

# Entrepreneurship, Inter-Generational Business Transmission and Aging\*

Alexandre Gaillard  
*Toulouse School of Economics*

Sumudu Kankanamge<sup>†</sup>  
*Toulouse School of Economics*

This version: February 10, 2019

First version: July 3, 2018

**Preliminary paper, please do not distribute.**

## Abstract

This paper introduces a quantitative stylized life-cycle model with entrepreneurship and endogenous business selling, buying and founding decisions. Using a new dataset on the small business sale market as well as the SSBF, the SBO and the PSID datasets, we document the importance of the buying, selling and founding margins for entrepreneurs and find large mismatches on the business sale market. The data also reveal a key role for age and life-cycle dynamics for entrepreneurial entry and exit decisions. Using the model, we find that the combination of (i) illiquid business assets, (ii) frictions on the business sale market and (iii) the life-cycle components of entrepreneurship are key to reproducing our empirical finding. Finally, we simulate a large demographic event akin to the baby-boomers generation reaching peak retirement age and evaluate the macroeconomic outcome of such a change.

**Keywords: Entrepreneurship, Business Selling and Buying Frictions, Aging**

**JEL classification: E21, L26, J11**

---

\*For helpful comments and suggestions, we are grateful to Fabrice Collard, Patrick Fève and Christian Hellwig. We also thank participants at the macroeconomic workshop in Toulouse School of Economics. We finally thank the University of Science of Toulouse for providing us access to the supra-calculator OLYMPE under the CALMIP project.

<sup>†</sup>Corresponding author: [sumudu.kankanamge@tse-fr.eu](mailto:sumudu.kankanamge@tse-fr.eu).

# 1 Introduction

There is little doubt that the entrepreneurial sector of most modern economies is a major contributor to key macroeconomic indicators such as growth and employment. In this paper, we identify a number of elements that signal that both the demographic evolution and a number of frictions in entrepreneurial markets could be a concern for macroeconomic outcomes. On the one hand, well known demographic trends show that over the next decades we will reach the peak of retiring baby-boomers. A growing literature documents the substantial shifts in the labor market that this will create but surprisingly, very little attention has been given so far to a pivotal implication of the above evolution: the aging of entrepreneurs. In 2016, 30% of entrepreneurs were aged 60 years and more compared to only 22% in 2004 and 18% in 1989. If we look at the population aged 55 and more, the corresponding numbers are 47% in 2016, 34% in 2004 and 26% in 1989. Evidences indicate quite predictably that the aging of the entrepreneurial population will only become more severe in the upcoming years<sup>1</sup> all the while this age group is a principal provider of entrepreneurial production and employment and preeminent holder of business assets.<sup>2</sup> On the other hand, we document substantial frictions on the entrepreneurial business sale market impacting business transmissions. As compared to the already illiquid housing market, selling a business takes close to twice as much time, to an average of seven months over the 2017-2018 period. Moreover using a new dataset, we find that the vast majority of businesses listed for sale remains unsold. At the other end, on the business creation or purchasing front, potential new entrepreneurs are subject to important credit market frictions. The cumulative effects of business sale market frictions and entrepreneurial aging might be quite significant, specially since entrepreneurs hold very undiversified portfolios ([Moskowitz and Vissing-Jørgensen \(2002\)](#)), which is of crucial importance for individuals planning to retire. For this reason, the main contribution of this paper is to provide a theoretical model able to fully capture the important frictions appearing on entrepreneurial markets as well as the age dynamics. We support the above model by first reporting a large number of micro-data based new empirical facts on business creation and acquisition, the small business sale market and the age and life-cycle aspects of entrepreneurship. To the best of our knowledge, our paper is the first to tackle this agenda.

Our baseline economy is a stylized life-cycle incomplete markets model with heterogeneous agents and occupational choices. We introduce endogenous business selling, buying and founding decisions into this framework as an important and novel mechanism for entrepreneurial entry and exit. Households age probabilistically through four different stages

---

<sup>1</sup>See for instance [Pugsley et al. \(2016\)](#) on the decrease of young entrepreneurs starting businesses.

<sup>2</sup>For instance, 2016 ASE data reveal that the fraction of entrepreneurs aged 55 to 64 years and 65 years and more employ respectively around 30% and 20% of total entrepreneurial employment

of life designed to concisely capture the phases of adult professional life and retirement as well as the key saving, occupational and entrepreneurial decisions. The model finely describe business selling and buying mechanisms. Each period an incumbent entrepreneur might need to sell (voluntarily or involuntarily) her business and will face an endogenous selling price as well as a selling probability. Absent a sale opportunity, the incumbent will be forced to dismantle the activity and liquidate the entrepreneurial assets or remain in this activity. Conversely, a non-entrepreneur might enter an entrepreneurial activity by endogenously choosing a firm size and either finding an available business to buy or founding a new business. Each of these decisions are subject to credit constraints. Moreover, illiquid capital adjustments are subject to adjustment costs. We argue that all the above ingredients are key in reproducing the business selling, buying/founding and age related frictions appearing in the empirical data.

We use data from the Annual Survey of Entrepreneurs (ASE), the Survey of Business Owners (SBO), the Survey of Small Business Finance (SSBF) and the Panel Study of Income Dynamics (PSID) to document entrepreneurial acquisition and ceasing characteristics. We find that about two-thirds of businesses are newly created whereas about a quarter are purchased from the pool of businesses for sale. We document that financial constraints are a key element that differ between founded and purchased businesses: both credit limits on loans and interest rate conditions substantially favor purchased businesses. Moreover, we find that business outcomes are about 5 times larger with purchased firms. This reveals that the maturity of a business in its respective market is an important driver of credit conditions and entrepreneurial earnings. Based on that buyers should focus on purchasing an existing business. Not only do we find that it is not the case in the data, but younger and older individuals favor founding more than middle-aged individuals. This reveals the existence of important credit constraints on the business buying side. Our findings thus show that the decision to purchase an existing business as opposed to founding a new one is a crucial dimension for entry into entrepreneurship. On the business selling side, we document that retirement and selling are the leading reasons for ceasing a business activity as opposed to operating or financial reasons, and this is specially true for older firms. To better understand the characteristics of the business sale market, we build our own dataset by collecting data on the online marketplace platform BizBuySell (BBS) over an extensive period. Using this dataset, we find that, depending on the period, between 10% and 20% of businesses listed for sale are actually sold. Between 2017 and 2018, the time on the market (TOM) of a business for sale was on average around 7 months. But behind this number, the distribution of the TOM displays considerable variation. Interestingly, we also find that the ratio of list price to market price of these businesses display considerable variance. Based on these evidences, we argue that the business sale market is subject to substantial frictions and mismatches that condition entrepreneurial exit and could lead to important economic losses. Finally, we document that age and life-cycle dynamics are a major factor that explains

entrepreneurial entry and exit. Using Current Population Survey (CPS) data, we find that the mean age at which workers enter entrepreneurship increases from age 43 to 48 between 1996 and 2016. Also, according to the Survey of Consumer Finances (SCF) data, we find a considerable shift to the right of the entrepreneurial distribution by age between 1989 and 2016, whereas the corresponding distributions for workers is fairly stable. We also show that old entrepreneurs are the principal provider of total entrepreneurial business value and employment. This indicates that together with business acquiring and selling frictions, the aging of the entrepreneurial population is a major macroeconomic concern.

We simulate a calibrated version of our benchmark economy. Prior to calibration, we test the exact effect of the type of business acquisition (buying versus founding) on the interest rate charged on the businesses as well as on the credit limit using SSBF data. Our estimation results find a significant relationship between the type of acquisition and the interest rate. Recent owners (with less than 5 years in operation) having purchased an existing business are charged, on average, an interest rate 1.59% lower than those who founded their businesses. Purchased businesses can also borrow 242 000 dollars more than founded businesses over the sample of all firms. Our simulation results show that our model and calibration are able to reproduce the key elements that we pointed out in our empirical findings. We match the distribution of founders and purchasers as well as the corresponding life-cycle dynamics. Moreover, individuals beyond the legal retirement age hold large portions of illiquid entrepreneurial capital and are an important fraction of the business sale market. We also find that business sale market mismatches, together with the life-cycle setup are key in reproducing the empirical data. In particular, we find that the common assumption of illiquid capital adjustment costs, although important, is not sufficient to reproduce the actual entrepreneurial entry and exit dynamics appearing in the data.

We detail the impact of key ingredients of our model on the entrepreneurial pool, aggregate production and capital and the characteristics of the business sale market. To this end, we first quantify the effects of improving the matching between business buyers and sellers. We find that the fraction of businesses sold and bought increases in equilibrium, lowering the overall entrepreneurial capital destruction through liquidation. As a result, entrepreneurial capital significantly increases as well as production. Second, we explore the impact of lowering the interest rate on business loans. Measuring this channel is of some importance for a number of active labor market policies aiming to help self-employment. These policies often target the borrowing costs, specially to promote early stage entrepreneurship. We find that the effects of this channel are large on the equilibrium. The share of entrepreneurs significantly increases and especially the ratio of existing businesses to new comers change in favor of the latter. The business sale market is also significantly impacted with a simultaneous decrease of selling and buying, as the tradeoff between purchasing and founding a business changes in favor the latter.

Third we also measure the impact of illiquid capital adjustment costs by symmetrically increasing the cost of both capital increases and decreases. We find that the impact of this channel is very significant on the business sale market as the share of business buyers and specially the share of sellers increase remarkably. This is a clear sign that the business sale market is a preferred alternative to costly illiquid capital liquidation, a feature that models with only capital adjustment costs can not capture. Finally, we change the population structure by artificially increasing the fraction of agents of age  $[51, 65[$  to favor on average richer individuals in the life-cycle. This change has a significant impact on entrepreneurial and total capital, that both increase along with production. This also leads to an increase of business buyers and sellers: because a larger fraction are older, more of them switch to retirement, increasing the fraction of sellers while the fraction of buyer increases endogenously increase with age. Overall, we find our setup to be both appropriate and tractable to investigate a rich entrepreneurial environment. On tractability, despite the fact that the model includes a number of computationally challenging elements, we provide an algorithm that tackle both the kinks in the value function and allows for a fast computation time by combining the insight in recent papers by [Iskhakov et al. \(2017b\)](#) and [Drue Dahl and Jørgensen \(2017\)](#).

Finally, we intend to use the model to simulate a demographic change similar to the consequences of the baby-boom generation reaching the peak of retirement. To this end, we modify the stationary assumption of our model and simulate a specific transition. This contribution is still in progress.

**Related literature** The literature on entrepreneurship is vast and quite a number of papers consider the subject in combination of incomplete markets [Bewley \(1983\)](#); [Huggett \(1993\)](#); [Aiyagari \(1994\)](#) type models. Entrepreneurship has been shown to generate differential saving rates and high returns to capital contributing in generating the high concentration of wealth, in line with what is observed in the data ([Cagetti and De Nardi \(2006\)](#) for the US). This literature has been extended to answer broad questions related to misallocation of capital ([Buera and Shin \(2013\)](#)), tax policies ([Kitao \(2008\)](#); [Cagetti and De Nardi \(2009\)](#) among others) or policies that aim to promote entrepreneurship ([Fairlie et al. \(2011\)](#); [Mankart and Rodano \(2015\)](#); [Gaillard and Kankanamge \(2018\)](#)). Relative to this literature, our paper adds an empirically realistic model that account for the life-cycle property of entrepreneurship, as well as explicitly modeling the inter-generational transmission of business capital. The introduction of a stylized life-cycle model can account for the observed hump-shaped relationship between age and entrepreneurship, which is crucial for understanding the small and middle scale business dynamics and the buying/founding and the selling/liquidating margins. To the best of our knowledge, we are the first to focus on the transmission of business capital over generations in an otherwise quite standard incomplete markets framework.

Recent papers study the decline in the entrepreneurship rate observed in the US since the 1980s and relates this to the demographic change. [Pugsley et al. \(2016\)](#) argue that the lower population growth rate and the aging of the population has led to a lower start-up rate in the US. To recover this, we calibrate our model while specifically accounting for the demographic change by targeting the (possibly) non-stationary 2007 US economy. We thus study the consequences of the ongoing aging of entrepreneurs both on the transmission of business capital and aggregate outcomes.

Finally, our paper contributes to the growing literature trying to understand the implications of illiquid assets versus liquid assets. In our model, entrepreneurs face adjustment costs when investing and disinvesting in business assets, which is crucial in generating the opportunity to buy an existing business. However, the two assets structure of the model in addition to occasionally binding constraints make the problem particularly difficult to solve. We use the recent state-of-the-art *DC-EGM* computational techniques in [Iskhakov et al. \(2017a\)](#); [Druedahl and Jørgensen \(2017\)](#) that extend the Endogenous Grid Method (EGM) initially introduced by [Carroll \(2006\)](#) to the combination of multiple assets with discrete and continuous endogenous variables. This is needed because the presence of occupational choices generates kinks and as a result, first order conditions are no longer sufficient, while still necessary. In addition to achieve some notable computational advantages, this method also allows for *IID extreme value taste shocks* which is of particular interest since, as argued [Hurst and Pugsley \(2011, 2015\)](#), unobservable heterogeneity in non-pecuniary benefits is of first order importance for the decision to become an entrepreneur.<sup>3</sup>

The rest of the paper proceeds as follows. Section 2 documents facts related to entrepreneurship, life-cycle and the business sale market. Section 3 lays out our framework and Section 4 describes how we take the model to the data. We discuss the results of the model in Section 5. In Section 6, we simulate the implications of a demographic shock on occupational choice and aggregate outcomes. Finally, section 7 concludes.

## 2 Stylized facts

In this section, we document a number of empirical facts concerning entrepreneurial business acquisition and selling. We explore the purchasing versus founding dimensions of entrepreneurial entry, the nature of the business sale market and the profile of sellers. We also report a number of findings about the age profile and life-cycle aspects of entrepreneurship.

---

<sup>3</sup>In particular, they have shown that a significant fraction of entrepreneurs choose to own businesses due to non-pecuniary reasons and never intend to grow their firms. In line with this, our model allow for non-pecuniary self-employment benefits which is reflected by direct utility gain and idiosyncratic taste shocks over household's occupational choice.

## 2.1 Data origin

We use various US survey data to identify business sellers and buyers together with their individual demographic as well as wealth and income related characteristics. We use the Annual Survey of Entrepreneurs (ASE) and the micro panel of the 2007 Survey of Business Owners (SBO) to document entrepreneurial acquisition and ceasing reasons. We obtain detailed information on the buyers' side using the 2003 Survey of Small Business Finance (SSBF) and on the sellers' side using the Panel Study of Income Dynamics (PSID)<sup>4</sup>. Finally, we build our own dataset using the online business selling platform *BizBuySell* to provide empirical evidences on the business sale market. The data correspond to 92900 observations of sold and for sale businesses records obtained from continuously scrapping public data available on this website over a number of years. Appendix B provides further details on the datasets and the variables that we use.

In order to remain consistent across datasets, we define an entrepreneur as an active self-employed business owner<sup>5</sup>.

## 2.2 Purchasing or founding a small business

The literature on entrepreneurship has long been interested in the behavior of incumbent entrepreneurs but has been somewhat silent on how businesses came to be in the first place. Using the micro sample of the SBO 2007, we find that 65.6% of the entrepreneurs founded a new business, 23.3% purchased an existing business, 7.8% received their business as a gift and 3.2% inherited the family business. Moreover, among those who founded and started a business, 62.8% already had experienced a self-employment situation against 72.6% of those who purchased a business.

