Optimal Fiscal Consolidation in a Currency Union*

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Abstract

This paper studies, in the context of a New Keynesian open-economy model, the optimal response of fiscal policy to a risk premium shock for a country in a currency union. First, I show that the planner should *not* use government spending to stimulate the economy. Instead of distorting the provision of public goods, it is optimal to use simple tax instruments, as consumption, sales, and payroll tax, to achieve stabilization goals. Second, it is optimal to *front-load* taxes, i.e., the overall level of taxes increase in response to a positive risk premium shock, and it declines over time. The composition of taxes is also time-varying. Consumption tax is increasing, while either VAT or payroll taxes decline over time after an initial increase. Under downward nominal wage rigidities, it is optimal to implement a form of *fiscal appreciation*, a decline in the VAT accompained by an increase in the payroll tax. Government debt is smaller under the optimal policy than under a passive fiscal policy where the government does not react to the shock. Under some circumstances, it may be optimal to stabilize the government debt at its pre-shock level. Therefore, under the optimal policy, there is no necessary trade off between stabilization policy and fiscal consolidation.

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1 Introduction

Sudden stops are costly episodes, as financial outflows and the resulting increase in the country's interest rates oftentimes are followed by a severe recession, increase in unemployment, and worsening of government finances. Sudden stops are especially costly for countries in a currency union, as became evident in the recent European sovereign debt crisis, where peripheral countries suffered a substantial decline in economic activity and increase in government debt as their borrowing costs diverged from the one in Germany, as shown in figure 1. In this context, fiscal policy becomes a first line of defense.

However, the appropriate design of fiscal policy became the center of a contentious debate, with one side pointing out the need for fiscal stimulus, given the severity of the recession, and the other defending the need for fiscal consolidation, given the deterioration of government finances. Since the proper design of fiscal policy is particularly relevant in this context, and in face of conflicting views on the topic, a formal evaluation of the appropriate fiscal response becomes especially important. In this paper, I provide such formal evaluation, by studying the optimal fiscal policy in response to interest rate shocks in a currency union.

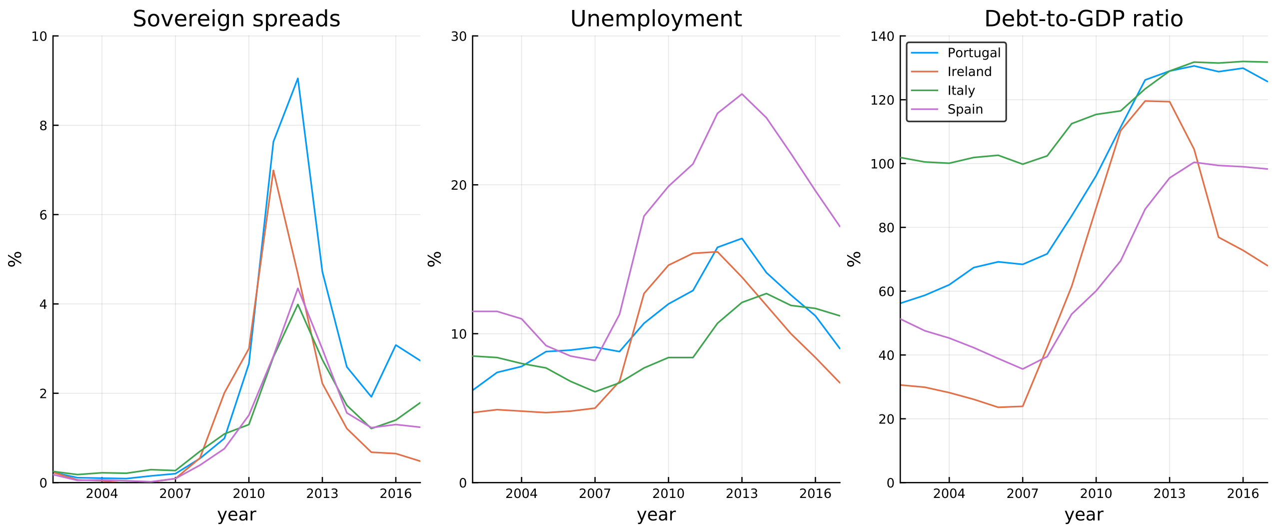
The environment is based on the continuous-time version of the open economy New Keynesian model of Gali and Monacelli (2005), as proposed by Farhi and Werning (2012). The continuous-time setting adds tractability to the analysis, allowing for a sharper characterization of the optimal policy. The monetary union consists of a continuum of countries sharing a common currency. Households consume a combination of domestically produced goods and an aggregate of goods produced in each country of the union. The degree of openness of an economy, measured by the share of expenditure on domestic goods, and the elasticity of substitution between foreign goods will be important determinants of the optimal fiscal policy. Firms are monopolistically competitive and subject to pricing frictions à la Calvo. As an extension, I will consider the role of downward nominal wage rigidities.

In contrast to previous work on the topic, I take the point of view of an individual country when solving for the optimal policy, instead of assuming the planner maximizes welfare for the whole monetary union. This will be important as it will capture the fact that, while countries in the periphery of Europe are in a currency union, they are not in a fiscal union. Another important aspect regards the availability of instruments. I assume the country has access to a small set of simple fiscal instruments, such as government spending, consumption, sales (VAT), and payroll taxes. However, the government cannot use import tariffs or export subsidies, as these countries are typically limited by trade agreements on the use of such instruments. It turns out that, in the absence of such instruments, it is impossible to separate the design of domestic policies from international considerations.

The nominal interest rate in each country is determined by two components. First, the interest rate chosen by the centralized monetary authority, which is taken as given by any individual member of the union. Second, a country-specific sovereign spread or risk premium. I will focus on the case of an exogenous risk premium. This will allow us to isolate the impact of stabilization policies on government finances, abstracting from any feedback from government debt to spreads. As we will see, there will be a role for fiscal consolidation even if the government is completely unable to reduce spreads through the choice of policies.[1]

[1] Movements in sovereign spreads that are independent of government debt levels can be interpreted as movements in

Figure 1: The costs of sudden stops: spreads, unemployment, and government debt



Source: Eurostat. Sovereign spreads denote the difference on 10-year nominal rates between each country and Germany.

The model is able to capture the main features of a sudden stop as documented, for instance, in Mendoza (2010). In response to a risk premium shock, consumption falls, the real exchange rate depreciates, while net exports increase. Government debt increases with the rise in borrowing costs. Under sticky prices, the economy falls into a recession and, under downward nominal wage rigidities, unemployment increases.

Our first main result regards the use of government spending as a stabilization tool. Regardless of the degree of price or wage stickness, it is *not* optimal to deviate from purely cost-benefit considerations in the provision of public goods in order to stimulate the economy. In particular, it is never optimal to engage in wasteful spending, even if no other type of spending is available. This is in contrast with the Keynesian idea that "filling old bottles with banknotes, bury them at suitable depths", just so they could be dug up again, would be beneficial in a depressed economy given a positive spending multiplier. However, despite any multiplier effect, it is always optimal to use taxes to achieve any stabilization goal. The logic is similar to the Principle of Targeting, commonly used in international trade.[2] In the presence of static and intertemporal distortions, it is optimal to use tax instruments that directly act on these margins, instead of only indirectly affecting them by distorting the provision of public goods.

The burden of the stabilization policy will then fall on tax policy. Our second main result is that it is optimal to *front-load* taxes, i.e., the overall level of taxes, the sum of consumption, sales, and payroll taxes, increase on impact and decline over time. Moreover, the composition of taxes is time-varying. Consumption tax is increasing while sales or payroll taxes decline over time, depending on which instrument is being adopted. The optimal policy problem typically does not pin down individual

the level of risk aversion in international financial markets, as in Rey (2015), or from changes in the capital of international arbitrageurs, as in Maggiori (2017).

[2]The Principle of Targeting was developed in Bhagwati and Ramaswami (1963). See Dixit (1985) for a discussion.

taxes, as there are different combinations of taxes that implement the optimal allocation. However, it uniquely determines *wedges*. Two wedges will be of interest: the *labor wedge*, the gap between the marginal rate of substitution and transformation between consumption and labor, and the *intertemporal wedge*, the gap between the marginal rate of substitution and transformation between consumption in different dates. The labor wedge will be associated with the sum of taxes, while the intertemporal wedge will determine the change in consumption taxes. Importantly, the labor wedge can be interpreted as an effective mark-up, as it will be equal to the ratio of relative price to the (pre-tax) real marginal cost.

Under flexible prices, the labor wedge is determined by the balancing of two effects. First, an *international monopolist effect*, where the planner has an incentive to tax domestic firms to keep prices high and extract more revenue from foreigners, as in the standard optimal tariff argument. Second, a *domestic distortion effect*, where the planner has an incentive to subsidize domestic firms to eliminate the effect of market power. Depending on the relative importance of each effect, the planner may on average subsidize or tax firms. However, in response to a risk premium shock, it is always optimal to raise taxes. As domestic consumption declines in response to the increase in interest rates, foreign demand becomes more important, strengthening the international monopolist effect and leading to the increase in taxes. As consumption slowly increases, the domestic distortion effect regain its relevance, and taxes will tend to fall over time.[3]

A high labor wedge means that the cost of domestic goods is relatively low, as the marginal utility of an additional good exceeds the marginal disutility of providing it. The planner then has an incentive to shift resources to periods where the labor wedge is positive. Hence, a time-varying labor wedge will induce intertemporal distortions. As the labor wedge increases on impact, the planner has an incentive to shift demand to the present by having an increasing consumption tax, which creates inflation expectations and reduces the real interest rate. Therefore, it is optimal to partially offset the risk premium shock even under flexible prices. One particular implementation of the optimal policy is to have an increasing consumption tax, a VAT that increases on impact and declines faster than the increase in the consumption tax, such that the sum of taxes is decreasing.

The behavior of the consumption tax is similar under sticky prices. As before, the planner responds to a temporary increase in the labor wedge by shifting consumption to the present. But now the increase in the labor wedge is coming in part from an increase in mark-ups, as prices do not follow the decline in costs during a downturn. Hence, it is again optimal to have an increasing consumption tax.

It will be optimal to front-load taxes under sticky prices provided that international demand is sufficiently inelastic. Notice that the pricing friction prevents an immediate depreciation of the real exchange rate, so a natural policy response would be to reduce taxes to accelerate the process of internal devaluation and stimulate the economy. However, the planner acts in the direction of *appreciating* the terms of trade. The reason is that the only way of achieving a depreciation of the real exchange rate is by a period of deflation. However, deflation is costly with staggered price setting. If the cost of deflation is sufficiently high, it is optimal for the planner to achieve a more stable terms of trade and limit the internal devaluation caused by the recession. The more inelastic the international de-

[3]Such dynamic effects are in contrast to standard optimal tariff results. If international demand is constant, as assumed, then the optimal tariff should be constant, despite the presence of shocks.

mand, the smaller the threshold for the cost of inflation/deflation such that it is optimal to increase taxes in response to shock.[4] If international demand is sufficiently inelastic, then it is always optimal to front-load taxes.[5]

The front-loading of taxes will have an impact on the evolution of government debt. First, we show that the fiscal needs of the government fall under the optimal policy, compared to a passive policy where the government does not react to the shock. Moreover, under some circumstances, the path of government debt is always smaller under the optimal policy than under the passive policy. This is indeed the case under the baseline calibration. Moreover, it is optimal to stabilize government debt at roughly the pre-shock level. Hence, by properly designing tax and spending policy, the government is able to stimulate the economy and achive fiscal consolidation.

Finally, I consider the economy under downward nominal rigidities. In contrast to the economy with sticky prices, the planner now needs all three taxes to implement the optimal policy. This allow us to evaluate the optimality of a *fiscal devaluation* policy, where the government increases the VAT tax and reduces the payroll tax. It turns out it is optimal to implement a fiscal *appreciation*, as the VAT decreases on impact, while the payroll tax is increased. By decreasing the VAT and increasing the payroll tax, the planner is able to shift up the labor demand and shift down the labor supply, reducing unemployment. By combining these instruments with the consumption tax, the planner is able to achieve the unconstrained flexible prices optimum.

Literature. The analysis of fiscal policy in currency unions can be traced all the way back to Kenen (1969). Recent contributions focusing on optimal policy can be found in Gali and Monacelli (2008) and Ferrero (2009). In contrast to their work, who focus on the case of a centralized fiscal authority, I analyze the case of decentralized fiscal decisions. In the context of constrained monetary policy in a closed economy, Werning (2011) analyzed the optimal government spending in the absence of tax instruments and found that stimulus spending, deviations from a pure cost-benefit analysis, is quantitatively small. The behavior of consumption taxes echoes the unconventional fiscal policy of Correia et al. (2013). However, in contrast to the closed-economy analysis, it is not optimal to keep producer price inflation at zero, as the planner has incentives to manipulate the terms of trade. Moreover, the intensity of the intervention depends on the degree of openness. In particular, as the economy becomes fully open, it is optimal not to use consumption taxes, despite of the recession. Terms of trade manipulation has been recently used in the study of capital controls, Costinot et al. (2011), and foreign exchange interventions, Fanelli and Straub (2016). A recent literature has focused on the impact of fiscal consolidations, see, for instance, De Mooij and Keen (2012) and Alesina et al. (2017). Farhi et al. (2013) studies the implementation of fiscal devaluations, but not its optimal design, and Correia et al. (2011) provided an early discussion of the policy.

Organization. The paper is organized as following. Section 2 describes the environment and the equilibrium conditions. Section 3 presents the non-linear Ramsey problem. Section 4 considers the linear-quadratic problem under flexible prices and section 5 under stick prices. Section 6 discusses the case of downward nominal wage rigidities and the last section presents the conclusion.

[4]The threshold is smaller with more inelastic demand because the international monopolist effect gets stronger in this case, strengthening the incentive to keep the terms of trade appreciated.

[5]This will be the case for unitary elasticity, as in the Cole-Obstfeld economy commonly studied in the literature.

2 Fiscal Policy in a Currency Union

The economy consists of a continuum of countries, indexed by $i \in [0, 1]$, sharing a common currency. Preferences and technology are symmetric across countries. I will focus attention in an specific country which will be called "Home", with index $i = H$.

I will consider a perfect foresight equilibrium. Since most of the analysis will focus on a first-order approximation of the equilibrium conditions, certainty-equivalence holds and there is no further loss on abstracting from ongoing uncertainty.

2.1 Environment

Households

Preferences are given by

$$\int_0^\infty e^{-\rho t} \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \log G_t - \frac{N_t^{1+\phi}}{1+\phi} \right] dt \tag{1}$$

Consumer derives utility from an aggregate of consumption goods C_t, an aggregate of government purchases G_t, and there is a disutility from supplying labor N_t.

Aggregate consumption C_t is a Cobb-Douglas composite of domestic and foreign goods:

$$C_t = \left(\frac{C_{H,t}}{1-\alpha} \right)^{1-\alpha} \left(\frac{C_{F,t}}{\alpha} \right)^{\alpha}$$

where $C_{F,t}$ is a composite of foreign goods:

$$C_{F,t} = \left(\int_0^1 \Lambda_{i,t}^{\frac{1}{\gamma}} C_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

and $C_{i,t}$ is an index of goods produced in country i:

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

and for $i = H$ we obtain the composite of domestic goods.

The parameter α controls the degree of home-bias. As α goes to zero, there is extreme home-bias and the economy barely trades with the rest of the world. As α goes to one, there is no home-bias, and households in all countries will consume the same basket of goods. The parameter ϵ represents the price elasticity of goods within a given country and γ represents the elasticity of substitution between goods from different countries. The term $\Lambda_{i,t}$ represents an export demand shock for country i.

The per-period budget constraint is given by

$$\dot{B}_t = i_t B_t + (1 - \tau_t^l) W_t N_t + \Pi_t + \tilde{T}_t - (1 + \tau_t^c) \left[\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di \right]$$

where i_t represents the nominal interest rate, B_t nominal assets, and Π_t aggregate nominal profits.

Note that the consumer faces two different types of taxes: a labor income tax τ_t^l, and a tax on consumption τ_t^c. Importantly, the consumption tax is the same for foreign and domestic goods. The consumer also possibly receives some lump-sum transfers from the government $\tilde{T}_t$.

Households are subject to the usual No-Ponzi condition:

$$\lim_{t\to\infty} e^{-\int_0^t i_s ds} B_t \geq 0$$

Terms of Trade and Real Exchange Rate

Let's define now the price indexes associated with the bundles the consumer demands. I will define the *before-tax* price levels. The producer price index for country i is defined as

$$P_{i,t} = \left[\int_0^1 P_{i,t}(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$$

and we obtain the domestic producer price index (PPI) for $i = H$.

The price index for the aggregate of foreign goods is

$$P_{F,t} = \left(\int_0^1 \Lambda_{i,t} P_{i,t}(j)^{1-\gamma} dj\right)^{\frac{1}{1-\gamma}}$$

The consumer price index (CPI) is given by

$$P_t = P_{H,t}^{1-\alpha} P_{F,t}^{\alpha} \tag{2}$$

I will focus on the case of a symmetric rest of the world, where $P_{i,t} = P_{j,t}$ for $i, j \neq H$. Hence, the price level for any foreign country and the price index of imported goods are equal to each other, denoted by P_t^*. Following Gali and Monacelli (2008), I define the terms of trade as the ratio of import prices to export prices

$$S_t = \frac{P_{F,t}}{P_{H,t}} = \frac{P_t^*}{P_{H,t}}$$

As all countries share the same currency, the nominal exchange rate is fixed and equal to one. The real exchange rate is then given by

$$\mathcal{Q}_t = \frac{P_t^*}{P_t}$$

The assumption of unit elasticity of substitution between domestic and foreign goods gives a simple relationship between the real exchange rate and the terms of trade:

$$\mathcal{Q}_t = S_t^{1-\alpha}$$

I will then refer to movements in the terms of trade or the real exchange rate interchangeably.

Firms

Each differentiated good is produced using labor as the only input:

$$Y_t(j) = A_t N_t(j)^{\frac{1}{\varphi}}$$

where $\varphi \geq 1$.

The problem of the firm will depend on the degree of price flexibility. For the case of sticky prices, I assume Calvo pricing, i.e., the periods that firms are allowed to reset their prices are determined by a Poison arrival with intensity ρ_δ.

The problem of the firm is given by:

$$\max_{\overline{P}_t(j)} \int_0^\infty e^{-\int_0^s (i_{t+z}+\rho_\delta)dz} \left[(1-\tau^v_{t+s})\overline{P}_t(j)Y_{t+s|t} - W_{t+s}\left(\frac{Y_{t+s|t}}{A_{t+s}}\right)^\varphi \right] ds$$

where the firm faces a sales tax τ^v_t, $Y_{t+s|t}$ represents the demand function the producer faces at period $t+s$:

$$Y_{t+s|t} = \left(\frac{\overline{P}_t(j)}{P_{H,t+s}}\right)^{-\epsilon} Y_{t+s}$$

and Y_t denotes aggregate demand for domestic goods.

