Rational Confusion:
Persistent Responses to Transitory Shocks

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This Draft: February 15, 2019
Preliminary draft – Please do not circulate

Abstract

In an analytical framework, we study how an economy with rationally inattentive firms responds to supply and demand shocks. Firms optimally choose to ignore the nature of shocks in favor of having a better estimate of how those shocks affect their prices. As a result, in our economy, when firms get signals that they should increase their price, they are confused about whether the underlying shock is a positive demand shock or a negative supply shock. This has significant implications for monetary policy: we prove that if monetary policy shocks are not persistent enough, every expansion caused by a positive monetary policy shock will lead to a recession as firms would interpret it as a negative supply shock. This favors policies such as interest rate smoothing.

1 Introduction

The dynamics of price-setters' expectations about the shocks in the economy is central to their pricing decisions, and therefore to the propagation of those shocks into aggregate output and price. While macroeconomic models are mostly built on the full-information rational expectations hypothesis, a new growing empirical literature on price-setters' expectations shows that price-setters are largely affected by informational frictions.[1] These new empirical findings call for new theories that incorporate these frictions into our models and study their implications for the dynamics of aggregate variables. This paper studies a fundamental question within this domain: how do expectations of rationally inattentive price setters evolve, when they face independent demand and supply shocks?

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[1]See for instance, Coibion, Gorodnichenko, and Kumar (2015); Kumar, Afrouzi, Coibion, and Gorodnichenko (2015).

I show that price-setters optimally choose to have correlated beliefs about exogenously independent shocks. In other words, they do not pay attention to how every single shock affects their profits, and instead focus on the resultant effect on their optimal pricing strategy. This result has drastic implications for how firms behave: firms perceive every demand shocks as a potential supply shock and vice versa. I apply this insight to a general equilibrium analysis of aggregate price and output in a setting with monetary policy and firm-specific productivity shocks, and show that every expansion caused by a transitory monetary policy shock is followed by a recession, as if there was a negative aggregate productivity shock in the economy.

This result hinges on firms' incentives in having correlated beliefs about the two shocks. When a transitory monetary policy shock happens, firms focus their attention on how their optimal price has changed, but they do not directly observe whether the change was due to shift in their supply or their demand. In dynamics, this affects the evolution of firms' expectations. Since firms assign some probability to a negative permanent productivity shock when a transitory monetary policy shock happens, they continue to charge higher than average prices even after the transitory shock to demand disappears, until their posteriors converge to the truth after they get more signals over time. Therefore, after a short term expansion due to the exogenous increase in demand, the economy enters a period of recession until firms fully recognize that the change in their optimal price had nothing to do with a productivity shock.

I show that three factors are important in determining the depth and the relative length of a recession that follows an expansionary monetary policy shock, the first one of which is the degree of rational inattention on the part of the price-setters. If firms have infinite capacity to process information, then fully observing their optimal price is equivalent to observing all independent shocks and calculating their resultant effect on their price. The latter, of course, assumes a finer information set for the firms but induces the same pricing strategy as the former. However, when the capacity of processing information is finite, then firms' confusion about the origins of shocks comes into play. In presence of the noisy signals that firms observe every period, they have to refer to their prior about how their optimal price has evolved, and what the contribution of each shock at each point of time has been on that price.

The second factor is the relative persistence of monetary policy shocks to productivity shocks. Firms keep their prices higher than usual when a transitory monetary policy shock happens because they are confused about the origin of the shock. The more persistent the productivity shocks are relative to the demand shocks, the longer it will take for prices to fall to their normal level.

The third factor is the relative volatility of monetary policy shocks to productivity shocks. Firms' posteriors about the origin of a change in their optimal prices depend on the relative contribution of each shock to those optimal prices. When productivity shocks are more volatile than monetary policy shocks, firms assign a higher probability to the possibility that a change in their optimal price is due to supply shocks, and continue to keep their prices higher than average for a longer period of time.

The paper also contributes to the literature by developing a new method for solving dynamic rational inattention models in continuous time environments, which allows me to derive analytical solutions to the general equilibrium model.

