Interest Rates, Moneyness, and the Fisher Equation

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ABSTRACT

The Euler equation of a representative consumer – or its long-run counterpart, the Fisher equation – is at the heart of modern macroeconomics. But in empirical applications, it is badly misapplied: it prices a bond that is short-term, perfectly safe, yet perfectly illiquid. Such a bond does not exist. Real-world safe assets are highly tradable or pledgeable as collateral, hence their prices reflect their *moneyness* as much as their dividends. Indeed, I estimate the return on a hypothetical illiquid bond, for the postwar United States, via inflation and consumption growth, and show that it behaves very differently from the return on safe and liquid assets. I also argue that this distinction helps resolve a great number of puzzles associated with the Euler/Fisher equation, and points to a better way of understanding how monetary policy affects the economy.

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1 Money and interest rates

What is the opportunity cost of holding money? Economists give two answers to this question. First, money is a way to save, and the alternative to saving is to consume right away. Thus, the holding cost of money reflects patience and the changing value of consumption over time (summarized in a *stochastic discount factor*). Second, there are many ways to store wealth, so another alternative to holding money is to buy an asset – perhaps a Treasury Bill or a book entry in a deposit account – which may pay interest. If so, then this *interest rate* is another opportunity cost of holding money.

The natural conclusion is that the two answers are one and the same: interest rates must equal the inverse of the expected discount factor (including inflation, in case of nominal assets). If there is a representative consumer who consumes c, pays price p, and discounts the future by $\beta < 1$, this conclusion becomes the famous Euler equation:

$$1 + i_t = \left(\mathbb{E}_t \left\{ \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}} \right\} \right)^{-1} \tag{1}$$

Although its interpretation differs across schools of thought (more on which in Section 5 below), this equation is at the heart of modern macroeconomics. This becomes a problem when empirical applications identify i_t with the monetary policy instrument, or a similar short-term rate. For instance, models estimated with U.S. data generally use the Federal Funds Rate [43], 3-month T-bill rate [41], or commercial paper rate [45, 32] as the empirical counterpart of i_t.[1] Yet all of these rates price *highly liquid* assets that have as much in common with money as they have with long-term saving. Consider what the literature has identified as making for a liquid, "money-like", asset:

(a) Serving as a medium of exchange [ancient];
(b) Serving as collateral for a loan when money is needed [48];
(c) The ability to sell it on a secondary market when money is needed [19];
(d) The expectation that it turns into money soon, before that money is needed [22].

Any medium of exchange is money in its most direct sense (a). But a collateralizable or saleable asset is *indirect* money, because people who give up money to buy such an asset know that they have a way to get their money back if they need it badly enough (b,c). This channel is weaker if the collateral is subject to a large haircut or if selling the asset at a good price takes time, certainly, but it is always present, and surely so for U.S. Treasuries that can be pledged or sold in a day. Finally, if one needs money in two months, then a bond that

[1] Not everybody does this. The list of papers that explicitly identify the main monetary policy instrument with the rate on a liquid bond is growing by the minute, and now includes [4, 2, 15, 20, 8, 3]. Others model monetary policy as implemented via open-market operations of a (partially or fully) liquid bond, but do not explicitly identify its yield as the *main* policy instrument [49, 27, 40].

matures in one month is as good as money (d). So which asset could in practice be perfectly safe, short-term, yet perfectly illiquid?[2]

No, Equation (1) cannot possibly price the monetary policy instrument, which in most countries is an interest rate on debt as short-term and liquid as one can imagine. And there is a better alternative. Suppose we add to our models a bond which does not serve as a medium of exchange (hence it is distinct from "money", which does) but which can be sold or used as collateral whenever money is needed. Let us denote the nominal interest rate on this bond by i_t^P ("**policy rate**"). In contrast, consider a hypothetical illiquid bond whose payout must be used for buying consumption in the future; its nominal interest rate, i_t^F, satisfies Equation (1) by construction.[3] Since that equation is the Fisher equation in the long run, Geromichalos and Herrenbrueck [20] proposed to call i_t^F the **Fisher interest rate**.

As people can always hold money (which pays no interest), the policy rate i_t^P cannot be less than zero. And since the liquid bond cannot be less valuable than the illiquid one, i_t^P cannot exceed i_t^F:

$$i_t^P \in [0, i_t^F] \tag{2}$$

Every fact about monetary policy makes more sense once we identify short-term monetary policy with i_t^P, rather than i_t^F, as the rest of this paper will demonstrate.

2 An informal model

"Different types of general equilibrium models are needed for different purposes. For exploration and pedagogy, the criteria should be transparency and simplicity and, for that, toy models are the right vehicles." Blanchard, 2018 [10]

In Appendix A.1, I present a formal intertemporal model of i_t^F and i_t^P, but its essential insights can be explained informally. First, abstract from shocks, risk, and second-order terms, and suppose: (i) people expect consumption to grow at rate g and prices to grow at rate π; (ii) the marginal utility of consumption is proportional to $c^{-\sigma}$, so that σ is the inverse elasticity of intertemporal substitution; (iii) people discount the future at rate $\rho \equiv 1/\beta - 1$; and (iv) the yield on the liquid bond is a known function G of i^F (the opportunity cost of holding money) and the relative supply of liquid bonds to money, B/M, increasing in both arguments.[4] Thus, the two rates i^F and i^P must satisfy:

[2] Two things. First, U.S. Treasuries are quite liquid but still not perfect substitutes to U.S. money [35], and of course they can never be perfect substitutes to *non-U.S.* money. Second, safe assets do tend to be more liquid than risky ones, although the association is not perfect [31, 39, 21].

[3] [11] also estimate the right-hand side of various Euler equations, and suggest that the spread against money market rates may be due to the "liquidity services" of the latter.

[4] A good number of formal models indeed yield something like this function G in reduced form; among them, [18, 49, 40, 4, 28, 20], and more.

$$i^P = G\left(i^F, \frac{B}{M}\right) \tag{3}$$

$$i^F = \rho + \sigma g_C + \pi \tag{4}$$

We again see the problem with treating i^F as the policy instrument. First, we can graph the policy rate against measures of expected consumption growth plus inflation (see Section 3). For any value of σ, the two look nothing alike, and attempting to make them fit implies a negative ρ. (Are people really that patient?) Second, how would a desired level of i^F even be implemented by a monetary authority? Through expected inflation? (No; any measure of expected inflation is basically uncorrelated with policy rates, at least in countries and times where these rates stay in single digits.) Through expected consumption growth? (Ditto.) Through shocks to ρ conveniently timed to coincide with monetary policy announcements? (No, says Occam's razor.)

