Liquidity and Price Dispersion in Markets with Information Heterogeneity

Qi Li*
Robert Shimer †

Preliminary and Incomplete

Abstract

We consider asset markets where quality is heterogeneous and buyers have different ability to evaluate quality. In particular, we allow sellers to set up markets indexed by asking prices and buyers direct their search into different markets. We construct an equilibrium that features pooling on the sellers' side and separation on buyers' side. A buyer without expertise in asset evaluation will search in a high-price market that contains more good assets. On the other hand, a buyer with expertise will search in a low-price market. The low-price market will be polluted with more bad assets but the buyer can use his expertise to pick out the good assets and reject the bad ones. In equilibrium, the probability for a seller to meet a buyer decreases in asking price, and sellers with a bad asset are further rationed due to rejections by expert buyers. The model predicts that a larger share of bad assets is associated with a worsening of asset liquidity and an exacerbation of price dispersion. (JEL D82, G12, G32)

1 Introduction

This paper studies the equilibrium of an asset market where quality is heterogeneous and buyers have different ability to evaluate quality. What is the range of asking

*Affiliation: Pennsylvania State University. Email: qili@psu.edu.

†Affiliation: University of Chicago. Email: shimer@uchicago.edu.

prices? Are good assets matched with expert buyers? Is the equilibrium efficient? How is market liquidity at different price levels?

One answer to these questions comes from Kurlat (2016). Assuming non-exclusive trading, Kurlat (2016) shows that the equilibrium reduces to a one price equilibrium. In his model, assets of different quality and asset buyers with different levels of expertise pool in the same market. i.e., there is no pre-trade sorting between buyers and sellers. Moreover, all sellers are matched with buyers and good assets are sold with ease at the marketing clearing price.

The assumption of non-exclusive trading seems plausible in markets where sellers can secretly double list an asset at different price points. In other markets, double-listing is not feasible for most sellers. In the real estate market, assets are identifiable and therefore cannot be secretly double listed. In the stock market, brokerage firms usually prevent their customers to submit two limit orders for the same stock so that their customers do not run the risk of short selling. Few theoretical works have been done to study trading patterns in exclusive markets with heterogeneously informed buyers.

Moreover, the assumption of non-exclusive trading precludes sellers using posted prices to signal quality, which is highlighted by Guerrieri et al (2010), Delacroix and Shi (2013), etc. In these papers, sellers commit to the prices they posted even if they fail to sell. In particular, the tradeoff between the posted price and liquidity presents a way to separate good and bad assets because, everything else being equal, a seller of a good asset has a higher value absence of trading and thus values liquidity relatively less. The negative association between quality and liquidity also presents in the data. For example, Adelino et al. (2018) show that it takes longer to sell privately securitized mortgages with good ex-post performance.

In this paper, we adopt the competitive search framework and assume assets markets are exclusive. In the model, sellers set up markets indexed by asking prices. Importantly, sellers cannot double list the same asset at different prices. On the other side, buyers direct their search into different markets. Within a market, buyers and sellers are matched according to a piecewise linear matching function: the short side trades with certainty while the long side is proportionally rationed.

We show that the equilibrium of the model features pooling among sellers and separation among buyers. Compared to Kurlat (2016), the equilibrium in our model features many active markets that differ in posted prices, asset mixes and liquidity. Buyers without expertise search in a high-price market, knowing that they will more likely to encounter good assets. Buyers with expertise, on the other hand, search in low-price market. The low-price market is polluted with more bad assets, but buyers can use their expertise to evaluate assets and reject the likely bad ones. Compared to other models with competitive search, adverse selection, and homogeneously informed buyers, active markets in our model can pool good and bad assets together. Still, quality and liquidity can be negatively associated as a larger share of good assets is offered in the illiquid high-price market.

Our framework can be applied to analyze liquidity and price dispersion caused by information heterogeneity. In particular, we show that fundamental shocks such as an increase in the proportion of bad asset can generate negative comovements in price dispersion and asset liquidity. This prediction is consistent empirical evidence in OTC markets (Jankowitsch et al. (2011)).

Related Work

This paper closely relates to the literature that examines adverse selection in exclusive markets. Gale (1992) proposes a Walrasian approach to provide a general characterization of equilibrium with one-sided private information. Guerrieri, Shimer, and Wright (2010), instead, lay out a search theoretical framework to study the same topic. These models of one-sided private information feature full separation. In equilibrium, prices reveal quality and sellers do not cross-subsidize each other. Guerrieri and Shimer (2014) develop a dynamic model of asset markets where sellers have private information on quality. Chang (2018) considers multi-dimensional private information on the sellers' side. Although the equilibrium does not separate sellers fully[1], it separates sellers up to their no-trade values. Unlike these papers above, our model has two-sided private information which results in pooling on the sellers' side. Apart from one-sided private information, Gale (1992, 2001) also dis-

[1] In her model, the dimension of signaling device is smaller than the dimension of private information.

cusses two-sided private information. However, he focuses on deriving the conditions for full separation while we present an equilibrium that features pooling on the sellers' side.

The environment of information heterogeneity is extensively studied in Kurlat (2016), Kurlat and Scheuer (2017). Kurlat (2016) proposes a novel competitive equilibrium approach to study asset markets with adverse selection and heterogeneously informed buyers. He shows that fire sales naturally arise in this setting, but not when buyers are equally uninformed. Kurlat and Scheuer (2017) extend the contract space to allow for signaling. They show that with the presence of expert buyers, sellers may forgo signaling and pool in the same market. By assuming exclusive trading, we allow sellers to use posted prices to signal quality. In this sense, our paper is closer to Kurlat and Scheuer (2017). However, our result is different: in Kurlat and Scheuer (2017), there is at most one market where sellers pool; in our paper, there are many seller-pooling markets, each with different seller composition.