2 Model

This section introduces a general equilibrium model with rational inattention on the part of firms. In this section, I assume that there are only monetary policy shocks in the economy and derive closed form solutions for dynamics of inflation and output in the economy. The intuition provided in this section is helpful in interpreting the results in later sections where I extend the model to an environment with both monetary policy and productivity shocks.

2.1 Households and Monetary Policy

Households have full information rational expectations, and taking prices as given, form demand for a continuum of weakly substitutable goods in the economy, indexed by $i \in [0,1]$. In addition, they also choose to supply labor N_t in a competitive labor market with a given wage W_t, and have demand for a set of nominal bonds B_t in the economy given the net nominal interest rate i_t. Formally, the representative household's problem is

$$\begin{aligned}
&\max \mathbb{E}_0^f [\int_0^\infty e^{-\rho t}(\log(C_t) - N_t)dt] \\
s.t. &\frac{dB_t}{dt} = -\int_0^1 P_{i,t}C_{i,t}di + W_tN_t + i_tB_t + Tax \\
&C_t = \left[\int_0^1 C_{i,t}^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}.
\end{aligned}$$

The term Tax in the budget constraint refers to a lump sum constant tax over time that is used to finance a hiring subsidy for firms in order to eliminate the steady state inefficiencies of imperfect competition in the economy.

The implied demand of household for variety i is

$$C_{i,t} = C_t(\frac{P_{i,t}}{P_t})^{-\eta}.$$

Also, the Euler and labor supply equations are

$$i_t = \rho - \frac{\mathbb{E}_t[d\ln(Q_t)]}{dt},$$
$$W_t = Q_t,$$

where $Q_t \equiv P_t C_t$ is the nominal aggregate demand. The direct relationship that the Euler equation establishes between i_t and Q_t implies that the monetary policy can be formulated in terms of a law of evolution for the nominal aggregate demand. In particular, I assume that $\ln(Q_t)$ follows an Ornstein-Uhlenbeck process:

$$d\ln(Q_t) = -\theta \ln(Q_t)dt + \sigma_q dW_{q,t},$$

where $W_{q,t}$ is a Wiener process.

2.2 Firms

There is a measure of monopolistically competitive firms indexed by $i \in [0,1]$. They demand labor $(N_{i,t})_{i=0}^{1}$ from the household side given the nominal wage W_t, and produce with a linear production technology, $Y_{i,t} = N_{i,t}$. Firms are rationally inattentive and at any given increment dt of time, they cannot process more than $\tilde{\kappa}dt$ bits of information. At every point in time firms make two decisions in order to maximize the net present value of their lifetime profits: first they choose a signal $s_{i,t}$ to observe about the shocks in the economy subject to their rational inattention constraint and then set their price given their information set $S_{i,t} = \{s_{i,\tau}, \tau \leq t\}$ and their demand from the household side. They then hire enough labor to produce their implied demand given their chosen price.

Formally, firm i's problem is

$$\max_{(P_{i,t}(S_{i,t}),s_{i,t})_{t=0}^{\infty}} \mathbb{E}[\int_0^\infty e^{-\int_0^s i_s ds}(P_{i,t} - (1-\bar{\mathfrak{s}})W_t)C_t P_{i,t}^{-\eta} P_t^\eta dt | S_{i,0}]$$

$$s.t. \lim_{dt \to 0} \frac{\mathcal{I}(s_{i,t}, \ln(Q_t) | S_i^{t-dt})}{dt} \leq \tilde{\kappa}$$

$$S_{i,t} = \{s_{i,\tau}, \tau \leq t\}$$

$$S_{i,0} \text{ given.}$$

Here $\mathcal{I}(.;.)$ is Shannon's mutual information function, and $\bar{\mathfrak{s}}$ is the constant hiring subsidy per unit of labor that eliminates the steady state inefficiency of imperfect competition. The following lemma derives a closed form for rational inattention constraint.

Lemma 1. *Let $\sigma_{i,t}^2 \equiv var(\ln(Q_t)|S_{i,t})$ be the subjective variance of $\ln(Q_t)$ from firm i's perspective at time t. The attention constraint is given by*

$$\frac{d\sigma_{i,t}^2}{dt} \geq \sigma_q^2 - (\kappa + 2\theta)\sigma_{i,t}^2,$$

where $\kappa \equiv 2\ln(2)\tilde{\kappa}$ is a normalization of the capacity of firms in processing information.

