters).

2.2 The Financial Crisis: An Application

To quantify the amplifying effect of time-varying risk aversion on the economic impact of uncertainty shocks, we use the estimates from Eq. (1) to generate the fitted values of output and investment when the risk premium channel is turned on and off. In other words, we construct fitted values with the state multiplier, $\beta_{1,h}$, set to either the estimated value (high RA) or to zero (average RA). Specifically, we

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[15] Our measure of leverage is based on market prices (market leverage). In the model of Santos and Veronesi (2016), the debt-to-wealth ratio is monotonically decreasing in the surplus consumption ratio (see their Corollary 13), which can be seen as the inverse of risk aversion.

examine output and investment declines during the financial crisis using the post 2007Q4 sample and choose the forecast horizon, h, to be one quarter.

Figure 7 presents the time series plots of realized and fitted values of output, while Figure 8 presents the same plots for investment. RA is proxied by either the (inverse of) surplus consumption in subplot (a) or the consumption-price ratio in subplot (b).[16] Focusing on Figure 7, we see the one-quarter ahead forecast of output (dashed line) from the SLP matches the realized path of output (solid line) in both subplots well. In particular, the forecasted maximal drop in output appears within four quarters of the actual minimal output during the financial crisis. Next, we set the state multiplier to zero and repeat the forecast while keeping all other coefficient estimates from the SLP. The resulting fitted output path (square-dashed line) is plotted along the original forecast for counter-factual analysis. Figure 7 subplot (a) shows that relative to the level of output at the onset of the crisis, the maximal decline in output due to uncertainty is exacerbated by more than 300% when the risk premium channel is active (dashed vs. square-dashed lines), where risk aversion is proxied by surplus consumption. In subplot (b), the amplifying effect is roughly 41% when the risk premium channel is turned on, and risk aversion is approximated by the consumption-price ratio.

[Insert Figures 7 and 8 about here.]

Quantitatively, heightened risk aversion during the financial crisis generates significantly larger decline in investment due to uncertainty. Figure 8 presents the realized (solid line), the actual one-quarter ahead forecast (dashed line), and the counter-factual forecast (square-dashed line) of investment when risk aversion is zeroed. Similar to Figure 7, the SLP forecast of investment matches reasonably well with the realized path, especially in subplot (a) when surplus consumption is used as

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[16] In the interest of space, we report only the case where uncertainty is proxied by EPU. Please refer to Appendix D.2 for the corresponding figures using financial uncertainty as the uncertainty proxy.

the risk aversion proxy. Relative to the level of investment at the end of 2007, Figure 8 shows the maximal forecasted decline in investment is 28% and 38% greater, in subplots (a) and (b) respectively, when we allow for risk-aversion-dependence of uncertainty in the SLP.

Overall, the economic significance of time-varying risk aversion on macroeconomic dynamics cannot be overlooked. Our results from applying the SLP methodology to examine the financial crisis can perhaps be viewed in one of two ways. First, in the absence of the risk premium channel, the econometrician cannot decipher the true impact of uncertainty shocks on economic aggregates. Second, conversely, intensified risk aversion aggravated the depth of the recession by causing uncertainty shocks to be more effective through general equilibrium mechanism.

In the next section we present our DSGE model which we then use in Section 4 to revisit the interaction between risk aversion and uncertainty in relation to the macroeconomy using model simulated data.

3 A Theoretical Model of Uncertainty Shocks in the Presence of Time-Varying Risk Aversion

In this section, we examine the interaction of risk aversion and uncertainty using a variant of the standard New-Keynesian model. We nest the Campbell and Cochrane (1999) external habit process in Epstein-Zin-Weil recursive preferences such that the surplus consumption ratio becomes a key state variable that delivers time variation in risk aversion. In what follows, we refer to our baseline model as the “NK-EZ-Habit” model.

3.1 A New-Keynesian Model with Recursive Preferences and Habit

We build a small-scale dynamic stochastic general equilibrium model with monopolistic competition and sticky prices. Since the model economy is rather standard, we highlight, in particular, the two key ingredients for our analysis: the source of risk and the preference specification of the agent.

3.1.1 Uncertainty

We consider intermediate goods-producing firms i with the same constant returns to scale Cobb-Douglas production function, subject to a fixed cost of production Φ and their level of productivity Z_t:

$$Y_t(i) = \left(K_t(i)U_t(i)\right)^{\alpha} \left(Z_t N_t(i)\right)^{1-\alpha} - \Phi,$$

where α is the capital share, $U_t(i)$ is the rate of utilization of their installed physical capital, $K_t(i)$ is capital subject to the usual law of motion with adjustment cost, $N_t(i)$ is labor, and $Y_t(i)$ is the intermediate good. The technological process Z_t evolves according to a first-order autoregressive process with stochastic volatility:

$$Z_{t+1} = \left(1 - \rho_z\right) \bar{Z} + \rho_z Z_t + \sigma_{z,t+1}\varepsilon_{z,t+1} \tag{2}$$

$$\sigma_{z,t+1} = \left(1 - \rho_\sigma\right) \bar{\sigma_z} + \rho_{\sigma_z}\sigma_{z,t} + \sigma_{\sigma_z}\varepsilon_{\sigma,t+1} \tag{3}$$

with $\varepsilon_{z,t} \sim \text{i.i.d.}N(0,1)$ and $\varepsilon_{\sigma,t} \sim \text{i.i.d.}N(0,1)$. The innovations $\varepsilon_{z,t+1}$ and $\varepsilon_{\sigma,t+1}$ are assumed to be mutually independent at all leads and lags. In words, two independent innovations affect the level of productivity. The first innovation, $\varepsilon_{z,t+1}$, changes the level of productivity itself, while the second innovation, $\varepsilon_{\sigma,t+1}$, determines the spread of values for the productivity level.

3.1.2 Time-varying risk aversion

To generate endogenous time-varying risk aversion, we augment the Epstein and Zin (1989) recursive preference specification with Campbell and Cochrane (1999) external habit. Specifically, the Epstein-Zin utility function is defined over habit-adjusted consumption rather than consumption itself:

$$U_t = \left[\left(C_t^{h\,\eta} (1 - N_t)^{1-\eta} \right)^{\frac{1-\gamma}{\theta}} + \beta \left(E_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} , \tag{4}$$

where $\theta \equiv (1-\gamma)/(1-1/\psi)$, γ is the coefficient of risk aversion, ψ is the elasticity of intertemporal substitution, β is the subjective discount factor, η determines the Frisch elasticity of labor supply, and C_t^h is consumption with external habit such that $C_t^h = C_t \times S_t$. Consistent with Campbell and Cochrane (1999), we define S_t as the surplus consumption ratio, which evolves according to the following process:

$$\log(S_t) = (1-\rho_s)\bar{s} + \rho_s log(S_{t-1}) + \lambda_t^h log\left(\frac{C_t}{C_{t-1}} \right) . \tag{5}$$

This is a generalized AR(1) process with mean $\bar{s}$ and autoregressive coefficient ρ_s. λ_t^h is the sensitivity function of the surplus consumption ratio to consumption growth. It introduces non-linearity in the original Campbell and Cochrane (1999) model and it is key for generating time-varying equity risk premium within their endowment framework. Denoting $\log(S_t)$ as s_t, we define λ_t^h to be $\sqrt{\frac{1-\rho_s}{\gamma\sigma_z^2}}\sqrt{1-2(s_t-\bar{s})} - 1$.[17] Campbell and Cochrane (1999) further show that the time-varying local risk aversion coefficient is equal to the inverse of the surplus consumption ratio. In other words, when S_t is high, risk aversion of the representative agent is low and vice versa.

The corresponding real stochastic discount factor (SDF) of the NK-EZ-Habit

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[17] In the Campbell and Cochrane (1999) setting, this particular parameterization of λ_t^h allows to achieve a constant risk free rate in the model. This is no longer the case in our DSGE model with Epstein-Zin preferences. Nonetheless, we rely on the same parameter values in our calibration as can be seen from Table 1.

model is defined as:

$$M_{t,t+1} = \beta \left(\frac{{C^h_{t+1}}^{\eta}(1-N_{t+1})^{1-\eta}}{{C^h_t}^{\eta}(1-N_t)^{1-\eta}} \right)^{(1-\gamma)/\theta} \left(\frac{C^h_{t+1}}{C^h_t} \right)^{-1} \left(\frac{U^{1-\gamma}_{t+1}}{\mathbb{E}_t[U^{1-\gamma}_{t+1}]} \right)^{1-1/\theta}, \qquad (6)$$

where habit-adjusted consumption is, again, constructed over raw consumption and the surplus consumption ratio. As is common to habit driven SDFs, $M_{t,t+1}$ can be further decomposed into a consumption growth term, a term containing the growth rate of surplus consumption ratio, and, in this case, the continuation utility term resulting from recursive preferences.

When household labor is not fixed (as in our setting), the measure of risk aversion needs to be suitably modified. In particular, Swanson (2018) derives that the coefficient of absolute wealth-gamble risk aversion when the household has generalized recursive preferences and can vary its labor supply is given by:

$$R^a_t = \frac{-\mathbb{E}_t[U^{-\gamma}_{t+1}U''_{t+1} - \gamma U^{-\gamma-1}_{t+1}{U'_{t+1}}^2]}{\mathbb{E}_t[U^{-\gamma}_{t+1}U'_{t+1}]},$$

where U'_{t+1} and U''_{t+1} denote the first and second derivatives of the utility function with respect to total wealth. In Appendix B we show that with our preference specification, similar to Campbell and Cochrane (1999), wealth-gamble risk aversion is a function of the inverse of the surplus consumption ratio.

3.1.3 Intermediate Goods Producers

There is a continuum of intermediate goods producers that rent labor from households. The intermediate goods market is monopolistically competitive and producers face each period quadratic costs ϕ_P when changing their nominal price $P_t(i)$. The firms own their capital stocks $K_t(i)$ and face convex costs ϕ_K of changing the quantity of capital installed. In addition to prices, firms choose the rate of utilization of their installed physical capital $U(i)$ which affects its depreciation rate. Firm i chooses labor input $N_t(i)$, investment $I_t(i)$, and prices $P_t(i)$ to maximize cash flows

$D_t(i)/P_t(i)$ given aggregate demand Y_t and the aggregate goods price index P_t. Further, the intermediate goods firms all have the same constant-returns-to-scale Cobb-Douglas production function, subject to a fixed production cost Φ.

Each firm maximizes the discounted cash flows:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} M_{t,t+s} \frac{D_{t+n}(i)}{P_{t+s}} \tag{7}$$

subject to the production function,

$$\left[\frac{P_t(i)}{P_t}\right]^{-\theta_\mu} Y_t \leq \left[K_t(i)U_t(i)\right]^{\alpha} \left[Z_t N_z(i)\right]^{1-\alpha} - \Phi, \tag{8}$$

and subject to the capital accumulation equation,

$$K_{t+1}(i) = \left(1 - \delta\left(U_t(i)\right) - \frac{\phi_K}{2}\left(\frac{I_t(i)}{K_t(i)} - \delta\right)^2\right) K_t(i) + I_t(i) \ . \tag{9}$$

The cash flows are defined as follows:

$$\frac{D_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t}\right]^{1-\theta_\mu} Y_t - \frac{W_t}{P_t} N_t(i) - I_t(i) - \frac{\phi_P}{2}\left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1\right]^2 Y_t, \tag{10}$$

and capital depreciation is non-linearly determined by utilization:

$$\delta(U_t(i)) = \delta + \delta_1\left(U_t(i) - U\right) + \left(\frac{\delta_2}{2}\right)\left(U_t(i) - U\right)^2 \ . \tag{11}$$

Finally, each intermediate goods firm finances a fraction ν of its capital stock each period with one-period risk-less bonds which pay the one-period real risk-free interest rate. Therefore, total firm cash flows are split into payments to bond and equity holders:

$$\frac{D_t^E(i)}{P_t} = \frac{D_t(i)}{P_t} - \nu\left(K_t(i) - \frac{1}{R_t^R} K_{t+1}(i)\right) \ . \tag{12}$$

3.1.4 Final Goods Producers

The representative final goods uses $Y_t(i)$ units of each intermediate good, where $i \in [0,1]$ to assemble the final good. The market for final goods is perfectly competitive which results in zero profits in equilibrium. The aggregate goods price index P_t is defined as:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_\mu} di\right]^{\frac{1}{(1-\theta_\mu)}} \ . \tag{13}$$

3.1.5 Monetary Policy

The monetary authority sets the nominal interest rate, r_t, to stabilize inflation and output growth. Doing so, the monetary policy is in accordance with the Taylor rule:

$$r_t = r + \rho_\pi(\pi_t - \pi) + \rho_y \Delta y_t \ , \tag{14}$$

where $r_t = ln(R_t)$, $\pi_t = ln(\Pi_t)$, and $\Delta_t = ln(Y_t/Y_{t-1})$. The gross nominal interest rate R_t is further pinned down by the standard Euler equation:

$$1 = R_t \mathbb{E}_t \left[M_{t,t+1} \frac{1}{\Pi_{t+1}}\right] \ . \tag{15}$$

3.1.6 Equilibrium

In the symmetric equilibrium, all intermediate goods firms choose the same price, employ the same amount of labor, the same amount of capital and the same utilization rate. As a result, all intermediate goods firms have the same cash flows that are financed with the exogenously determined mix of bonds and equity. Intuitively, one can interpret the continuum of firms as one representative intermediate goods producer.

3.2 The Interaction of Risk Aversion with Risk

We solve the model using third order perturbation method. The results are robust to solution methods that keep the higher order terms in a Taylor series expansion, such as fifth order perturbation. In this section, we provide some intuition behind the link between risk aversion and stochastic volatility in a perturbation setting. Furthermore, we demonstrate the importance of terms higher than second-order order for this link to be present.

Consider a simple neoclassical growth model (see Appendix C.1 for a detailed description of the economy) where productivity z_t follows an autoregressive process with stochastic volatility:

$$\begin{aligned} Z_t &= \rho Z_{t-1} + \sigma_t \varepsilon_t \\ \sigma_t &= (1-\rho_\sigma)\,\bar{\sigma} + \rho_\sigma \sigma_{t-1} + \Lambda \eta u_t \end{aligned}$$

i.e., the z_t process is affected by two innovations, an innovation to technology, ε_t, and an innovation to the standard deviation of technology, u_t. Λ is called the perturbation parameter. Intuitively, if we set $\Lambda = 0$, we eliminate the sources of uncertainty in the model and the economy will (asymptotically) settle down at the steady state. For this model economy, we can define the state vector as

$$s_t = \left(\widehat{k}_t, Z_{t-1}, \widehat{\sigma}_{t-1}, \varepsilon_t, u_t; \Lambda\right)$$

where k_t is capital, and the hat sign denotes deviation of a variable from its steady state value, and we have incorporated the perturbation parameter Λ as a pseudo-state (where the pseudo is emphasized by the use of a semicolon to separate it from the pure states). With this notation, the third-order perturbation solution for consumption, c_t,[18] is of the form (see Appendix C.2–C.3 for a derivation):

$$\begin{aligned}\widehat{c}_t = & \sum_{i=1}^{5} c_{i,ss} s_{i,t} + \frac{1}{2}\sum_{i=1}^{5}\sum_{j=1}^{5} c_{ij,ss} s_{i,t} s_{j,t} + \frac{1}{2} c_{\Lambda\Lambda,ss} \\ & + \sum_{i=1}^{5}\sum_{j=1}^{5}\sum_{l=1}^{5} c_{ijl,ss} s_{i,t} s_{j,t} s_{l,t} + \frac{3}{6}\sum_{i=1}^{5} c_{i\Lambda\Lambda,ss} s_{i,t} \Lambda^2 + \text{H.O.T.}\end{aligned} \tag{16}$$

where we represent all the higher-order terms by "H.O.T.".

Under second order approximation (given by the first row in Eq. (16)), Binsbergen et al. (2012a) and Caldara et al. (2012) show that in models with Epstein-Zin-Weil preferences, the parameter of relative risk aversion (γ) only enters the solution in $c_{\Lambda\Lambda,ss}$, generating a constant in the policy rule that can be interpreted as the precautionary behavior toward risk.

In a third order approximation, risk aversion enters into those coefficients in the policy function that load on the square of the perturbation parameter (Λ^2), such as $c_{\sigma\Lambda\Lambda}$ and $c_{u\Lambda\Lambda}$. As a result, γ directly influences the effect of stochastic volatility on the endogenous variable. For a given value of $\widehat{\sigma}_{t-1}$ or u_t, higher γ amplifies the dynamics of the endogenous variable by making the coefficients $c_{\sigma\Lambda\Lambda}$ and $c_{u\Lambda\Lambda}$ more positive or more negative. Previously, we document that the link between risk aversion and risk is indeed present and significant in the data via estimated conditional impulse response functions. Next, we explore theoretically the implications of these interaction terms and show that they are quantitatively important.

4 Model Analysis

We calibrate the model and examine the dynamic response of the macroeconomy to uncertainty shocks. We perform the analysis in two ways. First, we shut down the habit channel and compare the impulse responses under a low RA specification and

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[18] We can proceed in analogous ways for all other variables and derive the appropriate formulas.

a high RA specification by changing the parameter governing risk aversion. Second, we turn on external habit and study the impulse responses when risk aversion is effectively time-varying. We show that risk aversion is a crucial determinant of the impact of uncertainty shocks on economic aggregates. A calibration featuring high coefficient of risk aversion and a model of time-varying risk aversion produce quantitatively similar amplifying effects on shocks to SV.

4.1 Calibration

To calibrate the baseline model we largely rely on parameter values from existing literature. For example, we set the intertemporal elasticity of substitution, ψ, is to 0.95 which is in line with papers that find it to be less than one, see e.g. Hall (1988). The risk aversion parameter $\gamma = 75$ lies between the values from Binsbergen et al. (2012b) (risk aversion of 66) and Rudebusch and Swanson (2012) (risk aversion of 110). Moreover, our calibration of the surplus consumption dynamics follows Campbell and Cochrane (1999). Finally, we choose the steady-state of hours worked and $\eta = 0.26$ such that the model has a Frisch labor supply elasticity of three.

Various firm related parameters are chosen to match corresponding quantities in the data: The capital share α is set to one third, the capital discount rate $\delta = 0.025$ is chosen to match the investment/output ratio and the leverage ratio in the economy is set to 0.2. Further, $\delta_1 = 0.031$ follows from the first-order condition for the capacity utilization in steady state. We then set the price adjustment cost parameter to $\Phi_P = 120$ which implies a Calvo parameter of 0.76 or, put differently, firms can reset prices about once a year. The capital adjustment costs $\Phi_K = 10$ are calibrated such that the model response of investment to uncertainty shocks compares well to the data.

Monetary policy related parameters such as reaction to inflation ($\rho_\pi = 1.5$) and output growth ($\rho_y = 0.25$) are standard. We further assume that the annualized inflation target equals two percent.

Lastly, parameters that determine the aggregate exogenous productivity process are set to the most conservative values found in the existing literature. For example, the persistence of productivity $\rho_z = 0.983$ and the volatility of the productivity volatility shock $\sigma_{\sigma_z} = 0.006$ are taken from Kung (2015). Finally, the steady-state volatility of the productivity level shock $\bar{\sigma}_z = 0.006$ is consistent with Schmitt-Grohé and Uribe (2012) and a lower bound to Andreasen (2012).

[Insert Tables 1 and 2 about here.]

Panel A of Table 2 compares model-implied moments against the data. Overall, our model performs reasonably well in terms of moment-matching. For example, the volatilities of output, consumption, and investment are comparable in terms of magnitude with those in the data. Conversely, the variation of hours worked in the data is larger than in the model. Panel B of Table 2 presents the model implied variance decomposition for the NK-EZ-Habit model. Despite the fact that level shocks are the main driver of variation in macro variables we observe that, thanks to the endogenous counter-cyclical variation of risk aversion, uncertainty shocks too explain a large part of the variation in macro variables. As we will see in Section 4.3, the model is capable of generating IRFs that are quantitatively in line with their empirical counterparts, particularly for investment.

4.2 Uncertainty Shocks, Risk Aversion and Macroeconomic Dynamics

To start we consider a special case of our model where there is no external habit, and the representative household chooses sequences of consumption, C_t, and labor, N_t, to maximize the standard Epstein-Zin-Weil utilities:

$$U_t = \left[\left(C_t^{\eta} (1 - N_t)^{1-\eta} \right)^{\frac{1-\gamma}{\theta}} + \beta \left(E_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \tag{17}$$

where γ is the key coefficient controlling risk aversion.[19] We refer to this model as to “NK-EZ”.