Outline

The remainder of this paper proceeds as follows. Section 2 develops a competitive search model of asset markets with adverse selection and heterogeneously informed buyers. Section 3 defines equilibrium and states our preferred refinement criterion. Section 4 presents our main result: the equilibrium of our model features pooling on the sellers' side and separating on the buyers' side. We also discuss some testable implications of the model on market liquidity and price dispersion.

2 Model

In this section, we lay out a model of decentralized asset markets with adverse selection and heterogeneously informed buyers. The economy lasts for two periods. There are two types of risk-neutral agents in the economy.[2] There is a unit mass of sellers who discount consumption at date 2 with a factor β, $\beta < 1$, and there is a

[2] One can relax risk neutrality by introducing stochastic discount factors. Since risk preference is not a focus of this paper, we choose not to do so here.

unit mass of buyers who are more patient than natural sellers. The discount factor of buyers is normalized to 1.[3]

Each seller is endowed with 1 unit of asset that can be either good or bad: a good asset pays one unit of consumption good at date 2; bad assets pay no dividend.[4] At the beginning of date 1, seller i privately observes the quality of his asset, either good or bad. We denote the type of seller i as a^i, and $a^i \in A = \{0, 1\}$ represents the dividend amount. The distribution of a^i in the population of sellers is $\bar{\gamma}$.

Each buyer is endowed with w units of consumption good at date 1 (numeraire). Buyers have inferior information compared to sellers in the sense that they can mistake bad assets as good ones. Moreover, buyers have different levels of expertise. We denote the type of buyer i as h^i, and h^i represents the false positive rate that associates with buyers evaluation. Specifically, h^i is the probability for buyer i to misidentify an asset as good when in fact it is bad. A lower h^i means the sellers is less prone to make false positive error and thus more sophisticated. The distribution of h^i in the population of sellers is $\bar{\mu}$. The support of $\bar{\mu}$ is assumed to be $H = [\underline{h}, \overline{h}]$ with $0 < \underline{h} < \overline{h} < 1$.

Since sellers are less patient than buyers, there will be gains from trade when future cash flow is transferred from sellers to buyers. In the model, this is done through the trading of assets. Decentralized markets for assets open up at date 1, after sellers observe their signals. A seller sets up a market by paying a setup cost c per unit of asset and committing to a posted price p.[5] A buyer directs his search into a market by agree to the posted price. We allow a seller to set up many markets and a buyer to search in many markets. However, trading is non-exclusive: each asset or numeraire can only be brought to one market.

In a market, when demand equals supply, both sides of the market can trade with

[3]The preference of seller i, $i \in [0, 1]$, can be summarized by $u^i = c_1^i + \beta c_2^i$, where c_t^i is seller i's consumption at period t, $c_t^i \geq 0$. Similarly, the preference of buyer i, $i \in [0, 1]$, can be summarized by $u^i = c_1^i + c_2^i$.

[4]These assets can be thought of as defaultable one-period discount bond with a face value of one.

[5]Later we will assume that the setup cost is minimal: $c < w\mathbb{E}_{\bar{\mu}}[h] \min\{1, \frac{1-\beta}{\overline{h}}\}$. This assumption insure that good and bad sellers will earn positive profit in equilibrium.

certainty. Otherwise, the long side of a market will be rationed probabilistically.[6] We use $\theta^s(p)$ and $\theta^b(p)$ to denote agents' beliefs on matching probabilities for sellers and buyers in market p, and we interpret $\theta^s(p)$ as asset liquidity since it measures the selling probability of good assets. Upon matched with a seller, a buyer can evaluate the asset and determine if it will be accepted. If the asset is accepted, the buyer pays the agreed upon price.

Denote P as the set of eligible prices. Without loss of generality, let $P = [\beta, 1]$. A seller decides how much to sell at any given price; i.e., he chooses a Borel measure (say, φ) on P. Sellers cannot double list the assets. This is captured by the sellers' feasibility constraint:

$$\int_P d\varphi(p) \leq 1. \tag{1}$$

At the same time, a buyer decides how much to buy at any given price, i.e., they also choose a Borel measure on P (say, σ) . Buyers cannot double pledge the numeraire: they face feasibility constraint

$$\int_P pd\varphi(p) \leq w. \tag{2}$$

We refer to Borel measures chosen by sellers and buyers as supply and demand schedules, respectively. [7]

At date 1, buyers who are matched with sellers and accept their assets pay consumption good in exchange for assets, and agents consume their remaining consumption good. At the beginning of date 2, good assets generate dividend, and agents consume. Conditional on being matched, the payoff of listing a bad asset at price p is $\mathbb{E}_{\mu(p)}[h]p$, where $\mu(p, \cdot)$ represents the distribution of buyers in the market p. This

[6] Different markets operate rationing independently: whether agent i is rationed in one market is independent from whether he is rationed in the other markets in which he participates.