This lemma shows that how rational inattention introduces a lower bound on how much uncertainty can be reduced at a given point in time. To see this, notice that if the agent does not learn any new information about the process of its optimal price, their subjective variance changes by $\sigma_q^2 - 2\theta\sigma_{i,t}^2$. Here, σ_q^2 is the variance that is introduced by the new increment of the Wiener process at that time. $-2\theta\sigma_{i,t}^2$, on the other hand, is the variance about past increments of the Wiener process that is excluded due to the transitory nature of them. Given this change in the variance that comes through the evolution of optimal price in time, the rational inattention constraint requires that the agent cannot reduce this variance by more than $\kappa\sigma_{i,t}^2$ so that the total slope of this uncertainty has to be larger than $\sigma_q^2 - 2\theta\sigma_{i,t}^2 - \kappa\sigma_{i,t}^2$.

From now on, I consider a second order approximation to the problem of firms. Given this approximation, firm's objective in choosing its prices becomes

$$\min_{(p_{i,t}(S_{i,t}))_{t=0}^{\infty}} \int_0^\infty e^{-\rho t}\mathbb{E}[(p_{i,t}(S_{i,t}) - p_t^*)^2 | S_{i,0}]dt,$$

where $p_{i,t}(S_{i,t}) \equiv \ln(P_{i,t}(S_{i,t}))$ and $p_t^* \equiv \ln(Q_t)$.

2.3 The Optimal Information Structure of Firms

It is straight forward to show that the optimal pricing strategy of the firm, given an arbitrary information set, is

$$p^*_{i,t}(S_{i,t}) = \mathbb{E}[p^*_t | S_{i,t}].$$

Hence, the information choice of the firm simply becomes:

$$\min_{\sigma^2_{i,t}} \int_0^\infty e^{-\rho t} \sigma^2_{i,t} dt$$

$$s.t. \frac{d\sigma^2_{i,t}}{dt} \geq \sigma^2_q - (\kappa + 2\theta)\sigma^2_{i,t},$$

$$\sigma^2_{i,0} \text{ given.}$$

The solution to this problem is trivial. Every firms' problem is to minimize its subjective variance of their optimal price over time, subject to the fact that the slope of this uncertainty has to always be larger than $\sigma^2_q - (\kappa + \theta)\sigma^2_{i,t}$. The firm's optimal behavior is to make this constraint always bind. This leads to a first order differential equation in $\sigma^2_{i,t}$. It is straight forward to show that under the optimal behavior of firms

$$\sigma^2_{i,t} = \sigma^2_{i,0} e^{-(\kappa+2\theta)t} + \frac{\sigma^2_q}{\kappa + 2\theta}(1 - e^{-(\kappa+2\theta)t}). \tag{1}$$

This law of motion shows that under rational inattention a firm's subjective variance of its optimal price evolves away from their initial uncertainty towards a steady state value that only depends on the key parameters of the model:

$$\lim_{t \to \infty} \sigma^2_{i,t} = \frac{\sigma^2_q}{\kappa + 2\theta}.$$

This steady state variance, which is proportional to firms' losses in profits from imperfect information, increases with the intensity of the innovations to their optimal price, σ_q. Moreover, a lower persistence in the process of this optimal price, which translates into a larger θ, decreases these losses as it makes the shocks to the process more transitory. Finally, a higher capacity to process information reduces this loss because firms are able to reduce their uncertainty at a higher rate.

In addition to their effect on the steady state subjective variance, κ and θ also affect the speed of transition to this steady state from any initial variance. A firm with a higher capacity to process information, or facing a process with more transitory shocks, reaches its steady state subjective variance faster.