If prices are completely flexible, the problem of the firm collapses to

$$\max_{\overline{P}_t(j)} \left\{ (1-\tau^v_t)\overline{P}_t(j)Y_{t|t} - W_t\left(\frac{Y_{t|t}}{A_{t+s}}\right)^\varphi \right\}$$

Government

Government consumption is an aggregate of domestically produced goods:

$$G_t = \left(\int_0^1 G_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

Government purchase of each individual good is made in order to minimize costs, given the aggregate amount G_t. The government flow budget constraint is given by

$$\dot{D}^g_t = i_t D^g_t + P_{H,t}G_t + P_{H,t}T_t - \tau^v_t P_{H,t}Y_t - \tau^c_t P_t C_t - \tau^l_t W_t N_t$$

where D^g_t denotes government debt and $T_t = \tilde{T}_t / P_{H,t}$ denotes real transfers.

The government is also subject to a No-Ponzi condition

$$\lim_{t\to\infty} e^{-\int_0^t i_s ds} D^g_t \leq 0$$

2.2 Equilibrium Conditions

We can divide the equilibrium conditions into two blocks: a demand and a supply block.

Demand Block

The solution of the consumer problem involves an intratemporal condition:

$$C_t^{\sigma} N_t^{\phi} = \frac{(1-\tau_t^l)W_t}{(1+\tau_t^c)P_t} \tag{3}$$

and an intertemporal condition[6]

$$\frac{\dot{C}_t}{C_t} = \sigma^{-1}\left(i_t - \pi_t - \dot{\hat{\tau}}_t^c - \rho\right)$$

and the transversality condition

$$\lim_{t\to\infty} e^{-\int_0^t i_s ds} B_t = 0$$

Notice the level of the labor income tax and the consumption tax affects the labor supply condition, while only changes in the consumption tax affect the Euler equation.

An analogous Euler equation holds at the foreign countries:

$$\frac{\dot{C}_t^*}{C_t^*} = \sigma^{-1}\left(i_t^* - \pi_t^* - \rho\right)$$

where an asterisk indicates foreign variables and foreign taxes are assumed to be constant.

In order to connect the Euler equation in the home country to the Euler equation in the rest of the world, we need to specify how domestic and foreign interest rates are related:

$$i_t = i_t^* + \psi_t$$

I allow for possible deviations from the uncovered interest parity (UIP). The term ψ_t captures in a reduced-form way changes in risk premium. For instance, an increase in ψ_t may indicate a reduction of the ability of foreign investor to hold domestic assets due to an increase in risk aversion or balance sheet problems.

Taking the difference between the two Euler equations and integrating the resulting expression, we get the so-called Backus-Smith condition:

$$C_t = \Theta_t C_t^* S_t^{\frac{1-\alpha}{\sigma}} \tag{4}$$

where

$$\Theta_t = \Theta_0 \Psi_t \mathcal{T}_t; \qquad \Psi_t = e^{\frac{1}{\sigma}\int_0^t \psi_s ds}; \qquad \mathcal{T}_t = \left(\frac{1+\tau_0^c}{1+\tau_t^c}\right)^{\frac{1}{\sigma}} \tag{5}$$

for some constant Θ_0.

Equation (4) captures the *intertemporal* effect of real exchange rates on domestic consumption. An increase in $S_t^{1-\alpha}$, a more depreciated real exchange rate, implies higher real interest rates on average, and faster consumption growth. In the absence of risk premium shocks and with constant consump-

[6]The term $\dot{\hat{\tau}}_t^c$ is the time derivative of $\log(1+\tau_t^c)$.

tion taxes, the term Θ_t is constant as in the complete markets economy of Backus and Smith (1993), where Θ corresponds to the Pareto weight in the planner's problem. It is common in the literature with incomplete markets to refer to Θ_t as a (pseudo) Pareto weight, even though it is time-varying. Movements in Θ_t reflect the effect of risk premium shocks and consumption taxes on the real interest rate the household faces. A positive risk premium shock or a reduction in consumption taxes leads to an increase in the real interest rate and in consumption growth.

The demand for domestic goods is given by $Y_t = C_{H,t} + G_t + C^*_{H,t}$ and can be written as

$$Y_t = (1-\alpha)S_t^{\alpha}C_t + G_t + \alpha\Lambda_{H,t}S_t^{\gamma}C_t^* \tag{6}$$

using the fact that $S_t^{-\alpha} = \frac{P_{H,t}}{P_t}$.

The first term represents the demand of domestic agents for domestic output, the second term represents government demand and the third term corresponds to exports. This equation captures the *intratemporal* effects of movements in the real exchange rate. An increase in the terms of trade, a depreciation of the real exchange rate, will induce an *expenditure-switching* effect, as domestic and foreign households shift consumption to domestic goods.

Real net exports are given by

$$NX_t \equiv \frac{P_{H,t}C^*_{H,t} - P_{F,t}C_{F,t}}{P_t} = S_t^{-\alpha}\left[\alpha\Lambda_{H,t}S_t^{\gamma}C_t^* - \alpha S_t^{\alpha}C_t\right]$$

using the demand for domestic goods to eliminate $C^*_{H,t}$.

Let the amount of external debt of the home country be denoted by $E_t \equiv D_t^g - B_t$. The external solvency condition relates the external debt to the present discounted value of net exports:

$$\frac{C_0^{-\sigma}E_0}{P_0(1+\tau_0^c)} = \int_0^{\infty} e^{-\rho t}\frac{C_t^{-\sigma}}{1+\tau_t^c}S_t^{-\alpha}\left[\alpha\Lambda_{H,t}S_t^{\gamma}C_t^* - \alpha S_t^{\alpha}C_t\right]dt \tag{7}$$

Similarly, the government solvency constraint relates government debt to the present discount value of primary surpluses

$$\frac{C_0^{-\sigma}D_0^g}{P_0(1+\tau_0^c)} = \int_0^{\infty} e^{-\rho t}\frac{C_t^{-\sigma}}{1+\tau_t^c}S_t^{-\alpha}\left[\tau_t^v Y_t + \tau_t^c S_t^{\alpha}C_t + \tau_t^l\frac{W_tN_t}{P_{H,t}} - G_t - T_t\right]dt \tag{8}$$

where I used the Euler equation to eliminate the real interest rate from both conditions.

Supply Block

Aggregate demand for labor is given by

$$N_t = \Delta_t\left(\frac{Y_t}{A_t}\right)^{\varphi} \tag{9}$$

where

$$\Delta_t \equiv \int_0^1\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon\varphi}dj \tag{10}$$

The term Δ_t captures the effect of price dispersion on aggregate labor demand (and production).

Under sticky prices, the optimal price setting condition is given by

$$\int_0^\infty e^{-(\rho+\rho_\delta)s} \frac{C_{t+s}^{-\sigma}}{P_{t+s}(1+\tau_{t+s}^c)} \left[(1-\tau_{t+s}^v) \frac{\overline{P}_t(j)^{-\epsilon}}{P_{H,t+s}^{-\epsilon}} Y_{t+s} - \frac{\varphi\epsilon}{\epsilon-1} W_{t+s} \frac{\overline{P}_t(j)^{-\varphi\epsilon-1}}{P_{H,t+s}^{-\varphi\epsilon}} \left(\frac{Y_{t+s}}{A_{t+s}}\right)^{\varphi} \right] ds = 0 \tag{11}$$

Equation above indicates the firm will choose prices as an weighted average of all future marginal costs. In the appendix, I derive the aggregate supply condition that shows how current inflation is determine by current and future costs.

In the limit $\rho_\delta \to \infty$, the expression above collapses to the flexible price supply condition:

$$(1-\tau_t^v)\frac{P_{H,t}}{P_t} = \frac{\varphi\epsilon}{\epsilon-1}\frac{W_t}{P_t}\left(\frac{Y_t}{A_t}\right)^{\varphi}\frac{1}{Y_t} \tag{12}$$

where I used the fact that $P_{H,t}(j) = P_{H,t}$ under flexible prices.

This completes the description of the equilibrium. An equilibrium is a sequence of allocations (C_t, C_t^*, N_t, Y_t), prices $(W_t, S_t, \mathcal{Q}_t)$, and government policy $(G_t, \tau_t^c, \tau_t^l, \tau_t^v, T_t)$ such that conditions (3)-(8) and (9)-(11) are satisfied.

3 The Optimal Policy Problem

In this section, I will consider the optimal fiscal policy, under flexible and fully rigid prices, in the context of the non-linear Ramsey problem. Section 4 will provide explicit solutions using linear methods for the case of flexible prices. The role of sticky prices and downward nominal wage rigidity will be considered in sections 5 and 6.

It will be convenient to adopt a primal approach, where the planner chooses the equilibrium quantities $(Y_t, G_t, \Theta_t, S_t)$ instead of choosing taxes directly. Under flexible prices, the planner must satisfy two conditions. First, market clearing condition for goods

$$Y_t = \left[(1-\alpha)\Theta_t S_t^{\xi} + \alpha\Lambda_{H,t} S_t^{\gamma}\right] C_t^* + G_t \tag{13}$$

where $\xi \equiv \frac{1-\alpha}{\sigma} + \alpha$ is the (general equilibrium) terms of trade elasticity of domestic consumption.

Second, the external solvency constraint

$$\frac{E_0}{P_0^*} = \int_0^\infty e^{-\rho t}\Psi_t^{-\sigma}\left(\frac{C_t^*}{C_0^*}\right)^{-\sigma} \alpha \left[\Lambda_{H,t}S_t^{\gamma-1} - \Theta_t S_t^{\xi-1}\right] C_t^* dt \tag{14}$$

Conditions (13) and (14) are sufficient for determining a competitive equilibrium, in the following sense. If $(Y_t, G_t, \Theta_t, S_t)$ satisfy the conditions above, we can always find taxes and prices that support this allocation as part of an equilibrium. Therefore, the Ramsey problem consists of maximizing utility subject to conditions (13) and (14). If prices are fully rigid, the planner faces the additional constraint $S_t = \overline{S}$.

3.1 The Ramsey Problem

The Ramsey problem can be written as

$$\max_{\{\Theta_t, S_t, Y_t, G_t\}} \left\{ \int_0^\infty e^{-\rho t} \left[\frac{1}{1-\sigma} \left(\Theta_t C_t^* S_t^{\xi-\alpha} \right)^{1-\sigma} + \chi \log G_t - \frac{1}{1+\phi} \left(\frac{Y_t}{A_t} \right)^{\varphi(1+\phi)} \right] dt \right\}$$

subject to (13), (14), and $S_t = \overline{S}$ if prices are fully rigid.

Notice the problem takes the perspective of the domestic country, so the optimal policy can be implemented without international cooperation. This contrasts with previous work on fiscal policy in currency unions (see Ferrero (2009) and Gali and Monacelli (2008)) where it was taken the perspective of the whole union, implicitly assuming coordination of both monetary and fiscal policy.

Taxes and wedges

The first order condition with respect to output is

$$\lambda_t = \varphi \left(\frac{Y_t}{A_t} \right)^{\varphi(1+\phi)} \frac{1}{Y_t} = \frac{N_t^\phi}{\frac{1}{\varphi} A_t N_t^{\frac{1}{\varphi}-1}} \tag{15}$$

where λ_t indicates the Lagrange multiplier on the goods market clearing condition.

The Lagrange multiplier λ_t captures the cost of producing an additional unity of domestic output. It is given by the ratio of the marginal disutility of work and the marginal product of labor. In an open economy, this represents only a fraction of the cost of providing domestic consumption, as households also demand foreign goods. The optimality condition for Θ_t shows the planner balances the marginal benefit of consumption with the cost of obtaining domestic and foreign goods

$$C_t^{-\sigma} = (1-\alpha)\lambda_t S_t^\alpha + \alpha \Gamma \Psi_t^{-\sigma} S_t^{\alpha-1} \tag{16}$$

where Γ is the multiplier on the external solvency condition (14), and we assumed $C_t^* = C_0^*$.

Condition (16) determines the intertemporal allocation of consumption. The marginal benefit of buying a consumption bundle at period t is given by the left-hand side. The right-hand side is the marginal cost of providing the domestic and foreign goods that constitute the consumption bundle. A fraction $1-\alpha$ of the total expenditure goes to buying $S_t^\alpha = \frac{P_t}{P_{H,t}}$ units of domestic production. A fraction α of the expenditure goes to buying $S_t^{\alpha-1} = \frac{P_t}{P_t^*}$ units of the foreign good. Importantly, the cost of obtaining foreign goods at future dates is reduced in the presence of positive risk premium shocks. Hence, the planner has an incentive to back-load consumption, a factor that will be important in the optimal design of fiscal policy.

Notice that it is optimal for the planner to set $S_t^{-\alpha} = \lambda_t C_t^\sigma$ for an economy that barely trades with the rest of the world, i.e., for $\alpha = 0$. Since $\lambda_t C_t^\sigma$ represents firms' real marginal cost in the absence of taxes and $S_t^{-\alpha}$ their relative price, this condition corresponds to the laissez-faire outcome under perfect competition. The term $S_t^{-\alpha}/(\lambda_t C_t^\sigma)$ then represents the optimal mark-up and it will be tightly connected to the optimal level of taxes. Following Chari et al. (2007), I will refer to this ratio as the *labor* *wedge*:

$$1+\omega_t^L \equiv \frac{S_t^{-\alpha}}{\lambda_t C_t^\sigma}$$

Similarly, let's define the intertemporal wedge as the marginal rate of substitution between consumption in t and 0 adjusted by the marginal rate of transformation, given by the real interest rate over the same period:

$$1+\omega_t^I \equiv \frac{e^{-\rho t}(C_t/C_0)^{-\sigma}}{e^{-\int_0^t (i_s-\pi_s)ds}}$$

A positive intertemporal wedge means that consumption at the given date is more expensive than under a laissez-faire economy with the same path of real interest rates. Both wedges will play an important role during all of our analysis. Notice these wedges would be zero under no intervention and perfect competition. However, in general, the optimal policy imply non-zero wedges.

Proposition 1 (Wedges)**.** *Consider the solution to the Ramsey problem.*

i. ***Labor wedge****: if prices are flexible, then*

$$\omega_t^L = \frac{\alpha \Lambda_{H,t} S_t^\gamma}{(\gamma-1)\Lambda_{H,t} S_t^\gamma + (1-\alpha) S_t^\xi \Theta_t} \tag{17}$$

ii. ***Intertemporal wedge****: if prices are flexible or fully rigid, and if $\alpha > 0$, then*

$$1+\omega_t^I = \left[\frac{\alpha+\omega_t^L}{1+\omega_t^L}\right]^{-\sigma} \left[\frac{\alpha+\omega_0^L}{1+\omega_0^L}\right]^{\sigma} \tag{18}$$

and $1+\omega_t^I = \Psi^\sigma$, if $\alpha = 0$.

Expression (17) shows that the optimal mark-up will in general vary over time, depending on the relative importance of foreign and domestic private demand. This contrasts with the private calculation, where the mark-up is constant and given by $\epsilon/(\epsilon-1)$.

In order to gain intuition for this result, it is useful to consider two extreme cases. First, assume that $\alpha = 1$, i.e., no home-bias. In this case, all domestic production goes to exports and all consumption comes from imports. The planner implements a constant mark-up $1+\omega_t^L = \gamma/(\gamma-1)$ (assuming $\gamma > 1$), but instead of using ϵ as the relevant elasticity, it uses the elasticity of substitution between foreign goods γ. The reason is that if an individual firm attempts to increase its price, consumers will substitute towards its domestic competitors according to the elasticity ϵ. The planner internalizes that if all firms raise their price at the same time, they can act as a monopolist in international markets, where the relevant elasticity of demand is γ. I will refer to this as the *international monopolist effect*, an application of the optimal tariff argument to our dynamic economy. The second case is $\alpha \to 0$, i.e., extreme home bias. Transactions with the rest of the world are now negligible and the optimal labor wedge is $\omega_t^L = 0$. Since the planner cannot extract revenue from foreigners, it will simply correct for the inefficiency created by the mark-up of domestic firms. I will refer to this as the *domestic distortion* effect.

In the intermediate case, $0 < \alpha < 1$, the planner will balance the desire to act as an international monopolist and the need to correct domestic distortions. Therefore, the optimal labor wedge will be in general time-varying, positive, and less than the pure international monopolist mark-up. Moreover, it will be closer to the monopolist mark-up when exports are relatively more important than domestic demand. The next proposition shows that this is the result of a missing instrument problem.

Proposition 2 (Optimal tariff)**.** *Suppose the planner could impose a tariff on imported goods. The optimal policy under flexible prices sets (one plus) the tariff to* $\frac{\gamma}{\gamma-1}$ *and the labor wedge to zero.*

In the presence of an import tariff, or equivalently export taxes, there is no trade-off between the international monopolist and the domestic distortion effects.[7] The planner can eliminate the domestic distortion and obtain the desired relative price in its foreign transactions. Moreover, given the CES structure, the optimal tariff is constant. The dynamic effects obtained here are then not a direct application of the standard optimal tariff argument, but the result of a trade-off between international and domestic objectives.

A time-varying labor wedge creates incentives for intertemporal distortions. Consumption is relatively cheap to provide in periods where the labor wedge is large. Hence, an increase in the labor wedge should lead the planner to reduce the price of consumption, as indicated in equation (18). In contrast, a constant labor wedge will not lead to any intertemporal distortion when $\alpha > 0$. When $\alpha = 0$, the cost of foreign goods is not relevant for domestic consumption, so it is optimal to insulate households from risk premium shocks.

In an economy with sticky prices, a similar trade-off between the international monopolist and domestic distortion effects emerge. However, the nature of the domestic distortion is different. In an economy with flexible prices, monopoly power is the only distortion and the only reason for a positive labor wedge. Sticky prices provides an additional distortion and creates the potential for a positive and time-varying labor wedge during a recession.

The behavior of the labor and intertemporal wedges are of importance since they determine the evolution of taxes. In order to implement a given labor wedge, taxes must satisfy the following condition in the case of flexible prices.

$$1 + \omega_t^L = \frac{\epsilon}{\epsilon - 1} \frac{1 + \tau_t^c}{(1 - \tau_t^v)(1 - \tau_t^l)}$$

The intertemporal wedge will pin down the change in consumption tax.

$$1 + \omega_t^I = \frac{1 + \tau_t^c}{1 + \tau_0^c}$$

The intertemporal wedge determines the evolution of consumption taxes, but not its level. The labor wedge can be implemented by a combination of different taxes. For concreteness, we will typically assume that $\tau_0^c = 0$ and allow for a constant lump-tax to guarantee the government solvency constraint.

[7]The equivalence between the import tariff and the export tax is an application of the so-called "Lerner Symmetry Theorem". See Costinot and Werning (2017) for a modern treatment.

Optimal government spending and the principle of targeting

The next proposition characterizes the optimal government spending.

Proposition 3 (Optimal spending)**.** *Consider an economy under either flexible or fully rigid prices.*

i. ***Wasteful government spending****: if $\chi = 0$, then it is optimal to set $G_t = 0$.*

ii. ***Non-wasteful government spending****: if $\chi > 0$, then it is optimal to set*

$$\frac{G_t}{Y_t} = \frac{\chi}{\varphi}\left(\frac{Y_t}{A_t}\right)^{-\varphi(1+\phi)} \tag{19}$$

A striking implication of proposition 3 is that the optimal rule for government spending does not depend on which extreme of the price flexibility spectrum the economy is. Even though it is clear that the government should not engage in wasteful spending when prices are fully flexible and the economy is in "full employment", it is much less obvious this is the case when prices cannot adjust. Indeed, the optimality of wasteful spending have been explored previously in the literature in related contexts and, of course, the idea can be traced all the way back to Keynes in the General Theory.[8] As we are going to see, the difference lies, in general, in the instruments available to the government.