Concerning the financing source, 63% (resp. 83%) of purchasers (resp. founders) declare using personal assets and 65% (resp. 49%) declare using diverse loans. As shown in figure 1, purchasing a business requires prospective entrepreneurs to invest a large initial amount of capital<sup>6</sup>. Using the SSBF (table 1), we find that entrepreneurs who just purchased businesses are substantially richer (around 1.5 times richer on average) but make also substantially more

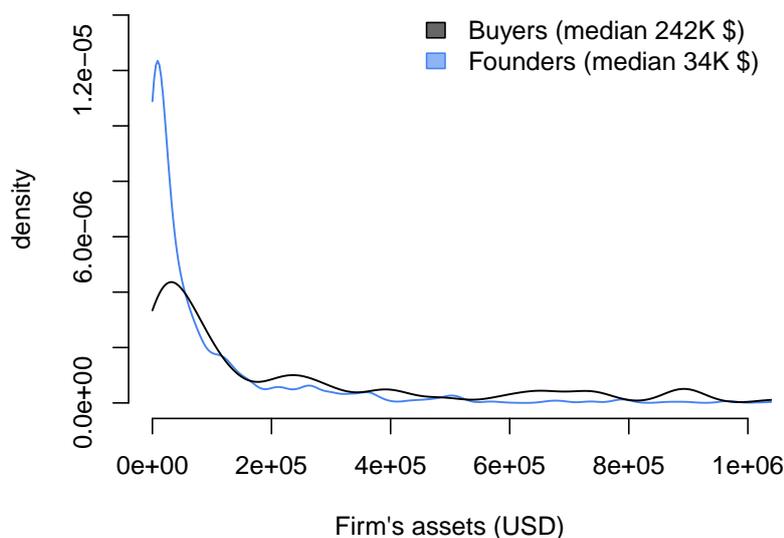
---

<sup>4</sup>The SBO and the PSID contain a number of information concerning the age of an entrepreneur, her education level, past experiences, as well as detailed information regarding individual income and family wealth. This lets us draw a more precise picture of who are the businesses sellers and buyers in the US.

<sup>5</sup>This is possible for all the surveys that we use. Unfortunately, we can not control for active self-employment on the BizBuySell website. We therefore capture some business owners not actually entrepreneurs.

<sup>6</sup>We take early stage firms in order to capture the amount of startup capital between founders and purchasers. In appendix, we also use the SBO - 2007 and we draw the reported bracketed startup capital distribution between acquisition types.

profit (around 5 times larger)<sup>7</sup>.



**Figure 1.** Distribution of firm's assets of early stage firms (less than 5 years) in SSBF - 2003.

|                 | Profit (K\$) |        | Firm's equity (K\$) |        | Owner's networth (K\$) |        |
|-----------------|--------------|--------|---------------------|--------|------------------------|--------|
|                 | mean         | median | mean                | median | mean                   | median |
| Founded/started | 104          | 8      | 140                 | 10     | 635                    | 260    |
| Purchased       | 505          | 50     | 555                 | 37     | 1196                   | 390    |
| Ratio           | 4.8          | 6.1    | 3.7                 | 3.9    | 1.5                    | 1.9    |

**Table 1.** Firm's and owner's characteristics depending on acquisition type. SSBF (2003), using main owner and ESB firms.

We also find that purchased businesses tend to be less financially constrained. Using the same SSBF data, we find that among those who needed a credit during the preceding three years, 27.5% of the entrepreneurs who started a new business did not apply because they thought the application would be turned down. This number goes down to 21.5% for buyers. Regarding recent loan applications<sup>8</sup>, 73.5% were always approved and 19.3% were always denied for founders, against respectively 83.6% and 11.6% for purchasers. If the magnitude of denied loan applications are different between acquisition types, the reasons for it are also quite different. For founded businesses, the most recurrent reasons for denied loan applications are

<sup>7</sup>Regarding the distribution of profits, we find large disparities among purchasers and founders as shown in Figure 16 in Appendix, with a substantial fraction with losses. This is in line with many papers arguing that a substantial number of entrepreneurs experience losses but still stay in their entrepreneurial situation (see for instance Hamilton (2000)).

<sup>8</sup>Unfortunately, we can not control for loan size. Those numbers have therefore to be taken with caution.

that the firm was not in business long enough (28.7%), that credit history is not favorable (22%) and that the firm has poor balance sheet or financial situation (19.8%). For purchased businesses, credit history is the main reason (25.7%) followed with no particular reason (25%) and insufficient collateral or no guarantees available (19%). This means that purchased businesses are not only less risky in terms of income stream (higher profit), but they can take advantage of favorable credit conditions because the history of firm's performance is long enough for a bank to evaluate the quality of a business project.

The above illustrate that the decision to purchase an existing business as opposed to founding a new one is an important dimension for entry into entrepreneurship. This translates to many facets such as the initial wealth level of the prospective entrepreneur but also to future credit conditions (credit limits and interest rates) for the business. This is linked to the maturity of a business on its respective market: it is possible to purchase a mature firm with good credit prospects and expected outcomes by investing more whereas although it might be less costly to found a new business, the credit and earning prospects are quite different.

### **2.3 Selling a small business**

In Table 2, we document the main reasons explaining why entrepreneurs ceased their activity in 2016. As compared to earlier comparable results, entrepreneurs who declared ceasing their activity due to retirement and because they sold their businesses were respectively 9.9% and 11% in the SBO - 2007, whereas they were respectively 18.9% and 17.9% in the ASE - 2016<sup>9</sup>. These two reasons constitute, together with inadequate cash flow or sales, the three main reasons to cease a business<sup>10</sup>. Those statistics depend on the number of operating years. Younger firms tend to cease more often, and do so mainly because of business risks and financial constraints translated into inadequate cash flows or sales, lack of business and personal loans/credits and turnover (i.e. starting another business). On the contrary, retirement and selling the business account for almost 50% of the reasons to cease for older firms (more than 5 years of operation), whereas reasons linked to business risks are less predominant, suggesting that selling the firm are likely to be an important exit option for businesses in operation for a long period<sup>11</sup>.

To understand the characteristics of sold businesses, business sellers and the small business sale market in general, we compiled a new dataset using public data available on the online

---

<sup>9</sup>Those two datasets are comparable since the Annual Survey of Entrepreneurs is the continuation of the Survey of Business Owners.

<sup>10</sup>An individual can indicate multiple choices for the reason to cease a business. For instance, an individual could retire and sell her business.

<sup>11</sup>Notice that the number of ceasing business with paid employee for retirement and selling purposes is 2% per year, whereas the fraction of entrepreneurs declaring having paid advice services for exit planning program is close to 5.9% each year; suggesting that a large fraction of entrepreneurs do not succeed in exiting their occupation.

|  | All firms   | < 2 years   | > 5 years   |
|--|-------------|-------------|-------------|
| Ceasing per year (%)                           | 6           | 8.6         | 5.5         |
| <b>Operations ceased due to</b>                |             |             |             |
| inadequate cash flow or sales (%)              | <b>19.4</b> | <b>27.7</b> | <b>14.6</b> |
| owner(s) retired (%)                           | <b>18.9</b> | <b>3.5</b>  | <b>27.3</b> |
| owner(s) sold the business (%)                 | <b>17.9</b> | <b>11.0</b> | <b>21.0</b> |
| owner(s) illness, injury or died (%)           | 6.0         | 4.0         | 7.5         |
| owner(s) started another business (%)          | 5.1         | 6.6         | 4.1         |
| lack of business and personal loans/credit (%) | 3.3         | 7.4         | 1.9         |
| another reason (%)                             | 29.4        | 36.5        | 23.0        |

**Table 2.** Entrepreneurs' reasons for ceasing their activity. ASE 2016.

marketplace platform *BizBuySell* (BBS) over an extensive period. BBS is one of the oldest and largest platform dedicated to facilitate business selling transactions in the US either directly or through brokers. Our dataset contains information on small businesses sold and for sale in the US from 2011 to 2018<sup>12</sup>. On the representativity of this datasets as compared to surveys, we estimate that among sold businesses, 23% to 25% were sold because the owner(s) retired, against 19% in ASE - 2016. Concerning the distribution of selling price in BBS, we find a mean value of 579K USD and a median value of 190K USD against respectively 682K USD and 95K USD in the PSID<sup>13</sup>. From this, we conclude that BBS displays a distribution of middle-sized small businesses, which are comparable to those observed in the data<sup>14</sup>.

We find important frictions on the business sale market. First, we find that only 10% to 20% of businesses for sale are actually sold. Second, regarding the distribution of the time on market (TOM), we find that the mean and the median TOM are respectively 232 days and 170 days for businesses sold between 2017 and 2018 as shown on the left panel of Figure 2. We also find (left panel of Figure 2) that the distribution of list price to market price displays considerable variation<sup>15</sup>. We further investigate the relationship between the probability to sell a business and the ratio of the list price to the market price and we find a decreasing and

<sup>12</sup>Available variables are selling/listing prices, cash-flows, gross-revenues, ebitda, number of employees, inventory, date of the first listing, date when the business was sold and main reason to sell. The oldest data had to be recovered through archives and brokers.

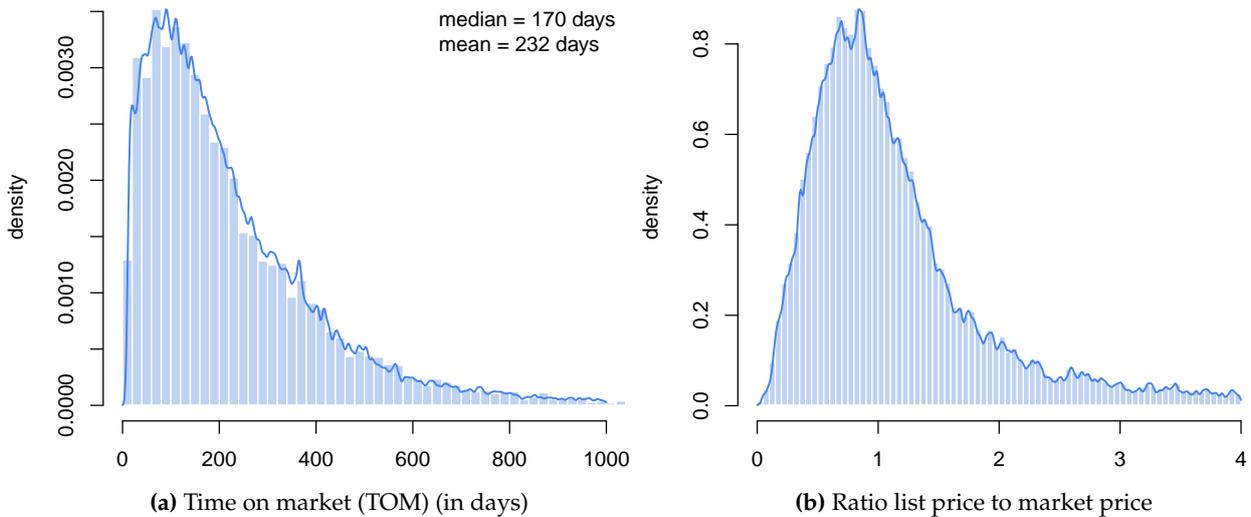
<sup>13</sup>Notice that we can not control for an entrepreneur's role on BBS platform. We therefore compare with all the business owners that sold a business in the PSID. Results are similar if we count only entrepreneurs (self-employed business owners). Finally, the comparison with the PSID is indicative, since the number of observations concerning sold businesses is 357 in the PSID (1990 to 2015) against more than 60000 in BBS.

<sup>14</sup>The low number of very-small businesses in the BBS data can be explained by the fact that entrepreneurs have to pay a monthly premium membership to list their ad, which might be constraining for very small businesses.

<sup>15</sup>We provide in the Appendix details on how we estimate the market price of businesses. Our  $R^2$  is 0.62.

convexe relationship<sup>16</sup>.

Based on the evidence above, we argue that the business sale market is subject to substantial frictions and mismatches that condition entrepreneurial exit and could lead to important economic losses. They are also important to explain the large fraction of new entrepreneurs that actually prefer to found a new business instead of purchasing an existing one. We argue that in their absence, all prospective entrepreneurs would prefer to purchase an existing business, which is less risky and provide larger returns. These frictions and mismatches are an important dimension of exit from entrepreneurship through selling the business.



**Figure 2.** Distribution of TOM and listing price in BBS.

## 2.4 Entrepreneurial entry and exit and age dynamics

Over the last decades, the entrepreneurship rate shows a gradual decline in the US. Some reasons, according to Pugsley et al. (2016), are the lower start-up rate of the younger generations who are the most likely to select into entrepreneurship and the changes in the growth rate of the working age population. Indeed, following the recent work of Liang et al. (2014), selection into entrepreneurship follows a hump-shaped pattern by age and peaks around 35 years old. This pattern is driven by two main forces. On the one hand, young individuals (less than 30 years old) tend to be more financially constrained than older ones and can not invest sufficient amount of wealth in a business. On the other hand, older (more than 55 years old) individuals find it less profitable to start a business due to the associated loss of accumulated worker advantages (work experience, firm specific promotions, responsibility status, etc.) and the potential risk of failure inherent to any businesses. However, using the Current Population Survey (CPS), we find that the mean age at which workers select into entrepreneurship tends

<sup>16</sup>We provide estimation results and fitted values in Appendix.

to increase from age 43 in 1996 to age 48 in 2016<sup>17</sup>. Concerning the question of purchasing or founding a business, table 3 shows that younger and older individuals are more likely to start businesses whereas middle-aged individuals tend to purchase an existing business more often. This U-shaped pattern of the likelihood of founding a business over the life-cycle could be again explained by financial constraints limiting credit availability for younger and older individuals.<sup>18</sup>

| Age        | Founded/started | Purchased | Inherited | Gift/Received |
|------------|-----------------|-----------|-----------|---------------|
| under 25   | 78.5            | 9.8       | 2.4       | 9.3           |
| 25 to 34   | 69.6            | 19.4      | 1.5       | 9.5           |
| 35 to 44   | 61.8            | 27.6      | 1.4       | 9.2           |
| 45 to 54   | 62.3            | 28.5      | 1.7       | 7.5           |
| 55 to 64   | 67.2            | 23.9      | 2.1       | 6.9           |
| 65 or over | 72.9            | 16.4      | 4.7       | 6.0           |

**Table 3.** Type of business acquisition and age of the entrepreneur. SBO 2007 using only early established business (ESB) (less than 3 years in operation).

The rise in the mean entry age and the lower start-up rate of younger generations together with the aging of entrepreneurs in the baby-boom cohort contribute to an overall aging of the entrepreneurial population. In figure 3, we display the mass of entrepreneurs (left panel) and workers (right panel) between 1989 to 2016 estimated from the SCF<sup>19</sup>. As compared to the density per age of workers, that remains roughly stable during that period, the fraction of entrepreneurs of age 60 and more rise from 18% in 1989 to 22% in 2004 and 30% in 2016<sup>20</sup>. Taking entrepreneurs of age 54 and more, those numbers drastically increase from 26% in 1989, to 34% in 2004 and 47% in 2016. That is, around half of the total population of entrepreneurs will retire in the next two decades. Using month-to-month CPS data, Figure 4 shows that the fraction of entrepreneurs of age 65 and more almost doubled in the 10 years since 2007.

