

On the Political Economy of Financial Regulation*

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Abstract

Loose financial regulation encourages some banks to adopt a risky strategy of specializing in residential mortgages. In the event of an adverse aggregate housing shock, these banks fail. When banks do not fully internalize the losses from such failure (due to limited liability or deposit insurance), they offer mortgages at less than actuarially fair interest rates. This opens a door to home-ownership for some young low net-worth individuals. In turn, the additional demand from these new home-buyers drives up house prices. All of this leads to non-trivial distribution of gains and losses from lax regulation amongst the households. Renters and individuals with large non-housing wealth suffer from the fragility of the banking system induced by the lax regulation. On the other hand, some young middle-income households are able to get a mortgage and buy a house, thus benefiting from the lax regulation. Furthermore, the current (old) homeowners benefit from the increase in the price of their houses. If the latter two groups constitute a majority of the population, then regulatory failure can be an outcome of a democratic political process.

*The views expressed here are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia, the Federal Reserve System, or the Bank of Canada.

1 Introduction

Many have argued that the “bubble” in the housing market, whose burst led to the 2008 financial crisis, was fueled by “irresponsible” lending practices of mortgage lenders (e.g., [Acharya et al. \(2011\)](#), [Brunnermeier \(2009\)](#), [Dell’Ariccia, Igan, and Laeven \(2012\)](#), and [Mian and Sufi \(2009\)](#)). This excessive risk-taking by lenders (banks) was permitted by the regulatory environment of the early 2000s (see, for example, [Bernanke \(2010\)](#) and [Zingales \(2008\)](#)). In this paper, we point out that democratic political economy mechanism may lead to insufficient banking regulation, which permits excessive risk-taking on mortgage loans and inflates a house-price “bubble.” This occurs when the beneficiaries of such policy (existing home owners plus “subprime” buyers who would not be able to borrow under a tighter regulation) outnumber its opponents (buyers who could afford a house under tight regulation and renters, who get exposed to the fragility of the financial system).

The economic mechanism underlying the political considerations is an intuitive one. Loose financial regulation permits some banks to adopt a “gambling” strategy of specializing in *risky* residential mortgages. In the event of an adverse aggregate housing shock, these banks fail. Since these banks do not fully internalize the losses from such failure (due to limited liability or deposit insurance), they offer mortgages at less than actuarially fair interest rates. This opens a door to home-ownership for some young low net-worth individuals. In turn, the additional demand from these new home-buyers drives up house prices. All of this leads to non-trivial distribution of gains and losses from lax regulation amongst the households. Renters and individuals with large non-housing wealth suffer from the fragility of the banking system induced by the lax regulation. On the other hand, some young middle-income households are able to get a mortgage and buy a house, thus benefiting from the lax regulation. Furthermore, the current (old) homeowners benefit from the increase in the price of their houses. If the latter two groups constitute a majority of the population, then regulatory failure can be an outcome of a democratic political process.

To capture this key mechanism, we present a parsimonious two-period model populated by overlapping generations of heterogeneous households. In the first period, initial old people are endowed with houses, while initial young are endowed with heterogeneous wealth. The young derive utility from owning a house and are thus interested in buying a house from the initial old or from the newly constructed stock. Construction technology is subject to decreasing returns to scale. Young households can finance a house purchase from their initial wealth and by taking on defaultable non-recourse mortgages from banks (or foreign investors). We abstract from idiosyncratic uncertainty, so the only source of mortgage defaults in the second period is a possible adverse realization of an aggregate house-value shock. Thus, mortgage portfolios are subject to aggregate risk. Since domestic households are risk-averse and the mortgage risk is non-diversifiable, risk-neutral foreign investors have a natural advantage at holding mortgages. Yet, domestic banks, which hold all the households’ deposits, are subject to limited liability and have an incentive to invest in risky mortgages at actuarially fair interest rates. This is the moral hazard in banking, which needs prudential regulation to address it. Prudential regulation in the model takes the form of a

risk-weighted capital requirement. We will refer to regulation as effective as long as it precludes the risk of bank failure (which is defined as depositors not being repayed in full). In contrast, regulation that does permit bank failure with positive probability in equilibrium will be referred to as lax.

The key finding of the paper is that a lax regulation, while failing to prevent systemic risk in the banking system, may actually be preferred to an effective regulation by *majority* of agents in the economy. This majority is composed of two distinct groups — low-wealth young buyers, who benefit from resulting lax lending standards and are thus able to finance purchasing a home, and old home-owners, who benefit from the increased house prices bid up by the additional buyers. While the finding is rather intuitive, it is worth noting that it demands some key ingredients from the model. For example, we would not get our main result without a decreasing returns-to-scale construction industry. If the stock of houses is fixed, then the only beneficiaries of lax regulation are the old home-owners, as the house price increase in equilibrium of that model absorbs all of the gains from lax lending standards, and the pool of equilibrium home-buyers remains unchanged. Linear construction technology, on the other hand, would preclude house prices from rising in response to lax banking regulation, thus limiting gains from such regulation to poorer young home-buyers. Our model is thus a minimal environment needed to generate the *coalition* of young (poor home-buyers) and old (home-sellers) that favours lax regulation.

This research contributes to the nascent literature on the political economy of the mortgage crisis (Mian, Sufi, and Trebbi (2010), Mian, Sufi, and Trebbi (2013), Igan, Mishra, and Tressel (2012)).¹ But whereas these papers focus on the effects of lobbying (by financial institutions), we emphasize a more directly democratic mechanism by focusing on which voters stood to gain from lax regulation.² Our analysis further highlights the importance of heterogeneity (pointed to by Rajan (2010)). Yet again, the heterogeneity we are focusing is of a more “democratic” nature. It has little to do with the concentration of wealth and income at the top but rather highlights large categories of population who stand to win (and lose) from deregulation-induced housing boom. This paper is also clearly related to the ample literature on regulatory failure (ranging from rational, as in Brusco and Castiglionesi (2007), to ones driven by time-inconsistency of policy makers, as in Chari and Kehoe (2008) for example, to ones arising from the policy maker’s own moral hazard, as in D’Erasmus, Livshits, and Schoors (2019)). Unlike the latter, we abstract from the agency problem of the regulator, and focus instead on the possible *democratic* political roots of the regulatory failure.

The rest of the paper is organized as follows. Section 2 presents the model environment. Section 3 presents individual agents’ problems and defines the economic equilibrium for a given set of policies (regulation). Section 4 characterizes the economic equilibrium in two key benchmark settings — laissez-

¹While literature on the mortgage crisis in general is too vast to list here, we do want to single out Gupta (2018), who highlights how decisions of (large) lenders to extend risky mortgages affected house prices.

²We are offering a complementary political economy mechanism for the laxness of financial regulation of the mortgage industry in the early 2000s. Notably, these two political mechanisms (lobbying and popular support) may interact in interesting ways. For example, where lobbying efforts directed mostly at congress members from districts with disproportionate fraction of “winners” from the lax regulation (e.g., larger share of young subprime borrowers), which would be more easily swayed by such lobbying? Or were lobbying efforts directed more at the other districts, as parliamentarians from “winning” districts would be expected to vote for the lax regulation regardless?

fare and effective regulation. Section 5 highlights the key finding of the paper and analyzes political economy aspects of the model. Section 6 discusses an important additional insight that arises in a fully dynamic extension of the model — it takes into account the effect of the forward-looking regulation (of future banks) on solvency of the current banking sector, which works through the current house prices and their effects on the current default rate of (old) incumbent homeowners. Section 7 concludes.

2 Environment

We formulate a parsimonious almost-static model built to highlight the key economic mechanism. The model economy lasts for two periods. In the first period, it is populated by measure 1 of old people, of whom fraction λ_0 own houses (and land), and measure $(1 + n)$ of young people, who live for two periods. Besides these individuals, the economy has bankers, who live for two periods, and construction firms, who operate in the first period. All agents in the economy are risk-neutral. Besides houses, which do not depreciate, there is a single perishable consumption good per period.

2.1 Households

Individuals consume perishable good when old, and derive utility u from home-ownership.³ Young households receive idiosyncratic income y (of perishable good) in the beginning of the first period.⁴ It is drawn from distribution $F(\cdot)$. The endowment can be spent on purchasing a house or invested in a bank. Households have no other investment technology.

2.2 Banks and Foreign Investors

There a continuum of foreign investors who can freely enter the domestic banking sector. Investors are risk-neutral and are endowed with initial wealth, which can serve as a capital if they open a bank. Some investors have strictly positive wealth while others have zero wealth. Besides domestic banking sector, these investors have access to storage technology with rate of return \bar{r} . You can simply think of \bar{r} as the world interest rate. We will normalize \bar{r} to 0.

2.3 Construction

New houses are produced by measure 1 of competitive construction firms with identical decreasing returns-to-scale production functions. The firms are owned by old households.⁵ The cost of producing I

³All of the analysis goes through under the assumption of risk-neutrality. In order to maximize the simplicity of the model, we maintain that assumption, though results would, of course, be more striking if households were risk-averse. In the current benchmark, the downside to any excessively risky behaviour of the banks is a lower rate of return on savings, rather than misallocation of risk.

⁴This can be thought of as random endowment of efficiency units of labor, which is inelastically supplied to a production sector that transforms a unit of labor into w unit of the perishable consumption good. This labor is also used in construction.

⁵Alternatively, we can assume that the construction firms are foreign-owned.

units of housing is $k(I) = \kappa I^2/2$.

2.4 Houses

Houses are subject to an aggregate valuation shock in the second period. The house value (price) is either high, v_H , with probability p , or low, v_L , with probability $(1 - p)$. We set v_L to 0.⁶

2.5 Investments and Financial Markets

Banks can invest in two types of assets: safe projects with deterministic rate of return \bar{r} , and mortgages, which are defaultable. We will denote investments in the safe assets by s , and investments in mortgages by m . Then bank i 's balance sheet can be simply put as $s_i + m_i = e_i + d_i$, where e_i is the bank's equity and d_i is the amount of deposits in bank i .

Mortgages are non-recourse, which immediately implies that they will be repayed in equilibrium only if the value of the house (weakly) exceeds the face value of the mortgage. All "underwater" mortgages are defaulted on. The mortgage interest rates reflect the implied risk of default.

2.6 Regulation

A regulatory authority can impose capital requirements on banks operating in the economy (by "operating" we mean accepting deposits). A capital requirement prescribes that bank(er)s must own fraction α of a bank's risk-weighted total assets:

$$e_i \geq \alpha(\omega_s s_i + \omega_m m_i), \quad (1)$$

where e_i is bank i 's equity; s_i and m_i are the bank's investments into safe assets and mortgages, respectively; ω_s and ω_m are the risk weights assigned to the two assets. We will focus on the setting with $\omega_s = 0$ and $\omega_m = 1$.

3 Economic Equilibrium

This section spells out individual agents problems and defines the economic equilibrium of the model for any given regulation. For now, the slightly messy statements of the individuals' problems are relegated to the Appendix A.

For any given regulation, the equilibrium in this economy is characterized by

- House prices and land prices in the first period;
- Interest rates on deposits and on mortgages;

⁶This is more than a normalization, as this implies zero recovery for the lenders in the event of foreclosure.