The basic intuition for the potential beneficial effect of wasteful spending is the following. When prices are sticky, a drop in demand would cause a decline in consumption and output which is not compensated by a decline in prices, creating an inefficiently high labor wedge. An increase in spending, to the extent it has a multiplier effect on the rest of the economy, it would reduce the inefficiency created by sticky prices. Keynes clearly recognized the role of the labor wedge when he says "when involuntary unemployment exists, the marginal disutility of labour is necessarily less than the utility of the marginal product".[9] These periods of high labor wedge would be periods where government spending are particularly beneficial, the case of wasteful spending being only a clarifying extreme case.

The main reason it is not optimal to distort the optimal provision of public goods to stimulate the economy in this environment is that spending is a *dominated instrument*. In the presence of an inefficient labor wedge, it is better to use a tax instrument that acts directly at the relevant margin, instead of distorting other economic decisions. This is essentially an application of the *Principle of Targeting*, commonly used in the analysis of trade or environmental policy.[10] In this case, it is optimal to use sales and consumption taxes to affect the labor and intertemporal wedges and keep the decision of government spending undistorted. In some extreme cases, as the fully rigid prices considered here, it would not be optimal to use spending for stimulus purposes even in the absence of taxes, as there is no multiplier effect in this case. However, this is no longer the case as soon as we allow for some degree of price flexibility, highlighting this is essentially a missing instrument problem.

When government spending is non-wasteful, it is determined by a pure cost-benefit analysis, i.e., a comparison of the marginal benefit of spending with the cost of increasing one unit of domestic

[8]Ferrero (2009) and Gali and Monacelli (2008) cover the open-economy case and Werning (2011) the liquidity trap closed-economy case.

[9]Book 3, chapter 10, of Keynes' "The General Theory of Employment, Interest, and Money".

[10]See Dixit (1985) for a discussion and Kopczuk (2003) for a general treatment.

prodution. Since the marginal cost of providing domestic goods falls with output, it is optimal to have countercyclical government spending, regardless of the degree of price flexibility or the level of domestic distortions.

4 Optimal Fiscal Policy in a Sudden Stop

Let's consider now the optimal response of fiscal policy to a risk premium shock ψ_t under flexible prices. In order to obtain an explicit solution, I will consider a first-order approximation of the optimal allocation and prices. The approximation can be obtained by considering a second-order expansion of the objective function subject to linear constraints approximated around a stationary solution of the Ramsey problem.[11] I will denote log deviations from the stationary solution by a lowercase, for instance, $c_t \equiv \log C_t - \log \overline{C}$, where $\overline{C}$ is the consumption level at the stationary solution.

The linear quadratic problem is

$$\min_{[\theta_t, s_t, g_t, y_t, c_t]} \left\{ \frac{1}{2} \int_0^\infty e^{-\rho t} \left[\varrho_c \left(c_t - \varrho_{cs} s_t\right)^2 + \varrho_g g_t^2 + \varrho_y y_t^2 + \varrho_s s_t^2 + 2\varrho_\Psi \left((\gamma - \xi)s_t - \theta_t\right) \hat{\Psi}_t \right] dt \right\} \tag{20}$$

subject to

$$c_t = \theta_t + (\xi - \alpha)s_t \tag{21}$$

$$y_t = \varsigma_c \theta_t + \varsigma_g g_t + (\varsigma_c \xi + \varsigma_x \gamma)s_t \tag{22}$$

$$0 = \int_0^\infty e^{-\rho t} \left[(\gamma - \xi)s_t - \theta_t\right] dt \tag{23}$$

where $\hat{\Psi}_t = \frac{1}{\sigma} \int_0^t \psi_s ds$ and the coefficients, defined in the appendix, are positive if $\gamma \geq \xi$.

The constraints are the linearized versions of the Backus-Smith condition (4), the market clearing condition for domestic goods (13), and the external solvency constraint (14). The coefficients ς_c, ς_g, and ς_x are the steady-state shares in domestic production of private domestic consumption, government consumption, and foreign demand, respectively. The term $(\gamma - \xi)s_t - \theta_t$ is the log-linear version of net exports around a steady state with zero net foreign debt.

The loss caused by deviations of consumption, government spending, and output from their steady-state levels are analogous to the one in the closed-economy analysis. Three new terms are specific to our open economy setting. First, an interaction term between consumption and the terms of trade, indicating a smaller loss of having positive consumption when the terms of trade is more depreciated.[12] Second, a term penalizing variations in the terms of trade. The coefficient ϱ_s is zero for $\alpha = 0$ and $\gamma(1 - \varsigma_g)$ for $\alpha = 1$, so it captures the effect of international demand. Finally, an interaction between the net exports and the risk premium shock. For the case of a positive risk premium shock, $\hat{\Psi}_t$ is positive and increasing, so the cost of having positive net exports is larger in future dates. This will create an incentive for the planner to reduce external borrowing in response to risk premium shocks.

[11] The stationary solution corresponds to the optimal policy when $[C_t^*, \Lambda_{H,t}, A_t]$ are constant and $\psi_t = 0$. For simplicity, I will focus on the case where the steady state level of net foreign debt and the consumption tax are equal to zero.

[12] This reflects the fact that, given the level of consumption, the labor wedge is larger in periods where s_t is more depreciated, given our restriction on parameters.

Given the solution to the linear quadratic problem, we can find the taxes necessary to implement the optimal allocation from the condition for the evolution of θ_t

$$\dot{\theta}_t = \frac{\psi_t - \dot{\hat{\tau}}_t^c}{\sigma} \tag{24}$$

and the log-linear pricing condition

$$-s_t = \sigma\theta_t + (\varphi(1+\phi) - 1)y_t + (\hat{\tau}_t^v + \hat{\tau}_t^l + \hat{\tau}_t^c) \tag{25}$$

where $\hat{\tau}_t^v = -\log \frac{1-\tau_t^v}{1-\bar{\tau}^v}$, $\hat{\tau}_t^l = -\log \frac{1-\tau_t^l}{1-\bar{\tau}^l}$, and $\hat{\tau}_t^c = \log \frac{1+\tau_t^c}{1+\bar{\tau}^c}$.

4.1 A Sudden Stop Episode

It will be useful to consider as a benchmark the case of *passive fiscal policy*, i.e., the fiscal authority sets $\hat{\tau}_t^v = \hat{\tau}_t^l = \hat{\tau}_t^c = g_t = 0$. In this case, the government does not affect directly any of the equilibrium conditions (21)-(25).[13] This will allow us to characterize the behavior of a sudden stop in the absence of government intervention and it will serve as a basis of comparison for the optimal policy. The next proposition characterizes the equilibrium allocation under passive fiscal policy.

Proposition 4. *Suppose $\gamma > \xi$, $\psi_t \geq 0$ for every $t \geq 0$, and fiscal policy is passive.*

i. ***Evolution of θ_t:***

$$\theta_t = \frac{1}{\sigma}\left[\int_0^t \psi_s ds - \int_0^\infty e^{-\rho s}\psi_s ds\right]$$

ii. ***Equilibrium allocation:***

$$c_t = \kappa_c^P\theta_t; \qquad y_t = -\kappa_y^P\theta_t; \qquad nx_t = -\kappa_{nx}^P\theta_t; \qquad s_t = -\kappa_s^P\theta_t$$

where the coefficients κ_c^P, κ_y^P, κ_{nx}^P, and κ_s^P are all positive.[14]

iii. ***Short-run vs long-run:***

$$\begin{array}{llll} c_0 < 0; & y_0 > 0; & nx_0 > 0; & s_0 > 0 \\ \lim_{t\to\infty} c_t > 0; & \lim_{t\to\infty} y_t < 0; & \lim_{t\to\infty} nx_t < 0; & \lim_{t\to\infty} s_t < 0 \end{array}$$

The Pareto weight θ_t is increasing, initially negative, and eventually positive. This reflects the incentive of households to cut consumption and accumulate assets in response to the higher interest rate. The equilibrium dynamics reflects the behavior of θ_t, since output and the terms of trade are determined by the static conditions (22) and (25), for any given θ_t. The drop in domestic demand leads to an internal devaluation as the terms of trade increase. Net exports increase on impact since the drop in θ_t reduces imports and the depreciation of the terms of trade has a positive impact on net exports when $\gamma > \xi$. Such assumption, a version of the so-called Marshall-Lerner condition, is also

[13] The government may still satisfy the solvency condition by using a lump-sum tax.

[14] The superscript P stands for "passive fiscal policy".

sufficient (but not necessary) to guarantee that output *increases* in response to the risk premium shock. This prediction is inconsistent with the evidence in Mendoza (2010) regarding sudden stops and it will be an important difference with respect to the sticky prices economy. In the long-run, as the country accumulates external assets, the equilibrium behavior is reversed. Consumption increases, output and net exports becomes negative, and the terms of trade appreciate.

The Marshall-Lerner condition $\gamma > \xi$ will also be important to guarantee the linear quadratic problem is convex and a solution to the Ramsey problem exists, so this assumption will be maintained for the rest of the analysis. Proposition 5 characterizes the equilibrium allocation under the optimal policy.

Proposition 5. *Suppose $\gamma > \xi$. The solution to problem (20) satisfies the following conditions:*

i. ***Evolution of θ_t:***

$$\theta_t = \frac{\kappa_\theta}{\sigma}\left[\int_0^t \psi_s ds - \int_0^\infty e^{-\rho s}\psi_s ds\right] = \kappa_\theta \theta_t^P$$

where $\kappa_\theta \in (0,1)$.

ii. ***Optimal allocation:***

$$c_t = \kappa_c\theta_t; \qquad y_t = -\kappa_y\theta_t; \qquad nx_t = -\kappa_{nx}\theta_t; \qquad s_t = -\kappa_s\theta_t; \qquad \hat{\omega}_t^L = -\kappa_L\theta_t$$

where κ_c, κ_y, κ_{nx}, κ_s, and κ_L are all positive constants.

iii. ***Optimal policy vs. passive policy:*** *For $\gamma - \xi$ sufficiently small, we obtain*

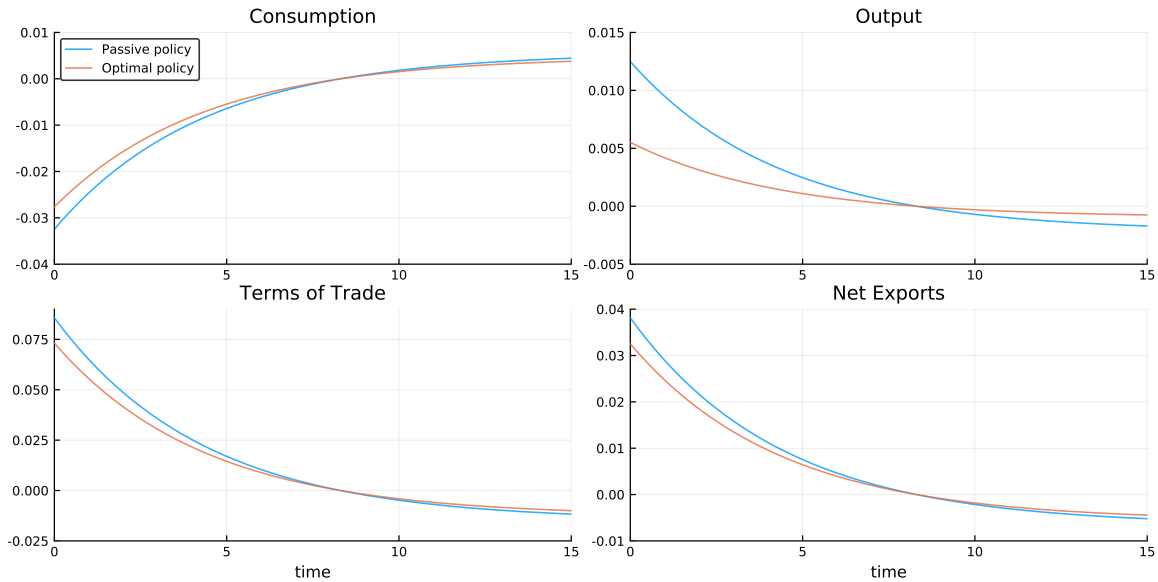
$$\frac{s_t}{s_t^P} < 1; \qquad \frac{y_t}{y_t^P} < 1; \qquad \frac{nx_t}{nx_t^P} < 1; \qquad \frac{c_t}{c_t^P} \lesseqgtr 1$$

where the superscript P denotes the allocation under passive fiscal policy.

As in the case of passive policy, θ_t initially jumps down, it is increasing and eventually becomes positive. However, movements on θ_t are attenuated. In the presence of a positive risk premium shock, θ_0 falls by less than under a passive policy. Therefore, the planner reacts only partially to the shock. Importantly, even though it is feasible to completely undo the risk premium shock, it is not optimal to do so. As a fraction of the households consumption comes from abroad and the cost of future imported goods falls with the increase in interest rates, the planner has an incentive to shift consumption from the present to the future. Hence, it is optimal for the planner to react to the interest rate shocks. The reason it reacts by less than individual private agents is that the planner has an incentive to keep the terms of trade initially more appreciated, i.e., to generate a positive labor wedge, as foreign demand becomes more important and the international monopolist effect stronger. The relatively high labor wedge at the initial date creates an incentive to front-load consumption, counteracting the effect of the risk premium shock.

The attenuation of the terms of trade is obtained only under the assumption of relatively inelastic international demand. For γ sufficiently large, the real exchange rate ends up more depreciated under the optimal policy. Due to countercyclical spending, output reacts less strongly to movements

Figure 2: A sudden stop episode: passive and optimal policy



in the real exchange rate in the case of the optimal solution, which would then require a larger initial depreciation. In order to obtain a more appreciated real exchange rate, the government must actively use its tax instruments, as reflected by the positive labor wedge $\hat{\omega}_t^L$. When γ is large, the international monopolist effect gets weaker, so smaller will be the pull towards appreciating the real exchange rate caused by the tax policy.

Figure 2 shows the quantitative differences between the optimal and passive fiscal policy in the case of a exponentially decaying risk premium shock, $\psi_t = e^{-\rho_\psi t}\psi_0$. The calibration follows closely Gali and Monacelli (2008): $\phi = 3$, $\epsilon = 6$, $\rho = 0.04$, $\sigma = 1.5$, $\gamma = 1.2$, and $\alpha = 0.35$. Average price duration is set to three quarters. The share of government spending on total demand is set to $\varsigma_g = 0.19$, the average for Portugal, Italy, Greece, and Spain. Debt-to-GDP ratio is calibrated to 0.77 to match the average for the same group of countries in 2007, prior to the spike in spreads and rise in debt in the recent European sovereign debt crisis. The half-life of risk premium shock is 3 years and the initial shock is 4%, close to the value for Italy and Spain and smaller than the value for Portugal or Greece.[15]

The effects of the sudden stop on consumption, terms of trade, and net exports are large, while the effect on output is relatively small.[16] Outcomes under the optimal policy look like an attenuated version of the passive policy allocation. In general, the response of consumption is ambiguous, as the increase in θ_0 pushes consumption up, while the appreciation of the real exchange rate pushes consumption down. The net effect depends on the relative strength of these two forces. For the baseline calibration, the first effect is the dominant one.

[15]The change in spreads for Greece was much larger than for the other countries and possibly reflected to a larger extent changes in the probability of default which are not captured here.

[16]In all figures, consumption, government, and net exports are expressed as fraction of steady-state GDP.

4.2 Optimal Fiscal Consolidation

Let's consider now the behavior of the fiscal variables. The next proposition characterizes the optimal path of taxes and spending.

Proposition 6 (Optimal fiscal policy)**.** *Suppose $\gamma > \xi$ and $\hat{\tau}^l_t = 0$.*

i. ***Optimal taxation****:*

$$\hat{\tau}^v_t + \hat{\tau}^c_t = -\kappa_\tau \theta_t; \qquad \dot{\hat{\tau}}^c_t = \kappa_{\tau^c}\psi_t; \qquad \dot{\hat{\tau}}^v_t = -\kappa_{\tau^v}\psi_t;$$

where $\kappa_\tau, \kappa_{\tau^c}$, and κ_{τ^v} are positive constants.

ii. ***Optimal government spending****:*

$$g_t = -\left(\varphi(1+\phi) - 1\right) y_t \tag{26}$$

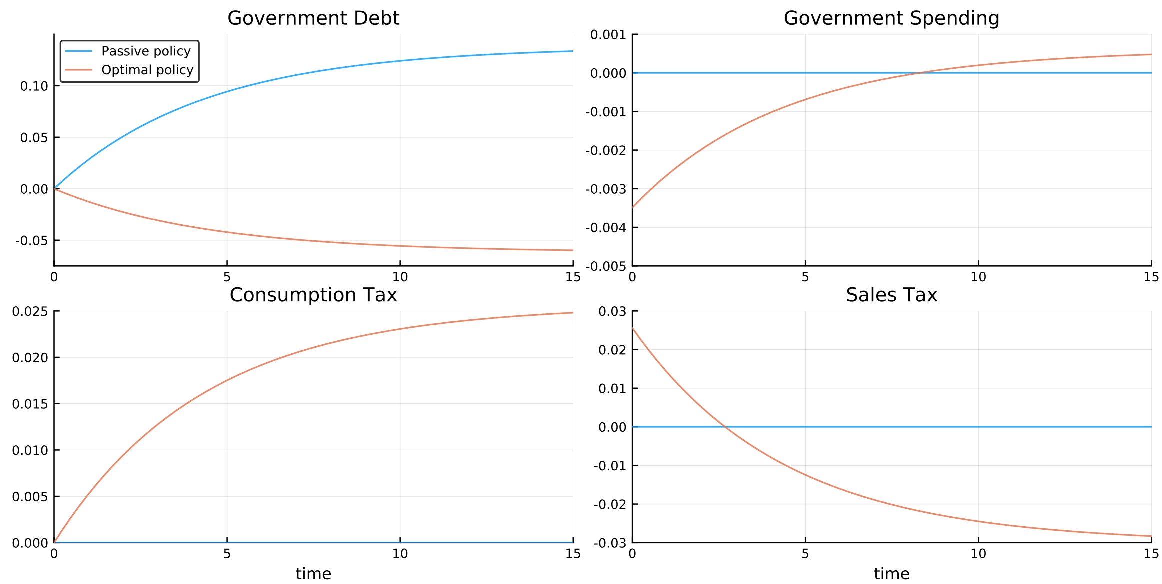
The solution to the optimal policy problem pins down the labor wedge $\hat{\tau}^v_t + \hat{\tau}^c_t + \hat{\tau}^l_t$ and the intertemporal wedge $\dot{\hat{\tau}}^c_t$, leaving indeterminate how the government solvency constraint will be satisfied. For concreteness, I will set $\hat{\tau}^l_t = 0$ and $\hat{\tau}^c_0 = 0$ and assume the government solvency constraint is satisfied by a constant lump-sum tax. Similar results are obtained if the government does not use lump-sum transfers or if $\hat{\tau}^v_t = 0$. The labor wedge increases on impact and declines over time. Hence, it is optimal for the government to *raise* taxes in response to a risk premium shock. The reason is that foreign demand becomes relatively more important as domestic consumption falls, increasing the role of the international monopolist effect. Consumption taxes are increasing in order to reduce the real interest rate faced by households, shifting consumption to periods where the labor wedge is relatively large. Hence, the consumption tax does not move on impact, by assumption, and it increases over time. The sales tax jumps up and it is declining over time, fast enough such the sum of taxes is also declining.

