Beliefs, Aggregate Risk, and the U.S. Housing Boom*

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Abstract

This paper investigates the quantitative importance of the interaction of beliefs with credit conditions in explaining the run-up of house prices during the U.S. housing boom. To allow for interacting beliefs and credit conditions while maintaining computational tractability, I will introduce adaptive expectations into a general equilibrium life-cycle model with aggregate risk, incomplete markets, and defaultable debt. I will compare results from the model solved under adaptive expectations derived from ZIP code level house price data to results solved under rational expectations. Although house prices grew by 40 percent relative to their pre-boom level in the data, positive income shocks only generate a 5 percent increase in house prices under rational expectations in the model.

Keywords: housing boom; aggregate risk; heterogeneous agents; adaptive expectations
JEL Codes: E20, E3, C68, R21

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1 Introduction

After growing an average of 4 percent per year historically, U.S. house prices averaged 10 percent yearly gains from 2000 to 2006 as shown in Figure (1).[1] What is the quantitative importance of the interaction of beliefs with credit conditions in explaining this run-up? Understanding the source of excessive house price growth in the 2000s may guide policy efforts aimed at preventing a repeat of the damaging events of the 2007-2009 financial crisis. While shifts in credit conditions and other model fundamentals such as income, interest rates, or preferences have mixed success[2] in generating the boom in house prices, models relying on non-fundamentals like beliefs have more success.[3] Kaplan et al. (2017) specify beliefs as news shocks orthogonal to shocks in credit conditions which allows for fully rational expectations and computational tractability in a general equilibrium macro model with household heterogeneity and aggregate risk. Assuming orthogonality, however, neglects a feedback channel between the two potential sources of higher house prices established by reduced form empirical evidence.[4]

I develop the first framework that allows for interacting beliefs and credit conditions in a general equilibrium model with aggregate risk, incomplete markets, and defaultable debt. To maintain computational tractability, I will introduce adaptive expectations derived from ZIP code level house price data to capture beliefs about future house prices. Strategies that instead entwine credit conditions with beliefs while maintaining rationality –such as the Bayesian updating framework of Boz and Mendoza (2014)– would be intractable in a model with borrower heterogeneity and aggregate risk suggesting adaptive expectations as a practical workaround. Maintaining borrower heterogeneity, however, disciplines the influence of credit conditions as a source of booming house prices. Because constrained agents can put upward pressure on house prices in response to looser aggregate credit conditions, borrower heterogeneity can help limit the number of constrained agents to match the fraction observed in the data.

I will compare results from the model solved under adaptive expectations to results solved under rational expectations. Under rational expectations, the model generates a 5 percent increase in house prices in response to positive income shocks which undershoots the 40 percent increase observed in the data. Allowing borrowers to default on mortgage debt worsen

[1]Real house prices followed a similar pattern over the same periods.

[2]See Favilukis et al. (2017),Greenwald (2018),Garriga et al. (2019),Justiniano et al. (2017),Liu et al. (2013).

[3]See Burnside et al. (2016) Piazzesi and Schneider (2009),Kaplan et al. (2017),Gelain et al. (2016)

[4]In assessing the impact of beliefs and credit conditions on house prices, Mian and Sufi (2009, 2016) find that changes in credit conditions increased optimism about future house prices while Adelino et al. (2017),Foote et al. (2012), Albanesi et al. (2017) find the opposite.

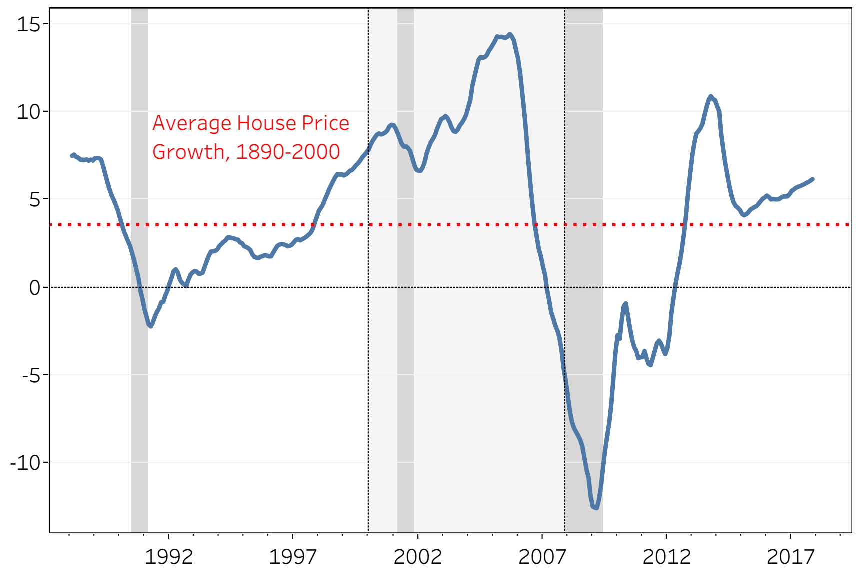

Figure 1: Year-over-year U.S. house price growth, percent. Light shaded area indicates the housing boom period and dark shaded areas indicate NBER-dated recessions. House price data is the CoreLogic Case-Shiller National House Price Index from S&P Dow Jones Indices accessed via Bloomberg (SPCSUSS INDEX). The data on long-term average house price growth is from Knoll et al. (2017).

the ability of the model to generate a boom in house prices under rational expectations. Even after a string of positive income shocks, agents know that future downturns will lead to defaults making these future shocks still salient. Kaplan et al. (2017) similarly find that shocks to income alone are insufficient to match house prices during the boom.

An extensive literature has examined the role of shifts in mortgage finances in generating the housing boom. Under rational expectations, Favilukis et al. (2017) and Liu et al. (2013) link looser loan-to-value constraints to higher house prices. Greenwald (2018), Justiniano et al. (2017), and Kaplan et al. (2017) caution that this mechanism may lead to counterfactual movements in mortgage interest rates or require a counterfactually large number of constrained borrowers. Boz and Mendoza (2014), Adam and Marcet (2012), and Caines (2015) also link shifts in mortgage finance to higher house prices by requiring agents to learn whether or not the changes were permanent. The ability of Bayesian updating and adaptive learning to generate higher house prices suggests an important interaction between beliefs and financial conditions that may not be fully exploited under full information rational expectations.

Piazzesi and Schneider (2009) and Burnside et al. (2016) depart from non-rational expectations with heterogeneous beliefs and show that optimistic investors can push up house prices. While Piazzesi and Schneider (2009) use a search model to highlight the importance of optimists, Burnside et al. (2016) model expectations directly by introducing social dynamics where agents may change beliefs after interacting with other agents. Allowing each agent to forecast their own expected future value of house prices would be ideal in this paper, but impossible because such data is not available.[5] To proxy for household variation in house price forecasts, I will rely on variation in ZIP Code level house price data.

Ferreira and Gyourko (2017) and Charles et al. (2015) show that the boom in house prices varied in timing and intensity across MSAs. Figure (2) further illustrates disperse house price growth across ZIP Codes. At the start of the boom in 2001, California, Florida, and the East Coast were already experiencing house price growth of over 10 percent per year. The Carolinas, Texas, the Midwest showed more muted house price growth that was slightly negative in some regions. Other areas of the country were experiencing house price *declines* in excess of 10 percent per year.

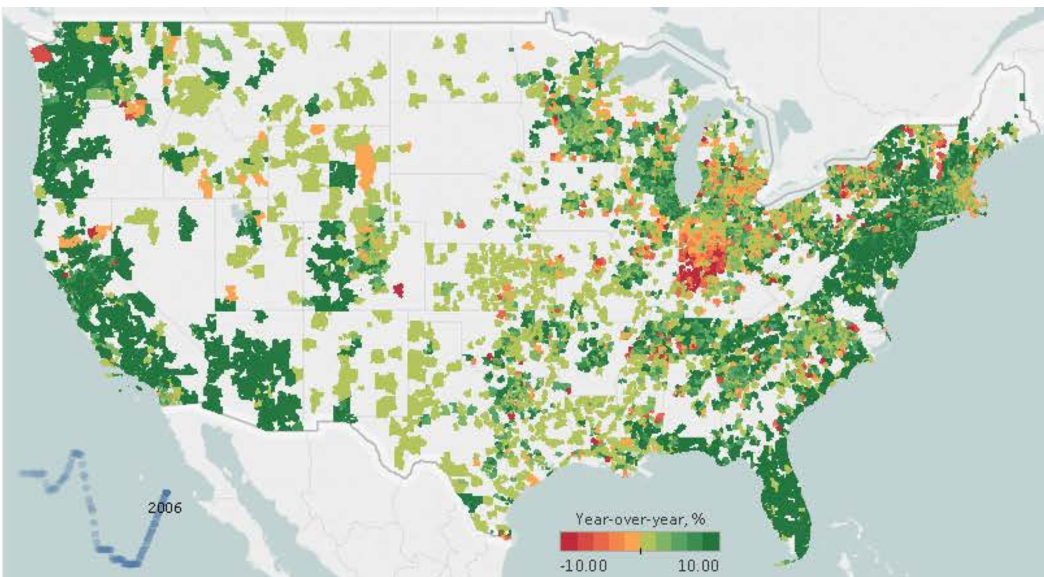

Figure 2: Federal Funds Rate and year-over-year house price growth by ZIP code. Sources: Zillow and Federal Reserve Board. Click the ▷ button to play animation if running Windows operating systems. Animation is unavailable for Mac or Linux operating systems.

While results from the model are obtain under rational expectations, future versions of this paper will use AR(1) forecasts from ZIP Code level house price data to generate adaptive expectations. Under adaptive expectations, this paper will assess the quantitative importance of the interaction of beliefs with credit conditions.