But the equations also present an opportunity by offering i^P as a better model of the policy rate. Why does the policy rate correlate so weakly with $\sigma g_C + \pi$? Because i^F varies less over the business cycle than i^P does. How is a desired level of i^P implemented? Through open-market operation that adjust the money supply in the background: a lower M/B ratio causes higher interest rates, moving up the "money demand curve" which appears in every undergraduate textbook. Assuming G is invertible, we can write:

$$\frac{B}{M} = H\left(i^F, i^P\right) \tag{5}$$

$$i^F = \rho + \sigma g_C + \pi \tag{6}$$

Now, i^F and i^P are logically independent instruments. They may still be correlated in the data depending on the nature of shocks (across big fluctuations in inflation the inequality $i^P \leq i^F$ will assert itself) or the rules governing policy (e.g., a Taylor rule). But the tight link between interest rates and inflation, implied by Equation (1), is broken.

To take this reasoning further, call the spread between i^F and i^P the *aggregate liquidity premium*:

$$\ell \equiv i^F - i^P \tag{7}$$

Suppose we are pricing an asset X that can be liquidated whenever money is needed, in the same way as the liquid bond, but only with probability $\eta \in [0,1]$. (Or, almost equivalently, one can use it as collateral but subject to a haircut $1-\eta$.) Suppose this asset also depreciates at rate δ (which, again almost equivalently, could stand in for default risk or second-order risk premia). Then, the nominal rate of return on asset X will be:

$$r = \delta + (1-\eta)i^F + \eta i^P$$

$$\leftrightarrow \qquad = \delta + i^F - \eta\ell$$

$$\leftrightarrow \qquad = \delta + i^P + (1-\eta)\ell \tag{8}$$

This equation is a no-arbitrage condition among assets with varying real properties (δ, η). It illustrates how financial liquidity imbues an asset with *indirect moneyness*, lowers its return (relative to the fundamental) by an asset-specific liquidity premium $\eta\ell$, and integrates its return with the monetary policy rate i^P. The direct pass-through from i^P to r is always less than 1, but only if we hold all else (inflation expectations, risk premia, asset tradability) fixed, which may not be the case for all real-world assets. However, what *is* the case for almost all real world assets is that η is not zero.

3 A look at the data

"Our results suggest that the problem is fundamental: alternative specifications of preferences can eliminate the excessive volatility, but they yield an Euler equation rate that is strongly negatively correlated with the money market rate." Canzoneri *et al*, 2007 [11]

"To test the Fisher Equation one should not compare [inflation and the nominal rate on liquid bonds], but [inflation and] the nominal rate on an illiquid asset. That may be hard to implement empirically since most assets have some degree of liquidity." Rocheteau *et al*, 2018 [40]

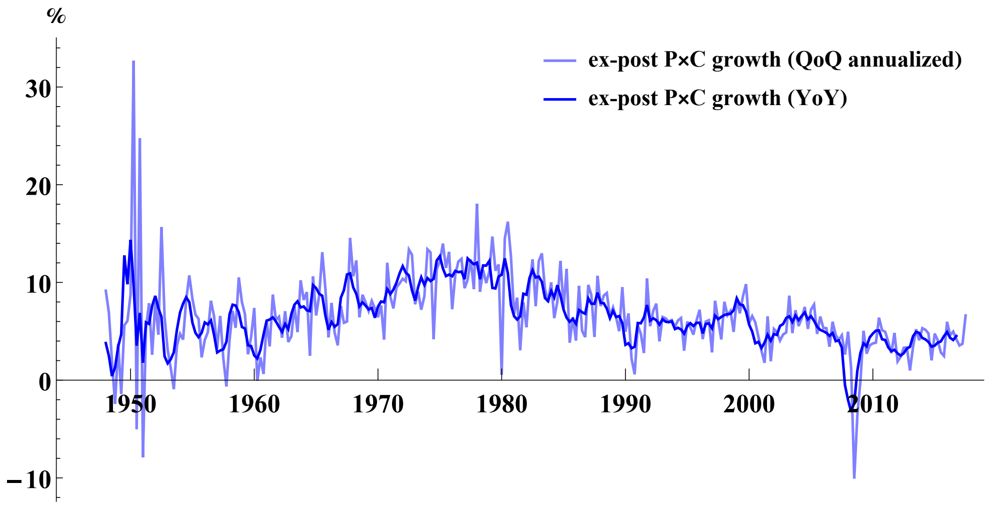
Indeed! Trying to match up the right-hand side of Equation (1) with short-term interest rates results in a spectacularly bad fit. This is true for any plausible elasticity of intertemporal substitution, but to avoid complicating things I assume for now that this elasticity is 1 – corresponding to logarithmic utility of consumption. Thus, the variable to look at is the growth rate of nominal consumption, $p_t \cdot c_t$. At this stage, I do not divide by the size of the population; every model of long-run growth (whether neoclassical or OLG) implies a positive effect from population growth to interest rates. (The pass-through elasticity is generally either 1 or σ, depending on the details of how people care about future generations, but with logarithmic utility we have $\sigma = 1$ anyway.)

Postulating a log-linear trend for time preference, $\rho_t \equiv -\log(\beta_t) = \rho_0 + \rho_1 t$,[5] and picking (ρ_0, ρ_1) to provide the best fit between ex-post consumption growth and a short term interest rate on highly liquid bonds, we obtain the following result, illustrated in Figure 1:[6]

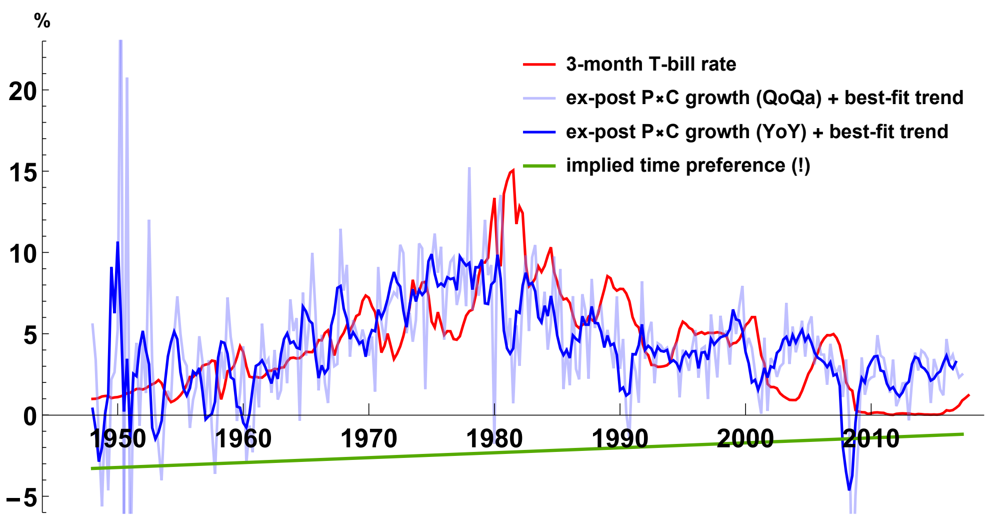
$$\hat{\rho}_{1947} \approx -3\% \qquad \text{linearly increasing to} \qquad \hat{\rho}_{2017} \approx -1\% \qquad \text{(annually)}$$