The dynamics of the consumption tax is an example of the so-called *unconventional fiscal policy*. Correia et al. (2013) analyzed such policies in the context of a closed economy. D'Acunto et al. (2016) provided evidence of the effectiveness of unconventional fiscal policy in the case of Germany. In contrast to the closed-economy case, the sales/VAT tax is not used simply to offset the use of the consumption tax. The time-varying labor wedge is specific to the open economy and it requires the VAT to more than compensate for the increase in consumption taxes.

Figure 3 shows the evolution of taxes and government spending for both optimal and passive fiscal policy. The sales tax increases on impact by around 2.5%, while the consumption tax is kept constant. The government changes the composition of taxes by increasing the consumption tax and reducing the sales tax. The consumption tax ends up 2.5% higher than its initial level, while the sales tax ends up 3.0% below its initial level. Hence, the overall level of taxes is smaller in the long-run. As discussed in section 3, optimal government spending is countercyclical, so it declines in response to the shock, as output increases. The decline in government is less than 0.5% of the initial GDP.

The next proposition shows how the government debt evolves under the optimal policy.

Figure 3: Taxes, spending and debt: passive and optimal policy



Proposition 7 (Optimal fiscal consolidation)**.** *Suppose $\gamma > \xi$, $\psi_t \geq 0$, and $\overline{D} > 0$.*

i. ***Fiscal needs of the government****: the lump-sum tax necessary to finance the government is smaller under the optimal fiscal policy than under passive fiscal policy.*

ii. ***Debt dynamics****: There exists d^* such that debt is increasing over time if and only if $\frac{\overline{D}}{\overline{P}_H \overline{Y}} > d^*$.*

iii. ***Fiscal consolidation****: For γ sufficiently large, government debt is smaller under optimal policy than under passive fiscal policy for all dates $t > 0$.*

In the case of both optimal and passive fiscal policy, the fiscal needs of the government, as measured by the constant lump-sum tax, are given by $-\sigma\theta_0 \frac{\rho\overline{D}}{\overline{P}_H \overline{Y}}$. Hence, the fiscal needs of the government are smaller under the optimal policy, given the attenuation of θ_0. Notice that even though taxes increase on impact, the present discounted value of the additional revenue is zero, since θ_t is on average zero under the optimal and passive fiscal policy. The reduction in the lump-sum tax is coming from the fact that, by increasing inflation expectations and reducing the real rate, the planner is able to reduce its financial costs. The second part of the proposition shows that government debt increases in response to a risk premium shock for relatively indebted economies. Importantly, the threshold depends on the path of the wedges. For our given calibration, figure 3 shows that, while government debt increases under the passive policy, it decreases for the optimal fiscal policy. This is specific to the assumed level of debt-to-GDP ratio of 80%, which is closer to the value of Portugal. A value of 100%, as in Italy or Greece, would be enough for the government debt to increase under the optimal policy as well, but still by much less than the one for the passive policy. For large values of γ, so the dampening of output is limited, we are able to show analytically that debt is always smaller under the optimal policy. This is the result of the front-loading of the labor wedge, as the sum of taxes increase on impact.

5 Optimal Policy under Sticky Prices

We have considered so far the case of flexible prices. As seen above, the optimal allocation involves an immediate depreciation of the real exchange rate in response to an increase in the risk premium. Since the nominal exchange rate is fixed, this is achieved by a coordinated, simultaneous, discrete cut in prices. As in the classical argument of Friedman (1953), it may be hard to achieved in practice this coordinated immediate reduction in prices, and the actual process of adjustment may take a long time. In order to capture this process, I will assume that prices are sticky and reconsider how fiscal policy should be designed. I will first consider the opposite extreme of flexible prices, fully rigid prices, then proceed to the more complex case of sticky prices.

5.1 Fully Rigid Prices

The optimal policy can be obtained by solving the following linear quadratic problem:

$$\min_{[\theta_t, g_t, y_t]} \left\{ \frac{1}{2} \int_0^\infty e^{-\rho t} \left[\varrho_c \theta_t^2 + \varrho_g g_t^2 + \varrho_y y_t^2 - 2\varrho_\Psi \theta_t \hat{\Psi}_t \right] dt \right\}$$

subject to

$$y_t = \varsigma_c \theta_t + \varsigma_g g_t$$
$$0 = \int_0^\infty e^{-\rho t} \theta_t dt$$

where I used the fact that $c_t = \theta_t$ and $s_t = 0$ for all $t \geq 0$.

Let's start by characterizing the optimal allocation:

Proposition 8 (Optimal allocation: rigid prices)**.** *Suppose $\gamma > \alpha$ and prices are fully rigid.*[17]

i. ***Optimal allocation****:*

$$\theta_t = \frac{\kappa_\theta^R}{\sigma} \left[\int_0^t \psi_s ds - \int_0^\infty e^{-\rho s} \psi_s ds \right]$$

$$y_t = \frac{\varsigma_c}{1 + (\varphi(1+\phi) - 1)\varsigma_g} \theta_t; \qquad c_t = \theta_t; \qquad nx_t = -\varsigma_x \theta_t; \qquad \hat{\omega}_t^L = -\sigma \left[1 + (\xi - \alpha)\mathcal{G} \right] \theta_t$$

where $\kappa_\theta^R \in (0, 1)$ and $\mathcal{G} \equiv \frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}$.

ii. ***Optimal policy vs passive policy****:*

$$\frac{\theta_t}{\theta_t^P} < 1; \qquad \frac{y_t}{y_t^P} < 1; \qquad \frac{c_t}{c_t^P} < 1; \qquad \frac{nx_t}{nx_t^P} < 1; \qquad \frac{\hat{\omega}_t^L}{\hat{\omega}_t^{L,P}} < 1$$

where the superscript P indicates the allocation under the passive fiscal policy.

[17] Under rigid prices, the weaker condition $\gamma > \alpha$ is enough to guarantee the problem is well behaved.

iii. ***Role of openness and international demand:***

$$\lim_{\alpha \to 0} \kappa_\theta^R = 0; \qquad \lim_{\alpha \to 1} \kappa_\theta^R = 1; \qquad \lim_{\gamma \to \alpha} \kappa_\theta^R = 1; \qquad \frac{\partial \kappa_\theta^R}{\partial \gamma} < 0; \qquad \lim_{\gamma \to \infty} \kappa_\theta^R > 0$$

As before, θ_t is increasing, initially negative and eventually positive, reflecting the incentive to shift consumption into the future due to the higher interest rates. The planner reacts by attenuating movements in consumption by setting $\kappa_\theta^R < 1$, which is implemented by effectively reducing the real interest rate to households via rising consumption taxes. Attenuation in consumption leads to an attenuation of the movements in the remaining equilibrium variables. The main difference with respect to the flexible prices case is that the risk premium shock now creates a recession, as output falls on impact in the absence of a depreciation of the real exchange rate. The recession will manifest itself into the labor wedge $\hat{\omega}_t^L = -\sigma\theta_t - (\varphi(1+\phi) - 1)y_t$, which will be positive on impact under both the optimal and passive fiscal policy.

When prices are fully rigid, the international monopolist effect does not have dynamic implications, as it is not possible to manipulate the terms of trade. However, the degree of openness and how competitive international markets are still play a determinant role on how fiscal policy should be designed. In particular, the planner's incentive to shift demand to periods where the economy is depressed, i.e., the labor wedge is high, is primarily determined by such considerations. This can be seen by linearizing expression (18):

$$\hat{\omega}_t^I = -\sigma \frac{(\gamma - \alpha)(1-\alpha)}{\alpha(\gamma + 1 - \alpha)} \left[\hat{\omega}_t^L - \omega_0^L\right]$$

For an economy that barely trades with the rest of the world, the planner reacts strongly to even small fluctuations in the labor wedge, completely eliminating the recession. For a fully open economy, it is optimal not to react to the shocks, as the increase in interest rates reflect entirely the cost of providing consumption goods. Hence, even though the economy is in a recession, it is a fully efficient one. Another important determinant of the optimal reaction of the fiscal authority is the elasticity of international demand. When demand is very inelastic, $\gamma \to \alpha$, the labor wedge will be very high in steady-state and the initial cost of providing domestic goods very low. This captures an environment where taxes are initially very high as the planner attempts to keep international prices high. Even before the shock hits, the economy is so depressed that the first-order effects of movements in the labor wedge are negligible. As demand gets more elastic, the labor wedge in steady-state is reduced, and movements in the cost of providing domestic goods gain in importance, so the planner will react more strongly to the recession created by the risk premium shock.

Fiscal policy is determined by conditions analogous to the ones in flexible prices:

$$g_t = -(\varphi(1+\phi) - 1)y_t; \qquad\qquad \dot{\tau}_t^c = (1 - \kappa_\theta^R)\psi_t$$

The optimal rule for government spending is exactly the same as the one under flexible prices (26). The actual behavior of government spending will be different under rigid prices though, since the decline in output on impact imply that government spending should increase. This will contribute to the attenuation of the decline in output to the extent government spending is socially valuable, as captured by ς_g. The consumption tax are determined by the intertemporal wedge. Hence, consumption taxes is increasing over time and it responds more strongly to shocks when the economy is relatively closed or when international demand is more elastic. In contrast to the flexible price economy, the sales taxes is not determined, even after normalizations. Since prices are fully rigid, there is no pricing condition to pin down the level of taxes. This will not be the case anymore in a sticky prices economy where firms adjust prices infrequently.

5.2 Sticky Prices

Under sticky prices, the optimal policy can be obtained by solving the linear quadratic problem:

$$\min_{[\theta_t, s_t, g_t, y_t, c_t, \pi_{H,t}]} \left\{ \frac{1}{2} \int_0^\infty e^{-\rho t} \left[\varrho_c \left(c_t - \varrho_{cs} s_t \right)^2 + \varrho_g g_t^2 + \varrho_y y_t^2 + \varrho_s s_t^2 + 2\varrho_\Psi \left((\gamma - \xi) s_t - \theta_t \right) \hat{\Psi}_t + \frac{\epsilon}{\kappa} \pi_{H,t}^2 \right] dt \right\}$$

subject to

$$\begin{aligned} c_t &= \theta_t + (\xi - \alpha) s_t \\ y_t &= \varsigma_c \theta_t + \varsigma_g g_t + (\varsigma_c \xi + \varsigma_x \gamma) s_t \\ 0 &= \int_0^\infty e^{-\rho t} \left[(\gamma - \xi) s_t - \theta_t \right] dt \\ \dot{s}_t &= -\pi_{H,t} \end{aligned}$$

where $s_0 = 0$.

Allowing for sticky prices introduces a new term in the loss function, corresponding to the standard cost of inflation caused by price dispersion with staggered price setting. There is also a new constraint in the problem. Given the price stickness, the price level cannot jump in response to shocks, so the initial value of the terms of trade is predetermined at its steady state level. An implication of this constraint is that a period of internal deflation is necessary to obtain a depreciation of the terms of trade. Hence, a quick adjustment of the real exchange rate will come at a cost of a period of large deflation, while maintaining inflation close to its target implies a long period of adjustment for the real exchange rate. As we will see, how the planner will resolve this trade-off will depend on the relative importance of the cost of inflation and of external demand.

As before, taxes can be inferred from the solution to the Ramsey problem. The change in consumption tax is given by (24) and the level of taxes will be determined by the open-economy version of the New Keynesian Phillips curve:

$$\dot{\pi}_{H,t} = \rho \pi_{H,t} - \kappa \left[(\varphi(\phi + 1) - 1) y_t + \sigma c_t + \alpha s_t + \hat{\tau}_t^v + \hat{\tau}_t^c \right]$$

Optimal allocation

The next proposition characterizes the optimal allocation under sticky prices:

Proposition 9 (Optimal allocation: sticky prices)**.** *Suppose* $\gamma > \xi$ *and* $\psi_t = e^{-\rho_\psi t} \psi_0 > 0$.

i. ***Initial and average effect****:*

$$\theta_0 < 0; \qquad c_0 < 0; \qquad y_0 < 0; \qquad \pi_{H,0} < 0; \qquad nx_0 > 0; \qquad \hat{\omega}_0^L > 0$$

$$\int_0^\infty \theta_t dt < 0; \qquad \int_0^\infty c_t dt < 0; \qquad \int_0^\infty y_t dt < 0; \qquad \int_0^\infty s_t dt < 0; \qquad \int_0^\infty \hat{\omega}_t^L dt > 0$$

ii. ***Terms of trade****:*

$$s_0 = 0; \qquad \dot{s}_0 > 0; \qquad \lim_{t\to\infty} s_t < 0$$

iii. ***Dynamics****:*

$$\theta_t = \theta_0 - \kappa_{\theta,s} s_t + \kappa_{\theta,\Psi} \hat{\Psi}_t; \qquad nx_t = nx_0 + \kappa_{nx,s} s_t - \kappa_{nx,\Psi} \hat{\Psi}_t;$$
$$c_t = c_0 + \kappa_{c,s} s_t + \kappa_{c,\Psi} \hat{\Psi}_t; \qquad y_t = y_0 + \kappa_{y,s} s_t + \kappa_{y,\Psi} \hat{\Psi}_t;$$

where all coefficients are positive, except for $\kappa_{c,s}$ whose sign is ambiguous.

As in the case of rigid prices, consumption and output fall on impact, while net exports increase. The recession ends up reflected in a positive labor wedge. When prices are allowed to adjust, the drop in demand caused by the risk premium shock leads to deflation. Hence, the real exchange rate becomes more depreciated via an internal devaluation. In the long-run, the real exchange rate ends more appreciated than before the shock, consistent with a smaller level of external debt generated by the increase in net exports. The dynamics of the equilibrium variables in part inherits the hump-shaped dynamics of the terms of trade and in part reflects the pattern of intertemporal substitution, as captured by the $\hat{\Psi}_t$.[18]

An important difference with both the flexible and rigid prices economies is that average consumption and output are both negative. In the two extremes of price flexibility, consumption is on average at its steady-state level, with the shock affecting only the intertemporal distribution of consumption. The effect of a risk premium shock is then maximized for intermediary levels of price flexibility. This illustrates how constraints on monetary policy caused by a currency union are essentially different from a closed-economy environment with constrained monetary policy, like the zero lower bound, where price flexibility tends to monotonically amplify the effect of shocks.

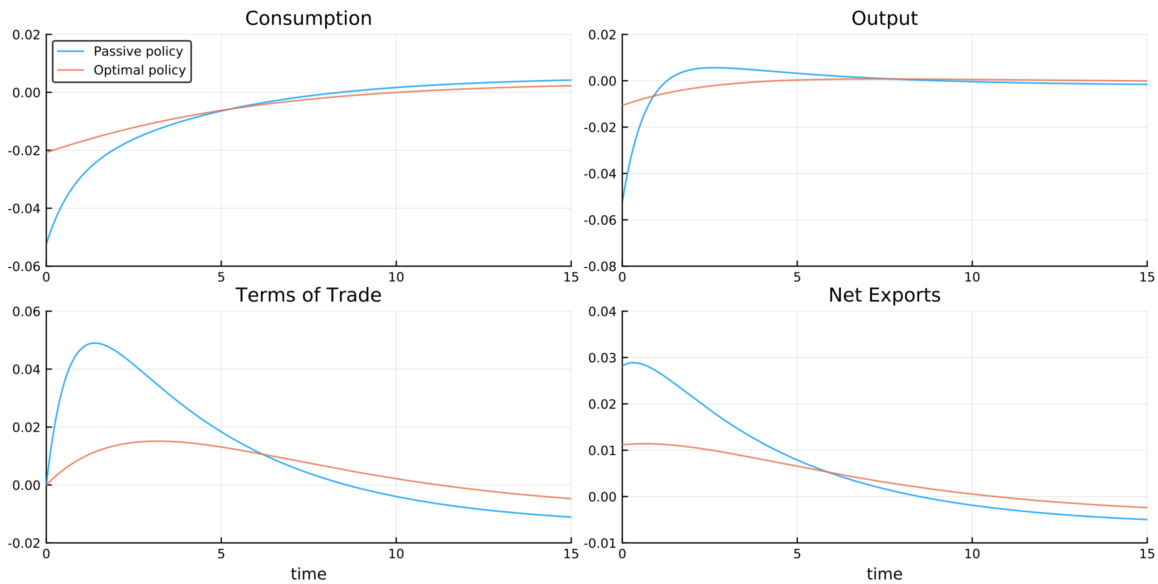
The economy under passive fiscal policy has a similar qualitative behavior to the one under optimal policy, however, the quantitative implications are severely different.[19] As can be seen from figure 4, optimal fiscal policy is able to turn a significant depression into a mild recession. In contrast, while the recovery lasts less than a year under passive fiscal, due a large deflation and consequent internal devaluation, output takes more than two and a half years to return to its pre-shock level under optimal fiscal policy.

Notice that, under flexible prices, the economy avoids a recession by a coordinated decline in prices across all domestic producers. The resulting depreciation of the real exchange rate supports the level

[18] For instance, since $\hat{\Psi}_t = \frac{1}{\sigma}\int_0^t \psi_s ds$ is positive and increasing over time, a positive $\kappa_{\theta,\Psi}$ implies consumption will tend to increase over time after the initial drop.

[19] See the appendix for the analytical characterization of the allocation under passive fiscal policy.

Figure 4: Equilibrium under sticky prices: passive and optimal policy



of demand in the economy. Interestingly, in the case of sticky prices the planner acts in the direction of *reducing* the depreciation of the real exchange rate in the short-run, but keeping the real exchange rate more depreciated in the long-run. This reflects in part the costs of large fluctuations in inflation, captured by the elasticity of substitution among domestic goods ϵ. Figure 5 shows the behavior of the terms of trade, output, and taxes for different values of the elasticity ϵ. When ϵ is relatively low, there is a larger deflation initially, the real exchange rate is more depreciated, and the recession is shorter. When ϵ is large, the recession lasts longer and internal devaluation is more limited. In the long-run, the behavior is reversed: the economy with a shorter recession ends up with a more appreciated real exchange rate and smaller output.

Another important factor determining the optimal response is the elasticity of international demand γ. Countries who face a more inelastic demand will have a more limited depreciation of the real exchange rate and a slower recovery, as can be seen in figure 6. This is a consequence of the international monopolist effect discussed in section 3. The more inelastic the demand, the larger the mark-up an international monopolist would like to charge on its products. The planner internalizes this effect and it acts less intensively to eliminate the positive labor wedge caused by the recession, leading to a relatively more appreciated real exchange rate in the short run.