[5] Although there is an extensive line of research that explores housing expectations from surveys, the results are often conflicting as to whether or not respondents over or underestimate house price growth. See Case and Shiller (2004), Landvoigt (2016), Niu and van Soest (2014), and Gelain and Lansing (2013).

2 Model

Environment

Time is discrete where next period quantities are denoted by $'$ and aggregate quantities are denoted by $\tilde{}$.

Preferences

The economy is populated by infinitely lived risk-neutral representative lenders (l), housing constructions firms (h), and final goods firms (c) as well as a continuum of measure one finitely lived borrowers (b). Borrowers live for $j = 1, \ldots, J$ periods where they work from $j = 1$ to J^{ret-1} and retire at J^{ret} until the end of their lives which occurs with certainty in at J. Borrowers have preferences over final goods consumption and next period housing $\{C_{b_j}, H'_{b_j}\}_{j=1}^{J}$ with final goods consumption as the numeraire.

$$u_j(H'_b, C_b) = e_j \frac{[(1-\phi)C_b^{1-\gamma} + \phi H_b'^{1-\gamma}]^{\frac{1-\sigma}{1-\gamma}} - 1}{1-\sigma}$$

Where ϕ is the taste for housing relative to consumption, $1/\gamma$ is the elasticity of substitution between housing and nondurable consumption, and σ is the intertemporal elasticity of substitution. A deterministic equivalence scale $\{e_j\}_{j=1}^{J}$ adjusts consumption for changes in household size over the life-cycle. Borrowers' expected life-time utility is given as:

$$\mathbb{E}_0 \left[\sum_{j=1}^{J} \beta^{j-1} u_j(H'_{b_j}, C_{b_j}) + \beta^J v(\flat) \right]$$

Where the warm-glow bequest motive at the end of life J has the functional form:

$$v(\flat) = \psi \frac{(\flat + \underline{\flat})^{1-\sigma} - 1}{1-\sigma}$$

The bequest motive prevents borrowers from counterfactually ramping up their borrowing and drawing down their housing as they near the end of their lives. The strength of the bequest motive is given as ψ and $\underline{\flat}$ measures the extent to which bequests are luxuries.

Income Endowments and Aggregate Risk

While working, borrowers receive an endowment of log income comprised of an aggregate stochastic component $\Theta(S)$ and a deterministic life cycle component χ_j:[6]

$$\log Y = \Theta(S) + \chi_j, \quad \text{when} \quad j < J^{ret}$$

In retirement, borrowers receive

$$\log Y = \chi_{J^{ret}-1}, \quad \text{when} \quad J^{ret} \leq j \leq J$$

Aggregate income $\Theta(S)$ evolves according to a two state Markov Chain. There are two states of the economy, a good state ($S = good$) and a bad state ($S = bad$) where,

$$\Theta(S = good) > \Theta(S = bad)$$

With the transition matrix:

$$\begin{bmatrix} \pi(S' = good|S = good) & \pi(S' = bad|S = good) \\ \pi(S' = good|S = bad) & \pi(S' = bad|S = bad) \end{bmatrix} = \begin{bmatrix} \pi_{good,good} & \pi_{good,bad} \\ \pi_{bad,good} & \pi_{bad,bad} \end{bmatrix}$$

Markets and Individual State Space

Incomplete markets prevent borrowers from insuring against aggregate risk. Borrowers cannot trade amongst themselves and can only obtain one period loans L'_{b_j} from lenders at price q_{b_j}. There is not short selling of loans ($0 \leq L'_{b_j}$) and borrowing is limited to a fraction of the value of existing house collateral. Borrowers can only purchase houses H'_{b_j} from the housing construction sector at price p which also prevents borrowers from using housing as a mechanism to risk share amongst themselves.

Borrowers receive an initial endowment of loans L_{b_j} and an initial endowment of housing H_{b_j}. Initial endowments are presently drawn from independent normal distributions (see table 2 in section 3.1 for more details). Newly born agents begin life with no debt and inherit the housing stock from the agent they are replacing. Individual borrowers have the state space $\{L_{b_j}, H_{b_j}\}$.

[6]Future versions will develop idiosyncratic risk as well.

Aggregate State Space

Incomplete markets and aggregate risk make the distribution of agents across individual borrower states μ a necessary state variable for agents to correctly forecast next period prices. $\Gamma_\mu(\mu; Y, Y')$ is the equilibrium law of motion of the measure of agents such that $\mu' = \Gamma_\mu(\mu; Y, Y')$. μ_{b_j} is the measures of borrowers agents over housing H_{b_j} and loans L_{b_j} where $\sum_{j=1}^{J} \mu_{b_j} = 1$. The aggregate state space of the economy is thus $\{Y, \mu\}$.

Default

Default is presently exogenous and driven by income. Assuming exogenous default abstracts away from the details of borrowers' decisions to default and overstates both the default rates observed in the data and the cross-sectional composition of defaulters. Exogeneity, however, allows for computational tractability and a simplified loan pricing structure discussed in more detail in section 2.2. Moreover, Foote and Willen (2017) explain that prior to the 2007-2009 financial crisis –the period studied in this paper– very few borrowers defaulted on mortgages and did so only when faced with both negative equity and an adverse life event such as job loss. Future versions may relax the exogenous default assumption and instead incentivize borrowers to default only when they have negative equity. Allowing borrowers to strategically default is beyond the scope of this paper.

Borrowers are allowed to default at zero cost with probability $\nu_b(Y) \in [0, 1]$. If borrower income is lower then some threshold $Y < \underline{Y}$, then $\nu_b(Y) = 1$ and borrowers will default. Otherwise, borrowers have infinite default cost with probability $1 - \nu_b(Y)$ and will not default if $Y \geq \bar{Y}$. Borrowers discharge debts in default and must foreclose their housing assets to lenders.

2.1 Borrowers' Problem

See Appendix B.1 for the full recursive borrowers' problem. Borrowers receive an endowment of current period housing stock H_{b_j} and choose next period's housing stock H'_{b_j} valued at price $p(\mu, Y)$. They borrow loans $L'_{b,j}$ costing $q_{b_j}(\mu, Y)$ and repay lenders either the full amount of the loan L_{b_j} if they do not default or a portion of the current value of their housing stock $\alpha p(Y, \mu) H_{b_j}$ if they default. The borrowers' budget constraint is given as:[7]

$$C_{b_j} = Y + p(Y, \mu)[(1 - \delta_h) H_{b_j} - H'_{b_j}] + q_{b_j}(Y, \mu) L'_{b_j} - \begin{cases} L_{b_j}, & \text{no default} \\ \alpha p(Y, \mu) H_{b_j}, & \text{default} \end{cases}$$

[7]Int the final period of life, borrowers leave their housing stock in bequest $\flat = p'(\mu; Y, Y') H'_{b_j}$.

In default, borrowers do not pay back their loans and surrender a fraction $\alpha \in [0,1]$ of their housing stock to the bank in foreclosure.[8] Although $\alpha < 1$ may not be entirely realistic because homeowners cannot lose a fraction of their house in foreclosure, α can instead be interpreted as the cost of returning to previous levels of housing and final goods consumption after default. If $\alpha = 1$, borrowers surrender their entire housing stock to the lender making default costly. As α decreases, the default burden becomes less costly so that if $\alpha = 0$, borrowers discharge their debts without further penalty allowing them to increase both their housing stock and consumption. Allowing for $\alpha < 1$ helps with with computational tractability. With a high α, goods consumption C_b may be negative for a large part of the parameter space making the borrowers' problem undefined. I am presently trying to verify that the equilibrium prices and quantities will never visit these undefined areas which is more challenging with a higher α. As α increases, the solution jumps around regions of the parameter space as the economy transitions between good and bad states with a higher α.

Borrowing L'_{b_j} is limited to a fraction θ of the value of the housing stock:

$$L'_{b_j} \leq \theta p(\mu, Y) H'_{b_j}$$

As the house price $p(\mu, Y)$ or the loan-to-value limit θ increase, borrower b aged j will be able increase the amount they borrow L'_{b_j}.

2.2 Lenders

See Appendix B.2 for the full recursive lenders' problem. The representative lender lends L'_{b_j} to borrower b aged j at price $q_{b_j}(\mu, Y)$ and receives L_{b_j} if the individual borrower does not default and $\alpha p(\mu, Y) H'_{b_j}$ otherwise. Because future profits and losses[9] depend on the repayment rate of loans, the representative lender will charge a risk premium based on the future default probability of each borrower. With only aggregate risk and exogenous default, the loan market will clear loan-by-loan, but each borrower will be charged the same

[8] Although I am assume all defaults lead to foreclosure and use the two terms interchangeably, this is not necessarily true [see Lutz et al. (2018)]. A default is a failure to meet the terms of a mortgage contract. A defaulted mortgage is foreclosed on when the homeowner's rights to the property are eliminated. See Fannie Mae's glossary. https://www.knowyouroptions.com/find-resources/information-and-tools/glossary .

[9] Although lenders are competitive, uninsurable aggregate may induce profits and losses along the equilibrium path of prices. I assume that lenders are owned by non-modeled foreign agents with deep pockets. This assumption also allows the representative lender's problem to be discounted at the risk-free rate $1/(1+r) = \beta$

risk premium.[10] The loan pricing function is:

$$q_{b_j}(\mu, Y) = \beta \mathbb{E}_{Y'|Y}[(1 - \nu_b(Y'))] \tag{1}$$

Where $(1-\nu_b(Y'))$ is the probability that borrowers have infinite default cost and do not default. Default cost uncertainty results in a lower effective stochastic discount relative to an economy without default uncertainty where the probability of costless default will always be zero, $\nu_b(Y') = 0$ for all Y' .