- Measure of houses constructed;
- Total equity of banks (deposit-taking institutions) that are safe;
- Total equity of banks that are risky (i.e., default on deposits in the second period if house values fall);
- Amount of deposits in risky banks;
- Amount of deposits in safe banks;
- Home-ownership rate amongst the young;

such that

- Bankers' allocation solves their maximization problem. That implies:
 - Expected return to the owners of a risky bank does not exceed \bar{r} . If there are risky banks, then this expected return is equal to \bar{r} .
 - Expected return to the owners of a safe bank does not exceed \bar{r} . If there are safe banks, then this expected return is equal to \bar{r} .
 - If there are banks without equity, they earn zero profits.
- Construction firms' allocation solves their maximization problem and yields 0 profit.
- Households' allocation solves their maximization problem.
- Markets clear:
 - Housing market clears.
 - Mortgage market clears.
 - Deposit market clears.

Using notation defined in the Appendix, this can be restated as

Definition 1. A competitive equilibrium in this economy consists of prices $(\hat{q}_1, \hat{r}, \hat{i}, \hat{\tau})$, individual strategies $(\lambda(\cdot; \hat{r}, \hat{i}), \gamma(\cdot; \hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}))$, and aggregate quantities $(\hat{I}, \hat{H}_1, \hat{M}, \hat{D})$ such that

1. $(M^S(\hat{r}, \hat{i}), D^S(\hat{r}, \hat{i}))$ is consistent with $\lambda(\cdot; \hat{r}, \hat{i}) \in \Lambda(\cdot; \hat{r}, \hat{i})$.
2. $(H_1^D(\hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}), M^D(\hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}), D^D(\hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}))$ is consistent with $\gamma(\cdot; \hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}) \in \Gamma(\cdot; \hat{q}_1, \hat{r}, \hat{i}, \hat{\tau})$.
3. $H_1^S(\hat{q}_1)$ is consistent with the profit maximization of construction firms.

4. Housing market clears:

$$\hat{H}_1 = H_1^D(\hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}) = H_1^S(\hat{q}_1). \quad (2)$$

5. Financial markets clear:

$$\hat{M} = M^D(\hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}) = M^S(\hat{r}, \hat{i}), \quad (3)$$

$$\hat{D} = D^D(\hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}) = D^S(\hat{r}, \hat{i}). \quad (4)$$

4 Equilibrium Outcomes

In this section we characterize equilibrium outcomes for two key regulatory settings — laissez-faire and effective regulation — as well as the efficient allocation.

4.1 Laissez-Faire Allocation

We begin by establishing a few preliminary points that will prove convenient. Several proofs are relegated to Appendix B. We first characterize the set of interest rates for which all banks make zero expected profits and the markets for both mortgage loans and deposits are served by some banks.

Lemma 1. (i) $r = i \in [0, r^*]$ if and only if (a) $\Pi(e; r, i) = 0$ for all $e \in \mathbb{R}_+$, (b) there exists $(m, s, d, e) \in \mathbb{R}_+^4$ such that $(m, s, d) \in \Lambda(e; r, i)$ and $m > 0$, and (c) there exists $(m, s, d, e) \in \mathbb{R}_+^4$ such that $(m, s, d) \in \Lambda(e; r, i)$ and $d > 0$.

(ii) Consider $(m, s, d, e) \in \mathbb{R}_+^4$ such that $(m, s, d) \in \Lambda(e; r, i)$. Then: (a) If $r = i = r^*$, then $d = 0$ or $s = 0 < d$; (b) If $r = i \in (0, r^*)$, then $m = d = 0$ for $e > 0$ and $s = 0$ for $e = 0$; (c) If $r = i = 0$, then $m = 0$ for $e > 0$.

Corollary 1. For a given $(r, i) \in \mathbb{R}_+^2$, let $M^S(r, i)$ and $D^S(r, i)$ be the aggregate amounts of mortgage loans and deposits supplied by profit-maximizing banks. Then

$$M^S(r, i) - D^S(r, i) \begin{cases} \in \mathbb{R}_+, & \text{for } r = i = r^*, \\ = 0, & \text{for } r = i \in (0, r^*), \\ \in \mathbb{R}_-, & \text{for } r = i = 0. \end{cases}$$

Proof. If $r = i = r^*$, then the profit-maximizing strategies are $d = 0$ or $s = 0 < d$. For all banks with $d = 0$, $m - d = m \geq 0$. For all banks with $s = 0 < d$, $m - d = e - s = e \geq 0$. Therefore, $m - d \geq 0$ holds for all banks.

If $r = i \in (0, r^*)$, then the profit-maximizing strategies are $m = d = 0$ for $e > 0$ and $s = 0$ for $e = 0$. Since $s = m - d$ for $e = 0$, $m - d = 0$ holds for all banks.

If $r = i = 0$, then $m = 0$ is the profit maximizing strategy for $e > 0$. Therefore, $m - d = -d \leq 0$ for $e > 0$. Moreover, for $e = 0$, $m - d = -a \leq 0$. Thus, $m - d \leq 0$ holds for all banks. \square

Note that $(1+i)(1-p_L\tau) \geq p_H(1+r)$ holds when $r = i \in [0, r^*]$, because $(1+i)(1-p_L\tau) - p_H(1+r) = (1+r)(1-p_L\tau - p_H) = p_L(1+r)(1-\tau) \geq 0$.

Lemma 2. Consider $(c_L, c_H, d, h, m, y, q_1, r, i, \tau)$ such that $(y, q_1, r, i) \in \mathbb{R}_+^2$, $\tau \in [0, 1]$, and $(c_L, c_H, d, h, m) \in \Gamma(y; q_1, r, i, \tau)$. Then: (i) If $q_1 > \bar{q}_1(r, i, \tau)$ or $y < \underline{y}(q_1, r, i, \tau)$, then $h = m = 0$; (ii) If $q_1 < \bar{q}_1(r, i, \tau)$ and $y > \underline{y}(q_1, r, i, \tau)$, then $h = 1$; (iii) If $(1+i)(1-p_L\tau) > p_H(1+r)$, then $m = \bar{m}(r)$ for $h = 1$; (iv) If $(1+i)(1-p_L\tau) < p_H(1+r)$, then $m = \max\{q_1 - y, 0\}$ for $h = 1$.

Corollary 2. For $(q_1, r, i) \in \mathbb{R}_+^3$ and $\tau \in [0, 1]$, let $H^D(q_1, r, i, \tau)$, $M^D(q_1, r, i)$, and $D^D(q_1, r, i)$ be the aggregate amounts of housing, mortgage loans, and deposits demanded by utility-maximizing households. Then

$$H_1^D(q_1, r, i, \tau) \begin{cases} = 0 & \text{for } q_1 > \bar{q}_1(r, i, \tau) \\ \in \left[0, 1 - F\left(\underline{y}(q_1, r, i, \tau)\right)\right] & \text{for } q_1 = \bar{q}_1(r, i, \tau) \\ = 1 - F\left(\underline{y}(q_1, r, i, \tau)\right) & \text{for } q_1 < \bar{q}_1(r, i, \tau) \end{cases}$$

and

$$M^D(q_1, r, i) - D^D(q_1, r, i) = q_1 H_1^D(q_1, r, i, \tau) - Y.$$

Moreover, $M_1^D(q_1, r, i, \tau) = 0$ for $q_1 > \bar{q}_1(r, i, \tau)$. For $q_1 \leq \bar{q}_1(r, i, \tau)$,

$$M^D(q_1, r, i) \begin{cases} = \int_{\underline{y}(q_1, r, i, \tau)}^{q_1} (q_1 - y) dF(y) & \text{if } p_H(1+r) > (1+i)(1-p_L\tau) \\ \in \left[\int_{\underline{y}(q_1, r, i, \tau)}^{q_1} (q_1 - y) dF(y), H_1^D(q_1, r, i, \tau) \bar{m}(r) \right] & \text{if } p_H(1+r) = (1+i)(1-p_L\tau) \\ = H_1^D(q_1, r, i, \tau) \bar{m}(r) & \text{if } p_H(1+r) < (1+i)(1-p_L\tau) \end{cases}$$

Proposition 1. (i) $(\hat{I}, \hat{M}, \hat{D}) \in \mathbb{R}_{++}^3$; (ii) $\hat{q}_1 \in (0, \bar{q}_1(\hat{r}, \hat{i}, \hat{\tau})]$; (iii) $\hat{r} = \hat{i} \in [0, r^*]$; (iv) $\hat{\tau} = 1$ for $\hat{r} = \hat{i} \in (0, r^*]$ and $\hat{\tau} = \hat{M}/\hat{D} \in (0, 1]$ for $\hat{r} = \hat{i} = 0$.

Proof.

Lemma 3. $\Pi(e; \hat{r}, \hat{i}) = 0, \forall e \in \mathbb{R}_+$.

Proof. Suppose that $\Pi(e; \hat{r}, \hat{i}) > 0$ for some $(m, s, d, e) \in \mathbb{R}_+^4$ such that $(m, s, d) = \lambda(e; \hat{r}, \hat{i})$. Then it must be the case that $m > 0$ or $d > 0$, because $m = d = 0$ would give a zero expected profit. However, $m > 0$ or $d > 0$ implies $M^S(\hat{r}, \hat{i}) \rightarrow \infty$ or $D^S(\hat{r}, \hat{i}) \rightarrow \infty$ due to an infinite measure of foreign investors, which violates (3) or (4). \square

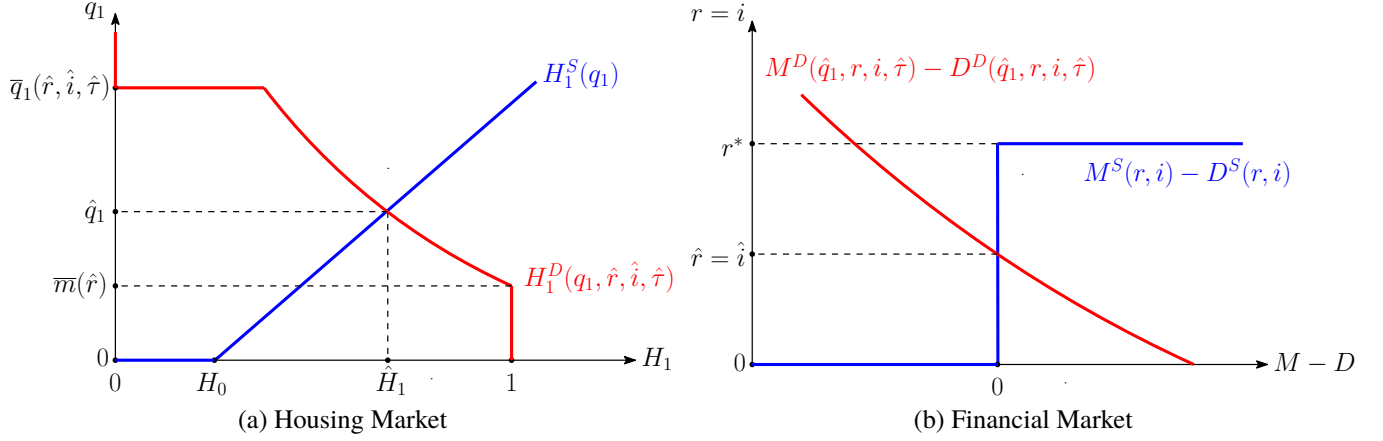


Figure 1: Equilibrium

Lemma 4. $\hat{i} \geq 0$, $\hat{i} \geq r$, and $\hat{r} \leq r^*$.