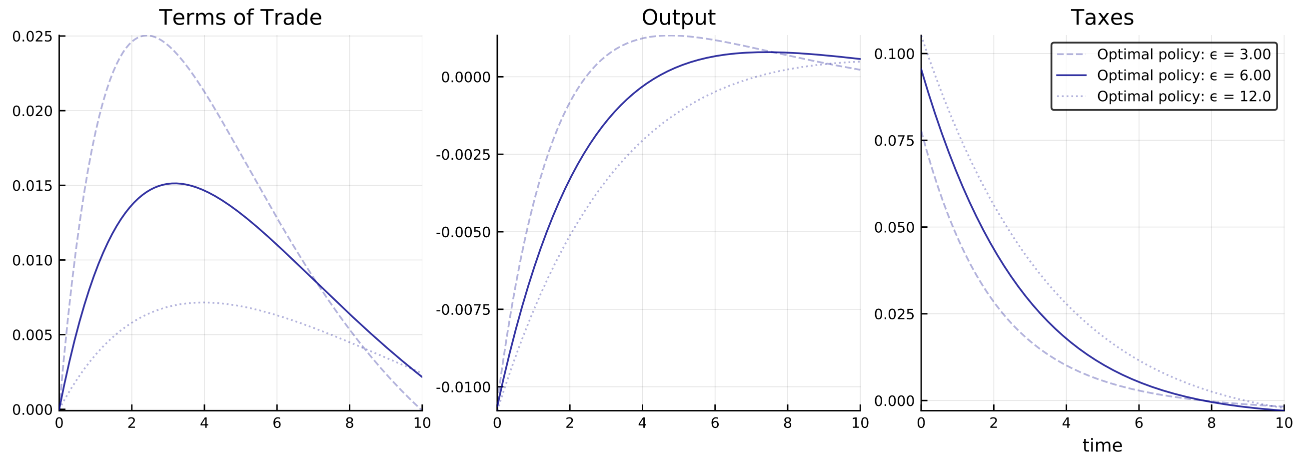
The planner have then incentives to avoid large real exchange rate depreciations when the cost of inflation is high, high ϵ, or when international demand is relatively inelastic, low γ. The balancing of these two forces will be the key determinant of the optimal level of taxes.

Optimal front-loading of taxes

The next proposition characterizes the behavior of optimal fiscal policy.

Proposition 10 (Optimal fiscal policy)**.** *Suppose* $\gamma > \xi$.

Figure 5: The role of the cost of inflation: effect of changes in ϵ



i. ***Optimal taxes****:*

$$\hat{\tau}_t^c = -\kappa_{\tau^c,\pi}\pi_{H,t} + \kappa_{\tau^c,\psi}\psi_t$$

where $\kappa_{\tau^c,\pi} > 0$ *and* $\kappa_{\tau^c,\psi} \in (0,1)$*.*

$$\hat{\tau}_t^v + \hat{\tau}_t^c = \kappa_{\tau,s}s_t + \kappa_{\tau,\theta}\theta_t$$

where $\kappa_{\tau,\theta} < 0$ *if* $\gamma(1-\varsigma_g) < \epsilon$ *and* $\kappa_{\tau,\theta} > 0$ *if* $\gamma(1-\varsigma_g) \gg \epsilon$*.*

ii. ***No home-bias limit****: As* $\alpha \to 1$*, we obtain*

$$\begin{aligned}
\kappa_{\tau^c,\pi} &= \kappa_{\tau^c,\psi} = 0 \\
\kappa_{\tau,\theta} &= -\left(\epsilon - \gamma(1-\varsigma_g)\right)\frac{\sigma}{\epsilon} \\
\kappa_{\tau,s} &= -\left(\epsilon - \gamma(1-\varsigma_g)\right)\left(\frac{1+\gamma\mathcal{G}}{\epsilon}\right)
\end{aligned}$$

where $\mathcal{G} = \frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}$.

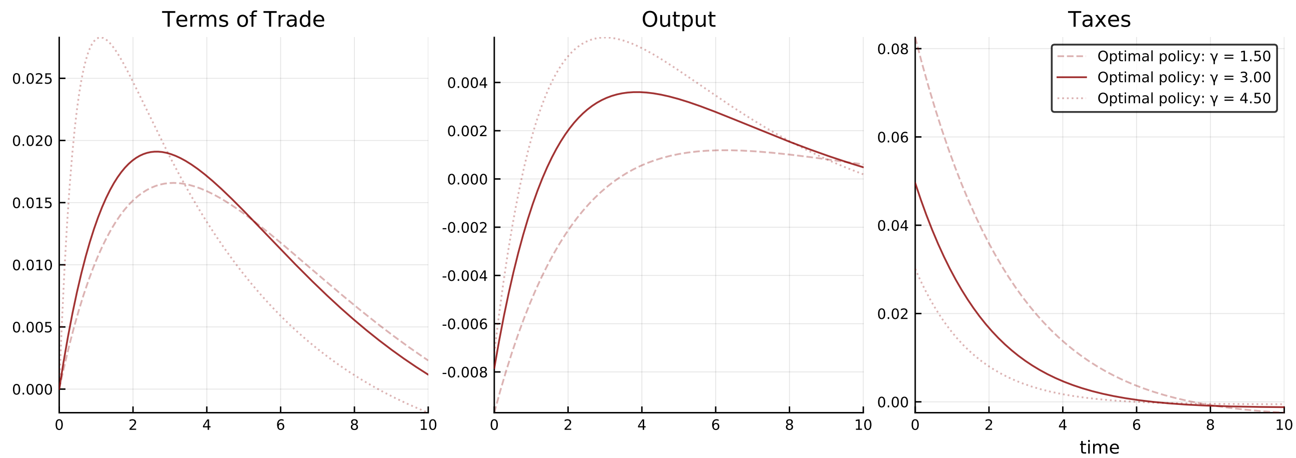
The behavior of the consumption tax is similar to the one under rigid prices. The planner reacts partially to the shock in order to shift demand to periods of high labor wedge. The new aspect is the determination of the sum of taxes $\hat{\tau}_t^v + \hat{\tau}_t^c$. Since $\theta_0 < 0$, it is optimal to increase taxes on impact if the following condition is satisfied

$$\gamma(1-\varsigma_g) < \epsilon \tag{27}$$

An increase in taxes will lead to a more appreciated real exchange rate, as we can see from figures 5 and 6. In the absence of any cost of inflation, $\epsilon \approx 0$, the optimal response would be simply an attempt of replicating the flexible prices economy.[20] The government should give massive tax breaks

[20]Notice, however, that for the firm's problem to be well defined we must have $\epsilon > 1$, which puts a positive lower bound on the cost of inflation coefficient.

Figure 6: The role of international demand: effect of changes in γ



to companies, so they have an incentive to cut prices, achieving the required internal devaluation to support domestic production. In this case, it would be optimal to back-load taxes, as there is an initial cut in taxes which is slowly phased out. In contrast, when the cost of inflation is relatively high, it is optimal for the planner to limit the extent of the deflation by increasing taxes and creating inflationary pressures. In this case, it is optimal to *front-load* taxes, as the planner initially increases taxes and slowly reduce them over time.

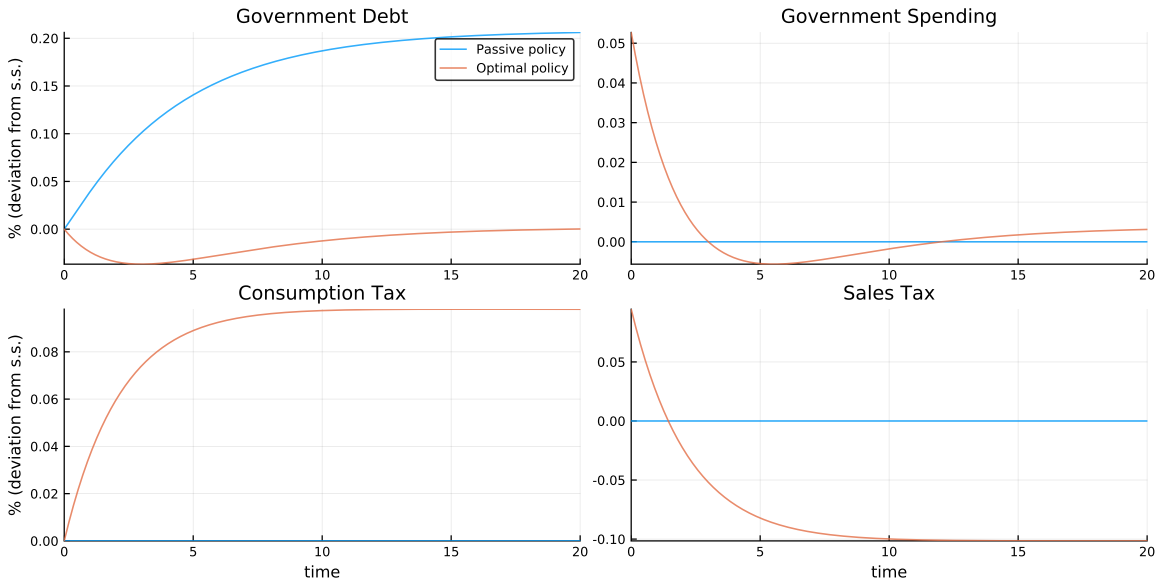
How large the cost of inflation needs to be in order to be optimal to front-load taxes depends on the strength of the international monopolist effect. When international demand is relatively inelastic, then even a modest cost of inflation will be sufficient to guarantee it is optimal to raise taxes in response to a risk premium shock. An inelastic international demand means the optimal mark-up is larger and deviations from the desired mark-up are less costly. Since government spending consists entirely of domestic goods, only a fraction $1 - \varsigma_g$ of demand is sensitive to changes in the real exchange rate.

From figure 5, we can see that the effect of the domestic elasticity on the level of taxes is relatively modest. Even if $\epsilon = 3$, half of the baseline value of $\epsilon = 6$, the level of taxes will fall from around 10.5% to a still sizeable level of 9.5%. In contrast, the level of taxes is very sensitive to changes in international demand. A change in γ from 1.2, the baseline value, to 1.5 is enough to bring taxes down to around 8.0%. Going all the way to 3.0 or 4.5 will reduce taxes to around 5.0% and 3.0%, respectively. However, even for substantial departures from the standard calibration adopted by the literature, it is still optimal for the planner to front-load taxes in response to an increase in interest rates. In particular, condition (27) is always satisfied in the commonly adopted Cole-Obstfeld case $\gamma = \sigma = 1$.

Optimal fiscal consolidation

Figure 7 shows the dynamic path of taxes, spending, and government spending under optimal and passive policy. As usual, the initial level of consumption tax is not pinned down and we normalize it to zero. The government solvency constraint is satisfied by a constant lump-sum tax. Optimal policy consists of an initial increase in the sales tax and slowly increasing the consumption while the sales tax

Figure 7: Taxes, spending and debt: passive and optimal policy



is reduced. The sum of both taxes is positive at first and decreasing over time. Government spending follows the fiscal rule (26) and it is essentially a mirror image of the behavior of output. Spending initially increases by 1.0% of steady-state GDP and it reaches its pre-shock level in about three years.

The front-loading of taxes has important implications for the dynamics of government debt. Under passive fiscal policy, debt increases substantially over time, consistent with the experience of the peripheral economies that experienced a substantial increase in borrowing costs. In contrast, the government debt *declines* under the optimal policy and stabilizes at around the same level it started. Notice government debt declines despite the increase in spending. The front-loading of taxes initially compensates for the increase in borrowing costs, stabilizing the level of debt. Moreover, the lump-sum transfers required to finance the government are smaller under the optimal policy than under the passive fiscal policy. Hence, it is not only the timing of revenue that it is affected by the optimal policy. By reducing the real interest rate and increasing the overall level of economic activity, the government is able to reduce its need for additional revenues.

6 Downward Nominal Wage Rigidities

Let's consider now an economy subject to *downward nominal wage rigidities*, as this will allow the consideration of a few new issues.[21] First, it will not be possible anymore to normalize either the VAT or the payroll tax to zero, as both taxes will be necessary for the implementation of the optimal policy. This will allow us to consider the optimality of a *fiscal devaluation*, i.e., the reduction in the payroll tax accompained of an increase in the VAT tax, a policy that have received significant attention in the liter-

[21] For empirical evidence of downward nominal wage rigidities in the European case, see Babecký et al. (2010). See Schmitt-Grohé and Uribe (2011) for a discussion of the impact of downward nominal rigidities under fixed exchange rates in a related model.

ature recently.[22] Second, it will be possible now to account for the increase in unemployment observed in the economies exposed to increase in interest rates in the recent European sovereign debt crisis. The inability of wages to go down will limit the market clearing process in the labor market, creating the possibility of unemployment. Finally, the internal devaluation obtained in the sticky prices economy is in part generated by a drop in nominal wages, so considering the case of sticky wages allow us to evaluate the robustness of the conclusions under sticky prices.

Assume that prices are completely flexible, but nominal wages are downward rigid. Consider the effect of a positive risk premium shock and, to keep the analysis as simple as possible, assume $\gamma = \xi$. Consider first the case of passive fiscal policy. The decline in consumption in response to the shock induces a positive income effect on labor supply that, when wages are not allowed to decline, will lead to unemployment. As consumption increases over time, this will tend to reduce labor supply and eventually the economy will return to full employment. The next proposition characterizes the unemployment and full employment periods induced by the shock.

Proposition 11. *Suppose $\xi = \gamma$, fiscal policy is passive, and the risk premium shock is positive ($\psi_t \geq 0\ \forall t$). Define T_0^P as the time period such that $\theta_t^P = 0$.*

- ***Unemployment period**: for $t \leq T_0^P$, nominal wage satisfy $w_t = 0$ and*

$$c_t = \overline{\kappa}_c^P \theta_t^P; \qquad y_t = \overline{\kappa}_y^P \theta_t^P; \qquad nx_t = -\overline{\kappa}_{nx}^P \theta_t^P; \qquad s_t = -\overline{\kappa}_s^P \theta_t^P; \qquad u_t = -\overline{\kappa}_u^P \theta_t^P$$

  where $\overline{\kappa}_c^P$, $\overline{\kappa}_y^P$, $\overline{\kappa}_{nx}^P$, $\overline{\kappa}_s^P$, $\overline{\kappa}_u^P$ and are all positive constants.

- ***Full employment period**: for $t > T_0^P$, unemployment satisfy $u_t = 0$ and*

$$c_t = \kappa_c^P \theta_t; \qquad y_t = -\kappa_y^P \theta_t; \qquad nx_t = -\kappa_{nx}^P \theta_t; \qquad s_t = -\kappa_s^P \theta_t; \qquad w_t = \kappa_w^P \theta_t$$

  where κ_c, κ_y, κ_{nx}, and κ_s are all positive constants.

As in the sticky prices economy, and in contrast to the case of flexible prices and wages, output responds positively to θ_t in the unemployment period. Hence, under downward nominal wage rigidity, a positive risk premium shock generates a recession. Since output responds negatively to θ_t under the full employment period, then $y_t \leq 0$ for all t. Hence, the shock affects not only the intertemporal allocation of output, but also its average value. Unemployment shoots up on impact and slowly decreases as consumption and output recovers. As in the case of flexible prices, the terms of trade depreciate mitigating in part the effect of the shock.

How should fiscal policy respond to the interest rate shock under downward wage rigidity? It turns out the optimal policy can do as well as in the case of flexible prices, but the fiscal policy required to implement the optimal is different. In particular, labor income tax (or payroll tax) becomes important.

Proposition 12. *(Optimal fiscal policy) Suppose $\xi = \gamma$, $\hat{\tau}_0^c = 0$, and the risk premium shock is positive ($\psi_t \geq 0\ \forall t$).*

[22] See Farhi et al. (2013) for a discussion of fiscal devaluations.

- ***Optimal government spending***:

$$g_t = -\left(\varphi(1+\phi) - 1\right) y_t$$

- ***Optimal taxation***:

$$\hat{\tau}^v_t + \hat{\tau}^c_t + \hat{\tau}^l_t = -\sigma\frac{\alpha}{\gamma}\theta_t; \qquad \hat{\tau}^v_t = \sigma\frac{\gamma-\alpha}{\gamma}\theta_t; \qquad \dot{\hat{\tau}}^c_t = \kappa_{\tau^c}\psi_t; \qquad \dot{\hat{\tau}}^l_t = -\psi_t$$

  where $\kappa_{\tau^c} \in (0,1)$ *and* $\hat{\tau}^l_0 > 0$.

- ***Fiscal needs of the government***: *For* $\alpha \to 1$*, the lump-sum tax necessary to finance the government is smaller under the optimal fiscal policy than under passive fiscal policy.*

As before, it is not optimal to deviate from the optimal provision of public goods. By the Principle of Targeting, it is better to use instruments that directly affect labor demand and labor supply when an economy is facing unemployment instead of indirectly by changing government spending. The optimal allocation involves no unemployment, as the planner shifts labor demand upward by reducing the VAT tax. Consumption tax is increasing over time in order to partially undo the effects of the interest rate shock on consumption. Given the other taxes, the payroll tax is chosen in order to guarantee the terms of trade will not depreciate beyond the optimal level. As in the case of flexible prices and sticky prices under inelastic international demand, it is optimal to front-load taxes. As in the case of flexible and rigid prices, the fiscal needs of the government decline under the optimal policy, as the amount of taxes necessary to guarantee the government budget constraint is satisfied is smaller under the optimal policy.

As downward nominal wage rigidity requires an additional instrument to be implemented, it is possible to explicitly discuss the optimality of a fiscal devaluation policy, i.e., an increase in the VAT tax accompained with a reduction in the payroll tax. As we have seen, it turns out to be optimal to implement a *fiscal appreciaton*, i.e., a decline in the VAT tax and an increase in the payroll tax. This is partially due to the international monopolist effect, where as before it explains the increase in the sum of taxes. However, this effect vanishes as $\gamma \to \infty$, since it is optimal to have a zero mark-up. In this special case, $\hat{\tau}^c_t = 0$ for all $t \geq 0$, the VAT decreases and the payroll tax increases by the same magnitude. The decline in the VAT shifts the labor demand, while the payroll tax shifts the labor supply, eliminating the unemployment. Hence, the fiscal appreciation is a natural response to the downward nominal wage rigidity friction, instead of being entirely driven by the international monopolist effect.

7 Conclusion

In this paper, I studied the optimal design of fiscal policy of an economy subject to risk premium shocks in a currency union. The first main result is that it is not optimal to deviate from the optimal provision of public goods to stimulate the economy, regardless of the degree of price or wage stickness. In particular, wasteful spending is never optimal. This is an application of the Principle of Targeting, as it better to use tax instruments that act directly in the inefficiencies of the economy. The second main result is the front-loading of taxes. Under flexible prices, as domestic consumption declines in response to the risk premium shock, foreign demand gains in importance, and it is optimal for the planner to keep the real exchange rate relatively more appreciated to extract more revenue from foreigners. The same result applies under sticky prices, provided the cost of inflation is sufficiently high or international demand is sufficiently inelastic, as the planner limits the deflation caused by the drop in economic activity. Similarly, it is optimal to implement a fiscal appreciation when there is downward nominal wage rigidity. Finally, the optimal policy implies a form of fiscal consolidation, as the government debt is reduced under the optimal policy. Depending on parameters, it may be optimal to completely stabilize the level of government debt.

Fiscal policy is especially important for a member of a currency union, as the country loses the ability of using monetary policy to stabilize the economy. Hence, the importance of analyzing how fiscal policy should be designed in such environment, in particular given the debates over the relative importance of fiscal stimulus and fiscal consolidation. A formal analysis of the trade-off allowed us to find ways of achieving the desired level of stimulus without sacrificing the soundness of government finances.