When lenders obtain housing in foreclosure they immediately sell the entire housing stock back to borrowers which helps ensure that lenders always prefer to be repaid in loans instead of housing.[11]

2.3 Final Goods and Construction Sectors

See Appendix B.3 for the full recursive final goods and housing construction firms' problems.[12] The competitive final goods sector has a linear constant returns to scale technology:

$$Y = \Theta(S)\tilde{N}_c$$

Where $\tilde{N}_c$ is the unit of labor services. With inelastic labor supply, profit maximization delivers the wage as:

$$w(\mu, Y) = \Theta(S)$$

A competitive construction sector has the production technology of houses $\tilde{H}'_h = (\Theta(S)\tilde{N}_h)^{\epsilon}\bar{L}^{1-\epsilon}$ where $\tilde{N}_h$ is labor services and $\bar{L}$ is the amount of newly available buildable land.
Using the equilibrium condition $w(\mu, Y) = \Theta(S)$ from the final goods firms, the supply of housing is given as:

$$\tilde{H}'_h \equiv [\epsilon p(\mu, Y)]^{\frac{\epsilon}{1-\epsilon}}\bar{L} \tag{2}$$

The profit maximization of the representative construction firm pins down one house price $p(\mu, Y)$ via the aggregate housing stock housing stock $\tilde{H}$ and the aggregate productivity shock $\Theta(S)$.

[10]If the risk premium is a function of individual borrower states, then the loan pricing function will become more complicated.

[11]If lenders were instead allowed to hold housing in their portfolio, their buying and selling of housing may complicate housing market dynamics and house price determination.

[12]With aggregate risk the firms, like lenders, may take profits and losses along the equilibrium path. I also assume they are owned by foreign agents with deep pockets.

2.4 Recursive Competitive Equilibrium

- A *recursive competitive equilibrium* consists of:
  - A sequence of stochastic productivity endowments $\Theta(S)$ and the borrower default threshold $\underline{Y}$ and housing lost in default α
  - Prices for houses, wages, and loans to borrowers, : $\{p(\mu, Y), w(\mu, Y), q_{b_j}(\mu, Y)\}$
  - Government policies for the loan-to-value limit and land permits: $\{\theta, \bar{L}\}$
  - Perceived laws of motion for the state space $\mu = \Gamma_\mu(\mu, Y, Y')$
- Value functions $\{V^d_{b_j}, V^n_{b_j}\}$ and policy functions for consumption, loan demand and housing demand $\{C_{b_j}, L'_{b_j}, H'_{b_j}\}$ solve the individual borrowers' problem
- Value function V_l and the policy function for loan originations $\tilde{L}'_l$ solve the representative lender's problem. The loan market clears loan-by-loan with pricing function $q_{b_j}(\mu, Y)$

$$\int_{\mathcal{L}\times\mathcal{H}\times\mathcal{J}} L'_{b_j} d\mu_{b_j} = \tilde{L}'_l$$

- Firms in the construction sector maximize profits with policy functions $\{\tilde{N}_h, \tilde{H}'_h\}$ which delivers a single housing price $p(Y, \mu)$ that clears the housing market

$$\int_{\mathcal{L}\times\mathcal{H}\times\mathcal{J}} (H'_{b_j} - (1-\delta_h)H_{b,j}) d\mu_{b_j} = \tilde{H}'_h$$

- Final goods firms maximize profits so that the labor market clears at $\Theta(S) = w(\mu, Y)$ and labor demand is equal to

$$\tilde{N}_c = 1 - \tilde{N}_h$$

- The aggregate resource constraint is satisfied

$$\int_{\mathcal{L}\times\mathcal{H}\times\mathcal{J}} C_{b_j} d\mu_{b_j} + \tilde{L}_l - \sum_{b_j} q_{b_j}(\mu, Y) L'_{b_j} = Y$$

- Consistency is satisfied and perceived laws of motion of the state space $\mu' = \Gamma_\mu(\mu, Y, Y')$ is consistent with individual behavior

3 Computational Solution

The solution method is adapted from Kaplan et al. (2017) and Favilukis et al. (2017) who use a variation of the Krusell and Smith (1998) algorithm. See Appendix C.1 for details on the algorithm and its accuracy. To forecast future prices, agents must keep track of the infinite dimensional distribution over individual borrower states μ. Computation of the law of motion of the distribution $\mu' = \Gamma(\mu; Y, Y')$ is thus infeasible due to its dimensionality. Approximating the distribution with house prices $\mu \approx \log p(Y)$ allows agents enough information to forecast future prices. With loan prices $q_{b_j}(\mu, Y)$ pinned down by the profit maximization of the risk-neutral representative lender, agents only need to forecast future house prices $p'(\mu; Y, Y')$.

I posit a linear forecasting rule for house prices $p(Y)$[13] conditional on realizations of the aggregate state to compute an approximate equilibrium under the assumption of bounded rationality.[14] The aggregate state space (μ, Y) is thus approximated as $(p(Y), Y)$ allowing agents to forecast house prices p' conditional on realizations Y and Y'.

$$\log p'(p(Y); Y, Y') = a^0_{Y,Y'} + a^1_{Y,Y'} \log p(Y) \quad \Longleftrightarrow \quad \mu' = \Gamma_\mu(\mu; Y, Y') \tag{3}$$

The algorithm is initialized by guessing coefficients $\{a^i_{Y,Y'}\}$ for $i = 0, 1$ and then solving the individual borrowers' problems on a grid for prices p in addition to grids for individual states. Given an initial distribution of housing, loans, and ages, the individual quantities are simulated across time for each grid point p. Aggregating individual housing choices delivers a demand schedule for aggregate housing. Solving for zero excess yields a market clearing price $p^*_t(Y_t)$. The individual policy functions are then interpolated at $p^*_t(Y_t)$ to deliver optimal quantities which are then re-aggregated. With the new time series $p^*_t(Y_t)$, new coefficients $\{a^{i,new}_{Y,Y'}\}$ are computed. These steps are repeated until the coefficient converge such that $\{a^i_{Y,Y'}\} \approx \{a^{i,new}_{Y,Y'}\}$. The grid in prices has 8 grid points and the individual state grids have 30 grid points while their choice grids have 100 grid points.

3.1 Parameterization

The parameterization of the model largely follows that of Kaplan et al. (2017) and is adjusted to assure computational convergence and tractability. The parameters are chosen to resemble the U.S. economy in the late 1990s, the period preceding the housing boom and bust. Each model period is equal to two years.

The economy has one aggregate shock $\Theta(S)$ where $S = \{good, bad\}$ which follows a two-state Markov chain that follows an approximation of linearly de-trended U.S. total labor productivity. Default parameters are set so that all borrowers default in the bad state and have a low default burden $\alpha = 0.3$ which helps with computational tractability.

[13] As noted by Kaplan et al. (2017), the aggregate quantities of loans and houses are not pre-determined making it difficult to include moments of these quantities in the price forecasting equation (3).

[14] Assessing whether or not the equilibrium is unique is beyond the scope of this paper. With bounded rationality, multiple equilibria are more likely.

Demographics		
Maximum Age	J	30
Retirement Age	J^{ret}	22
Preferences		
Discount Factor	β	0.96
Inverse elasticity of substitution (C_b, H_b')	γ	0.8
Risk aversion	σ	2
Strength of bequest motive* (key for market clearing)	ψ	550
Extent of bequest as a luxury	$\underline{b}$	7.7
Taste for housing	ϕ	0.12
Housing		
Depreciation rate of housing	δ_H	0.015
Housing supply elasticity	$\epsilon/(1-\epsilon)$	1.5
New land permits	$\bar{L}$	0.311

Table 1: Parameter values. All values from Kaplan et al. (2017) unless*, then supplied for computational tractability

Aggregate Parameters		
Loan-to-value ratio	θ	0.8
Aggregate productivity	$\{\Theta(good), \Theta(bad)\}$	$\{1.135, 1.065\}$
Deterministic income	$\{\chi_j\}$	Kaplan and Violante (2014)
Default threshold*	$\underline{Y}$	All borrowers default in bad state
Default loss*	α	0.3
Transition matrix	$\begin{bmatrix}\pi_{g,g} & \pi_{g,b} \\ \pi_{b,g} & \pi_{b,b}\end{bmatrix}$	$\begin{bmatrix}0.9 & 0.1 \\ 0.1 & 0.9\end{bmatrix}$
Initial distributions		
Age distribution*	$j_0 \in [1, 30]$	Uniform
Loan distribution*	$L_0 \in [0, 0.5]$	Normal
Housing distribution*	$H_0 = [\min\{\tilde{H}_h'\}, \max\{\tilde{H}_h'\}]$	Normal

Table 2: Aggregate risk parameters. All values from Kaplan et al. (2017) unless*, then supplied for computational tractability

3.2 Simulating the Housing Boom

Following Kaplan et al. (2017), I will simulate the housing boom-bust with labor productivity from 1997-2007. In the future I will add the relaxation of borrowing constraints and adaptive expectations. The exercise bellow compares the cases when default or no default are allowed.[15]

The solution was obtained through Fortran code solved on Indiana University's Karst

[15]Figure (3) was chosen from the simulation of the income processes to match the number of periods the economy was in the good state.

high-throughput computing cluster. With initial guess for coefficients obtained from smaller versions of the model and parallelization on four cores, the coefficients converge in nine iterations in 47 minutes.