[5] Letting β_t vary over time in an unrestricted way is obviously a dead end. But we do not need to go to the other extreme and assume that it was constant over the entire postwar era, either. If we did, though, we would estimate $\beta \approx 1.02$ – yes, exceeding unity.

[6] I take the 3-month T-bill secondary market rate as representative, but other choices are reasonable: federal funds, commercial paper, Libor, etc. Aside from a short period around 1980, these rates never differed by more than a percentage point.



(a) U.S. nominal consumption growth, raw



(b) Best fit of U.S. nominal consumption growth on T-bill rate

Figure 1: Ex-post realized nominal consumption growth versus short-term interest rates (3-month T-bills, secondary market rate). Consumption growth is calculated as: $[(p_{t+1}c_{t+1})/(p_tc_t)]^4 - 1$ and $(p_{t+4}c_{t+4})/(p_tc_t) - 1$, measured in percentage points. Best-fit estimates (throughout this paper) are computed after log points transformation $x \mapsto \log(1+x)$.

Best fit it may be, but it flies in the face of received wisdom in many ways:[7]

(a) Implied time preference is negative throughout (the risk-free rate puzzle);
(b) Implied time preference has *increased* over time (what about secular stagnation?);
(c) The fit is atrocious: the correlation is only 0.33 in levels, and -0.15 in first differences.

Of course, these problems are well known [23, 25, 11, 34]. But they are usually understood as problems with "the Euler equation": short-term interest rates do not fit well with ex-post consumption growth, so it is the equation that must be misspecified, and solutions are to be found via a distinction between long-run and short-run risk (the Epstein-Zin literature), the particular interests of wealthy investors [33], or various other models of stochastic discount factors [34]. However, in an exhaustive analysis Canzoneri et al. [11] showed that the problem persists for any common model of preferences. This suggests that the source of the problem is not the equation, but the choice of which data it is estimated with. Equation (1) prices an *illiquid* bond, one which never, ever, relaxes any liquidity constraint at all. But no such bond exists. Real-world interest rates never had any business fitting Equation (1) in the first place.

And it is not hard to do better.

3.1 Proposed new approach: distinguish i^F and i^P

Abstracting away from issues of risk and Jensen's inequality, which are likely of secondary importance in postwar U.S. asset returns, I use a loglinear representation of Equation (1) to define the ex-post counterpart of the Fisher rate (and multiply by 4 to convert to annual frequency):

$$x_{t+1} \equiv -\log(\beta) + 4\big[\log(p_{t+1}c_{t+1}) - \log(c_t p_t)\big] \tag{9}$$

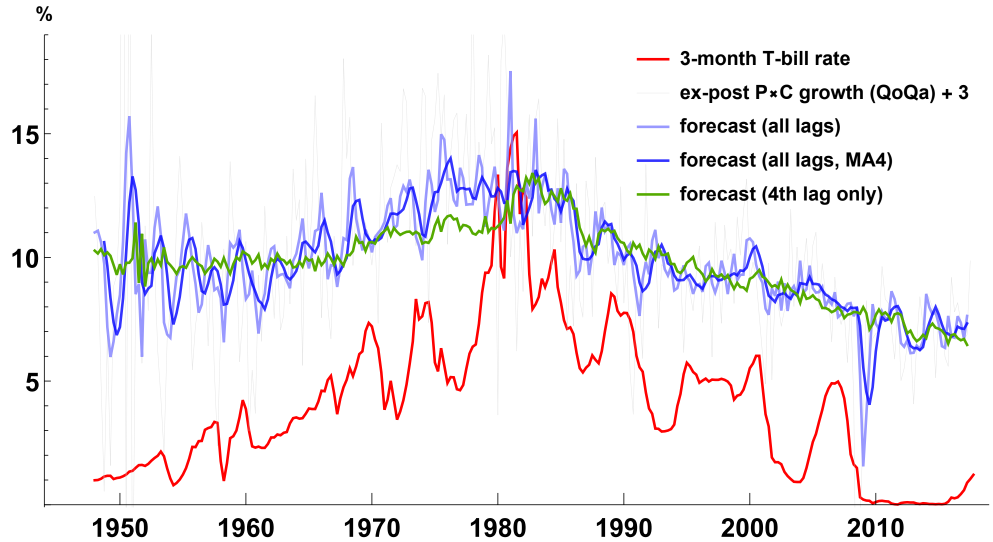
and define the Fisher interest rate as a forecast of x_{t+1}: $\log(1 + i_t^F) \equiv \mathbb{E}_t x_{t+1}$.

I compute three versions of the forecast: first, via a regression of x_t on one-quarter lags of itself, of the 3-month T-bill rate, and of the Moody's AAA corporate bonds rate; second, via a four-quarter moving average of the first estimate (where the estimates at time $t-3$, $t-2$, $t-1$, and t are averaged so as to provide a smoother forecast of x_{t+1}); third, via a regression of x_{t+1} on *one-year* lags of itself, of the 3-month T-bill rate, and of the Moody's AAA corporate bonds rate. This method yields the smoothest forecast of the three, but one that perhaps does not react enough to short-term movements in the variables.