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A Derivations for section 3

A.1 The non-linear Ramsey problem

The Ramsey problem can be written as

$$\max_{\{\Theta_t, S_t, Y_t, G_t\}} \left\{ \int_0^\infty e^{-\rho t} \left[\frac{1}{1-\sigma} \left(\Theta_t C_t^* S_t^{\xi-\alpha} \right)^{1-\sigma} + \chi \log G_t - \frac{1}{1+\phi} \left(\frac{Y_t}{A_{H,t}} \right)^{\varphi(1+\phi)} \right] dt \right\} \tag{28}$$

subject to

$$Y_t = \left[(1-\alpha)\Theta_t S_t^{\xi} + \alpha \Lambda_{H,t} S_t^{\gamma} \right] C_t^* + G_t \tag{29}$$

$$\frac{E_0}{P_0^*} = \alpha \int_0^\infty e^{-\rho t} \Psi_t^{-\sigma} \left(\frac{C_t^*}{C_0^*} \right)^{-\sigma} \left[\Lambda_{H,t} S_t^{\gamma-1} - \Theta_t S_t^{\xi-1} \right] C_t^* ds \tag{30}$$

The first-order conditions are

$$\chi G_t^{-1} = \lambda_t \tag{31}$$

$$\varphi \left(\frac{Y_t}{A_{H,t}} \right)^{\varphi(1+\phi)} \frac{1}{Y_t} = \lambda_t \tag{32}$$

$$C_t^{1-\sigma} - \lambda_t (1-\alpha) \Theta_t S_t^{\xi} C_t^* = \alpha \Gamma \Psi_t^{-\sigma} \left(\frac{C_t^*}{C_0^*} \right)^{-\sigma} S_t^{\xi-1} \Theta C_t^*$$

$$(\xi-\alpha) C_t^{1-\sigma} - \lambda_t \left[\xi (1-\alpha) \Theta_t S_t^{\xi} + \gamma \alpha \Lambda_{H,t} S_t^{\gamma} \right] C_t^* = -\alpha \Gamma \Psi_t^{-\sigma} \left(\frac{C_t^*}{C_0^*} \right)^{-\sigma} \left[(\gamma-1) \Lambda_{H,t} S_t^{\gamma-1} - (\xi-1) S_t^{\xi-1} \Theta_t \right] C_t^*$$

A.2 Proof of proposition 1

i. **Labor wedge**: The optimality condition for Θ_t can be written as

$$C_t^{-\sigma} = (1-\alpha) \lambda_t S_t^{\alpha} + \alpha \Gamma \Psi_t^{-\sigma} \left(\frac{C_t^*}{C_0^*} \right)^{-\sigma} S_t^{\alpha-1} \tag{33}$$

Combining the optimality condition for Θ_t and S_t, we get

$$\alpha \Gamma \Psi_t^{-\sigma} \left(\frac{C_t^*}{C_0^*} \right)^{-\sigma} S_t^{\alpha-1} = \alpha \lambda_t \frac{\left[\gamma \Lambda_{H,t} S_t^{\gamma} + (1-\alpha) \Theta_t S_t^{\xi} \right] S_t^{\alpha}}{(\gamma-1) \Lambda_{H,t} S_t^{\gamma} + (1-\alpha) S_t^{\xi} \Theta_t} \tag{34}$$

Combining the last two conditions, we get

$$\frac{C_t^{-\sigma} S_t^{-\alpha}}{\lambda_t} = \frac{(\gamma - 1 + \alpha) \Lambda_{H,t} S_t^{\gamma} + (1-\alpha) S_t^{\xi} \Theta_t}{(\gamma-1) \Lambda_{H,t} S_t^{\gamma} + (1-\alpha) S_t^{\xi} \Theta_t} = 1 + \frac{\alpha \Lambda_{H,t} S_t^{\gamma}}{(\gamma-1) \Lambda_{H,t} S_t^{\gamma} + (1-\alpha) S_t^{\xi} \Theta_t} \tag{35}$$

Hence, we obtain the expression in the text for the labor wedge.

ii. **Intertemporal wedge**: Rearranging the optimality condition for Θ_t, we obtain

$$1 = \frac{1-\alpha}{1+\omega_t^L} + \alpha\Gamma\Psi^{-\sigma}(C_0^*)^{\sigma}\Theta_t^{\sigma} \tag{36}$$

If $\alpha > 0$, then

$$\Theta_t = \Theta_0 \Psi_t \left[\frac{1 - \frac{1-\alpha}{1+\omega_t^L}}{1 - \frac{1-\alpha}{1+\omega_0^L}} \right]^{\sigma} \tag{37}$$

Using the fact that $C_t = \Theta_t C_t^* (1+\omega_t^I)^{-\frac{1}{\sigma}}$, we can write the intertemporal wedge as follows

$$1+\omega_t^I = \left[\frac{\alpha + \omega_t^L}{1+\omega_t^L} \right]^{-\sigma} \left[\frac{\alpha + \omega_0^L}{1+\omega_0^L} \right]^{\sigma} \tag{38}$$

If $\alpha = 0$, then $1 = \lambda_t C_t^{\sigma}$. The market clearing condition for goods can be written as

$$\lambda_t Y_t = \lambda_t C_t + \chi \tag{39}$$

Combining the equations above, we obtain

$$Y_t \left(1 - \frac{\chi}{\varphi} \left(\frac{Y_t}{A_{H,t}} \right)^{-\varphi(1+\phi)} \right) = \left[\varphi \frac{Y_t^{\varphi(1+\phi)-1}}{A_{H,t}^{\varphi(1+\phi)}} \right]^{-\frac{1}{\sigma}} \tag{40}$$

There is a unique solution to the expression above and it is independent of Ψ_t. Hence, consumption is also independent of Ψ_t and the intertemporal wedge is given by $1+\omega_t^I = \Psi_t^{\sigma}$. □

A.3 Proof of proposition 2

Suppose the government introduces now a tariff on imported goods, so the total tax on foreign goods paid by consumers is $(1+\tau_t^c)(1+\tau_t^T) - 1$ while consumers pay τ_t^c on domestic goods. Since foreign consumers don't pay the tariff to import the domestic goods, then they face a relative price of domestic to foreign goods different than the one faced by domestic consumers. Let $\tilde{S}_t = \frac{P_t^*}{P_{H,t}}$ and $S_t = (1+\tau_t^T)\tilde{S}_t$ denote the relative price faced by foreign and domestic consumers, respectively. The Backus-Smith condition will now be given by $C_t = \Theta_t C_t^* \tilde{S}_t^{\frac{1-\alpha}{\sigma}} \left(\frac{\tilde{S}_t}{S_t} \right)^{\frac{\alpha}{\sigma}}$. Net exports will be given by

$$\frac{P_{H,t}C_{H,t}^* - P_{F,t}C_{F,t}}{P_t} = \alpha \left(\frac{\tilde{S}_t}{S_t} \right)^{\alpha} \tilde{S}_t^{\gamma-\alpha} \Lambda_{H,t} C_t^* - \alpha \frac{\tilde{S}_t}{S_t} C_t \tag{41}$$

The external solvency constraint will be given by

$$\frac{E_0}{P_0^*} = \alpha \int_0^\infty e^{-\rho t} \Psi_t^{-\sigma} \left(\frac{C_t^*}{C_0^*} \right)^{-\sigma} \tilde{S}_t^{\alpha-1} \left(\frac{\tilde{S}_t}{S_t} \right)^{-\alpha} \left[\alpha \left(\frac{\tilde{S}_t}{S_t} \right)^{\alpha} \tilde{S}_t^{\gamma-\alpha} \Lambda_{H,t} C_t^* - \alpha \frac{\tilde{S}_t}{S_t} C_t \right] ds \tag{42}$$

$$= \alpha \int_0^\infty e^{-\rho t} \Psi_t^{-\sigma} \left(\frac{C_t^*}{C_0^*} \right)^{-\sigma} \left[\alpha \tilde{S}_t^{\gamma-1} \Lambda_{H,t} C_t^* - \alpha S_t^{\alpha-1} C_t \right] ds \tag{43}$$

Finally, the goods market clearing condition can be written as $Y_t = (1-\alpha) S_t^\alpha C_t + G_t + \alpha \Lambda_{H,t} \tilde{S}_t^\gamma C_t^*$. The Ramsey problem can then be written as

$$\max_{\{C_t, S_t, \tilde{S}_t, Y_t, G_t\}} \left\{ \int_0^\infty e^{-\rho t} \left[\frac{1}{1-\sigma} C_t^{1-\sigma} + \chi \log G_t - \frac{1}{1+\phi} \left(\frac{Y_t}{A_{H,t}} \right)^{\varphi(1+\phi)} \right] dt \right\} \tag{44}$$

subject to

$$Y_t = (1-\alpha) S_t^\alpha C_t + G_t + \alpha \Lambda_{H,t} \tilde{S}_t^\gamma C_t^* \tag{45}$$

$$\frac{E_0}{P_0^*} = \alpha \int_0^\infty e^{-\rho t} \Psi_t^{-\sigma} \left(\frac{C_t^*}{C_0^*} \right)^{-\sigma} \left[\alpha \tilde{S}_t^{\gamma-1} \Lambda_{H,t} C_t^* - \alpha S_t^{\alpha-1} C_t \right] ds \tag{46}$$

Notice it is more convenient now to choose C_t instead of Θ_t. The first-order conditions are

$$\chi G_t^{-1} = \lambda_t$$

$$\varphi \left(\frac{Y_t}{A_{H,t}} \right)^{\varphi(1+\phi)} \frac{1}{Y_t} = \lambda_t$$

$$(1-\alpha)\lambda_t S_t^\alpha + \alpha \Gamma \Psi_t^{-\sigma} \left(\frac{C_t^*}{C_0^*} \right)^{-\sigma} S_t^{\alpha-1} = C_t^{-\sigma}$$

$$\alpha(1-\alpha)\lambda_t S_t^{\alpha-1} C_t = \alpha(1-\alpha)\Gamma \Psi_t^{-\sigma} \left(\frac{C_t^*}{C_0^*} \right)^{-\sigma} S_t^{\alpha-2} C_t$$

$$\gamma \alpha \lambda_t \tilde{S}_t^{\gamma-1} \Lambda_{H,t} C_t^* = \alpha(\gamma-1)\Gamma \Psi_t^{-\sigma} \left(\frac{C_t^*}{C_0^*} \right)^{-\sigma} \tilde{S}_t^{\gamma-2} \Lambda_{H,t} C_t^*$$

Combining the conditions above, we obtain

$$\frac{S_t}{\tilde{S}_t} = \frac{\gamma}{\gamma-1}; \qquad 1 = \frac{C_t^{-\sigma} S_t^{-\alpha}}{\lambda_t} \tag{47}$$

Hence, the optimal tariff is constant and the labor wedge is equal to zero. □

A.4 Implementation

Combining the labor supply condition (3) and the pricing condition (12), we obtain

$$\frac{(1-\tau_t^v)(1-\tau_t^l)}{1+\tau_t^c} \frac{P_{H,t}}{P_t} = \frac{\epsilon}{\epsilon-1} C_t^\sigma \varphi \left(\frac{Y_t}{A_t} \right)^{\varphi(1+\phi)} \frac{1}{Y_t} \tag{48}$$

rearranging we obtain the expression for the labor wedge

$$\frac{S_t^{-\alpha} C_t^{-\sigma}}{\lambda_t} = \frac{\epsilon}{\epsilon - 1} \frac{1 + \tau_t^c}{(1 - \tau_t^v)(1 - \tau_t^l)} \tag{49}$$

using the definitions of S_t and λ_t.

Combining the definition of the intertemporal wedge and Euler equation, we obtain

$$1 + \omega_t^I = e^{\int_0^t \dot{\tau}_s^c dc} = \frac{1 + \tau_t^c}{1 + \tau_0^c} \tag{50}$$

A.5 Proof of proposition 3

The first-order conditions for the Ramsey problem under rigid prices are the same as under flexible prices, except for the optimality condition for S_t which is replaced by $S_t = \overline{S}$. If $\chi = 0$, then the only effect of an increase in G_t is to increase the desutility of work, so it is optimal to have $G_t = 0$. If $\chi > 0$, then the first-order condition is

$$\frac{\chi}{G_t} = \varphi \left(\frac{Y_t}{A_{H,t}} \right)^{\varphi(1+\phi)} \frac{1}{Y_t} \Rightarrow \frac{G_t}{Y_t} = \frac{\chi}{\varphi} \left(\frac{Y_t}{A_{H,t}} \right)^{-\varphi(1+\phi)} \tag{51}$$

□

B Derivations for section 4

B.1 Stationary solution

Consider a stationary solution for the Ramsey problem. Assume $\overline{E} = 0$, i.e., $\overline{\Theta} = \Lambda_H \overline{S}^{\gamma - \xi}$.

Combining conditions (31) and (32):

$$\frac{\overline{G}}{\overline{Y}} = \frac{\chi}{\varphi} \left(\frac{\overline{Y}}{\overline{A}_H} \right)^{-\varphi(1+\phi)} \tag{52}$$

rearranging

$$\chi = \varphi \overline{N}^{1+\phi} \varsigma_g \tag{53}$$

From the optimality condition for Θ_t and S_t, we get

$$\overline{C}^{-\sigma} = \overline{\lambda}(1 - \alpha)\overline{S}^{\alpha} + \alpha \frac{\Gamma}{\overline{C}^*} \overline{S}^{\alpha - 1} \tag{54}$$

and

$$(\xi - \alpha)\overline{C}^{-\sigma} = \overline{\lambda} \left[(1 - \alpha)\xi + \alpha\gamma \right] \overline{S}^{\alpha} + \alpha \frac{\Gamma}{\overline{C}^*} (\xi - \gamma) \overline{S}^{\alpha - 1} \tag{55}$$

Eliminating Γ, we obtain the labor wedge

$$\frac{\overline{C}^{-\sigma}\overline{S}^{-\alpha}}{\overline{\lambda}} = \frac{\gamma}{\gamma - \alpha} \tag{56}$$

From the market clearing condition, we have $\overline{Y} = \overline{C}\overline{S}^{\alpha}/(1-\varsigma_g)$. The Lagrange multiplier $\overline{\lambda}$ is then given by

$$\overline{\lambda} = \frac{\varphi}{\overline{A}^{\varphi(1+\phi)}(1-\varsigma_g)^{\varphi(1+\phi)-1}}\left(\overline{C}\overline{S}^{\alpha}\right)^{\varphi(1+\phi)-1} \tag{57}$$

Using $\overline{C} = \Lambda_H \overline{C}^* \overline{S}^{\gamma-\alpha}$ and the expression for the labor wedge, we can solve for $\overline{S}$:

$$\overline{S} = \left[\frac{\gamma-\alpha}{\alpha}\frac{\overline{A}^{\varphi(1+\phi)}(1-\varsigma_g)^{\varphi(1+\phi)-1}}{\varphi(\Lambda_H \overline{C})^{\varphi(1+\phi)+\sigma-1}}\right]^{\frac{1}{(\gamma-\alpha)(\varphi(1+\phi)+\sigma-1)+\alpha\varphi(1+\phi)}} \tag{58}$$

Given $\overline{S}$, we can obtain $\overline{C}, \overline{Y}, \overline{N}$ using the conditions

$$\overline{C} = \Lambda_H \overline{C}^* \overline{S}^{\gamma-\alpha}; \qquad \overline{Y} = \frac{\overline{C}\overline{S}^{\alpha}}{1-\varsigma_g}; \qquad \overline{N} = \left(\frac{\overline{Y}}{\overline{A}}\right)^{\varphi} \tag{59}$$

Combining the conditions above and the expression for $\overline{C}^{1-\sigma}$, we get

$$\overline{C}^{1-\sigma} = \frac{\gamma\varphi}{\gamma-\alpha}\overline{N}^{1+\phi}\left(1-\varsigma_g\right) \tag{60}$$

This expression will be useful on the derivation of the quadratic approximation below.

In order to support the level of output above, taxes must satisfy the condition:

$$\frac{(1-\overline{\tau}^v)(1-\overline{\tau}^l)}{1+\overline{\tau}^c} = \frac{\varphi\epsilon}{\epsilon-1}\overline{N}^{1+\phi}\frac{\overline{C}^{\sigma}\overline{S}^{\alpha}}{\overline{Y}} = \frac{\epsilon}{\epsilon-1}\frac{\gamma-\alpha}{\gamma} \tag{61}$$

Finally, consider the government budget constraint. Let's focus in a steady state where $\hat{\tau}^c = 0$. The government budget constraint is then given by

$$\frac{\overline{D}}{\overline{P}_H\overline{Y}} = \left(\frac{\overline{\tau}^v - \varsigma_g}{\rho}\right) + \frac{1}{\rho}\left[\overline{\tau}^p\left(1-\overline{\tau}^v - \frac{\overline{W}\overline{N}}{\overline{P}_H\overline{Y}}\right) + \overline{\tau}^l\frac{\overline{W}\overline{N}}{\overline{P}_H\overline{Y}} - \frac{\overline{T}}{\overline{Y}}\right] \tag{62}$$

The wage bill can be written as

$$\frac{\overline{W}\overline{N}}{\overline{P}_H\overline{Y}} = \frac{1}{1-\overline{\tau}^l}\frac{\overline{N}^{1+\phi}}{\overline{Y}}\overline{C}^{\sigma}\overline{S}^{\alpha} = (1-\overline{\tau}^v)\frac{\epsilon-1}{\varphi\epsilon} \tag{63}$$

The tax rate on labor income can be written

$$\overline{\tau}^l = 1 - \frac{1}{1-\overline{\tau}^v}\frac{\epsilon}{\epsilon-1}\frac{\gamma-\alpha}{\gamma} \tag{64}$$

The revenue from the labor tax is then given by

$$\overline{\tau}^l \frac{\overline{WN}}{\overline{P}_H\overline{Y}} = (1-\overline{\tau}^v)\frac{\epsilon-1}{\varphi\epsilon} - \frac{\gamma-\alpha}{\varphi\gamma} \tag{65}$$

If we set the tax on profits equal to zero, we get

$$\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} = \overline{\tau}^v\left(1-\frac{\epsilon-1}{\varphi\epsilon}\right) - \varsigma_g + \left(\frac{\epsilon-1}{\varphi\epsilon} - \frac{\gamma-\alpha}{\varphi\gamma}\right) - \frac{\overline{T}}{\overline{Y}} \tag{66}$$

B.2 An approximate welfare criterion

Utility from consumption can be written as

$$\frac{C_t^{1-\sigma}}{1-\sigma} = \frac{\overline{C}^{1-\sigma}}{1-\sigma}e^{(1-\sigma)c_t} \approx \frac{\overline{C}^{1-\sigma}}{1-\sigma} + \varphi\overline{N}^{1+\phi}(\varsigma_c+\varsigma_x)\frac{\gamma}{\gamma-\alpha}\left[c_t + \frac{1-\sigma}{2}c_t^2\right] \tag{67}$$

where I used the fact $\overline{C}^{1-\sigma} = \varphi\overline{N}^{1+\phi}(\varsigma_c+\varsigma_x)\frac{\gamma}{\gamma-\alpha}$.