[7] Notice that the concepts of supply and demand schedules in our paper are different from that in a Walrasian auction. In a Walrasian auction, there is only one market clearing price. To determine the total supply of a seller, we only need to look at one point on his supply schedule, which is the point associated with market clearing price. In the competitive search setting, a seller in principle can supply at multiple prices. To determine the seller's total supply, we need to integrate his supply schedule over the range of prices he supplies.

is because a bad asset will be accepted only when the buyer makes a false positive mistake. The payoff of listing a good asset at price p does not depends on who buyers are. It is the same $p - \beta$ as long as the seller is matched.[8] Conditional on being matched, the payoff of buying in market p is $\gamma(p, 1)(1-p) - \gamma(p, 0)hp$, where $\gamma(p, \cdot)$ represents the distribution of buyers in the market p. Agents understand that they are infinitesimal: they take $\theta^b, \theta^s, \gamma, \mu$ as given when they make their optimal selling or buying decision.

3 Equilibrium Definition

In this subsection, we develop the concept of the decentralized equilibrium with information heterogeneity. Going beyond the work of Guerrieri, Shimer, and Wright (2010), our definition allows for heterogeneity and private information on both sides of the market.

Definition 1. An *equilibrium* is $\varphi_h : \mathcal{B}(P) \to \mathbb{R}_+$, $\sigma_a : \mathcal{B}(P) \to \mathbb{R}_+$, $\theta^b : P \to [0, 1]$, $\theta^s : P \to [0, 1]$, $\mu : P \to \Delta(H)$, $\gamma : P \to \Delta(A)$ that satisfies conditions C1-C3: [9]

C1. Optimal Buying: given θ^b and γ, for all $h \in H$, $\varphi(h)$ solves the buyer's problem

$$
\begin{aligned}
v(h) = \max_{\varphi} & \int \theta^b(p)[\gamma(p, 1)(1-p) - \gamma(p, 0)hp] d\varphi(p) \\
\text{s.t.} & \int p d\varphi(p) \le w
\end{aligned}
$$

C2. Optimal Selling: given θ^s and μ, $\sigma(0)$ solves the bad seller's problem

$$
\begin{aligned}
u_0 = \max_{\sigma} & \int [\theta^s(p)\mathbb{E}_{\mu(p)}[h]p - c] d\sigma(p) \\
\text{s.t.} & \int d\sigma(p) \le 1
\end{aligned}
$$

[8] All payoffs are gross payoffs net of agents' utilities in autarky.

[9] Θ and $\hat{z}$ are Borel functions.

while $\sigma(1)$ solves the good seller's problem

$$u_1 = \max_{\sigma} \int [\theta^s(p)(p-\beta) - c] d\sigma(p)$$
$$\text{s.t.} \quad \int d\sigma(p) \leq 1$$

C3. Consistency: Let $\bar{\varphi} = \mathbb{E}_{\bar{\mu}}[\varphi(h)]$ and $\bar{\sigma} = \mathbb{E}_{\bar{\gamma}}[\sigma(a)]$.
(a) $\theta^s(p) = \min\{\frac{d\bar{\varphi}(p)}{d\bar{\sigma}(p)}, 1\}$ and $\theta^b(p) = \min\{\frac{d\bar{\sigma}(p)}{d\bar{\varphi}(p)}, 1\}$
(b)

$$\gamma(p, a) = \frac{d\sigma_a(p)}{d\bar{\sigma}(p)}$$

(c)

$$\mu(p, h) = \frac{d\varphi_h(p)}{d\bar{\varphi}(p)}$$

The equilibrium conditions C1-C3 are natural. C1 requires that buyers maximize their payoff by optimal search in markets subject to their feasibility constraints, given their belief on matching probability and seller composition in each market. C2 requires that sellers maximize their payoff by optimally setting up market subject to their feasibility constraint, given their belief on matching probability and buyer composition in each market. C3 requires that when a market is not empty, the matching probabilities conjectured by agents must coincide with the actual demand and supply in markets, and the beliefs about buyer and seller composition must be Bayesian consistent with agents' optimal strategies.

4 The Robust Equilibrium

In this section, we construct an equilibrium that features separation on buyers' side. We show that a buyer-separating equilibrium exists and it predicts a continuum of active markets with different asset composition and liquidity. Sellers, on the other side, pool in all but one market, which distinguish our result from other papers that examine adverse selection in exclusive markets, such as Gale (1992, 2001) and Guerrieri, Shimer, and Wright (2010).

Define $H(p, \theta^b(p), \gamma(p))$ as the set of buyer types who find market p optimal and $H^+(p, \theta^b(p), \gamma(p))$ the set of buyer types who find it strictly optimal, if they believe their matching probability is $\theta^b(p)$ and the composition of the other side is $\gamma(p)$. $A(p, \theta^s(p), \mu(p))$ as the set of sellers who find market p optimal and $A^+(p, \theta^s(p), \mu(p))$ the set of seller types who find it strictly optimal, if they believe their matching probability is $\theta^s(p)$ and the composition of the other side is $\mu(p)$.

Definition 2. An *equilibrium* is robust if, for any market $p \in P$, we have

R1

$$\max\{\theta^s(p), \theta^b(p)\} = 1$$

R2(a)

$$H(p, \theta^b(p), \gamma(p)) = \emptyset \Rightarrow \theta^b(p) = 1 \text{ and } A^+(p, \theta^s(p), \mu') = \emptyset \text{ for all } \mu' \text{ on } H$$
$$H(p, \theta^b(p), \gamma(p)) \neq \emptyset \Rightarrow \mu(p) \text{ is on } H(p, \theta^b(p), \gamma(p))$$

R2(b)

$$A(p, \theta^s(p), \mu(p)) = \emptyset \Rightarrow \theta^s(p) = 1 \text{ and } H^+(p, \theta^b(p), \gamma') = \emptyset \text{ for all } \gamma' \text{ on } A$$
$$A(p, \theta^s(p), \mu(p)) \neq \emptyset \Rightarrow \gamma(p) \text{ is on } A(p, \theta^s(p), \mu(p))$$

Without additional restrictions on beliefs in inactive market, many equilibrium can arise in markets with information asymmetry. In particular, we can shut down any market if we assign sufficiently pessimistic beliefs. Analogous to Banks and Sobel (1987) and Guerrieri, Shimer, and Wright (2010), the robustness conditions here jointly ensure that beliefs are sufficiently optimistic and in doing so they resolve the multiplicity problem.

