From Malthus to Malthus

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Abstract

Underneath a veneer of common structural transformation patterns, countries exhibit vast differences in their growth and allocation of human capital. In the one extreme, countries such as Korea, Taiwan, and, more recently, some segments of China, have transitioned, within just two generations, from prototypical agrarian Malthusian economies to fast-growing, high-skill-intensive modern service economies. In the other extreme, quite a few countries in Latin America, Africa and Asia have transitioned from similar initial Malthusian economies to urban, slow-growing, pseudo-Malthusian economies that are intensive in low-skill service jobs. We argue that a key determinant for such divergent paths lies in the direction of education policies of countries. We show that in the data, those countries that prioritize subsidies to university education in detriment of elementary and secondary education are those with large segments of the population with low skills, i.e., what we call the pseudo-Malthusian state. In contrast, countries that emphasize subsidizing high quality elementary and secondary education, possibly in detriment of higher education are the ones that have converged to the group of modern economies, which are mostly populated by high-skilled workers. In our model, we enhance the traditional quantity-quality fertility-vs-education model with households with multiple skill levels and multiple choices for the education of their children, ranging from no investment at all, to primary, secondary and up to tertiary education. The model replicates quite naturally that countries that emphasize lower levels of education eventually converge to a modern, skill intensive economy, while those which emphasize financing higher education will transition from a Malthusian economy to another.

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1 Introduction

During the last sixty years, the vast majority of developing countries underwent a dramatic transformation of their economies, transitioning from majorly producing agricultural goods to producing manufactures and services. Yet, underneath the veneer of common structural transformation patterns, the countries exhibit vast differences in their growth and allocation of human capital, and, ultimately, in the degree in which they have caught up with developed countries. In the one extreme, countries such as Korea, Singapore, Taiwan, and, more recently, some segments of China, have transitioned –within just two generations– from prototypical agrarian Malthusian economies to fast-growing, high-skill-intensive modern service economies. In the other extreme, quite a few countries in Latin America, Africa and Asia have transitioned from similar initial Malthusian economies to urban, slow-growing, pseudo-Malthusian economies that are intensive in low-skill service jobs.

In this paper, we argue the education policies of countries are a crucial determinant for such divergent development paths. More than the total share of income used in education, we emphasize the direction in which those resources are allocated between elementary and secondary education vs higher education. The data shows that those countries that prioritize subsidies to university education in detriment of elementary and secondary education are those with large segments of the population with low skills, i.e., what we call the pseudo-Malthusian state. In contrast, countries that emphasize subsidizing high quality elementary and secondary education, possibly in detriment of higher education are the ones that have converged or are in rout to converge to the group of modern economies, which are mostly populated by high-skilled workers.

We construct a structural transformation model, enhanced with education decisions and policy differences and use the model to understand the role of those policies in generating the drastic differences in the development experience of the countries. We enhanced the traditional quantity-quality fertility-vs-education model with households with multiple skill levels and multiple choices for the education of the children. Instead of simply having low- and high-skilled households deciding whether to have low- or high-educated children –and how many to have—we consider households with multiple skills, and education choices that range from very low to very high investments. Moreover, instead of simply having high- or low-education subsidies, the government of the different countries can opt to subsidize the lower and/or the higher level of education. These extensions are crucial to capture the observed differences in the education strategies of the different countries as observed in the data. And, as we argue in this paper, they are also crucial to actually capture the quantity-quality trade-offs of families in developing countries.

Our model emphasizes the role of education policies for the fertility/education decisions of households to explain the education and demographic patterns observed in the data. The model also emphasizes the role of human capital on the direction of countries, whether they move from 'Malthus to Solow,' as observed in Japan, South Korea, Singapore and Taiwan, or from 'Malthus to Malthus,' as observed in Brazil and many other countries in Latin America, Africa and Asia. While the model is highly stylized, it replicates quite naturally our observation that countries that emphasize lower levels of education eventually converge to a modern, skill intensive economy, while those which emphasize financing higher education will transition from a Malthusian economy to another.

Consider a developing country that heavily subsidizes education at the university level, but not at the elementary or secondary level. As shown in Second 2, this is an accurate description of Brazil, Libya and many other poor countries in Africa and Latin America. Being poor, the country is largely populated by lowly educated households, for most of whom the high subsidy for the college education of their children is all but completely irrelevant: To reach those subsidies they would need to finance themselves the primary and secondary education of their children. Aside from market incompleteness, wealth effects and the relative higher costs keep those households from high education goals for their children. Given that the elementary and secondary schools receive little resources, the poor households in those countries remain in a corner of low education, even if they also opt for low fertility. For the few highly educated households in the country, education subsidies for college education are relevant. These households pay for the cost of elementary and secondary education necessary for their children to enter college –and benefit from the subsidies. Overall, two simple forces preclude the aggregate population from moving towards the higher education groups. First, college subsidies are redundant, i.e. most rich, highly educated parents would invest in college for their children without those subsidies. Second, if a fraction of those children do not attain high skills, their progeny will not recover. In those economies, downward social mobility is dominant and eventually the population of those countries will be dominated by low-skilled individuals. In those countries, structural transformation and urbanization, if and when it happens, would be directed towards low-skill service jobs, as widely observed in Brazil and many other Malthus to Malthus countries.

Consider now a developing country that heavily subsidizes education at the elementary or secondary level. As shown in Second 2, this is an accurate description of not only South Korea, Singapore, and Taiwan, but also quite a few of European countries in the aftermath of WWII, when their education attainment was very low. Highly-subsidized/high-quality elementary and secondary education would drive households of the country, including the lowest educated, towards lower fertility and, more importantly, towards higher education. Within a generation, the population of the country would be dominated by medium skilled households, rich enough to contemplate the option of investing the additional resources to send their children to college. If either the returns to college are high enough and/or there are government subsidies to that end, then, within another generation we would observe the population of those countries moving in the direction of a higher share of the population with high levels of human capital. In those economies, the structural transformation towards services will not only happen, but will also be directed towards the high skill service sectors, as observed in Korea, Singapore and other successful transitions from Malthus to Solow.

In Section 2 of the paper, we revisit a number of development patterns for a large number of countries from 1950 onwards. As widely known, structural transformation is prevalent among the vast majority of countries. From 1950 to 1980, most countries moved away from agriculture and employment became dominant in manufactures and services. Despite substantial dispersion, the movement away from agriculture substantially reinforced growth, as the non-agriculture productivity was substantially higher and more skill-intensive. More interestingly, we observed that from 1980 to 2010, the non-agriculture productivity stagnate in many countries, and was often negative. Thus, a significant fraction of countries were modernizing but towards low-skill sectors. We argue that those countries are the ones that failed to increase the education attainment of the lower end of the skill distribution of households.