Figures 9(a) and 9(b) show the IRFs for a level and a volatility shock to productivity, respectively.[20] We present the impulse responses under third order perturbation for two calibrations: baseline RA and high RA. In the baseline RA, the coefficient of risk aversion is set to 75, and it is increased to 150 in the high RA calibration. Our analysis shows that the amplification effect of risk aversion is present only for impulse responses to a volatility shock in productivity; responses following a level shock do not display any sensitivity to the risk aversion parameter, see Figure 9(a). Focusing on the impulse responses to a positive uncertainty shock, Figure 9(b) shows that a higher level of risk aversion generates a more pronounced decline in output, consumption, and hours in response to an uncertainty shock.

[Insert Figure 9 about here.]

This evidence clearly shows that risk aversion has a role in influencing the dynamics of the theoretical model. However, within the NK-EZ model, we are limited to address the evidence by means of comparative statics, i.e. by exogenously changing the risk aversion parameter γ. To endogenize the risk aversion implication, we next turn to our “NK-EZ-Habit” model.

4.3 Model with Habit-Induced Time-Varying Risk Aversion

With habit-adjusted consumption in the utility function, the representative agent’s risk aversion (RA) displays time variation driven by the surplus consumption ratio (see Eq. (5), and discussion in Appendix B). In particular, when consumption growth is low (bad times), S_t is also low, making RA high. We show here that this

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[19] See Binsbergen et al. (2012b) and Swanson (2018).

[20] Appendix E discusses the technical details behind the computation of IRFs and variance decompositions in general equilibrium models featuring stochastic volatility.

endogenous counter-cyclicality of RA interacts with uncertainty shocks to generate large movements in economic variables.

Figure 10 shows the IRFs to a positive productivity uncertainty shock in our model. Figure 10(a) compares the model implied responses to the empirical responses. The subplot demonstrates that the NK-EZ-Habit model does a reasonable job in producing impulse responses that are largely consistent with those obtained from Smooth Local Projection (SLP) employing EPU as a proxy for uncertainty. In particular, we observe large falls in output, consumption, investment, and hours following an uncertainty spike in line with data. This provides us with some confidence that the calibrated model is performing as expected. Figure 10(b) contrasts the impulse responses of the NK-EZ-Habit model to those in the baseline NK-EZ model, which has a fixed value of RA of 75. We see that when RA exhibits habit-induced time variation, the quantitative impact of the uncertainty shock is significantly larger. The declines in output and the real rate more than double when external habit is active, the fall in investment more than quadruples, and the drop in hours worked more than triples.

[Insert Figure 10 about here.]

In fact, comparing Figures 9(b) and 10(b), it is noticeable that the quantitative impact of time-varying RA in the NK-EZ-Habit model is on par with that of the NK-EZ case featuring high RA ($\gamma = 150$). Furthermore, if we focus on the negative effect of productivity uncertainty on investment, we observe that doubling the coefficient of risk aversion in the NK-EZ model is not nearly enough (roughly -0.15%) to replicate the drop of about -0.5% observed in the data (see top right panel in Figure 10). On the contrary, the NK-EZ-Habit model is able to produce a decline in investment of more than -0.4% following a positive shock to SV in productivity.

We conclude that counter-cyclical RA is a powerful mechanism that intensifies the effectiveness of productivity uncertainty shocks. The interaction of risk aversion with risk generates quantitatively large movements in the New-Keynesian model that is consistent with those in the data without resorting to extreme calibrations of volatility shock size.

4.4 Model Implied Conditional Responses to Uncertainty Shocks

The NK-EZ-Habit setup allows us to study the endogenous response of the economy to productivity uncertainty shocks conditional on the level of risk aversion displayed by the representative agent. To do so, we use the model to simulate economies that span 204 periods (to match the data used in our empirical exercise in Section 2.1).

As we did in the data, within the model we proxy for risk aversion with either the inverse of the surplus consumption ratio or the consumption-price ratio. Indeed, Appendix B shows that with our preference specification, wealth-gamble risk aversion is a function of the inverse of the surplus consumption ratio. Moreover, Appendix B.1 shows that the behavior of the surplus consumption ratio in the model is well captured by the price-consumption ratio.[21] We then use the simulated series of output, consumption, investment, consumption-price ratio, and the surplus consumption ratio and perform the SLPs as outlined above.

Figure 11 plots the SLP results from simulated data where we employ the surplus consumption ratio as a proxy for risk aversion. Similar to Figure 1−4, the left column shows the conditional IRFs of output, consumption, and investment following a positive one standard deviation shock to productivity volatility, and the right column shows the state multipliers from estimating equation (1), $\beta_{1,h}$, over horizons $h \in \{1, \ldots, 24\}$ for the same three variables. There are two main takeaways in Figure 11. First, conditional on high risk aversion (surplus consumption ratio is low and RA_t is high), output, consumption, and investment react more negatively

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[21] These two results extend the Campbell and Cochrane (1999) observation that the price-dividend ratio is nearly linear in the surplus consumption ratio (see their Figure 3) to a production economy.

to a positive uncertainty shock relative to the scenario where risk aversion is neutral ($RA_t = 0$). This is consistent with the corresponding empirical IRFs shown in Figures 1 and 3. Second, the estimated state multipliers are significantly negative for output, consumption, and investment in the simulated data causing high risk aversion ($RA_t = 1$ and $\beta_{1,h} RA_t < 0$) to generate stronger declines in those variables following an increase in uncertainty.

[Insert Figures 11 and 12 about here.]

Figure 12 shows the results when we use the consumption-price ratio as a proxy for risk aversion. In line with Figure 11, the IRFs in Figure 12 suggest that conditional on a high consumption-price ratio relative to its average (risk aversion is high), output, consumption, and investment drop more after a positive uncertainty shock. Conversely, when consumption-price ratio is relatively low, the economic responses to the same shock are significantly milder. The similarities between Figures 11 and 12 provide some assurance that the consumption-price ratio is indeed a credible instrument to proxy for the level of risk aversion in the data. Furthermore, our model is doing a reasonable job in capturing the amplifying effect of risk aversion on the impact of uncertainty shocks.

Next, we proxy financial uncertainty with model-implied market variance (i.e. the expected conditional volatility of the return on firm equity). This aligns the SLP estimations on simulated data to the empirical analysis in Figure 2 and 4 as closely as possible. Figures 13 and 14 show that our conclusion continues to hold when we replace productivity uncertainty with the model-implied market variance.

[Insert Figures 13 and 14 about here.]

Overall, the SLP results using simulated data presented here serve as validation bridging the NK-EZ-Habit model in Section 3 and the empirical findings in Section 2.1. First, we verify the theoretical implication from the model that risk aversion is a significant state variable in determining the impact of uncertainty shocks to macroeconomic aggregates. Second, similarities of the SLP plots using simulated data between the inverse of the surplus consumption ratio (Figure 11) and the model-implied consumption-price ratio (Figure 12) provide us with some confidence that the consumption-price ratio is indeed a credible proxy to risk aversion in the data.

5 Conclusion

Our study shows that not only risk matters to equilibrium outcomes, but more importantly, the degree of risk aversion determines the magnitude of these outcomes.

The effects we document here are quantitatively important: after a positive shock to uncertainty, the higher the risk aversion, the larger and more prolonged the decline in economic activity. Empirically, we show that conditional on the fact that risk aversion was elevated during the 2008 crisis, the fall in output and investment driven by uncertainty was deepened by 41% and 28%, respectively. From a theoretical perspective, a New-Keynesian model with endogenous time variation in risk aversion caused by habit is able to reproduce our empirical evidence using simulated data.

Our results could be relevant for policymakers to consider stochastic volatility, and its interplay with financial quantities via risk aversion, when implementing fiscal and monetary policy.

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Tables

Table 1: **Model Parameters**: This table reports the calibrated parameters for the baseline model. The parameters are organized in 4 subgroups that relate them to preferences, monetary policy, firms or shocks.

Preferences:			*Firms:*		
β	time discount parameter	0.994	α	capital share	0.333
γ	risk aversion parameter	75.000	δ	capital discount rate	0.025
ψ	intertemporal elasticity of substitution	0.950	δ_1	capital discount rate	0.031
η	consumption share	0.261	δ_2	capital discount rate	0.0003
ρ_s	AR(1) surplus consumption ratio	0.970	Φ_K	capital adjustment cost parameter	10.000
$\bar{s}$	steady-state surplus consumption	-3.016	Φ_P	price rigidity parameter	120.000
Shocks:			θ_u	elasticity across goods	6.000
ρ_z	AR(1) technology	0.983	ν_u	leverage ratio	0.200
$\bar{\sigma_z}$	steady-state volatility technology	0.006	*Monetary Policy:*		
ρ_{σ_z}	AR(1) volatility technology	0.826	Π	steady-state inflation	1.005
σ_{σ_z}	volatility of volatility technology	0.006	ρ_r	AR(1) short rate	0.750
			ρ_π	TR coefficient inflation gap	1.500
			ρ_y	TR coefficient output gap	0.250

Table 2: **Model Moments and Variance Decomposition**

Panel A reports empirical and model-implied unconditional moments. Empirical moments are obtained from HP-filtered data series from 1986:Q2 to 2014:Q4. Model-implied moments are obtained from simulating the model 200 times for 120 periods. The table reports the median model-implied moments along with the 95% confidence bands (in brackets). Panel B shows the variance decomposition for the different structural shocks in the model. The first two columns report data and model moments with all shocks active. The third and fourth columns show model moments with only level and volatility shocks to technology, respectively.

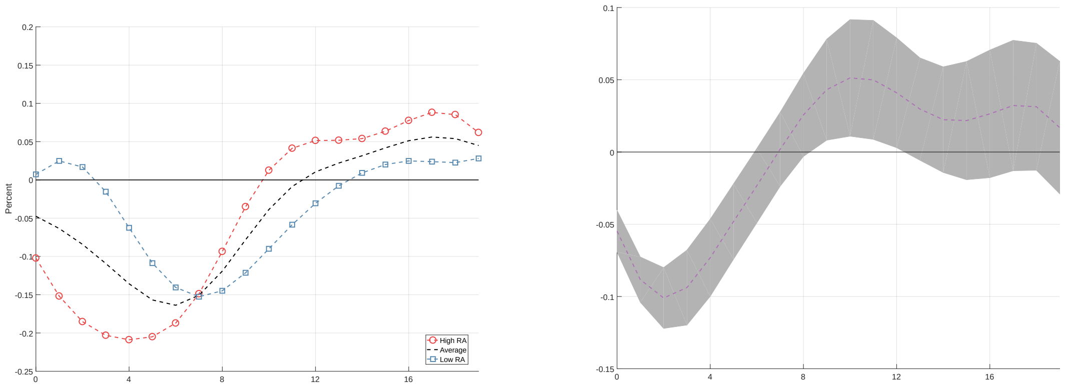
Panel A: Model Moments

	Standard Deviation		Autocorrelation	
	Data	Model	Data	Model
Output	1.11	1.53 [0.84,2.46]	0.88	0.77 [0.62,0.89]
Consumption	0.71	0.73 [0.44,1.06]	0.88	0.72 [0.56,0.89]
Investment	3.79	3.76 [2.05,6.36]	0.93	0.79 [0.66,0.93]
Hours Worked	1.41	0.40 [0.22,0.78]	0.93	0.84 [0.72,0.94]

Panel B: Variance Decomposition

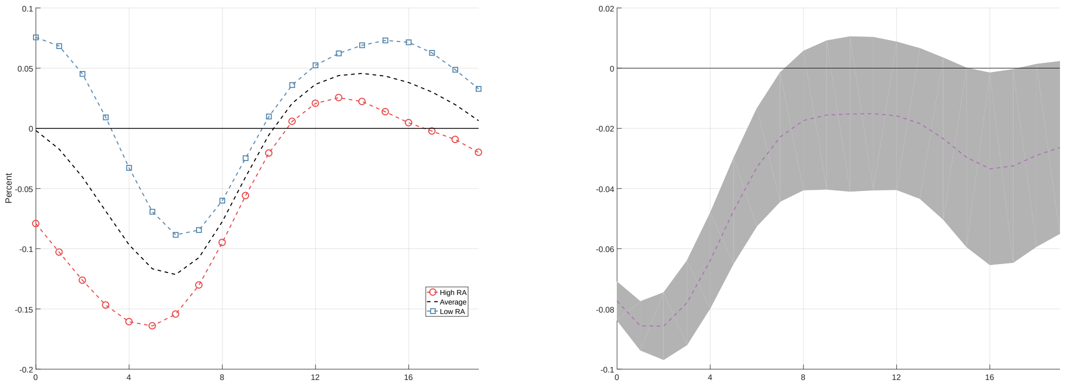
	Data	All Shocks	Level only	Volatility only
Output	1.11	1.53 [0.84,2.46]	0.69 [0.48,0.93]	0.27 [0.21,0.37]
Consumption	0.71	0.73 [0.44,1.06]	0.32 [0.23,0.41]	0.18 [0.14,0.25]
Investment	3.79	3.76 [2.05,6.36]	1.71 [1.11,2.38]	0.52 [0.42,0.71]
Hours Worked	1.41	0.40 [0.22,0.78]	0.17 [0.11,0.29]	0.19 [0.16,0.24]

Figures



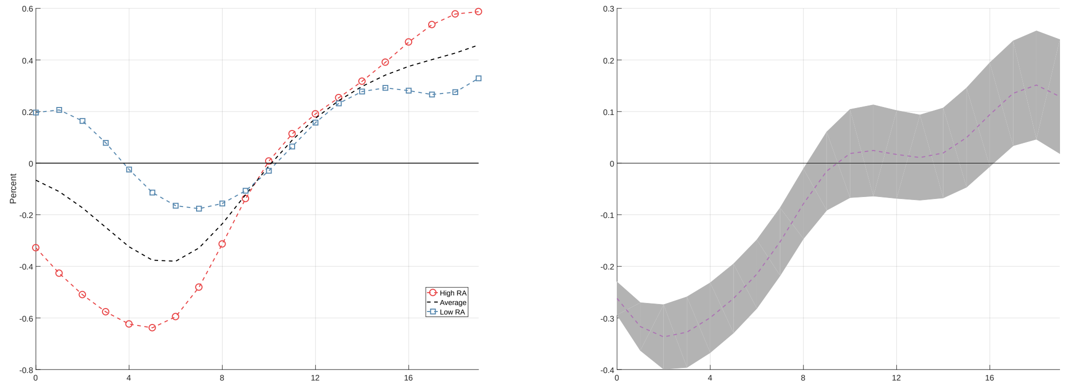
(a) Output.

(b) State Multiplier – Output.



(c) Consumption.

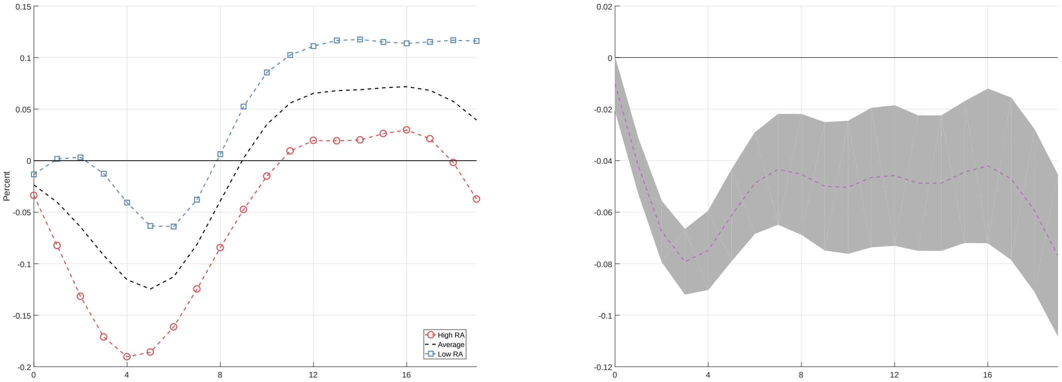
(d) State Multiplier – Consumption.



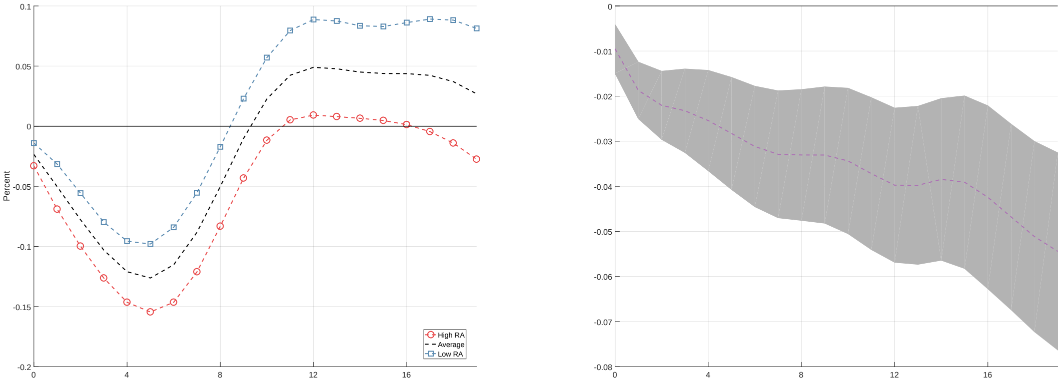
(e) Investment.

(f) State Multiplier – Investment.

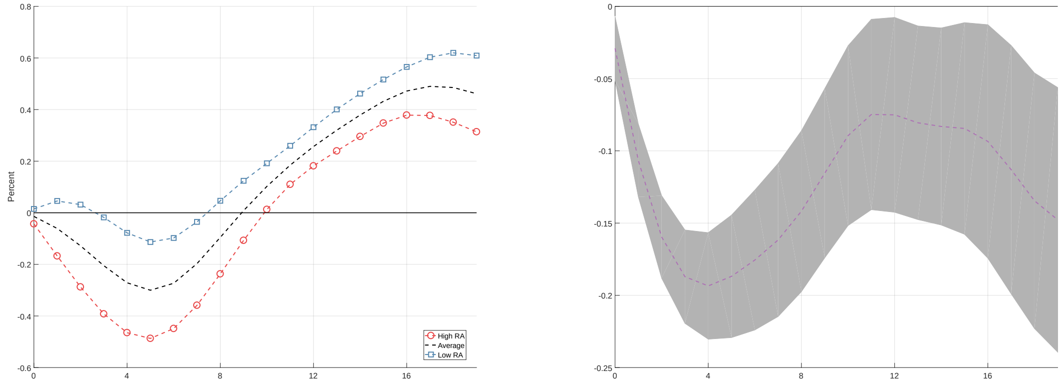
Figure 1: **State-dependent (surplus consumption) IR to an uncertainty shock (EPU)**: This figure plots the empirical impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the economic policy uncertainty (EPU) by Baker, Bloom and Davis (2016). Our proxy for risk aversion is the surplus consumption proxy, $\sum_{j=1}^{40} \phi^j \Delta c_{t-j}$. We standardize the risk aversion proxy to have zero-mean and unit variance. The figure shows the response when the state variable RA_t is one standard deviation below (low risk aversion; blue line with squares), at (black line), or above (high risk aversion; red line with circles) its average value. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column displays the state multiplier of the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to an uncertainty shock. The shaded areas denote 90% confidence intervals.



(a) Output. (b) State Multiplier – Output.

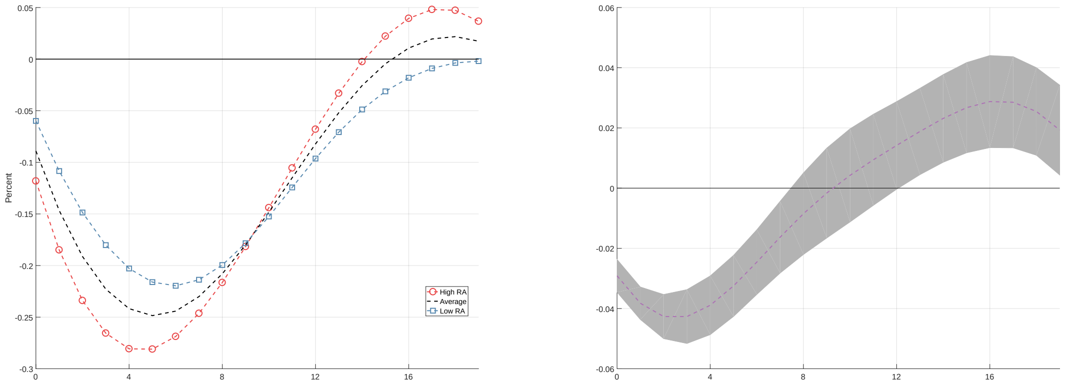


(c) Consumption. (d) State Multiplier – Consumption.