Fix an inactive market. R1 says that belief is orderly: either sellers or buyers believe they will be matched with certainty. This is a reasonable restriction since only the long side in a market will be rationed. R2(a) says that if all buyers strictly prefer no to enter the market, the conjectured matching probability of buyers has to be one; moreover, sellers must not have strict incentive to enter in all possible buyer composition. Alternatively, if some buyers are indifferent between entering the market or not, sellers' belief must concentrate on the set of indifferent buyers: they believe that if they were to sell here, they would be randomly matched with a subset of the indifferent buyers.

What is underlying R2(a)? When a seller considers deviating to an inactive market, he initially imagines that buyers' matching probability is one. If the market is unable to attract any buyers, he is free to contemplate any possible buyer composition, and we require that he has no strict incentive to deviate in all possible scenarios. Otherwise, he thinks that some buyers would enter, which pulls down buyers' matching probability. In the process, some sellers leave as they no longer find the market attractive. When the adjustment is over, buyers' matching probability will make some buyers indifferent between deviating or not, and only these indifferent buyers can possibly stay until the end. Therefore, we require sellers' belief must concentrate on the set of indifferent buyers

R2(b) repeats the same requirement for the other side.

4 establishes the existence and payoff-uniqueness of the robust equilibrium under certain assumptions on primitives. In equilibrium, assets are simultaneously traded at many markets with different posted prices. Good and bad sellers pool together in almost all of these markets. This feature is distinctive. In an economy with homogeneously informed buyers, when good sellers are indifferent between two markets because the decline of liquidity exactly offsets the increase in price, bad sellers tend to prefer a low-price high-liquidity market because they derive low value from keeping their assets. In our model, however, both good and bad sellers are indifferent among a range of posted prices. This is because bad sellers are more likely to be rejected in the low-price market due to the presence of expert buyers. In fact, buyer expertise creates a wedge in selling probabilities between good and bad assets, which makes seller-pooling equilibrium possible. On the other side, an expert buyer has the comparative advantage to search in the low-price low-quality market precisely because they can utilize their expertise to reject bad assets.

Proposition. *Under certain assumptions on primitives, the robust equilibrium exists and it is payoff-unique. The equilibrium satisfies the following properties:*

(a) Buyers use monotone pure strategies: type h buyers search solely in the market with posted price $\frac{\beta\kappa}{\kappa-h}$, where κ is positive constant. Qualitatively, less sophisticated

buyers search in markets of higher prices.

(b) Sellers use mixed strategies. The proportion of good asset in market p is

$$\gamma(p,1) = [\frac{(1-\hat{p}) + \kappa(\hat{p}-\beta)}{(1-p) + \kappa(p-\beta)} \frac{p}{\hat{p}}]^{1/(1-\beta\kappa)} \tag{3}$$

which is strictly positive and increases with the posted price.

(c) In an active market, buyers are always matched while sellers can left unmatched. In particular, a seller's matching probability is

$$\theta^s(p) = \frac{u_1 + c}{p - \beta} \tag{4}$$

which decreases with the posted price.

Proof. Consider a candidate equilibrium of the following form.

(1) Buyers use monotone pure strategy: $\varphi(h)$ is an atom at $s(h)$ of size $\frac{w}{s(h)}$, where

$$s(h) = \frac{\beta\kappa}{\kappa - h}.$$

while sellers use mixed strategy:

$$d\sigma_0(p) = [1 - \gamma(p,1)]\theta^s(p)^{-1}\frac{w}{p}d\overline{\mu}(s^{-1}(p))$$

$$d\sigma_1(p) = \gamma(p,1)\theta^s(p)^{-1}\frac{w}{p}d\overline{\mu}(s^{-1}(p))$$

(2) In active markets, we have

$$\theta^b(p) = 1$$

$$\theta^s(p) = \frac{u_1 + c}{p - \beta}$$

$\mu(p)$ is a singleton at $s^{-1}(p)$ and $\gamma(p)$ satiesfies (3) and $\hat{p} = s(\overline{h})$.

(3) Market clearing:

$$\bar{\gamma}(1) = \int_{\underline{h}}^{\overline{h}} \gamma(s(h), 1)\theta^s(s(h))^{-1} \frac{w}{s(h)} d\overline{\mu}(h) \tag{5}$$

$$1 = \int_{\underline{h}}^{\overline{h}} \theta^s(s(h))^{-1} \frac{w}{s(h)} d\overline{\mu}(h) \tag{6}$$

(4) For inactive markets, we have

$$\theta^b(p) = \begin{cases} 1 & \text{if } p \geq \underline{\underline{p}} \\ \frac{p}{1-p} v(\overline{h}) & \text{otherwise} \end{cases}$$

$$\theta^s(p) = \begin{cases} 1 & \text{if } p < \underline{\underline{p}} \\ \frac{u_0+c}{\overline{h}p} & \text{if } p \in [\underline{\underline{p}}, \underline{p}] \\ \frac{u_1+c}{p-\beta} & \text{if } p > \hat{p} \end{cases}$$

$\mu(p)$ is a singleton at $\overline{h}$ and

$$\gamma(p) = \begin{cases} \text{singleton at } 0 & \text{if } p \in [\underline{\underline{p}}, \underline{p}] \\ \text{singleton at } 1 & \text{otherwise} \end{cases}$$

where $\underline{p} = s(\underline{h})$ and $\underline{\underline{p}} = \theta^s(\underline{p})\underline{p}\underline{h}/\overline{h}$.