Proof. If these conditions are not satisfied, all banks can make strictly positive expected profits, as shown in the proof of Lemma 1. \square

Lemma 5. $\hat{D} > 0$.

Proof. Suppose that $\hat{D} = 0$, which is the case when all households hold zero deposits. This is consistent with utility maximization only if $(1 + \hat{i})(1 - p_L \hat{\tau}) \leq 0$. However, $(1 + \hat{i})(1 - p_L \hat{\tau}) > 0$ holds since $\hat{i} \geq 0$ and $\hat{\tau} \in [0, 1]$. \square

Lemma 6. $1 + \hat{r} > 0$.

Proof. Suppose that $1 + \hat{r} \leq 0$. Then any value of $m \geq 0$ satisfies (12). Since $(1 + \hat{i})(1 - p_L \hat{\tau}) > 0$, households keep increasing utility by raising m . Therefore, the solution to Problem 2 does not exist, that is, $\Gamma(y; \hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}) = \emptyset$. \square

Lemma 7. $\hat{H}_1 > 0$, $\hat{I} > 0$, and $\hat{q}_1 \in (0, \bar{q}_1(\hat{r}, \hat{i}, \hat{\tau})]$.

Proof. $\hat{H}_1 = 0$ or $\hat{I} = 0$ holds only if $\hat{q}_1 \leq 0$. First, note that $\bar{q}_1(\hat{r}, \hat{i}, \hat{\tau}) > 0$ holds due to $(1 + \hat{i})(1 - p_L \hat{\tau}) > 0$ and $1 + \hat{r} > 0$. Moreover, $\hat{q}_1 \leq 0$ implies $\underline{y}(\hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}) < 0$. Therefore, when $\hat{q}_1 \leq 0$, $H^D(\hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}) = 1$ and the housing market does not clear. Moreover, $\hat{q}_1 > \bar{q}_1(\hat{r}, \hat{i}, \hat{\tau})$ does not hold, because it implies $H^D(\hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}) = 0 < H^S(\hat{q}_1)$. \square

Lemma 8. $\hat{M} > 0$.

Proof. Suppose that $\hat{M} = 0$. Since $\hat{H}_1 > 0$, it must be the case that $(1 + \hat{i})(1 - p_L \hat{\tau}) \leq p_H(1 + \hat{r})$ holds; otherwise, $M^D(\hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}) = \hat{H}_1 \bar{m}(\hat{r}) > 0$. However, $(1 + \hat{i})(1 - p_L \hat{\tau}) = p_H(1 + \hat{r})$ does not hold, as it

implies

$$M^D(\hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}) \geq \int_{\hat{q}_1 - \bar{m}(\hat{r})}^{\hat{q}_1} (\hat{q}_1 - y) dF(y) > 0,$$

where the last inequality holds due to $1 + \hat{r} > 0$. Therefore, $(1 + \hat{i})(1 - p_L \hat{\tau}) < p_H(1 + \hat{r})$ must hold, which also implies $\hat{i} < \hat{r}$ due to $\hat{\tau} \leq 1$. However, as shown by Lemma 4, $\hat{i} < \hat{r}$ implies banks can make strictly positive expected profits. \square

Lemma 9. $\hat{r} = \hat{i} \in [0, r^*]$.

Proof. Lemma 1, along with $\Pi(e; \hat{r}, \hat{i}) \geq 0 \forall e \in \mathbb{R}_+$, $\hat{D} > 0$, and $\hat{M} > 0$ gives the result. \square

Lemma 10.

$$\hat{\tau} = \frac{\hat{i} + [1 - p_H(1 + \hat{r})] \frac{\hat{M}}{\hat{D}}}{p_L(1 + \hat{i})}. \quad (5)$$

Lemma 11. $\hat{\tau} = \min\{\hat{M}/\hat{D}, 1\}$.

Proof. Suppose that $\hat{r} = \hat{i} = r^*$. Then $\hat{M} - \hat{D} = M^S(\hat{r}, \hat{i}) - D^S(\hat{r}, \hat{i}) \geq 0$. Since $\hat{D} > 0$, $\hat{M}/\hat{D} \geq 1$. Moreover, $p_H(1 + r^*) = 1$ holds, which implies $\hat{\tau} = r^*/[p_L(1 + r^*)] = 1$.

Suppose that $\hat{r} = \hat{i} \in (0, 1)$. Then $\hat{M} - \hat{D} = M^S(\hat{r}, \hat{i}) - D^S(\hat{r}, \hat{i}) = 0$, which implies $\hat{\tau} = [1 + \hat{i} - p_H(1 + \hat{r})]/[p_L(1 + \hat{i})] = 1$.

Suppose that $\hat{r} = \hat{i} = 0$. Then $\hat{M} - \hat{D} = M^S(\hat{r}, \hat{i}) - D^S(\hat{r}, \hat{i}) \leq 0$, which implies $\hat{\tau} = \hat{M}/\hat{D} \in (0, 1]$. \square

Corollary 3. (i) If $Y > u + p_H q_{2,H}$, then $\hat{r} = \hat{i} \in [0, r^*]$; (ii) If $Y > (u + p_H q_{2,H})/p_H$, then $\hat{r} = \hat{i} = 0$.

4.2 Efficient Allocation

Define the ‘fundamental value’ of a house

$$q_1^* \equiv u + p_H q_{2,H}.$$

Proposition 2. A competitive equilibrium is Pareto optimal if and only if $\hat{q}_1 \leq q_1^*$.

4.2.1 Fundamental Assumptions for Inefficiency

A competitive equilibrium is inefficient if and only if $\hat{q}_1 > q_1^*$, which requires (i) $\bar{q}_1(\hat{r}, \hat{i}, \hat{\tau}) > q_1^*$ (households are willing to pay more than q_1^*) and (ii) $H^D(q_1^*, \hat{r}, \hat{i}, \hat{\tau}) > H^S(q_1^*)$ (house demand exceeds supply at q_1^*).

Lemma 12. (i) $\bar{q}_1(\hat{r}, \hat{i}, \hat{\tau}) \geq q_1^*$; (ii) $\bar{q}_1(\hat{r}, \hat{i}, \hat{\tau}) = q_1^*$ if and only if $\hat{r} = \hat{i} = r^*$.

Proof. When $\hat{r} = \hat{i} \in (0, r^*]$, $\hat{\tau} = 1$ and

$$\bar{q}_1(\hat{r}, \hat{i}, \hat{\tau}) = \frac{u + p_H q_{2,H}}{p_H(1 + \hat{r})} \geq u + p_H q_{2,H} = q_1^*,$$

where the inequality holds as equality if and only if $\hat{r} = \hat{i} = r^*$. When $\hat{r} = \hat{i} = 0$, $\hat{\tau} > 0$ and

$$\bar{q}_1(\hat{r}, \hat{i}, \hat{\tau}) = \frac{u + p_H q_{2,H}}{(1 - p_L \hat{\tau})} > u + p_H q_{2,H} = q_1^*.$$

□

Assumption 1. $Y > u + p_H q_{2,H}$.

Lemma 13. If Assumption 1 holds, then $\hat{r} = \hat{i} < r^*$.

Proof. Suppose that $\hat{r} = \hat{i} = r^*$. Then $\bar{q}_1(\hat{r}, \hat{i}, \hat{\tau}) = q_1^*$ and $\hat{M} - \hat{D} \geq 0$. However,

$$\hat{M} - \hat{D} = M^D(\hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}) - D^D(\hat{q}_1, \hat{r}, \hat{i}, \hat{\tau}) = \hat{q}_1 \hat{H}_1 - Y \leq \hat{q}_1 - Y \leq \bar{q}_1(\hat{r}, \hat{i}, \hat{\tau}) - Y = q_1^* - Y < 0,$$

where the last inequality is due to Assumption 1. Therefore, $\hat{r} = \hat{i} = r^*$ contradicts Assumption 1. □

Assumption 2.

$$H_0 + \frac{u + p_H q_{2,H}}{\kappa} \leq 1 - F(u).$$

Proposition 3. If Assumptions 1 and 2 hold, then a competitive equilibrium is not Pareto optimal.

Proof. Lemma 12 and Lemma 13 show that, if Assumption 1 holds, then $\hat{r} = \hat{i} < r^*$ and $\bar{q}_1(\hat{r}, \hat{i}, \hat{\tau}) > q_1^*$ and households are willing to pay more than the fundamental price q_1^* . Assumption 2 further implies that the housing demand exceeds the housing supply at the fundamental price, therefore it is below the equilibrium price:

$$H^S(q_1^*) = H_0 + \frac{q_1^*}{\kappa} \leq 1 - F\left(q_1^* - \frac{q_{2,H}}{1 + r^*}\right) < 1 - F\left(q_1^* - \frac{q_{2,H}}{1 + \hat{r}}\right) = H^D(q_1^*, \hat{r}, \hat{i}, \hat{\tau}),$$

where the first inequality follows from Assumption 2 and the second inequality is due to $\hat{r} < r^*$. □

4.3 Economic Equilibrium with Regulation

Attaching zero risk weight to the safe investment, the risk-based capital requirement is

$$e \geq \alpha m, \tag{6}$$

where $\alpha \in [0, 1]$.

When $s = 0$, this can be written as

$$d \leq \left(\frac{1 - \alpha}{\alpha} \right) e,$$

which is equal to the fixed capital requirement.