To explore these patterns, Section 2 looks at the alternative quantity-quality trade-offs for raising children as observed in the different countries. As argued above, on one side, we observed the Malthus-to-Solow countries such as Singapore, Korea and Taiwan, in which peasant/low-educated parents in 1950 give raise to technician/middle-educated parents in 1980, which, in turn, would rear a high share of college educated adults. On the other side, we also observe the Malthus-to-Malthus countries such as Brazil, Bolivia and quite a few others in which the peasant/illiterate parents in 1950 give raise to at most, primarily educated low-skill workers of 1980, which may also raise a high share of high-school dropouts that work in low-skill service jobs in 2010. Section 2 also documents the drastic cross-country differences in the ratio of resources per-pupil. While a handful of very poor countries such as Libya and Brazil spend orders of magnitude more in college students, relative to primary and secondary, other countries such as Japan, Switzerland and of course Korea, subsidize education at almost the same levels.

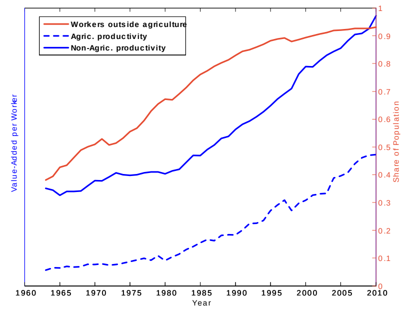
In Section 3, we construct our model. We consider a number of standard aspects of existing structural transformation models, e.g. non-homothetic preferences and low-skill intensity of agricultural goods, into an infinity horizon, altruistic model, with multiple education decisions and government policies. Rearing children costs are in terms of foregone earnings, while their education is in terms of high skill labor. Producing non-agriculture goods can be done using two different technologies: One basic that uses medium and low skills, and one advanced that uses high-and medium skills. Thus, the model can replicate the movement away from agriculture into two different paths: towards basic non-agriculture (Malthus-to-Malthus) or towards advanced non-agriculture (Malthus-to-Solow.) We show that feeding the model with different policies can naturally lead to the drastically divergent patterns of developed as discussed above. In particular, two otherwise identical economies can behave like Singapore, Korea or Taiwan, or like Brazil, Bolivia, Kenya and Mexico, entirely driven by differences in the share of education expenditures directed to elementary and secondary education and not to college. To illustrate this simple but important point, in this preliminary version we illustrate a simple calibration of the model to Brazil and South Korea. In the example, educational policies explains a large parte of the growth difference of these countries after 1980. Had South Korea the same policies observed in Brazil, its growth would be halfed.

2 Stylized facts

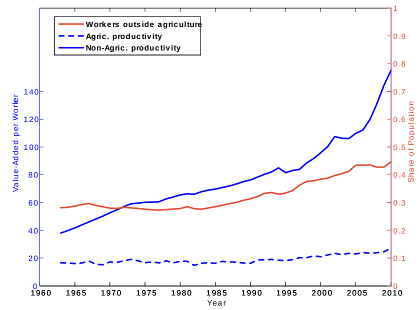
2.1 Structural Transformation and Productivity

At certain point of their development process countries experience structural transformation. The share of workers in agriculture declines, and so does its share of value added. At the same time, the share of the labor force in service and manufacturing activities increase. Advanced economies that, in most cases, have already completed this process end up with less than 5% of their workers in agriculture.

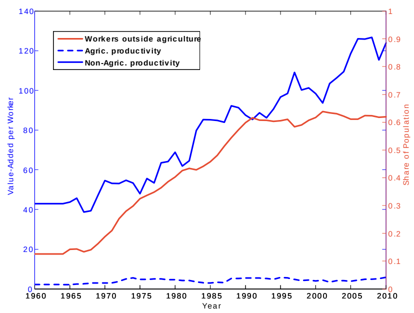
Moreover, services tend to be dominant and in the U.S., for instance, they comprise more than 80% of the total labor force. In this country, and in many more, a sizable share of the workforce end up in high skill services sectors, such as T.I., telecommunications, finance, etc. As workers move from low productive agriculture jobs to high productive urban jobs these economies experience an acceleration of their aggregate growth rate.



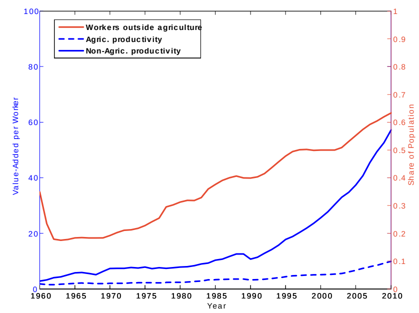
South Korea



India



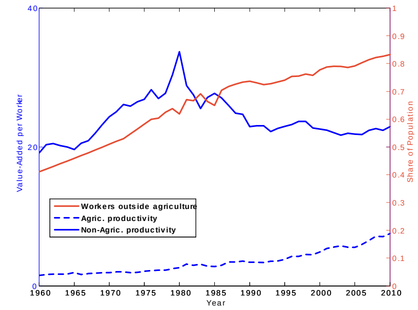
Botswana



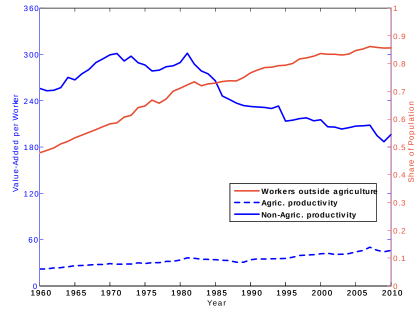
China

The four countries above follow this standard road map. South Korea, for instance, had in 1960 around 40% of their workforce outside agriculture, but fifty years later this figure jumped to more than 90%. At the same time, non-agriculture productivity increased by 200% in the period. Although in different stages of their development experiences, India, China and Botswana are other examples of structural transformation and steady growth.

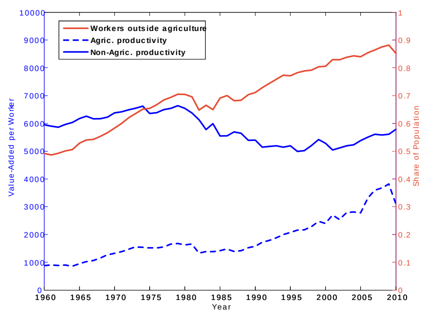
Many other countries have, however, followed a different path. Take for instance Brazil, displayed in the figure below. The share of non-agriculture workforce went from around 40% in 1960 to 84% in 2010, close to the South Korea trend. The productivity path, however, was very different. Until 1980, when most of the switch from agriculture to non-agriculture activities took place, aggregate productivity grew very fast, at above 2.5% per year. After this year, however, productivity per worker fell, at a rate of 0.5% per year. As we will argue below, the major driver for this decline is a composition effect in the labor force, as the expansion of employment outside agriculture has been driven by an increase in the mass of workers with low levels of schooling and skills. Mexico, Costa Rica, Kenya, also depart from the standard view of structural transformation, as displayed in the figure below.