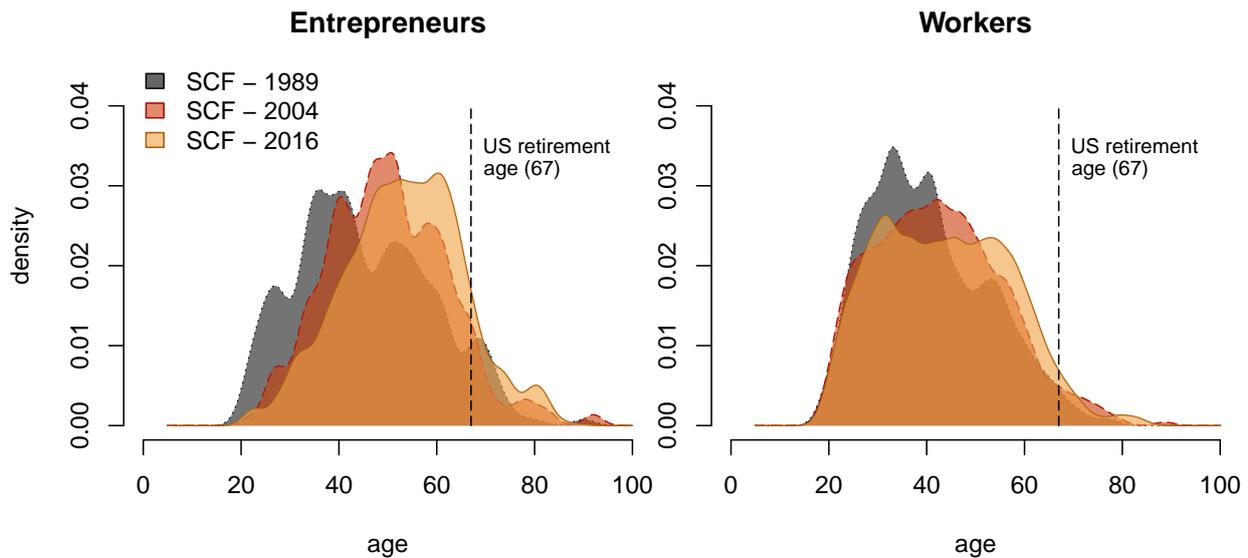
Finally, as stressed above, retirement has become a leading reason to cease the entrepreneurial activity. In figure 5, we display the age distribution of entrepreneurs who sold a business between 1990 to 2015, as observed in the PSID. We observe two peaks: one around age 45 and the

<sup>17</sup>Figure 13 in Appendix displays the density of workers selecting into entrepreneurship as observed in the CPS for these two periods. We also display results from the ASE dataset corroborating this feature.

<sup>18</sup>Entrepreneur's borrowing constraint as long been debated in the academic literature (see, among others, [Quadrini \(1999\)](#)). Here, we suggest that younger are on average poorer than middle aged individuals, which may limit the amount of collateral that they could engage in their businesses. For older, however, bank might be less likely to borrow due to the higher death risk.

<sup>19</sup>In the Appendix, we provide the corresponding distributions using the CPS.

<sup>20</sup>We also look at the mass of individuals not in the labor force (NILF). We find that the fraction of NILF at each age remains roughly stable between 1989, 2004 and 2016.



**Figure 3.** Density of entrepreneurs (left) and workers (right). *Source:* SCF (1989, 2004, 2016)

other around age 65, with the latter mostly explained by the retirement decision.

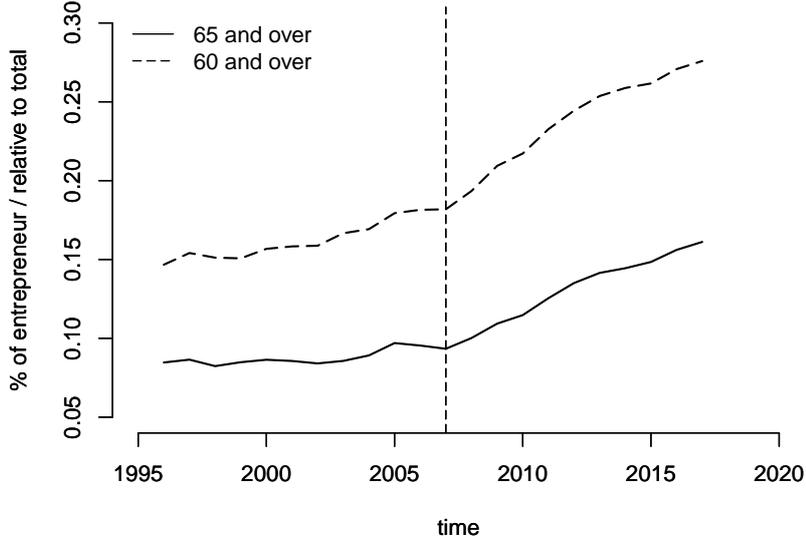
From the above findings, we conclude that the age and life-cycle dimensions are major factors in understanding entrepreneurial entry and exit dynamics and how businesses are transmitted, specially between generations, in the US. It is worth noting that if the fraction of entrepreneurs of age 60 and over represents between 28% and 30% of the total according to various datasets, they hold 45% of the total business value (SCF - 2016). Moreover, using 2016 ASE data, the fraction of entrepreneur of age 55 to 64 years and 65 years and more are respectively responsible for around 30% and 20% of total entrepreneurial employment. Together with business acquiring and selling frictions, the aging of the entrepreneurial population is a major macroeconomic concern.

### 3 Model

In this paper, a life-cycle incomplete markets heterogenous agents model with occupational choice and endogenous entrepreneurial business selling, buying and founding decisions is developed. Time is discrete and there is no aggregate uncertainty. The economy consists of a corporate sector, a unit measure of heterogenous agents, a business sale market and a government. In the following, the decisions of each type of agent in the model are described.

#### 3.1 Corporate Sector

The economy has two production sectors: a corporate one presented here and an entrepreneurial one discussed later. The two sectors share the available labor and capital supplies. The corporate output  $Y$  is produced by a single competitive representative firm using a Cobb-Douglas



**Figure 4.** Fraction of entrepreneurs at a given age. *Source:* basic monthly CPS.

technology with capital share  $\alpha \in (0, 1)$  and total factor productivity  $A$ , capital level  $K_c$  and labor  $L_c$ , such that:  $Y = F(K_c, L_c) = AK_c^\alpha L_c^{1-\alpha}$ . Capital depreciates at rate  $\delta$  in both sectors.

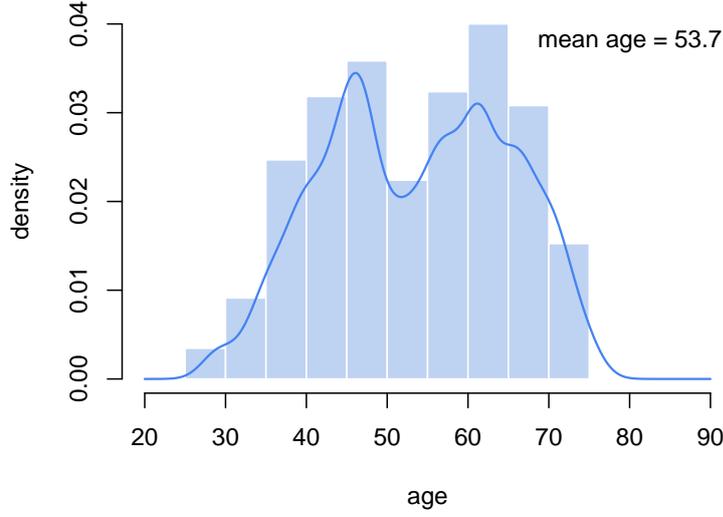
### 3.2 Agents

We use a stylized life-cycle setup with multi-generational probabilistic aging. Households live through four stages of life and the total population, of unit mass, is divided among 4 generations. The generations, indexed with  $j \in \{F, J, S, R\}$ , are called the Freshmen ( $F$ ), the Juniors ( $J$ ), the Seniors ( $S$ ) and the Retirees ( $R$ ). Each generation retains striking features of their counterparts in an actual life cycle economy. Only  $F$ ,  $J$  and  $S$  groups have access to the labor market while the  $R$  group are beyond the retirement age but can still manage a business. Each period, households in generation  $j \in \{F, J, S\}$  face a constant probability of aging and reaching the next generation  $P(j'|j)$ . At the same time, we assume that a fraction  $p_{die}$  of the retirees deceases and exit the model.<sup>21</sup>

Over the life-cycle, households belong in an occupation  $o \in \{o_e, o_w, o_r\}$ . Households in groups  $\{F, J, S\}$  can be entrepreneurs ( $o_e$ ) or occupied/unoccupied in the workforce ( $o_w$ ) whereas households at the retirement stage ( $j = R$ ) are either entrepreneur or retired ( $o_r$ ).

Households have preferences over consumption  $c$  and their utility is decreasing in the job search effort  $s_w$ . Retired households also face an additional utility cost  $u_R$  when operating a business, in order to translate the difficulty of still being in activity in old age. Finally, following the recent paper by [Hurst and Pugsley \(2015\)](#), we allow for non-pecuniary benefits on top of

<sup>21</sup>This assumption is widely used in the literature, see [Sommer and Sullivan \(2017\)](#) for an application in housing models or [Cagetti and De Nardi \(2006\)](#) in a related literature.



**Figure 5.** Distribution of businesses sold by age of the entrepreneur. *Source:* PSID 1990 - 2015.

the business activity taking the form of an additional utility component  $u_E$ .<sup>22</sup> We define:

$$u(c, j, o, s_w) = \mathcal{U}(c, j, o) - v(s_w)$$

$$\text{with } \mathcal{U}(c, j, o) = u(c) + \mathbb{1}_{o=o_e}(u_E - \mathbb{1}_{j=R}u_R)$$

where  $u(\cdot)$  satisfies the usual assumptions and  $v(\cdot)$  is a decreasing and convex function.

Depending on its occupation, a household can possess liquid and/or illiquid assets. Liquid assets are akin to savings and are noted  $a$ . Illiquid business assets, noted  $k$ , are used to produce the consumption good with the entrepreneurial technology.

The state space for an entrepreneur are saving  $a$ , business capital  $k$ , and  $\mathbf{x}_e = \{j, m, z\}$ , where  $m = \{0, 1\}$  indicates whether the business is mature,  $z$  is an idiosyncratic business shock and  $j$  is the entrepreneur's age group. Entrepreneurs are not permitted to possess multiple firms. Similarly, the state space for a worker is  $a$ , and  $\mathbf{x}_w = \{j, y, u, \iota\}$ , with  $y$  the worker's productivity,  $u$  the employment status ( $u = 1$  for unemployed,  $u = 0$  for employed) and  $\iota$  an indicator of whether workers have an entrepreneurial ability. We use the generic term *worker* to designate any individual in the workforce.

An agent's income depends on her occupation. We note  $\mathcal{Y}(u, j, y)$  a worker's income. The entrepreneurial income is denoted  $\pi(z, k)$  and is defined as follows:

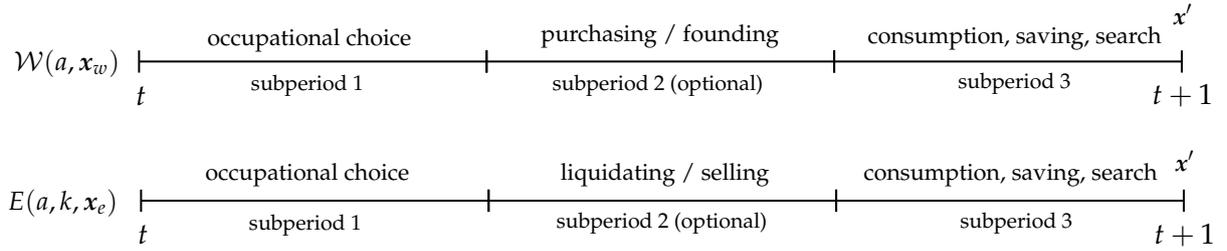
$$\pi(z, k) = \max_n f(z, k, n) - \omega n - \delta k \quad (1)$$

where  $n$  is the entrepreneurial labor demand and  $\omega$  denotes the wage level in the economy.

<sup>22</sup>This component is used to match the data and based on the fact that individuals have a preference for being self-employed. In a related model, [Chen et al. \(2018\)](#) also incorporate such preference.

### 3.3 Dynamic Problem

We decompose an agent's intra-period decision process into a sequence of three subperiods. In the last subperiod, the consumption-saving-search and investment problems are tackled. In the middle subperiod, the buying and selling problems are addressed contingent on occupational changes and the maturity of a business.<sup>23</sup> Finally, in the first subperiod occupational choices are made. Given that  $\mathcal{W}(a, x_w)$  and  $E(a, k, x_e)$  are respectively the general value function of a worker and an entrepreneur, Figure 6 summarizes this decomposition. Following this decomposition, the problem is solved backwards and we detail below the problem in each subperiods.



**Figure 6.** Intra-period timing of an individual.

#### 3.3.1 The last subperiod: Consumption-Saving-Search (CSS) problem

Depending on choices made in the previous subperiods, CSS decisions can be split into 4 types in the last subperiod: those of workers continuing or exiting their activity and those of entrepreneurs continuing or exiting their activity. Continuing unemployed workers and exiting entrepreneurs search for a job with effort  $s_w$  and succeed with probability  $\pi(s_w)$ . Contrastingly, employed workers are subject to an exogenous lay off rate  $\eta$ , such that  $P(u = 1 | u = 0) = \eta$ .<sup>24</sup> For simplicity, continuing workers are subject to a no-borrowing constraint. An exiting worker quits the workforce for an entrepreneurial activity and, similarly to a continuing entrepreneur, can borrow in order to invest in their business asset  $k$  as long as a minimum amount  $\theta k$  is invested using their own wealth. Thus those entrepreneurs are subject to the following borrowing constraint:

$$a' \geq -(1 - \theta)k \quad (2)$$

An indebted entrepreneur faces an interest  $r_b(m)$  that depends on the maturity of her business,  $m$  such as:  $\tilde{r}(m) = \mathbb{1}_{a' \geq 0} r_s - \mathbb{1}_{a' < 0} r_b(m)$

<sup>23</sup>Only mature firms ( $m = 1$ ) can be sold on the business sale market. This is to reflect the fact that the average age of sold businesses is much higher than the average age of all firms.

<sup>24</sup>For the sake of brevity, we include both the job finding and the lay off probabilities in the agent's expectation and do not detail it further.

**Continuing entrepreneurs** An entrepreneur keeping her business chooses the next period illiquid capital  $k'$  and saving  $a'$  given her income  $\pi(z, k)$ . Investing or disinvesting in the illiquid capital are subject to a capital adjustment cost noted  $\mathcal{C}(k, k')$ . The CSS problem of this entrepreneur is thus:

$$E^c(a, k, \mathbf{x}_e) = \max_{c > 0, a' \geq -(1-\theta)k', k' \geq 0} \mathcal{U}(c, j, o_e) + \beta \mathbb{E}_{z', m', j' | z, m, j} E(a', k', \mathbf{x}'_e) \quad (3)$$

$$s.t. \quad c + a' + k' = (1 + \tilde{r}(m))a + (1 - \tau_e)\pi(k, z) + k - \mathcal{C}(k, k') \quad (4)$$

with  $E^c$  the subperiod specific value function of this entrepreneur and  $\tau_e$  the entrepreneurial income tax rate. Note that such an entrepreneur solves a portfolio choice problem.