The expected profits are

$$\begin{cases} -[1 - p_H(1 + r)]m - id & \text{if } s \geq (1 + i)d \\ p_H(rm - id) - (1 - p_H)e & \text{if } s < (1 + i)d \end{cases}$$

4.3.1 Sufficient Regulation

Lemma 14. *If $\alpha = 1$, then $\hat{r} = r^*$, $\hat{i} = 0$, and $\hat{\tau} = 0$.*

Proof. Guess that no banks fail in equilibrium. Then $\hat{r} = r^*$, $\hat{i} = 0$, and $\hat{\tau} = 0$ must hold. The guess is verified since $s \geq (1 + \hat{i})d$ is satisfied due to $\hat{i} = 0$ and $m \leq e$. \square

4.3.2 Insufficient Regulation

Lemma 15. *Suppose that $\alpha \in (0, 1)$. (1) $r = \alpha r^* + (1 - \alpha)i \in [\alpha r^*, r^*]$ if and only if (i) $\Pi(e; r, i) = 0$ for all $e \in \mathbb{R}_+$, (ii) there exists $(m, s, d, e) \in \mathbb{R}_+^4$ such that $(m, s, d) \in \Lambda(e; r, i)$ and $m > 0$, and (iii) there exists $(m, s, d, e) \in \mathbb{R}_+^4$ such that $(m, s, d) \in \Lambda(e; r, i)$ and $d > 0$. (2) Consider $(m, s, d, e) \in \mathbb{R}_+^4$ such that $(m, s, d) \in \Lambda(e; r, i)$. Then: (i) If $r = i = r^*$, then $d = 0$ or $s = 0 < d = (1 - \alpha)/\alpha e$; (ii) If $r = \alpha r^* + (1 - \alpha)i$ and $i \in (0, r^*)$, then $m = d = 0$ or $s = 0 < d = (1 - \alpha)/\alpha e$; (iii) If $r = \alpha r^*$ and $i = 0$, then $m = 0$ or $m = e/\alpha$.*

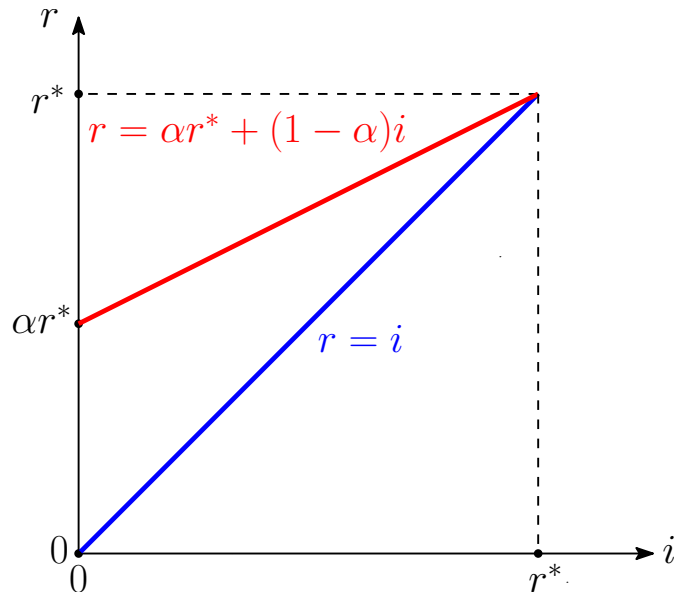


Figure 2: Interest Rates for Banking Equilibrium with Regulation

4.4 Economic Equilibrium with Sufficient Demand for Deposits

We consider a special case when the demand for deposit is large enough.

Assumption 3. $Y > (u + p_H q_{2,H})/p_H$.

Notice that Assumption 3 is stronger than Assumption 1. Therefore, Assumption 3, along with Assumption 2, ensures that the economic equilibrium without regulation (i.e., $\alpha = \beta = 0$) is not Pareto efficient, justifying the implementation of some regulation. Assumption 3 also helps pin down equilibrium deposit interest rate, as shown by the following proposition.

Proposition 4. *If $\alpha = 1$ or Assumption 3 holds, then $\hat{i} = 0$ and $\hat{r} = \alpha r^*$.*

5 Politico-Economic Equilibrium

Under Assumption 3, $\hat{i} = 0$ and $\hat{r} = \alpha r^*$ hold, so choosing $\alpha \in [0, 1]$ is equivalent to choosing $\hat{r} \in [0, r^*]$. Therefore, we focus on economic equilibria corresponding to different levels of r (let's omit hat now). For the rest of the paper, we impose Assumptions 2 and 3.

5.1 Economic Equilibrium Conditional on a Policy

Definition 2. Let $(\hat{q}_1(r), r, i, \hat{\tau}(r), \hat{H}_1(r), \hat{M}(r), \hat{D}(r), \hat{V}(r, y))$ be the equilibrium quantities, prices, and indirect utility satisfying $i = 0$ and

$$\hat{H}_1(r) = H_1^D(\hat{q}_1(r), r, i, \hat{\tau}(r)) = H_1^S(\hat{q}_1(r)), \quad (7)$$

$$\hat{M}(r) = M^D(\hat{q}_1(r), r, i, \hat{\tau}(r)) = M^S(r, i), \quad (8)$$

$$\hat{D}(r) = D^D(\hat{q}_1(r), r, i, \hat{\tau}(r)) = D^S(r, i), \quad (9)$$

$$\hat{V}(r, y) = V(y; \hat{q}_1(r), r, i, \hat{\tau}(r)). \quad (10)$$

Lemma 16. (i) $\hat{q}_1(r)$ and $\hat{H}_1(r)$ are weakly decreasing in r ; (ii) $\hat{\tau}(r)$, $\hat{M}(r)$, and $\hat{D}(r)$ are strictly decreasing in r ; (iii) $\hat{q}_1(r) - \bar{m}(r)$ is strictly increasing in r .

5.2 Preferred Policies

To simplify notation, define minimum downpayment requirement and expected return on deposits:

$$\begin{aligned} x(r) &\equiv \hat{q}_1(r) - \bar{m}(r), \\ z(r) &\equiv 1 - p_L \hat{\tau}(r). \end{aligned}$$

Then

$$\hat{V}(r, y) = \begin{cases} z(r)y, & \text{for } y < x(r), \\ u + z(r)[y - x(r)], & \text{for } y \geq x(r). \end{cases}$$

Lemma 17. For (r, r') such that $0 \leq r < r' \leq r^*$,

$$\hat{V}(r', y) - \hat{V}(r, y) \begin{cases} > 0, & \text{for } y < x(r), \\ < 0, & \text{for } y \in \left[x(r), \frac{z(r')x(r') - z(r)x(r)}{z(r') - z(r)} \right), \\ = 0, & \text{for } y = \frac{z(r')x(r') - z(r)x(r)}{z(r') - z(r)}, \\ > 0, & \text{for } y > \frac{z(r')x(r') - z(r)x(r)}{z(r') - z(r)}, \end{cases}$$

where

$$\frac{z(r')x(r') - z(r)x(r)}{z(r') - z(r)} > x(r').$$

Moreover, $\hat{V}(r', y') - \hat{V}(r, y') > \hat{V}(r', y) - \hat{V}(r, y)$ if $y < y' < x(r)$ or $x(r) \leq y < y'$.

Proof. First, consider the case $y < x(r)$. Since $x(r)$ is strictly increasing in r , $y < x(r')$ also holds. Since $\hat{\tau}(r)$ is strictly decreasing in r , $\hat{V}(r, y)$ is strictly increasing in r .

Next, for those with $y \in [x(r), x(r'))$,

$$\begin{aligned}
\hat{V}(r', y) - \hat{V}(r, y) &= z(r')y - u - z(r)[y - x(r)] \\
&= [z(r') - z(r)]y - u + z(r)x(r) \\
&< [z(r') - z(r)]x(r') - u + z(r)x(r) \\
&< -u + z(r')x(r') \\
&\leq 0,
\end{aligned}$$

where the first inequality is due to $y < x(r')$ and the second inequality uses the fact that $x(r)$ is strictly increasing in r . The last inequality is due to $\hat{q}_1(r) \leq \bar{q}_1(r, i, \hat{\tau}(r)) = u/z(r) + \bar{m}(r) = u/z(r) - x(r) + \hat{q}_1(r)$ for all r .

Finally, for those with $y \geq x(r')$,

$$\begin{aligned}
\hat{V}(r', y) - \hat{V}(r, y) &= z(r')[y - x(r')] - z(r)[y - x(r)] \\
&= [z(r') - z(r)]y - z(r')x(r') + z(r)x(r),
\end{aligned}$$

which is positive if and only if

$$y \geq \frac{z(r')x(r') - z(r)x(r)}{z(r') - z(r)},$$

where

$$\frac{z(r')x(r') - z(r)x(r)}{z(r') - z(r)} > x(r')$$

holds because $x(r)$ is strictly increasing in r .

Note that, $\hat{V}(r', y) - \hat{V}(r, y)$ is differentiable with respect to y except at points $x(r)$ and $x(r')$, with derivative given by

$$\frac{d[\hat{V}(r', y) - \hat{V}(r, y)]}{dy} = z(r') - z(r) > 0.$$

Moreover,

$$\begin{aligned}
\lim_{y \uparrow x(r')} [\hat{V}(r', y) - \hat{V}(r, y)] &= [z(r') - z(r)]x(r') - u + z(r)x(r) \\
&\leq [z(r') - z(r)]x(r') - z(r')x(r') + z(r)x(r) \\
&= \hat{V}(r', x(r')) - \hat{V}(r, x(r')).
\end{aligned}$$

Therefore, $\hat{V}(r', y') - \hat{V}(r, y') > \hat{V}(r', y) - \hat{V}(r, y)$ if $y < y' < x(r)$ or $x(r) \leq y < y'$. □

Proposition 5. Let $R(y) \equiv \operatorname{argmax}_{r \in [0, r^*]} \hat{V}(r, y)$ be the set of preferred policies for an individual with income y . Then: (i) $R(y) = \{r^*\}$ for $y < x(0)$; (ii) $R(y) = \{0\}$ for $y = x(0)$; (iii) $R(y)$ is increasing in y for $y \geq x(0)$. That is, for any (y, y', r, r') such that $x(0) \leq y < y'$, $r \in R(y)$, and $r' \in R(y')$, we have $r \leq r'$.

Proof. Part (i): For those with $y < x(0)$, $\hat{V}(r, y) = z(r)y$ for all $r \in [0, r^*]$, which is strictly increasing in r . Therefore, $R(y) = r^*$.

Part (ii): By Lemma 17, $\hat{V}(r, x(0)) > \hat{V}(0, x(0))$ for all $r \in (0, r^*]$. Therefore, $R(y) = r^*$ for $y = x(0)$.

Part (iii): Suppose that, for (y, y', r, r') such that $x(0) \leq y < y'$, $r \in R(y)$, and $r' \in R(y')$, we have $r > r'$. Since $r \in R(y)$, $\hat{V}(r, y) \geq \hat{V}(r', y)$, and by Lemma 17, $x(r) \leq y$ also holds, which implies $x(r') \leq y$. Moreover, Lemma 17 also suggest that $\hat{V}(r, y) \geq \hat{V}(r', y)$ implies $\hat{V}(r, y') > \hat{V}(r', y')$, which contradicts $r' \in R(y')$. Therefore, $r \leq r'$ must hold. \square

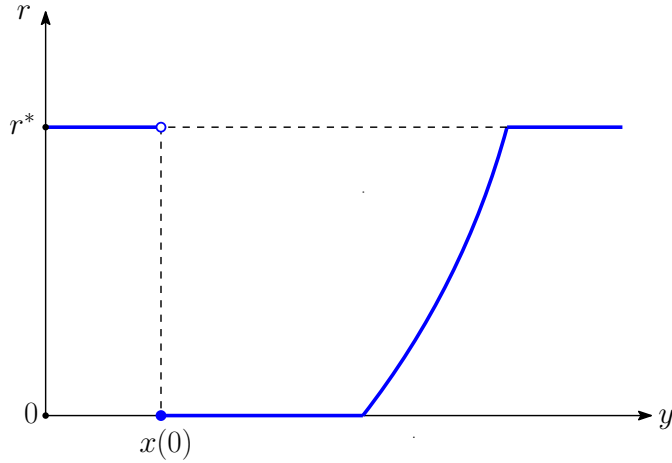


Figure 3: Preferred Mortgage Interest Rates

5.3 Majority Voting Equilibrium

Suppose that only potential home buyers vote. Clearly, owners of existing houses and construction firms prefer $r = 0$.