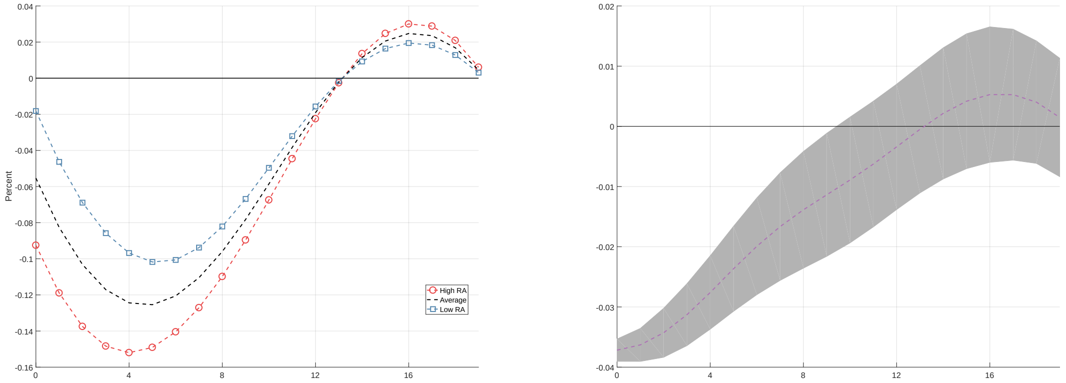
(e) Investment. (f) State Multiplier – Investment.

FIGURE 2: **State-dependent (consumption-price ratio) IR to an uncertainty shock (EPU)**: This figure plots the empirical impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the economic policy uncertainty (EPU) by Baker, Bloom and Davis (2016). Our proxy for risk aversion is the consumption-price ratio. We standardize the risk aversion proxy to have zero-mean and unit variance. The figure shows the response when the state variable RA_t is one standard deviation below (low risk aversion; blue line with squares), at (black line), or above (high risk aversion; red line with circles) its average value. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column displays the state multiplier of the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to an uncertainty shock. The shaded areas denote 90% confidence intervals.



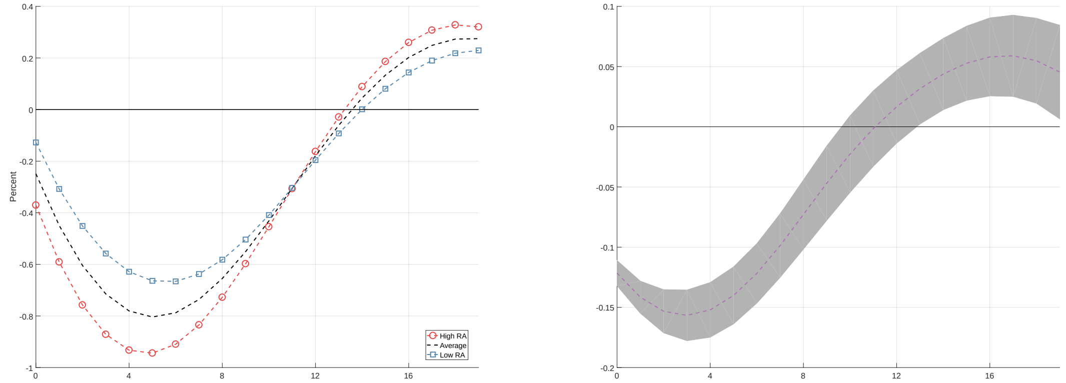
(a) Output.

(b) State Multiplier – Output.



(c) Consumption.

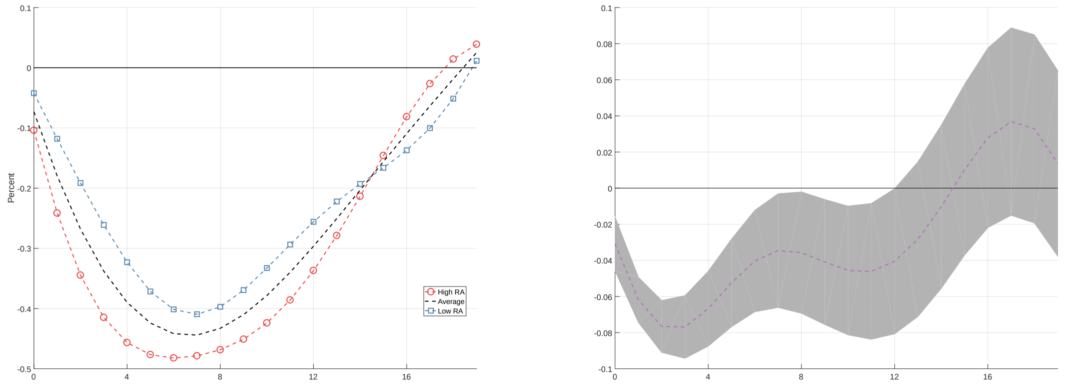
(d) State Multiplier – Consumption.



(e) Investment.

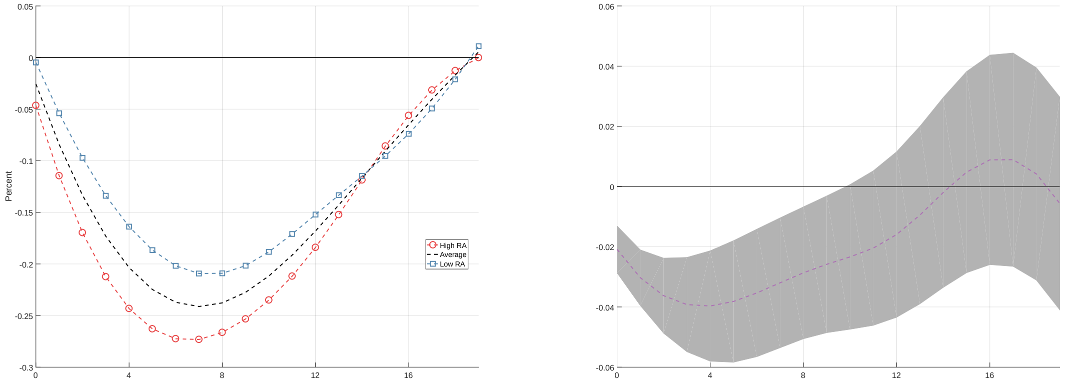
(f) State Multiplier – Investment.

FIGURE 3: **State-dependent (surplus consumption) IR to an uncertainty shock (JLN)**: This figure plots the empirical impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the financial uncertainty series by Jurado, Ludvigson and Ng (2015). Our proxy for risk aversion is the surplus consumption proxy, $\sum_{j=1}^{40} \phi^j \Delta c_{t-j}$. We standardize the risk aversion proxy to have zero-mean and unit variance. The figure shows the response when the state variable RA_t is one standard deviation below (low risk aversion; blue line with squares), at (black line), or above (high risk aversion; red line with circles) its average value. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column displays the state multiplier of the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to an uncertainty shock. The shaded areas denote 90% confidence intervals.



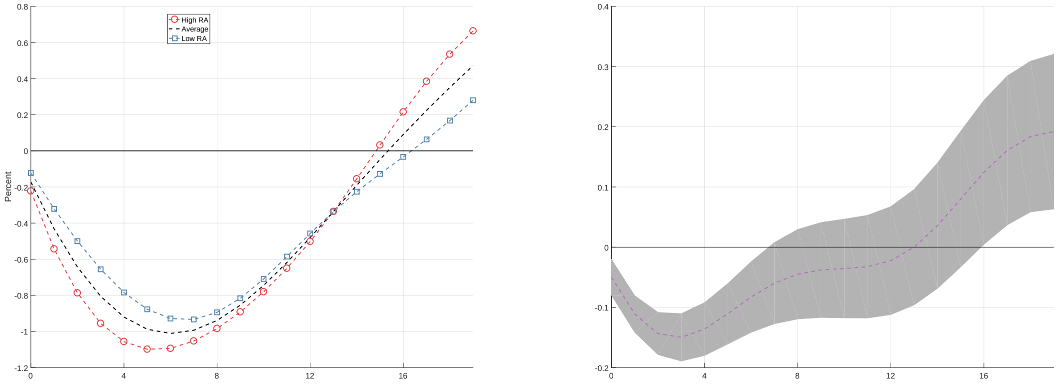
(a) Output.

(b) State Multiplier – Output.



(c) Consumption.

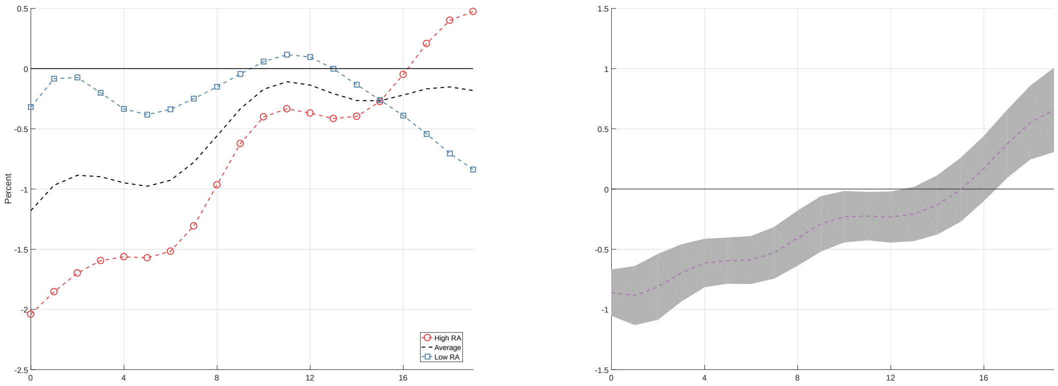
(d) State Multiplier – Consumption.



(e) Investment.

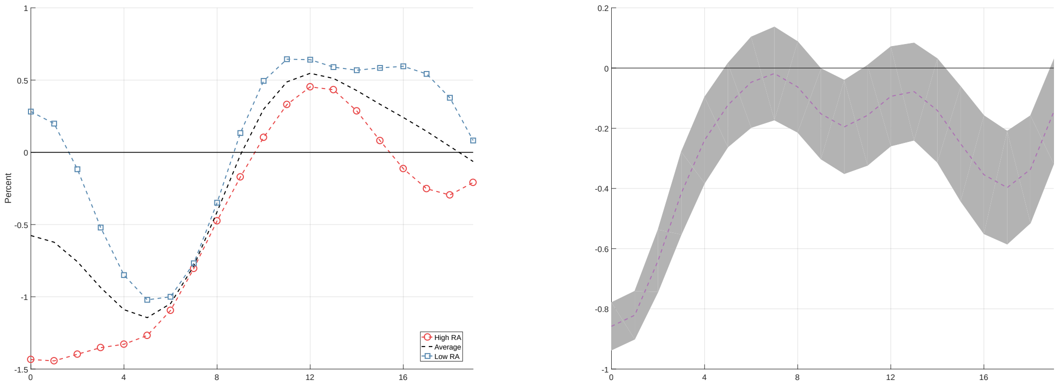
(f) State Multiplier – Investment.

FIGURE 4: **State-dependent (consumption-price ratio) IR to an uncertainty shock (JLN)**: This figure plots the empirical impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the financial uncertainty series by Jurado, Ludvigson and Ng (2015). Our proxy for risk aversion is the consumption-price ratio. We standardize the risk aversion proxy to have zero-mean and unit variance. The figure shows the response when the state variable RA_t is one standard deviation below (low risk aversion; blue line with squares), at (black line), or above (high risk aversion; red line with circles) its average value. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column displays the state multiplier of the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to an uncertainty shock. The shaded areas denote 90% confidence intervals.



(a) Stock prices and Surplus Consumption.

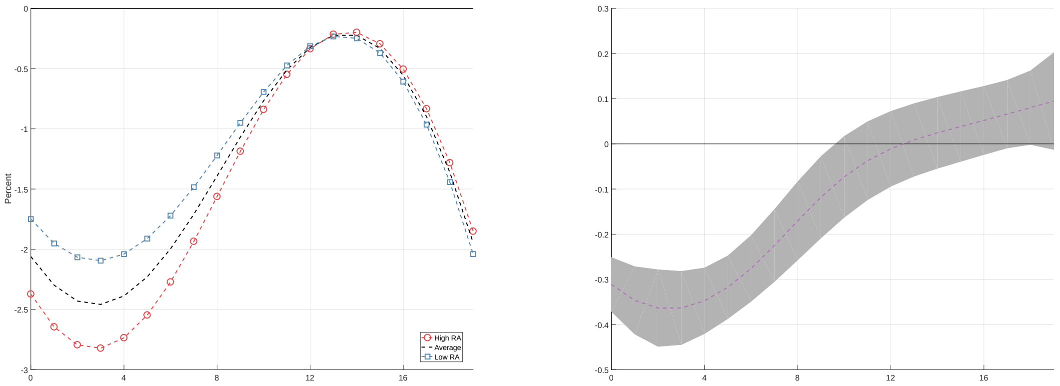
(b) State Multiplier – Stock prices.



(c) Stock prices and Consumption-Price ratio.

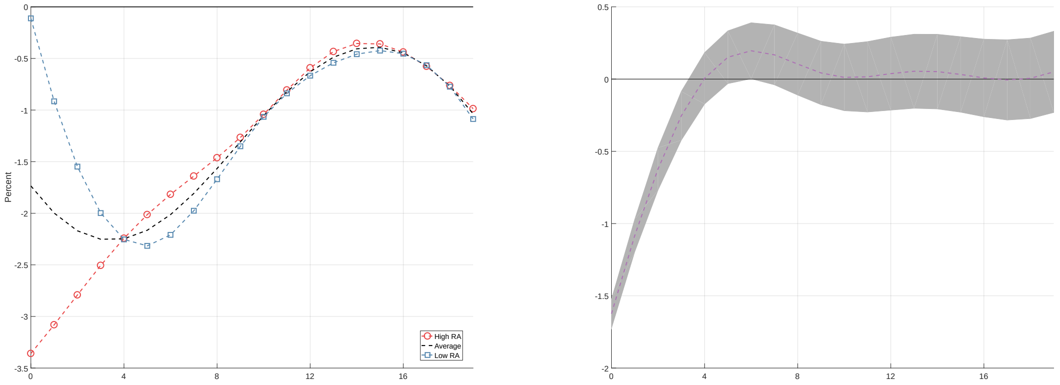
(d) State Multiplier – Stock prices.

FIGURE 5: **State-dependent IR of stock prices to an uncertainty shock (EPU)**: This figure plots the empirical impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the economic policy uncertainty (EPU) by Baker, Bloom and Davis (2016). To proxy for risk aversion we use either the surplus consumption proxy $\sum_{j=1}^{40} \phi^j \Delta c_{t-j}$ in Panel (a) and (b), or the consumption-price ratio in Panel (c) and (d). We standardize the risk aversion proxy to have zero-mean and unit variance. The figure shows the response when the state variable RA_t is one standard deviation below (low risk aversion; blue line with squares), at (black line), or above (high risk aversion; red line with circles) its average value. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column displays the state multiplier of the state-dependent IR of stock prices to an uncertainty shock. The shaded areas denote 90% confidence intervals.



(a) Stock prices and Surplus Consumption.

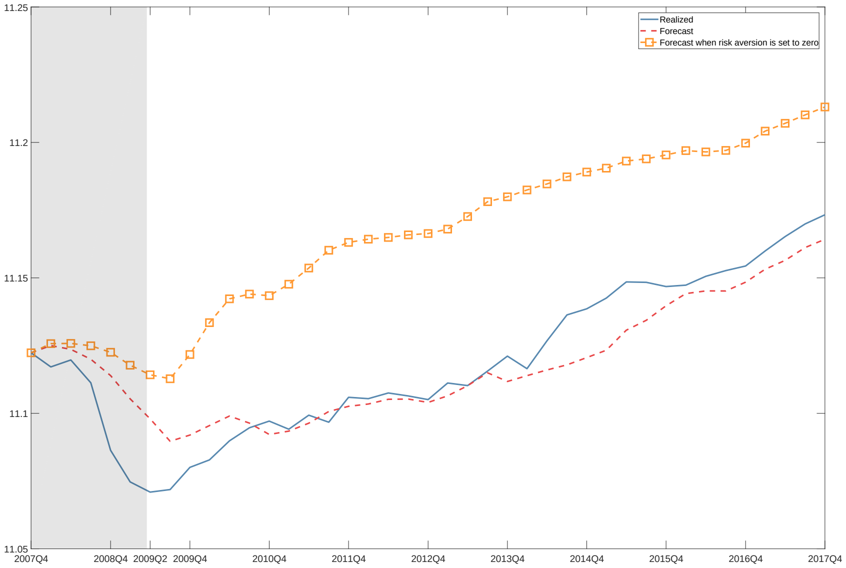
(b) State Multiplier – Stock prices.



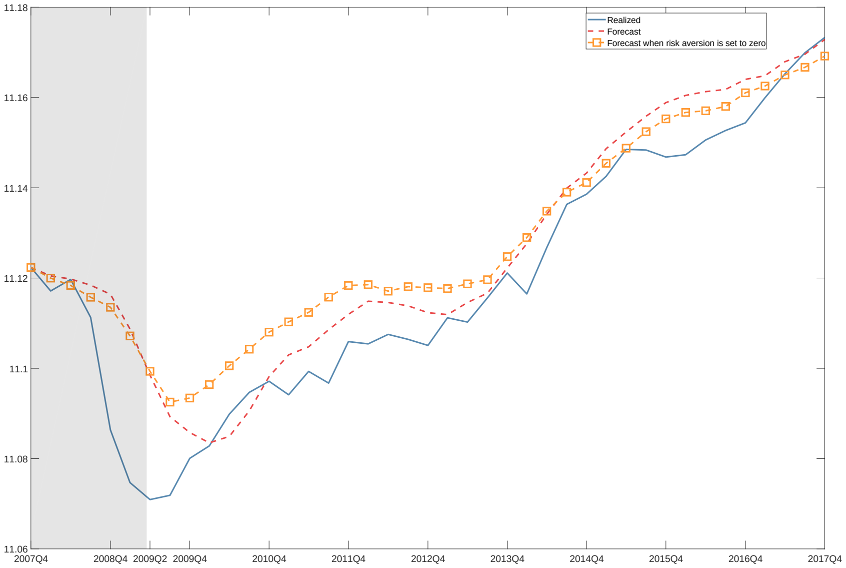
(c) Stock prices and Consumption-Price ratio.

(d) State Multiplier – Stock prices.

FIGURE 6: **State-dependent IR of stock prices to an uncertainty shock (JLN)**: This figure plots the empirical impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the financial uncertainty series by Jurado, Ludvigson and Ng (2015). To proxy for risk aversion we use either the surplus consumption proxy $\sum_{j=1}^{40} \phi^j \Delta c_{t-j}$ in Panel (a) and (b), or the consumption-price ratio in Panel (c) and (d). We standardize the risk aversion proxy to have zero-mean and unit variance. The figure shows the response when the state variable RA_t is one standard deviation below (low risk aversion; blue line with squares), at (black line), or above (high risk aversion; red line with circles) its average value. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column displays the state multiplier of the state-dependent IR of stock prices to an uncertainty shock. The shaded areas denote 90% confidence intervals.

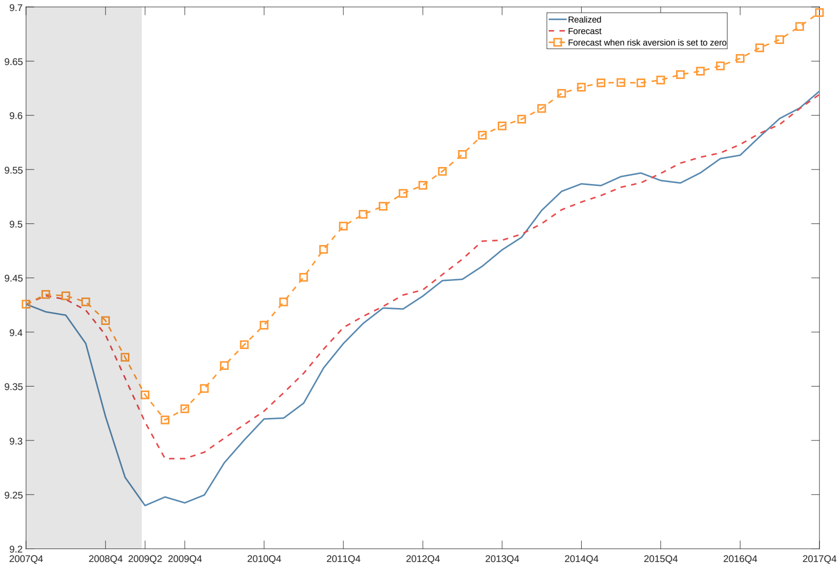


(a) Output and Surplus Consumption.

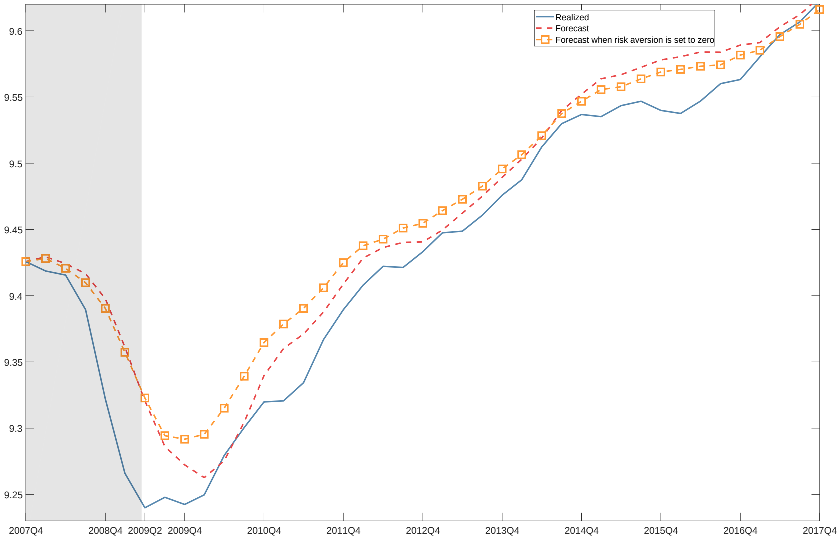


(b) Output and Consumption-Price ratio.