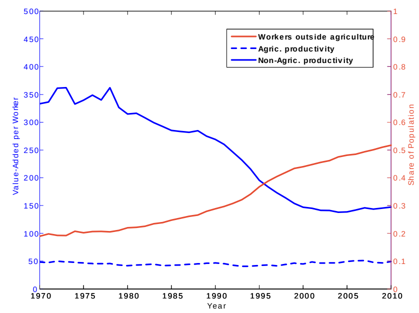
Brazil



Mexico

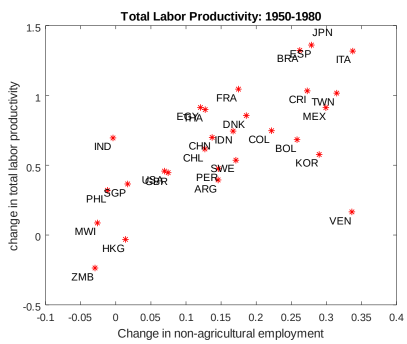


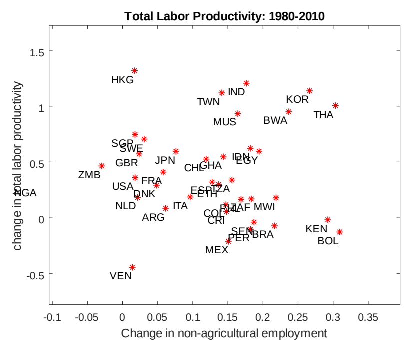
Costa Rica



Kenya

The two figures below show that these paths were not specific to this group of countries. In the horizontal axis of both figures we have the growth of the share of non agriculture workers and in the vertical axis the growth of labor productivity. The figure to the left - for the 1950-1980 period - is our "common sense" structural transformation story: aggregate labor productivity grows as the share of labor in agriculture declines. In almost all cases as this process advances countries grow continuously and in many cases productivity more than doubled.





The figure to the right (for the 1980-2010 period), tells a different story. For this period, there are different patters and there is clearly a group of economies that at the same time that the process of structural transformation continued, either experienced negative growth or grew very slowly. Among them, in addition to the four countries above, Peru, Malawi, South Africa and Bolivia. For this group structural transformation is associated to stagnation. In contrast, there is another group in which structural transformation lead to growth, in many cases at high rates. Examples here are South Korea, Indonesia, Thailand, and China, among others.

The classic view from Lewis of development and sector transformation, apparently, is less general than usually considered, and there are many countries that do not fit well in it. In this case, the reduction of the importance of agriculture did not lead to growth and continuous development.

2.2 Skills and education

As we just saw, after 1950 Brazil and South Korea experienced similar processes of structural transformation. But, while South Korea grew steadily in the period, the Brazilian product per worker fell after 1980. Many other economies followed Brazil as their growth slowed down at certain moment. The table below that uses data from the SEA displays one possible reason for this disparity.

Table 1: Skill Distribution in Services, Brazil and South Korea (%, 2009)

	Low Skill	**Medium Skill**	**High Skill**
Services - Brazil	33.0	43.7	23.3
Services - S. Korea	8,2	40.3	51.4
Retail - Brazil	38.4	52.0	9.5
Retail - S. Korea	3.1	44.2	52.8
Transportation - Brazil	42.9	45.6	11.4
Transportation - S. Korea	15.6	51.7	32.7
Finance - Brazil	5.8	32.8	61.4
Finance - S. Korea	17.2	40.8	42.0
Personal Services - Brazil	57.2	36.3	6.5
Personal Services - S. Korea	9.5	49.0	41.6
Public Services - Brazil	19.5	44.7	35.7
Public Services - S. Korea	4.7	21.0	74.3

Source: Social Economics Accounts

While both economies have similar share of workers in service, the skill composition is very different. In Brazil, the share of high skill workers in the sector was just 23% in 2009, in contrast to 51% in South Korea. In the latter there were only 8% low skill workers in the sector, but they were one-third in Brazil. In certain sub-sectors the contrast is even greater, there is virtually no low skill workers in retail in South Korea, but the figure in Brazil is close to 40%. And the share of high skill labor in this sector in South Korea is five times larger than in Brazil. Similar patterns is observed in almost all subsectors[1].

Hence, Brazil and other "Malthus" economies specialized in services but their distribution of skills is clearly concentrated in low skill workers, as opposed to fast growing economies such as South Korea that also became a service economy, but specialized in high skill services. The evolution of schooling, in these two groups, is behind these two very different patterns. The table below displays the evolution of secondary education for these two economies and also USA, Mexico and Chile, from 1950 to 2010:

[1] Finance is an exception, but the numbers of workers in both countries in this sub-sector is a very small share of the total labor force in services.

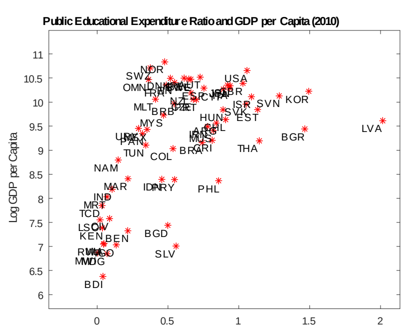
Table 2: Secondary Education (% population aged 25 and over)

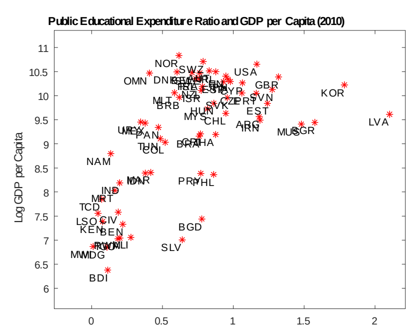
	South Korea	Brazil	USA	Mexico	Chile
1950	4.2	3.9	35.2	2.8	11.8
1955	5.2	4.3	39.1	2.9	12.5
1960	8.4	4.7	43.5	3.0	13.1
1965	11.2	5.4	53.2	4.1	14.6
1970	15.6	5.7	62.0	5.5	17.3
1975	19.9	6.6	71.1	7.3	18.9
1980	27.6	7.9	77.1	9.9	22.2
1985	36.1	10.0	78.5	13.5	29.4
1990	46.8	12.6	80.0	16.9	38.0
1995	57.3	16.3	82.2	22.0	42.3
2000	63.5	21.2	84.1	25.2	46.7
2005	69.5	29.3	89.1	30.3	50.8
2010	74.2	35.9	93.2	32.5	52.9

Source: Barro and Lee Database (2015)

The share of adult population with at least secondary education in Brazil and Mexico was very low in 1950, less than 4%. Moreover, it grew very slowly in the following thirty years. In both cases, the gap with respect to the U.S. - where the proportion of adults with secondary education more than doubled - increased markedly in the period. The path in Chile, and specially in South Korea, was very different. In these economies secondary education expanded very fast. South Korea went from figures not very different from Brazil and Mexico in 1950 to almost four times as much by 1980 . Hence, human capital accumulation paths were very different in stagnant and fast growing economies[2].