**Exiting entrepreneurs** When exiting, an entrepreneur has to choose savings  $a'$  subject to the no-borrowing constraint. Depending on whether the entrepreneur is exiting entrepreneurship by selling ( $s = 1$ ) or liquidating ( $s = 0$ ) the business, we have:

$$E^e(a, k, \mathbf{x}_e, s) = \max_{c > 0, a' \geq 0} \left[ \mathcal{U}(c, j, o_e) + \max_{s_w} \left\{ -v(s_w) + \beta \mathbb{E}_{y', j', u' | j, s_w} \mathcal{W}(a', \mathbf{x}'_w) \right\} \right] \quad (5)$$

$$s.t. \quad c + a' = (1 + \tilde{r}(m))a + (1 - \tau_e)\pi(k, z) + sq(k) + (1 - s)(k - \mathcal{C}(k, 0)) \quad (6)$$

with  $E^e$  the subperiod specific value function of this entrepreneur. Liquidating is identical to adjusting the business capital to zero by fully paying the corresponding adjustment cost  $\mathcal{C}(k, 0)$ .<sup>25</sup> Alternatively, by successfully selling the business the entrepreneur recovers the total amount  $q(k)$ .

**Continuing workers** Such a worker has to choose savings  $a'$  subject to the no-borrowing constraint and solves:

$$\mathcal{W}^c(a, \mathbf{x}_w) = \max_{c \geq 0, a' \geq 0} \left[ \mathcal{U}(c, j, o_w) + \max_{s_w} \left\{ -v(s_w u) + \beta \mathbb{E}_{u', j', y' | s_w, j, y} \mathcal{W}(a', \mathbf{x}'_w) \right\} \right] \quad (7)$$

$$s.t. \quad c + a' = (1 + r_s)a + \mathcal{Y}(u, j, y)(1 - \tau_w) \quad (8)$$

with  $\mathcal{W}^c$  the subperiod specific value function of this worker and  $\tau_w$  the worker income tax rate. Note that the search search efforts here is only relevant for an unemployed worker ( $u = 1$ ).

**Exiting workers** An exiting worker enters entrepreneurship with business assets  $k'$  in the next period, either by purchasing an existing mature business ( $b = 1$  and  $m' = 1$ ) and paying

---

<sup>25</sup>For simplicity, we assume the same cost structure here as in the disinvestment case. The model can easily be extended to account for a different cost structure between disinvestment and liquidation.

the total amount  $q(k')$  or by founding a new business ( $b = 0$  and  $m' = 0$ ) and paying the adjustment cost  $\mathcal{C}(0, k')$ .<sup>26</sup> This worker solves:

$$\mathcal{W}^e(a, k', \mathbf{x}_w, b) = \max_{c \geq 0, a' \geq -(1-\theta)k'} \mathcal{U}(c, j, o_w) + \beta \mathbb{E}_{z', j' | j} E(a', k', \mathbf{x}'_e) \quad (9)$$

$$s.t. \quad c + a' = (1 + r_s)a + \mathcal{Y}(u, j, y)(1 - \tau_w) - bq(k') - (1 - b)(k' + \mathcal{C}(0, k')) \quad (10)$$

with  $\mathcal{W}^e$  the subperiod specific value function.

### 3.3.2 The middle subperiod: acquisition and selling problems

**The selling problem** An entrepreneur with a mature business ( $m = 1$ ) can try to sell it on the business sale market. A buyer is found with probability  $P_s$ . Otherwise, the entrepreneur chooses whether to liquidate or to continue the business. In the end, the following problem is solved:

$$\mathcal{S}(a, k, \mathbf{x}_e) = P_s E^e(a, k, \mathbf{x}_e, 1) + (1 - P_s) \max \left\{ E^e(a, k, \mathbf{x}_e, 0), E^c(a, k, \mathbf{x}_e) \right\} \quad (11)$$

with  $\mathcal{S}(a, k, \mathbf{x}_e)$  the subperiod specific value function for this problem.

**The founding and purchasing problem** Depending on whether a worker is currently buying ( $b = 1$ ) or founding ( $b = 0$ ) a business, the problem of the future entrepreneur is to choose the optimal capital size  $k'$ :

$$\widetilde{\mathcal{W}}(a, \mathbf{x}_w, b) = \max_{k'} \mathcal{W}^e(a, k', \mathbf{x}_w, b) \quad (12)$$

A worker trying to buy a business has a probability  $P_b$  of finding a seller. Otherwise, she chooses whether to found a new business or to continue being a worker. In the end, the following problem is solved:

$$\mathcal{B}(a, \mathbf{x}_w) = P_b \widetilde{\mathcal{W}}(a, \mathbf{x}_w, 1) + (1 - P_b) \left\{ \widetilde{\mathcal{W}}(a, \mathbf{x}_w, 0), \mathcal{W}^c(a, \mathbf{x}_w) \right\} \quad (13)$$

with  $\mathcal{B}(a, \mathbf{x}_w)$  the subperiod specific value function for this problem.

### 3.3.3 The first subperiod: occupational choice and exit strategy

**Worker** A worker starts the period with states  $\{a, \mathbf{x}_w\}$  and, provided she has an entrepreneurial ability (i.e.  $\iota = 1$ ), chooses whether to try to purchase an existing business, to found a new business or to stay a worker.<sup>27</sup> Trying to purchase a business implies the utility cost of searching a

<sup>26</sup>As for the disinvestment and liquidating case, we assume for simplicity that the cost of founding a new business is similar to the adjustment cost of investing.

<sup>27</sup>The entrepreneurial ability follows a first order Markov process with two states: possessing ( $\iota = 1$ ) or not possessing ( $\iota = 0$ ) the ability. An alternative specification would be to set an exogenous probability of drawing an entrepreneurial ability each period. However, our specification allows for a persistent entrepreneurial ability that generates higher saving rates and brings the model closer to the data.

business to buy  $u_b$ . In the end, the following problem is solved:

$$\mathcal{W}(a, \mathbf{x}_w) = (1 - \iota)\mathcal{W}^c(a, \mathbf{x}_w) + \iota \max \left\{ \mathcal{B}(a, \mathbf{x}_w) - u_b, \widetilde{\mathcal{W}}(a, \mathbf{x}_w, 0), \mathcal{W}^c(a, \mathbf{x}_w) \right\} \quad (14)$$

**Entrepreneur** An entrepreneur starts the period with states  $\{a, \mathbf{x}_e\}$  and decides to sell her business, liquidate the business or continue the business. In the end, the following problem is solved:

$$E(a, k, \mathbf{x}_e) = \max \{ \mathcal{S}(a, k, \mathbf{x}_e) - u_s, E^e(a, k, \mathbf{x}_e, 0), E^c(a, k, \mathbf{x}_e) \} \quad (15)$$

with  $u_s$  the utility cost of searching for a buyer.

### 3.4 Demography and bequest

Upon death, an individual can leave a bequest to its descendant. An individual in the Retiree group has a probability  $p_{die} = P(j' = F | j = R)$  of dying and value her descendant's value function with a degree of altruism,  $\Lambda$ . We assume the descendant is reborn in the model as an unemployed worker ( $u = 1$ ) with productivity  $y$  and entrepreneurial ability  $\iota$  drawn from their invariant distribution. Finally, bequests are taxed with the estate tax rate  $\tau_a$ . Hence, for an individual with next period value  $V(\mathbf{x})$  in the last age bracket, we define:

$$V(\mathbf{x}) = (1 - p_{die})V(\mathbf{x}') + p_{die}\Lambda\mathbb{E}_{y', \iota'} W(a(1 - \tau_a), j = F, y', 1, \iota') \quad (16)$$

### 3.5 Government

The government collects revenue from income taxes on labor earnings, entrepreneurial earnings, pensions and unemployment compensation, as well as from the product of estate taxation. Government expenditures are an exogenous government spending  $G$  proportional to aggregate output,  $G = \bar{G}Y$ , unemployment compensation and pensions, such that government budget is:

$$\int_{\mathbf{x}_w} \mathcal{Y}(u, y, j) \tau_w d\Gamma(\mathbf{x}_w) + \int_{\mathbf{x}_e} \pi(z, k) \tau_e d\Gamma(\mathbf{x}_e) = \bar{G}Y + \int_{\mathbf{x}_w} \mathbb{1}_{u=1, j \in \{F, J, S\}} \rho w y j + \mathbb{1}_{j=R} w j d\Gamma(\mathbf{x}_w) \quad (17)$$

with  $\Gamma$  a measure over agents of the specified type.

### 3.6 Business sale market

On the business sale market, business sale brokers behave competitively and earn zero profit. The behavior of the business sale broker is fairly mechanical in our model and is used as a modeling device to equate the quantity of businesses sold to the quantity of businesses purchased. To do so, the business broker buys all businesses sold by entrepreneurs and sells them to buyers.<sup>28</sup> In our baseline model, we make the simplifying assumption that the total sale value  $q(k)$

<sup>28</sup>This assumes that once sold, a business can be broken into sub-businesses or, alternatively, can be merged with other businesses.

of a business is proportional to the business capital  $k$ , such that  $q(k) = qk$ .<sup>29</sup>

### 3.7 Equilibrium

**Definition.** A Stationary Recursive Equilibrium in this economy consists of a set of agent's decision rules, factor prices  $\{w, r\}$ , tax rate  $\tau_w$ , a level of business sale value,  $q$ , and a distribution  $\Phi(\mathbf{x})$  of agents, such that:

- The decision rules  $a'(\mathbf{x}), k'(\mathbf{x}), b(\mathbf{x}_w), s(\mathbf{x}_e), s_w(\mathbf{x}), n(\mathbf{x}_e)$ , solve the agent's optimization problem.
- The distribution  $\Phi(\mathbf{x})$  is stationary.
- The government budget constraint in (17) is balanced with  $\tau_w$ .
- The labor market clears and total labor demand by the corporate sector and the entrepreneurial sector equate households' labor supply. Wage is determined by the marginal productivity of labor in the corporate sector.

$$L_c + \int_{\mathbf{x}} \mathbb{1}_{\{o=e\}} n(\mathbf{x}) d\Gamma(\mathbf{x}) = \int_{\mathbf{x}} \mathbb{1}_{\{o=w\}} d\Gamma(\mathbf{x}) \quad (18)$$

- The capital market clears. Corporate capital and the total entrepreneurial capital equate total agent's networth in the economy. The interest rate is determined by the marginal productivity of capital in the corporate sector.<sup>30</sup>

$$K_c + \int_{\mathbf{x}} \mathbb{1}_{\{o=e\}} k(\mathbf{x}) d\Gamma(\mathbf{x}) = \int_{\mathbf{x}} a(\mathbf{x}) d\Gamma(\mathbf{x}_{o_w}) \quad (19)$$

- The business sale market clear, total capital sold in the economy equate total capital purchased. Average business sale value  $q$  clears the market.<sup>31</sup>

$$\int_{\mathbf{x}} \mathbb{1}_{b(\mathbf{x})=1} k(\mathbf{x}) d\Gamma(\mathbf{x}) = \int_{\mathbf{x}} \mathbb{1}_{s(\mathbf{x})=1} k(\mathbf{x}) d\Gamma(\mathbf{x}) \quad (20)$$

- By Walras' Law, the goods market also clears at the equilibrium.

This problem has no analytical solution and has to be solved numerically. Two major problems arise in our setup. First, the dimensionality of the problem with two-assets is itself a challenge and one would refer to fast optimization method in order to solve the model. Second, due to the presence of both discrete (occupational choice) and continuous choices, FOC

<sup>29</sup>An alternative specification would be to assume a submarket for each capital size  $k$  and to clear all the submarkets. However, we prefer to keep the model as simple as possible.

<sup>30</sup>Profit maximization implies:  $r = A\alpha \left(\frac{L_c}{K_c}\right)^{1-\alpha} - \delta$  and  $w = A(1-\alpha) \left(\frac{K_c}{L_c}\right)^\alpha$ , with  $w$  and  $r$  the wage and interest rates, which by a no arbitrage condition are identical in the entrepreneurial sector.

<sup>31</sup>Alternatively, we could let  $q(k)$  varies with size  $k$  (i.e.  $\mathbf{q} \equiv \{q(k_1), q(k_2), \dots, q(k_K)\}$ ). However, this substantially increases the computational complexity of the model while this does not seem particularly relevant for the main findings of the paper.

are no longer sufficient (while still necessary). To encompass these issues, we modified the recent implementation of *DC-EGM* as introduced in [Iskhakov et al. \(2017b\)](#) and extended it to multiple assets as in [Drue Dahl and Jørgensen \(2017\)](#). While making the model tractable, this also substantially increases the speed of the whole algorithm.<sup>32,33</sup>

**Taste shocks** Following [Iskhakov et al. \(2017b\)](#), we introduce taste shocks to tackle the issue of kinks in the value function. Denoting the discrete worker's choice  $d \in D \equiv \{d_b, d_f, d_c\}$  of either purchasing a business, founding a business or continuing as a worker, we define the perturbed value function of a worker as:

$$W_\sigma(a, \mathbf{x}_w, \epsilon_w) = \max_{d \in D} \left\{ \underbrace{\mathcal{B}(a, \mathbf{x}_w) - u_b}_{w_\sigma(d_b, a, \mathbf{x}_w)} + \sigma_w \epsilon_w(d_b), \underbrace{\tilde{W}(a, j, 0)}_{w_\sigma(d_f, a, \mathbf{x}_w)} + \sigma_w \epsilon_w(d_f), \underbrace{W^c(a, \mathbf{x}_w)}_{w_\sigma(d_c, a, \mathbf{x}_w)} + \sigma_w \epsilon_w(d_c) \right\} \quad (21)$$

where  $\epsilon_w$  is a *vector* of IID extreme value type I taste shock across the discrete choices and  $\sigma_w$  is a scale parameter of the Type I Extreme value distribution, proportional to the variance of the taste shock. We furthermore assume that taste shocks are contemporaneously independent across different discrete choices but are also intertemporally independent. Under the perturbed problem, the expected value of  $W_\sigma(a, \mathbf{x}_w, \epsilon_w)$ , before the realization of the taste shock, can be derived in a closed form as

$$\mathbb{E}[W_\sigma(a, \mathbf{x}_w)] = \sigma_w \log \left( \sum_{d \in D} \exp \left\{ \frac{w_\sigma(d, a, \mathbf{x}_w)}{\sigma_w} \right\} \right) \quad (22)$$

Similarly for an entrepreneur, we define the set of discrete choice  $g \in \mathcal{G} \equiv \{g_s, g_l, g_c\}$  over selling or liquidating the business or pursuing the entrepreneurial activity. We write the perturbed problem of an entrepreneur as:

$$E_\sigma(a, k, \mathbf{x}_e, \epsilon_e) = \max_{g \in \mathcal{G}} \left\{ \underbrace{\mathcal{S}(a, k, \mathbf{x}_e) - u_s}_{e_\sigma(g_s, a, k, \mathbf{x}_e)} + \sigma_e \epsilon_e(g_s), \underbrace{E^e(a, k, \mathbf{x}_e, 0)}_{e_\sigma(g_l, a, k, \mathbf{x}_e)} + \sigma_e \epsilon_e(g_l), \underbrace{E^c(a, k, \mathbf{x}_e)}_{e_\sigma(g_c, a, k, \mathbf{x}_e)} + \sigma_e \epsilon_e(g_c) \right\} \quad (23)$$

And as for a worker, we can derive the closed form for the expected value as:

$$\mathbb{E}[E_\sigma(a, k, \mathbf{x}_e, \epsilon_e)] = \sigma_e \log \left( \sum_{g \in \mathcal{G}} \exp \left\{ \frac{e_\sigma(g, a, k, \mathbf{x}_e)}{\sigma_e} \right\} \right) \quad (24)$$

Under this specification, taste shocks play two roles. First, it facilitates the numerical computation by smoothing the next period expected value over discrete choice and limit the propagation of the kinks over the grids of assets. Second, it allows for unobservable heterogeneity in

<sup>32</sup>Our model equilibrium is solved on average in ten minutes for reasonably fine grids for both illiquid capital and liquid net worth using a reasonably fast desktop computer.