Since the information problem of the firms reduces to a path for $\sigma_{i,t}^2$, a signal structure is optimal if and only if it induces this path. Moreover, since this is a linear-quadratic rational inattention problem, we know that the optimal signals are Gaussian processes.[2] Given this, it is straight forward to show that the following signal structure is optimal:

$$s_{i,t}^* = p_t^* dt + \sigma_{s,t} dW_{i,s,t},\ dW_{i,s,t} \sim \mathcal{N}(0, dt),$$

where $\sigma_{s,t}^2 \equiv \kappa^{-1}\sigma_{i,t}^2$, where $\sigma_{i,t}^2$ is given by Equation (1) .[3]

2.4 Characterization of the General Equilibrium

Definition 1. A general equilibrium for this economy is an allocation for $(C_{i,t}, B_t, N_t)_{i=0}^1$ for household, an information structure for firms $(S_{i,t})_{t=-\infty}^{\infty}, \forall i \in [0,1]$, a pricing function for firms $P_{i,t} : S_{i,t} \to \mathbb{R}, \forall i, t$ along with an allocation $(N_{i,t}, Y_{i,t})_{t=0}^{\infty}, \forall i$, and an stochastic process for aggregate prices, wage and nominal rates, $(P_t, W_t, i_t)_{t=0}^{\infty}$, such that

1. Given prices, household's allocation solves their problem.
2. Given the stochastic process for aggregate price and wage, $(S_{i,t}, P_{i,t}(S_{i,t}))_{t=0}^{\infty}$ solves firm's problem given $S_{i,0}, \forall i$ such that $Y_{i,t} = N_{i,t}$ satisfies firms demand at any point in time.
3. Markets clear, and aggregate demand follows the monetary policy.

The following proposition characterizes this general equilibrium.

Proposition 1. ***(Equilibrium output and prices)*** *Let $y_t \equiv \ln(Y_t)$ and $p_t \equiv \ln(P_t)$ denote the logs of output and aggregate price. In the equilibrium*

1. *Log of output follows an Ornstein-Uhlenbeck process:*

$$dy_t = -(\kappa + \theta) y_t dt + \sigma_p dW_{q,t},$$

and is given by

$$y_t = \sigma_p \int_{-\infty}^{t} e^{-(\kappa+\theta)(t-\tau)} dW_{q,\tau}.$$

[2]See Cover and Thomas (2012).

[3]To see this, let $S_{i,t}^* = \{s_{i,\tau}^* | \tau < t\}$, and $\sigma_{i,t}^{*2} \equiv var(p_t^* | S_{i,t})$. Note that $d\sigma_{i,t}^{*2}/dt = \sigma_q^2 - \theta\sigma_{i,t}^{*2} - \sigma_{i,t}^{*4}/\sigma_{s,t}^2$. Set $\sigma_{s,t}^2 = \kappa^{-1}\sigma_{i,t}^{*2}$ and observe that this implies

$$\frac{d\sigma_{i,t}^{*2}}{dt} = \sigma_q^2 - (\theta + \kappa)\sigma_{i,t}^{*2}.$$

2. *Log of aggregate price follows*

$$dp_t = -(\theta + \kappa) p_t dt + \kappa q_t dt,$$

and is given by

$$p_t = \sigma_p \int_{-\infty}^{t} e^{-\theta(t-\tau)} (1 - e^{-\kappa(t-\tau)}) dW_{q,\tau}.$$

This proposition shows that how rational inattention leads to monetary non-neutrality in this economy, and how prices adjust slowly and sluggishly to a monetary policy shock. Given the analytical solutions for the evolution of the aggregate price and output, the following corollary derives the impulse response functions of these variables to monetary policy shocks over time.

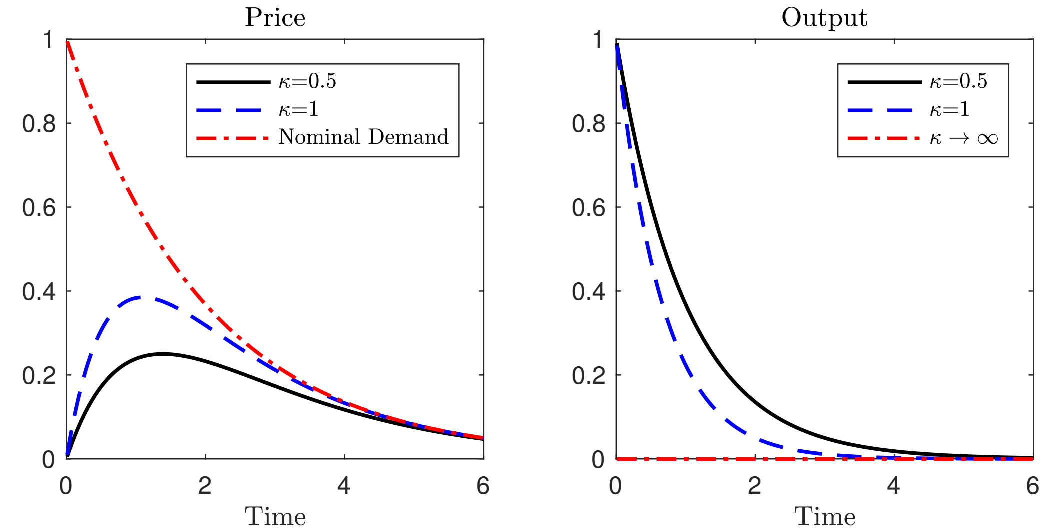
Corollary 1. *There is monetary non-neutrality under rational inattention, and output responds persistently to a shock to aggregate demand. Formally, the impulse response of output and price to a one standard deviation shock to $dW_{q,t}$ at time zero is given by*