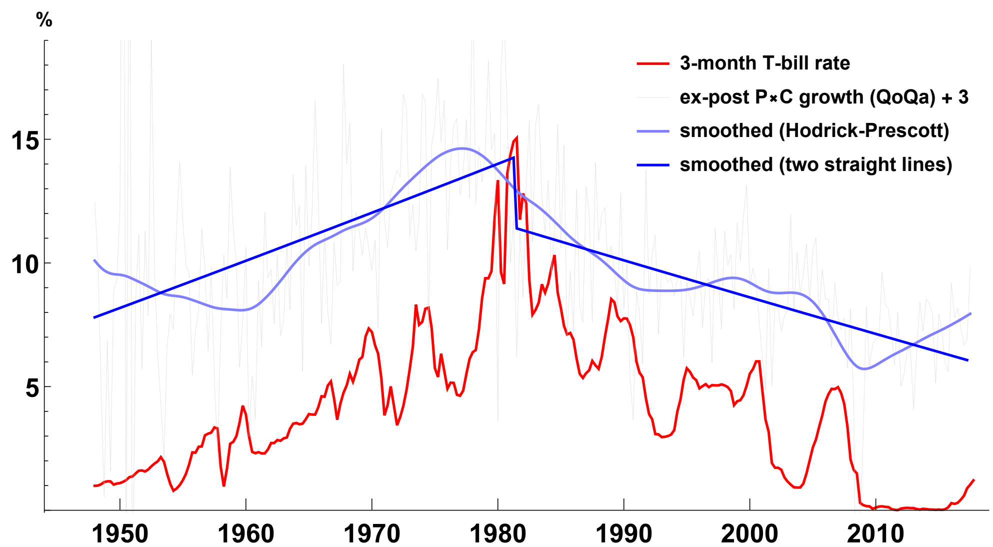
These three measures of i_t^F, along with ex-post x_{t+1} and the 3-month T-bill rate as an estimate of i_t^P, are shown in the top panel of Figure 2.[8] Clearly, while the forecast measures

[7] These facts survive even if we look at per-capita consumption. Implied ρ is then between -2% and 0.

[8] Other plausible estimates of i_t^P include the federal funds rate, short-term commercial paper rate, or short-term money market rate. They all track the T-bill rate closely.



(a) i_t^F estimated via forecasting regression



(b) i_t^F estimated via smoothing

Figure 2: Policy rate i_t^P versus Fisher rate i_t^F, estimated in various ways. $\rho = 3\%$ is assumed.

differ a bit in the short run, they agree on the following big-picture facts: (1) i_t^F does not match up with i_t^P, both in terms of slope and intercept; (2) the i_t^F-series does appear to cover the i_t^P-series in the shape of a tent, which is consistent with the model from Section 2 that only imposed $i_t^P \in [0, i_t^F]$.

The reader will notice that this is not a sophisticated forecast. But since the raw ex-post outcome x_t is also shown, the reader may agree that a more sophisticated forecast is unlikely to change the message. In fact, I am more sympathetic to the opposite argument: given all the uncertainty in what the right measure is, and what the right forecast horizon is, what would it look like if we simply *smoothed* the x_t series, in order to focus on its long-term movements? In the bottom panel of Figure 2, I show two such series: first, the trend component of a standard Hodrick-Prescott filter applied to x_t; second, the best fit of two straight lines, one before 1980 (when the policy rate peaked) and one after. Strikingly, the message does not change: (1) i_t^F does not match up with i_t^P, both in terms of slope and intercept, and (2) the Fisher rate covers the policy rate like a tent, consistent with the model proposed in Section 2.

One problem has not been addressed yet: what to do about β (or the rate of time preference, $\rho \equiv -\log(\beta)$)? This important parameter is no longer identified. Standard practice in macroeconomics has been to estimate time preference via observed interest rates, but now an interest rate must *also* provide information about the liquidity of that particular asset. Unless we could confidently identify the associated liquidity η, we cannot trust any particular real-world interest rate to identify ρ. Future research will revisit this issue in a dataset with many assets, where plausible assumptions on the distribution of their relative liquidities can be made. For now, I adopt $\rho = 3\%$ as a working hypothesis; it is the minimal ρ that is consistent with the constraint $i^P \leq i^F$ being satisfied throughout the dataset.[9] Experimental and field studies that directly measure time preferences tend to find much larger values ($\rho \geq 10\%$) [1, 6]; if these numbers are correct (and representative of the population), then the implied Fisher rate – the opportunity cost of storing wealth via money – would be much larger.

3.2 Long-run relationship between i^F and i^P

As explained earlier, the correlation between first differences of x_t and i_t^P is actually negative. But x_t is measured ex-post – what about its forecastable component, i_t^F? Its correlation with i_t^P, in first differences, is still nearly zero. The long run is another matter, however: the Fisher equation (4) is a steady-state relationship and, as such, is only supposed to hold in the long run. In order to investigate this possibility, I split the dataset into 4-year bins and average i_t^P and i_t^F (now identified as x_t, without any attempt at a short-term forecast) in each bin. Figure 3 shows the result.

[9] There is a tiny bit of overlap at the beginning of the Volcker disinflation that can be explained with noisy measurement and, perhaps, slow adjustment of expectations. Or, perhaps ρ really is time-varying and was unusually high then. In that case, a lower value of $\rho = 1\%$ or 2% in the 2010s may be defensible.

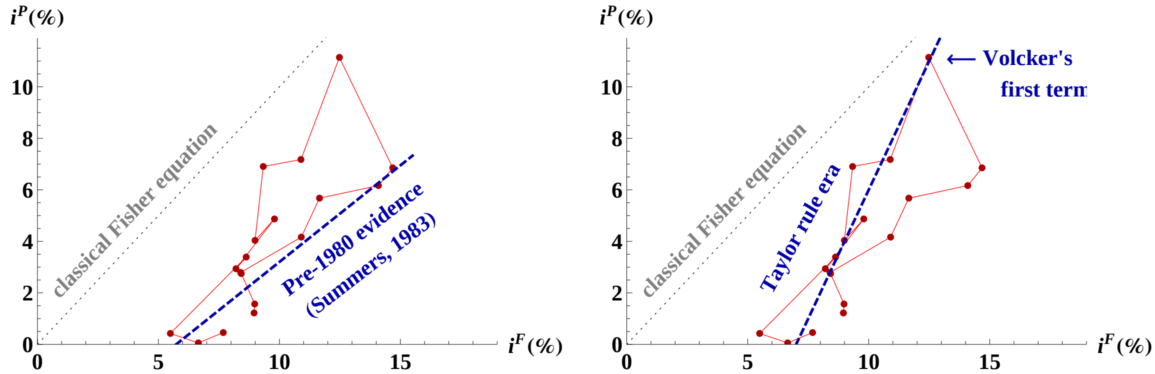


Figure 3: i_t^P versus i_t^F averaged in 4-year bins. $\rho = 3\%$ is assumed. Trend lines are drawn with slope 0.75 (left panel) and 2 (right panel).

Of course, with only 17 data points, a serious econometric analysis is impossible. But even so, we can gain some insight. First: the "classical" Fisher equation (4) still does not fit the data. If estimated as a regression of i^P on i^F, the classical hypothesis implies a slope of 1 and an intercept of 0. With only 17 data points, a slope coefficient of 1 cannot be rejected, but an intercept of 0 is clearly ruled out – liquid bonds carry a liquidity premium.