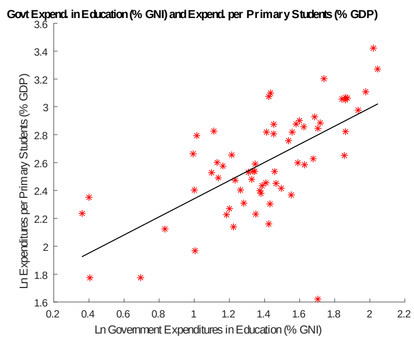
These economies followed very different educational policies and the quality of education were also very diverse across them. One very important policy is the distribution of public resources across educational level. There is clearly a positive relationship between the ratio of public expenditures in basic (primary and secondary) education to tertiary education and output per capita. Governments in poor countries tend to spend relatively more funds in college than in secondary or primary education, when comparing to richer economies. The figure to the left displays the ratio of primary to tertiary expenditures per student and the figure to the right presents the secondary to tertiary expenditures per student, both in 2010 and both only consider public expenses. In both cases, larger expenditures in basic education relative to college is associated to higher GDP per capita. This relationship seems to be permanent as it is also valid for different years that there are data available (e.g., 1998).

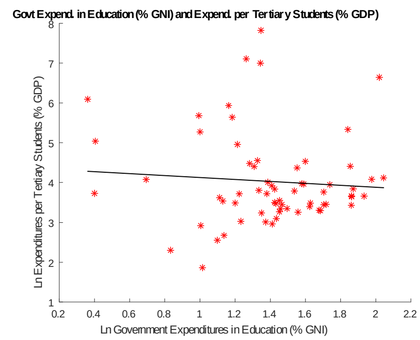




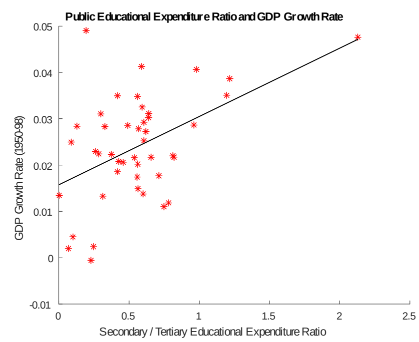
[2] This is also the case for other slow growing countries such as Costa Rica and Tunisia.

The figure to the left below displays the relationship between government expenditures as percentage of GDP and public expenses per primary student, also as percentage of GDP per capita. To the right we substitute primary by tertiary. There is a positive relationship (with high correlation) between public expenditures in education and how much you spend in primary students: economies that tend to spend proportionally more in education, spends also proportionally more with basic education. As for tertiary students the relationship, if any, is negative but the correlation is very small (-0.06).



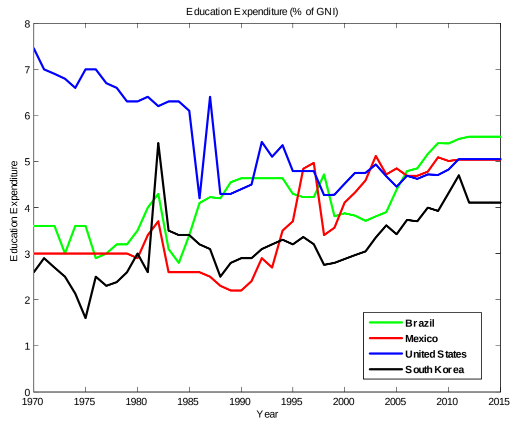


Moreover, as one can see from the figure below, countries that expend relatively less in tertiary education grew faster from 1950 to 1998. Hence, there is a level effect - economies with high basic to tertiary education expenditures tend to be richer - and a growth effect: they also tend to grow faster. Moreover, countries that tend to expend relatively more on education concentrate these expenses in basic education.



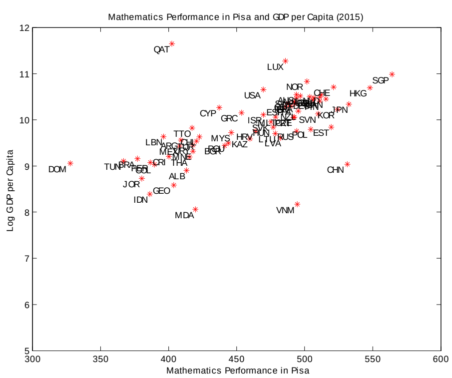
Note from the figure below that from 1970 to mid-eighties, the U.S. spent significantly more in education as proportion of GDP then Brazil, Mexico and South Korea. At that moment, those were mid to poor income economies, and they spent around the same in education. From the nineties on, Brazil and Mexico increased their expenditures, reaching the U.S. But in 1998, Brazil and Mexico spent relatively less in primary and secondary education than in tertiary. They were below the sample median in these dimensions and the indirect evidence is that this was also true before this date. South Korea, in contrast, concentrated its expenditures in basic education.

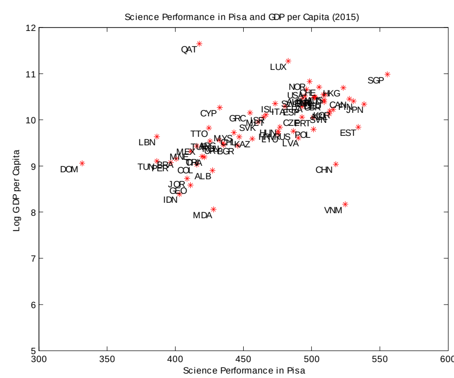
Hence, one can observe similar ratios of education expenditures to GDP in different countries, but the distribution across educational levels can be very diverse. If this distribution impacts growth and long run income, the level of education expenditures could be misleading. Of course, everything else the same, it is better to spend more than less in education. But the point is that "everything else is not the same," the distribution of these expenditures across levels of education is very different across countries.