FIGURE 7: **The interaction between Uncertainty and Risk aversion in Output**: The solid line displays (log) per capita, real GDP for our sample. The dashed line is the 1-quarter ahead forecast from direct regressions that allow for an interaction between risk aversion and uncertainty. The line with squares is the 1-quarter ahead forecast from direct regression with no interaction between risk aversion and uncertainty. Shaded areas indicate NBER recession dates. We measure uncertainty using the economic policy uncertainty (EPU) by Baker, Bloom and Davis (2016). To proxy for risk aversion we use either the surplus consumption proxy $\sum_{j=1}^{40} \phi^j \Delta c_{t-j}$, (Panel A), or the consumption-price ratio (Panel B).

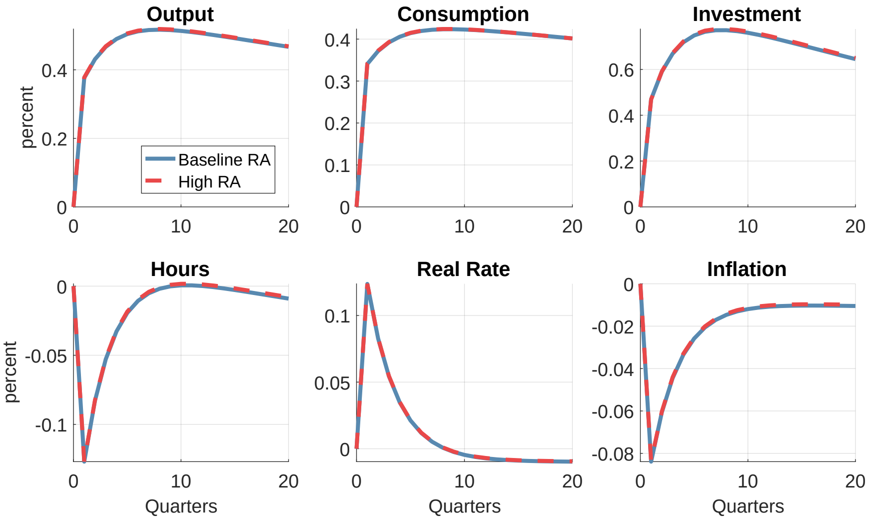


(a) Investment and Surplus Consumption.

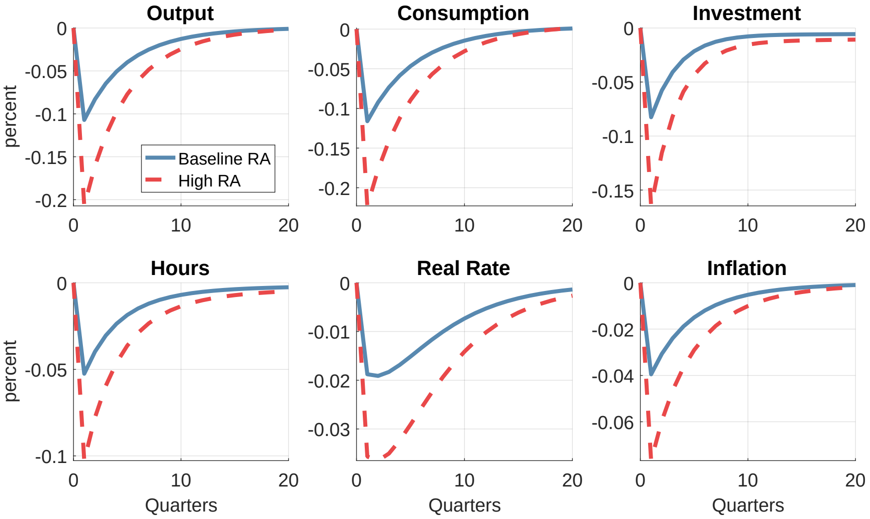


(b) Investment and Consumption-Price ratio.

FIGURE 8: **The interaction between Uncertainty and Risk aversion in Investment**: The solid line displays (log) per capita, real investment for our sample. The dashed line is the 1-quarter ahead forecast from direct regressions that allow for an interaction between risk aversion and uncertainty. The line with squares is the 1-quarter ahead forecast from direct regressions with no interaction between risk aversion and uncertainty. Shaded areas indicate NBER recession dates. We measure uncertainty using the economic policy uncertainty (EPU) by Baker, Bloom and Davis (2016). To proxy for risk aversion we use either the surplus consumption proxy, $\sum_{j=1}^{40} \phi^j \Delta c_{t-j}$, (Panel A), or the consumption-price ratio (Panel B).

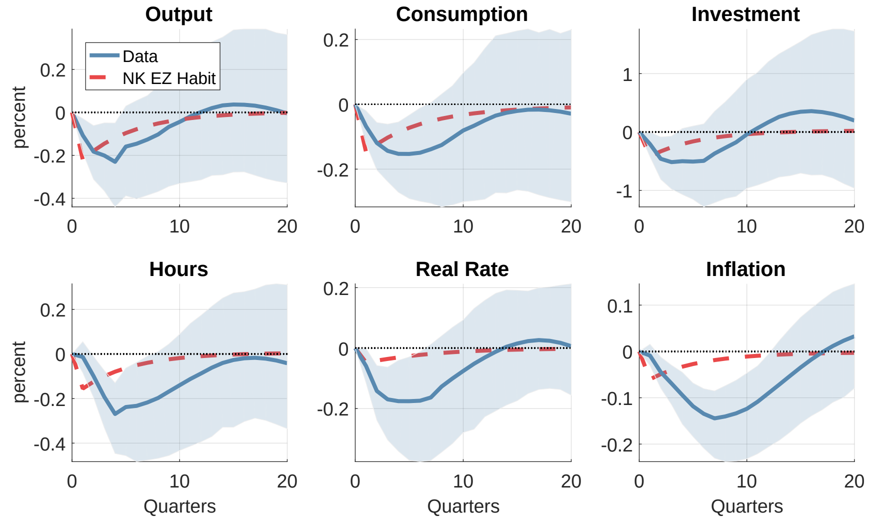


(a) Response to level shock.

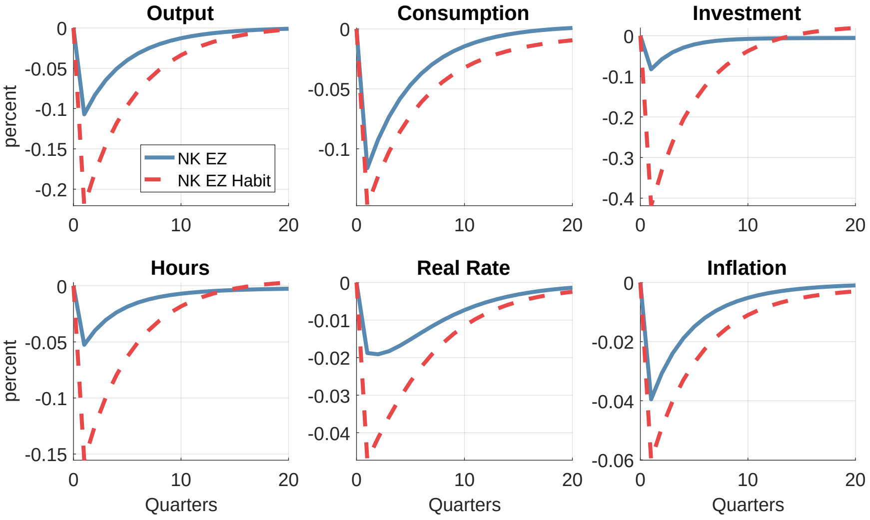


(b) Response to volatility shock.

Figure 9: **Impulse Response Function to Technology Shock – NK-EZ Model**: This figure plots the impulse responses for a one standard deviation shock to the (i) level, and (ii) volatility of technology. Impulse responses are for a one standard deviation shock when the model is approximated to the third order. To construct these responses, we set the exogenous shocks in the model to zero and iterate our third-order solution forward. After a sufficient number of periods, the endogenous variables of the model converge to a fixed point, which we denote the stochastic steady state. We then hit the economy with a one standard deviation uncertainty shock but assume the economy is hit by no further shocks. We compute the impulse response as the percent deviation between the equilibrium responses and the pre-shock stochastic steady state.

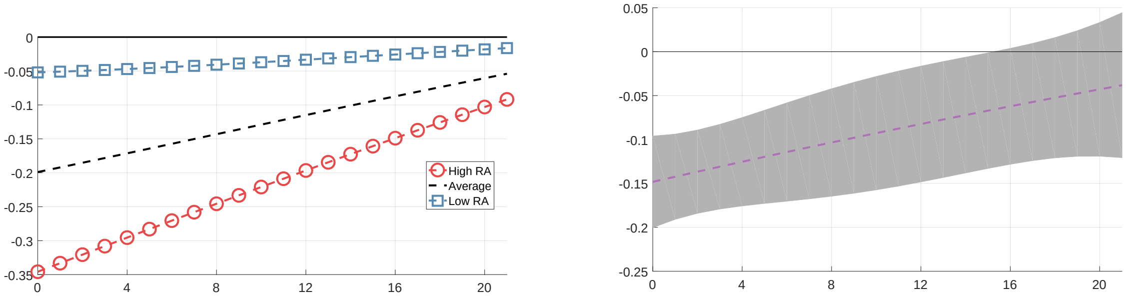


(a) Data vs Model.

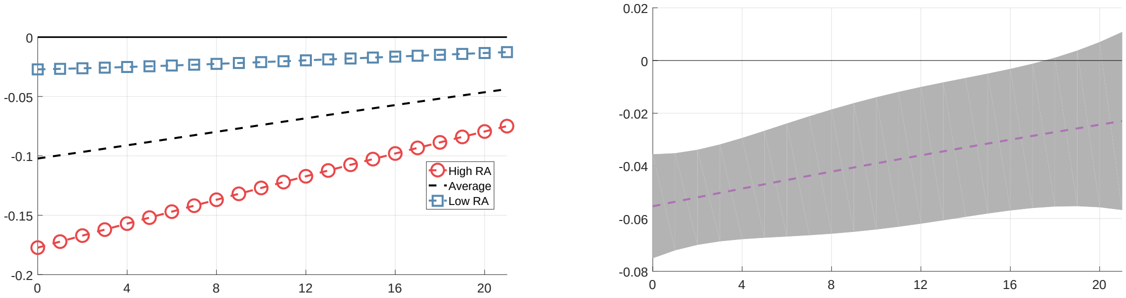


(b) Model Comparison.

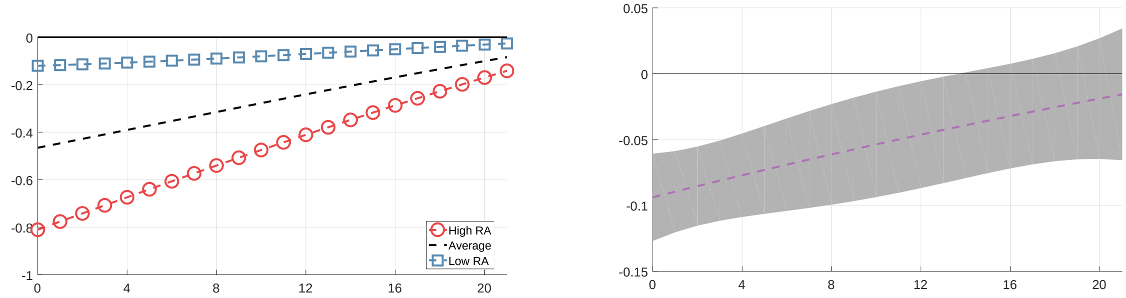
FIGURE 10: **Impulse Response Function to Technology Uncertainty Shock**: The top figure plots the impulse responses for a one standard deviation shock to the volatility both in data and model. The empirical responses are obtained from a four lags SLP that includes economic policy uncertainty (EPU), GDP, consumption, investment, hours worked, the GDP deflator, the M2 money stock, and the Wu and Xia (2016) shadow rate. Model-implied impulse responses are for a one standard deviation shock when the model is approximated to the third order. We compute the impulse response as the percent deviation between the equilibrium responses and the pre-shock stochastic steady state. The bottom figure plots the impulse responses to a one standard deviation shock to volatility of technology both for the NK-EZ and NK-EZ-Habit models.



(a) Output. (b) State Multiplier – Output.

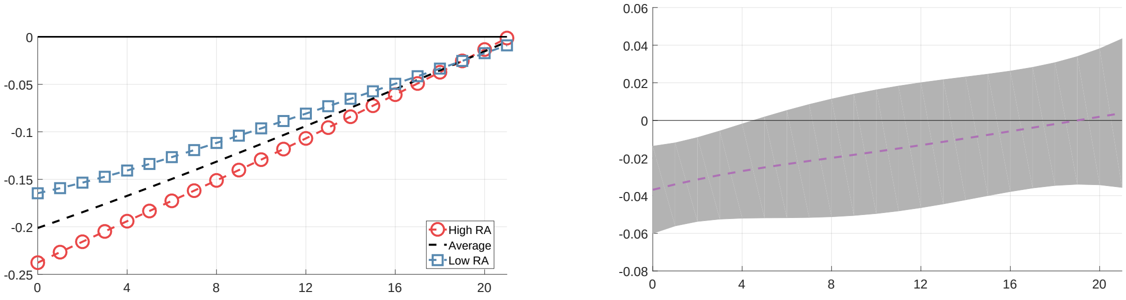


(c) Consumption. (d) State Multiplier – Consumption.

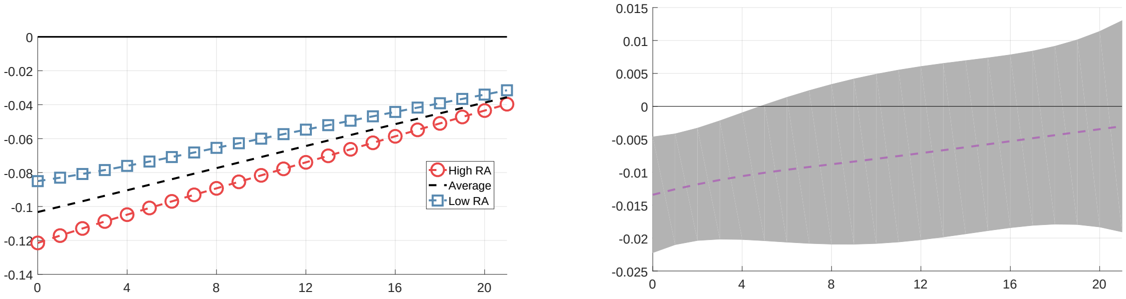


(e) Investment. (f) State Multiplier – Investment.

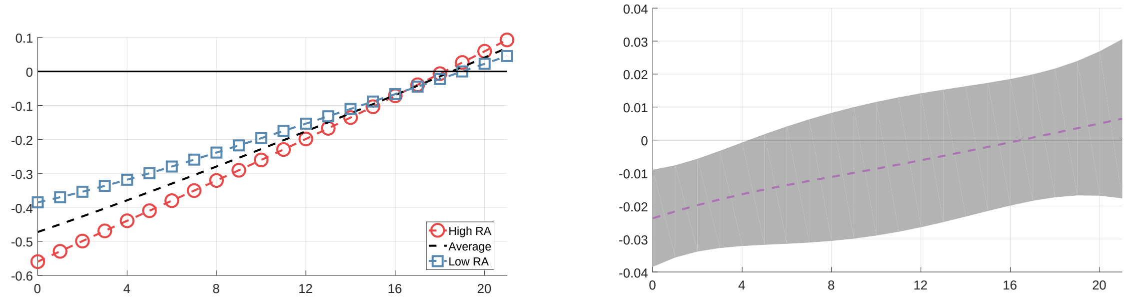
FIGURE 11: **Model implied state-dependent (RA) IR to a productivity uncertainty shock**: This figure plots the model-implied impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the productivity volatility time series from model simulations. Risk aversion is the inverse of the consumption surplus ratio. The state variable RA_t takes values -.25 (low risk aversion; blue line with squares), 0 (black line), and +.25 (high risk aversion; red line with circles) units of $\sigma(RA_t)$. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column reports the state multiplier of the state-dependent IR of GDP (top), consumption (mid),and investment (bottom) to a volatility shock. The shaded areas denote 90% confidence intervals.



(a) Output. (b) State Multiplier – Output.

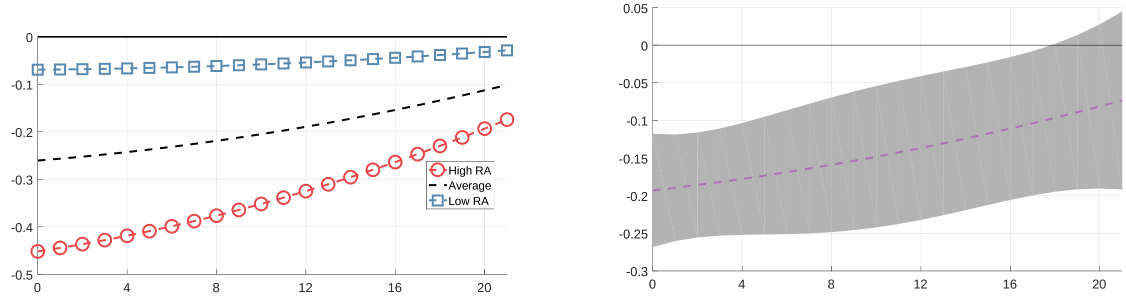


(c) Consumption. (d) State Multiplier – Consumption.

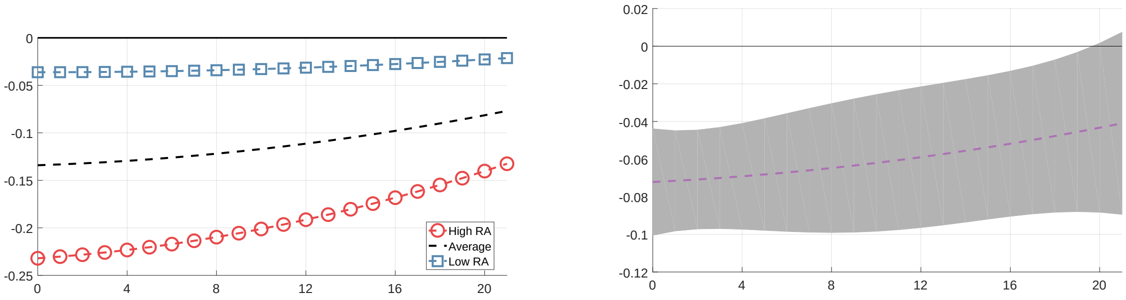


(e) Investment. (f) State Multiplier – Investment.

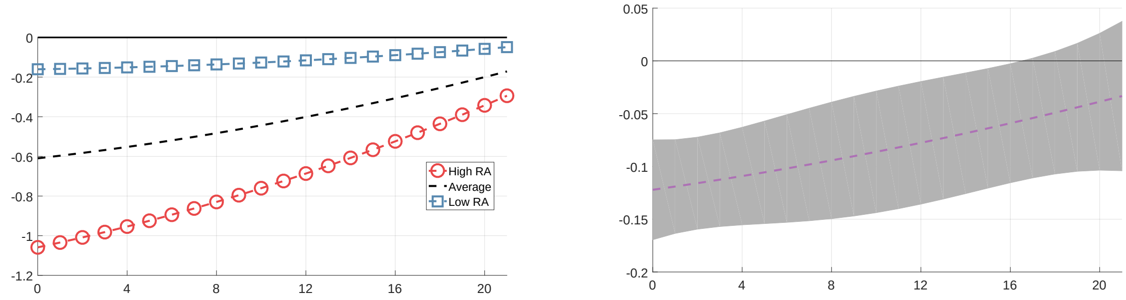
FIGURE 12: **Model implied state-dependent (consumption-price) IR to a productivity uncertainty shock**: This figure plots the model-implied impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the productivity volatility time series from model simulations. Our proxy for risk aversion is the model-implied log dividend-price ratio. The state variable RA_t takes values -1 (low risk aversion; blue line with squares), 0 (black line), and +1 (high risk aversion; red line with circles) units of $\sigma(RA_t)$. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column displays the state multiplier of the state-dependent IR of GDP (top), consumption (mid),and investment (bottom) to a volatility shock. The shaded areas denote 90% confidence intervals.