Step 1: We want to show that in any robust equilibrium buyers will follow the strategies described above: an buyer of type h searches solely in the market with posted price $\frac{\beta\kappa}{\kappa-h}$. Suppose not, then from Lemma 4, he has to search in a market $p > \hat{p}$. If no sellers find $\hat{p}$ optimal, buyers are free to believe any seller composition at $\hat{p}$ and in particular, $\gamma'(1) = 1$. But then buyer h will strictly prefer to deviate to $\hat{p}$, a contradiction. If some sellers find $\hat{p}$ optimal, We want to show that $\gamma(\hat{p}, 1) = 1$. Otherwise, no buyers will find $\hat{p}$ optimal and sellers are free to believe any buyer composition. If bad sellers find $\hat{p}$ optimal, good sellers will strictly prefer to deviate to $\hat{p}$ with a belief such that $\mathbb{E}_{\mu'}[h] = \underline{h}$, a contraction. If only good sellers find $\hat{p}$ optimal, we have $\gamma(\hat{p}, 1) = 1$ by definition. With $\gamma(\hat{p}, 1) = 1$, buyer h still strictly prefers to deviate to $\hat{p}$, a contradiction.

Step 2: We want to show that (2) holds in any robust equilibrium. The result for buyers' matching probability directly comes from Lemma 2. The result for sellers' matching probability comes from good sellers' indifference condition. The result on buyer composition comes from buyers' equilibrium strategy. The result on seller composition comes from Lemma 5 and Lemma 6.

Step 3: We want to show that in any robust equilibrium sellers will follow the strategies described above. This is is directly from C3.

Step 4: We want to show that u_0 and u_1 are uniquely determined by two market clearing conditions. We can rewrite (5) as

$$\bar{\gamma}(1) = \frac{w}{u_0 + c} \int_{\underline{h}}^{\overline{h}} \gamma(s(h), 1) h d\overline{\mu}(h)$$

Similarly, rewrite (6) as

$$1 = \frac{w}{u_0 + c} \int_{\underline{h}}^{\overline{h}} h d\overline{\mu}(h) \tag{7}$$

Taking the ratio, we have

$$\bar{\gamma}(1) = \frac{\int_{\underline{h}}^{\overline{h}} \gamma(s(h), 1) h d\overline{\mu}(h)}{\int_{\underline{h}}^{\overline{h}} h d\overline{\mu}(h)} \tag{8}$$

From (7), we have $u_0 = w\mathbb{E}_{\overline{\mu}}[h] - c$. The RHS of (8) strictly increases in κ because

$$\gamma(s(h), 1) = (\frac{(1-\beta)\kappa - \overline{h} + \beta\kappa\overline{h}}{(1-\beta)\kappa - h + \beta\kappa h})^{1/(1-\beta\kappa)}$$

strictly increases in κ, so κ is uniquely determined. Then $u_1 = \frac{u_0+c}{\kappa} - c$ is also uniquely determined.

Finally, We need to check the participation constraints are satisfied for all agents.

First, we need $u_0, u_1 \geq 0$ which requires $w\mathbb{E}_{\overline{\mu}}[h] \geq c$ and $w\mathbb{E}_{\overline{\mu}}[h] \geq c\kappa$. Secondly, we need $v(\overline{h}) \geq 0$ which requires $s(\overline{h}) \leq 1$ or equivalently, $(1-\beta)\kappa \geq \overline{h}$. Finally, we need to make sure $\theta^s(\underline{p}) = \frac{u_1+c}{\underline{p}-\beta} \leq 1$, which is guaranteed under $w\mathbb{E}_{\overline{\mu}}[h] \leq \beta\underline{h}$. Combining all these inequalities, we need $c \leq w\mathbb{E}_{\overline{\mu}}[h] \leq \beta\underline{h}$ and $\frac{\overline{h}}{1-\beta} \leq \kappa \leq \frac{w\mathbb{E}_{\overline{\mu}}[h]}{c}$. To ensure the latter, we need additional restrictions on primitives: $w\mathbb{E}_{\overline{\mu}}[h] \geq c\frac{\overline{h}}{1-\beta}$ and $\bar{\gamma}(1) \in [g(\frac{\overline{h}}{1-\beta}), g(\frac{w\mathbb{E}_{\overline{\mu}}[h]}{c})]$, where

$$g(\kappa) = \int_{\underline{h}}^{\overline{h}} (\frac{(1-\beta)\kappa - \overline{h} + \beta\kappa\overline{h}}{(1-\beta)\kappa - h + \beta\kappa h})^{1/(1-\beta\kappa)} h d\overline{\mu}(h)/\mathbb{E}_{\overline{\mu}}[h]$$

Under the assumptions above, it is easy to show the candidate equilibrium constructed above satisfy all equilibrium and robustness conditions. Step 1-4 also establishes payoff-uniqueness. □

Lemma 1. *In a robust equilibrium, buyers' strategy is weakly monotone in expected purchase of bad assets.*

Proof. Take any $h_0, h_1 \in H$ with $h_0 < h_1$. Suppose in equilibrium h_0 finds p_0 optimal, and h_1 finds p_1 optimal. We want to show $\theta^b(p_0)\gamma(0,p_0) \geq \theta^b(p_1)\gamma(0,p_1)$.