$$y_t|_{dW_{q,0}=1} = \sigma_p e^{-(\kappa+\theta)t},$$
$$p_t|_{dW_{q,0}=1} = \sigma_p e^{-\theta t}(1 - e^{-\kappa t}).$$

Moreover, the half-life of output response is $\frac{\ln(2)}{\kappa+\theta}$.

This corollary summarizes the heart of this very simple model. While there is complete price flexibility in this economy and firms ideally want to set their prices equal to q_t, in presence of rational inattention, it takes time for them to fully recognize a shock to aggregate demand and adjust their nominal prices accordingly. Meanwhile, during the time that it takes for firms to do this, the economy experiences an increase in real output. Therefore, the duration of the boom, or equivalently the persistence of output response, is a direct function of the fundamental persistence of the aggregate demand shock, and also the capacity of firms in processing information, as it is shown in the expression for its half-life. Figure (I) depicts a numerical example of these impulse response functions for different values of κ, and shows how higher capacity leads to a more transitory response of output in the equilibrium.

Figure I: Impulse Responses in the Simple Model for Different Values of κ



The figure depicts the impulse response functions of aggregate price and output to a 1 standard deviation shock to the nominal aggregate demand, for different values of κ. Prices adjust sluggishly to an expansionary shock under rational inattention, which leads to an transitory increase in output. Prices adjust faster and output response is less persistent when firms' capacity to process information is higher. For this exercise, I have set $\theta = 0.5$ and $\sigma_q = 1$. See Corollary (1) for an analytical expression of these functions.

2.5 A Model with Productivity Shocks

This section builds on the simple model of the previous section by introducing productivity shocks to the firm side of that model. The goal is to investigate the informational choices of rationally inattentive firms in presence of aggregate demand shocks and productivity shocks.

The setup of the model is the same as in the simple model, with only one difference in the production function of firms. Every firm $i \in [0, 1]$ now produces with $Y_{i,t} = Z_{i,t} N_{i,t}$, where $Z_{i,t}$ is an exogenous productivity shock to the firm. I assume that $z_{i,t} \equiv \ln(Z_{i,t})$ is a Brownian motion,

$$dz_{i,t} = \sigma_z dW_{z,i,t}, \; dW_{z,i,t} \sim \mathcal{N}(0, dt).$$

where $dW_{z,i,t}$'s are independent across firms at any given point in time. Firms are again rationally inattentive and now pay attention to both productivity and aggregate demand shocks. Formally, the firms problem is

$$\max_{(P_{i,t}(S_{i,t}),s_{i,t})_{t=0}^{\infty}} \mathbb{E}[\int_0^\infty e^{-\int_0^s i_s ds}(P_{i,t} - (1-\bar{s})\frac{W_t}{Z_{i,t}})C_t P_{i,t}^{-\eta} P_t^{\eta} dt | S_{i,0}]$$

$$s.t. \lim_{dt \to 0} \frac{\mathcal{I}(s_{i,t}, \begin{bmatrix} q_t \\ z_{i,t} \end{bmatrix} | S_i^{t-dt})}{dt} \leq \tilde{\kappa}$$

$$S_{i,t} = \{s_{i,\tau}, \tau \leq t\}$$

$$S_{i,0} \text{ given.}$$

With a similar second order approximation as before, the objective of the firm i becomes