(a) Output. (b) State Multiplier – Output.

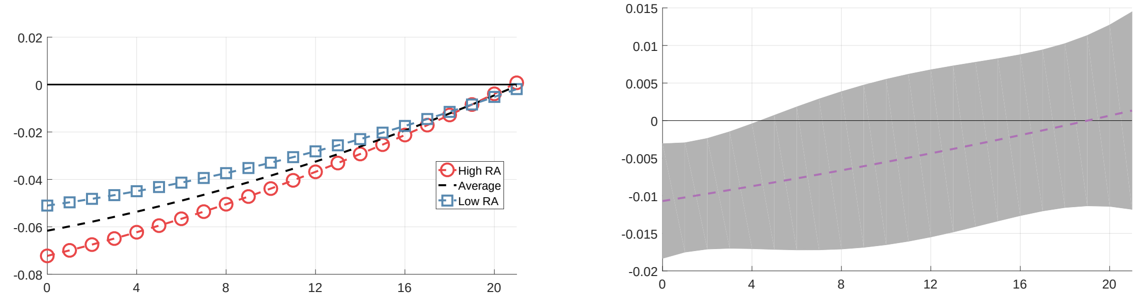


(c) Consumption. (d) State Multiplier – Consumption.



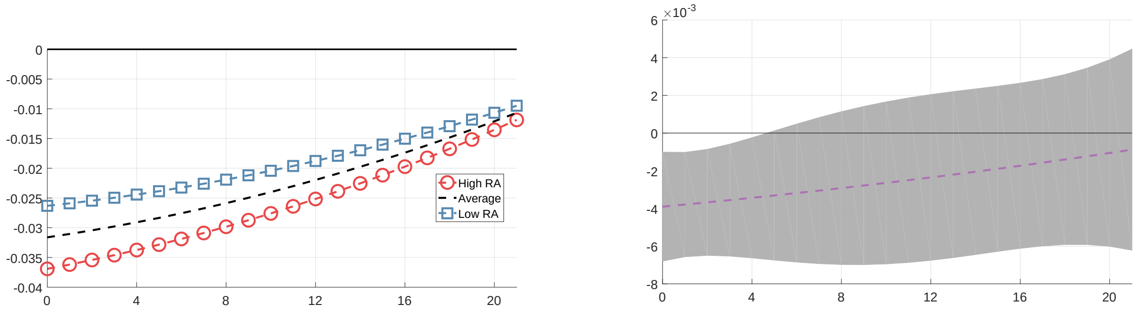
(e) Investment. (f) State Multiplier – Investment.

FIGURE 13: **Model implied state-dependent (RA) IR to a stock market uncertainty shock**: This figure plots the model-implied impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the model-implied expected conditional volatility of the return on firm equity. The state variable RA_t takes values -.25 (low risk aversion; blue line with squares), 0 (black line), and +.25 (high risk aversion; red line with circles) units of $\sigma(RA_t)$. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column reports the state multiplier of the state-dependent IR of GDP (top), consumption (mid),and investment (bottom) to a volatility shock. The shaded areas denote 90% confidence intervals.



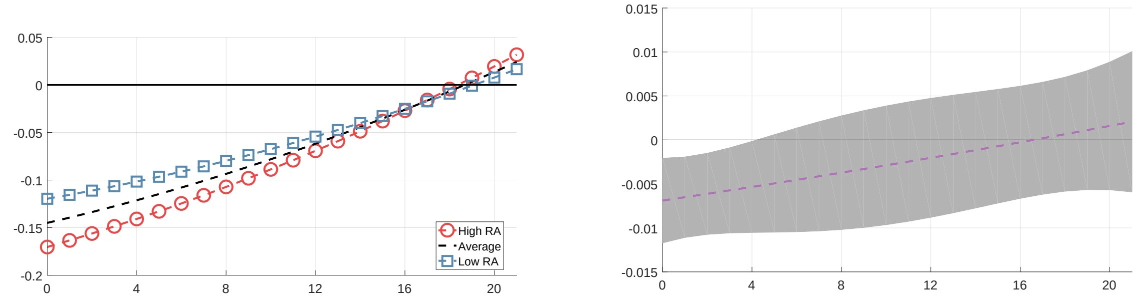
(a) Output.

(b) State Multiplier – Output.



(c) Consumption.

(d) State Multiplier – Consumption.



(e) Investment.

(f) State Multiplier – Investment.

FIGURE 14: **Model implied state-dependent (dividend-price) IR to a stock market uncertainty shock**: This figure plots the model-implied impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the model-implied VXO index which equals the expected conditional volatility of the return on firm equity. Our proxy for risk aversion is the model-implied log dividend-price ratio. The state variable RA_t takes values -1 (low risk aversion; blue line with squares), 0 (black line), and +1 (high risk aversion; red line with circles) units of $\sigma(RA_t)$. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column displays the state multiplier of the state-dependent IR of GDP (top), consumption (mid),and investment (bottom) to a volatility shock. The shaded areas denote 90% confidence intervals.

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A Data Construction

This section provides additional details on the data construction for the empirical evidence from Section 2 of the main text. We estimate our baseline Smooth Local Projection (SLP) using data on the uncertainty, real GDP, real consumption, real investment, hours worked, the GDP deflator, the Standard & Poors 500 Stock Price Index, and the Wu and Xia (2016) shadow rate. We use the Wu and Xia (2016) shadow rate series as our indicator of monetary policy since the Federal Reserve hit the zero lower bound on nominal interest rates at the end of 2008. Away from the zero lower bound, this series equals the federal funds rate. But at the zero lower bound, the shadow rate uses information from the entire yield curve to summarize the stance of monetary policy. To match the concept in the model, we measure consumption in the data as the sum of non-durable and services consumption. Then, we use the sum of consumer durables and private fixed investment as a measure of investment in our baseline empirical model. We convert output, consumption, investment, and hours worked to per capita terms by dividing by population (Civilian Noninstitutional Population: 16 Years and Over). Except for the shadow rate, all other variables enter the SLP in log levels. The series of output, consumption, investment are from NIPA.

To compute the price-consumption ratio, we measure stock market wealth by the quarter-end market capitalization of the CRSP value-weighted index, expressed in real per capita terms for comparability to the consumption data.

The measure of economic policy uncertainty (EPU) is based on Baker et al. (2016). We use the News-Based Policy Uncertainty Index available on EPU's web site for the US. The series is monthly and spans the period 1985:1−2017:12. We convert it to quarterly values by taking the end of the quarter value. To go further back in time, we merge the News-Based Policy Uncertainty Index series with the US Index, a longer series available from the Historical EPU's web site.

The measure of financial uncertainty is an updated version of data used in Jurado et al. (2015) and Ludvigson et al. (2017). It is available on Sydney Ludvigson's home page. We use their 3-month ahead uncertainty series. The series spans the period 1960:07−2017:12. We take the end of the quarter value to construct qaurterly series.

As additional robustness checks, we re-estimated our IRFs excluding the Standard & Poor's 500 Stock Price Index and our conclusions remain unaltered. We have also re-estimated the IRFs when we replace the Standard & Poors 500 Stock Price Index with the M2 money stock. Once again results are not affected by these changes.

B Consumption Surplus and Risk Aversion

From Swanson (2018), wealth-gamble risk aversion with recursive preferences can be written as:

$$R^a(a_t;\theta_t) = \frac{-\mathbb{E}_t[U(a^*_{t+1};\theta_{t+1})^{-\gamma}U_{11}(a^*_{t+1};\theta_{t+1}) - \gamma U(a^*_{t+1};\theta_{t+1})^{-\gamma-1}U_1(a^*_{t+1};\theta_{t+1})^2]}{\mathbb{E}_t[U(a^*_{t+1};\theta_{t+1})^{-\gamma}U_1(a^*_{t+1};\theta_{t+1})]},$$

where a^*_{t+1} is optimal wealth through the budget constraint $a^*_{t+1} \equiv (1+R_t)a_t + W_tN^*_t + D_t - C^*_t$. θ_t represents the set of exogenous shocks driving the dynamics of the economy. Subscripts 1 and 11 denote the first and second derivatives, respectively, with respect to future wealth.

Assuming within-period power utility over habit adjusted consumption only, the Epstein-Zin value function has the following form:

$$U(a^*_{t+1};\theta_{t+1}) = \left[\frac{{C^h_{t+1}}^{1-\psi}}{1-\psi} + \beta\mathbb{E}_{t+1}\left[U(a^*_{t+2};\theta_{t+2})^{1-\gamma}\right]^{\frac{1-\psi}{1-\gamma}}\right]^{\frac{1}{1-\psi}}.$$

Recall that $C^h_{t+1} = C^*_{t+1}S_{t+1}$.

We can apply the chain rule to calculate the derivative of $U(a^*_{t+1};\theta_{t+1})$ with respect to a^*_{t+1} such that

$$\frac{\partial U(a^*_{t+1};\theta_{t+1})}{\partial a^*_{t+1}} = \frac{\partial U(a^*_{t+1};\theta_{t+1})}{\partial C^h_{t+1}}\frac{\partial C^h_{t+1}}{\partial C^*_{t+1}}\frac{\partial C^*_{t+1}}{\partial a^*_{t+1}} = \frac{\partial U(a^*_{t+1};\theta_{t+1})}{\partial C^h_{t+1}}S_{t+1}(1+R_{t+1}),$$

and

$$\begin{aligned}\frac{\partial U(a^*_{t+1};\theta_{t+1})}{\partial C^h_{t+1}} &= \frac{1}{1-\psi}\left[U(a^*_{t+1};\theta_{t+1})^{1-\psi}\right]^{\frac{1}{1-\psi}-1}{C^h_{t+1}}^{-\psi} \\ &= \frac{1}{1-\psi}U(a^*_{t+1};\theta_{t+1})^{\psi}{C^h_{t+1}}^{-\psi}.\end{aligned}$$

The second derivative of $U(a^*_{t+1};\theta_{t+1})$ with respect to a^*_{t+1} can be found by repeated application of the product rule to the expression $\frac{\partial U(a^*_{t+1};\theta_{t+1})}{\partial C^h_{t+1}}S_{t+1}(1+R_{t+1})$ and notice that $\frac{\partial S_{t+1}}{\partial a^*_{t+1}} = 0$ and $\frac{\partial(1+R_{t+1})}{\partial a^*_{t+1}} = 0$. Thus we have:

$$\frac{\partial^2 U(a^*_{t+1};\theta_{t+1})}{\partial a^*_{t+1}\partial a^*_{t+1}} = \frac{\partial^2 U(a^*_{t+1};\theta_{t+1})}{\partial C^h_{t+1}\partial C^h_{t+1}}S^2_{t+1}(1+R_{t+1})^2,$$

and

$$\begin{aligned}\frac{\partial^2 U(a^*_{t+1};\theta_{t+1})}{\partial C^h_{t+1}\partial C^h_{t+1}} &= \frac{1}{1-\psi}\Big[-\psi U(a^*_{t+1};\theta_{t+1})^{\psi}{C^h_{t+1}}^{-\psi-1} \\ &\quad +\psi U(a^*_{t+1};\theta_{t+1})^{\psi-1}\frac{\partial U(a^*_{t+1};\theta_{t+1})}{\partial C^h_{t+1}}{C^h_{t+1}}^{-\psi}\Big] \\ &= \frac{1}{1-\psi}\Big[-\psi U(a^*_{t+1};\theta_{t+1})^{\psi}{C^h_{t+1}}^{-\psi-1} \\ &\quad +\psi U(a^*_{t+1};\theta_{t+1})^{\psi-1}\frac{1}{1-\psi}U(a^*_{t+1};\theta_{t+1})^{\psi}{C^h_{t+1}}^{-\psi}{C^h_{t+1}}^{-\psi}\Big] \\ &= \frac{\psi}{\psi-1}U(a^*_{t+1};\theta_{t+1})^{\psi}{C^h_{t+1}}^{-\psi}\Big[{C^h_{t+1}}^{-1} \\ &\quad -\frac{1}{1-\psi}U(a^*_{t+1};\theta_{t+1})^{\psi-1}{C^h_{t+1}}^{-\psi}\Big].\end{aligned}$$

Then, we can calculate the terms in R^a:

$$U(a^*_{t+1};\theta_{t+1})^{-\gamma}U_1(a^*_{t+1};\theta_{t+1}) = \frac{1}{1-\psi}U(a^*_{t+1};\theta_{t+1})^{\psi-\gamma}{C^h_{t+1}}^{-\psi}S_{t+1}(1+R_{t+1}),$$

$$\begin{aligned} U(a^*_{t+1};\theta_{t+1})^{-\gamma}U_{11}(a^*_{t+1};\theta_{t+1}) &= \frac{\psi}{\psi-1}U(a^*_{t+1};\theta_{t+1})^{\psi-\gamma}{C^h_{t+1}}^{-\psi}\Big[{C^h_{t+1}}^{-1} \\ &\quad -\frac{1}{1-\psi}U(a^*_{t+1};\theta_{t+1})^{\psi-1}{C^h_{t+1}}^{-\psi}\Big]S^2_{t+1}(1+R_{t+1})^2, \end{aligned}$$

$$U(a^*_{t+1};\theta_{t+1})^{-\gamma-1}U_1(a^*_{t+1};\theta_{t+1})^2 = \left(\frac{1}{1-\psi}\right)^2 U(a^*_{t+1};\theta_{t+1})^{2\psi-\gamma-1}{C^h_{t+1}}^{-2\psi}S^2_{t+1}(1+R_{t+1})^2,$$

such that

$$\begin{aligned} & R^a(a_t;\theta_t) \\ = \;& -\frac{\mathbb{E}_t\left[\frac{\psi}{\psi-1}U(a^*_{t+1};\theta_{t+1})^{\psi-\gamma}{C^h_{t+1}}^{-\psi}\left[{C^h_{t+1}}^{-1}-\frac{1}{1-\psi}U(a^*_{t+1};\theta_{t+1})^{\psi-1}{C^h_{t+1}}^{-\psi}\right]S^2_{t+1}(1+R_{t+1})^2\right]}{\mathbb{E}_t\left[\frac{1}{1-\psi}U(a^*_{t+1};\theta_{t+1})^{\psi-\gamma}{C^h_{t+1}}^{-\psi}S_{t+1}(1+R_{t+1})\right]} \\ & +\gamma\frac{\mathbb{E}_t\left[\left(\frac{1}{1-\psi}\right)^2U(a^*_{t+1};\theta_{t+1})^{2\psi-\gamma-1}{C^h_{t+1}}^{-2\psi}S^2_{t+1}(1+R_{t+1})^2\right]}{\mathbb{E}_t\left[\frac{1}{1-\psi}U(a^*_{t+1};\theta_{t+1})^{\psi-\gamma}{C^h_{t+1}}^{-\psi}S_{t+1}(1+R_{t+1})\right]} \\ \approx \;& \psi\left[{C^h_{t+1}}^{-1}-\frac{1}{1-\psi}U(a^*_{t+1};\theta_{t+1})^{\psi-1}{C^h_{t+1}}^{-\psi}\right]S_{t+1}(1+R_{t+1}) \\ & +\gamma\left(\frac{1}{1-\psi}\right)U(a^*_{t+1};\theta_{t+1})^{\psi-1}{C^h_{t+1}}^{-\psi}S_{t+1}(1+R_{t+1}) \\ = \;& \psi\left[{C^*_{t+1}}^{-1}+\frac{1}{\psi-1}U(a^*_{t+1};\theta_{t+1})^{\psi-1}{C^*_{t+1}}^{-\psi}S^{1-\psi}_{t+1}\right](1+R_{t+1}) \\ & +\gamma\left(\frac{1}{1-\psi}\right)U(a^*_{t+1};\theta_{t+1})^{\psi-1}{C^*_{t+1}}^{-\psi}S^{1-\psi}_{t+1}(1+R_{t+1}), \end{aligned}$$

where the approximation relies on the fact that the risk involved in the wealth gamble is minuscule.

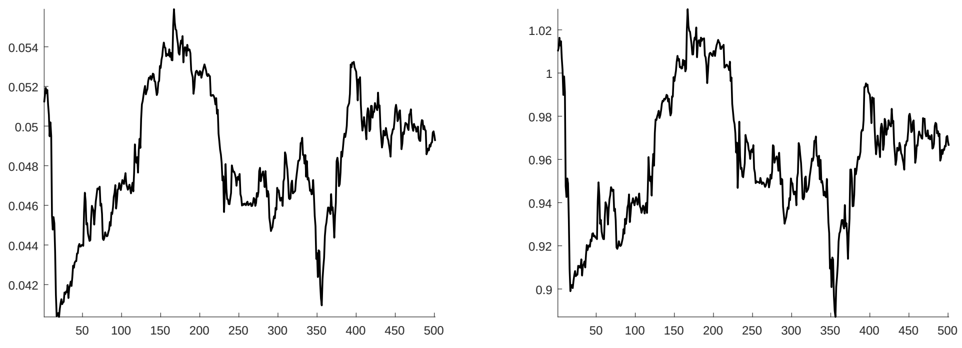
Recall that ψ is the inverse of the elasticity of intertemporal substitution. Under standard calibration, $\psi > 1$, and $R^a(a_t;\theta_t)$ can be written as:

$$\begin{aligned} R^a(a_t;\theta_t) &\approx \mathbb{E}_t\Bigg[\psi\left\{\frac{1}{C^*_{t+1}}+\frac{1}{\psi-1}U(a^*_{t+1};\theta_{t+1})^{\psi-1}\frac{1}{{C^*_{t+1}}^{\psi}S^{\psi-1}_{t+1}}\right\}(1+R_{t+1}) \\ &\qquad +\gamma\left(\frac{1}{1-\psi}\right)U(a^*_{t+1};\theta_{t+1})^{\psi-1}\frac{1}{{C^*_{t+1}}^{\psi}S^{\psi-1}_{t+1}}(1+R_{t+1})\Bigg] \\ &= \mathbb{E}_t\left[\left\{\frac{\psi}{C^*_{t+1}}+\frac{\psi-\gamma}{\psi-1}U(a^*_{t+1};\theta_{t+1})^{\psi-1}\frac{1}{{C^*_{t+1}}^{\psi}S^{\psi-1}_{t+1}}\right\}(1+R_{t+1})\right], \end{aligned}$$

which is an inverse function of S_{t+1}. In other words, when the consumption surplus ratio is high, wealth-gamble risk aversion is low and vice versa. The derivation with leisure preference is similar but slightly more involved.

B.1 Proxies for Risk Aversion: Simulations

Figure B.1 shows one simulation path for model variables that proxy for risk aversion: surplus consumption ratio, and wealth-consumption.



(a) Surplus Consumption Ratio.

(b) Price/Consumption Ratio.

Figure B.1: **Different Proxies for Risk Aversion within the Model**: This figure plots one simulation paths (1000 periods with a burn-in period of 5000) for the various proxies of risk aversion within our model environment, i.e. surplus consumption ratio and price-consumption ratio. The correlation between consumption surplus and wealth/consumption ratios 0.854. Note that the price-consumption ratio is defined as follows: $W/C = \left(E_t\left[U_{t+1}\right]/C_t\right)^{(1-1/\psi)}$.

Table B.1 shows that most of the variability in model-implied risk aversion proxies is driven by level shocks as opposed to volatility shocks.

TABLE B.1: **Variance Decomposition**

This table shows the variance decomposition of the model-implied proxies for risk aversion for the different structural shocks in the model. The first column reports model moments when all shocks are active. The second and third columns show model moments with only level and volatility shocks to technology, respectively.

Variance Decomposition

	All Shocks	Level only	Volatility only
Surplus Consumption Ratio	5.45	2.77	0.63
	[2.55,10.45]	[1.68,2.45]	[0.48,0.84]
Price/Dividend Ratio	10.26	1.62	0.92
	[1.16,19.44]	[0.68,3.03]	[0.68,1.27]
Wealth/Dividend Ratio	2.60	1.36	0.21
	[1.27,4.95]	[0.69,2.82]	[0.16,0.28]

C Illustrative Example

This section illustrates the interaction between risk aversion and volatility shocks within a simple prototype economy. Our description follows the line of Fernández-Villaverde and Rubio-Ramirez (2010).

C.1 Economic Environment

There is a representative household in the economy, whose preferences over stochastic sequences of consumption, c_t; and work, l_t, are representable by a utility function:

$$U = E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right]$$

where $\beta \in (0,1)$ is the discount factor and E_0 is the conditional expectation operator.
The household's budget constraint is given by:

$$c_t + i_t = \frac{b_{t+1}}{R_t} = w_t l_t + r_t k_t + b_t$$

where i_t is investment, R_t is the risk-free gross interest rate, b_t is the holding of an uncontingent bond that pays 1 unit of consumption good at time $t+1$, w_t is the wage, l_t is labor, r_t is the rental rate of capital, and k_t is capital. Capital is accumulated according to the law of motion $k_{t+1=(1-\delta)k_t+i_t}$ where δ is the depreciation rate.
The final good is produced by a competitive firm with a technology $y_t = e^{z_t} A k_t^{\alpha} l_t^{1-alpha}$ where z_t is the productivity level whose evolution we will describe momentarily and A is a constant. Thus, the economy must satisfy the aggregate resource constraint $y_t = c_t + i_t$.