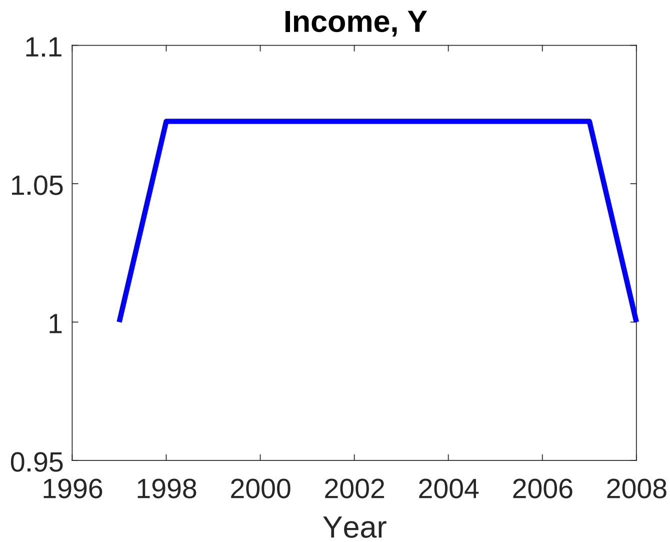

Figure 3: Labor Productivity from 1997-2008. All values are expressed as a ratio of their 1997 value at the start of the simulation.

3.3 Results: Rational Expectations, No Default

The loan pricing function is equal to the risk free rate $q_{b_j}(p(Y), Y) = 1/(1+r) = \beta$ for all borrowers in all income states when there is no default. The converged approximation of the law of motion of the distribution is computed as:

$$\begin{aligned} \log P'_{good,good} &= 0.10 + 0.88 \log p(Y_{good}), \quad R^2 = 0.77 \\ \log P'_{good,bad} &= 0.62 + 0.26 \log p(Y_{good}), \quad R^2 = 0.06 \\ \log P'_{bad,good} &= 0.61 + 0.25 \log p(Y_{bad}), \quad R^2 = 0.06 \\ \log P'_{bad,bad} &= 0.09 + 0.88 \log p(Y_{bad}), \quad R^2 = 0.77 \end{aligned}$$

Kaplan et al. (2017) and Favilukis et al. (2017) obtain R^2 close to 1 for all sample partitions, suggesting that my results fall short of the accuracy standards of similar models. Den Haan (2010) and Chipeniuk et al. (2019) propose alternate accuracy tests and caution that the R^2 can be a misleading statistic. I am working on these alternative accuracy tests which are detailed in Appendices C.2 and C.3.

Figure (4) shows that in the absence of default, borrowers increase final goods consumption along with income. Borrowers also increase their choice of housing in response to the positive income shocks because of non-separable preferences in housing and final goods. House prices rise along with housing choice. Although borrowing increases to finance higher housing choice in response to a positive productivity shock, borrowing does not stay elevated throughout the boom. Borrowers draw down their borrowing as the positive income shock persists because of an income effect. Higher incomes make borrowers wealthier which allows them to maintain higher levels of consumption with less debt.

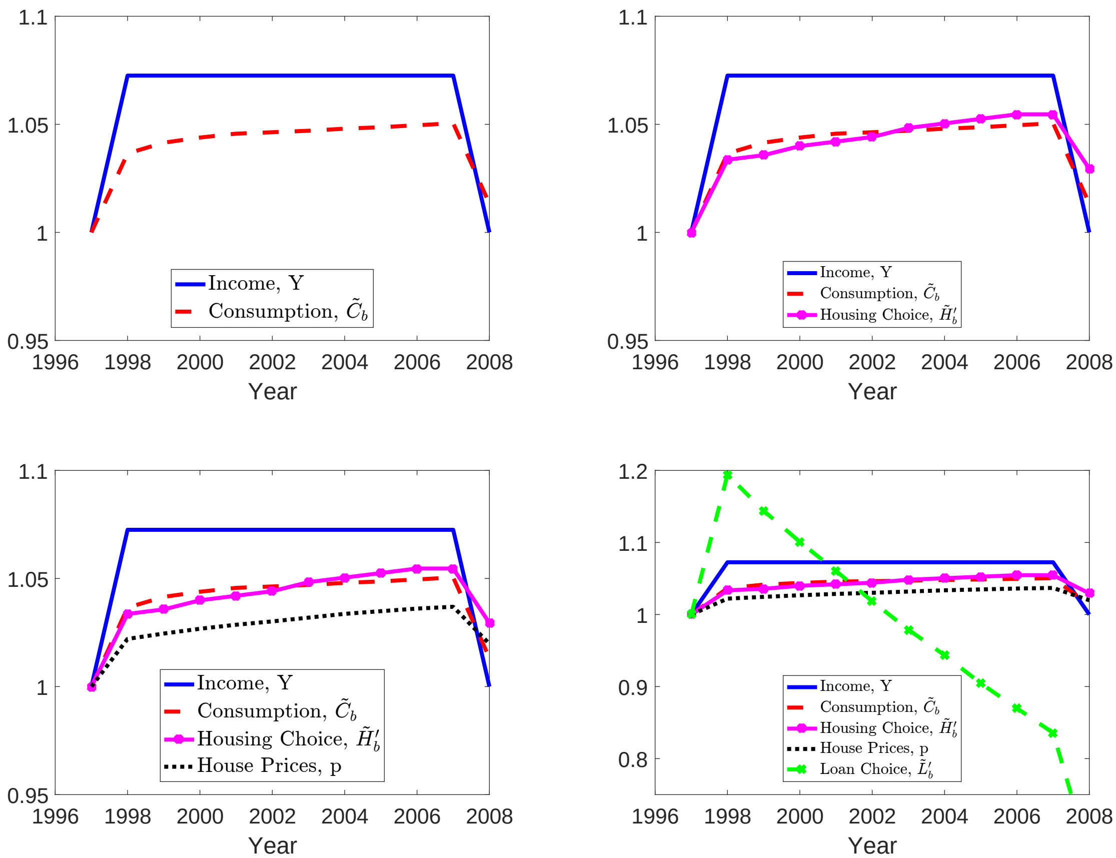

Figure 4: Borrower consumption, housing choice, house prices, and loan choice without default. All values are expressed as a ratio of their 1997 value which is a bad productivity state.

3.4 Results: Rational Expectations, Default

When borrowers default in the bad state, they surrender a portion of their housing stock $\alpha = 0.3$ in default and are charged a risk premium on loans.

$$q_{b_j}(p(Y),Y) = \beta \mathbb{E}_{Y'|Y}[(1-\nu_b(Y'))] = \begin{cases} 0.86, & \text{good state} \\ 0.10, & \text{bad state (default)} \\ 0.96, & \text{without default} \end{cases}$$

The price of loans is more expensive in both the good and bad states relative to the economy without default. The drop in $q_{b_j}(p(Y),Y)$ from 0.96 to 0.86 in the good state arises from the probability that the economy will enter the bad state and borrowers will default. With a high persistence of aggregate states $\pi_{good,good} = \pi_{bad,bad} = 0.9$, it becomes likely that the economy will stay in the bad state where borrowers will continue to default. Lenders do not want to take on this high default risk in the bad state and thus charge such a high risk premium $q_{b_j}(p(Y_{bad}), Y_{bad}) = 0.1$.

The solution to the model with default was also obtained on Indiana University's Karst high-throughput computing cluster with the same grids and number of grid points as the no default version of the model. With initial guess for coefficients obtained from the no default version of the model and parallelization on four cores, the coefficients converge in five iterations in 30 minutes. The converged approximation of the law of motion of the distribution is computed as:

$$\begin{aligned}
\log P'_{good,good} =& 0.12 + 0.80 \log p(Y_{good}), \quad R^2 = 0.65 \\
\log P'_{good,bad} =& 0.71 - 0.12 \log p(Y_{good}), \quad R^2 = 0.01 \\
\log P'_{bad,good} =& -0.06 - 0.13 \log p(Y_{bad}), \quad R^2 = 0.01 \\
\log P'_{bad,bad} =& -0.05 + 0.26 \log p(Y_{bad}), \quad R^2 = 0.07
\end{aligned}$$

The R^2 statistics are lower for all partitions once default is introduced. Forthcoming Den Haan (2010) and Chipeniuk et al. (2019) will help inform the accuracy of the solution.

Figure (5), also illustrates the evolution of prices and quantities throughout the housing boom relative to their pre-boom levels when borrowers can default on debt. Given that borrowers always default in the bad state, the beginning of the housing boom in 1998 is thus a transition from default to no-default. The downturn in 2008 is then a transition back to default. A more realistic simulation would start borrowers in the bad state with few defaults as was the case in 1997. After experiencing the good state of higher productivity from 1998 to 2007, borrowers would then transition to the bad state with higher defaults.