$$\min_{(p_{i,t}(S_{i,t}),s_{i,t})_{t=0}^{\infty}} \mathbb{E}[\int_0^\infty e^{-\rho t}(p_{i,t}(S_{i,t}) - p^*_{i,t})^2 dt | S_{i,0}],$$

where $p^*_{i,t} \equiv q_t - z_{i,t}$ is the firm's optimal price under full information. It is straight forward to show that the optimal pricing strategy of the firm is

$$p_{i,t}(S_{i,t}) = \mathbb{E}[p^*_{i,t} | S_{i,t}].$$

Afrouzi and Yang (2017) show that in the steady state of this problem firms strictly prefer to see one signal at every point in time. Similar to Afrouzi (2017), here I focus on a special case where $\rho \to \infty$, and prove the optimality of a single signal for this case. I argue in later sections that this case gives a lower bound on the results of this paper.

Proposition 2. *When $\rho \to \infty$, firms strictly prefer to see only one signal at any point in time. The optimal signal at time t is of the form*

$$s_{i,t} = p^*_{i,t} dt + \sigma_{s,i,t} dW_{s,i,t}$$

where $\sigma^2_{s,i,t} = \frac{var(p^*_{i,t}|S_{i,t})}{\kappa}$.

Proposition 2 shows that when firms are myopic, they focus on observing their optimal price, as precisely as allowed by their rational inattention constraint. In particular, firms do not intend to separate the aggregate demand shocks from the productivity shocks. They optimally decide to forgo differentiating these shocks from each other as doing so would consume a strictly positive amount of their attention. In other words, instead of consuming capacity to observe the shocks separately, they decide to see a more precise signal of how these shocks jointly affect their optimal price.

The next proposition analytically characterizes the equilibrium process of output under this optimal signal structure.

Proposition 3. ***(Equilibrium with Aggregate Demand and Productivity Shocks)*** *In the equilibrium,*

1. *Aggregate output follows a continuous time $AR(2)$ process:*

$$\frac{1}{dt^2}d^2y_t = -(\theta+\kappa)\frac{1}{dt}dy_t + \gamma y_t + \sigma_q \frac{1}{dt^2}d^2W_{q,t},$$

where $\gamma \equiv \theta\sqrt{(\theta\frac{\sigma_z^2}{\sigma_z^2+\sigma_q^2}+\kappa)^2 - \kappa^2\frac{\sigma_q^2}{\sigma_z^2+\sigma_q^2}} - \theta^2\frac{\sigma_z^2}{\sigma_z^2+\sigma_q^2}$.

2. *Moreover, the impulse response of output to a 1 standard deviation shock to aggregate demand is given by*

$$y_t|_{dW_{q,0}=1} = \sigma_q\left[\frac{\alpha_1}{\alpha_1-\alpha_2}e^{-\alpha_1 t} + \frac{\alpha_2}{\alpha_2-\alpha_1}e^{-\alpha_2 t}\right],$$

where $\alpha_1, \alpha_2 = \frac{\theta+\kappa\pm\sqrt{(\theta+\kappa)^2-4\gamma}}{2}$.[4]

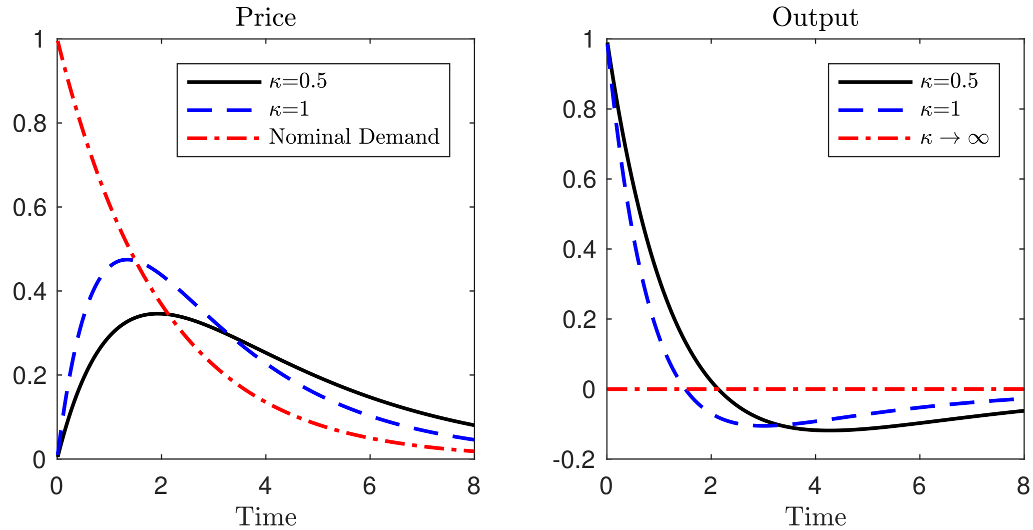
The next section discusses this result extensively, and studies its implications.