Productivity follows an autoregressive process with stochastic volatility, i.e.

$$z_t = \rho z_{t-1} + e^{\sigma_t} \varepsilon_t \tag{C.1}$$

$$\sigma_t = (1-\rho_\sigma)\,\bar{\sigma} + \rho_\sigma \sigma_{t-1} + \eta u_t \tag{C.2}$$

with $\rho < 1$, $\varepsilon_t, u_t \sim N(0,1)$, and where η denotes the standard deviation of the innovations to volatility.

The definition of competitive equilibrium of this model is standard.

C.2 Solution Method

To solve the model with time-varying volatility we rely on a higher-order perturbation, an approach that has been shown to be both accurate and fast, see Aruoba, Fernández-Villaverde and Rubio-Ramírez (2006). The main idea is to find a Taylor approximation of the decision rules around the steady state of the model. The first step tp doing so is to introduce a new parameter, called the perturbation parameter, Λ, and rewrite the stochastic process (C.1) and (C.2) as:

$$\begin{aligned} z_t &= \rho z_{t-1} + \Lambda e^{\sigma_t} \varepsilon_t \\ \sigma_t &= (1-\rho_\sigma)\,\bar{\sigma} + \rho_\sigma \sigma_{t-1} + \Lambda \eta u_t \end{aligned}$$

Then, if we make $\Lambda = 1$, we get back the original formulation of the problem. However, if we set $\Lambda = 0$, we eliminate the sources of uncertainty in the model and the economy will (asymptotically) settle down at the steady state.

The second step is to rewrite all variables in terms of deviations with respect to the steady state. Thus, we write $\widehat{x}_t = x_t - x$ for any arbitrary variable x_t with steady state x. Also, define an augmented state vector of the model

$$s_t = \left(\widehat{k}_t, z_{t-1}, \widehat{\sigma}_{t-1}, \varepsilon_t, u_t; \Lambda\right)$$

where we stack the states in deviations to the mean and innovations, and we have incorporated the perturbation parameter, Λ, as a pseudo-state (where the pseudo is emphasized by the use of a semicolon to separate it from the pure states). Then, the decision rules we are looking for are $\widehat{c}_t = c\left(s_t\right)$, $\widehat{l}_t = l\left(s_t\right)$, $\widehat{k}_{t+1} = k\left(s_t\right)$.

To approximate them, we search for the coefficients of the Taylor expansion of these decision rules evaluated at the steady state, $s = \mathbf{0}_{1\times 5}$. For example, for consumption,

$$\begin{aligned}\widehat{c}_t &= c\left(s_t\right)\\ &= c_{i,ss}s_t^i + \frac{1}{2}c_{ij,ss}s_t^i s_t^j\\ &+ \frac{1}{2}c_{66,ss}\\ &+ c_{ijl,ss}\frac{1}{6}s_t^i s_t^j s_t^l + \text{H.O.T.}\end{aligned}$$

where: (1) each term $c_{\ldots,ss}$ is a scalar equal to a derivative of the value function evaluated at the steady state; (2) we use the tensor notation (i.e. $c_{i,ss}s_t^i = \sum_{i=1}^5 c_{i,ss}s_{i,t}$, and $c_{ij,ss}s_t^i s_t^j = \sum_{i=1}^5 \sum_{j=1}^5 c_{ij,ss}s_{i,t}s_{j,t}$, and $c_{ijl,ss}s_t^i s_t^j s_t^l = \sum_{i=1}^6 \sum_{j=1}^6 \sum_{l=1}^6 c_{ijl,ss}s_{i,t}s_{j,t}s_{l,t}$) to eliminates the symbol $\sum_{j=1}^5$ when no confusion arises; (3) and where we represent all the higher-order terms by "H.O.T.". We can proceed in analogous ways for all other variables and derive the appropriate formulae.

In what follows we are explicit about the derivatives with respect to Λ, and denote them with $c_{\Lambda\Lambda,ss}$ rather than $c_{66,ss}$. Note also that we let $i = 1, \ldots, 5$ (and similarly for j) in first and second order since we single out the effect on Λ. Indeed, in a second-order approximation, the perturbation parameter, Λ, will only have a coefficient different from zero in the term where it appears in a square by itself, see Schmitt-Grohe and Uribe (2004). However we let $i = 1, \ldots, 6$ (and similarly for j and l) in third-order terms.

C.3 Risk Aversion and Volatility Shocks

Writing explicitly all the terms for the consumption solution we have

$$\begin{aligned}\widehat{c}_t &= c_{1,ss}\widehat{k}_t + c_{2,ss}z_{t-1} + c_{3,ss}\varepsilon_t\\ &+ c_{11,ss}\widehat{k}_t^2 + c_{12,ss}\widehat{k}_t z_{t-1} + c_{14,ss}\widehat{k}_t\varepsilon_t + c_{22,ss}z_{t-1}^2 + c_{24,ss}z_{t-1}\varepsilon_t + c_{44,ss}\varepsilon_t^2 + c_{45,ss}\times\varepsilon_t u_t + c_{34,ss}\varepsilon_t\widehat{\sigma}_{t-1}\\ &+ \frac{1}{2}c_{\Lambda\Lambda,ss}\\ &+ c_{ijl,ss}\frac{1}{6}s_t^i s_t^j s_t^l + \text{H.O.T.}\end{aligned}$$

where the first and second rows follow respectively from Theorem 1 and 2 in Fernández-Villaverde et al. (2015).

Importantly for our purpose, in a second-order approximation innovations to the volatility shocks, u_t, appear with coefficients different from zero when they show up with the innovation to their own structural shock ε_t, see $c_{45,ss}\times\varepsilon_t u_t$. Also, Koijen et al. (2009) demonstrate that the risk aversion parameter does not affect the value of any of the coefficients except $c_{\Lambda\Lambda,ss}$, and that $c_{\Lambda\Lambda,ss} \neq 0$.

Next, we focus on the third-order terms. Using Theorem 1 in Andreasen (2012) we have that $c_{\Lambda ij} = 0$ for $i, j = 1, \ldots, 5$. Moreover, if all innovations have symmetric distributions, $c_{\Lambda\Lambda\Lambda} =$

$h_{\Lambda\Lambda\Lambda} = 0$. So only the terms

$$c_{ijl,ss} s_t^i s_t^j s_t^l \equiv \sum_{i=1}^{5} \sum_{j=1}^{5} \sum_{l=1}^{5} c_{ijl,ss} s_{i,t} s_{j,t} s_{l,t}$$

and

$$c_{\Lambda\Lambda i,ss}\ s_t^i \times \Lambda^2 = \Lambda^2 \times \sum_{l=1}^{5} c_{\Lambda\Lambda l,ss} s_{i,t}$$

may be non-zero. Eventually we have

$$\begin{aligned}
\widehat{c}_t &= c_{1,ss}\widehat{k}_t + c_{2,ss} z_{t-1} + c_{3,ss}\varepsilon_t \\
&\quad + c_{11,ss}\widehat{k}_t^2 + c_{12,ss}\widehat{k}_t z_{t-1} + c_{14,ss}\widehat{k}_t\varepsilon_t + c_{22,ss} z_{t-1}^2 + c_{24,ss} z_{t-1}\varepsilon_t + c_{44,ss}\varepsilon_t^2 + c_{45,ss} \times \varepsilon_t u_t + c_{34,ss}\varepsilon_t\widehat{\sigma}_{t-1} \\
&\quad + \frac{1}{2} c_{\Lambda\Lambda,ss} \\
&\quad + c_{ijl,ss} s_t^i s_t^j s_t^l + \frac{3}{6} c_{\Lambda\Lambda i,ss} s_t^i \Lambda^2 + \text{H.O.T.}
\end{aligned}$$

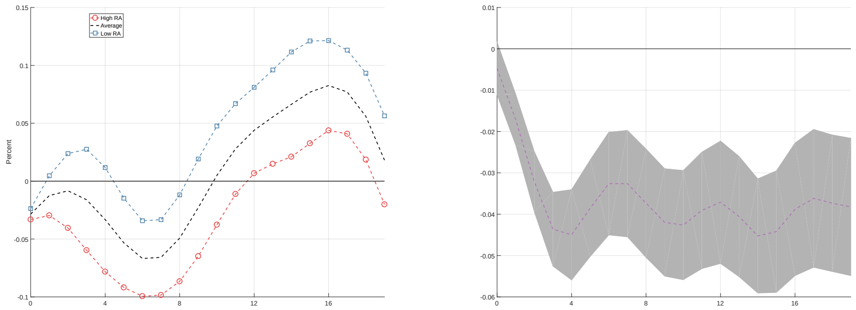
Importantly for our paper, in the third-order approximation, all of the terms on functions of Λ^2 depend on the coefficient of risk aversion.

D Local Projections: Additional Results

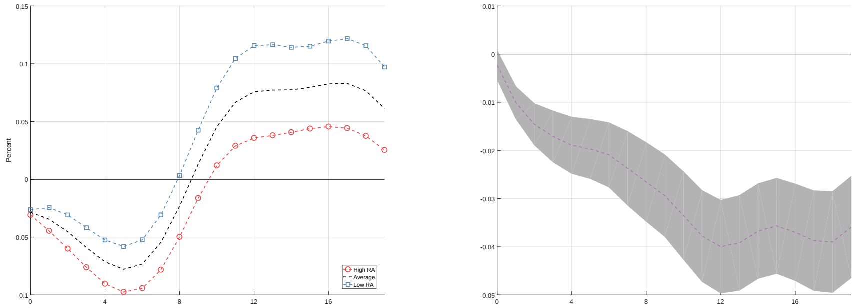
D.1 Alternative Measure of Uncertainty and Risk Aversion

This section shows that our results are robust to alternative measures of uncertainty and risk aversion.

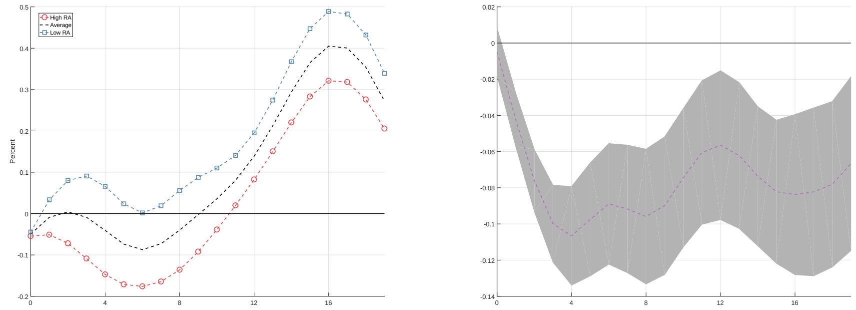
First, Figures D.1 and D.2 show the results when we employ the financial intermediary leverage as measured by He, Kelly and Manela (2017) as our proxy for risk aversion. This is motivated by the work of Santos and Veronesi (2016): in this model the debt-to-wealth ratio is monotonically decreasing in the surplus consumption ratio (see their Corollary 13), which can be seen as the inverse of risk aversion. In Figure D.1 we proxy uncertainty with the economic policy uncertainty by Baker, Bloom and Davis (2016). In Figure D.2 we proxy uncertainty with the aggregate uncertainty index proposed by Jurado, Ludvigson and Ng (2015).



(a) Output. (b) State Multiplier – Output.

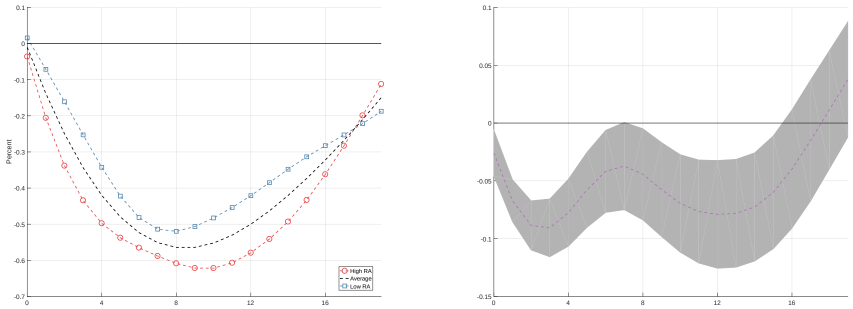


(c) Consumption. (d) State Multiplier – Consumption.



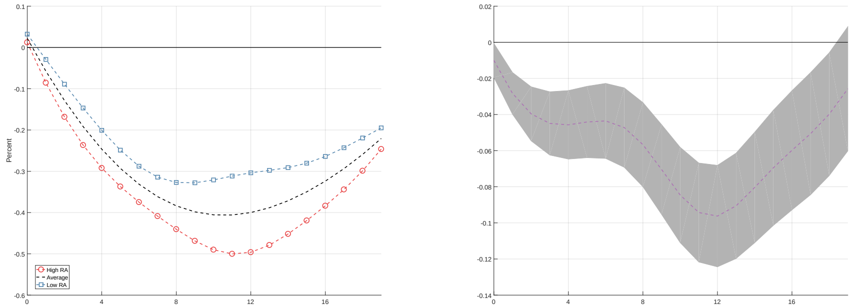
(e) Investment. (f) State Multiplier – Investment.

FIGURE D.1: **State-dependent (leverage) IR to an uncertainty shock (EPU)**: This figure plots the empirical impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the economic policy uncertainty (EPU) by Baker, Bloom and Davis (2016). Our proxy for risk aversion is intermediary leverage by He, Kelly and Manela (2017). We standardize the risk aversion proxy to have zero-mean and unit variance. The figure shows the response when the state variable RA_t is one standard deviation below (low risk aversion; blue line with squares), at (black line), or above (high risk aversion; red line with circles) its average value. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column reports the state multiplier of the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The shaded areas denote 90% confidence intervals.



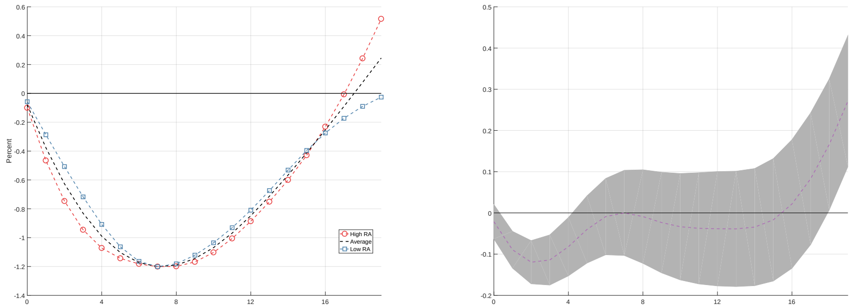
(a) Output.

(b) State Multiplier – Output.



(c) Consumption.

(d) State Multiplier – Consumption.



(e) Investment.

(f) State Multiplier – Investment.

FIGURE D.2: **State-dependent (leverage) IR to an uncertainty shock (JLN)**: This figure plots the empirical impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the financial uncertainty series by Jurado, Ludvigson and Ng (2015). Our proxy for risk aversion is intermediary leverage by He, Kelly and Manela (2017). We standardize the risk aversion proxy to have zero-mean and unit variance. The figure shows the response when the state variable RA_t is one standard deviation below (low risk aversion; blue line with squares), at (black line), or above (high risk aversion; red line with circles) its average value. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column reports the state multiplier of the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The shaded areas denote 90% confidence intervals.

Second, we replace UNC_t and RA_t in Eq. (1) with the proxies to risk aversion and economic uncertainty proposed by Bekaert, Engstrom and Xu (2017).[22] Since this series is available only starting from 1986/06 to 2015/02 we include two instead of four lags in the estimation of the local projections. Also, we find results to be noisy when we use the raw measure of risk aversion. We therefore let RA_t take value equal to 1 when the risk aversion proxy is above the 75th percentile, RA_t is equal to -1 when it is below the 25th percentile, and RA_t is set to zero otherwise. We then standardize the variable RA_t.

Figure D.3 shows the results, and it confirms the evidence obtained when we proxied risk aversion with the surplus consumption (see Figures 1 and 3), and the consumption-price ratio (see Figures 2 and 4).

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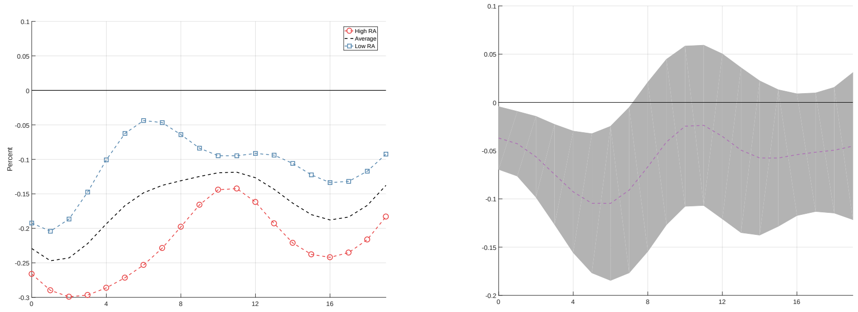
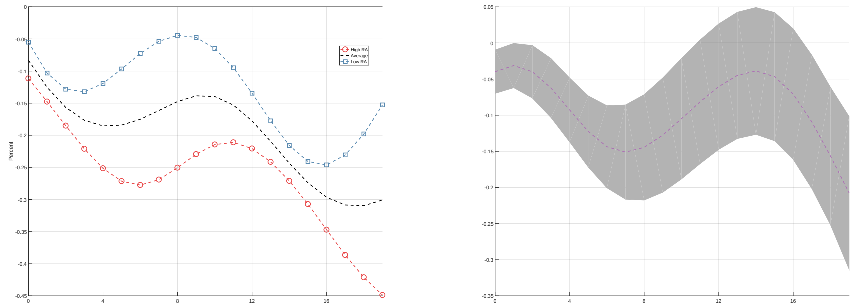
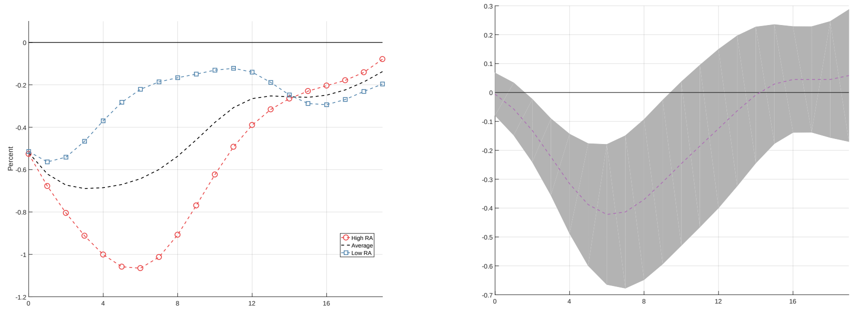
[22] The authors kindly thank Nancy Xu for sharing with us the risk aversion and uncertainty indices.



(a) Output. (b) State Multiplier – Output.

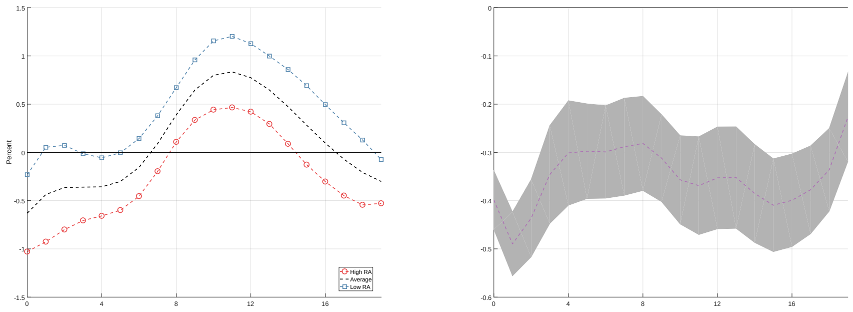


(c) Consumption. (d) State Multiplier – Consumption.

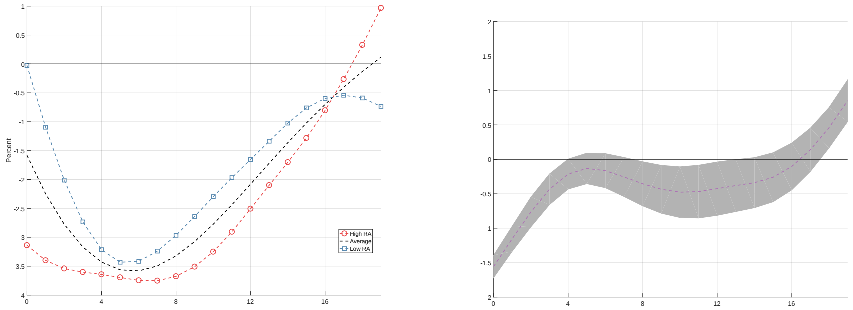


(e) Investment. (f) State Multiplier – Investment.