Notice $p_0 \succeq_{h_0} p_1$ implies

$$\theta^b(p_0)[\frac{\gamma(1,p_0)}{p_0} - \gamma(1,p_0) - \gamma(0,p_0)h_0] \geq \theta^b(p_1)[\frac{\gamma(1,p_1)}{p_1} - \gamma(1,p_1) - \gamma(0,p_1)h_0]$$

□

and $p_0 \preceq_{h_1} p_1$ implies

$$\theta^b(p_0)[\frac{\gamma(1,p_0)}{p_0} - \gamma(1,p_0) - \gamma(0,p_0)h_1] \leq \theta^b(p_1)[\frac{\gamma(1,p_1)}{p_1} - \gamma(1,p_1) - \gamma(0,p_1)h_1]$$

Taking the difference, we have

$$\theta^b(p_0)\gamma(0,p_0)(h_1 - h_0) \geq \theta^b(p_1)\gamma(0,p_1)(h_1 - h_0)$$

By assumption $h_1 - h_0 > 0$, thus

$$\theta^b(p_0)\gamma(0, p_0) \geq \theta^b(p_1)\gamma(0, p_1) \tag{9}$$

Lemma 2. *In a robust equilibrium, buyers will not be rationed.*

Proof. We want to show that when market $p_0 \in P$ is optimal for some sellers, we have $\theta^b(p_0) = 1$.

If p_0 is optimal for bad sellers only, R2(b) says $\gamma(p_0, 1) = 0$. Buyers' payoff of searching in market p_0 is $-\theta^b(p_0)h$, and they will not find p_0 optimal [Need to add a tie breaking rule to rule out $\theta^b(p_0) = 0$]. Thus $\theta^b(p_0) = 1$. Now assume p_0 is optimal for good sellers.

Suppose $\theta^b(p_0) < 1$, then $\theta^s(p_0) = 1$. Since good sellers can trade with certainty at p_0, any $p_1 < p_0$ is strictly dominated for good sellers. Thus, either no sellers find p_1 optimal, or only bad sellers find p_1 optimal. In the latter case, we have $\gamma(p_1, 1) = 0$ and no buyers find p_1 optimal. Thus, there is no active market below p_0.

Maintain the assumption that $\theta^b(p_0) < 1$ and $\theta^s(p_0) = 1$. If $p_0 = 1$, all sellers will go to p_0. Thus $\gamma(p_0, 1) = \bar{\gamma}(1) < 1$. Additionally, D3(a) implies some buyers will also go to p_0. But this means they earn negative profit in equilibrium, a contradiction.

Maintain the assumption that $\theta^b(p_0) < 1$ and $\theta^s(p_0) = 1$. Consider the case where $p_0 < 1$. Let p_2 be an arbitrary contract and $p_2 \in (p_0, 1)$. If $\theta^s(p_2) = 1$, since good sellers can trade with certainty in both markets, they strictly prefer the market with a higher price, that is p_2. This contradicts the assumption in paragraph 1 that p_0 is optimal for good sellers. Thus $\theta^b(p_2) = 1$.

When p_2 is sufficiently close to p_0, some sellers have to find p_2 weakly optimal. Otherwise, buyers are free to contemplate any seller composition for market p_2 and in particular, $\gamma'(1) \geq \gamma(p_0, 1)$. In this case, buyers strictly prefer to search in the market with a slightly higher price, p_2, in exchange for the certainty of trade. But this contradicts R2(b). If only good sellers find p_2 optimal, we have $\gamma(p_2, 1) = 1 \geq \gamma(p_0, 1)$, buyers still strictly prefer p_2. Then from R2(a), we have $\theta^b(p_0) = 1$, which contradicts our earlier assumption. Thus, bad sellers must find p_2 optimal, i.e.,

$$\theta^s(p_2)\mathbb{E}_{\mu(p_2)}[h]p_2 \geq \theta^s(p_0)\mathbb{E}_{\mu(p_0)}[h]p_0$$

In addition, R2(a) says some buyers have to find p_2 optimal. From Lemma 1, we have $\mathbb{E}_{\mu(p_2)}[h] \leq \mathbb{E}_{\mu(p_0)}[h]$. But then we have

$$\theta^s(p_2)(p_2 - \beta) \geq \theta^s(p_0)(p_0 - \beta)$$

meaning good sellers strictly prefer p_2 to p_0, a contradiction. □

Lemma 3. *In a robust equilibrium, exactly one buyer type will search in any active market.*

Proof. We want to show for any active market p_0, $\mu(p_0)$ is a singleton.

First, we want to show that either $\gamma(p_0, 1) < 1$ or $\theta^s(p_0) < 1$. From paragraph 2 of the proof of Lemma 2, if $\theta^s(p_0) = 1$, p_0 is the lowest active market. Also Lemma 2 says buyers are not rationed in p_0. If, in addition, $\gamma(p_0, 1) = 1$, buyers strictly prefer p_0 to any higher market. Then p_0 is the only active market. Bad sellers strictly prefer to post p_0, which is inconsistent with $\gamma(p_0, 1) < 1$.