However, just saying that "a null-hypothesis slope of 1 cannot be rejected statistically" is not good enough, because we have theoretical reasons to believe in alternative hypotheses. In particular, monetary policy before 1980 was *not* conducted by setting interest rates; instead, a variety of money supply targeting schemes were employed, and interest rates were left free to adjust. This is consistent with Equation (3) of the informal model: expected inflation (via i^F) and tightness of credit (via B/M) were varying autonomously and i^P was responding. In most formal models of liquid bonds, which the function G represents in reduced form, the pass-through of inflation to interest rates is of slope *less than 1*. Indeed, the best-fit slope for the pre-1980 data (shown in the left panel of Figure 3) is 0.75 (close to the estimate by Summers [46]), not 1.

Something changed, however, just as Summers' study was published: in his first term as Fed Chair, Paul Volcker raised policy interest rates ($i^P\uparrow$) via restrictive open-market operations ($B/M\uparrow$ in Equation 3), which brought down inflation and, in the short term, even GDP growth ($i^F\downarrow$). This regime shift is shown in the right panel of Figure 3. In subsequent years, the U.S. Federal Reserve's policy has been consistent with a Taylor rule whereby it adjusts the policy rate strongly (slope ≈ 1.5) in response to inflation movements and modestly (slope ≈ 0.5) in response to GDP movements. We see the signature of this policy regime in Figure 3, as well: i^F reflects the sum of inflation and consumption growth, the long-term pass-through of GDP growth to consumption growth is 1, and indeed the post-1980 pass-through of i^F to i^P was almost exactly 2.

4 Questions answered and puzzles resolved

"It can hardly be supposed that a false theory would explain, in so satisfactory a manner as does [my theory], the several large classes of facts above specified." Darwin, 1872 [13]

"Attempts to explain [asset pricing puzzles] involve narrowly specialized structural assumptions that would limit the applicability of any model in other areas of macroeconomics." Azariadis, 2018 [7]

A long list of questions and puzzles is resolved when we properly distinguish between the yields on liquid and illiquid bonds: how monetary policy can set interest rates in the first place, the lack of a Fisher effect in the short run, the post-2008 persistence of low inflation despite low interest rates, the risk-free rate puzzle, the equity premium puzzle, the UIP puzzle, and more.

Yes, each of these puzzles has stimulated a big literature with plausible tweaks to the standard model, all of which may be important in explaining a particular effect. But few of these tweaks have found applications – or even empirical support – when applied to other areas of macroeconomics, outside of the specific puzzle they were constructed to explain. And how likely can it be a coincidence that so many different puzzles are resolved by the same simple principle: price *all* assets for their substitutability with money?

(1) How exactly is the monetary authority able to set interest rates?

As in Macro 101: via open-market operations in secondary asset markets. This works because the monetary authority controls the money stock (at the margin; private agents can create money competitively but they cannot outcompete the monetary authority). Thus, they can achieve any $i^P \in [0, i^F]$ without needing to obey the Fisher equation.

(2) The awkward coexistence of the Fisher effect and a short-run liquidity effect

Resolution: the Fisher effect applies to i^F and the liquidity effect applies to i^P. Specifically:

Fisher effect: higher money supply ⇒ higher inflation ⇒ higher i^F

Liquidity effect: higher money supply ⇒ bonds are scarce ⇒ lower i^P

In the long run, the two rates i^F and i^P do tend to move together due to an inflation-targeting monetary policy (Taylor rule); we wouldn't expect the same correlation to hold under, say, a gold standard. And in the short run, any correlation is possible, depending on which shocks are hitting the economy.

(3) The "lowflation" puzzle

The coexistence of low interest rates and low but positive inflation is a puzzle in the standard New Keynesian model, which predicts either accelerating inflation (in the standard equilibrium) or deflation (in the liquidity trap equilibrium), unless the "natural" real interest rate has cratered (the secular stagnation hypothesis). Not so here: since Equations (3)-(4) hold even in steady state, coexistence between low i^P and any $i^F \geq i^P$ is no problem. Given i^F, different i^P's just correspond to different B/M ratios (relative to demand for these assets).

(4) Has the U.S. been running the Friedman rule in 2009-2014?

Certainly not. See Figures 2-3: the policy rate was zero in that time but the Fisher rate was far from it. We cannot be sure what ρ was, but even disregarding it altogether implies $i^F_{2014} \approx 4\%$. And if $\rho = 3\%$ (over which macroeconomists seem to have a fairly strong prior, and which rationalizes $i^F \approx i^P$ during their peaks around 1980), then $i^F_{2014} \approx 7\%$; lower than ever, but still far from the Friedman rule.

(5) The risk-free rate puzzle

Obvious, since a liquidity premium necessarily reduces the yields on liquid bonds below their fundamental levels.

(6) The equity premium puzzle

Explained by Lagos [31]: even if equity is *almost* as liquid as bonds, a small difference is enough to account for the equity premium.

(7) The positive term premium

Explained by Geromichalos et al. [22]: short-term assets are inherently more liquid because they turn into money when they mature. Long-term assets must be liquidated instead, which is subject to delays and/or transaction costs.

(8) The prominence of a "liquidity factor" in empirical asset pricing

Observed by Liu [37], and explained by Equation (8).

(9) The uncovered interest parity puzzle

The UIP condition states that interest rate hikes should cause an immediate appreciation of a country's currency, followed by expected depreciation. Yet it fails in the data. This apparent puzzle can be explained by the differential liquidity of the bonds involved [36, 30], and exchange-rate movements more broadly also support the liquidity story [16].

(10) The fact that interest rates do not forecast consumption growth well

Noted by Hall [23], and many others [26]. The Euler equation for money from the formal model (Equation A.2) suggests two main reasons:

(i) The fact that $\lambda_t > 0$ on average (liquidity is valued)

(ii) The possibility that λ_t is positively correlated with interest rates and/or consumption growth (it may even be driving the cycle), which soaks up explanatory power

(11) Low elasticity of c_{t+1} in estimates of the Euler equation

Again, Equation (A.2) suggests two technical explanations: procyclical λ_{t+1} (frequency of liquidity shocks) or countercyclical q_{t+1} (wedge between use value and purchase price of output). The former might or might not be exogenous (it is even plausible as a driver of the business cycle), the latter definitely is not. So more research is needed on this question.