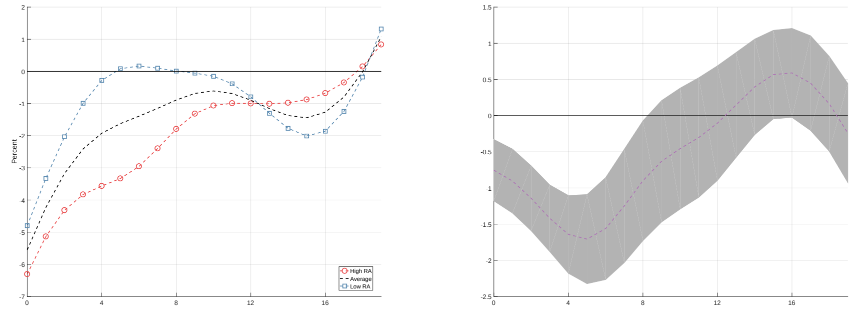
FIGURE D.3: **State-dependent IR to an uncertainty shock**: This figure plots the empirical impulse responses to an uncertainty shock for different levels of risk aversion. We measure risk aversion and uncertainty using the financial proxies provided by Bekaert et al. (2017). The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column reports the state multiplier of the state-dependent IRs. The shaded areas denote 90% confidence intervals.



(a) Stock prices, Leverage and EPU. (b) State Multiplier – Stock prices.



(c) Stock prices, Leverage and JLN. (d) State Multiplier – Stock prices.

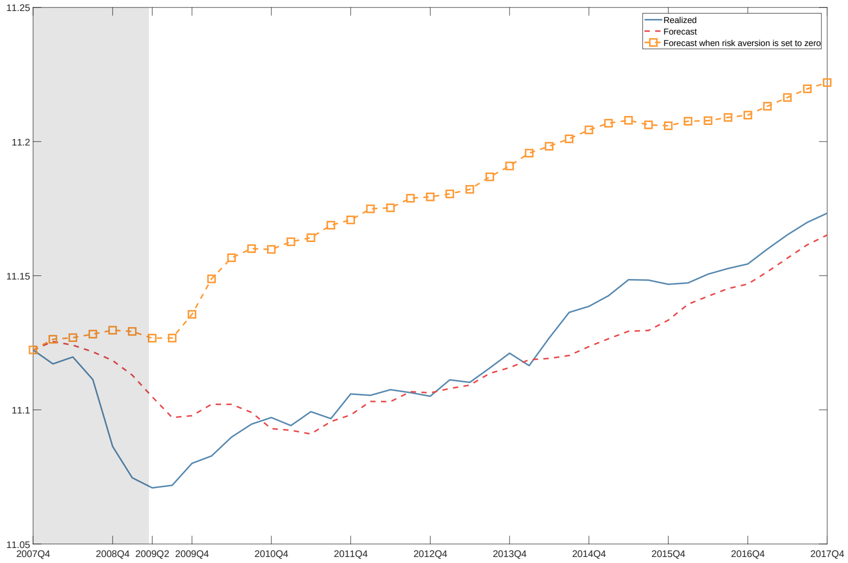


(e) Stock prices and Bekaert et al. (2017). (f) State Multiplier – Stock prices.

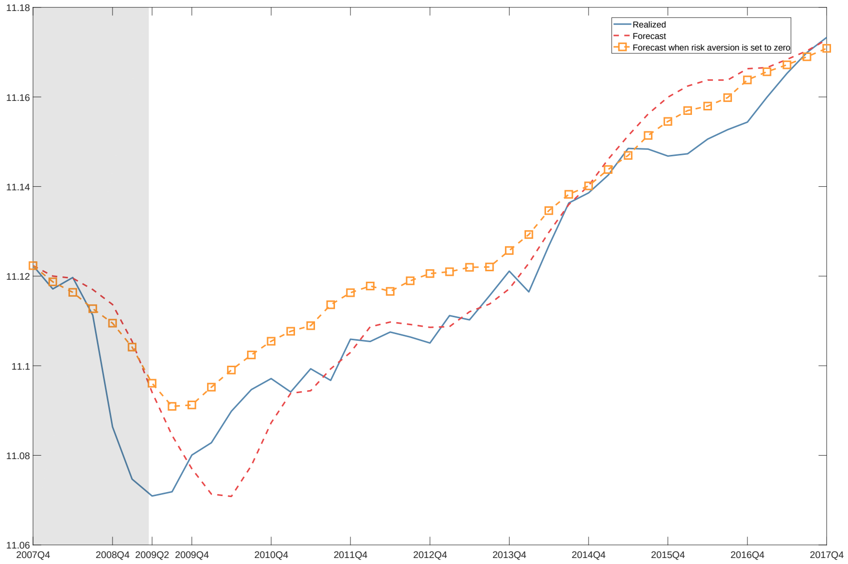
FIGURE D.4: **State-dependent IR of stock prices to an uncertainty shock**: In Panel (a)-(d) risk aversion is proxied with intermediary leverage by He, Kelly and Manela (2017). In Panel (a)-(b) we measure uncertainty using the economic policy uncertainty (EPU) by Baker et al. (2016). In Panel (c)-(d) we measure uncertainty using the financial uncertainty series by Jurado et al. (2015). We standardize the risk aversion proxy to have zero-mean and unit variance. In Panel (e) and (f) we measure risk aversion and uncertainty using the financial proxies provided by Bekaert et al. (2017).The figure shows the response when the state variable RA_t is one standard deviation below (low risk aversion; blue line with squares), at (black line), or above (high risk aversion; red line with circles) its average value. The left column shows the state-dependent IR (estimated using SLP) of stock prices to an uncertainty shock for different levels of risk aversion. The right column displays the state multiplier of the state-dependent IR of stock prices to an uncertainty shock. The shaded areas denote 90% confidence intervals.

D.2 Local Projections: Financial Crisis Results Using Alternative Measures of Uncertainty

Figure D.5 presents the time series plots of realized and fitted values of output when RA is proxied by the surplus consumption (Panel A) or the consumption-price ratio (Panel B). Figure D.6 presents the time series plots of realized and fitted values investment when RA is proxied by the surplus consumption (Panel A) or the consumption-price ratio (Panel B). In both figures, to proxy for uncertainty we use the financial uncertainty index of Jurado, Ludvigson and Ng (2015).

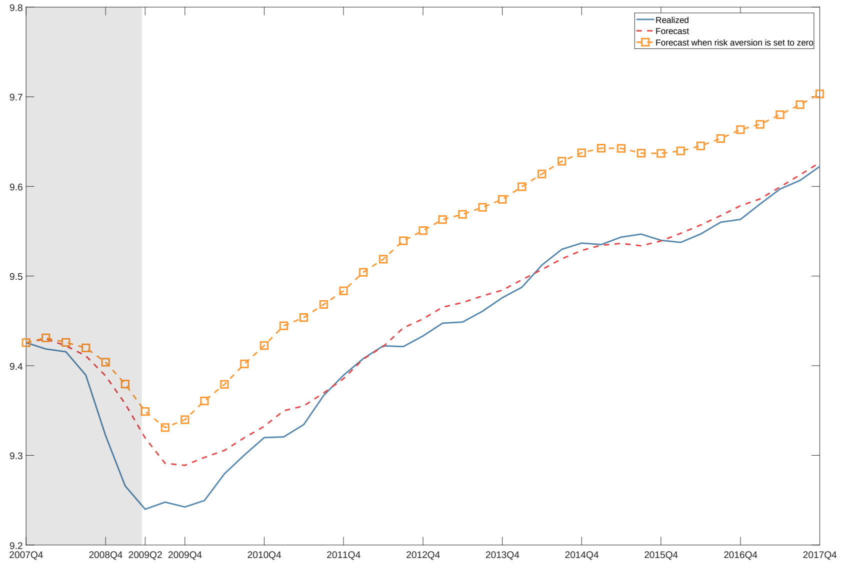


(a) Output and Habit.

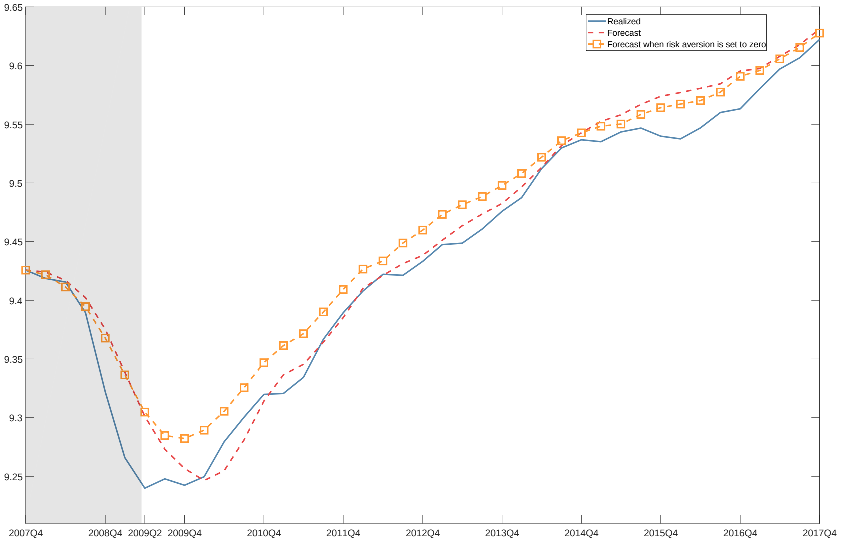


(b) Output and Consumption-Price ratio.

FIGURE D.5: **The interaction between Uncertainty and Risk aversion in Output**: The solid line displays (log) per capita, real GDP for our sample. The dashed line is the 1-quarter ahead forecast from direct regressions that allow for an interaction between risk aversion and uncertainty. The line with squares is the 1-quarter ahead forecast from direct regression with no interaction between risk aversion and uncertainty. Shaded areas indicate NBER recession dates. We measure uncertainty using the financial uncertainty series by Jurado, Ludvigson and Ng (2015). To proxy for risk aversion we use either the surplus consumption proxy $\sum_{j=1}^{40} \phi^j \Delta c_{t-j}$, (Panel A), or the consumption-price ratio (Panel B).



(a) Investment and Habit.

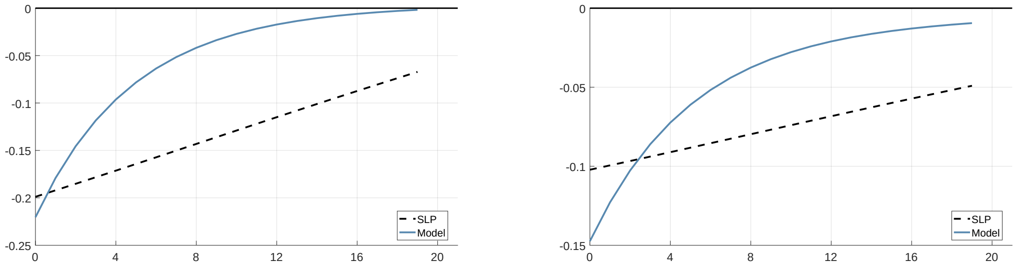


(b) Investment and Consumption-Price ratio.

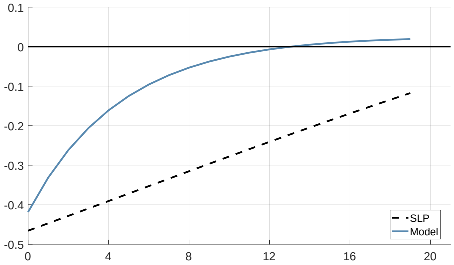
FIGURE D.6: **The interaction between Uncertainty and Risk aversion in Investment**: The solid line displays (log) per capita, real investment for our sample. The dashed line is the 1-quarter ahead forecast from direct regressions that allow for an interaction between risk aversion and uncertainty. The line with squares is the 1-quarter ahead forecast from direct regressions with no interaction between risk aversion and uncertainty. Shaded areas indicate NBER recession dates. We measure uncertainty using the financial uncertainty series by Jurado, Ludvigson and Ng (2015). To proxy for risk aversion we use either the surplus consumption proxy, $\sum_{j=1}^{40} \phi^j \Delta c_{t-j}$, (Panel A), or the consumption-price ratio (Panel B).

D.3 Local Projections Vs. Generalized IRFs

In this section we investigate whether the SLP method is able to recover the unconditional (Generalized) IRFs to technology uncertainty shocks from the NK-EZ-Habit model. We compare the unconditional responses with the SLP-estimated IRFs recovered from simulated economies that span 116 periods. Further, the technology uncertainty shock in the SLP exercise is re-scaled such that the SLP and unconditional responses of output coincide on impact. As can be seen from figure D.7 below, the two sets of IRFs are indeed very similar on impact but also at longer horizons.



(a) Output. (b) Consumption.



(c) Investment.

Figure D.7: **Unconditional Model IR vs SLP IR to a uncertainty shock**: This figure plots the unconditional impulse responses of output, consumption, and investment to an uncertainty shock in the NK-EZ-Habit model (blue line) against the median impulse responses recovered from SLP on model simulated data with risk aversion being neutral, i.e. $RA_t = 0$ (black dashed line).

E Perturbation Methods and Generalized Impulse Response Function

This appendix includes a more detailed discussion of the solution of the model and the explanation of how we compute the IRFs and the variance decomposition of the model. We refer the interest reader to the Born and Pfeifer (2014) Appendix for an exhaustive discussion of the use of perturbation and pruning techniques, and their implications for simulation and IRFs.

To judge the importance of volatility shocks for business cycle moments, and their interaction with risk aversion, our analysis relies on perturbation methods. Perturbation methods were first extensively applied to dynamic stochastic models by Judd (1998).

Our investigation faces a number of computational challenges. First, we are interested in the implications of a volatility increase while keeping the level of the variable constant. We thus have to consider a third-order Taylor expansion of the solution of the model, see e.g. Schmitt-Grohe and Uribe (2004), Fernández-Villaverde et al. (2011) and Fernández-Villaverde et al. (2015). Indeed, in a first-order approximation, stochastic volatility would not even play a role, since the policy rules of the representative agent follow a certainty equivalence principle. In the second-order approximation, only the product of the two innovations appears in the policy function. Only in the third-order approximation do the innovations to volatility play a role by themselves.[23]

Second, higher order perturbation solutions tend to explode due to the accumulation of terms of increasing order. For example, in a second order approximated solution, the quadratic term at time t will be raised to the power of two in the quadratic term at $t+1$, thus resulting in a quartic term, which will become a term of order 8 at $t+2$ and so on. As a solution, we adopt the pruning scheme described in Andreasen et al. (2016). This pruning scheme augments the state space to keep track of first to third order terms and uses the Kronecker product of the first and second order terms to compute the third order term. In contrast, Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014) use a IRF-pruning scheme were all higher order terms are based on the first-order terms. Also, whereas in Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014) the IRF-pruning scheme differs from the scheme used for simulations, we use the same pruning for both IRFs and simulations.

Third, computing IRFs in a nonlinear environment is somewhat involved, since the IRFs are not invariant to rescaling and to the previous history of shocks. To circumvent this problem, we consider the generalized impulse response function (GIRF) proposed by Koop et al. (1996). In particular, we follow Fernández-Villaverde et al. (2011), Born and Pfeifer (2014) and Basu and Bundick (2017), and we start the IRFs at the ergodic mean in the absence of shocks (EMAS).

Fourth, to judge the importance of risk shocks for business cycle moments, it is instructive to consider a variance decomposition. However, computing a variance decomposition is complicated because, with a third-order approximation to the policy function and its associated nonlinear terms, we cannot neatly divide total variance among the shocks as we would do in the linear case. Thus, to gauge the relative importance of shocks we follow Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014), and we simulate the model with only a subset of the shocks. In particular, we set the realizations of one or two of the shocks to zero and measure the volatility of the economy with the remaining shocks. The agents in the model still think that the shocks are distributed by the law of motion that we specified: it just happens that their realizations are zero in the simulation.

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[23] Recently, de Groot (2016) shows that to risk-correct the constant term for the standard deviation of stochastic volatility innovations (a.k.a. vol of vol) a fourth (or sixth, depending on the functional form of the volatility process) order expansion is further needed. de Groot (2016) shows that this risk-correction has important consequences for the bond and equity risk premia as well as for understanding the welfare cost of business cycle fluctuations.

F Impact of Risk Aversion on Uncertainty Shocks in an Open Economy Model

To complement the analysis of the New-Keynesian model with uncertainty in productivity from section 3, we study the open economy model from Fernández-Villaverde et al. (2011) (FGRU) in this section. We show that the effects of volatility shocks on the real economy are intertwined with the magnitude of risk aversion in a model with no rigidities and in which conditional volatility affects the real interest rate. To study how uncertainty shocks and risk aversion jointly determine business cycle moments, we consider impulse response functions (IRFs) and variance decompositions in the analysis. We refer interested readers to Appendix E for the technical details behind the computation.

F.1 The FGRU Model

The FGRU model is a standard small open economy business cycle model calibrated to match data from four emerging economies: Argentina, Brazil, Ecuador, and Venezuela. The small open economy is populated by a representative household.[24] In contrast to Fernández-Villaverde et al. (2011), we model the preferences of the household with a recursive utility function similar to equation 17 (see Epstein and Zin (1989) and Weil (1990)). We do so because we want to separate the effect of risk aversion from that of intertemporal substitution. Trivially, when risk aversion equals the inverse of the elasticity of substitution we obtain exactly the same results as Fernández-Villaverde et al. (2011).The household can invest in two types of assets: the stock of physical capital, K_t, and an internationally traded bond, D_t. Firms maximize profits by equating wages and the rental rate of capital to marginal productivities. Thus,

$$Y_t - C_t - I_t = D_t - \frac{D_{t+1}}{1+r_t} + \frac{\Phi_D}{2}\left(D_{t+1} - D_t\right)^2$$

where $\Phi_D > 0$ is a parameter that controls the costs of holding a net foreign asset position.

The model is calibrated to monthly frequency. Following the original approach, we construct quarterly simulated data, and we report results on a quarterly basis. Finally FGRU takes the real interest rate, r_t, as an exogenously defined process. We now turn to describe these dynamics.

F.1.1 Stochastic Volatility in Real Interest Rate

The real interest rate, r_t, a country faces on loans denominated in US dollars is decomposed as the international risk-free real rate plus a country–specific spread:

$$r_t = r + \varepsilon_{tb,t} + \varepsilon_{r,t}$$

where r is the mean of the international risk-free real rate plus the mean of the country spread; the term $\varepsilon_{tb,t}$, equals the international risk-free real rate subtracted from its mean, and $\varepsilon_{r,t}$ equals the country spread subtracted from its mean. Both $\varepsilon_{tb,t}$ and $\varepsilon_{r,t}$ follow AR(1) processes described by

$$\begin{aligned} \varepsilon_{tb,t} &= \rho_{tb}\varepsilon_{tb,t-1} + e^{\sigma_{tb,t}}u_{tb,t} \\ \varepsilon_{r,t} &= \rho_{r}\varepsilon_{r,t-1} + e^{\sigma_{r,t}}u_{r,t}\ , \end{aligned}$$

where both $u_{r,t}$ and $u_{tb,t}$ are normally distributed random variables with mean zero and unit variance. Importantly, the process for interest rates displays stochastic volatility. In particular, the

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[24] For the interested reader, a detailed derivation of the model equations, and steady states is available in Fernández-Villaverde et al. (2011), and hence not repeated here.

standard deviations $\sigma_{tb,t}$ and $\sigma_{r,t}$ follow an AR(1) process:[25]

$$\sigma_{tb,t} = (1 - \rho_{\sigma_{tb}})\,\sigma_{tb} + \rho_{\sigma_{tb}} \sigma_{tb,t-1} + \eta_{tb} u_{\sigma_{tb},t} \tag{F.1}$$

$$\sigma_{r,t} = (1 - \rho_{\sigma_{r}})\,\sigma_{r} + \rho_{\sigma_{r}} \sigma_{r,t-1} + \eta_{r} u_{\sigma_{r},t}, \tag{F.2}$$

where both $u_{\sigma_{tb},t}$ and $u_{\sigma_r,t}$ are normally distributed random variables with mean zero and unit variance.

Each of the components of the real interest rate is affected by two innovations. For instance, $\varepsilon_{tb,t}$ is hit by $u_{tb,t}$ and $u_{\sigma_{tb},t}$. The first innovation, $u_{tb,t}$, changes the rate, while the second innovation, $u_{\sigma_{tb},t}$, affects the standard deviation of $u_{tb,t}$. The innovations $u_{r,t}$ and $u_{\sigma_r,t}$ have a similar reading.