<sup>33</sup>Appendix A provides detail on the equilibrium definition and the implemented numerical algorithm.

the choice of becoming a worker (resp. an entrepreneur) that fit recent finding in the literature, as already pointed out. The larger are  $\sigma_w$  and  $\sigma_e$  and the larger are the variances of the taste shocks and the smoother are the value functions.

## 4 Parameterization

We have three broad goals in choosing the parameters for the model. We first need to replicate the occupational choice between the two alternatives of self-employment and paid-employment. Second, we seek a calibration of the exogenous stochastic process for labor earnings, which fits the lifetime pattern as observed in the data. Finally, in order to obtain quantitatively realistic business dynamics, our model must generate realistic exit transition from entrepreneurship. Of particular importance are the distributions of business buyers and business sellers, as well as the fraction of entrepreneurs stopping their activity each year.

**Preferences** We use a CRRA utility over consumption with parameter  $\sigma = 1.5$ . We use a power function to describe the disutility of search with search elasticity equals to 2. Finally, we set the altruistic parameter  $\Lambda = 1$  while the two preference parameters  $u_R$  and  $u_E$  are estimated endogenously. We use:

$$u(c, j, o, s_w) = \mathcal{U}(c, j, o) - v(s_w) = \frac{(c^{1-\sigma} - 1)}{1 - \sigma} + \mathbb{1}_{o=o_e}(u_E - \mathbb{1}_{j=R}u_R) - s_w^2 \quad (25)$$

**Labor income and labor frictions** In the model, labor income allows workers to accumulate savings at different rates, especially important for the decision to switch toward entrepreneurship.<sup>34</sup> We model labor income as

$$\mathcal{Y}(u, j, y) = \mathbb{1}_{j \in \{F, J, S\}}((1 - u) + u\rho)wyh(j) + \mathbb{1}_{j=R}h(j) \quad (26)$$

where  $\rho$  is the unemployment replacement rate that we set to 0.4 as in [Shimer \(2005\)](#) and  $h(R)$  defines the retirement pension that we fix to 40% of the average income. We define labor earnings as a function of an age-dependent component, the wage level and a persistent stochastic process for labor productivity, such that:

$$\log(\text{earnings}_{i,t}) = \log(w_t) + \log(y_{i,t}) + \log(h(j)_{i,t}) \quad (27)$$

$$\log(y_{i,t}) = \rho_y \log(y_{i,t-1}) + \epsilon_{i,t}^y; \quad \epsilon_{i,t}^y \sim \mathcal{N}(0, \sigma_y) \quad (28)$$

We model the age-dependent component in order to replicate the average lifetime earning profile within each earning percentile as observed in the PSID. Table 4 displays the values for  $h(j)$ .<sup>35</sup>

<sup>34</sup>Particularly, two main saving motives arise in the model. A precautionary saving due to the inherent productivity risk and an accumulation motive in order of becoming a future entrepreneur.

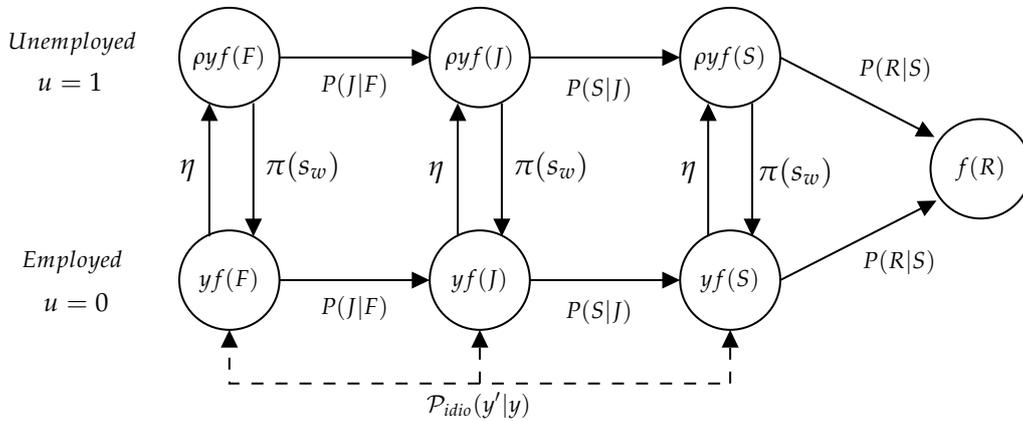
<sup>35</sup>This is also in line with the paper by [Karahan and Ozkan \(2013\)](#), among other paper in the literature.

| Age bracket | [20:35[ | [35:50[ | [50:65[ | [65:+[          |
|-------------|---------|---------|---------|-----------------|
| $j$         | F       | J       | S       | R               |
| $h(j)$      | 0.75    | 1.0     | 1.15    | 40% avg. income |

**Table 4.** Age-dependent labor income component.

The probability of aging from  $j$  to  $j'$  for any  $j \neq R$  is set to 0.067, and the probability of dying  $p_{die}$  is set to 0.05. The logarithm of productivity  $y$  follows an AR(1) process with autocorrelation  $\rho_y$  and standard deviation  $\sigma_y$ . We discretize the process by fixing the value of  $\rho_y = 0.96$  and adjusting the variance of  $\sigma_y$  to generate an earning Gini of 0.36.

Unemployment in the model is the result of labor frictions. Unemployed individuals have to search for a job opportunity with effort  $s_w$  and find a job with probability  $P(u' = 0|u = 1, s_w) = \pi(s_w)$ , with  $\pi(s_w) = 1 - e^{-\kappa s_w}$ . Where  $\kappa$  is a matching parameter that translate frictions in the labor market. Because  $\rho < 1$ , unemployed workers are assumed to switch to paid-employment once a job is found. In the model, job search intensity react to a change in the wage level  $\omega$ , generating a correlation between unemployment rate and wages (possibly negative or positive). This allows us to study the quantitative effect of varying labor demand on the equilibrium unemployment rate. Finally, employed workers are subject to an exogenous average lay off rate  $P(u' = 1|u = 0) = \eta$  of 2.5% per year. Figure 7 summarizes the labor process over the life-cycle depending on the employment status.



**Figure 7.** Labor income process of a worker, either unemployed or employed.

**Business income** We discretize business shocks with 4 states and we assume that entrepreneurial income and labor income evolve independently. We set the first value to  $z_1 = 0$  and we use the Tauchen's method to set the remaining 3 values and the corresponding transition matrix.<sup>36</sup> The probability of switching from a state  $z$  to another state  $z'$  is weighted by the probability of

<sup>36</sup>This state  $z_1$  is treated as an exogenous shock faced by the entrepreneur that forces her to exit.

not switching to the state  $z_1$ , such that

$$P(z'|z) = (1 - P(z_1|z))\Pi(z'|z) \quad (29)$$

where  $\Pi(z'|z)$  is the probability transition induced by the business shock persistency  $\rho_z$  and its variance  $\sigma_z$ . We set the value  $\rho_z = 0.9$  and choose endogenously the value for the variance  $\sigma_z$  and the probability  $P(z_1|z) = p_{z_1} \forall z$ .

The maturity of the business,  $m$ , affects the interest rate charged on the entrepreneurs. Moreover, only mature businesses can be sold in our economy, which generates a link between the maturity  $m$  and the type of acquisition. We use the 2003 SSBF data in order to estimate the effect of the type of acquisition on the interest rate charged on the businesses as well as on the credit limit. To do so, we run the following regression on the sample of early stage firms (owned in the last 5 years) and all firms:

$$int.rate_i = \alpha + \beta D(purchased)_i + X_i + u_i \quad (30)$$

where  $D(purchased)_i$  is a dummy variable indicating whether a business has been purchased by the current owner or founded.  $X_i$  is a vector of controls for both the business (employment and capital size, sector etc.) and the owners (net worth, age, education, sex, entrepreneurial experience, past default history etc.) and  $u_i$  is an error term. Table 5 summarizes the results.

|                | Interest rate (%)  |                          | Credit limit                 |                           |
|----------------|--------------------|--------------------------|------------------------------|---------------------------|
|                | All firms<br>(1)   | Young firms (<5y)<br>(2) | All firms<br>(3)             | Young firms (<5y)<br>(4)  |
| Purchased      | -0.58***<br>(0.13) | -1.59***<br>(0.28)       | 242,690.10***<br>(53,245.24) | 58,917.96<br>(208,220.70) |
| Controls       | Yes                | Yes                      | Yes                          | Yes                       |
| N              | 8,919              | 1,811                    | 7,212                        | 1,285                     |
| R <sup>2</sup> | 0.12               | 0.31                     | 0.16                         | 0.44                      |

Notes: p<0.1; \*\*p<0.05; \*\*\*p<0.01. In parenthesis: std. deviation.

**Table 5.** Interest rate, credit limit and type of acquisition.

We find a significant relationship between the type of acquisition and the interest rate. Recent owners (with less than 5 years of operation) having purchased an existing business are charged, on average, an interest rate 1.59% smaller than those who founded their business. Taking the sample of all firms, we consistently find that purchasers are charged an interest rate smaller than founders, but the effect is lower than previously found. We interpret these findings in the following way: financial intermediary lower the premium charged on their loans when they are able to infer business characteristics over a longer period. On the credit limit, we also find similar results: taking the sample of all firms, purchased businesses can, on

average, borrow 242 thousand dollars more than founded businesses, but this is not statistically significant for businesses recently owned.

Following these findings, we set the value for the interest rate charged on the businesses as follows:  $r_b(m) = r_s + \phi_s + \phi_m \mathbb{1}_{m=0}$ , where  $\phi_s$  is a wedge common to all businesses, while  $\phi_m$  is the interest rate charged on immature business. We set  $\phi_s = 2\%$  and, according to the point estimate, we set  $\phi_m = 1.6\%$ .<sup>37</sup>

Finally, we specify the entrepreneurial technology as

$$f(k, z, n) = z(k^\gamma(1+n)^{1-\gamma})^\nu \quad (31)$$

where we fix,  $\nu = 0.84$ , and,  $\gamma = 0.88$ . The depreciation rate  $\delta$  is assumed to be the same as in the corporate sector and is equal to 0.06.

**Adjustment costs** In the model, adjustment cost plays a crucial role, since it determines the tradeoff between buying or founding a new business, as well as liquidating the business assets or selling the business, together with the matching frictions imposed by  $P_s$  and  $P_b$ . For the sake of simplicity, we make the assumption of non-convex fixed costs in our baseline economy, for both investment and disinvestment.<sup>38</sup> We use the following cost structure:

$$\begin{cases} \mathcal{C}(k, k') = \tau_{up}(k' - k) & \text{if } k' > k \\ \mathcal{C}(k, k') = \tau_{down}(k - k') & \text{if } k' < k \end{cases} \quad (32)$$

where we fix the value  $\tau_{up} = 0.04$  and we calibrate endogenously the value of  $\tau_{down}$ .<sup>39</sup> We perform sensitive analysis on the effect of such parameters in appendix.

**Matching probabilities** We assume a simple matching structure on the business sale market which is summarized by two key probabilities:  $P_s$ , the probability that a business seller find a buyer, and  $P_b$  the probability that a buyer find a seller. Those probability could feature matching frictions arising during the transaction and the bargaining processes, or the frictions arising due to mismatch between buyer's entrepreneurial idea and seller's business activity.

As our model period is the year, we set the value of  $P_s$  to the fraction of reported business for sale on the *BizBuySell* platform that are not sold. We estimate that around 20% - 50% of

---

<sup>37</sup>In the baseline model, we do not take in account different credit limits. However, an extended version of the model accounting for this features could generate additional misallocation effects concomitantly to the destruction of older mature businesses.

<sup>38</sup>We extend our model to account for convex non linear adjustment costs in an alternative model specification.

<sup>39</sup>The non-symmetric cost structure that we choose is motivated by the observation made by [Tan \(2017\)](#) using the KFS data. In his paper,  $\tau_{up} = 0.034\%$  and  $\tau_{down} = 0.32$ . However, his model does not account for matching frictions that can also limit the entry of new entrepreneurs. As we will fix the value of  $P_s$ , the value of  $\tau_{down}$  helps to match the fraction of entrepreneurs who sold their business in equilibrium.

the businesses for sale are sold after 1 year, depending on the business sector. We therefore set  $P_s = 0.4$ . As we can not directly observe the matching friction on the buyer's side, we endogenously calibrate the value of  $P_b$ .

**Other parameters** It remains the calibration of parameters related to the production technology in the corporate sector. We set the capital share  $\alpha = 0.33$ . The tax parameters related to entrepreneurial business income  $\tau_e$  is set to 11%, the estate taxation to 16% and government spending share  $\bar{G} = 0.19$ , following [Cagetti and De Nardi \(2009\)](#). Concerning the transition probability of the entrepreneurial ability process  $\iota$ , we restrict,  $P(\iota' = 1 | \iota = 0) = 1 - p_{z_1}$ , and,  $P(\iota' = 0 | \iota = 1) = p_{z_1}$ . The borrowing constraint parameter  $\theta$  is also picked to match model moments to data. Finally, we choose the variance of the type I extreme value shocks to  $\sigma_w = \sigma_e = 0.01$ . [Table 6](#) summarizes the value of the fixed parameters.

| Parameter                     | Symbol                        | Value                | Source   |
|-------------------------------|-------------------------------|----------------------|--|
| Preferences                   | $\sigma, \Lambda$             | 1.5, 1               | Normalization  |
| Earning process               | $h(j), \rho_y, \sigma_y$      | Table 4, 0.96, 0.5   | <a href="#">Karahan and Ozkan (2013)</a> , Earnings Gini 0.38 (PSID) |
| Lay off rate                  | $\eta$                        | 2.5%                 | Avg. rate in CPS (2001-2015)   |
| Replacement rate              | $\rho$                        | 40%                  | <a href="#">Shimer (2005)</a>  |
| Business shock process        | $\rho_z$                      | 0.9                  | Assumption   |
| Interest rate wedges          | $\phi_s, \phi_m$              | 2%, 1.6%             | Borrowing wedge of 2%, SSBF (2003)                                   |
| Production technologies       | $\nu, \gamma, \delta, \alpha$ | 0.84, 0.88, 6%, 0.33 | <a href="#">Cagetti and De Nardi (2006)</a> , US capital share       |
| Government parameters         | $\tau_e, \tau_a, \bar{G}$     | 11%, 16%, 19%        | <a href="#">Cagetti and De Nardi (2009)</a>                          |
| Taste shocks                  | $\sigma_e, \sigma_w$          | 0.01, 0.01           | Assumption   |
| Investment adj. cost          | $\tau_{up}$                   | 4%                   | <a href="#">Tan (2017)</a>   |
| Seller's matching probability | $P_s$                         | 40%                  | fraction of business sold (per y.)                                   |