(12) The long-run volatility of the risk-free rate of return

The risk-free rate of return is more volatile than the risky one on a decade-by-decade basis, and it is also more variable between countries [29]. This is a puzzle for standard asset pricing theory where the risky rate equals the safe one plus a risk premium (plus, possibly, forecast errors). It is no problem for the liquidity-augmented model, however, as Equation (8) shows: the return on safe-and-liquid assets (i^P) is governed by monetary policy, whereas the return on risky-and-illiquid assets (r^X with $\eta \to 0$) is governed by fundamentals (the Fisher rate plus the risk premium). So no matter how volatile the risk premium and growth expectations are in the short term, this volatility is averaged out in the long run. But changes in the stance of monetary policy (and differences between countries) can be slow-moving and persistent.

5 Background

"Maybe next time [we update our model], we can finally get rid of the... Euler Equation."
Larry Christiano, 2014, quoted in [44]

5.1 The New-Keynesian / Neo-Wicksellian approach

In the New Keynesian model, Equation (1) is called the "expectations-augmented IS curve" and interpreted as follows. i_t is set by monetary policy while expectations of future consumption and inflation are (to a first approximation) given; thus it is c_t, consumption today, that adjusts to make the equation true. This is the main way monetary policy affects the economy: higher interest rates reduce output via consumption demand.

However, even apart from questions of empirical fit of the IS curve itself, the idea that monetary policy can "set" i_t is problematic. In simple NK models this is just left as a black box. In models that open the box, such as [12] where i_t is set via fiscal interventions in households' budgets, the fiscal implications run counter to the facts.

In empirical estimation of a New Keynesian DSGE model, researchers sometimes allow for a liquidity premium on Treasuries; for example [14], who call it a "convenience yield". However, this convenience yield is an independently varying noise term; the pass-through from the monetary policy rate to the rate in the consumer's Euler equation is still 1 after the noise is averaged out. So the resemblance between the convenience yield and the aggregate liquidity premium in my model is only superficial: I argue that the monetary policy rate can be set independently of the Fisher rate (it may have an effect on the latter, but the short-term pass-through is probably zero or negative), and that the liquidity premium is best understood as the residual $i^F - i^P$.

5.2 Monetarism, old and new

In most of monetary theory, by contrast, the deep implementation of monetary policy is through money growth which determines prices and inflation.[10] These affect the economy via a real balance effect where expected inflation makes people seek to avoid holding money; generally, this hurts any kind of economic activity where cash is used. Again, higher interest rates reduce output, but the mechanism is very different. Changing interest rates may be how monetary policy is *communicated*, but the true causal variable is expected inflation.

This approach is problematic since expected inflation does not vary much over the business cycle in developed countries [24]. Inflation does vary decade by decade, and across countries, which is how we know that the inflation tax exists and is significant [9]. But modeling a 25 basis points cut in the policy rate via a 25 basis points reduction in expected inflation is a non-starter.

Certainly, monetary economists have known for a long time that some bonds are liquid, and these have a lower yield because of a liquidity premium [18]. However, the full implications of this fact have not been appreciated, as evidenced by the dozens of papers that still identify i_t^F with the monetary policy instrument. Part of the reason is that in some models (e.g., [48]), the bond liquidity premium is a constant markdown $\bar{\mu}$ such that:

$$i_t^P = (1 - \bar{\mu}) i_t^F$$

[10] Generally, any monetary theory paper since 1967 [42] can be presumed to use money growth as the main policy variable, unless explicitly stated otherwise in the abstract – and sometimes even then [41].

So even though i^F and i^P are now distinct rates, as may be accounted for in a calibration, it is still the case that policy changes to i_t^P must correspond one-to-one to changes in i_t^F. This does not help much. The risk-free rate puzzle is resolved (liquid bond rates are low because of the liquidity premium), but other puzzles remain (e.g., the weak link between i_t^P and inflation). The solution lies in models where i_t^P also depends on the quantity of liquid bonds relative to money, so it can be implemented via open-market operations *independently* from the evolution of i_t^F (see Sections 2 and A.1).

5.3 The natural rate of interest

This is a related concept, since the Fisher rate i^F and the "natural rate" represent similar things: time preference, expected growth, expected inflation, and risk aversion. But there are three big differences. First, the Fisher rate is the *theoretical upper bound* on the policy rate (and on all short-term safe rates), whereas the natural rate is supposed to be the *average* of actual rates over time. Second, the natural rate is supposed to be the interest rate that would prevail if prices were flexible; here, prices are already flexible, and the reason actual rates differ from the Fisher rate is due to the moneyness of real-world bonds. Third, more than just being the average, the natural rate is supposed to be an *attractor* of policy rates, at least in a determinate model (given "active" policy, e.g. the Taylor principle being satisfied). Not so here: through altering the money supply (relative to bonds and relative to demand), the monetary authority can pick any $i^P \in [0, i^F]$. The Fisher rate is just a bound, not an attractor.

6 Summary

Back to the question that opened this paper: what is the opportunity cost of holding zero-interest cash? The answer depends on what we compare it to. If the alternative is spending the cash on consumption right now, then the opportunity cost equals the return i^F on a virtual, illiquid bond, which must be estimated rather than just observed. This return can be split into two spreads which affect the economy in different ways. First, the opportunity cost of holding money versus a liquid bond, which is the interest rate i^P. Second, the opportunity cost of holding a liquid bond versus buying consumption right away, which is the aggregate liquidity premium ℓ. Both of these spreads – the spread from 0 to i^P and the spread from i^P to i^F – *combine to define the stance of monetary policy*.

I conclude this note with a remark on the term "liquidity premium" to resolve some possible confusion about its plausible size. The way this term is used in most of the finance literature, it refers to spreads between the yields on otherwise similar assets which can be clearly attributed to liquidity differences. For instance, the 30-60 basis points on-the-run premium [47], or the 40 basis points gap between Treasuries and TIPS returns that remains after