In comparison with the country spread, the international risk-free real rate has both lower average standard deviation of its innovation (σ_{tb} is smaller than σ_r for all four countries) and less stochastic volatility ($\eta_{tb,t}$ is smaller than $\eta_{r,t}$ for all four countries). These relative sizes justify why in our analysis we concentrate only on the innovation to the volatility of the country spread, $u_{\sigma_{tb},t}$, and forget about shocks to the international risk-free real rate. For simplicity, we refer to the innovation $u_{\sigma_{tb},t}$ as the stochastic volatility shock.

F.1.2 Volatility Shocks, Risk Aversion and Macro Dynamics

We examine the impulse response functions (IRFs) of the model to shocks in the productivity, country spread, and country spread volatility. We report the results only for the model calibrated to Argentina. We consider both the original calibration of Fernández-Villaverde et al. (2011) and the re-calibrated model of Born and Pfeifer (2014).[26] The IRFs for a positive one standard deviation shock are reported in Figure F.1. We plot the IRFs of output (first row of panels), consumption (second row), investment (third row) to the three shocks (columns).

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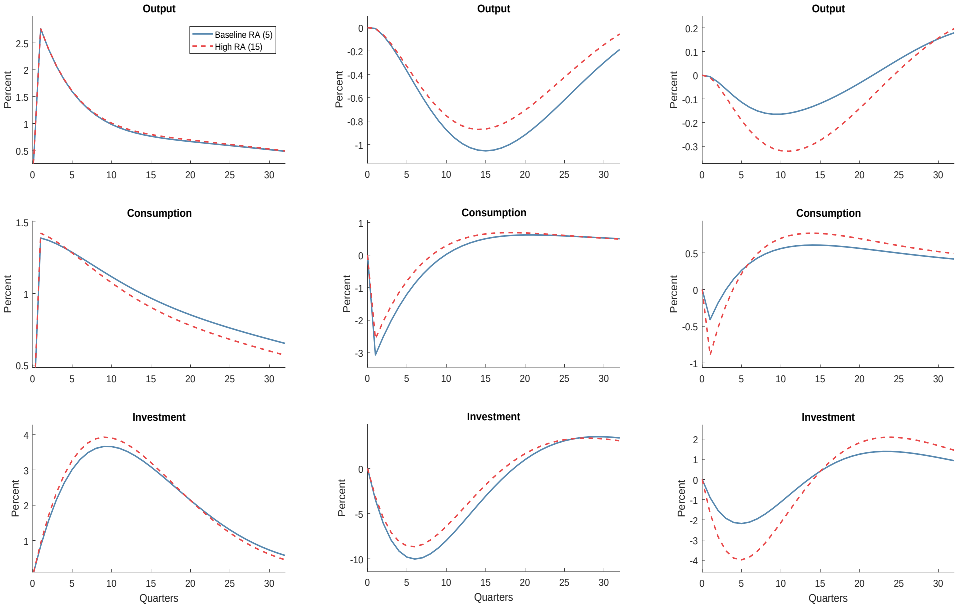
[25] This specification has been adopted by Justiniano and Primiceri (2008) among others.

[26] We use the same parameters as in Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014).



(a) Technology level shock. (b) Country spread level shock. (c) Country spread vol shock.

FIGURE F.1: **Impulse Response Functions – FGRU**: This figure plots the impulse responses for a one standard deviation shock to the (i) technology level (ii) interest rate (iii) conditional volatility in interest rate. Impulse responses are for a one standard deviation shock when the model is approximated up to third order. The cost of debt is set to $\Phi_D = 0.001$. To construct these responses, we set the exogenous shocks in the model to zero and iterate our third-order solution forward. After a sufficient number of periods, the endogenous variables of the model converge to a fixed point, which we denote the stochastic steady state. We then hit the economy with a one standard deviation to e.g. country spread shock but assume the economy is hit by no further shocks. The IRFs must be interpreted as the percentage deviations from the ergodic mean in absence of shocks, EMAS.

The amplifying effect of risk aversion on the macroeconomic dynamics is apparent only following uncertainty shocks. The third column plots the IRFs to a one-standard-deviation shock to the volatility of the Argentinean country spread, $u_{\sigma_{tb},t}$. This column shows that there is a large effect of risk aversion on macro dynamics. The first two columns of Figure F.1 show that IRFs to shocks in the level are hardly affected. In response to a positive volatility shock, output, consumption, and investment fall more in the case of high risk aversion than in the case of low risk aversion. For example, after a shock to volatility, consumption drops 0.5% upon impact when risk aversion equals to 5; the contraction is larger (1.1% percent at impact) when risk aversion equals to 15. Similarly, we observe a slow fall in output (after 10 quarters, it falls 0.16 percent) when risk aversion is low. However, for high risk aversion, the fall is deeper and more persistent (after 11 quarters years, it falls 0.32 percent). The implication is the same for investment. Columns $(2) - (4)$ in Table F.1 display the drop in macroeconomic variables, and the length of the recovery phase, for alternative values of risk aversion within FGRU.

Table F.1: **IRF Analysis - The Effect of a Volatility Shocks in FGRU**

This table reports the drops in macroeconomic variables, and the length of the recovery phase, for alternative values of risk aversion. Results are shown for both the model by Fernández-Villaverde et al. (2011) and the recalibrated corrected model by Born and Pfeifer (2014). "Recovery time" is defined as time (closest quarter) it takes for a variable to revert back to its unconditional mean. An example: With a risk aversion of 15, it takes 4 quarters for consumption to revert back to its mean level after a one standard deviation shock in volatility. See also Figure F.2.

	Fernández-Villaverde et al. (2011)			Born and Pfeifer (2014)		
(1) Risk Aversion	(2) Largest Drop	(3) Time	(4) Recovery Time	(5) Largest Drop	(6) Time	(7) Recovery Time
Panel A: Output						
5	-0.165	10	22	-0.287	10	26
15	-0.321	11	25	-0.753	11	26
20	-0.481	11	26	-1.2209	11	26
25	-0.642	12	27	-1.6886	11	26
Panel B: Consumption						
5	-0.410	1	4	-1.125	1	8
15	-0.896	1	4	-2.757	1	7
20	-1.381	1	5	-4.390	1	7
25	-1.866	1	5	-6.022	1	7
Panel C: Investment						
5	-2.183	5	14	-4.230	5	15
15	-3.979	5	15	-9.778	5	14
20	-5.774	5	15	-15.353	4	14
25	-7.570	5	16	-20.957	4	14

Table F.2 shows the variance decomposition of output, consumption, and investment.Each column corresponds to a specific simulation: (1) the benchmark case with all three shocks (productivity, the country spreads and its volatility); (2) when we have a shock only to productivity; (3) when we have a shock to productivity and to the interest rate (with volatility fixed at its unconditional value); (4) when we have shocks to interest rate and to volatility; (5) when we have a shock only to the interest rate level; and (6) when we have shocks only to interest rate volatility. The last column shows that volatility alone makes a relatively important contribution to the fluctuations of consumption (the standard deviation is 0.75) and investment (standard deviation of 3.08). Again, increasing the risk aversion almost doubles these contributions.

TABLE F.2: **Variance Decomposition FGRU - The Effect of Structural Shocks**

This table reports the variance decomposition for the different structural shocks in the model of Fernández-Villaverde et al. (2011) with stochastic volatility. First column: 200 simulations of the model; second column: TFP shocks only; third column: without volatility shocks to spread and T-bill rate; fourth column: without TFP shocks; fifth column: only level shocks to the spread and the T-bill rate; sixth column: only shocks to the volatility of spread and the T-bill rate.

	(1)	(2)	(3)	(4)	(5)	(6)
					Interest Rate	
	All Shocks	TFP only	w/o volatility	w/o TFP	Level only	Volatility only
Panel A: $\gamma = 5$, Pruning, 200 Replications						
σ_Y	5.25	5.02	5.09	1.10	0.64	0.16
σ_C	7.60	2.63	4.70	7.11	4.00	0.75
σ_I	20.63	5.00	12.71	19.90	11.63	3.08
Panel B: $\gamma = 15$, Pruning, 200 Replications						
σ_Y	5.23	5.01	5.07	1.07	0.57	0.31
σ_C	7.42	2.73	4.38	6.86	3.53	1.48
σ_I	20.42	5.42	11.86	19.57	10.54	5.89

F.2 An Alternative Calibration: Born and Pfeifer (2014)

We next consider an alternative calibration of the FGRU model. In particular, Born and Pfeifer (2014) note an error in the time aggregation of flow variables, and they show that the model of Fernández-Villaverde et al. (2011) must be recalibrated. Figure F.2 compares the IRFs for the recalibrated model with the original IRFs in Fernández-Villaverde et al. (2011). The figure shows that a one standard deviation positive volatility shock now leads to a larger drop in macro quantities than originally reported in Fernández-Villaverde et al. (2011). The difference between the two calibrations is further magnified under high risk aversion in the right column of the figure. Table F.1 columns (5) – (7) display the drop in macroeconomic variables, and the length of the recovery phase, for varying degrees of risk aversion.

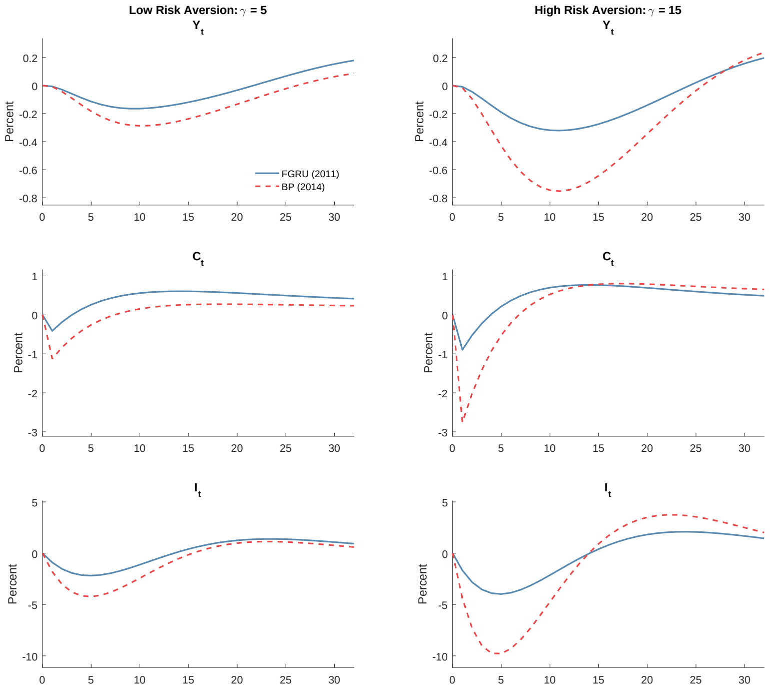


Figure F.2: **Impulse Response Function to a Volatility Shock Interest Rates – Fernández-Villaverde et al. (2011) vs Born and Pfeifer (2014) Calibrations**

Impulse responses are for a one standard deviation shock to the conditional volatility in interest rate when the model is approximated up to third order. The IRFs must be interpreted as percentage deviations from the theoretical mean based on the third-order pruned state space of Andreasen et al. (2016).

Table F.3 shows the variance decomposition for the alternative calibration proposed by Born and Pfeifer (2014). First, we find that in the re-calibrated model that corrects for the time-aggregation, the contribution of volatility shocks to business cycle volatility increase, and more so the higher the risk aversion.[27] Second, by comparing Table F.3 with Table F.2, an important insight emerges: risk aversion amplifies not only the simulation with volatility shocks only in column (6), but also the simulation where both level and volatility shocks are active in column (4). For example, in the Born and Pfeifer (2014) re-calibrated economy, investment raises by about 28% (18.11/14.19) when risk aversion raises from 5 to 15. On the other hand, the original Fernández-Villaverde et al. (2011) calibration does not show any sensitivity of investment to risk aversion in column (4) of Table F.2. This makes us conclude that: (1) volatility shocks are amplified by the magnitude of risk aversion; (2) the amplification effect of risk aversion in a simulation where both level and volatility shocks are active depends on the specific calibration of the model. The next section digs deeper into this issue and highlights the key role played by the cost of debt parameter Φ_D (which is higher in Fernández-Villaverde et al. (2011) and lower in Born and Pfeifer (2014))in determining the interaction between risk aversion and the level shock to interest rates.

TABLE F.3: **Variance Decomposition FGRU - The Effect of Structural Shocks**

This table reports the variance decomposition for the different structural shocks relying on the recalibration of Born and Pfeifer (2014) with stochastic volatility. First column: 200 simulations of the model; second column: TFP shocks only; third column: without volatility shocks to spread and T-bill rate; fourth column: without TFP shocks; fifth column: only level shocks to the spread and the T-bill rate; sixth column: only shocks to the volatility of spread and the T-bill rate.

	(1) All Shocks	(2) TFP only	(3) w/o volatility	(4) w/o TFP	(5) Interest Rate Level only	(6) Interest Rate Volatility only
	Panel A: $\gamma = 5$, Pruning, 200 Replications					
σ_Y	4.59	4.46	4.49	0.75	0.40	0.27
σ_C	4.39	2.10	2.72	3.81	1.76	1.40
σ_I	15.51	5.84	9.78	14.19	7.80	5.46
	Panel B: $\gamma = 15$, Pruning, 200 Replications					
σ_Y	4.61	4.44	4.47	0.93	0.38	0.66
σ_C	5.24	2.40	2.65	4.63	1.20	3.38
σ_I	19.54	6.68	9.85	18.11	7.11	13.10

F.2.1 Level Shock to the Country Spread

Examining the variance decomposition in Table F.2, it is striking that as risk aversion increases from Panel A to Panel B, the unconditional volatilities of macroeconomic aggregates actually drop in columns (1), (3), (4) and (5). This is driven by the level shock to country spread in column (5), as columns (2) and (6) show that the level shock to TFP and the volatility shock to country spread generate higher economic volatilities with increasing risk aversion. This implies that risk aversion can dampen the macroeconomic response to some shocks while strengthening the response to others.

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[27] When compared with the benchmark case with all three shocks (column 1), volatility shocks alone (column 6) account for 6 percent of output volatility and 35 percent of investment volatility. By increasing risk aversion, the contribution of volatility shocks to output and investment raises is a remarkable 35% and 67%, respectively.

To understand the mechanism causing elevated risk aversion to attenuate output, consumption and investment volatilities following level shocks to country spread, we focus on the Euler equation specific to the open economy model of FGRU (2011):

$$\frac{1}{1+r_t} = \Phi_D(D_{t+1} - D) + \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\nu} \right].$$

Here, like the original model, we assume CRRA utilities for the ease of exposition. To start with, assume the debt adjustment parameter, Φ_D, is zero. A positive level shock to r_t increases the country spread and lowers the price ($\frac{1}{1+r_t}$) of the internationally traded bond. Under the low risk aversion calibration, $\nu = 5$ for example, lower bond price today translates into higher expected consumption growth between today and tomorrow in the Euler equation. As a result, the representative agent optimally decides to borrow more today and invest less in capital. As risk aversion increases, to $\nu = 15$ for example, a level shock of the same magnitude to country spread does not raise consumption growth expectation as significantly. To see this, rewrite the Euler equation in logs while keeping $\Phi_D = 0$:

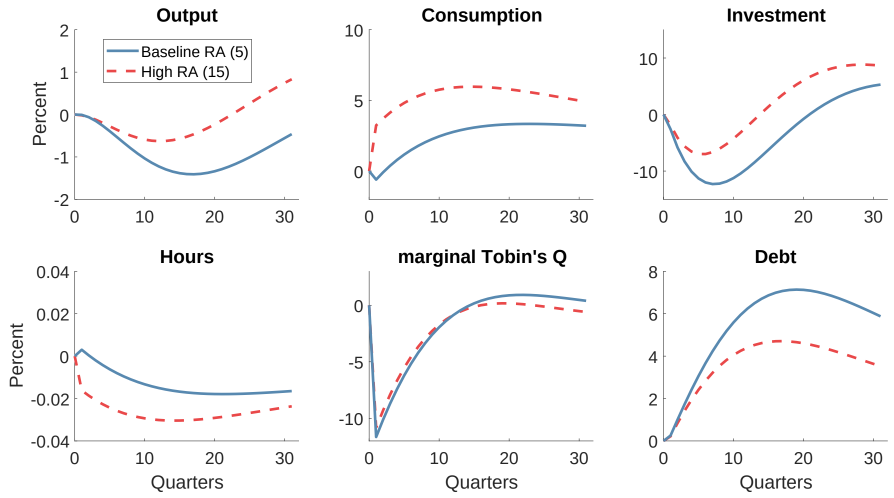
$$e^{-r_t} = \beta \mathbb{E}_t \left[e^{-\nu(c_{t+1} - c_t)} \right].$$

Holding the increase in r_t constant, larger ν means smaller $(c_{t+1} - c_t)$. As consumption growth expectation is tempered due to high risk aversion, the representative agent does not adjust borrowing and investment after the level shock is realized as dramatically relative to the case when risk aversion is low. Taken together, high risk aversion attenuates the dynamic response of macroeconomic variables with respect to level shocks through the consumption growth expectations.

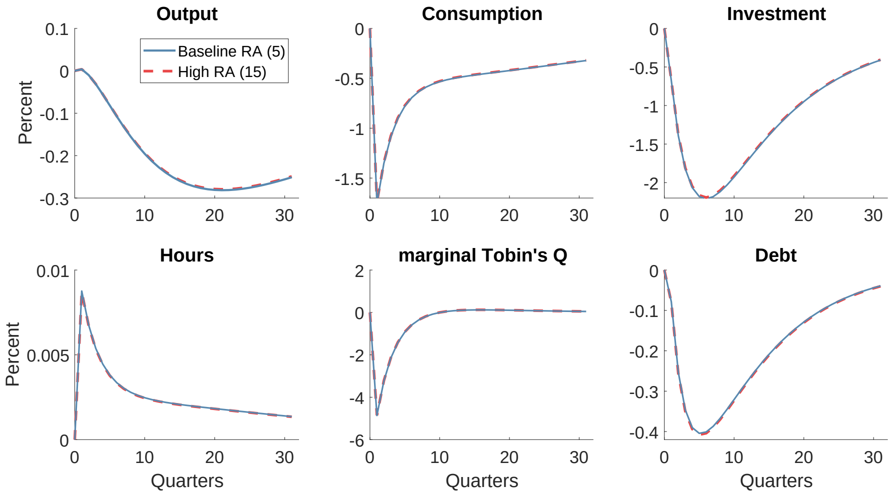
In the benchmark case, however, Φ_D is not zero, and the debt adjustment cost term enters the Euler equation. Under this scenario, a positive level shock to country spread lowers the price of the internationally traded bond and causes debt level to decline (since $\Phi_D > 0$). Furthermore, because the debt adjustment term partially absorbs the price drop, consumption growth expectation does not alter between high and low risk aversion calibrations as much compared to the scenario when Φ_D is zero. Therefore, the representative agent assuages the disinvestment to similar degrees regardless of high or low risk aversion. In other words, debt adjustment cost renders the impact of risk aversion on the debt and investment responses to level shocks to country spread ineffective.

Figure F.3 demonstrates the FGRU model implied impulse response functions for output, consumption, investment, hours, q and debt following a positive level shock to country spread under low and high risk aversion calibrations. Panel A presents the IRFs when the debt adjustment cost parameter is set to close to zero ($\Phi_D = 0.0001$), while Panel B contains the IRFs when the same parameter is set to 0.1. When the adjustment term is small in Panel A, higher r_t causes output, investment, hours and q to decline. At the same time, consumption and borrowing increase due to a rise in consumption growth expectation, consistent with the mechanism described above. Furthermore, as risk aversion is elevated from 5 to 15 in Panel A, the drops in output and investment and the increase in borrowing are less exaggerated.

In Panel B of Figure F.3, we set the debt adjustment term to be large. Three takeaways are immediate. First, in comparison to Panel A, the positive level shock to country spread leads to moderate declines in consumption and borrowing as the adjustment cost absorbs most of the “good news” generated by lower price of debt. Second, as consumption growth expectation is mitigated by the cost of changing the debt level, investment only dips mildly in contrast to when $\Phi_D = 0$. Finally, the differential impact of the country spread shock under low and high risk aversion is completely nullified in Panel B in the presence of debt adjustment cost. These implications are in line with our priors formed by examining the Euler equation of the FGRU model and provide us with insights on the interaction between risk aversion and first moment shocks to the real interest rate.



(a) Response to country spread level shock when cost of debt is low.



(b) Response to country spread level shock when cost of debt is high.

FIGURE F.3: **Impulse Response Function to Country Spread Level Shock for different values of the holding cost of debt**: This figure plots the impulse responses for a one standard deviation shock to the country spread level shock when the (i) cost of debt $\Phi_D = 0.0001$, and (ii) cost of debt $\Phi_D = 0.1$. Impulse responses are for a one standard deviation shock when the model is approximated up to third order. To construct these responses, we set the exogenous shocks in the model to zero and iterate our third-order solution forward. After a sufficient number of periods, the endogenous variables of the model converge to a fixed point, which we denote the stochastic steady state. We then hit the economy with a one standard deviation country spread shock but assume the economy is hit by no further shocks.