Definition 3. A policy r is an outcome of majority voting if, for all $r' \in [0, r^*]$,

$$\int \mathbb{I}_{\hat{V}(r, y) \geq \hat{V}(r', y)} dF(y) \geq \frac{1}{2}$$

Let y_m be the median income, that is, $F(y_m) = 1/2$. The preferences of voters feature “ends-against-the-middle” property (Epple and Romano, 1996a,b). Because of non-single-peaked preferences, $r \in R(y_m)$ is not necessarily a majority voting outcome.

Proposition 6. (i) If r is a majority voting outcome, then $r \geq \max R(y_m)$; (ii) If $r^* \in R(y_m)$, then r^* is the unique majority voting outcome; (iii) If there exists \tilde{y} such that $0 \in R(\tilde{y})$ and $F(\tilde{y}) - F(x(0)) \geq 1/2$, then $r = 0$ is a majority voting outcome. Moreover, if $R(\tilde{y}) = \{0\}$, then $r = 0$ is the unique outcome.

Proof. Part (ii): If $y_m < x(0)$, then $R(y_m) = \{r^*\}$. So r^* has more than majority support against all $r < r^*$ because $R(y) = \{r^*\}$ for all $y < x(0)$ with measure $F(x(0)) > F(y_m) = 1/2$. If $y_m > x(0)$ and $r^* \in R(y_m)$, then by Lemma 17, it must be the case that $y_m \geq x(r^*)$; otherwise, $r < r^*$ such that $y_m \geq x(r)$ is strictly preferred to r^* by y_m . Therefore, by Lemma 17, $R(y) = \{r^*\}$ for all $y > y_m$ and r^* has more than majority support against all $r < r^*$.

Part (i): It is enough to consider the case $\max R(y_m) \in (0, r^*)$. First, note that it must be the case that $y_m \geq x(\max R(y_m))$; otherwise, $r < \max R(y_m)$ such that $y_m \geq x(r)$ is strictly preferred to $\max R(y_m)$ by y_m . Then $\max R(y_m)$ has more than majority support against all $r < \max R(y_m)$ because those with $y > y_m$ and $y < x(r)$ strictly prefer $\max R(y_m)$ to r .

Part (iii): Lemma 17 implies that, $0 \in R(y)$ for all $y \in [x(0), \tilde{y}]$. Therefore, $r = 0$ has majority support against all $r' > 0$. To show the uniqueness, suppose that $\hat{V}(r', \tilde{y}) - \hat{V}(0, \tilde{y}) < 0$ holds for all $r' > 0$, that is, $R(\tilde{y}) = \{0\}$. Since $\hat{V}(r', y) - \hat{V}(0, y)$ is right-continuous in y , there exists $\tilde{y}' > \tilde{y}$ such that $\hat{V}(r', \tilde{y}') - \hat{V}(0, \tilde{y}') < 0$ holds. Therefore, $r = 0$ has more than majority support against all $r' > 0$. □

One possible issue with political equilibrium discussed above is the possibility of multiple equilibria and even of Condorcet paradox. One option for eliminating these issues is making political choice binary — focusing on the choice between effective (strict) regulation and a single option of lax regulation.

5.4 Comparative Statics

If we do choose to model the political choice as a binary one, then any comparative static analysis may require “smoothing” of the decision rules. one possible way to achieve that is to resort to probabilistic voting.

6 Dynamic Model

The key mechanism described above extends to a dynamic version of the model with over-lapping generations. Such extension yields one additional important insight. By changing the house prices, current financial regulation affects *current* mortgage default rate as well as the forward-looking risk-exposure of the banking sector, which was emphasized above.

6.1 Response to (Housing) Shock

This dynamic model is well-suited to analyze equilibrium policy response to an (MIT-type) housing shock, which is meant to mimic the events of the 2008 housing bust.

7 Conclusion

We have put forward a parsimonious model that captures one key intuitive message — there are two groups of households/voters who may have benefitted from lax regulation of mortgage lending industry. First such group are borrowing-constrained young households who would have been priced out of the mortgage market under effective regulation. Under lax regulation, banks are willing to advance risky mortgages to these households, charging less than actuarially fair interest rates (these banks will themselves fail in the event of a negative aggregate house-value shock, and thus do not demand adequate compensation for the non-repayment risk in that aggregate state). The added demand from the marginal young home-buyers, whose entry into the market is facilitated by the regulatory failure, pushes up the price of existing (as well as newly constructed) houses. This generates the second group of households who benefit from lax regulation — incumbent (old) home owners. This key economic insight translates into a simple political economy implication. If the coalition of old home-owners and young borrowing-constrained potential home-buyers is large enough, then regulation failure can arise as an outcome of democratic process.

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A Individual Problems

A.1 Banks' Problem

A bank with capital $e \in \mathbb{R}_+$ takes deposits d and invests in safe assets (or storage technology) s and makes risky mortgage loans m . The net return on the safe asset is zero and the contracted net interest rates on deposits and mortgages are i and r , respectively. Mortgage loans are risky in the sense that they are not paid back when the housing market bursts, which occurs with probability $p_L \in (0, 1)$. Therefore, the actuarially fair mortgage interest rate is

$$r^* \equiv \frac{p_L}{p_H},$$

where $p_H \equiv 1 - p_L$. When the bank cannot deliver the contracted interest rates on deposits, it fails and the money left (if any) is distributed to the depositors.

The bank's profit maximization problem is as follows.

Problem 1. Taking $(r, i) \in \mathbb{R}^2$ as given, a bank with equity $e \in \mathbb{R}_+$ solves

$$\begin{aligned} \Lambda(e; r, i) \equiv \operatorname{argmax}_{(m, s, d) \in \mathbb{R}_+^3} & \left\{ p_L \max \{s - (1+i)d, 0\} + p_H \max \{(1+r)m + s - (1+i)d, 0\} - e \right\} \\ & \text{subject to } m + s = d + e. \end{aligned}$$

Let $\Pi(e; r, i)$ be the maximized expected profit. Since banks have an option not to operate (i.e., $m = d = 0$), in which case they earn a zero expected profit, $\Pi(e; r, i) \geq 0$ must hold for all (e, r, i) .

A.2 Households' Problem

There exists a measure 1 of households with heterogeneous income y , continuously distributed with full support on \mathbb{R}_+ according to a distribution function $F(\cdot)$. Let $Y \equiv \int y dF(y)$ be the aggregate income.

Let $\tau \in [0, 1]$ be the fraction of deposit lost in the bad state and let $q_{2,L} \in \mathbb{R}_+$ and $q_{2,H} \in \mathbb{R}_+$ be the exogenous house price in period 2 satisfying $q_{2,L} < q_{2,H}$. For now, also assume $q_{2,L} = 0$.

Problem 2. Taking $(q_1, r, i, \tau) \in \mathbb{R}^4$ as given, households with income $y \in \mathbb{R}_+$ solves

$$\begin{aligned} \Gamma(y; q_1, r, i, \tau) \equiv \operatorname{argmax}_{(c_L, c_H, d, h, m)} & \left\{ uh + p_L c_L + p_H c_H \right\} \\ & \text{subject to } h \in \{0, 1\}, \\ & d + q_1 h \leq y + m, \\ & c_L \leq (1+i)(1-\tau)d + \max \{q_{2,L}h - (1+r)m, 0\}, \\ & c_H \leq (1+i)d + \max \{q_{2,H}h - (1+r)m, 0\}, \\ & (c_L, c_H, d, m) \in \mathbb{R}_+^4, \end{aligned} \tag{11}$$

where $u \in \mathbb{R}_{++}$ is the utility value of housing and let $V(y; q_1, r, i, \tau)$ be the value function.

$$q_{2,H}h \geq (1+r)m, \quad (12)$$

which can be rewritten as (when $1+r \neq 0$)

$$m \leq \bar{m}(r)h$$

where $\bar{m}(r) \equiv q_{2,H}/(1+r)$.

Define

$$\bar{q}_1(r, i, \tau) \equiv \frac{u}{(1+i)(1-p_L\tau)} + \frac{p_H q_{2,H}}{\min\{(1+i)(1-p_L\tau), p_H(1+r)\}}. \quad (13)$$

and

$$\underline{y}(q_1, r, i, \tau) \equiv \begin{cases} q_1 - \bar{m}(r) & \text{if } (1+i)(1-p_L\tau) \geq p_H(1+r) \\ \max\left\{q_1 - \bar{m}(r), \frac{p_H(1+r)q_1 - (1+i)(1-p_L\tau)\bar{q}_1(r, i, \tau)}{p_H(1+r) - (1+i)(1-p_L\tau)}\right\} & \text{if } (1+i)(1-p_L\tau) < p_H(1+r) \end{cases}$$

A.3 Construction Firms' Problem

Suppose that there exists H_0 units of housing at the end of period 0. At the beginning of period 1, construction firms build I units. Thus, the total units in period 1 is $H_1 = H_0 + I$. Assume $H_0 \in [0, 1]$ so that the stock is not enough for everyone, which ensures the existence of equilibrium.

Let $k(I) = \kappa I^2/2$ be the cost function, where $\kappa \in \mathbb{R}_{++}$. The construction firm's problem is

$$\max_{I \in \mathbb{R}_+} \{q_1 I - k(I)\}$$

For $q_1 \in \mathbb{R}_+$, the first order condition is

$$q_1 = k'(I) = \kappa I$$

which gives the supply curve:

$$H_1^S(q_1) \begin{cases} = 0 & \text{for } q_1 \in \mathbb{R}_- \\ \in [0, H_0] & \text{for } q_1 = 0 \\ = H_0 + \frac{q_1}{\kappa} & \text{for } q_1 \in \mathbb{R}_+. \end{cases}$$

B Proofs

B.1 Proof of Lemma 1

Proof. Part (i): The necessity is implied by the proof of part (ii). We prove the sufficiency by showing that, when $r = i \in [0, r^*]$ does not hold, some of the conditions (a)–(c) are violated.

Suppose that $i < 0$. A strategy $(m, s, d) \in \mathbb{R}_+^3$ with $m = 0$ and $s = d + e$ gives the expected profit $-id$, which is strictly increasing in d . Therefore, all banks can make strictly positive profits.

Suppose that $r > i$. A strategy $(m, s, d) \in \mathbb{R}_+^3$ with $s = 0$ and $m = d + e$ gives the expected profit $p_H(r - i)d + [p_H(1 + r) - 1]e$, which is strictly increasing in d . Therefore, all banks can make strictly positive profits.

Suppose that $r < 0$ and that there exists $(m, s, d, e) \in \mathbb{R}_+^4$ such that $(m, s, d) \in \Lambda(e; r, i)$ and $m > 0$. Reducing m and increasing s strictly increases the expected profit, because it would strictly increase $\max\{(1 + r)m + s - (1 + i)d, 0\} = (1 + r)m + s - (1 + i)d$ and weakly increase $\max\{s - (1 + i)d, 0\}$, contradicting the assumption $(m, s, d) \in \Lambda(e; r, i)$.