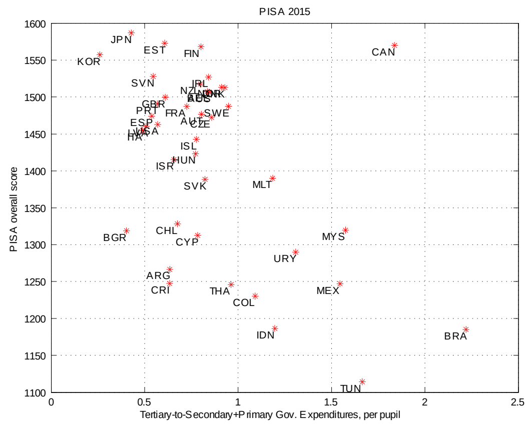
2.3 Quality of education

There is a clear positive relationship between quality of education as measured by Pisa results and output per capita. This, of course, is a correlation not implying causation. However, the numbers are staggering. The average math score of the richest quarter is 25% larger than the poorest quarter. The score in some of the slow growing economies - Brazil, Costa Rica, Mexico, Tunisia, for instance - are among the lowest in the group. For instance, the median of the entire group is 28% higher than Brazil´s score and the maximum score is 50% above of Brazilian results. The outcome is similar when considering the Science and Reading tests.





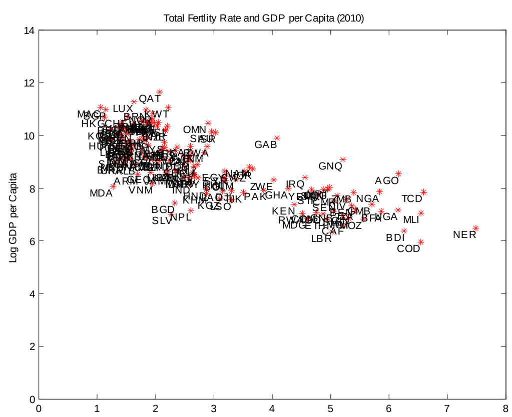
Also very important is the relationship between quality of education and the distribution of public expenditures across educational levels, as displayed by the figure below :



As one can see, there is a very strong negative relationship between the ratio of public expenditures per student in college to secondary and primary education and the quality of education. The latter is also represented by PISA test results, in this case by the sum of the three tests. Hence, the more a country spend with college students relative to basic education, the worst the education quality.

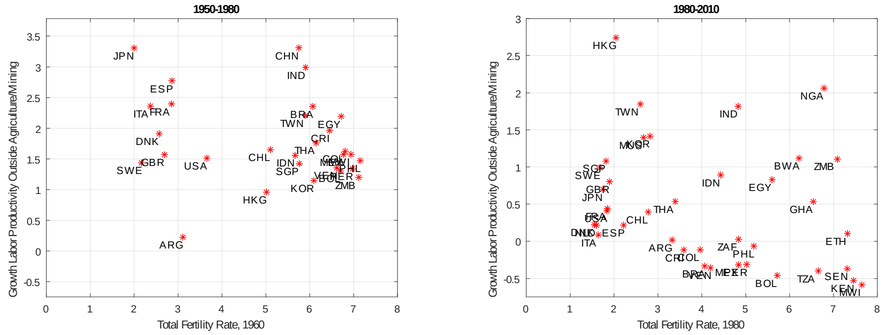
2.4 Fertility

The figure below presents the well known negative relationship between fertility rate and income per capita. In rich (poor) countries fertility rates tend to be smaller (larger).



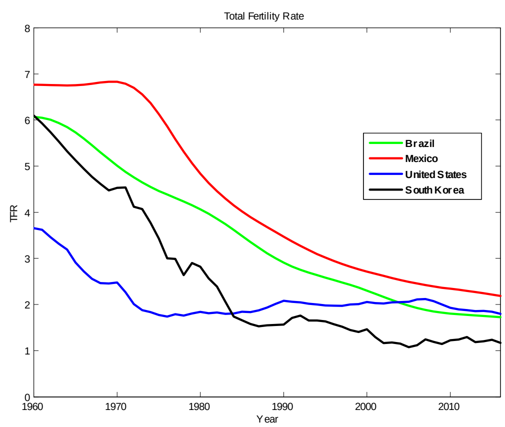
As for the relationship between fertility rates and growth, the relationship seems to be negative too but it is less clear. The two figures below present the link between fertility rate in the initial year (1960 and 1980) and growth. In the first case, for the 1950-1980 period, and in the second for 1980-2010 period[3].

[3] For lack of data we could not use, in the first case, 1950 fertility rates.



In the figure to the right there is a slight negative relationship: countries with lower (higher) fertility rates in 1980 tended to growth faster (slower) in the following 3 decades. In the figure to the left this relationship is less clear, but for the subgroup with fertility rates above 4 in 1960 the relationship seems to be negative.

In contrast, it is not entirely unexpected that fertility rates fell faster in South Korea, a fast growing country, than in Brazil, Mexico and other stagnant economies. From the figure below we see that in 1950 fertility rates were about the same in Brazil and South Korea. However, by 1985, it was twice as large in the former as in the latter. As we just saw, this was a period of very fast human capital accumulation in the Asian country, and very slow in Brazil.



3 The Model

3.1 The Environment

We consider an infinite horizon economy, populated by altruistic two-period lived individuals. Sectorial productivities and education provision policies are set exogenously, with proportional income taxes that are endogenously set to balance, period by period, the budget constraint of the government. Preferences are defined over the adults' own consumption of agricultural and agricultural goods. These preferences are non-homothetic, driving the structural transformation of countries over time. Similarly, the non-agricultural sector is modelled using nested CES to allow for the expansion of low- or high-skill sectors.

Time periods are indexed by $t = 0, 1, 2, ...$ There will be $e \in E = \{u, b, m, h\}$ education levels, $i \in I = \{U, B, M, H\}$ investment profiles in children.

Preferences: At time t, the utility of a household is given by

$$V_t = u\left(c_t^A, c_t^N\right) + \beta(n)\, nE_t\left[V_{t+1}\right],$$

where $u\left(c_t^A, c_t^N\right) = \ln\left[\left(c_t^A - \bar{c}^A\right)^{\alpha}\left(c_t^N\right)^{1-\alpha}\right]$, is the utility of consumption for the household, n is the number of children, $\beta(n) = \beta_0 n^{-\varepsilon}$ with $0 < \varepsilon < 1$, and $\beta_0 < 1$ define the discount factor, $E_t[\cdot]$ is the expectation over the possible realizations of the education attainment of children, and V_{t+1} is the attained utility of the children of the household[4]. Here, we assume that in equilibrium, it will always be the case that $\left(c_t^A - \bar{c}^A\right)^{\alpha}\left(c_t^N\right)^{1-\alpha} > 1$, so utilities are always positive.