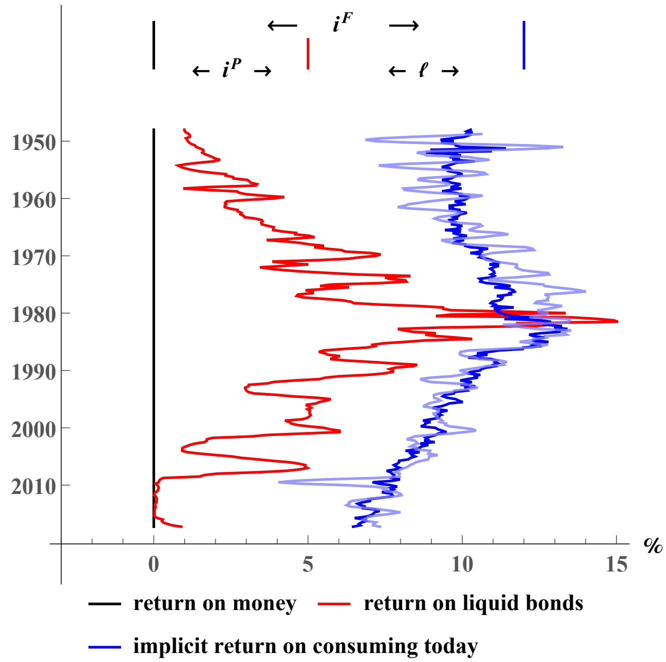


Figure 4: Black line: nominal return on the medium of exchange, which has traditionally been zero but may become something else if the future is cashless. Red line: 3-month U.S. T-bill rate. Blue lines: estimates of i_t^F as forecast of U.S. nominal consumption growth (details in Section 3).

stripping out inflation expectations [5].[11] Macroeconomists may perhaps feel that such relatively small spreads are not of first-order importance for *our* discipline. But in the language of the model from Section 2, what they measure is the relative liquidity of asset X to asset Y, times the aggregate liquidity premium:

$$(\eta^X - \eta^Y)\ell$$

Given $\ell \in (0, 10\%)$, this is on the order of basis points if η^X and η^Y are not too different – and as long as X and Y are traded often enough for their yields to be reliably measured, this is likely the case. So the above-mentioned small spreads are consistent with ℓ being on the order of several percentage points, and thus of first-order importance for the macroeconomy. Consider, again, money itself: it pays no interest yet people hold it even while inflation and consumption growth are robustly positive. If the liquidity premium on money can be 10%, surely the liquidity premium on the next-most liquid asset in the economy (3-month government bills, central bank reserve accounts, etc.) can be almost as large?

[11] Even Nagel [38], who explicitly seeks to estimate the "liquidity premium of near-money assets", identifies the opportunity cost of money with the yield on 3-month interbank repo loans. Not quite a perfectly illiquid asset, according to the definitions on page 2, which explains why his estimates of the aggregate liquidity premium stay between 0 and 1 percentage points.

Appendix

A.1 A formal model

This model is adapted from Geromichalos and Herrenbrueck [20], making two changes: the capital share α is set to zero (so capital is irrelevant, for simplicity), and instead I add a Lucas tree as the third asset in addition to money and bonds. Mathematical derivations, justification of modeling choices, and additional details (such as how to model fiscal and monetary policy separately) can be found in [20]. Here, I keep the exposition brief.

The economy consists of a unit measure of households and a consolidated government. Each household has two members: a worker and a shopper, who make decisions jointly to maximize the household's utility. All households are anonymous, therefore they cannot make long-term promises and trade must be quid-pro-quo.

Time $t = 0, 1, \ldots$ is discrete and runs forever. Each period is divided into three subperiods: an asset market (AM), a production market (PM), and a centralized market (CM). There are three assets: fiat money (supply M_t) and one-period nominal discount bonds (B_t) issued by the government, and a Lucas tree in exogenous supply X_t. Each period, a fraction δ of trees is destroyed; new trees are created exogenously and given equally to all households. Changes in the stock of money and bonds are implemented in two ways: either via open-market operations in the AM, or via lump-sum transfers in the CM.

A period proceeds as follows. At the beginning, a randomly selected fraction λ of shoppers learn that they will enter the PM where they can buy goods from workers; the other shoppers will not participate in the PM in that period. All workers participate in the PM, and each worker produces y_t units of consumption goods at zero cost (which is exogenous). Due to anonymity, credit is not feasible so shoppers must pay for the goods with a suitable medium of exchange. I assume that money is the only asset that can fulfill this role.[12] The CM is where consumption takes place, and consumption can come from two sources: produced goods y_t or the fruit of Lucas trees, d_t per tree. (Thus, the aggregate supply of consumption goods satisfies $c_t = y_t + d_t X_t$.) In the CM, all households can trade consumption goods among themselves, as well as all assets – hence, the CM is the 'primary' asset market.

At the beginning of a period, some shoppers learn that they will be active during the PM and that they will therefore need money, while other shoppers learn the opposite. They may want to trade with other households to rebalance their portfolios; they can do so in the AM, which is therefore the 'secondary' asset market. This is also the market where the government sets the policy rate, consistent with reality where monetary policy is implemented in markets where agents "borrow from and lend to each other overnight to meet short-term business needs" [17]. The timing is illustrated in Figure A.1.

[12] This is an assumption here, but see [20] for a discussion of the microfoundations behind it.

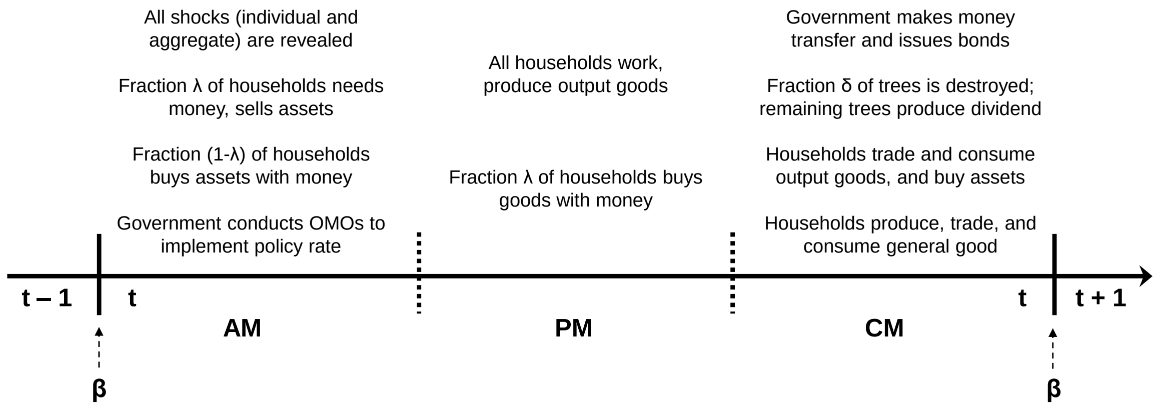


Figure A.1: Timing of events in the formal model.