Suppose that $i > r \geq 0$ and that there exists $(m, s, d, e) \in \mathbb{R}_+^4$ such that $(m, s, d) \in \Lambda(e; r, i)$ and $d > 0$. If $m > 0$, then reducing both d and m strictly increases the expected profit since it would strictly increase $\max\{(1 + r)m + s - (1 + i)d, 0\} = (1 + r)m + s - (1 + i)d$ and weakly increase $\max\{s - (1 + i)d, 0\}$, contradicting the assumption $(m, s, d) \in \Lambda(e; r, i)$. If $m = 0$, the expected profit is $s - (1 + i)d - e = -id$, which is strictly decreasing in d and thus $d > 0$ also contradicts $(m, s, d) \in \Lambda(e; r, i)$.

Finally, suppose that $r > r^*$. A strategy $(m, s, d) \in \mathbb{R}_+^3$ with $s = 0$ and $m = d + e$ gives the expected profit $[p_H(1 + r) - 1]e$, which is strictly positive. Therefore, all banks with strictly positive equity can make strictly positive profits.

Part (ii): It is useful to write the expected profit for two class of strategies:

$$\begin{cases} p_H(1 + r)m - e + s - (1 + i)d, & \text{for } s \geq (1 + i)d, \\ p_H(1 + r)m - e + p_H[s - (1 + i)d], & \text{for } s < (1 + i)d. \end{cases}$$

For $r = i = r^*$, consider a strategy $(m, s, d) \in \Lambda(e; r, i)$. If $s \geq (1 + i)d$, then the expected profit is $(m + s) - (1 + i)d - e = -id$, which implies $d = 0$ since $i > 0$. If $s < (1 + i)d$, then $r > 0$ implies $s = 0$ and the expected profit is $p_H[(1 + r)m - (1 + i)d] - e = p_H(r - i)d + [p_H(1 + r) - 1]e = 0$. Since both strategies are profit-maximizing, $d = 0$ or $s = 0 < d$ holds.

For $r = i \in (0, r^*)$, consider a strategy $(m, s, d) \in \Lambda(e; r, i)$. If $s \geq (1 + i)d$, then $p_H(1 + r) < 1$ implies $m = 0$ and the expected profit is $-e + s - (1 + i)d = -id$. Therefore, $d = 0$ due to $i > 0$. Next, if $s < (1 + i)d$, $r > 0$ implies $s = 0$ and the expected profit is $p_H[(1 + r)m - (1 + i)d] - e = p_H(r - i)d + [p_H(1 + r) - 1]e = [p_H(1 + r) - 1]e \leq 0$, where the inequality is strict if and only if $e > 0$. For $e = 0$, both strategies are profit-maximizing, so $s = 0 < m = d$ or $s = 0 = m = d$ holds. On the other hand, for $e > 0$, $m = d = 0$ is the optimal strategy.

For $r = i = 0$, consider a strategy $(m, s, d) \in \Lambda(e; r, i)$. If $s \geq (1 + i)d$, then $p_H(1 + r) < 1$ implies $m = 0$ and the expected profit is $-id = 0$. Next, if $s < (1 + i)d$, then the expected profit is $p_H(m + s - d) - e = -p_L e \leq 0$, where the inequality is strict if and only if $e > 0$. For $e = 0$, both strategies are profit-maximizing, so $m = 0$ or $m > 0$ holds. However, for $e > 0$, $m = 0$ is the optimal strategy. \square

B.2 Proof of Lemma 2

Proof. Let $V_h(y; q_1, r, i, \tau)$ be the value of choosing h . The value of not buying a house is

$$V_0(y; q_1, r, i, \tau) \equiv \max_{d \in [0, y]} \left\{ (1 + i)[p_L(1 - \tau) + p_H]d \right\} = (1 + i)(1 - p_L\tau)y.$$

Buying a house is only feasible for those with $y \geq q_1 - \bar{m}(r)$. Therefore, define $V_1(y; q_1, r, i, \tau) \equiv -\infty$ for $y < q_1 - \bar{m}(r)$. For $y \geq q_1 - \bar{m}(r)$, the value of buying a house is

$$V_1(y; q_1, r, i, \tau) \equiv \max_m \left\{ u + (1 + i)(1 - p_L\tau)(y - q_1) + p_H q_{2,H} + [(1 + i)(1 - p_L\tau) - p_H(1 + r)]m \right\}$$

subject to $m \geq \max\{q_1 - y, 0\}$, $m \leq \bar{m}(r)$.

Therefore, whether $m \geq \max\{q_1 - y, 0\}$ or $m \leq \bar{m}(r)$ is binding depends on the sign of $(1 + i)(1 - p_L\tau) - p_H(1 + r)$.

When $(1 + i)(1 - p_L\tau) > p_H(1 + r)$, $m \leq \bar{m}(r)$ is binding and

$$V_1(y; q_1, r, i, \tau) = u + (1 + i)(1 - p_L\tau)(y - q_1) + p_H q_{2,H} + [(1 + i)(1 - p_L\tau) - p_H(1 + r)]\bar{m}(r).$$

and the utility benefit of buying a house is

$$V_1(y; q_1, r, i, \tau) - V_0(y; q_1, r, i, \tau) = u - (1 + i)(1 - p_L\tau)q_1 + p_H q_{2,H} + [(1 + i)(1 - p_L\tau) - p_H(1 + r)]\bar{m}(r),$$

which is decreasing in q_1 while it does not depend on y . Therefore, among those with $y \geq q_1 - \bar{m}(r)$, the housing choice is independent of y and can be characterized by a threshold value of the house price above which households are never willing to pay. Let $\bar{q}_1(r, i, \tau)$ be the housing price that makes those with $y \geq q_1 - \bar{m}(r)$ indifferent between buying and not buying:

$$\bar{q}_1(r, i, \tau) = \frac{u}{(1 + i)(1 - p_L\tau)} + \frac{q_{2,H}}{1 + r},$$

When $q_1 > \bar{q}_1(r, i, \tau)$, $V_1(y; q_1, r, i, \tau) > V_0(y; q_1, r, i, \tau)$ and all households with $y \geq q_1 - \bar{m}(r)$ purchase a house. On the other hand, when $q_1 < \bar{q}_1(r, i, \tau)$, $V_1(y; q_1, r, i, \tau) < V_0(y; q_1, r, i, \tau)$ and nobody buys.

Next, when $(1 + i)(1 - p_L\tau) = p_H(1 + r)$, the utility benefit of buying is

$$V_1(y; q_1, r, i, \tau) - V_0(y; q_1, r, i, \tau) = u - (1 + i)(1 - p_L\tau)q_1 + p_H q_{2,H},$$

which implies the maximum willingness to pay

$$\bar{q}_1(r, i, \tau) = \frac{u + p_H q_{2,H}}{(1+i)(1-p_L\tau)}. \quad (14)$$

When $(1+i)(1-p_L\tau) < p_H(1+r)$, $m \geq \max\{q_1 - y, 0\}$ is binding and

$$V_1(y; q_1, r, i, \tau) - V_0(y; q_1, r, i, \tau) = u - (1+i)(1-p_L\tau)q_1 + p_H q_{2,H} + [(1+i)(1-p_L\tau) - p_H(1+r)] \max\{q_1 - y, 0\},$$

For those with $y \geq q_1$, the house purchase decision is still characterized by the maximum willingness to pay (14). For those with $y < q_1$, the utility benefit of buying a house is strictly increasing in y , which implies that those with higher income are more likely to buy a house. Therefore, for those with $y \in [q_1 - \bar{m}(r), q_1)$, the house purchase decision is characterized by a threshold value of income that makes households indifferent between buying and not buying:

$$\frac{p_H(1+r)q_1 - (u + p_H q_{2,H})}{p_H(1+r) - (1+i)(1-p_L\tau)} = \frac{p_H(1+r)q_1 - (1+i)(1-p_L\tau)\bar{q}_1(r, i, \tau)}{p_H(1+r) - (1+i)(1-p_L\tau)}. \quad (15)$$

Those with income above (15) buys a house while those below do not. \square

B.3 Proof of Lemma 10

Proof. It is useful to derive the goods market clearing conditions first. By aggregating the budget constraints of households, we get

$$\hat{q}_1 \hat{H}_1 + \hat{D} = Y + \hat{M} \quad (16)$$

$$\hat{C}_L = (1 + \hat{i})(1 - \hat{\tau})\hat{D} \quad (16)$$

$$\hat{C}_H = (1 + \hat{i})\hat{D} - (1 + \hat{r})\hat{M} + q_{2,H}. \quad (17)$$

By aggregating the budget constraints of banks, we get

$$\hat{M} + \hat{S} = \hat{D} + \hat{E}, \quad (18)$$

$$\hat{T}_{2,L} = \hat{S} - (1 + \hat{i})(1 - \hat{\tau})\hat{D},$$

$$\hat{T}_{2,H} = \hat{S} + \hat{M} - (1 + \hat{i})\hat{D},$$

where $\hat{T}_{2,L}$ and $\hat{T}_{2,H}$ are the aggregate amount paid to banks in each state of period 2.

The aggregate amount paid to the owners of construction firms and existing houses in period 1 is

$$\hat{T}_1 = \hat{q}_1 H_0 + [\hat{q}_1 \hat{I} - k(\hat{I})].$$

Summing the aggregate budget constraints over groups of agents gives the goods market clearing condi-

tions:

$$\begin{aligned}\hat{S} + k(\hat{I}) + \hat{T}_1 &= Y + \hat{E}, \\ \hat{C}_L + \hat{T}_{2,L} &= \hat{S}, \\ \hat{C}_H + \hat{T}_{2,H} &= \hat{S} + q_{2,H}\hat{H}_1.\end{aligned}$$

Because individual banks earn zero expected profits, aggregate expected profits in the banking sector must be also zero:

$$p_L\hat{T}_{2,L} + p_H\hat{T}_{2,H} - \hat{E} = 0.$$

Therefore, the expected aggregate household consumption in period 2 is

$$p_L\hat{C}_L + p_H\hat{C}_H = \hat{S} - \hat{E} + p_Hq_{2,H}\hat{H}_1 \quad (19)$$

Combining (16), (17), (18), and (19) gives the result. \square

B.4 Proof of Proposition 2

Proof. Let $j = F(y)$ be the quantile of households with income y . Let $(\hat{c}_L(j), \hat{c}_H(j), \hat{d}(j), \hat{h}(j), \hat{m}(j)) \in \gamma(F^{-1}(j); \hat{q}_1, \hat{r}, \hat{i}, \hat{\tau})$ be the equilibrium allocation for household j . Then the competitive equilibrium allocation satisfies the following resource constraints:

$$\begin{aligned}\hat{S} + k(\hat{I}) + \hat{T}_1 &= Y + \hat{E}, \\ \int_0^1 \hat{c}_L(j) dj + \hat{T}_{2,L} &= \hat{S}, \\ \int_0^1 \hat{c}_H(j) dj + \hat{T}_{2,H} &= \hat{S} + q_{2,H}(H_0 + \hat{I}), \\ \int_0^1 \hat{h}(j) dj &= H_0 + \hat{I}.\end{aligned}$$