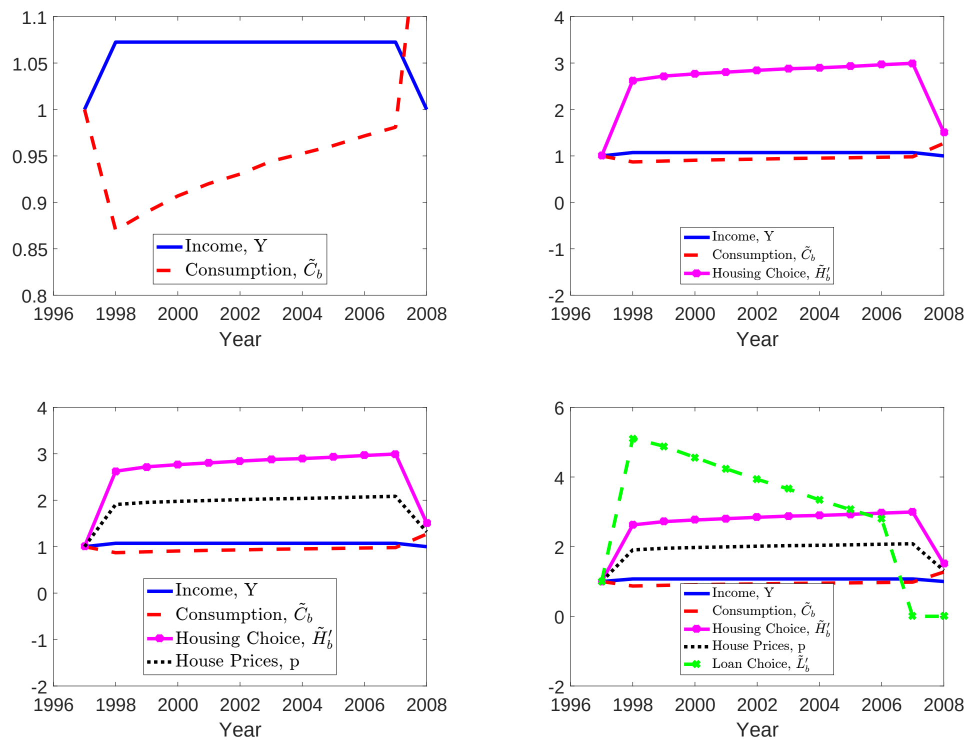

Figure 5: Borrower consumption, housing choice, house prices, and loan choice with default. All values are expressed as a ratio of their 1997 .

Figure (5) shows that final goods consumption falls in the bad state relative to the good state which is due to a low borrower default burden of $\alpha = 0.3$. Housing choice nearly triples in the good state due to the positive income shock and remains elevated throughout the boom. House prices also jumps, nearly doubling in the good state relative to the bad state. Similar to the economy without default, borrowing shoots up at the start of the boom. The increase in borrowing is larger in the economy with default because borrowers are more aggressively trying to replenish their housing stock. Borrowing in the bad state is near zero because loans become prohibitively expensive $q_{b_j}(p(Y_{bad}), Y_{bad}) = 0.1$ and quantities are constrained by a tight loan-to-value constraint arising from low housing choice and house prices.

The results are summarized in Table 3 and show little variation in average levels of consumption across the two states in both versions of the economy suggesting that borrowers are able to smooth consumption. The average housing stock is lower in the economy with default due to the confiscation of housing in foreclosure. Average borrowing is highest in the good state of the economy with default which suggests that agents borrow to replenish their housing stock to pre-default levels. House prices are lower and more volatile in the economy with default.

	No Default		Default	
	Good State	Bad State	Good State	Bad State
Average Consumption	0.36	0.34	0.31	0.33
Average Housing Choice	0.50	0.49	0.35	0.16
Average Loan Choice	0.09	0.10	0.23	≈ 0
Average House Price	2.30	2.24	0.94	1.90

Table 3: Results from the model solved under rational expectations with no default and default.

With house prices only rising about 5 percent in the economy with out default in the good state, it will be difficult to match the 40 percent increase in house prices observed throughout the housing boom. Although house prices double in the good state relative to the bad state with default, the bad state house price are dragged down by a counterfactually large number of defaulted borrowers.

4 Conclusion

(Forthcoming)

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A Appendix: Data

House price data are obtained at several aggregations of the ZIP code level. House price data are from Zillow's[16] Home Value Index which is a smoothed, seasonally adjusted measure of the median estimated home value across a given region and housing type for 4/1996 to 12/2017. Housing consists of single-family, condominium and co-operative homes with a county record and covers 95 percent of U.S. total housing stock by value.[17]

Following Hurst et al. (2016), I will also incorporate the loan-level data on mortgages securitized by Fannie Mae and Freddie Mac by 3-digit ZIP code.[18] The Fannie Mae Single Family Loan Performance Data[19] and Freddie Mac Single Family Loan-level Data Set[20] include borrower and loan information at origination along with monthly updates of loan performance. Average mortgage rates can be derived from this data as well as defaults. Although the housing boom was characterized by an expansion of mortgages that did not conform to GSE lending standards, mortgage originations eligible for Fannie Mae or Freddie Mac purchase[21] still represented 55 percent of all mortgage originations throughout the housing boom period.[22]

Zip Code level income data is available at intermittent intervals from 1998 till present from the IRS Individual Income Tax Statistics.[23] Data on individual consumption and balance sheets can be obtained at the individual level through the Survey of Consumer finances at 3 year intervals. The American Community Survey begins in 2001 and has annual data on demographics of individuals.

[16] https://www.zillow.com/research/data/

[17] An alternative to using the Zillow Home Value Index is using the Case-Shiller CoreLogic Index. The main differences between the two indices are their treatment of foreclosed re-sales and geographic coverage. Case-Shiller only includes homes that have sold at least twice in recent history, thereby excluding all new construction. Zillow excludes sales of foreclosed homes which is why it is lower than the Case-Shiller index after the 2007-2009 financial crisis. Zillow has a larger geographic coverage (95 percent vs. 71 percent of US housing stock by market value) which contributes to their index showing a more muted boom from 2000 to 2007 relative to the Case-Shiller index. Zillow argues that less populated areas experienced less of housing boom thereby biasing their index downward. Overall, Zillow forecast the Case-Shiller composition indices with a 0.1-0.2 percent margin of error. Zillow includes single family homes, condos, and co-ops while Case-Shiller is limited to single family homes only. See `https://wp.zillowstatic.com/3/ZHVI-InfoSheet-04ed2b.pdf`

[18] 3-digit ZIP codes are a superset of 5-digit ZIP codes.

[19] `http://www.fanniemae.com/portal/funding-the-market/data/loan-performance-data.html`

[20] `http://www.freddiemac.com/research/datasets/sf_loanlevel_dataset.html`

[21] Ginnie Mae is also a GSE but only purchases loans insured by government agencies such as those by the Federal Housing Administration, Department of Veteran Affairs, the Department of Housing and Urban Development's Office of Public and Indian Housing, and the Department of Agriculture's Rural Development. Loan level data is not available from Ginnie Mae to the extent is available from the other two GSEs.

[22] Calculation based on mortgage origination data from Inside Mortgage Finance, "The 2012 Mortgage Market Statistical Annual, Volume I".

[23] `https://www.irs.gov/statistics/soi-tax-stats-individual-income-tax-statistics-zip-code-data-soi`

B Appendix: Recursive Problems

B.1 Recursive Borrowers' Problem

The borrowers' problem is a stylized version of Gordon (2017). When not defaulting, borrowers' solve $V_{b_j}^n$ and when defaulting, they solve $V_{b_j}^d$. The probability of costless default is given as $\nu_b(Y) \in [0,1]$. All j subscripts have been dropped unless completely necessary.

If the income endowment of borrowers is equal to or above the default threshold $Y \geq \underline{Y}$, then the probability of costless default is $\nu_b(Y) = 0$ and borrowers do not default resulting in full repayment of their loan, $X_b = 1$. They solve V_b^n:

$$V_{b_j}^n(L_b, H_b; \mu, Y) = \max_{\{L_b', C_b, H_b'\}} \left\{ U_j(C_b, H_b') + \beta \mathbb{E}_{Y'|Y} \begin{bmatrix} (1 - \nu_b(Y'))\, V_{b_{j+1}}^n(L_b', H_b'; \mu', Y') + \dots \\ \dots \nu_b(Y') V_{b_{j+1}}^d(0, H_b'; \mu', Y') \end{bmatrix} \right\}$$

$$\begin{aligned} s.to. \quad C_b =& Y_b + q_{b_j}(\mu, Y) L_b' + p(\mu, Y)[H_b(1-\delta_h) - H_b'] - L_b X_b(Y), \quad X_b(Y) = 1 \\ L_b' \leq& \theta p(\mu, Y) H_b' \\ 0 \leq& L_b', H_b', C_b \\ \mu' =& \Gamma_\mu(\mu; Y, Y') \end{aligned}$$

If the income endowment of borrowers is sufficiently low such that $Y < \underline{Y}$, then the probability of costless default is $\nu_b(Y) = 1$ and borrowers default. In default, borrowers do not pay back their loans and instead repay lenders a fraction of their housing stock $L_b X_b(Y) = \alpha p(\mu, Y) H_b$ where $\alpha \in [0,1]$. If $\alpha = 1$ then the household surrenders their entire housing stock to the lender and if $\alpha = 0$ then the household makes no adjustment to their housing stock. Borrowers' value function in default $V_{b_j}^d$ is:

$$V_{b_j}^d(0, H_b; \mu, Y) = \max_{\{C_b, L_b', H_b'\}} \left\{ U(C_b, H_b') + \beta \mathbb{E}_{Y'|Y} \begin{bmatrix} (1 - \nu_b(Y'))\, V_{b_{j+1}}^n(L_b', H_b'; \mu', Y') + \dots \\ \dots \nu_b(Y') V_{b_{j+1}}^d(0, H_b'; \mu', Y') \end{bmatrix} \right\}$$

$$\begin{aligned} s.to. \quad C_b =& \theta_k Y_b + q_{b_j}(\mu, Y) L_b' + p(\mu, Y)[H_b(1-\delta_h) - H_b'] - L_b X_b(Y), \quad X_b(Y) = \frac{\alpha p(\mu, Y) H_b}{L_b} \\ L_b' \leq& \theta p(\mu, Y) H_b' \\ 0 \leq& L_b', H_b', C_b \\ \mu' =& \Gamma_\mu(\mu; Y, Y') \end{aligned}$$

If $H_b' = 0$ was also imposed in default so that borrowers were not allowed to adjust their choice of housing stock, then they would violate the Inada conditions on the utility function.