Child-Rearing Households choose n, i.e. how many children to have (which we consider to be a continuous variable) and how much to invest in their education. All children have a time cost ϕ_t^e, which we let it to be type and time varying since it would reflect the opportunity cost of the parents. We could model this as the cost of raising a children that don´t go to school as being ϕ_t^u,the cost of raising a child that only goes to primary school as $\phi_t^u + \phi_t^p$, and so on.

In addition of the time costs of children, parents can opt for different investment profiles:

- **Child Labor (U):** No schooling and making children work. Indeed, the household collects a fraction $0 \leq \psi_t \leq 1$ of the income of an unschooled child worker , y_t^u, and when adult, these children would attain the education level e with probability $0 \leq \pi_{U,e} \leq 1$, where $\sum_{e \in E} \pi_{U,e} = 1$.

- **Primary Education (B):** This is the basic investment. For each kid, parents bear the costs of not sending the kid to work and of the private costs (net of subsidies) of primary education. We assume that primary education requires $v_B > 0$ units of the schooling good S. When adult, these children attain the education level e with probability $0 \leq \pi_{B,e} \leq 1$, where $\sum_{e \in E} \pi_{B,e} = 1$.

- **Medium Education (M):** This is the medium investment. For each kid, parents bear the costs of not sending the kid to work, and the private costs (net of subsidies) of primary education and secondary education. In addition to the costs of primary education, secondary education requires requires $v_M > 0$ units of the schooling good units S. When adult, these children attain the education level e with probability $0 \leq \pi_{M,e} \leq 1$, where $\sum_{e \in E} \pi_{M,e} = 1$.

- **High Education (H):** This is the high investment. For each kid, parents bear the costs of not sending the kid to work, and the private costs (net of subsidies) of primary education, secondary and tertiary education. In addition to the costs of primary and secondary education, tertiary education requires requires $v_H > 0$ units of the schooling good units S. When adult, these children attain the education level e with probability $0 \leq \pi_{H,e} \leq 1$, where $\sum_{e \in E} \pi_{H,e} = 1$.

We make the natural assumption that the higher the investment, the higher the probabilities of a good realization:

$$\pi_{U,\cdot} \prec \pi_{B,\cdot} \prec \pi_{M,\cdot} \prec \pi_{H,\cdot},$$

where $\prec$ denotes dominance in the first order stochastic sense.

[4]Alternatively, we could assume imperfect altruism: $V = u\left(c_t^A, c_t^N\right) + \beta U(n, q)$, where q is the educational level of the children.

Policies: We consider exogenous education policies with endogenous income taxes to balance the period budget constraint of the government (BCG.) In particular, we assume that, in period t, the government covers a fraction σ_t^B of the costs of primary education, a fraction σ_t^M of the costs of secondary education and a fraction σ_t^H of the tertiary education. We take the sequence $\left\{\sigma_t^B, \sigma_t^M, \sigma_t^H\right\}_{t=0}^{\infty}$ as exogenously given, but a proportional income tax τ_t is set so that the tax collections exactly cover these subsidies, as detailed below.

With these assumptions the problem of the families is given by:

$$
\begin{aligned}
V_t^{e,i} &= \max_n \left\{ \ln\left[\left(c_t^A - \bar{c}^A\right)^{\alpha}\left(c_t^N\right)^{1-\alpha}\right] + \beta(n)\, n \sum_{e\in E} \pi_{i,e} V_{t+1}^e \right\}, \qquad (1)\\
\text{s.t.} &: \quad p_t^A c_t^A + p_t^N c_t^N + \sum_{i\in I}\left(1-\sigma_t^i\right)\eta_t^i n = \left(1-\tau_t\right) y_t^e \left(1-\phi_t^e n\right) + \psi_t y_t^u \\
\eta_t^i &= w_{t,i}^h N_{h,t,i}^s \text{ (for the time being...)}
\end{aligned}
$$

Technologies: The are two goods: agriculture and non-agriculture. Denote them by the superscripts A, N and S, respectively. The production functions are

$$
\begin{aligned}
Y_t^A &= X_t^A N_{l,t}^A, \\
Y_t^N &= X_t^N G_t\left[F_t^0\left(N_{l,t}^0, N_{M,t}^0\right), F_t^1\left(N_{M,t}^1, N_{H,t}^1\right)\right],
\end{aligned}
$$

here $G_t(\cdot)$, $F_t^0(\cdot)$,and $F_t^1(\cdot)$ are constant returns to scale CES production functions. For now, we will let them be time varying. The resource constraints are

$$
\begin{aligned}
N_{l,t} &= N_{l,t}^A + N_{l,t}^0, \\
N_{m,t} &= N_{m,t}^0 + N_{m,t}^1, \\
N_{h,t} &= N_{h,t}^1 + N_{h,t}^S.
\end{aligned}
$$

where $N_{h,t}^S$ are middle skill and high skill labor, respectively, used in schools.

Firms solve:

We assume the low-skill workers are given by the unschooled and those with only primary or less, i.e. $N_{l,t} = \omega N_{u,t} + N_{b,t}$, where $0 < \omega < 1$.

The problem of the firms are given by:

- – *Agricultural* firms maximize profits:

$$
\pi_t^A = \max_{N_{l,t}^A}\left\{p_t^A X_t^A N_{u,t}^l - w_t^l N_{l,t}^A\right\} \qquad (2)
$$

  – *Non-agricultural firms* maximize profits:

$$
\begin{aligned}
\pi_t^N = & \max_{N_{l,t}^0, N_{lmt}^0, N_{m,t}^1, N_{h,t}^1} p_t^N X_t^N G_t\left[F_t^0\left(N_{l,t}^0, N_{m,t}^0\right), F_t^1\left(N_{m,t}^1, N_{h,t}^1\right)\right] \qquad (3)\\
& - w_t^l N_{l,t}^0 - w_t^m\left(N_{m,t}^0 + N_{m,t}^1\right) - w_t^h N_{h,t}^1,
\end{aligned}
$$

3.2 Equilibrium

3.2.1 Definition

Given exogenous policies $\left\{\sigma_t^B, \sigma_t^M, \sigma_t^H\right\}_{t=0}^{\infty}$ and exogenous sectorial productivities $\left\{X_t^A \text{ and } X_t^N\right\}_{t=0}^{\infty}$ and an initial population, $\left(L_0^u,\, L_0^b,\, L_0^m,\, L_0^h\right)$, a **competitive equilibrium** is composed by (a) a *price system* $\left\{p_t^N,\, w_t^l,\, w_t^b, w_t^m, w_t^h\right\}_{t=0}^{\infty}$, (b) an *allocation* defined as (i) a sequence individual consumption and fertility/education choices choices $\left\{\left\{c_t^{A,e}, c_t^{N,e}\right\}_{e\in E}\right\}_{t=0}^{\infty}$, $\left\{\left\{n_t^e, i_t^e\right\}_{e\in E}\right\}_{t=0}^{\infty}$, (ii) attained values $\left\{\left\{V_t^e\right\}_{e\in E}\right\}_{t=0}^{\infty}$ and (iii) populations $\left\{\left\{L_t^e\right\}_{e\in E}\right\}_{t=0}^{\infty}$, and (c) proportional income taxes $\{\tau_t\}_{t=0}^{\infty}$, such that: (1) individuals choices are rational and attain values $\left\{\left\{V_t^e\right\}_{e\in E}\right\}_{t=0}^{\infty}$,(2) the populations $\left\{\left\{L_t^e\right\}_{e\in E}\right\}_{t=0}^{\infty}$ are consistent with initial conditions and individual fertility/education choices, (3) the budget constraint of the government is satisfied, and (4) goods and labor markets clear.