Part (i) Necessity: Suppose that $\hat{q}_1 > q_1^*$. Without loss of generality, assume $\hat{h}(j)$ is weakly increasing in j . Then there exists $\varepsilon \in (0, H_0 + \hat{I}]$ such that $\hat{h}(j) = 1$ for all $j \in [1 - \varepsilon, 1]$. Now, consider an alternative allocation $\tilde{I} = \hat{I} - \varepsilon$, $\tilde{S} = \hat{S} + k(\hat{I}) - k(\tilde{I})$, $(\tilde{T}_1, \tilde{T}_{2,L}, \tilde{T}_{2,H}) = (\hat{T}_1, \hat{T}_{2,L}, \hat{T}_{2,H})$, $(\tilde{h}(j), \tilde{c}_L(j), \tilde{c}_H(j)) = (\hat{h}(j), \hat{c}_L(j), \hat{c}_H(j))$ for $j \in [0, 1 - \varepsilon]$, and $(\tilde{h}(j), \tilde{c}_L(j), \tilde{c}_H(j)) = (0, \hat{c}_L(j) + z_L, \hat{c}_H(j) + z_H)$ for $j \in [1 - \varepsilon, 1]$, where

$$z_L = \frac{k(\hat{I}) - k(\tilde{I})}{\varepsilon}, \quad z_H = \frac{k(\hat{I}) - k(\tilde{I}) - q_{2,H}(\hat{I} - \tilde{I})}{\varepsilon}.$$

Under this alternative allocation, the resource constraints are satisfied and all agents are not worse off

compared to the original allocation. However, the households $j \in [1 - \varepsilon, 1]$ are strictly better off. Their utility gain is

$$[u\tilde{h}(j) + p_L\tilde{c}_{2,L}(j) + p_H\tilde{c}_{2,H}(j)] - [u\hat{h}(j) + p_L\hat{c}_{2,L}(j) + p_H\hat{c}_{2,H}(j)] = \frac{k(\hat{I}) - k(\hat{I} - \varepsilon)}{\varepsilon} - p_Hq_{2,H} - u,$$

which is strictly positive if and only if

$$\frac{k(\hat{I}) - k(\hat{I} - \varepsilon)}{\varepsilon} > u + p_Hq_{2,H}. \quad (20)$$

Since $\hat{q}_1 > q_1^* = u + p_Hq_{2,H}$ implies $k'(\hat{I}) = \lim_{\varepsilon \rightarrow 0} [k(\hat{I}) - k(\hat{I} - \varepsilon)]/\varepsilon > u + p_Hq_{2,H}$, there exists $\varepsilon > 0$ such that (20) holds. Therefore, the competitive allocation with $\hat{q}_1 > q_1^*$ is not Pareto optimal.

Part (ii) Sufficiency: Suppose that $\hat{q}_1 \leq q_1^*$ and the equilibrium allocation is not Pareto optimal. Then there exists a feasible alternative allocation with a household strategy $(\tilde{c}_L(\cdot), \tilde{c}_H(\cdot), \tilde{d}(\cdot), \tilde{h}(\cdot), \tilde{m}(\cdot))$ and aggregate quantities $(\tilde{I}, \tilde{S}, \tilde{E}, \tilde{T}_1, \tilde{T}_{2,L}, \tilde{T}_{2,H})$ such that

$$\begin{aligned} \tilde{S} + k(\tilde{I}) + \tilde{T}_1 &= Y + \tilde{E}, \\ \int_0^1 \tilde{c}_L(j) dj + \tilde{T}_{2,L} &= \tilde{S}, \\ \int_0^1 \tilde{c}_H(j) dj + \tilde{T}_{2,H} &= \tilde{S} + q_{2,H}(H_0 + \tilde{I}), \\ \int_0^1 \tilde{h}(j) dj &= H_0 + \tilde{I}. \end{aligned}$$

and

$$\tilde{T}_1 \geq \hat{T}_1, \quad (21)$$

$$\int_0^1 [u\tilde{h}(j) + p_L\tilde{c}_L(j) + p_H\tilde{c}_H(j)] dj \geq \int_0^1 [u\hat{h}(j) + p_L\hat{c}_L(j) + p_H\hat{c}_H(j)] dj, \quad (22)$$

$$p_L\tilde{T}_{2,L} + p_H\tilde{T}_{2,H} - \tilde{E} \geq p_L\hat{T}_{2,L} + p_H\hat{T}_{2,H} - \hat{E}, \quad (23)$$

where at least one of (21), (22), and (23) holds with a strict inequality.

Claim 1. $\tilde{I} > \hat{I}$.

Proof. The resource constraints, along with (21), (22), and (23) imply

$$(u + p_Hq_{2,H})\tilde{I} - k(\tilde{I}) > (u + p_Hq_{2,H})\hat{I} - k(\hat{I}). \quad (24)$$

Because $\hat{q}_1 = k'(\hat{I}) \leq q_1^* = u + p_Hq_{2,H}$ and $k(I)$ is strictly increasing and convex in I , (24) implies $\tilde{I} > \hat{I}$. \square

Claim 2.

$$\int_0^1 \left\{ [p_L \tilde{c}_L(j) + p_H \tilde{c}_H(j)] - [p_L \hat{c}_L(j) + p_H \hat{c}_H(j)] \right\} dj \leq p_H q_{2,H} (\tilde{I} - \hat{I}) - [k(\tilde{I}) - k(\hat{I})]. \quad (25)$$

Proof. Combining the resource constraints with (21) and (23) gives the result. \square

Claim 3. *There exists $\chi(\cdot)$ such that $\chi(j) \in \{0, 1\}$ for all $j \in [0, 1]$ and*

$$\int_0^1 \tilde{h}(j) [1 - \hat{h}(j)] \chi(j) dj = \tilde{I} - \hat{I}, \quad (26)$$

$$\int_0^1 \tilde{h}(j) [1 - \hat{h}(j)] [1 - \chi(j)] dj = \int_0^1 [1 - \tilde{h}(j)] \hat{h}(j) dj. \quad (27)$$

Proof. First, note that

$$\tilde{I} - \hat{I} = \int_0^1 [\tilde{h}(j) - \hat{h}(j)] dj = \int_0^1 \left\{ \tilde{h}(j) [1 - \hat{h}(j)] - [1 - \tilde{h}(j)] \hat{h}(j) \right\} dj, \quad (28)$$

which implies

$$\int_0^1 \tilde{h}(j) [1 - \hat{h}(j)] dj \geq \tilde{I} - \hat{I} > 0.$$

Therefore, it is possible to find $\chi(\cdot)$ that satisfies (26). Combining (26) and (28) gives (27). \square

Claim 4.

$$\begin{aligned} & \int_0^1 \left\{ [p_L \tilde{c}_L(j) + p_H \tilde{c}_H(j)] - [p_L \hat{c}_L(j) + p_H \hat{c}_H(j)] \right\} dj \\ & \geq \int_0^1 \left\{ [p_L \tilde{c}_L(j) + p_H \tilde{c}_H(j)] - [p_L \hat{c}_L(j) + p_H \hat{c}_H(j)] \right\} \tilde{h}(j) [1 - \hat{h}(j)] \chi(j) dj. \end{aligned} \quad (29)$$

Proof. First, define

$$g(j) \equiv \tilde{h}(j) \hat{h}(j) + [1 - \tilde{h}(j)] [1 - \hat{h}(j)] + \tilde{h}(j) [1 - \hat{h}(j)] \chi(j) + \tilde{h}(j) [1 - \hat{h}(j)] [1 - \chi(j)] + [1 - \tilde{h}(j)] \hat{h}(j).$$

Since $g(j) = 1$ for all $j \in [0, 1]$,

$$\int_0^1 \left\{ [p_L \tilde{c}_L(j) + p_H \tilde{c}_H(j)] - [p_L \hat{c}_L(j) + p_H \hat{c}_H(j)] \right\} dj = \int_0^1 \left\{ [p_L \tilde{c}_L(j) + p_H \tilde{c}_H(j)] - [p_L \hat{c}_L(j) + p_H \hat{c}_H(j)] \right\} g(j) dj.$$

Next, note that, for all $j \in [0, 1]$,

$$[p_L \tilde{c}_L(j) + p_H \tilde{c}_H(j)] - [p_L \hat{c}_L(j) + p_H \hat{c}_H(j)] \geq -u[\tilde{h}(j) - \hat{h}(j)].$$

Therefore

$$\int_0^1 \left\{ [p_L \tilde{c}_L(j) + p_H \tilde{c}_H(j)] - [p_L \hat{c}_L(j) + p_H \hat{c}_H(j)] \right\} \left\{ \tilde{h}(j) \hat{h}(j) + [1 - \tilde{h}(j)][1 - \hat{h}(j)] \right\} dj \geq 0.$$

and

$$\begin{aligned} & \int_0^1 \left\{ [p_L \tilde{c}_L(j) + p_H \tilde{c}_H(j)] - [p_L \hat{c}_L(j) + p_H \hat{c}_H(j)] \right\} \left\{ \tilde{h}(j)[1 - \hat{h}(j)][1 - \chi(j)] + [1 - \tilde{h}(j)]\hat{h}(j) \right\} dj \\ & \geq -u \int_0^1 \tilde{h}(j)[1 - \hat{h}(j)][1 - \chi(j)] dj + u \int_0^1 [1 - \tilde{h}(j)]\hat{h}(j) dj \\ & = 0, \end{aligned}$$

which proves the result. \square

Claim 5.

$$(1 + \hat{i})(1 - p_L \hat{\tau}) \left(k'(\hat{I}) - \frac{q_{2,H}}{1 + \hat{\tau}} \right) \leq k'(\hat{I}) - p_H q_{2,H}.$$

Proof. If $\hat{\tau} = \hat{i} \in (0, r^*]$, then $\hat{\tau} = 1$ and

$$(1 + \hat{\tau})p_H \left(k'(\hat{I}) - \frac{q_{2,H}}{1 + \hat{\tau}} \right) = (1 + \hat{\tau})p_H k'(\hat{I}) - p_H q_{2,H} \leq k'(\hat{I}) - p_H q_{2,H}$$

If $\hat{\tau} = \hat{i} = 0$, then

$$(1 + \hat{i})(1 - p_L \hat{\tau}) \left(k'(\hat{I}) - \frac{q_{2,H}}{1 + \hat{\tau}} \right) \leq (1 - p_L \hat{\tau}) \left(k'(\hat{I}) - q_{2,H} \right) < k'(\hat{I}) - p_H q_{2,H},$$

where the last inequality follows from $\hat{\tau} > 0$. \square

Claim 6.

$$\int_0^1 [p_L \hat{c}_L(j) + p_H \hat{c}_H(j)] \tilde{h}(j) [1 - \hat{h}(j)] \chi(j) dj < [k(\tilde{I}) - k(\hat{I})] - p_H q_{2,H} (\tilde{I} - \hat{I}). \quad (30)$$