Since p_0 is active, $\mu(p_0)$ is not empty. Suppose more than one buyer types post p_0. Let h_0, h_1 be the smallest/largest buyer types that post p_0, so $h_0 < \mathbb{E}_{\mu(p_0)}[h] < h_1$.

Let p_1 be an arbitrary contract that is very close to p_0 and $p_1 \in (0, p_0)$. First, we want to show that either bad sellers find p_1 optimal or no sellers find p_1 optimal. If only good sellers find p_1 optimal, Lemma 2 says $\theta^b(p_1) = 1$ and $\gamma(p_1, 1) = 1$. But then buyers will not search in p_0 because they are strictly better off buying at a lower price, p_1. This contradicts the assumption that p_0 is active.

Consider the case that bad sellers find p_1 optimal, i.e,

$$\theta^s(p_1)\mathbb{E}_{\mu(p_1)}[h]p_1 \geq \theta^s(p_0)\mathbb{E}_{\mu(p_0)}[h]p_0$$

Since bad sellers find p_1 optimal, Lemma 2 says $\theta^b(p_1) = 1$. Suppose no buyers find p_1 optimal. Sellers are free to contemplate any buyer composition. If $\mathbb{E}_{\mu(p_1)}[h] \leq h_0 < \mathbb{E}_{\mu(p_0)}[h]$ [problematic!!!what if $\mathbb{E}_{\mu(p_1)}[h] \geq h_0$], we have

$$\theta^s(p_1)(p_1 - \beta) \geq \theta^s(p_0)(p_0 - \beta)$$

meaning good sellers strictly prefer p_2 to p_0, a contradiction. Suppose some buyers find p_1 optimal. Since some buyer finds p_0 optimal, it has to be $\gamma(p_1, 1) < \gamma(p_0, 1)$. From Lemma 1, buyer $h > h_0$ will not post p_1, and $\mathbb{E}_{p_1}[h] \leq h_0 < \mathbb{E}_{p_0}[h]$. But good sellers still strictly prefer p_1 to p_0, a contradiction.

Consider the case that no sellers find p_1 optimal. We have $\theta^s(p_1) = 1$ from R2(b). Buyers are free to contemplate any seller composition at p_1, in particular a γ' such that $\gamma'(1) \geq \gamma(p_0, 1)$. If $\theta^s(p_0) < 1$, good sellers will deviate to p_1 from p_0, when p_1 is sufficiently close to p_0,because they are not rationed at p_1. Thus, it has to be that $\gamma(p_0, 1) < 1$ from the first paragraph. If no buyers find p_1 optimal, sellers are free to contemplate any buyer composition and in particular, a μ' such that $\mathbb{E}_{\mu'}[h] \geq h_1 > \mathbb{E}_{\mu(p_0)}[h]$. But then bad sellers strictly prefer to deviate to p_1 from p_0, when p_1 is sufficiently close to p_0, because they are less likely rejected in market p_1. Thus there has to be some buyer h who finds p_1 optimal. Since we assume no sellers find p_1 optimal, buyers are free to contemplate any seller composition. If $\gamma(p_1, 1) < 1$, buyer h strictly prefers to deviate to p_1 for any $\gamma'(1) > \gamma(p_1, 1)$. Thus $\gamma(p_1, 1) = 1$. Since $\gamma(p_0, 1) < 1$ from above, we have $h \geq h_1$ from Lemma 1. Thus, $\mathbb{E}_{\mu(p_1)}[h] \geq h_1 > \mathbb{E}_{p_0}[h]$. But then bad sellers strictly prefer to deviate to p_1 from p_0 because they are less likely to be rejected at p_1. □

Lemma 4. *In a robust equilibrium, almost all active buyers use monotone pure strategy.*

Proof. Good sellers must present in all markets with buyers; otherwise, some buyers will have negative payoffs. This requires good sellers to be indifferent among all markets with buyers, i.e.,

$$u_1 = \theta^s(p)(p - \beta) - c, \tag{10}$$

which implies sellers' matching probability has to decrease with the posted price as in 4. Moreover, bad sellers also have to present in all markets with buyers except possibly for the highest-price one; otherwise, the highest-price market with buyers will be a dominated choice for all buyers. Thus, bad sellers are also indifferent among almost markets with buyers, i.e.,

$$u_0 = \theta^s(p)\mathbb{E}_{\mu(p)}[h]p - c. \tag{11}$$

Combining the indifferent condition of both good and bad sellers, we have $\mathbb{E}_{\mu(p)}[h] = \kappa\frac{p-\beta}{p}$, with $\kappa = \frac{u_0+c}{u_1+c} > 0$. Since $\mu(p)$ is a singleton in all markets with buyers, we know buyers of type $\kappa\frac{p-\beta}{p}$ search in market p, except possibly for the highest-price market with buyers. Inversely, active buyers of type h either search in market $\frac{\beta\kappa}{\kappa-h}$, or they search in the highest-price market with buyers. □

Lemma 5. *In a robust equilibrium,* $\gamma(p,1)$ *satisfies ODE*

$$\gamma_p(p,1)/\gamma(p,1) = \frac{1}{p(1-p+ph)}\Big|_{h=\frac{\kappa(p-\beta)}{p}}$$

on $(\frac{\beta\kappa}{\kappa-\underline{h}}, \frac{\beta\kappa}{\kappa-\hat{h}})$*, where* $\hat{h}$ *is the most sophisticated buyer type that earns zero profit.*

Proof. $v(h)$ is continuous and weakly increasing in h. We have $\hat{h} = \sup\{h \in H | v(h) > 0\}$. Since type h buyers almost surely go to $p = \frac{\beta\kappa}{\kappa-h}$ and $\theta^b(p) = 1$, we have

$$\frac{\beta\kappa}{\kappa-h} = \arg\max_p \gamma(1,p)(1-p)/p - \gamma(0,p)h$$

First order condition gives us the desired result. □

Lemma 6. *In a robust equilibrium,* $\lim_{p\uparrow\hat{p}} \gamma(p,1) = 1$*, where* $\hat{p} = \frac{\beta\kappa}{\kappa-\hat{h}}$.