To complete this definition, we now lay out the individual problems, the law of motion for the population and the government budget constraint.

- **Individual Rationality:** Given prices:
  - **Individual households** solve:
  $$V_t^e = \max_{i\in I}\left\{V_t^{e,i}\right\},$$
  where $V_t^{e,i}$ solve problem (1)
  - **Firms** in agriculture solve problem (2) and firms in the non-agriculture sector solve (3).
- **The Government Budget Constraint:** For any time t, collected income taxes must finance education subsidies:
$$\tau_t\left[\sum_{e\in E} w_t^e L_t^e\left(1-\phi_t^e n_t^e\right)\right] = p_t^S \sum_{e\in E} L_t^e n_t^e\left[\sigma_t^B \chi_t^{e,B} + \sigma_t^M \chi_t^{e,M} + \sigma_t^H \chi_t^{e,H}\right].$$
- **Markets Clear:**
  - *Goods Markets:* For all t:
  $$\text{Agriculture:} \qquad \sum_{e\in E} L_t^e c_t^{e,A} = Y_t^A;$$
  $$\text{Non-agriculture} \;:\; \sum_{e\in E} L_t^e c_t^{e,N} = Y_t^N$$
  - *Labor Markets:* Assuming no indifference (i.e. h does not provide m and m does not provide b), then, for all t:
  $$\text{Low-skilled workers:} \qquad L_t^u\left(1-\phi_t^u n_t^u\right) + L_{b,t}^b\left(1-\phi_t^b n_t^b\right) = N_{l,t}^A + N_{l,t}^0,$$
  $$\text{Mid-skilled workers:} \qquad L_t^m\left(1-\phi_t^m n_t^m\right) = N_{M,t}^0 + N_{M,t}^1\{\{\{+N_{M,t}^S\}\}\},$$
  $$\text{Mid-skilled workers:} \qquad L_t^h\left(1-\phi_t^h n_t^h\right) = N_{H,t}^1 + N_{H,t}^S.$$

- **Law of Motion of the Population:** Let i_t^ε denote the investment choice by households $\varepsilon \in E$. Then, $\pi_{i_t^\varepsilon, e}$ denote the probability that a household type ε has children of type e

$$L_{t+1}^e = \sum_{\varepsilon \in E} \pi_{i_t^\varepsilon, e} \times n_t^\varepsilon L_t^\varepsilon, \text{ for all } e \in E.$$

4 A Simple Example

A first implementation of the model assumes that adults can be of only two discrite types, skilled, if they went to school and unskilled, if they worked as children and did not go to school. There are now three sectors:

Agricultural sector that uses unskilled labor only:

$$Y_A = A_A L_{AU}.$$

Manufacturing sector, that uses skilled labor only:

$$Y_M = A_M L_{MS}.$$

Services, that uses skilled and unskilled labor:

$$Y_{Se} = A_{Se}(L_{SeS})^\alpha (L_{SeU})^{1-\alpha}.$$

It is also assumed exogenous sectorial productivity:

$$A_j' = (1 + \gamma_j) A_j$$

Preferences are given by:

$$U(u, n_S, n_U) = u^\sigma + \beta(n_S + n_U)^{-\varepsilon}[n_S V_S' + n_U V_U'],$$

where the intratemporal utility u is given by:

$$u(c_A, c_M, c_{Se}) = v(c_A) + b\log(c_M) + (1-b)\log(c_{Se} + \bar{c}_{Se})$$

where:

$$v(c_A) = \begin{cases} -\infty & if \quad c_A < \bar{c}_A \\ min\{c_A, \bar{c}_A\} & if \quad c_A \geq \bar{c}_A \end{cases}$$

Adults allocate their time between working and raising their children.Raising a child takes a fraction $\phi > 0$ of the total time of an adult.

For a child to become a skilled adult she must attend school. This requires a fraction $\phi_S > 0$ of a teacher time.If the child do not attend school, she can ingress in the labor market performing unskilled tasks. Her work will be equivalent to a fraction $\phi_U < \phi$ of an unskilled adult.

There are two types of education policies: schooling subsidies & child labor restrictions.

Child labor policies: limit the number of hours that a child can work, which is equivalent to reductions on returns of child labor to ϕ_U^g: $0 \leq \phi_U^g < \phi_U$.

Education subsidies: The government subsidizes a fraction δ of the educational costs.

The government financing is given by a proportional income tax $\tau(X)$, so that its budget always balance.

4.1 Equilibrium

An adult of type $i \in \{S, U\}$, solves:

$$V_i(x) = \max_{c_A, c_M, c_{Se}, n_U, n_S \geq 0} \left\{ u^\sigma + \beta (n_S + n_U)^{-\varepsilon} [n_S V_S' + n_U V_U'] \right\},$$

subject to:

$$\sum_{j \in \{A,M,Se\}} p_j c_j + \phi(1-\tau(x)) w_i(x)(n_S + n_U) + (1-\delta)\phi_S w_S(x) n_S$$
$$\leq (1-\tau(x))(w_i(x) + \phi_U^g w_U(x) n_U).$$

For each adult, the values V_S' and V_U' given by the law of motion of X'

Labor market clearing conditions are:

$$L_{MS}(x) + L_{SeS}(x) = L_S(x) \quad ,$$
$$L_{AU}(x) + L_{SeU}(x) = L_U(x)$$

The laws of motion of population are:

$$N_S' = \lambda_{S,S} n_S(S,x) N_S + \lambda_{U,S} n_S(U,x) N_U,$$
$$N_U' = \lambda_{S,U} n_U(S,x) N_S + \lambda_{U,U} n_U(U,x) N_U,$$

where $\lambda_{i,j}(x)$ is the fraction of adults of type i who have children of type j and $n_{i,j}(x)$ is number of children of type j that have parents of type i. Finally, wages are given by the solution of the problem of the firms.