The log-linear version of the Backus-Smith condition holds exactly:

$$c_t = \theta_t + c_t^* + \frac{(1-\alpha)}{\sigma}s_t \tag{68}$$

The utility from government spending can be written as

$$\chi \log G_t = \chi \log \overline{G} + \varphi\overline{N}^{1+\phi}\varsigma_g g_t \tag{69}$$

where I used the fact that $\chi = \varphi\overline{N}^{1+\phi}\varsigma_g$.

The disutility from labor is given by

$$\frac{N_t^{1+\phi}}{1+\phi} = \frac{\overline{N}^{1+\phi}}{1+\phi}e^{\varphi(1+\phi)(y_t-a_{H,t})} \approx \frac{\overline{N}^{1+\phi}}{1+\phi} + \varphi\overline{N}^{1+\phi}\left[(y_t-a_{H,t}) + \frac{\varphi(1+\phi)}{2}(y_t-a_{H,t})^2\right] \tag{70}$$

The aggregate demand condition can be expressed as:

$$e^{y_t} = \varsigma_c e^{\theta_t+\xi s_t+c_t^*} + \varsigma_g e^{g_t} + \varsigma_x e^{\lambda_{H,t}+\gamma s_t+c_t^*} \tag{71}$$

up to second-order, we get

$$\begin{aligned} y_t + \frac{1}{2}y_t^2 = {} & \varsigma_c\theta_t + (\gamma\varsigma_x + \xi\varsigma_c)\,s_t + \varsigma_g g_t + (\varsigma_c+\varsigma_x)c_t^* + \varsigma_x\lambda_{H,t} \\ & + \frac{1}{2}\left[\varsigma_c(\theta_t+c_t^*+\xi s_t)^2 + \varsigma_g g_t^2 + \varsigma_x(\lambda_{H,t}+c_t^*+\gamma s_t)^2\right] \end{aligned}$$

Let $u_t \equiv \frac{C_t^{1-\sigma}}{1-\sigma} + \chi\log G_t - \frac{1}{1+\phi}N_t^{1+\phi}$, $\overline{u} \equiv \frac{\overline{C}^{1-\sigma}}{1-\sigma} + \chi\log\overline{G} - \frac{1}{1+\phi}\overline{N}^{1+\phi}$, and $\hat{u}_t \equiv \frac{u_t-\overline{u}}{\varphi\overline{N}^{1+\phi}}$. The term $\hat{u}_t$

can be written as

$$\hat{u}_t = \frac{\gamma(\varsigma_c+\varsigma_x)}{\gamma-\alpha}\left[c_t + \frac{1-\sigma}{2}c_t^2\right] + \varsigma_g g_t - \left[(y_t - a_{H,t}) + \frac{\varphi(1+\phi)}{2}(y_t - a_{H,t})^2\right]$$
$$= -\frac{1}{2}\left[\varsigma_c(\theta_t + c_t^* + \xi s_t)^2 + \varsigma_x(\lambda_{H,t} + c_t^* + \gamma s_t)^2 + \varsigma_g g_t^2 - y_t^2 + \varphi(1+\phi)(y_t - a_{H,t})^2\right]$$
$$- \frac{1}{2}\frac{\gamma(\sigma-1)(\varsigma_c+\varsigma_x)}{\gamma-\alpha}c_t^2 + \frac{\gamma\varsigma_x + \alpha\varsigma_c}{\gamma-\alpha}\left[\theta_t + (\xi-\gamma)s_t\right]$$

where I am ignoring terms independent of policy.

We can use the external solvency constraint to eliminate the linear terms:

$$0 = \rho\int_0^\infty e^{-\int_0^t(\rho+\psi_s)ds}e^{(1-\sigma)c_t^* + (\xi-1)s_t}\overline{\Theta S}^{\xi-1}\left[e^{\lambda_{H,t}+(\gamma-\xi)s_t} - e^{\theta_t}\right]dt \tag{72}$$

where I used the fact that $E_0 = 0$.

Taking a second order approximation, we get

$$\int_0^\infty e^{-\rho t}\left[\theta + (\xi-\gamma)s_t - \lambda_{H,t}\right]dt =$$
$$-\frac{1}{2}\int_0^\infty e^{-\rho t}\left[(\theta_t + (1-\sigma)c_t^* + (\xi-1)s_t - \sigma\hat{\Psi}_t)^2 - \left(\lambda_{H,t} + (\gamma-1)s_t + (1-\sigma)c_t^* - \sigma\hat{\Psi}_t\right)^2\right]dt$$

Let $\mathbb{U} = \int e^{-\rho t}\hat{u}_t dt$ be our measure of welfare. This can be written as

$$\mathbb{U} = -\int_0^\infty \frac{e^{-\rho t}}{2}\Big[\varrho_\theta(\theta_t + (1-\sigma)c_t^* + (\xi-1)s_t - \sigma\hat{\Psi}_t)^2 - \varrho_\theta\left(\lambda_{H,t} + (1-\sigma)c_t^* + (\gamma-1)s_t - \sigma\hat{\Psi}_t\right)^2 + \varrho_c c_t^2$$
$$+\varsigma_c(\theta_t + c_t^* + \xi s_t)^2 + \varsigma_x(\lambda_{H,t} + c_t^* + \gamma s_t)^2 + \varsigma_g g_t^2 - y_t^2 + \varphi(1+\phi)(y_t - a_{H,t})^2\Big]dt + t.i.p.$$

where $\varrho_\theta \equiv \frac{\gamma\varsigma_x + \alpha\varsigma_c}{\gamma-\alpha}$ and $\tilde{\varrho}_c \equiv \frac{\gamma(\sigma-1)}{\gamma-\alpha}(\varsigma_c+\varsigma_x) = (\sigma-1)(\varrho_\theta + \varsigma_x)$.

In the case there is only a risk premium shock, we get

$$\mathbb{U}_t = \varrho_\theta(\theta_t + (\xi-1)s_t - \sigma\hat{\Psi}_t)^2 - \varrho_\theta\left((\gamma-1)s_t - \sigma\hat{\Psi}_t\right)^2 + \tilde{\varrho}_c c_t^2 + \varsigma_c(\theta_t + \xi s_t)^2 + \varsigma_x\gamma^2 s_t^2 + \varsigma_g g_t^2 + \varrho_y y_t^2$$

where $\varrho_y \equiv \varphi(1+\phi) - 1$ and $\mathbb{U} = \int_0^\infty e^{-\rho t}\mathbb{U}_t dt$.

Expanding the expression above, we obtain

$$\mathbb{U}_t = \sigma(\varrho_\theta + \varsigma_c)\theta_t^2 + 2\left[\varrho_\theta(\xi-1) + (\sigma-1)(\varrho_\theta+\varsigma_c)(\xi-\alpha) + \varsigma_c\xi\right]\theta_t s_t - 2\varrho_\theta\left(\theta_t + (\xi-\gamma)s_t\right)\sigma\hat{\Psi}_t$$
$$+\left(\varrho_\theta(\xi-1)^2 - \varrho_\theta(\gamma-1)^2 + (\sigma-1)(\varrho_\theta+\varsigma_c)(\xi-\alpha)^2 + \varsigma_c\xi^2 + \varsigma_x\gamma^2\right)s_t^2 + \varsigma_g g_t^2 + \varrho_y y_t^2$$

simplifying

$$\mathbb{U}_t = \frac{\gamma}{\gamma-\alpha}\frac{\varsigma_c}{\xi-\alpha}\left[\theta_t^2 + 2(\xi-\alpha)\left(1-\frac{\alpha}{\gamma}\right)\theta_t s_t\right] - 2\varrho_\theta\left(\theta_t + (\xi-\gamma)s_t\right)\sigma\hat{\Psi}_t$$
$$+\frac{\gamma\varsigma_c}{\gamma-\alpha}\left(\xi - \alpha - 2\frac{\alpha}{\gamma}\xi - \frac{\alpha^2}{1-\alpha} - \gamma\frac{\alpha}{1-\alpha} + 2\frac{\gamma+1-\alpha}{1-\alpha}\alpha\right)s_t^2 + \varsigma_g g_t^2 + \varrho_y y_t^2$$

Completing the square, we obtain

$$\mathbb{U}_t = \frac{\gamma}{\gamma-\alpha}\frac{\varsigma_c}{\xi-\alpha}\left(c_t - \frac{\alpha}{\gamma}(\xi-\alpha)s_t\right)^2 - 2\varrho_\theta\left(\theta_t + (\xi-\gamma)s_t\right)\sigma\hat{\Psi}_t$$
$$+\frac{\gamma}{\gamma-\alpha}\alpha\varsigma_c\left[\frac{\gamma-\alpha}{1-\alpha}+\frac{\gamma-\alpha}{\gamma}+\frac{\gamma-\xi}{\gamma}+\frac{\xi-\alpha}{\gamma}\frac{\gamma-\alpha}{\gamma}\right]s_t^2 + \varsigma_g g_t^2 + \varrho_y y_t^2$$

In more concise notation, we get

$$\mathbb{U}_t = \varrho_c\left(c_t - \varrho_{cs}s_t\right)^2 + \varrho_g g_t^2 + \varrho_y y_t^2 + \varrho_s s_t^2 + 2\varrho_\Psi((\gamma-\xi)s_t - \theta_t)\hat{\Psi}_t \tag{73}$$

where

$$\begin{aligned}
\varrho_c &\equiv \sigma\frac{\gamma(1-\varsigma_g)}{\gamma-\alpha}\\
\varrho_{cs} &\equiv \frac{\alpha}{\gamma}(\xi-\alpha)\\
\varrho_g &\equiv \varsigma_g\\
\varrho_y &\equiv \varphi(1+\phi)-1\\
\varrho_s &\equiv \frac{\gamma}{\gamma-\alpha}\alpha\varsigma_c\left[\frac{\gamma-\alpha}{1-\alpha}+\frac{\gamma-\alpha}{\gamma}+\frac{\gamma-\xi}{\gamma}+\frac{\xi-\alpha}{\gamma}\frac{\gamma-\alpha}{\gamma}\right]\\
\varrho_\Psi &\equiv \sigma\frac{\gamma+1-\alpha}{\gamma-\alpha}\alpha(1-\varsigma_g)
\end{aligned}$$

where all coefficients are positive provided $\gamma \geq \xi$

B.3 Linear-quadratic problem

The first-order conditions for the linear quadratic problem are

$$\begin{aligned}
g_t &= \lambda_t\\
\left(\varphi(1+\phi)-1\right)y_t &= -\lambda_t\\
\varrho_c\theta_t + \varrho_c(\xi-\alpha)\frac{\gamma-\alpha}{\gamma}s_t - \rho_\Psi\hat{\Psi}_t + \Gamma &= \varsigma_c\lambda_t\\
(\xi-\alpha)\frac{\gamma-\alpha}{\gamma}\varrho_c\left[\theta_t + (\xi-\alpha)\frac{\gamma-\alpha}{\gamma}s_t\right] + \varrho_s s_t + \varrho_\Psi(\gamma-\xi)\hat{\Psi}_t - (\gamma-\xi)\Gamma &= (\varsigma_c\xi + \varsigma_x\gamma)\lambda_t
\end{aligned}$$

Combining the first two conditions, we get that government spending is countercyclical

$$g_t = -\left(\varphi(1+\phi)-1\right)y_t \tag{74}$$

Plugging the expression above into the aggregate demand condition

$$y_t = \frac{\varsigma_c\theta_t + (\varsigma_c\xi + \varsigma_x\gamma)s_t}{1 + \left(\varphi(1+\phi)-1\right)\varsigma_g} \tag{75}$$

The optimality condition for θ_t can be written as

$$\frac{\gamma}{\gamma-\alpha}\frac{\varsigma_c}{\xi-\alpha}\theta_t+\varsigma_c s_t-\varrho_\Psi\hat{\Psi}_t+\Gamma=\varsigma_c\lambda_t \tag{76}$$

The optimality condition for s_t can be written as

$$\varsigma_c\theta_t+\frac{\gamma}{\gamma-\alpha}\alpha\varsigma_c\left[\frac{\gamma-\alpha}{1-\alpha}+\frac{\gamma-\alpha}{\gamma}+\frac{\gamma-\xi}{\gamma}+\frac{\xi-\alpha}{\alpha}\frac{\gamma-\alpha}{\gamma}\right]s_t+\varrho_\Psi(\gamma-\xi)\hat{\Psi}_t-(\gamma-\xi)\Gamma=(\varsigma_c\xi+\varsigma_x\gamma)\lambda_t$$

Using the first-order condition for θ_t to solve for Γ, we obtain

$$\varsigma_c\theta_t+\frac{\gamma}{\gamma-\alpha}\alpha\varsigma_c\left[\frac{\gamma-\alpha}{1-\alpha}+\frac{\gamma-\alpha}{\gamma}+\frac{\gamma-\xi}{\alpha}+\frac{\xi-\alpha}{\alpha}\frac{\gamma-\alpha}{\gamma}\right]s_t=\gamma(\varsigma_c+\varsigma_x)\lambda_t$$

Combining the optimality condition for θ_t and s_t, we get

$$\frac{\varsigma_c\left[(\gamma-\alpha)^2+\alpha(\gamma-\xi)\right]}{(\gamma-\alpha)(\xi-\alpha)}\theta_t+\frac{\varsigma_c+\varsigma_x}{\gamma-\alpha}\left[\gamma(\gamma-\alpha)+\alpha(1-\alpha)(\gamma-\xi)\right]s_t=\gamma(\varsigma_c+\varsigma_x)\lambda_t \tag{77}$$

using the expression for λ_t, we get

$$s_t=-\kappa_s\theta_t \tag{78}$$

where

$$\kappa_s=\sigma\frac{(\gamma-\alpha)^2+\alpha(\gamma-\xi)+\gamma(\gamma-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}{\gamma(\gamma-\alpha)+\alpha(1-\alpha)(\gamma-\xi)+\gamma(\gamma-\alpha)((1-\alpha)\xi+\alpha\gamma)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} \tag{79}$$

A sufficient condition for the expression above to be positive is $\gamma\geq\xi$.

B.4 Proof of proposition 4

Solving (22) and (25) for a given θ_t under passive policy, we obtain the terms of trade

$$s_t=-\frac{\sigma+(\varphi(1+\phi)-1)\varsigma_c}{1+(\varphi(1+\phi)-1)(\varsigma_c\xi+\varsigma_x\gamma)}\theta_t\equiv-\kappa_s^P\theta_t \tag{80}$$

and output

$$y_t=-\frac{\sigma\varsigma_x(\gamma-\xi+1)}{1+(\varphi(1+\phi)-1)(\varsigma_c\xi+\varsigma_x\gamma)}\theta_t\equiv-\kappa_y^P\theta_t \tag{81}$$

where $\kappa_s^P>0$ and $\kappa_y^P>0$ is positive if $\gamma\geq\xi$.

Plugging the expression for s_t into (21), we obtain:

$$c_t=\frac{\alpha+(\varphi(1+\phi)-1)(\varsigma_c\alpha+\varsigma_x\gamma)}{1+(\varphi(1+\phi)-1)(\varsigma_c\xi+\varsigma_x\gamma)}\theta_t\equiv\kappa_c^P\theta_t \tag{82}$$

where $\kappa_c^P>0$.

Net exports (deflated by domestic CPI) are given by

$$NX_t = \frac{P_{H,t}(Y_t - G_t) - P_t C_t}{P_{H,t}} = \alpha \overline{C}^* \overline{S}^{\xi} e^{c_t^* + \xi s_t} \left[\Lambda_H \overline{S}^{\gamma - \xi} e^{(\gamma - \xi) s_t} - \overline{\Theta} e^{\theta_t} \right] \tag{83}$$

Let $nx_t \equiv \frac{NX_t}{\overline{Y}}$. Net exports can be written as

$$nx_t = \varsigma_x \left[(\gamma - \psi) s_t - \theta_t \right] \tag{84}$$

Using the expression for s_t, we get

$$nx_t = -\frac{1 + \sigma(\gamma - \xi) + (\varphi(1+\phi) - 1)\gamma(\varsigma_c + \varsigma_x)}{1 + (\varphi(1+\phi) - 1)(\varsigma_c \xi + \varsigma_x \gamma)} \theta_t \equiv -\kappa_{nx}^P \theta_t \tag{85}$$

where $\kappa_{nx}^P > 0$ if $\gamma \geq \xi$.

Plugging the expression for the net exports into the external solvency constraint (23), we obtain

$$0 = \int_0^\infty e^{-\rho t} \theta_t dt \Rightarrow 0 = \frac{\theta_0}{\rho} + \frac{1}{\sigma} \int_0^\infty \frac{e^{-\rho t}}{\rho} \theta_t dt \tag{86}$$

using the fact that $\theta_t = \theta_0 + \frac{1}{\sigma} \int_0^t \psi_s ds$.