**Table 6.** Fixed parameters.

Our calibration leaves us with a set of parameters  $\{u_R, u_E, \beta, \theta, P_b, \tau_{down}, \kappa, p_{z_1}\}$  that are picked to match some moments of interest observed in the data. We target: a share of entrepreneurs relative to the working age population of 10-11% (self-employed individuals), a share of new entrepreneurs buying their business of about 15-20%, a share of business sold relative to exiting entrepreneurs of about 10-15%, an entrepreneurial exit rate (per year) of 23% as observed in the CPS ([Gaillard and Kankanamge \(2018\)](#)), the fraction of entrepreneur relative to the population in the last age brackets corresponding to retirement of 0.75%, an average risk-free interest rate between 3% and 4%, an unemployment rate of 7% and a ratio of median entrepreneurs' net worth to workers' net worth between 7 and 8, as computed in the SCF (av-

eraged over the 2001, 2004 and 2007 waves). Our model is exactly identified, with 8 parameters used to pin down 8 moments. Table 7 display our calibration results.

| Calibrated parameter         | Symbol            | Value           |
|------------------------------|-------------------|-----------------|
| Preferences                  | $u_R, u_E, \beta$ | 0.17, 0.2, 0.97 |
| Collateral constraint        | $\theta$          | 40%             |
| Downsizing adjustment costs  | $\tau_{down}$     | 10%             |
| Buyer's matching friction    | $P_b$             | 5%              |
| Labor frictions              | $\kappa$          | 2.7             |
| Entrepreneurial ability rate | $p_{z_1}$         | 23%             |

**Table 7.** Calibrated parameters.

Note that while some parameters directly affect some moments (i.e.  $u_R$  on the fraction of entrepreneurs in the [65;+ ] age bracket), the whole equilibrium is affected. We seek to minimize the distance between observed moments and those generated through the model, taking identity matrix for weights. We globally search for the set of parameters that minimize the distance using a Control Random Search (CRS) algorithm. Table 8 shows the resulting targeted moments and compare them to the data.

| Targeted moment                                  | Model | Data    |
|--|-------|---------|
| Share of entrepreneurs                           | 11.7% | 11-12%  |
| Unemployment rate                                | 7.5%  | 7%      |
| Share of business buyers (rlv. to new ent.)      | 28%   | 15%-25% |
| Share of business sold (rlv. to ent. exit)       | 18%   | 10%-20% |
| Entrepreneurial exit rate                        | 23%   | 23%     |
| Share of entrepreneur in the [65,+ ] age bracket | 0.75% | 0.75%   |
| Interest rate                                    | 3.2%  | 3%-4%   |
| Ratio of median wealth (ent. to workers)         | 7.35  | 7-8     |

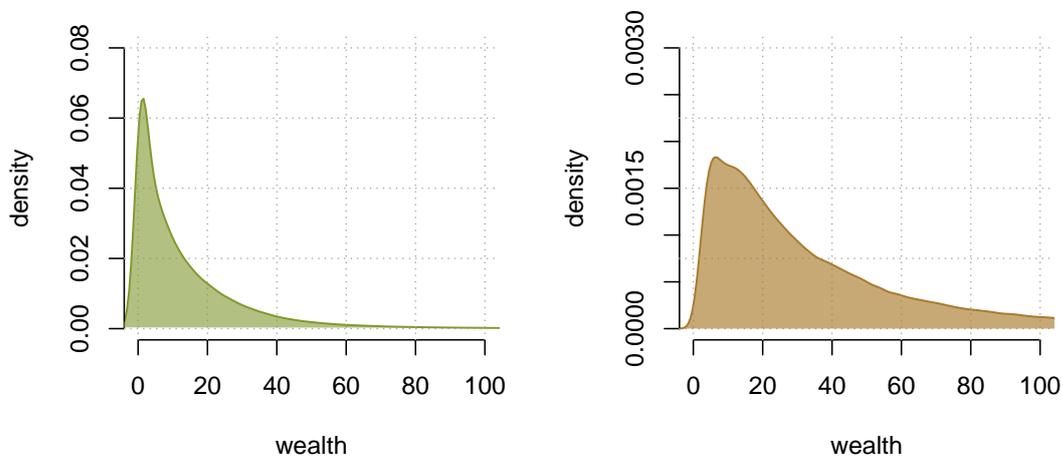
**Table 8.** Model fit.

## 5 Properties of the model

### 5.1 Evaluating our model-generated data against the actual data

We now compare some important features of the actual data for the US economy with the corresponding features of our model results.

**Occupational choice and wealth distribution** It is worth noting that we targeted the relative wealth between workers and entrepreneurs, but not with the whole population nor the whole distribution. Figure 8 compares the distribution of workers and entrepreneurs in the model. Our model exhibits some regularity already highlighted in the literature. In particular, our model does feature part of the wealth concentration as observed in the data. To this end, the particular role of the entrepreneurs and the bequests are crucial, as pointed out by [Cagetti and De Nardi \(2006\)](#). Our resulting wealth Gini of 0.7 is however under the value of the data, and the top 10% hold 40% of total wealth in the model, against 60% in the data. Concerning the ratio of median net worth of the entrepreneurs relative to the whole population, we find a value of about 6 in the SCF, against 5.7 in the model.



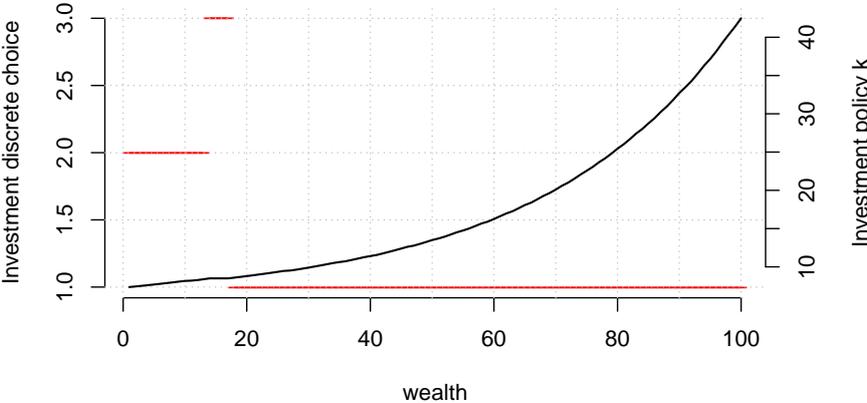
**Figure 8.** Wealth distribution for workers (left) and entrepreneurs (right).

With the presence of taste shock, the resulting choice between purchasing an existing business, founding a new business or continuing as worker is probabilistic. In Table 9, we show the effect on the decision to purchase a business or to stay a worker for different values of the variance of the taste shock,  $\sigma_w$ . As already mentioned, an interesting feature of this taste shock is that it provides a way to model the presence of non-pecuniary occupational benefits or unobserved heterogeneity in the data. We highlight here its specific consequences on equilibrium outcomes.

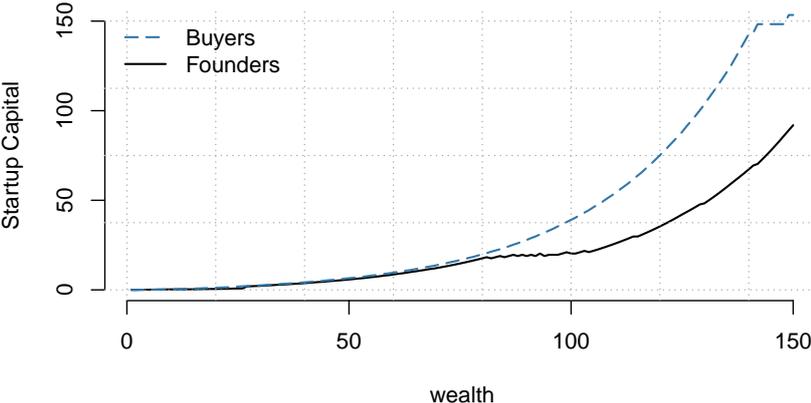
|                                    |      |      |
|------------------------------------|------|------|
| $\sigma_w$                         | 0.01 | 0.10 |
| share of entrepreneurs             | 11.7 | 10.9 |
| share of buyers (rlv. to entrants) | 28%  | 25%  |
| share of sellers (rlv. to exiters) | 18%  | 15%  |

**Table 9.** Fraction of buyers (relative to new entrants) in the model and the data.

**Agent’s investment decisions** With the presence of non-convex adjustment cost,  $\mathcal{C}(k, k')$  in the model, the decision to invest can be splitted in three cases: investing, disinvesting or nothing. In Figure 9, we display the investment strategy as a function of the entrepreneur’s wealth, when the current investment is equal to 15. Due to the presence of fixed costs, an inaction set arises around  $a \in [16 : 20]$ . In Figure 10, we show the startup capital investment made by workers who select into entrepreneurship as a function of wealth. We find that buyers also invest more wealth than founders, which is the resulting of the accrued advantages of purchasing an existing mature business as compared to founding a new one. Due to borrowing constraint and mismatches, it might be however not possible to buy, in such case, founding comes as the alternative.



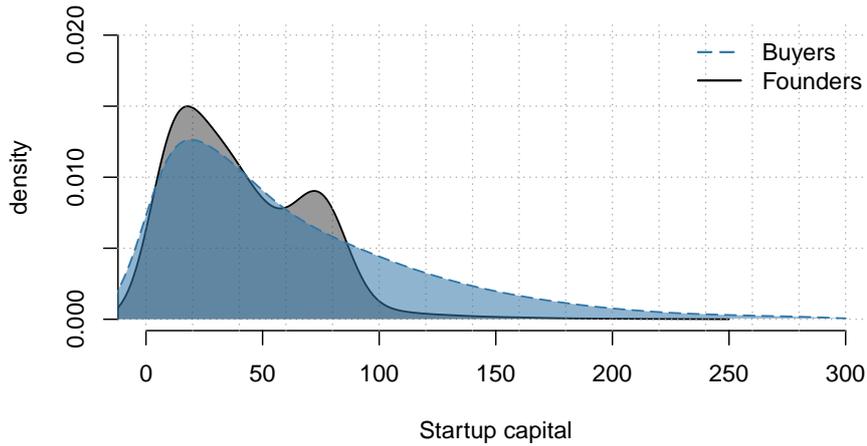
**Figure 9.** Investment decision with the presence of adjustment costs. In red: the discrete choice of doing nothing (=3), disinvesting (=2) and investing (=1).



**Figure 10.** Startup capital depending on the type of acquisition.

In Figure 11, we finally display the distribution of startup capital by type of acquisition in the model. In the data, the ratio of median startup capital between buyers and founders is about 6, against 4.5 in the model.

Finally, on the life-cycle side, we fit quite well the relationship between age and the type



**Figure 11.** Distribution of buyers and founders in the model.

of acquisition as shown in table 10. This is generated in the model since the younger (those in the [20:34] age bracket) do not have sufficient wealth in order to buy an existing business. The life-cycle property of the model seems particularly relevant to capture the existing tradeoff between purchasing versus founding a business.

| Age bracket        | [20:35[ | [35:50[ | [50:65[ |
|--------------------|---------|---------|---------|
| (%) buyers (Model) | 18      | 30      | 32      |
| (%) buyers (Data)  | 16      | 28      | 26      |

**Table 10.** Fraction of buyers (relative to new entrants) in the model and the data.

## 5.2 The role of matching technology, maturity, investment costs and life-cycle

In this section, we run four experiments in order to highlight the key mechanisms at play in our model. We explore the effect of: (i). improving the match between business buyers and sellers, (ii). lowering the interest rate faced by early stage firms, (iii). increasing the adjustment cost and (iv). the population aging. All those experiments are likely to affect the model equilibrium by specifically modifying the entry into entrepreneurship and the tradeoff between purchasing or founding a business, or between selling or liquidating the business. For each experiment, we focus on the steady-state comparison.

**(i). Improving the matching technologies** We first quantify the effect of improving the matching technology. In the model, such technology translates the easiness of a buyer (resp. a seller) to meet a corresponding seller (resp. buyer). As an experiment, we increase the matching probability  $P_b$  and  $P_s$  by 10%, while other parameters are unchanged.

**(ii). Lowering the business interest rate** Many policies around the world aim to promote early stage entrepreneurship by directly targeting the borrowing cost incurred by the entrepreneurs. In US, government small business loans are available under some guarantees by the Small Business Administration (SBA) and constitute one of the largest program to support entrepreneurship in the country. As a policy experiment, we investigate the effect of such policy by lowering the interest rate charged on early stage firm by 0.6%, while financing the program with labor income taxes.

**(iii). Increasing adjustment cost** We investigate the effect of increased business capital illiquidity on the equilibrium outcomes. In the model, the illiquid nature of business capital have major consequences on the margins of buyers and sellers. As an experiment, we let the costs  $\tau_{up}$  and  $\tau_{down}$  increase by 10%, reaching respectively 0.044 and 0.11.

**(iv). Age structure** We finally investigate the role of the age structure in the economy as a key element of the business sale market dynamic and the transmission of business capital over generations. To do that, we artificially change the life-cycle structure of the model, decreasing the probability of staying in the first two age brackets (corresponding to  $j = F$  and  $j = J$ ) compensated by an increase in the probability of staying in the third age bracket (corresponding to  $j = S$ ). As a consequence, the number of agents aged between [51,65] increases relative to the population aged between [20,50].<sup>40</sup>

We provide in table 11 the result for the above four experiments.

All the above experiments are shown to significantly affect the equilibrium outcomes. When improving the matching technologies, entrepreneurial capital significantly increases through an increased fraction of mature business that are sold on the business sale market. Both the fraction of businesses sold and purchased increase in equilibrium, lowering the overall capital destruction through liquidation.

Interestingly, providing support on the interest rate charged on early stage businesses significantly affect the equilibrium. Capital held by entrepreneurs increases by 5.5% and the fraction of mature businesses is reduced as compared to the baseline economy. By supporting the interest rate charged on the new businesses, the tradeoff between purchasing and founding a new business is affected, since part of the advantages of purchasing an existing business is lowered. As a result, equilibrium demand for mature business is reduced, and, *ceteris paribus*, this lowers the equilibrium business value  $q$  which in turn reduces the fraction of businesses sold in equilibrium.

---

<sup>40</sup>Of course, this also changes the expected average time of an age bracket in the model. We use this as a very simple experiment in order to describe the main mechanism at play in the model.

| Implementation            | baseline | (i)<br>$P_s, P_b$<br>+10% | (ii)<br>$\phi_m$<br>1% | (iii)<br>$\tau_{down}, \tau_{up}$<br>+10% | (iv)<br>$P(j' j)$<br>in text |
|---------------------------|----------|---------------------------|------------------------|---|------------------------------|
| share of entrepreneur (%) | 11.7     | 11.8                      | 12.1                   | 10.7                                      | 11.5                         |
| production                | 5.21     | 5.26                      | 5.28                   | 4.98                                      | 5.66                         |
| aggregate capital         | 15.53    | 15.68                     | 15.70                  | 15.05                                     | 16.95                        |
| entrepreneurial capital   | 5.26     | 5.36                      | 5.55                   | 4.51                                      | 5.62                         |
| share of sellers (%)      | 18.2     | 20.0                      | 17.3                   | 25.3                                      | 19.8                         |
| share of buyers (%)       | 28.3     | 30.8                      | 27.0                   | 32.6                                      | 28.7                         |
| mature businesses (%)     | 60.1     | 62.0                      | 59.7                   | 62.9                                      | 60.7                         |
| interest rate, $r_s$ (%)  | 3.211    | 3.186                     | 3.180                  | 3.289                                     | 3.145                        |
| business sale value, $q$  | 1.4961   | 1.4958                    | 1.4959                 | 1.4790                                    | 1.4963                       |

**Table 11.** Effects of the matching technology, maturity, investment cost, life-cycle and the retirement utility cost.

We find that illiquidity plays a crucial role in the model. When increasing by 10% the costs, the fraction of entrepreneurs, the production and aggregate capital fall, which increase the equilibrium interest rate. Importantly, due to the rising cost of founding a new business and the lowered value of liquidating business assets, the fractions of sellers and buyers significantly increase in equilibrium. A higher fraction of businesses are actually transmitted. As a result, the business value  $q$  falls.