3 Discussion of Results

Proposition 3 shows how the presence of productivity shocks fundamentally changes the response of aggregate price and output. This drastic change hinges on two factors: first, the difference in the persistence of nominal demand and productivity shocks, and second the inability of the firms in telling them apart due to their optimal informational choice given their rational inattention constraint.

[4] The process for aggregate price is implied by $p_t = q_t - y_t$.

Figure II: Impulse Responses of Aggregate Output and Price for Different Values of κ



The figure depicts the impulse response functions of aggregate price and output to a 1 standard deviation shock to the nominal aggregate demand, for different values of κ. Prices remain high even after the demand shock starts to disappear due to the confusion of firms about the origin of the shock. Prices adjust faster and output response is less persistent when firms' capacity to process information is higher. For this exercise, I have set $\theta = 0.5$ and $\sigma_q = \sigma_z = 1$. See Proposition 3 for analytical expressions of these impulse responses.

Figure (II) shows the impulse responses of aggregate price and output to a 1 standard deviation shock to nominal demand, for different values of κ. When a positive nominal aggregate demand shock hits the economy, the average firm starts observing signals whose values are larger than their long-run mean. Due to observing only one signal, the firm infers from these signals that either a positive demand shock has happened in the economy, or that the firm has been affected by a negative productivity shock. The response of prices, therefore, is a combination of how firms would react to each of these shocks.

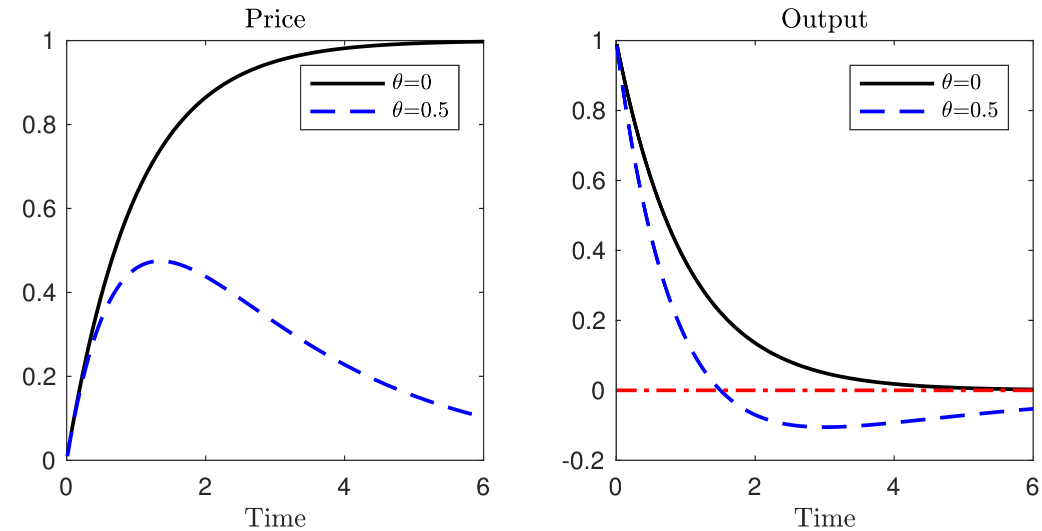
The key observation is that in contrast to the model with only demand shocks where prices were always below the nominal aggregate demand, in the model with both shocks prices overshoot the nominal aggregate demand in the transition path. The reason behind this is based on the higher persistence of the productivity shocks relative to demand shocks. As firms are not able to fully differentiate the source of the change in their prices, in every moment of the transition path, they assign a positive probability to the hypothesis that they are experiencing a negative productivity shock. Since productivity shocks live much longer than demand shocks, firms continue to keep their prices high even after the demand shock starts to disappear. Thus, after any positive demand shock, the economy enters a phase where prices are disproportionately higher than aggregate demand, and the economy enters a recession where output falls below its steady state level. The following proposition formalizes this argument.