The borrowers' problem yields the following objective functions with Lagrange multiplier λ on the budget constraint and $U_{C_b}\lambda^{LTV}$ on the loan to value constraint, and $U_{C_b}\lambda^S$ on the short sale constraint for loans L_b'. The short sale constraints for housing H_b' and consumption C_b will not bind due to the Inada conditions on the functional form of the utility function. For $i = n, d$:

$$V_{b_j}^i(L_b, H_b; \mu, Y) = \max_{C_b, L_b', H_b'} \Bigg\{ U(C_b, H_b') + \beta \mathbb{E}_{Y'|Y} \begin{bmatrix} (1-\nu_b(Y'))\, V_{b_{j+1}}^n(L_b', H_b'; \mu', Y') + \ldots \\ \ldots \nu_b(Y') V_{b_{j+1}}^d(0, H_b'; \mu', Y') \end{bmatrix}$$
$$\ldots - \mathbb{E}_{Y'|Y}\Bigg[\lambda(C_b - Y + L_b X(Y) - q_b(\mu, Y) L_b' + p(\mu, Y)\left[H_b' - (1-\delta_h)H_b\right]) \ldots$$
$$\ldots - U_{C_b}\lambda^{LTV}(L_b' - \theta p(\mu, Y) H_b') - U_{C_b}\lambda^S(-L_b')]\Bigg]\Bigg\}$$

$$\text{FOC}_{C_b}: \quad U_{C_b} = \lambda \tag{4}$$
$$\text{FOC}_{L_b'}: \quad \beta \mathbb{E}_{Y'|Y}[(1-\nu(Y'))\partial V_{b_{j+1}}^n(L_b', H_b'; \mu', Y')/\partial L_b'] + q_{b_j}(\mu, Y)\lambda = U_{C_b}[\lambda^S - \lambda^{LTV}] \tag{5}$$
$$\text{FOC}_{H_b'}: \quad \lambda p(\mu, Y) - U_{C_b}\lambda^{LTV}\theta p(\mu, Y) = U_{H_b'} + \ldots$$
$$\ldots \mathbb{E}_{Y'|Y}[(1-\nu(Y'))\partial V_{b_{j+1}}^n(L_b', H_b'; \mu', Y')/\partial H_b' + \nu(Y')\partial V_{b_{j+1}}^d(0, H_b'; \mu', Y')/\partial H_b'] \tag{6}$$
$$\text{Envelope}_{L_b}: \quad \partial V_{b_j}^n(L_b, H_b; \mu, Y)/\partial L_b = \lambda, \quad \text{since} \quad X(Y) = 1 \tag{7}$$
$$\text{Envelope}_{H_b}^n: \quad \partial V_{b_j}^n(L_b, H_b; \mu, Y)/\partial H_b = \lambda p(\mu, Y)(1-\delta_h), \quad \text{since} \quad X(Y) = 1 \tag{8}$$
$$\text{Envelope}_{H_b}^d: \quad \partial V_{b_j}^d(0, H_b; \mu, Y)/\partial H_b = \lambda p(\mu, Y)[(1-\delta_h) - \alpha], \text{ since } X(Y) = \frac{\alpha p(\mu, Y) H_b}{L_b} \tag{9}$$

Iterating forward the two envelope conditions for H_b (8) and (9) and substituting into the first order condition for housing H_b' (6) housing demand is given as:

$$p(\mu, Y)(1-\theta\lambda^{LTV}) = \frac{U_{H_b'}}{\lambda} + \frac{\beta}{\lambda}\mathbb{E}_{Y'|Y}\left[\lambda' p'(\mu', Y')\big((1-\nu(Y'))(1-\delta_h) + \nu(Y')(1-\delta_h-\alpha)\big)\right]$$

Letting $\mathcal{M}_b' \equiv \beta\lambda'/\lambda = \beta(U_{C_b'}/U_{C_b})$ and canceling terms:

$$p(\mu, Y)(1-\theta\lambda^{LTV}) = \frac{U_{H_b'}}{\lambda} + \mathbb{E}_{Y'|Y}[p'(\mu', Y')\mathcal{M}_b(1-\delta_h - \nu(Y')\alpha)]$$

Now let $\dfrac{U_{H_b'}}{\lambda} = \dfrac{U_{H_b'}}{\lambda'}\dfrac{\lambda'}{\lambda} = \dfrac{U_{H_b'}}{\lambda'}\mathcal{M}_b' = \mathcal{H}_b'\mathcal{M}_b'$

Then housing demand is given as:

$$p(\mu, Y)(1-\theta\lambda^{LTV}) = \underbrace{\mathbb{E}_{Y'|Y}[\mathcal{M}_b'\mathcal{H}_b']}_{\substack{\text{Intrinsic}\\ \text{Value}}} + \underbrace{\mathbb{E}_{Y'|Y}[\mathcal{M}_b' p'(\mu', Y')(1-\delta_h - \nu(Y')\alpha)]}_{\substack{\text{Expected}\\ \text{future price}}}$$

$$p(\mu, Y)(1-\theta\lambda^{LTV}) = \mathbb{E}_{Y'|Y}[\mathcal{M}_b'\mathcal{H}_b'] + \pi_{s,good}\mathcal{M}_b' p'(\mu', Y'_{good})(1-\delta_h - \underbrace{\nu(Y'_{good})}_{=0}\alpha) + \pi_{s,bad}\mathcal{M}_b' p'(\mu', Y'_{bad})(1-\delta_h - \underbrace{\nu(Y'_{bad})}_{=1}\alpha)$$

$$p(\mu, Y)(1-\theta\lambda^{LTV}) = \mathbb{E}_{Y'|Y}[\mathcal{M}_b'\mathcal{H}_b'] + \pi_{s,good}\mathcal{M}_b' p'(\mu', Y'_{good})(1-\delta_h) + \pi_{s,bad}\mathcal{M}_b' p'(\mu', Y'_{bad})(1-\delta_h - \alpha)$$

Which delivers the housing demand of individual borrower b:

$$(1-\theta\lambda^{LTV})p(\mu, Y) = \mathbb{E}_{Y'|Y}[\mathcal{M}_b'\mathcal{H}_b'] + \mathbb{E}_{Y'|Y}[\mathcal{M}_b' p'(\mu', Y')(1-\delta_h - \mathbb{1}_{[Y'_{bad}]}\alpha)]$$

The loan demand of borrower b can be obtained by iterating forward the envelope condition L_b (7) and substituting into the first order condition for L_b' yields:

$$q_b(\mu, Y) = \beta\mathbb{E}_{Y'|Y}[(1-\nu(Y'))\mathcal{M}_b'] + \lambda^{LTV} - \lambda^S$$

Let $\mathcal{M}_b' \equiv \beta\lambda'/\lambda = \beta(U_{C_b'}/U_{C_b})$ and re-writing the expectation:

$$\begin{aligned} q_{b_j}(\mu, Y) =& \pi_{s,good}\underbrace{(1-\nu(Y_{good}))}_{=1}\mathcal{M}_b' + \pi_{s,bad}\underbrace{(1-\nu(Y_{bad}))}_{0}\mathcal{M}_b' + \lambda^{LTV} - \lambda^S \\ q_{b_j}(\mu, Y) =& \pi_{s,good}\mathcal{M}_b' + \lambda^{LTV} - \lambda^S \end{aligned}$$

B.2 Recursive Lenders' Problem

Because lenders are owned by risk-neutral foreign agents with deep pockets, their problem is discounted at the risk free rate $1/(1+r) = \beta$.[24] Each loan is a one period contract with borrower b aged j consisting of a loan amount L_{b_j} and a price $q_{b_j}(\mu, Y)$. If the income endowment is equal to or above the default threshold $Y \geq \bar{Y}$, then the probability of costless default is $\nu_b(Y) = 0$ and borrowers do not default, repaying lenders $X_{b_j}(Y) = 1$. Otherwise, lenders obtain $X_{b_j}(Y) = \alpha p(\mu, Y) H_{b_j}/L_{b_j}$ in foreclosure. Let total lender wealth be given as $W_l = \sum_{b_j} L_{b_j} X_{b_j}(Y)$ where,

$$W_l = \begin{cases} \sum_{b_j} L_{b_j} = \tilde{L}_l, & \text{if no default} \\ \sum_{b_j} \alpha p(\mu, Y) H_{b_j} = \alpha p(\mu, Y)\tilde{H}_l, & \text{if default} \end{cases}$$

In the case of no default, the representative lender solves V_l^n:

$$V_l^n(\tilde{L}_l; \mu, Y) = \max_{\{L'_{b_j}\}} \left\{ \sum_{b_j} L_{b_j} - \sum_{b_j} q_{b_j}(\mu, Y) L'_{b_j} + \beta \mathbb{E}_{Y'|Y} \begin{bmatrix} (1-\nu_b(Y'))V_l^n(\tilde{L}'_l; \mu', Y') + \dots \\ \dots \nu_b(Y') V_l^d(\alpha p'(\mu; Y, Y')\tilde{H}'_l; \mu', Y') \end{bmatrix} \right\}$$

If borrowers default, the representative lender obtains housing stock $\alpha \sum_{b_j} H_{b_j}$ valued at $p(\mu; Y)$. Since lenders value total wealth, they are indifferent between being repaid in housing or loans absent any other frictions. To force lenders to immediately liquidate any housing stock obtained in foreclosure, they incur a utility penalty $-p(\mu, Y)\mathbb{1}_{[H_l>0]}\alpha\tilde{H}_l$ when solving V_l^d.[25]

$$V_l^d(\alpha p(\mu, Y)\tilde{H}_l; \mu, Y) = \max_{\{L'_{b_j}\}} \left\{ \sum_{b_j} \alpha p(\mu, Y) H_{b_j} - \sum_{b_j} q_{b_j}(\mu, Y) L'_{b_j} - p(\mu, Y)\mathbb{1}_{[H_l>0]} \sum_{b_j} \alpha p(\mu, Y) H_{b_j} + \dots \right.$$

$$\left. \dots \mathbb{E}_{Y'|Y} \begin{bmatrix} (1-\nu_b(Y'))V_l^n(\tilde{L}'_l; \mu', Y') + \dots \\ \dots \nu_b(Y') V_l^d(\alpha p'(\mu; Y, Y')\tilde{H}_l; \mu', Y') \end{bmatrix} \right\}$$

[24] An alternative approach taken by Gordon (2015) gives the lenders access to Arrow securities. The prices of these securities can then be used to discount future dividends. This approach shows how lenders risk share amongst themselves and allows for formal aggregation to a representative lender.