Finally, when modifying the age structure toward older individuals we find that the quantity of capital is largely increased. This results to a higher fraction of individuals having accumulated wealth over their lifetime. Because a larger fraction are older, more of them switch to retirement, increasing the fraction of sellers. Moreover, because the fraction of buyer increases with age, we also find that the total fraction of buyers in equilibrium goes up, which result in an increased business sale value.

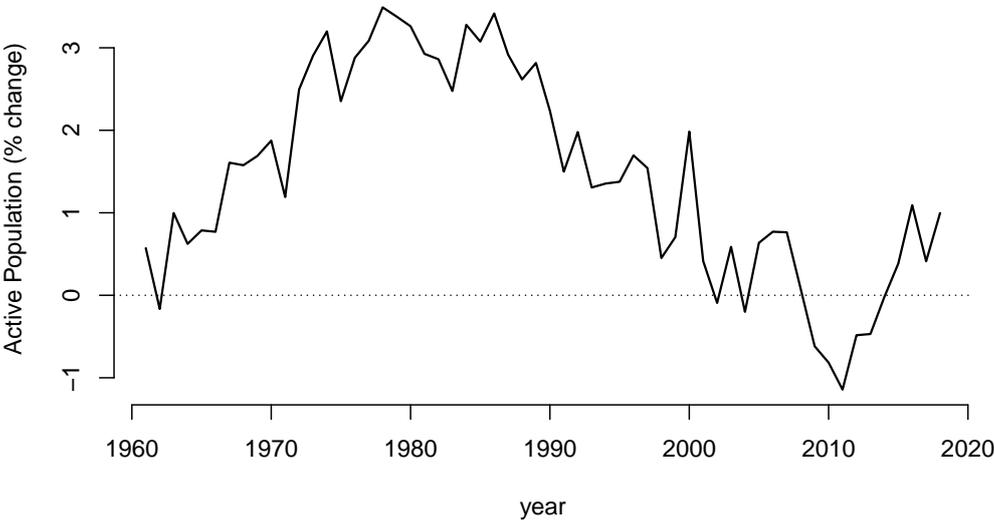
## 6 Demographic change and the aging of entrepreneurs

Our quantitative analysis so far is predicated on the assumption that the age structure of the economy is stationary. In this section, we begin studying the implications of our model when we relax the stationary assumption and try to match the demographic change that started in the 1970s with the so-called boomers generation. We thus try to match the non-stationary distribution as observed in 2007.

More specifically, the exercise we perform is as follows. We choose a set of parameters corresponding to the observed stationary 1970 distributions of occupation, age structure and wealth

as initial condition. We then shock the economy by increasing the number of new young individuals in the model during 10 periods. Such demographic shock propagates along the transition until date 2007 and, at such point in time, we compute the distance between the model generated moments and the data. We redo this experiment until finding the set of parameters that minimizes this distance. By doing so, we are able to answer the following question: "what was the effect of the demographic change started in the 1970s on the occupational choice?" and "what would be the implications of the ongoing aging of entrepreneurs on the economy, and more specifically, on the transmission of businesses?".<sup>41</sup>

The fundamental feature of the change in the age structure from 1970 to 2007, in the data, is the substantial entry of baby-boomers following the significant increase of birth rate from 1944 to 1964, where more than 65 million children were born in the United States. In this respect, we can obtain the baby boomers effects throughout our simplified life-cycle model by increasing the mass of individuals in our economy by 2.5% each year during 20 years. In this respect, we display in figure 12 the corresponding active population growth rate since the 1960s. Then, this mass slowly propagates over our life-cycle structure according to the aging probabilities. The economy recovers its stationary property when all the surplus of mass coming from the entry of baby boomers in the model die. We thus assume that some individuals do not reborn to recover a unit mass.<sup>42</sup>



**Figure 12.** Active population aged between 24 and 55 (annual percent change).

<sup>41</sup>Of course, our experiment assume that the structural parameters of the model remained unchanged between 1970 and 2007. Still, we think this experiment has the advantage to isolate the specific effect of the demographic change along the transition.

<sup>42</sup>We could also reduce slowly the probability of reborn, generating cycles in the demographic structure, but this is beyond the scope of this paper.

## 7 Conclusion

In this paper, we develop an incomplete markets heterogeneous agents general equilibrium model with stylized life-cycle dynamics and occupational choices. Importantly, our model lets incumbent entrepreneurs sell their businesses and prospective workers buy or found businesses, subject to an endogenous business price and adjustments costs. We also build a new dataset using data from a major online business selling platform and show that selling a business takes much time and that the vast majority of businesses are not sold. The model accounts for the main empirical features about business selling and buying frictions and age and life-cycle dynamics found in the empirical data. We show that illiquid business assets, frictions on the business sale market and the life-cycle components of entrepreneurship are key to reproducing our empirical findings. We show that our model is tractable and can be used to simulate the demographic changes appearing on the labor and entrepreneurial markets due to the aging entrepreneurial population.

## References

- Aiyagari, S., 1994. Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics* 109, 659–684.
- Bewley, T., 1983. A difficulty with the optimum quantity of money. *Econometrica* 51, 1485–1504.
- Buera, F.J., Shin, Y., 2013. Financial Frictions and the Persistence of History: A Quantitative Exploration. *Journal of Political Economy* 121, 221–272.
- Cagetti, M., De Nardi, M., 2006. Entrepreneurship, Frictions, and Wealth. *Journal of Political Economy* 114, 835–870.
- Cagetti, M., De Nardi, M., 2009. Estate taxation, entrepreneurship, and wealth. *American Economic Review* 99, 85–111.
- Carroll, C.D., 2006. The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics letters* 91, 312–320.
- Chen, D., Qi, S., Schlagenhaut, D., 2018. Corporate income tax, legal form of organization, and employment. *American Economic Journal: Macroeconomics* 10, 270–304.
- Druehdahl, J., Jørgensen, T.H., 2017. A general endogenous grid method for multi-dimensional models with non-convexities and constraints. *Journal of Economic Dynamics and Control* 74, 87–107.

- Fairlie, R.W., Kapur, K., Gates, S., 2011. Is employer-based health insurance a barrier to entrepreneurship? *Journal of Health Economics* 30, 146 – 162.
- Gaillard, A., Kankanamge, S., 2018. Entrepreneurship, Labor Market Mobility and the Role of Entrepreneurial Insurance. TSE Working Papers 18-929. Toulouse School of Economics (TSE).
- Hamilton, B.H., 2000. Does entrepreneurship pay? an empirical analysis of the returns to self-employment. *Journal of Political economy* 108, 604–631.
- Huggett, M., 1993. The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control* 17, 953–969.
- Hurst, E., Pugsley, B.W., 2011. What do small businesses do? Technical Report. National Bureau of Economic Research.
- Hurst, E.G., Pugsley, B.W., 2015. Wealth, tastes, and entrepreneurial choice. Technical Report. National Bureau of Economic Research.
- Iskhakov, F., Jørgensen, T., Rust, J., Schjerning, B., 2017a. The endogenous grid method for discrete-continuous dynamic choice models with (or without) taste shocks. *Quantitative Economics* 8, 317–365.
- Iskhakov, F., Jørgensen, T.H., Rust, J., Schjerning, B., 2017b. The endogenous grid method for discrete-continuous dynamic choice models with (or without) taste shocks. *Quantitative Economics* 8, 317–365.
- Karahan, F., Ozkan, S., 2013. On the persistence of income shocks over the life cycle: Evidence, theory, and implications. *Review of Economic Dynamics* 16, 452–476.
- Kitao, S., 2008. Entrepreneurship, taxation and capital investment. *Review of Economic Dynamics* 11, 44–69.
- Liang, J., Wang, H., Lazear, E.P., 2014. Demographics and Entrepreneurship. Working Paper 20506. National Bureau of Economic Research. URL: <http://www.nber.org/papers/w20506>, doi:10.3386/w20506.
- Mankart, J., Rodano, G., 2015. Personal bankruptcy law, debt portfolios, and entrepreneurship. *Journal of Monetary Economics* 76, 157–172.
- Moskowitz, T.J., Vissing-Jørgensen, A., 2002. The returns to entrepreneurial investment: A private equity premium puzzle? *American Economic Review* 92, 745–778.

Pugsley, B., Sahin, A., Karahan, F., 2016. Demographic origins of the startup deficit. 2015 Meeting Papers 1333. New York Fed, mimeo. URL: <https://ideas.repec.org/p/red/sed015/1333.html>.

Quadrini, V., 1999. The Importance of Entrepreneurship for Wealth Concentration and Mobility. *Review of Income and Wealth* 45, 1–19.

Shimer, R., 2005. The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review* 95, 25–49.

Sommer, K., Sullivan, P., 2017. Implications of u.s. tax policy for house prices, rents, and homeownership. *American Economic Review* .

Tan, E., 2017. Entrepreneurial investment dynamics and the wealth distribution .

## Appendix

### A Algorithm

### B Data

#### B.1 Public Micro Sample - Survey of Business Owners (SBO) (2007)

Provided by the U.S. Census Bureau, we use the SBO to recover information concerning both sellers' and buyers' side. The dataset contains information on whether owners established, purchased or acquired their businesses. The dataset also contain demographic variables such as age, education and past self-employment experience of business owners. On top of that, information concerning the amount of startup capital and how this capital was financed is provided. SBO 2007 includes variables for the four most important owners of a business. Following [Cagetti and De Nardi \(2006\)](#), we define an entrepreneur as a business owner that actually manage day-to-day the business or participate to the production process and whose the business constitute the primary source of personal income. Those individuals choose option 1 or 2 to question (1) and *Yes* in question (2)<sup>43</sup>:

1. In 2007, which of the following best represents Owner X's function(s) in this business? (multiple choices allowed) – Providing services and/or producing goods – Managing day-to-day operations – Financial control with the authority to sign loans, leases, and contracts – none of the above.

---

<sup>43</sup>We could also control for a minimum time spent in actually working for the business. This information is also contains in the survey questionnaire.

2. In 2007, did the business provide Owner X's primary source of personal income? – Yes – No.

Own robustness check on the definition of an entrepreneur provided similar results than those shown in the paper. Taking all business owners, without controlling for actively managing the business yield a respective fraction of business founded and purchased of about X% and Y%. Other results are also comparable.

In addition to this, the SBO 2007 public micro sample allows to match business owners' characteristics with businesses' characteristics. This allows us to recover information on the reasons to cease businesses along the life-cycle. We select early-stage businesses those who were established, purchased or acquired in the past two years (in 2006 and 2007).

## **B.2 Annual Survey of Entrepreneurs (ASE)**

Provided by the U.S. Census Bureau, we use the ASE, which contains 290,000 employer firms in the private, non-agricultural U.S. economy, in order to characterize recent small business dynamics concerning the evolution of reasons to cease a business and on the share of entrepreneurs that purchase or found a business. The dataset is not available as a micro public sample and we use the online API to recover informations<sup>44</sup>.

## **B.3 Panel Study of Income Dynamics (PSID)**

We use the PSID in order to get information on entrepreneurs' selling their businesses. Since 1989, the PSID contains variables that record whether or not an individual recently sold a business (available only every 5 years from 1989 to 1999 and every two years afterward). We use the panel by only considering the head of the family. In order to guarantee some consistency in our definition of entrepreneurship. We define an entrepreneur as someone owning a business and declaring being self-employed only (answering option 3 in question (1)).

1. On your main job, are you (HEAD) self-employed, are you employed by someone else, or what? – Someone else only – Both someone else and self – Self-employed only
2. Did you (or anyone else in the family there) own a business at any time in DATE or have a financial interest in any business enterprise? – Yes – No

Unfortunately, we could not control for actively managing a business/working in the business using the PSID<sup>45</sup>. We provide additional robustness on our results regarding our definition in the online appendix and we show that our main results are not very sensitive between business

---

<sup>44</sup>For more information: <https://www.census.gov/programs-surveys/ase.html>.

<sup>45</sup>Although, results from the SBO 2007 showed relatively few differences between these two definitions regarding the age profile of entrepreneurs and their propensity to sell businesses.

owners and self-employed business owners. Due to lack of business sold observations per year, we aggregate the number of business sold from 1989 to 2013 that we normalize using 2018 base prices. This provides an average profile of sellers in the US during the past 30 years.

#### **B.4 Survey of Consumer Finance (SCF)**

We use three SCF waves (1989, 2004, 2016) in order to draw the evolution of the entrepreneurs' characteristics over almost 30 years. As firstly used by [Cagetti and De Nardi \(2006\)](#), the SCF contains additional question as compared to the PSID that allows to better quantify the number of entrepreneurs in the economy. Indeed, as pointed by [Cagetti and De Nardi \(2006\)](#), total wealth reported in the SCF is close to the one reported for the US, making the comparison of moments close to aggregate data. We define an entrepreneur as a self-employed business owners owning at least a business of more than 5000\$. In order to be consistent with other datasets, we do not control for business management role. We use the SCF to calibrate the model. Among other moments, we compute the median net worth between workers and entrepreneurs, as well as different moments between generations of entrepreneurs.

#### **B.5 Current Population Survey (CPS)**

When used as a panel, the CPS from the U.S. Bureau of Labor Statistics provide exhaustive information on the transition rate between occupations as highlighted in [Gaillard and Kankanamge \(2018\)](#). In this paper, we use the CPS to compute the flow of entrepreneurs that retire each year, as well as to compute with an important number of observations, the share of entrepreneurs with more than 60 years old. CPS does not contain clear definition of entrepreneurship. We therefore construct a variable *holdhubus* that translates whether or not an entrepreneur was previously business owner. Using this variables, we define an entrepreneur as a self-employed person that already declared owning a business<sup>46</sup>.

#### **B.6 Survey of Small Business Finance (SSBF)**

Conducted by the U.S. Federal Reserve, the 2003's SSBF wave provides relevant information on how business were acquired (either purchased or founded) as well as information on business owners (age, net-worth etc.). The SSBF also provide a number of variable concerning small businesses' credit conditions. To ensure comparability with other dataset, we define an entrepreneur as a business owners who actually manages day-to-day his business. Finally, due to lack of observations, we define an early-stage business as a business who was acquired during the last five years.

---

<sup>46</sup>Some questions are not answered several times in the CPS, depending on the month of the survey.