Given preferences, high-low skilled children are perfect substitutes and indifference curves (n_S, n_U) are straight lines. Hence there are only corner solutions: $f_i = 0$ or $f_i = 1$. Hence, $n_{i,S}, n_{i,U}$ trivially determined by comparing solutions with only one type of child.

It is also show that parents of different skills cannot simultaneously be indifferent between the two types of children. We assume, in our calibration equilibrium with intergenerational upward mobility. Consequently, high-skilled parents choose only high-skilled children and low-skilled parents are indifferent between the two types of children.

4.2 Calibration

The model is calibrated to South Korea and Brazil.

4.2.1 *Preferences parameters*

Parameter	Value	Target/Source
β	0.132	Doepke (2004)
σ	0.5	"
ε	0.5	"
b	0.15	Herrendorf et al. (2011)
$\bar{c}_A$	by country/year	share labor in agriculture
$\bar{c}_{Se}$	by country/year	match C_{Se} =Y_{Se}

Note that β, σ and ε are consistent w/ fertility differential of 0.5 between skilled and unskilled parents (UK and US) and total fertility rate of 2.0.

4.2.2 ***Technology, Production, Education***

Parameter	**Value**		**Target/Source**
	Korea	**Brazil**	
α	0.40		PNAD
ϕ	0.155		Expenditure child/GDP
ϕ_S	0.04		Teachers/population
δ	0.5	0.0	Doepke(2004)
$\phi^g_{U,60-82}$	0.07	0.07	"
$\phi^g_{U,83-05}$	0	0.07	"

Sectorial productivity paths are set as in Duarte and Restuccia (2010), and skilled labor *defined* as *"some secondary education"* (otherwise too few in 1960).

4.3 Results

Growth Rates: the model matchs closely the growth rates of both countries and reproduces the large gaps in the Korea vs. Brazil growth rates.

Growth: Output per worker	**Brazil**	**Korea**
PWT: $\frac{avg.\ 83-05}{avg.\ 60-82}$	18%	210%
Model:	36%	232%

The model also do a good job in matching the allocation of workers across sectors in both countries. In the table below initial period stands for the years 1960-1982 and final period for the years 1983-2005.

Variable	Brazil				Korea			
	Initial		Final		Initial		Final	
	Data	Model	Data	Model	Data	Model	Data	Model
L_{AU}/L_U	0.64	0.63	0.35	0.36	0.65	0.65	0.27	0.28
L_{MS}/L_S	0.74	0.77	0.67	0.67	0.81	0.82	0.74	0.69
L_{SeU}/L_U	0.36	0.37	0.65	0.64	0.35	0.35	0.73	0.72
L_{SeS}/L_S	0.26	0.23	0.33	0.34	0.19	0.18	0.26	0.31

Note that the model matches the distribuition of skilled labor by sector in Korea and Brazil for both periods. It also matches shares of unskilled labor for both periods, and also follows closed structural chance, as seem by the fall in L_{AU}/L_U in both economies.

The model, however, overestimates the total share of skilled workers in the two countries, especially in the initial period, as shown by the table below:

Variable	Brazil				Korea			
	Initial		Final		Initial		Final	
	Data	Model	Data	Model	Data	Model	Data	Model
N_S	0.10	0.31	0.27	0.35	0.29	0.49	0.70	0.74

This is because there is only only skilled labor in manufacture. As a consequence, the model understimate total fertility rate - especially in Brazil - in the first period, although it matches very closely in the second period:

Total Fertility rate	**Brazil**		**Korea**	
	Data	**Model**	**Data**	**Model**
1982	3.8	2.3	2.4	2.2
2005	2.1	2.1	1.1	1.2

4.3.1 The Role of Educational Policy

In order to understand the role of educational policies on the growth trajectory of both economies, we ran an experiment in each we exchanged the policies of one country for that of the other. In other world we simulated a counterfactual growth path of Brazil (South Korea) with the restrictions of child labor and educational subsidies of South Korea (Brazil). We want to know how much of the stagnation of Brazil in the second period is due to its policies and, likewise, what is the contribuition of educational factors to the growth experience of South Korea. Results are displayed in the table below:

Brazil		**Korea**	
	Growth		Growth
Benchmark	36%	Benchmark	232%
Brazil: Korean policies	57%	Korea: Brazilian policies	112%

If South Korea had the Brazilian policies instead of their own, its growth rate between the two periods would be almost half as much as the observed: instead of a 232% growth this country would experience only a 112% growth. Brazil, in contrast, would grow 60% faster with South Korean policies.

There is still a big difference left to be explained - due to TFP differences, among other factors - but distance is now way smaller. In other words, in the benchmark simulation that reproduces closely the data, the growth difference between the two countries is 196 percentual points (232% - 36%). The distance now, with policies switched, felt to 55 percentual points (112% - 57%).

The channel for this result, as expected, is the share of skilled population that changed significantly, as displayed in the table below:

Variable	**Brazil**				**Korea**			
	Benchmark		Korean Policies		Benchmark		Brazilian Policies	
	Initial	Final	Initial	Final	Initial	Final	Initial	Final
TFR	2.3	2.1	2.5	1.2	2.2	1.2	2.1	1.9
N_H	0.31	0.35	0.38	0.60	0.49	0.74	0.37	0.42

Brazil, instead of having only 35% of its labor force with high skills, ends up with a share of 60%. South Korea, in its turn, would have with the Brazilian policies a share of skilled workers of only 42%, instead of 74% as observed. A large part of the stagnation of the Brazilian economy, hence, is due to its poor incentives and policies regarding education. Good incentives and policies, in contrast, explains much of the fast growth of South Korea, during and after structural change.

5 Concluding Remarks

We proposed a theory to explain why some countries, after a period of fast growth, stagnate. In our model, these economies spend few resources in education, which in general is of poor quality, and direct a disproportional part of their expenditures to tertiary education. As a consequence, schooling remains low and grows very slowly. Hence, structural transformation in these economies is such that they transit from the low productivity agriculture to low skill services.

In our model parents chose the education of their children (and the number of children) and government educational policy is a subsidy to the different levels of education. Child labor may be allowed or not. We simulated a simpler version of the model, with only two levels of education, and calibrate it to South Korea and Brazil. Both economies experienced structural change at the same time and at a similar pace, but the former country growth did not slow down, while Brazil´s stagnate after 1980. Schooling levels follow different paths, with human capital accumulation being way faster in the Asian economy. The simulations found that educational policy can explain a very large part of the growth difference between these economies and we show that South Korea growth with Brazil´s policies would be halved and the share of high-skill labor would be only 56% of the observed.