[25] Because borrowers have utility value of housing in addition to asset value, lenders will sell all of their housing to borrowers anyway. Absent a more formal investigation of this mechanism, the utility penalty assures that lenders will never hold houses even for very high house prices.

The first order and envelope conditions are given as:

$$\text{FOC}_{L'_{b_j}}: \quad q_{b_j}(\mu, Y) = \mathbb{E}_{Y'|Y}[(1-\nu_b(Y'))\partial V_l^n(\tilde{L}'_l;\mu', Y')/\partial L'_{b_j}] \tag{10}$$

$$\text{FOC}_{H'_{b_j}}: \quad 0 = \beta\mathbb{E}_{Y'|Y}[\alpha p'(\mu; Y, Y')\nu(Y')\partial V_l^d(\alpha\tilde{H}'_l p'(\mu; Y, Y'); \mu', Y')/\partial H'_{b_j}] \tag{11}$$

$$\text{Envelope}_{L_{b_j}}: \quad \partial V_l^n(\tilde{L}_l; \mu, Y)/\partial L_{b_j} = 1 \tag{12}$$

$$\text{Envelope}_{H_{b_j}}: \quad \alpha p(\mu, Y)\partial V_l^d(\alpha p(\mu, Y)\tilde{H}_l)/\partial H_{b_j} = 0 \tag{13}$$

Updating the envelope condition for housing obtained in default (13) and substituting into the first order condition for housing (11) ensures that lenders do not interfere with housing market dynamics and house price determination.

Updating the envelope condition for loans (12) and substituting into the first order condition for loans (10) delivers the loan pricing function:

$$q_{b_j}(\mu, Y) = \beta\mathbb{E}_{Y'|Y}[(1-\nu_b(Y'))]$$

B.3 Recursive Final Goods and Construction Firms' Problems

V_c denotes the value function of firms in the final goods sector.

$$\begin{aligned} V_c(\tilde{N}_c; \mu, Y) =& \max_{\tilde{N}_c} \left\{ Y - w(\mu, Y)\tilde{N}_c \right\} \\ s.to. \qquad Y =& \Theta(S)\tilde{N}_c \\ 0 \leq& \tilde{N}_c \\ \mu' =& \Gamma_\mu(\mu; Y, Y') \end{aligned}$$

Because final goods consumption C_b is the numeraire, the price for final goods has been normalized to one. Taking the first order condition with respect to labor $\tilde{N}_c$ pins down the equilibrium wage:

$$w(\mu, Y) = \Theta(S)$$

V_h denotes the value function of housing construction firms.

$$\begin{aligned} V_h(\tilde{N}_h; \mu, Y) =& \max_{\tilde{N}_h} \left\{ p(\mu, Y)\tilde{H}'_h - w(\mu, Y)\tilde{N}_h \right\} \\ s.to \qquad \tilde{H}'_h =& [\Theta(S)\tilde{N}_h]^\epsilon \bar{L}^{1-\epsilon} \\ 0 \leq& \tilde{N}_h \\ \mu' =& \Gamma_\mu(\mu; Y, Y') \end{aligned}$$

Taking the first order condition with respect to labor $\tilde{N}_h$ and using the above equilibrium expression for wage $w(\mu, Y)$ yields:

$$\epsilon\Theta(S)p(\mu, Y)[\Theta(S)\tilde{N}_h]^{\epsilon-1}\bar{L}^{1-\epsilon} = \Theta(S)$$

Canceling $\Theta(S)$ from both sides and re-arranging:

$$[\Theta(S)\tilde{N}_h]^{\epsilon-1}\bar{L}^{1-\epsilon} = [\epsilon p(\mu, Y)]^{-1}$$

Taking both sides to the $\frac{\epsilon}{\epsilon-1}$ power:

$$[\Theta(S)\tilde{N}_h]^{\epsilon}\bar{L}^{-\epsilon} = [\epsilon p(\mu, Y)]^{\frac{\epsilon}{1-\epsilon}}$$

Multiplying both sides by $\bar{L}$ delivers the expression for housing supply:

$$\underbrace{[\Theta(S)\tilde{N}_h]^{\epsilon}\bar{L}^{1-\epsilon}}_{\equiv \tilde{H}'_h} = [\epsilon p(\mu, Y)]^{\frac{\epsilon}{1-\epsilon}}\bar{L}$$

C Appendix: Computational Algorithm and Solution Checks

C.1 Full Algorithm

The algorithm is adapted from Kaplan et al. (2017) and Favilukis et al. (2017) who use a variation of the Krusell and Smith (1998) algorithm. All j subscripts have been dropped unless completely necessary.

1. Define grids over ages $j = 1, \ldots, J$, loans $L_b \in [0, 1]$, borrower housing stock $H_b \in [0, 1]$, borrower housing choice $H'_b \in [0, 1]$, aggregate income $Y \in \{Y_{good}, Y_{bad}\}$, and the housing price $p \in [0.8, 2.3]$.

2. With a risk-neutral representative lender, the loan price $q_{b_ij}(p(Y), Y)$ is pinned down by the lenders' loan supply equation:

$$q_{b_j}(p(Y), Y) = \beta[(1 - \nu_b(Y'))]$$

   Where $\nu_b(Y')$ is the probability of costless default where $\nu_b = \begin{cases} 0, & \text{when} \quad Y_{good} \\ 1, & \text{when} \quad Y_{bad} \end{cases}$

3. Guess coefficients $\left(a^0_{Y,Y'}, \quad a^1_{Y,Y'}\right)$ to forecast next period loan price $p'(p(Y); Y, Y')$:

$$\log p'(p(Y); Y, Y') = a^0_{Y,Y'} + a^1_{Y,Y'} \log p(Y) \tag{14}$$

   This will yield a total of $\{\#Y\}^2 = 4$ forecasting equations

4. Solve the individual borrowers' and problems at each grid point over $p(Y)$. I use value function iteration grid search with trilinear interpolation over $p'(p(Y); Y, Y')$, H'_b, and L'_b.

5. Simulate a long time series of Y_t for $t = 1, \ldots, T$ where $T = 2,000$ with a burn in period of 200.

6. Fix an initial distribution of borrowers' loans, housing, and ages $\mu_{b,0} \in \mathcal{L} \times \mathcal{H} \times \mathcal{J}$ for $N = 10,000$ borrowers. Where $\mathcal{L}$ is the set of all possible beginning of period loan realizations, $\mathcal{H}$ is the set of all possible beginning of period housing realizations, and $\mathcal{J}$ is the set of all possible beginning of period age realizations. Favilukis et al. (2017) note that there must be a large number of agents and I have noted that the larger the number of agents, the easier it is to assure period-by-period market clearing.

   Since aggregate loans must be in zero net supply $\tilde{L}_b = \tilde{L}_l$, one can obtain the initial endowment of loans for the representative lender by aggregating the borrowers' initial loan endowment. The grid for loans used in simulation is finer than that used to solve the individual borrowers' and lenders' problems.

$$\tilde{L}_{l,1} = \frac{1}{N} \sum_{b=1}^{N} L'_{b,i}(L_{b,1}, H_{b,1}, j_{b,1}; Y_1)$$

Some notes on the borrowers' distribution:

(a) Initial borrower loan endowments are drawn from a normal distribution $\ell_{b,1} \sim \mathcal{N}(0,1)$. Each $\ell_{b,1}$ is then re-scaled to lie in the grid for loans L: $L_{b,1} = 0.5 + \frac{||\lfloor \min \ell_1 \rfloor|| + \ell_{b,1}}{||\lfloor \min \ell_1 \rfloor|| + \lceil \max \ell_1 \rceil} * 0.5$

(b) Initial housing endowments are draw from a normal distribution $h_{b,1} \sim \mathcal{N}(0,1)$. Each $h_{b,1}$ is re-scaled to lie in the bounds of housing supply $H_{b,1} \in [\min \tilde{H}'_h, \max \tilde{H}'_h]$ $H_{b,1} = \max \tilde{H}'_h + \frac{||\lfloor \min h_1 \rfloor|| + h_{b,1}}{||\lfloor \min h_1 \rfloor|| + \lceil \max h_1 \rceil} * (\max \tilde{H}'_h - \min \tilde{H}'_h)$

(c) Initial borrower ages are drawn from a uniform distribution over integers, $j_{b,0} \sim \mathcal{U}\{1, \dots, J\}$

(d) After borrower b dies, i.e. $j_{b,t} = J$, a new borrower is reborn replacing that agent and they have age $j_{b,t+1} = 1$. They begin life with no debt $L_{b,t} = 0$ and inherit the housing stock of the agent they are replacing.