Proof. Suppose not. Let p_1 be an active market that is very close to $\hat{p}$ and $p_1 < \hat{p}$. If $\hat{p} = 1$, no buyers will find p_1 optimal, a contradiction.

Now consider the case where $\hat{p} < 1$. Let p_2 be an arbitrary contract that is very close to $\hat{p}$ and $p_2 \in (\hat{p}, 1)$. If $\theta^s(p_2) = 1$, good sellers strictly prefer to deviate p_2 from p_1. Thus, $\theta^b(p_2) = 1$. If $\gamma(p_2, 1) = 1$, buyers strictly prefer to deviate from p_1 to p_2. Thus $\gamma(p_2, 1) < 1$. Notice that buyers at p_1 will strictly prefer to deviate to p_2 if $\gamma'(1) = 1$ and $p_{1,}p_2$ are sufficiently close. Thus, some seller has to find p_2 optimal and these sellers include bad sellers.

Assume no buyer finds p_2 optimal. Sellers are free to contemplate any buyer composition and in particular, a μ' such that $\mathbb{E}_{\mu'}[h] \leq \hat{h}$. But then good sellers will strictly prefer to deviate from p_1 to p_2, which is a contradiction.

Assume some buyer h finds p_2 optimal. If $h > \hat{h}$, $\hat{h}$ can earn positive profit at p_2, which contradicts the definition of $\hat{h}$. Thus $h \leq \hat{h}$. But then good sellers will strictly prefer to deviate from p_1 to p_2, which is a contradiction.

So far, we prove that $\lim_{p \uparrow \hat{p}} \gamma(p, 1) = 1$. Combining the boundary condition with the ODE, we have

$$\gamma(p, 1) = [\frac{(1-\hat{p}) + \kappa(\hat{p} - \beta)}{(1-p) + \kappa(p-\beta)} \frac{p}{\hat{p}}]^{1/(1-\beta\kappa)}$$

□

5 Price Dispersion and Asset Liquidity

In this section, we investigate how information heterogeneity affects welfare, asset liquidity and price dispersion. For price dispersion, we show that it decreases in the overall fraction of good assets and increases in the dispersion of buyer expertise. For liquidity, we show that if the overall fraction of good assets decreases or the dispersion of buyer expertise increases, a buyer will offer less liquidity to sellers.

Corollary. *Keep the assumptions on primitives in 4. Everything else being equal, price dispersion, measured as the max-min ratio, increases in the fraction of bad asset and the dispersion of buyer expertise. Liquidity provided by a buyer type decreases with the fraction of bad asset and the dispersion of buyer expertise.*

Proof. κ increases with $\overline{\gamma}(1)$ and decreases after a mean-preserving spread of $\overline{\mu}$. The max-min ratio of active prices is $\frac{s(\overline{h})}{s(\underline{h})} = \frac{\kappa - \underline{h}}{\kappa - \overline{h}}$, which decreases in κ. Liquidity provided by buyer type h to sellers is $\theta^s(s(h)) = w\mathbb{E}_{\bar{\mu}}[h]\frac{\kappa - h}{\beta h \kappa}$, which increases in κ. □

6 Conclusion

To be added.

References

Adelino, M., K. Gerardi, and B. Hartman-Glaser (2018): "Are lemons sold first? Dynamic signaling in the mortgage market," *Journal of Financial Economics*. 1

Banks, J. S., and J. Sobel (1987): "Equilibrium Selection in Signaling Games," *Econometrica*, 55(3), 647–661. 4

Chang, B. (2017): "Adverse Selection and Liquidity Distortion," *The Review of Economic Studies*, forthcoming. 1

Delacroix, A., and S. Shi (2013): "Pricing and signaling with frictions," *Journal of Economic Theory*, 148(4), 1301–1332. 1

Gale, D. (1992): "A Walrasian Theory of Markets with Adverse Selection," *The Review of Economic Studies*, 59(2), 229–255. 1

——— (2001): "Signaling in Markets with Two-Sided Adverse Selection," *Economic Theory*, 18(2), 391–414. 1

Guerrieri, V., and R. Shimer (2014): "Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality," *American Economic Review*, 104(7), 1875–1908. 1

Guerrieri, V., R. Shimer, and R. Wright (2010): "ADVERSE SELECTION IN COMPETITIVE SEARCH EQUILIBRIUM," *Econometrica*, 78(6), 1823–1862. 1, 1, 3, 4

Jankowitsch, R., A. Nashikkar, and M. G. Subrahmanyam (2011): "Price dispersion in OTC markets: A new measure of liquidity," *Journal of Banking & Finance*, 35(2), 343–357. 1

Kurlat, P. (2016): "Asset Markets With Heterogeneous Information," *Econometrica*, 84(1), 33–85. 1, 1

Kurlat, P., and F. Scheuer (2017): "Signaling to Experts," Working Paper 23817, National Bureau of Economic Research. 1