Proof. Let $\underline{j} = 1 - H_0 - \hat{I}$ be the threshold for home ownership, satisfying $\hat{h}(j) = 0$ for all $j < \underline{j}$ and $\hat{h}(j) = 1$ for all $j > \underline{j}$. Since $\hat{\tau} = \hat{i} \in [0, r^*]$, $(1 + \hat{i})(1 - p_L \hat{\tau}) \geq p_H(1 + \hat{\tau})$ and

$$F^{-1}(\underline{j}) = \hat{q}_1 - \bar{m}(\hat{\tau}) = k'(\hat{I}) - \frac{q_{2,H}}{1 + \hat{\tau}}.$$

Thus, for all $j \in [0, 1]$ with $\hat{h}(j) = 0$,

$$p_L \hat{c}_L(j) + p_H \hat{c}_H(j) = (1 + \hat{i})(1 - p_L \hat{\tau}) F^{-1}(\underline{j}) \leq (1 + \hat{i})(1 - p_L \hat{\tau}) \left(k'(\hat{I}) - \frac{q_{2,H}}{1 + \hat{\tau}} \right).$$

Therefore,

$$\begin{aligned} \int_0^1 [p_L \hat{c}_L(j) + p_H \hat{c}_H(j)] \tilde{h}(j) [1 - \hat{h}(j)] \chi(j) dj &\leq (1 + \hat{i})(1 - p_L \hat{\tau}) \left(k'(\hat{I}) - \frac{q_{2,H}}{1 + \hat{\tau}} \right) (\tilde{I} - \hat{I}) \\ &\leq [k'(\hat{I}) - p_H q_{2,H}] (\tilde{I} - \hat{I}), \end{aligned}$$

which gives the result, because the convexity of $k(I)$ implies $k'(\hat{I}) < [k(\tilde{I}) - k(\hat{I})]/(\tilde{I} - \hat{I})$. \square

Equations (25), (29), and (30) imply

$$\begin{aligned} &\int_0^1 [p_L \tilde{c}_L(j) + p_H \tilde{c}_H(j)] \tilde{h}(j) [1 - \hat{h}(j)] \chi(j) dj \\ &\leq \int_0^1 [p_L \hat{c}_L(j) + p_H \hat{c}_H(j)] \tilde{h}(j) [1 - \hat{h}(j)] \chi(j) dj + p_H q_{2,H} (\tilde{I} - \hat{I}) - [k(\tilde{I}) - k(\hat{I})] \\ &< 0. \end{aligned}$$

Therefore, the alternative allocation is not feasible, suggesting that the equilibrium allocation is Pareto optimal. \square

B.5 Proof of Lemma 15

Proof. Part (1): The necessity is implied by the proof of part (2). We prove the sufficiency by showing that, when $r = \alpha r^* + (1 - \alpha)i \in [\alpha r^*, r^*]$ does not hold, some of the conditions (i)–(iii) are violated.

First, notice that $0 \leq i \leq r \leq r^*$ continues to hold even with regulation. Consider a strategy $(m, s, d) \in \mathbb{R}_+^3$ with $s < (1 + i)d$, which gives the expected profit $p_H(1 + r)m - e + p_H[s - (1 + i)d] = p_H[(r - i)d - rs] + [p_H(1 + r) - 1]e$. Since $r \geq i \geq 0$, $s = 0$ and $d = (1 - \alpha)/\alpha e$ maximize the expected profit

$$\left\{ p_H(1 + r) - 1 + p_H(r - i) \frac{1 - \alpha}{\alpha} \right\} e = \frac{p_H}{\alpha} [r - \alpha r^* - (1 - \alpha)i] e,$$

which is strictly positive for $e > 0$ if $r > \alpha r^* + (1 - \alpha)i$. Therefore, $r \leq \alpha r^* + (1 - \alpha)i$ must hold.

Suppose that $r < \alpha r^* + (1 - \alpha)i$. Then a strategy with $s < (1 + i)d$ gives a strictly negative expected profit for banks with $e > 0$. Therefore, consider a strategy $s \geq (1 + i)d$ for banks with $e > 0$. The expected profit is $p_H(1 + r)m - e + s - (1 + i)d = -[1 - p_H(1 + r)]m - id$. Since banks with $e = 0$ have $m = 0$ due to $\alpha > 0$, those with $e > 0$ must have $m > 0$. This implies $r = r^*$. However, $r < \alpha r^* + (1 - \alpha)i$ means $i > r^*$, which is a contradiction. Therefore, $r = \alpha r^* + (1 - \alpha)i$ must hold.

Part (2): Consider $(m, s, d, e) \in \mathbb{R}_+^4$ such that $(m, s, d) \in \Lambda(e; r, i)$. For $s \geq (1 + i)d$, $m = 0$ if $r < r^*$

and $d = 0$ if $i > 0$. For $s < (1 + i)d$, $m = e/\alpha$ due to $r > 0$. Then the expected profit is

$$p_H \left[(1 + r) \frac{e}{\alpha} + s - (1 + i)d \right] - e = \left(\frac{p_H(1 + r)}{\alpha} - 1 \right) e + p_H[s - (1 + i)d],$$

where

$$s = d - \left(\frac{1 - \alpha}{\alpha} \right) e \geq 0.$$

Therefore, the expected profit is

$$p_H \left[\left(\frac{r}{\alpha} - r^* \right) e - id \right].$$

Therefore, $s = 0$ and $d = (1 - \alpha)/\alpha e$ if $i > 0$. Since $r = \alpha r^* + (1 - \alpha)i$, this strategy gives a zero expected profit. \square

B.6 Proof of Proposition 4

Proof. Since the cases $\alpha = 1$ (i.e., sufficient regulation) and $\alpha = 0$ (i.e., no regulation) are already shown, suppose that $\alpha \in (0, 1)$. First, I show that $Y > (u + p_H q_{2,H})/p_H$ implies $\hat{M} < \hat{D}$. To see this,

$$\hat{M} - \hat{D} = \hat{q}_1 \hat{H}_1 - Y \leq \bar{q}_1(\hat{r}, \hat{i}, \hat{\tau}) - Y \leq \frac{u + p_H q_{2,H}}{p_H} - Y < 0,$$

where the first inequality is due to $\hat{q}_1 \leq \bar{q}_1(\hat{r}, \hat{i}, \hat{\tau})$ and the last inequality follows from (13), $\hat{\tau} \leq 1$, $\hat{i} \geq 0$, and $\hat{r} \geq 0$.

Next, I show that $\hat{i} > 0$ implies $\hat{M} \geq \hat{D}$, contradicting $Y > (u + p_H q_{2,H})/p_H$ and thus implying $\hat{i} = 0$. Suppose that $\hat{i} > 0$. There are two cases to consider: $\hat{r} = r^*$ and $\hat{r} \in (0, r^*)$.

If $\hat{r} = r^*$, then all non-failing banks do not accept deposits while they might invest in mortgages (i.e., $d = 0$ and $m \geq 0$). On the other hand, all failing banks take as much deposits as they can and invest everything in mortgages. That is, $s = 0$ and $d = (1 - \alpha)/\alpha e > 0$, which imply $m = d/(1 - \alpha)$. Therefore,

$$\hat{M} \geq \frac{\hat{D}}{1 - \alpha} > \hat{D},$$

where the first inequality holds from the fact that some non-failing banks might supply mortgages.

Similarly, if $\hat{r} \in (0, r^*)$, then all banks that operate take the maximum leverage and invest everything in mortgages (i.e., $m = d/(1 - \alpha)$). Therefore,

$$\hat{M} = \frac{\hat{D}}{1 - \alpha} > \hat{D}.$$

\square

B.7 Proof of Lemma 16

Proof. I first show $(1+i)(1-p_L\hat{\tau}(r)) > p_H(1+r)$. Note that, from (5), we have

$$\hat{\tau}(r) = \left(1 - \frac{r}{r^*}\right) \frac{\hat{M}(r)}{\hat{D}(r)}. \quad (31)$$

With $i = 0$, $(1+i)(1-p_L\hat{\tau}(r)) > p_H(1+r)$ can be written as

$$\hat{\tau}(r) \leq \frac{1-p_H(1+r)}{p_L} = \frac{p_L-p_Hr}{p_L} = 1 - \frac{r}{r^*},$$

which holds due to (31) and $\hat{M}(r) < \hat{D}(r)$.

Since $(1+i)(1-p_L\hat{\tau}(r)) > p_H(1+r)$, $\bar{q}_1(r, i, \hat{\tau}(r))$ is given by

$$\bar{q}_1(r, i, \hat{\tau}(r)) = \frac{u}{1-p_L\hat{\tau}(r)} + \frac{q_{2,H}}{1+r}.$$

Then $(\hat{q}_1(r), \hat{\tau}(r), \hat{H}_1(r), \hat{M}(r), \hat{D}(r))$ solves

$$\hat{q}_1(r) = \min \left\{ \bar{m}(r) + F^{-1}(1 - \hat{H}_1(r)), \bar{q}_1(r, i, \hat{\tau}(r)) \right\} \quad (32)$$

$$\hat{\tau}(r) = \left(1 - \frac{r}{r^*}\right) \frac{\hat{M}(r)}{\hat{D}(r)}, \quad (33)$$

$$\hat{H}_1(r) = H_0 + \frac{\hat{q}_1(r)}{\kappa}, \quad (34)$$

$$\hat{M}(r) = \hat{H}_1(r)\bar{m}(r), \quad (35)$$

$$\hat{D}(r) = Y - \hat{H}_1(r)(\hat{q}_1(r) - \bar{m}(r)), \quad (36)$$

First, suppose that $\hat{q}_1(r) < \bar{q}_1(r, i, \hat{\tau}(r))$. Then $\hat{q}_1(r)$ is the solution to the following equation:

$$1 - F(q_1 - \bar{m}(r)) = H_0 + \frac{q_1}{\kappa}, \quad (37)$$

Since the left hand side is weakly decreasing in q_1 and r (because it is constant when $q_1 < \bar{m}(r)$) and the right hand side is strictly increasing in q_1 , $\hat{q}_1(r)$ is weakly decreasing in r , which also implies that $\hat{H}_1(r)$ is weakly decreasing in r and $\hat{M}(r)$ is strictly decreasing in r . Then, from (37), it is also easy to see that $\hat{q}_1(r) - \bar{m}(r)$ is strictly increasing in r , which also implies $\hat{D}(r)$ is strictly decreasing in r .

Next, from (35) and (36),

$$\frac{\hat{M}(r)}{\hat{D}(r)} = \frac{\hat{H}_1(r)\bar{m}(r)}{\hat{H}_1(r)\bar{m}(r) + Y - \hat{H}_1(r)\hat{q}_1(r)} = \left(1 + \frac{Y - \hat{H}_1(r)\hat{q}_1(r)}{\hat{H}_1(r)\bar{m}(r)}\right)^{-1},$$

which is strictly decreasing in r because $\bar{m}(r)$ is strictly decreasing in r , and $\hat{H}_1(r)$ and $\hat{q}_1(r)$ are weakly

decreasing in r . Thus, $\hat{\tau}(r)$ is also strictly decreasing in r .

□