7. At $t = 1$, given Y_t, $\mu_{b,t}$, and policy functions, compute the aggregate schedule for housing:

$$\tilde{H}_{b,t+1}(p, Y_t) = \frac{1}{N} \sum\nolimits_{b=1}^{N} H'_b(L_{b,t}, H_{b,t}, j_{b,t}; Y_t, p)$$

Compute the excess demand for aggregate housing over the p grid and interpolate the excess demand function to find $p_t^*(Y_t)$ such that aggregate housing clears.

$$\tilde{H}_{b,t+1}(p_t^*(Y_t), Y_t) = \tilde{H}_{h,t+1}(p_t^*(Y_t), Y_t)$$

8. Find the values $\tilde{L}^*_{b,t+1}$ and $\tilde{H}^*_{b,t+1}$ by interpolating the individual policy functions L'_b and H'_b at $p_t^*(Y_t)$ to yield the equilibrium distribution $\mu^*_{b,j,t+1}$. Then aggregate across all borrowers

$$\tilde{L}^*_{b,t+1}(p_t^*(Y_t), Y_t) = \frac{1}{N} \sum\nolimits_{b=1}^{N} L'_b(L_{b,t}, H_{b,t}, j_{b,t}; p_t^*(Y_t), Y_t)$$
$$\tilde{H}^*_{b,t+1}(p_t^*(Y_t), Y_t) = \frac{1}{N} \sum\nolimits_{b=1}^{N} H'_b(L_{b,t}, H_{b,t}, j_{b,t}; p_t^*(Y_t), Y_t)$$

9. Repeat steps (7)-(8) for $t = 1, \ldots, T$

10. Using the pricing data $\{p_t^*(Y_t)\}_{t=1}^T$ generated above, partition the data by Y, Y' which will generate $\{\#Y\}^2 = 4$ sub-samples and compute new forecasting coefficients using linear regression for each sub-sample

$$p_{t+1}(p_t^*(Y_t), Y, Y') = a_{Y,Y'}^{0,new} + a_{Y,Y'}^{1,new} \log p_t^*(Y_t)$$

11. Iterate until coefficients converge

$$\left(a_{Y,Y'}^0, a_{Y,Y'}^1\right)' \approx \left(a_{Y,Y'}^{0,new}, a_{Y,Y'}^{1,new}\right)'$$

C.2 Den Haan Error Check

This alogirthm details a variant of the Den Haan (2010) accuracy check obtained from Algan et al. (2014, p. 310-311)

1. Given initial distributions of housing $H_{b,t}$ and ages j for borrower b at $t = 1$
2. Compute aggregate housing stock:

$$\tilde{H}_{b,t}^{Approx} = \frac{1}{N} \sum_{b=1}^{N} H_{b,t}$$

3. Substitute into the housing supply function[26] to obtain house prices for last period p_{t-1}^{Approx}

$$\tilde{H}_{b,t}^{Approx} = [\epsilon p_{t-1}^{Approx}]^{\frac{\epsilon}{1-\epsilon}} \bar{L}$$

4. Use the forecasting rule to compute current period house prices[27] $p_t^{Approx}(Y_t)$

$$\log p_t^{Approx}(p_{t-1}^{Approx}(Y_{t-1}), Y_{t-1}, Y_t) = \alpha_{Y,Y'}^0 + \alpha_{Y,Y'}^1 \log p_{t-1}^{Approx}(Y_{t-1})$$

5. Interpolate the individual housing demand functions at $p_t^{Approx}(p_{t-1}^{Approx}(Y_{t-1}), Y_{t-1}, Y_t)$ to obtain current period individual housing demand:

$$H_{b,t+1} = H_b'(L_{b,t}, H_{b,t}, j_{b,t}, p_t^{Approx}(p_{t-1}^{Approx}(Y_{t-1}), Y_{t-1}, Y_t))$$

6. Repeat steps 2-5 for $t = 2, \ldots, T-1$

[26] for $t = 1$, I am assuming $\tilde{H}_{l,1} = \tilde{H}_{b,1}$ such that $(1 - \delta_h)\tilde{H}_{l,0} = 0$

[27] In the first period $t - 1 = 0$ is taken as the average of the coefficients $0.5(a^i_{Y,Y_{good}} + a^i_{Y,Y_{bad}})$ where i=0,1

C.3 Chipeniuk et al. (2019) Solution Check

Under the assumption that

$$\log p'(p(Y); Y, Y') = a^0_{Y,Y'} + a^1_{Y,Y'} \log p(Y) \approx \mu' = \Gamma_\mu(\mu; Y, Y')$$

The state space individual borrowers becomes $\{L_b, H_b, Y, \mu\} = \{L_b, H_b, Y, p(Y)\}$ and the next period state space is thus $\{L_b,' H_b', Y', \mu'\} = \{L_b, H_b, Y, p'(p(Y); Y, Y')\}$.

Using the method of Chipeniuk et al. (2019), I take $p(Y)$ and the coefficients from the converged simulations as given. I will denote values obtained from simulation with a $_t$ subscript so that $p(Y)$ is written as $p^*_t(Y_t)$ and the coefficients are $a^{i,*}_{Y_t,Y'}$ for $i = 0, 1$. The state space of individual borrowers then becomes $\{L_b, H_b, Y, p^*_t(Y_t)\}$ and the next period state space $\{L_b', H_b', Y', p'(p^*_t(Y_t); Y, Y')\}$.

The algorithm for the Chipeniuk et al. (2019) solution check is given as:

1. Set grids for L_b,H_b,Y , $p'(p^*_t(Y_t), Y, Y') = \#Y' = 2$ for t=1.

$$\text{Where,} \quad \log p'(p^*_t(Y_t); Y_t, Y') = a^{0,*}_{Y_t,Y'} + a^{1,*}_{Y_t,Y'} \log p^*_t(Y_t)$$

With converged coefficients $\{a^{i,*}_{Y,Y'}\}$ for $i = 0, 1$

1.a For $j = J$ interpolate the continuation value $V_{j+1}(L_b', H_b'; Y', p'(p^*_t(Y_t); Y_t, Y'))$ from $V_j(L_b, H_b; Y', p'(p^*_t(Y_t); Y_t, Y'))$ using bilinear interpolation over $\{L_b', H_b'\}$

1.b Solve the borrowers' problem to obtain optimal policy functions $H'^*_j(L_b, H_b; Y, p^*(Y_t))$ and $L'^*_j(L_b, H_b; Y, p^*(Y_t))$ which will be used in simulation and an optimal policy function $V^*_j(L_b, H_b; Y, p^*_t(Y_t))$

1.c To solve the problem recursively, one needs to also obtain $V_{j+1}(L_b', H_b'; Y', p'(p'(p^*_t(Y_t); Y_t, Y'); Y_t, Y')$ via trilinear interpolation of $\{L_b', H_b, p'(p'(p^*_t(Y_t); Y_t, Y'); Y_t, Y')\}$ over $V_j(L_b, H_b; Y', p'(p^*_t(Y_t); Y_t, Y'))$

$$\text{Where,} \quad \log p'(p'(p^*_t(Y_t); Y_t, Y'); Y_t, Y') = a^{0,*}_{Y_t,Y'}(1 + a^{1,*}_{Y_t,Y'}) + (a^{1,*}_{Y_t,Y'})^2 \log p^*_t(Y_t)$$

In theory, one could have interpolated $\{L_b', H_b, p'(p'(p^*_t(Y_t); Y_t, Y'); Y_t, Y')\}$ over the optimal value function obtained in the previous step $V^*_j(L_b, H_b; Y, p^*(Y_t))$. However, since $p^*_t(Y_t)$ is a only one data point it is not possible to use interpolation in this dimension. Rather then expanding the p grid to $p = \{p^*_t(Y_t) - \epsilon, p^*_t(Y_t), p^*_t(Y_t) + \epsilon\}$ for $\epsilon > 0$ and interpolating $p'(p'(p^*_t(Y_t); Y_t, Y'); Y_t, Y')$ over this grid, approximation error can be reduced by defining the value function over $p'(p_t^*(Y_t);Y_t,Y')$ directly.

1.d Solve the borrowers' problem again using the continuation value $V_{j+1}(L_b',H_b';Y',p'(p'(p_t^*(Y_t);Y_t,Y');Y_t,Y')$ to obtain the $j-1$ continuation value $V_j(L_b,H_b;Y',p'(p_t^*(Y_t);Y_t,Y')$

1.e Repeat steps (1.a)-(1.d) for $j=J-1,\ldots,1$

2. Simulate and aggregate over individual policy functions for housing and loans

$$\tilde{L}_{b,t+1}^{new}(Y_t,p_t^*(Y_t)) = \frac{1}{N}\sum_{b=1}^{N} L_b'(L_{b,t},H_{b,t},j_{b,t};Y_t,p_t^*(Y_t))$$
$$\tilde{H}_{b,t+1}^{new}(Y_t,p_t^*(Y_t)) = \frac{1}{N}\sum_{b=1}^{N} H_b'(L_{b,t},H_{b,t},j_{b,t};Y_t,p_t^*(Y_t))$$

3. Compute excess demand for housing to find a new market clearing price $p_t^{*,new}$

$$\tilde{H}_{b,t+1}(Y_t,p_t^{*,new}(Y_t)) = \tilde{H}_{h,t+1}(Y_t,p_t^{*,new}(Y_t))$$

4. Repeat steps (1)-(3) for $t=2,\ldots T$

5. Using the pricing data $\{p_t^*(Y_t)^{new}\}_{t=1}^{T}$ generated above, partition the data by Y,Y' and compute new forecasting coefficients using linear regression for each sub-sample

$$p_{t+1}(p_t^*(Y_t)^{new};Y,Y') = a_{Y,Y'}^{0,new} + a_{Y,Y'}^{1,new}\log p_t^*(Y_t)^{new}$$

6. Iterate until coefficients converge

$$\left(a_{Y,Y'}^{0,*},a_{Y,Y'}^{1,*}\right)' \approx \left(a_{Y,Y'}^{0,new},a_{Y,Y'}^{1,new}\right)'$$