Proposition 4. *Every expansion caused by an aggregate demand shock is followed by a recession as long as $\theta > 0$ and $\sigma_z > 0$. In particular, the net effect of a demand shock on output is zero over time:*

$$\int_0^\infty y_t|_{dW_{q,0}=1} dt = 0.$$

Firms' confusion about the origin of the shock becomes irrelevant when the persistence of the two shocks are the same. Figure (III) shows that the two shocks are similarly persistent, output response is uniformly positive.

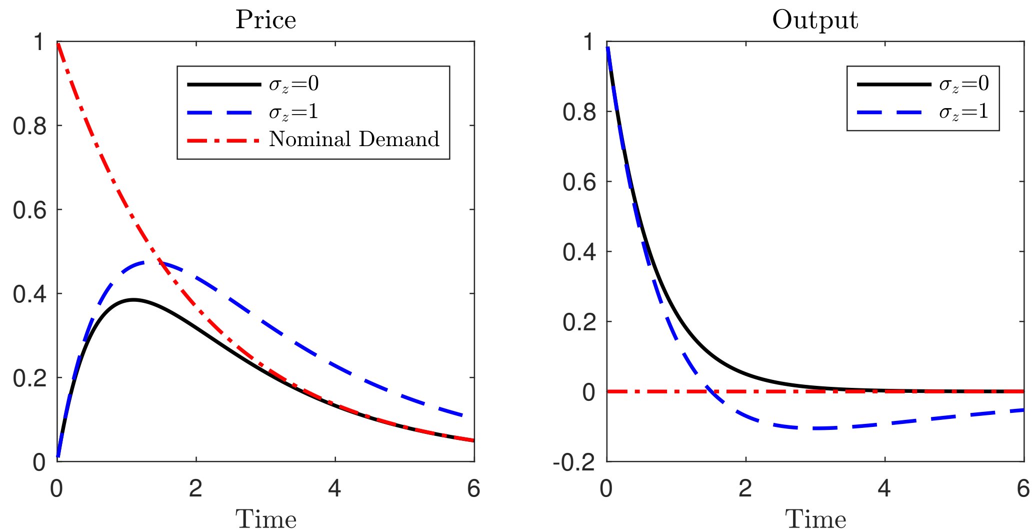
Figure III: Impulse Responses of Aggregate Output and Price for Different Values of θ



The figure depicts the impulse response functions of aggregate price and output to a 1 standard deviation shock to the nominal aggregate demand, for different values of θ. Firms' confusion about the origin of the shock becomes irrelevant when the two shocks are similarly persistent. For this exercise, I have set $\sigma_q = \sigma_z = 1$, and $\kappa = 1$. See Proposition 3 for analytical expressions of these impulse responses.

Moreover, the degree of firms' confusion about the origin of the shock directly relates to the relative intensity of innovations to productivity and demand shocks. For instance, in the extreme case when $\frac{\sigma_z}{\sigma_q} = 0$, then the firms know that any change in their optimal price is solely driven by demand shocks and this economy reverts back to the simple model of the previous section. However, as this ratio increases the relative contribution of demand shocks to firms' signals becomes smaller, and firms assign higher probabilities to a potential negative productivity shock in face of an aggregate demand shock. Accordingly, the contractionary effects of a positive demand shock is more pronounced when productivity shocks are more volatile. Figure (IV) depicts this effect.

Figure IV: Impulse Responses of Aggregate Output and Price for Different Values of σ_z



The figure depicts the impulse response functions of aggregate price and output to a 1 standard deviation shock to the nominal aggregate demand, for different values of σ_z. Firms' confusion about the origin of the shock disappears when $\frac{\sigma_z}{\sigma_q} = 0$. For this exercise, I have set $\theta = 0.5$, $\sigma_q = 1$ and $\kappa = 1$. See Proposition 3 for analytical expressions of these impulse responses.

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