The three assets differ in their "liquidity" as follows. Money is the most liquid: it can be traded for any other commodity, in any market. Bonds are the next most liquid: they can be freely traded in the AM and in the CM, but not in the PM. Trees are the least liquid: they can be traded freely in the CM, but only a fraction η of each tree can be traded in the AM.[13]

Finally, there exists a "general" consumption good, $g \in \mathbb{R}$, which households can produce, consume, and trade during the CM (negative values mean production and positive ones mean consumption). This good is in zero aggregate supply; its only function in the model is to induce linear preferences for an individual household, collapse the portfolio problem into something tractable, and create a representative household.[14] Households discount the future at rate $\beta < 1$ and have the following per-period utility function:

$$U_t(c_t, g_t) = u(c_t) + g_t,$$

where u is a twice continuously differentiable function that satisfies $u' > 0$ and $u'' < 0$.

Solving this model proceeds closely along the lines of [20], so here I skip straight to the solution. One important equilibrium object is $q_t \in (0, 1)$, which is the equilibrium ratio of real balances to the level of output sold in the PM. q_t also equals to the ratio between the real price of output in the PM and its marginal value in the CM, which must be less than 1 to compensate shoppers for bringing money into the goods market in the first place.

Other equilibrium objects include c_t, the aggregate level of consumption; p_t, the CM price of consumption goods in terms of money; p_t^B, the CM price of bonds in terms of money; p_t^X, the CM price of trees in terms of money; and i_t^P, the monetary policy rate. Consistent with

[13] Different combinations of δ, η, and the variance of the dividend d_t can flexibly accommodate most real-world assets: δ and $\text{var}(d_t)$ are negatively correlated with η across the span of real-world assets, but not perfectly so [21].

[14] Readers who dislike representative agents only need to take out the g-good and solve the model with the usual numerical methods, to obtain a liquidity-augmented *non*-representative agent asset pricing model.

reality, this is defined as the secondary market return on the liquid bond; equivalently, the AM price of bonds in terms of money is $1/(1+i_t^P)$.

Finally, the Euler equations for all assets become much simpler when written in terms of the *ex-post liquidity premium*:

$$\ell_t = \lambda_t \left(\frac{1}{q_t(1+i_t^P)} - 1 \right) \tag{A.1}$$

Then, the Euler equation for money is:

$$1 = \mathbb{E}_t \left\{ \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}} \cdot \left(\lambda_{t+1} \frac{1}{q_{t+1}} + (1-\lambda_{t+1})(1+i_{t+1}^P) \right) \right\}$$

$$\leftrightarrow \qquad 1 = \mathbb{E}_t \left\{ \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}} \cdot (1+\ell_{t+1})(1+i_{t+1}^P) \right\} \tag{A.2}$$

And the Euler equation for liquid bonds (in the primary market):

$$p_t^B = \mathbb{E}_t \left\{ \beta_t \frac{u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}} \cdot (1+\ell_{t+1}) \right\} \tag{A.3}$$

And the Euler equation for imperfectly liquid trees:

$$p_t^X = \mathbb{E}_t \left\{ \beta_t \frac{u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}} \cdot (1-\delta) \left(p_{t+1} d_{t+1} + p_{t+1}^X \right) \cdot (1 + \eta_{t+1} \ell_{t+1}) \right\} \tag{A.4}$$

A few useful approximations are immediately apparent. If we define the Fisher interest rate i_t^F via Equation (1), the Euler equation for money becomes:

$$(1+i_t^F) \approx (1+\ell_{t+1})(1+i_{t+1}^P) \qquad \Rightarrow \qquad i_t^F \approx \ell_{t+1} + i_{t+1}^P$$

And the Euler equations for bonds and trees become:

$$p_t^B \approx \frac{1+\ell_{t+1}}{1+i_t^F} \approx \frac{1}{1+i_{t+1}^P}$$

$$1 + r_{t+1}^X \approx \frac{1}{1-\delta} \cdot \frac{1+i_t^F}{1+\eta_{t+1}\ell_{t+1}} \qquad \Rightarrow \qquad r_{t+1}^X \approx \delta + i_{t+1}^P + (1-\eta_{t+1})\ell_{t+1}$$

where $1 + r_{t+1}^X \equiv (p_{t+1}d_{t+1} + p_{t+1}^X)/p_t^X$ is the ex-post *nominal* rate of return on trees.

The latter of these are exactly the equations from the informal model in the previous section. Thus, the informal model replicates the formal model in steady state, and provides a decent approximation outside of it.

Finally, we still need the structural form of the money demand functions, G and H from

Equations (3) and (5). These are provided by the AM clearing equation of the formal model:[15]

$$1 + i_t^P = \frac{\lambda_t}{1 - \lambda_t} \left[\frac{B_t}{M_t} + \eta_t \, \frac{(1-\delta)(p_{t+1} d_{t+1} + p_{t+1}^X) \, X_t}{M_t} \right] \tag{A.5}$$

Thus, the secondary market yield on liquid bonds depends on the demand for liquidity in that period ($\lambda_t/(1-\lambda_t)$), on the supply of liquid bonds relative to money (B_t/M_t), as well as the market value of other liquid assets relative to the money supply, times the liquidity of these assets ($\eta_t \cdot [\ldots]$).

We can invert Equation (A.5) to obtain the B/M-ratio required for a particular level of the policy rate:

$$\frac{B_t}{M_t} = \frac{1 - \lambda_t}{\lambda_t} \left(1 + i_t^P\right) - \eta_t \, \frac{(1-\delta)(p_{t+1} d_{t+1} + p_{t+1}^X) \, X_t}{M_t} \tag{A.6}$$

In the short run, the Fisher rate does not appear in either formula, but in the long run it does, because both the price of trees and the level of real balances depend on it. We impose steady states, substitute the price of goods via the PM clearing equation $M = qpy$, and solve the steady-state values of p^X/p and q using the Euler equations. After substituting, Equation (A.6) becomes:

$$\frac{B}{M} = \left(1 + i^P\right) \cdot \left[\frac{1-\lambda}{\lambda} - \eta \, \frac{(1-\delta)\left(1 + \frac{\ell}{\lambda}\right)}{1 - \beta(1-\delta)(1+\eta\ell)} \, \frac{d\,X}{y} \right], \tag{A.7}$$

where y is total output produced in the PM, and $\ell = (i^F - i^P)/(1+i^P)$ in steady state. Clearly, B/M (which defines the H-function from the informal model) is increasing in i^P and ℓ, so it must be an increasing function of the Fisher rate i^F for a given policy rate.

Thus, G and H can be solved in closed form and have the desired comparative statics. However, I claim without proof that similar qualitative relationships are true in most other models of liquid bonds as well.

[15] For these formulas to be correct, (M_t, B_t) must be defined as the quantities available at the *end of the AM* of period t, after the open-market operations have concluded. Generally, these quantities are different from those at the beginning or end of a full period.

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