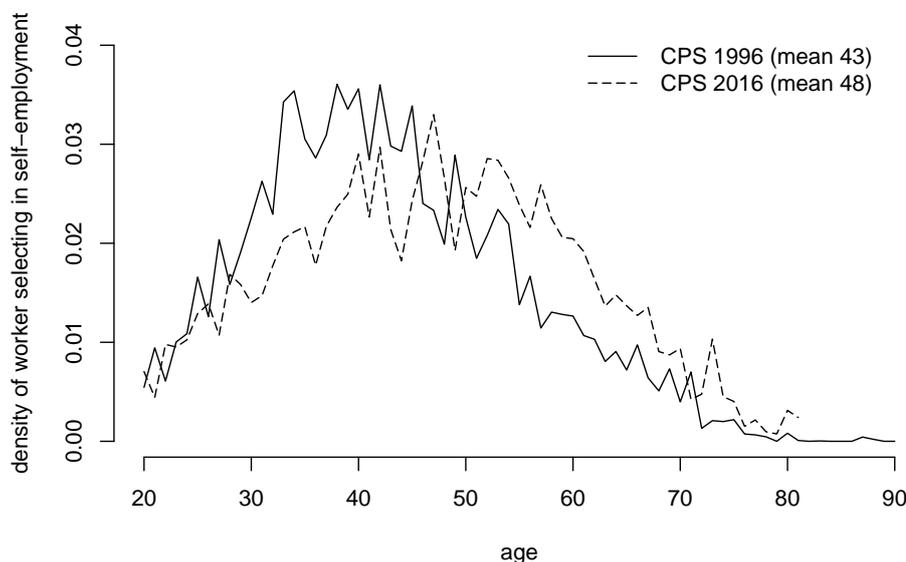
## B.7 Online platform BizBuySell (BBS)

We use data from a private company *BizBuySell.com* in order to get information related to the US business sale markets. BBS is currently one of the most active platform for selling/purchasing a business in US. We proceed a number of text mining in order to classify different variables. We first classify in four categories the reason to sell between *retirement*, *move to another location*, *family purposes* and *other reason*. We aggregate businesses into 20 main business types<sup>47</sup>. We observe the number of employees that are actually working in the business for sale, with additional information on the whether employees are full-time or part-time. We assign a weight 0.5 to part-time employees and 1 to full-time employees.

## C Supplement facts

### C.1 Demographic of entrepreneurs

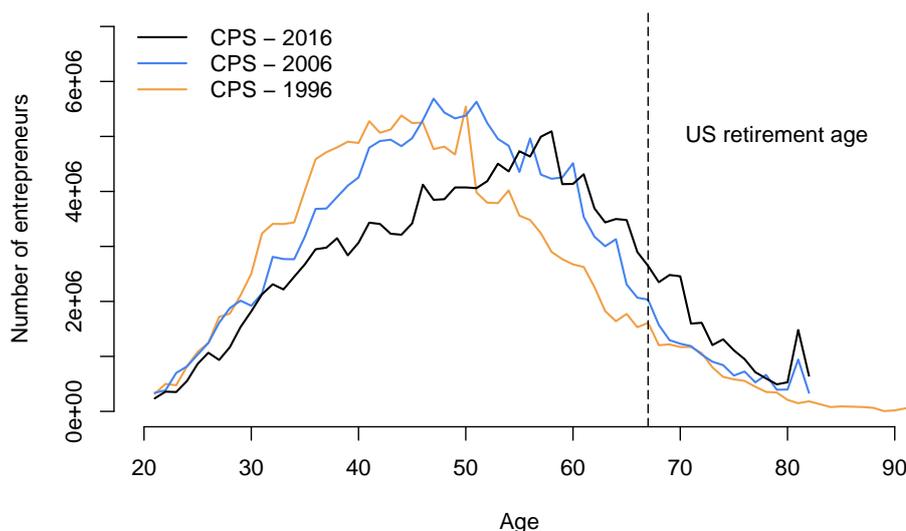
Figure 13 displays the density of workers selecting into entrepreneurship by age. As argued in the main text, we observe an increase in the mean age at which workers select into entrepreneurship, contributing to the overall aging of the entrepreneur's population. Figure 14 displays the evolution of the entrepreneur's density by age using the CPS.



**Figure 13.** Workers selecting into entrepreneurship by age. *Source:* CPS 1996 and 2016 from author's own calculation.

---

<sup>47</sup>Those types include: agriculture, automotive and boat, beauty and personal care, building and construction, communication and media, education and children, entertainment and recreation, financial services, food and restaurants, health care and fitness, manufacturing, non-classifiable establishments, online and technology, pet services, real estate, retail, service businesses, transportation and storage, travel, wholesale and distribution.



**Figure 14.** Entrepreneur’s density by age. *Source:* CPS 1996-2016 from author’s own calculation.

Table 12 also provide this evidence by comparing early stage firms (with less than two years of operation) with the entrepreneur’s population.

| Age             | 25 and less | 25 to 34 | 35 to 44 | 45 to 54 | 55 to 64 | 65 and more |
|-----------------|-------------|----------|----------|----------|----------|-------------|
| All firms       | 0.5         | 5.5      | 16.5     | 27.5     | 30.5     | 19.6        |
| Firms < 2 years | 1.4         | 16.6     | 29.6     | 27.6     | 17.8     | 7.0         |

**Table. 12.** Fraction of entrepreneurs for different age brackets. *Source:* ASE (2016).

In table 13 we report the increase of the business value and total wealth held by the older entrepreneurs.

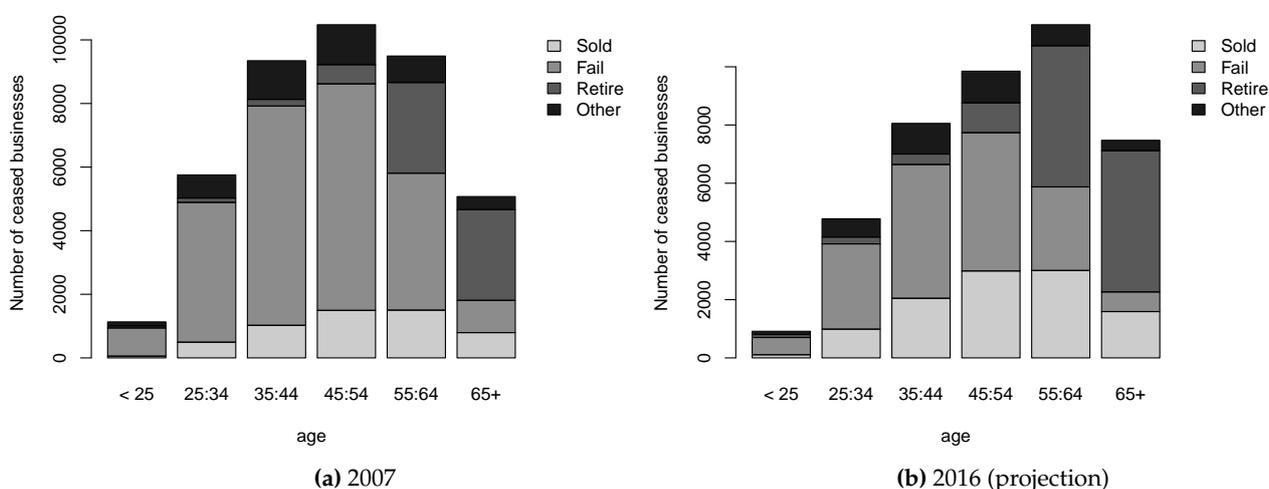
The reasons to cease a business because entrepreneurs sold or retire tends to increase a lots. We plot in figure 15 the projection of the 2016 distribution of reasons to cease given the increasing fraction of entrepreneurs who sold and retire (ASE - 2016). The underlying assumption is that we keep constant, for a given reason to cease, the share of individual in a given age bracket (for instance, in 2016 and in 2007, among those who sold their business, 27% were aged between 55 and 64 years).

In figure 16 we display the entrepreneur’s profit distribution by acquisition type.

In figure 17, we use the SBO - 2007 to draw the distribution of bracketed startup capital by type of acquisition.

| % of total entrepreneur's population | 1989 | 2004 | 2016 |
|--------------------------------------|------|------|------|
| <b>55 and over</b>                   | 26.1 | 33.7 | 47.1 |
| wealth                               | 49.3 | 49.4 | 65.2 |
| business value                       | 48.6 | 45.1 | 63.9 |
| <b>60 and over</b>                   | 17.3 | 22.2 | 31.8 |
| wealth                               | 36.7 | 33.7 | 46.8 |
| business value                       | 36.6 | 29.9 | 45.4 |

**Table 13.** Share of business capital and wealth relative to the entrepreneur's population.  
Source: SCF (1989,2004,2016).



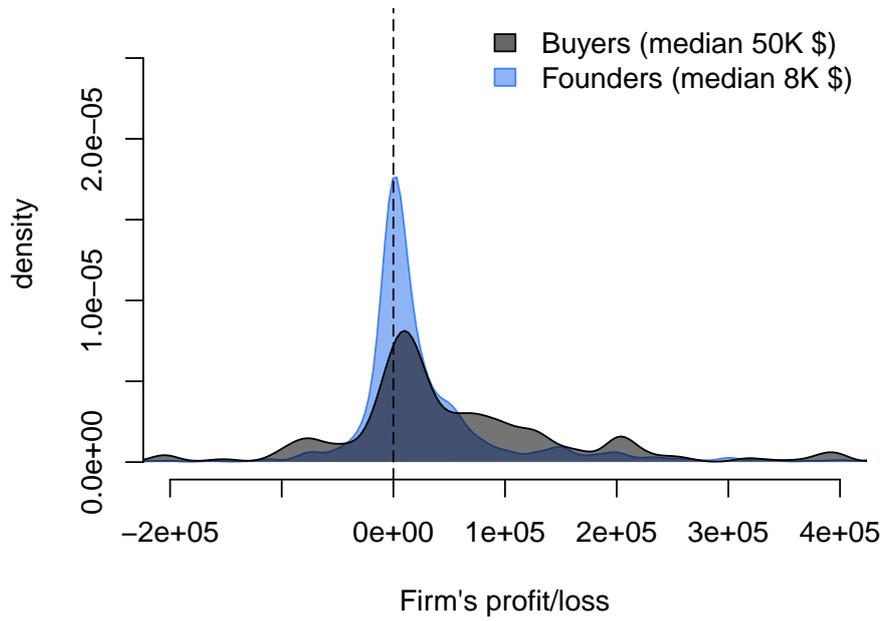
**Figure 15.** Age profile of ceasing entrepreneurs. SBO and projection using ASE.

## C.2 Small business sale market

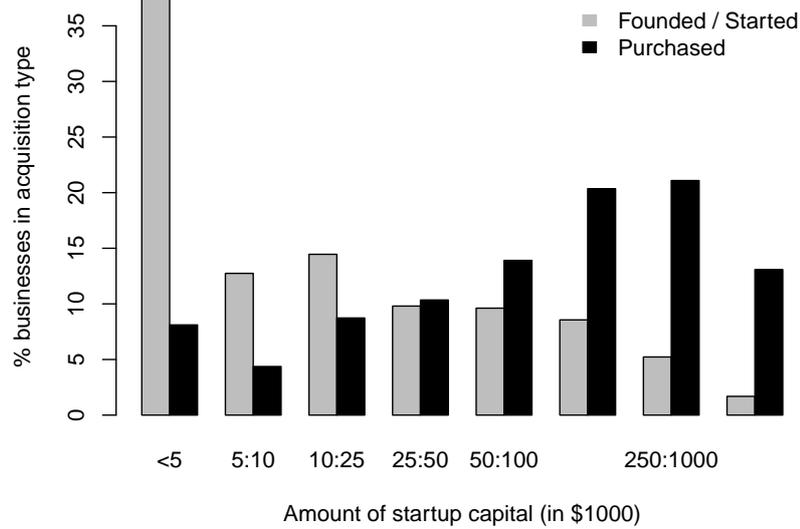
In order to estimate the market price of businesses, we conduct a hedonic regression with respect to cash-flow, gross-revenue, state, number of years in operation, number of employees. We estimate:

$$p_{j,s,q}^{sell} = \alpha_s + \alpha_q + cash.flow_j + gross.rev_j + inventory_j + employees_j + \epsilon_{j,s,q} \quad (33)$$

Finally, we also found that experienced worker tend to switch to entrepreneurship with lower probability, consistent with the model



**Figure 16.** Profit/loss by acquisition type. *Source:* SSBF 2003, taking into account early stage businesses (less than 5 years in operation).



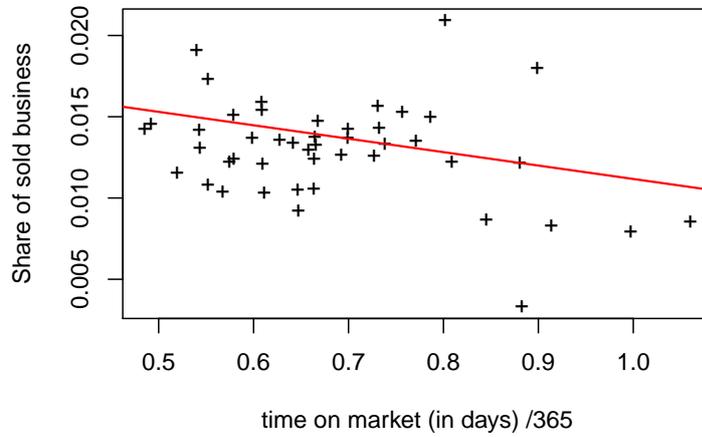
**Figure 17.** Distribution of startup capital invested by acquisition type. *Source:* SBO 2007 from author's own calculation.

|  | SBO     |           |      | BBS      |      |
|--|---------|-----------|------|----------|------|
|  | Founded | Purchased | Sold | For sale | Sold |
| Professional, Scientific, Technical Services | 20      | 7         | 9    | 3        | 13   |
| Construction                                 | 14      | 6         | 4    | 2        | 3    |
| Finance, Insurance, Real Estate              | 11      | 7         | 9    | 2        | 3    |
| Wholesale and Retail Trade                   | 11      | 27        | 26   | 23       | 19   |
| Health Care and Social Assistance            | 10      | 7         | 8    | 9        | 14   |
| Administrative and Support                   | 7       | 5         | 6    | 8        | 10   |
| Transportation and Warehousing               | 5       | 6         | 6    | 1        | <1   |
| Manufacturing                                | 3       | 6         | 6    | 4        | 2    |
| Accommodation and Food Services              | 2       | 12        | 10   | 27       | 17   |
| Other  | 17      | 17        | 16   | 21       | 19   |

**Table. 14.** Main category of businesses. *Sources:* SBO (2007) and *BizBuySell* (BBS, 2017).

|                                | share_sold_reported |
|--------------------------------|---------------------|
| Time on market (in days) / 365 | -0.008**<br>(0.003) |
| Constant                       | 0.018***<br>(0.002) |
| <i>N</i>                       | 46                  |
| <i>R</i> <sup>2</sup>          | 0.108               |

**Table. 15.** Share of sold businesses using ASE (2016) and time on market (divided by 365 days) in BBS, by US state. *Notes:*  $p < 0.1$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$ . In parenthesis: std. deviation.



**Figure 18.** Share of sold businesses and share of walk away as exit strategy using ASE (2016) and probability to sell a business (in a year) in BBS, by US state.

|   | Number (th.) | %          |
|---|--------------|------------|
| Walk away from the business                 | 494          | 11.2       |
| Liquidate business assets & repay bus. debt | 422          | 9.6        |
| Sell the business to employees or manager   | 281          | 6.4        |
| Sell ownership to external                  | 1159         | 26.2       |
| Sell ownership to another owner/family      | 806          | 18.2       |
| No exit strategy                            | 1085         | 24.6       |
| Other exit strategy                         | 171          | 3.9        |
| <b>Total report</b>                         | <b>4418</b>  | <b>100</b> |

**Table 16.** Exit strategy. *Source:* ASE (2016).