Solving for θ_0, we obtain

$$\theta_t = \frac{1}{\sigma} \left[\int_0^t \psi_s ds - \int_0^\infty e^{-\rho t} \theta_t dt \right] \tag{87}$$

where $\theta_0 < 0$ and $\lim_{t \to \infty} \theta_t > 0$ for $\psi_t \geq 0$ and it is assumed $\int_0^\infty \psi_t dt$ is well-defined. The short-run and long-run behavior for s_t, y_t, c_t, and nx_t is a direct consequence of the behavior of θ_t and the expressions above. □

B.5 Proof of proposition 5

i. **Evolution of θ_t**: The optimality condition for θ_t can be written as

$$\omega_\theta \theta_t = \varrho_\Psi \hat{\Psi}_t - \Gamma \tag{88}$$

where

$$\omega_\theta \equiv \frac{\gamma(\gamma + 1 - \alpha) + (1 - \alpha)\left[(\gamma - \alpha) + \alpha \frac{\gamma - \xi}{\gamma - \alpha}\right] + \gamma\left[(\gamma + 1 - \alpha)(\alpha\gamma + (1 - \alpha)\xi) + (1 - \alpha)(\xi - \alpha)\right] \frac{(\varphi(1+\phi) - 1)(1 - \varsigma_g)}{1 + (\varphi(1+\phi) - 1)\varsigma_g}}{\left[\alpha(1 - \varsigma_g)\sigma\right]^{-1} \left[\gamma(\gamma - \alpha) + \alpha(1 - \alpha)(\gamma - \xi) + \gamma(\gamma - \alpha)((1 - \alpha)\xi + \alpha\gamma) \frac{(\varphi(1+\phi) - 1)(1 - \varsigma_g)}{1 + (\varphi(1+\phi) - 1)\varsigma_g}\right]} \tag{89}$$

Differentiating with respect to time:

$$\dot{\theta}_t = \frac{\kappa_\theta}{\sigma} \psi_t \tag{90}$$

where $\kappa_\theta \equiv \varrho_\Psi / \omega_\theta$, or more explicitly.

$$\kappa_\theta = \frac{\frac{\gamma + 1 - \alpha}{\gamma - \alpha} \left[\gamma(\gamma - \alpha) + \alpha(1 - \alpha)(\gamma - \xi) + \gamma(\gamma - \alpha)((1 - \alpha)\xi + \alpha\gamma) \frac{(\varphi(1+\phi) - 1)(1 - \varsigma_g)}{1 + (\varphi(1+\phi) - 1)\varsigma_g}\right]}{\gamma(\gamma + 1 - \alpha) + (1 - \alpha)\left[(\gamma - \alpha) + \alpha \frac{\gamma - \xi}{\gamma - \alpha}\right] + \gamma\left[(\gamma + 1 - \alpha)(\alpha\gamma + (1 - \alpha)\xi) + (1 - \alpha)(\xi - \alpha)\right] \frac{(\varphi(1+\phi) - 1)(1 - \varsigma_g)}{1 + (\varphi(1+\phi) - 1)\varsigma_g}} \tag{91}$$

which is positive. The coefficient is less than one since $1-\kappa_\theta$ is also positive

$$1-\kappa_\theta \equiv \frac{(1-\alpha)\left[(1-\alpha)(\gamma-\alpha)+\alpha(\xi-\alpha)\right]+\gamma(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}{\gamma(\gamma+1-\alpha)+(1-\alpha)\left[(\gamma-\alpha)+\alpha\frac{\gamma-\xi}{\gamma-\alpha}\right]+\gamma\left[(\gamma+1-\alpha)(\alpha\gamma+(1-\alpha)\xi)+(1-\alpha)(\xi-\alpha)\right]\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} \tag{92}$$

The level of θ_t is given by

$$\theta_t = \theta_0 + \frac{\kappa_\theta}{\sigma}\int_0^t \psi_s ds \tag{93}$$

The value of θ_0 is determined by the external solvency constraint:

$$0 = \int_0^\infty e^{-\rho t}\left[(\gamma-\xi)s_t - \theta_t\right]dt \Rightarrow 0 = \int_0^\infty e^{-\rho t}\theta_t dt \tag{94}$$

Plugging the value for θ_t into the expression above, we get

$$\theta_0 = -\frac{\kappa_\theta}{\sigma}\int_0^\infty e^{-\rho t}\psi_t dt = \kappa_\theta \theta_0^P \tag{95}$$

Combining the expressions above, we solve θ_t

$$\theta_t = \frac{\kappa_\theta}{\sigma}\frac{\psi_0}{\rho_\psi}\left[\frac{\rho}{\rho+\rho_\psi} - e^{-\rho_\psi t}\right] \tag{96}$$

ii. **Optimal allocation**: From the optimality for government spending, the expression for s_t (78), and the demand condition (22), we obtain the expression for output

$$y_t = -k_y \theta_t \tag{97}$$

where $\kappa_y > 0$ and given by

$$\kappa_y \equiv \sigma \frac{(\gamma-\xi)\alpha\gamma(\gamma+1-\alpha)(1-\varsigma_g)\left(1+(\varphi(1+\phi)-1)\varsigma_g\right)^{-1}}{\gamma(\gamma-\alpha)+\alpha(1-\alpha)(\gamma-\psi)+\gamma(\gamma-\alpha)((1-\alpha)\xi+\alpha\gamma)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} \tag{98}$$

Notice that in the special case $\gamma=\xi$, which encompass the commonly used Cole-Obstfeld case, output should be zero. Consumption is given by

$$c_t = \theta_t + (\xi-\alpha)s_t = \left(1-(\xi-\alpha)\kappa_s\right)\theta_t = \kappa_c\theta_t \tag{99}$$

where $\kappa_c > 0$ and given by

$$\kappa_c = \frac{\alpha(\gamma-\alpha)(\gamma+1-\alpha)\left[1+\gamma\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]}{\gamma(\gamma-\alpha)+\alpha(1-\alpha)(\gamma-\xi)+\gamma(\gamma-\alpha)((1-\alpha)\xi+\alpha\gamma)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} \tag{100}$$

Consider now the behavior of net exports:

$$nx_t = (\gamma - \xi)s_t - \theta_t = -\left[(\gamma-\xi)\kappa_s + 1\right]\theta_t = -\kappa_{nx}\theta_t \tag{101}$$

where $\kappa_{nx} > 0$ and given by

$$\kappa_{nx} \equiv \frac{\gamma(\gamma-\alpha)\left[1+\sigma(\gamma-\xi)+\gamma\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]}{\gamma(\gamma-\alpha)+\alpha(1-\alpha)(\gamma-\xi)+\gamma(\gamma-\alpha)((1-\alpha)\xi+\alpha\gamma)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} \tag{102}$$

iii. **Optimal policy vs passive policy**: The ratio of the terms of trade under optimal policy and passive policy is given by

$$\frac{\kappa_s\kappa_\theta}{\kappa_s^P} = \frac{(\gamma+1-\alpha)\left[(\gamma-\alpha)+\alpha\frac{\gamma-\xi}{\gamma-\alpha}+\gamma(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]\frac{1+(\varphi(1+\phi)-1)(\varsigma_c\xi+\varsigma_x\gamma)}{1+(\varphi(1+\phi)-1)(\varsigma_c+\varsigma_x)(\xi-\alpha)}}{\gamma(\gamma+1-\alpha)+(1-\alpha)\left[(\gamma-\alpha)+\alpha\frac{\gamma-\xi}{\gamma-\alpha}\right]+\gamma\left[(\gamma+1-\alpha)(\alpha\gamma+(1-\alpha)\xi)+(1-\alpha)(\xi-\alpha)\right]\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} \tag{103}$$

We can see that the term above is not always less than one by taking the limit as γ goes to infinity:

$$\lim_{\gamma\to\infty}\frac{\kappa_s\kappa_\theta}{\kappa_s^P} = \frac{\left[1+(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]\frac{(\varphi(1+\phi)-1)\varsigma_x}{1+(\varphi(1+\phi)-1)(\varsigma_c+\varsigma_x)(\xi-\alpha)}}{\alpha\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} = \frac{1+(\varphi(1+\phi)-1)(\varsigma_g+(1-\varsigma_g)(\xi-\alpha))}{1+(\varphi(1+\phi)-1)(1-\varsigma_g)(\xi-\alpha)} > 1 \tag{104}$$

However, we are able to guarantee this ratio will be less than one if the difference $\gamma-\xi$ is sufficiently small. Notice the following inequality holds $1 \leq \frac{1+(\varphi(1+\phi)-1)(\varsigma_c\xi+\varsigma_x\gamma)}{1+(\varphi(1+\phi)-1)(\varsigma_c+\varsigma_x)(\xi-\alpha)} \leq \frac{(1-\alpha)\xi+\alpha\gamma}{\xi-\alpha}$. The upper bound becomes $\frac{\gamma}{\gamma-\alpha}$ in the case $\gamma=\xi$. For this case, we can write

$$\frac{\kappa_s\kappa_\theta}{\kappa_s^P} \leq \frac{\gamma(\gamma+1-\alpha)+\gamma^2(\gamma+1-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}{\gamma(\gamma+1-\alpha)+(1-\alpha)(\gamma-\alpha)+\left[\gamma^2(\gamma+1-\alpha)+(1-\alpha)\gamma(\gamma-\alpha)\right]\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} < 1 \tag{105}$$

Since the term above is strictly below one, by continuity, the inequality will be preserved for $\gamma-\xi$ sufficiently small. Let's consider output next:

$$\begin{aligned}\frac{\kappa_y\kappa_\theta}{\kappa_y^P} &= \frac{\gamma\frac{(\gamma-\xi)}{\gamma-\alpha}\frac{(\gamma+1-\alpha)^2}{\gamma-\xi+1}\frac{1+(\varphi(1+\phi)-1)(1-\varsigma_g)((1-\alpha)\xi+\alpha\gamma)}{1+(\varphi(1+\phi)-1)\varsigma_g}}{\gamma(\gamma+1-\alpha)+(1-\alpha)\left[(\gamma-\alpha)+\alpha\frac{\gamma-\xi}{\gamma-\alpha}\right]+\gamma\left[(\gamma+1-\alpha)(\alpha\gamma+(1-\alpha)\xi)+(1-\alpha)(\xi-\alpha)\right]\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}\\ &= \frac{\frac{(\gamma-\xi)}{\gamma-\alpha}\frac{(\gamma+1-\alpha)}{\gamma-\xi+1}\left[\gamma(\gamma+1-\alpha)\frac{1}{1+(\varphi(1+\phi)-1)\varsigma_g}+\gamma(\gamma+1-\alpha)((1-\alpha)\xi+\alpha\gamma)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]}{\gamma(\gamma+1-\alpha)+(1-\alpha)\left[(\gamma-\alpha)+\alpha\frac{\gamma-\xi}{\gamma-\alpha}\right]+\gamma\left[(\gamma+1-\alpha)(\alpha\gamma+(1-\alpha)\xi)+(1-\alpha)(\xi-\alpha)\right]\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}\end{aligned}$$

The expression above will be less than one if $\frac{(\gamma-\xi)}{\gamma-\alpha}\frac{(\gamma+1-\alpha)}{\gamma-\xi+1} < 1$. Since $\frac{(\gamma-\xi)}{\gamma-\alpha}\frac{(\gamma+1-\alpha)}{\gamma-\xi+1}$ is equal to zero at $\gamma=\xi$, it is increasing in γ, and converges to one when γ goes to infinity, then this term is always between zero and one.

The expression for consumption is given by

$$\frac{\kappa_c\kappa_\theta}{\kappa_c^P} = \frac{\left[\gamma(\gamma+1-\alpha)(1-\alpha)(\gamma-\alpha+1)+\gamma(\gamma+1-\alpha)^2\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]\frac{\alpha+(\varphi(1+\phi)-1)(1-\varsigma_g)\alpha((1-\alpha)\xi+\alpha\gamma)}{\alpha+(\varphi(1+\phi)-1)(1-\varsigma_g)((1-\alpha)\alpha+\alpha\gamma)}}{\gamma(\gamma+1-\alpha)+(1-\alpha)\left[(\gamma-\alpha)+\alpha\frac{\gamma-\xi}{\gamma-\alpha}\right]+\gamma\left[(\gamma+1-\alpha)(\alpha\gamma+(1-\alpha)\xi)+(1-\alpha)(\xi-\alpha)\right]\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} \tag{106}$$

The expression above is the product of a term that is bigger than one and a term that is smaller than one, the final result being ambiguous. Finally, consider net exports

$$\frac{\kappa_{nx}\kappa_{\theta}}{\kappa_{nx}^{P}} = \frac{\gamma(\gamma+1-\alpha)\left[1+\sigma(\gamma-\xi)+\gamma\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]\frac{1+(\varphi(1+\phi)-1)(1-\varsigma_g)((1-\alpha)\xi+\alpha\gamma)}{1+\sigma(\gamma-\xi)+(\varphi(1+\phi)-1)(1-\varsigma_g)\gamma}}{\gamma(\gamma+1-\alpha)+(1-\alpha)\left[(\gamma-\alpha)+\alpha\frac{\gamma-\xi}{\gamma-\alpha}\right]+\gamma\left[(\gamma+1-\alpha)(\alpha\gamma+(1-\alpha)\xi)+(1-\alpha)(\xi-\alpha)\right]\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} \tag{107}$$

which is smaller than one for $\gamma-\xi$ small. □

B.6 Proof of proposition 6

The expression for the optimal government spending is given in (74). Consider now the evolution of consumption taxes:

$$\dot{\theta}_t = \frac{1}{\sigma}\left[\psi - \dot{\tau}_t^c\right] \Rightarrow \dot{\hat{\tau}}_t^c = \psi_t - \sigma\dot{\theta}_t = (1-\kappa_\theta)\psi_t = \kappa_{\tau^c}\psi_t \tag{108}$$

where $\kappa_{\tau^c} \equiv 1-\kappa_\theta$.

Taxes are given by the supply condition (25):

$$(\hat{\tau}_t^v + \hat{\tau}_t^c) = (\kappa_s - \sigma)\theta_t - (\varphi(1+\phi)-1)y_t = -\kappa_\tau\theta_t$$

where I imposed $\hat{\tau}_t^l = 0$. $\kappa_\tau > 0$ and it is given by

$$\kappa_\tau \equiv \frac{\sigma\alpha\left[((1-\alpha)(\gamma-\xi)+\xi-\alpha)+\gamma(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]}{\gamma(\gamma-\alpha)+\alpha(1-\alpha)(\gamma-\xi)+\gamma(\gamma-\alpha)((1-\alpha)\xi+\alpha\gamma)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} \tag{109}$$

We can then determine the evolution of the VAT tax

$$\dot{\hat{\tau}}_t^v = -\left(\frac{\kappa_\tau\kappa_\theta}{\sigma} + \kappa_{\tau^c}\right)\psi_t \tag{110}$$

□

B.7 Proof of proposition 7

i. **Fiscal needs of the government**: Consider now the government solvency constraint:

$$\frac{C_0^{-\sigma}D_0^g}{P_0(1+\tau_0^c)} = \int_0^\infty e^{-\rho t}\frac{C_t^{-\sigma}}{1+\tau_t^c}\left[\tau_t^v S_t^{-\alpha}Y_t + \tau_t^c C_t + \tau_t^l\frac{W_t}{P_t}N_t - S_t^{-\alpha}G_t - T_t\right]dt \tag{111}$$

Linearizing the government solvency constraint:

$$[-\sigma c_0 - p_0]\frac{\overline{D}}{\overline{P}_H\overline{Y}} = \int_0^\infty e^{-\rho t}\left[\left(-\sigma c_t - \int_0^t \dot{\hat{\tau}}_s^c ds\right)\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} + \hat{\tau}_t^v + \overline{\tau}^v(-\hat{\tau}_t^v - \alpha s_t + y_t) + \frac{\overline{PC}}{\overline{P}_H\overline{Y}}((1+\overline{\tau}^c)\hat{\tau}_t^c + \overline{\tau}^c c_t)\right.$$
$$\left. + \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\left(\overline{\tau}^l\left(\hat{\tau}_t^c + \hat{\tau}_t^l + \sigma c_t + \varphi(1+\phi)y_t\right) + (1-\overline{\tau}^l)\hat{\tau}_t^l\right) - \frac{\overline{G}}{\overline{Y}}(-\alpha s_t + g_t) - \frac{\overline{T}}{\overline{Y}}(-\alpha s_t + \hat{T}_t)\right]dt$$

Using the pricing condition to eliminate s_t, we obtain

$$-\sigma\theta_0\frac{\overline{D}}{\overline{P}_H\overline{Y}} = \int_0^\infty e^{-\rho t}\left[-\int_0^t \dot{\hat{\tau}}_s^c ds\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} + \hat{\tau}_t^v + \overline{\tau}^v(\varphi(1+\phi)y_t + \hat{\tau}_t^l + \hat{\tau}_t^c) + \frac{\overline{PC}}{\overline{P}_H\overline{Y}}((1+\overline{\tau}^c)\hat{\tau}_t^c + \overline{\tau}^c(1-\sigma)c_t)\right.$$
$$\left. + \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\left(\overline{\tau}^l\left(\hat{\tau}_t^c + \varphi(1+\phi)y_t\right) + \hat{\tau}_t^l\right) - \left(\frac{\overline{G}}{\overline{Y}} + \frac{\overline{T}}{\overline{Y}}\right)((\varphi(1+\phi)-1)y_t + \hat{\tau}_t^v + \hat{\tau}_t^l + \hat{\tau}_t^c) - \frac{\overline{G}}{\overline{Y}}g_t - \frac{\overline{T}}{\overline{Y}}\hat{T}_t\right]dt$$

using $p_0 = -(1-\alpha)s_0$.

Simplifying the expression above, we get

$$-(\sigma\theta_0 + \hat{\tau}_0^c)\frac{\overline{D}}{\overline{P}_H\overline{Y}} = \int_0^\infty e^{-\rho t}\left[\left((\varphi(1+\phi)-1)\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} + \overline{\tau}^v + \overline{\tau}^l\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\right)y_t + (1-\varsigma_g)(\hat{\tau}_t^v + \hat{\tau}_t^c + \hat{\tau}_t^l) + \right.$$
$$\left.(1-\varsigma_g)\overline{\tau}^c((1-\sigma)c_t - (\varphi(1+\phi)-1)y_t) - \left((1-\overline{\tau}^v) - \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\right)\hat{\tau}_t^l - \varsigma_g g_t - \frac{\overline{T}}{\overline{Y}}\hat{T}_t\right]dt$$

Using $\hat{\tau}_t^l = \overline{\tau}^c = \hat{\tau}_0^c = 0$ and the fact that θ_t is equal to zero on average, we obtain

$$\tilde{T}_t = -\sigma\theta_0\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} \tag{112}$$

where $\tilde{T}_t \equiv -\frac{\overline{T}}{\overline{Y}}\hat{T}_t$ is the lump-sum tax, instead of transfer, in terms of steady-state GDP.

A similar calculation would show that the same expression holds for the case of passive policy, but with θ_0^P instead of θ_0. Since $\theta_0 = \kappa_\theta\theta_0^P$, the required amount of taxes required to satisfy the budget under the optimal policy is smaller than under the passive policy.

ii. **Debt dynamics**: The value of debt at period t is given by

$$\left(-(\sigma(\theta_t-\theta_0) + \hat{\tau}_t^c) + \hat{D}_t^g\right)\frac{\overline{D}}{\overline{P}_H\overline{Y}} = -\left[\left(\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} + \varsigma_g\right)(\varphi(1+\phi)-1)\kappa_y + \left(\overline{\tau}^v + \overline{\tau}^l\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\right)\kappa_y + (1-\varsigma_g)\kappa_\tau\right]\int_t^\infty e^{-\rho(s-t)}\theta_s ds \tag{113}$$

where I used the fact that $y_t = -\kappa_y\theta_t$ and the expression for $\tilde{T}_t$.

The first term on the left-hand side is given by

$$\sigma(\theta_t-\theta_0) + \hat{\tau}_t^c = \sigma(\theta_t-\theta_0) + \kappa_{\tau^c}\int_0^t \psi_s ds = (\kappa_\theta + \kappa_{\tau^c})\int_0^t \psi_s ds = \int_0^t \psi_s ds = \frac{1-e^{-\rho_\psi t}}{\rho_\psi}\psi_0 \tag{114}$$

where the last equality uses the assumption $\psi_t = e^{-\rho_\psi t}\psi_0$.

Similarly, using the decaying risk premium shock, we get

$$\int_t^\infty e^{-\rho(s-t)}\theta_s ds = \frac{\theta_t}{\rho} + \frac{\kappa_\theta}{\sigma}\frac{\int_t^\infty e^{-\rho(s-t)}\psi_s ds}{\rho} = \kappa_\theta\frac{1-e^{-\rho_\psi t}}{\rho_\psi(\rho+\rho_\psi)}\psi_0 \tag{115}$$

Combining the expressions above, we get

$$\hat{D}_t^g = \frac{1-e^{-\rho_\psi t}}{\rho_\psi}\psi_0\left[1 - \left(\frac{\overline{D}}{\overline{P}_H\overline{Y}}\right)^{-1}\left[\left(\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} + \varsigma_g\right)(\varphi(1+\phi)-1)\kappa_y + \left(\overline{\tau}^v + \overline{\tau}^l\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\right)\kappa_y + (1-\varsigma_g)\kappa_\tau\right]\frac{\kappa_\theta}{\rho+\rho_\psi}\right] \tag{116}$$

rearranging

$$\hat{D}_t^g = \frac{1-e^{-\rho_\psi t}}{\rho_\psi}\psi_0\left[\left(1-(\varphi(1+\phi)-1)\frac{\rho\kappa_y\kappa_\theta}{\rho+\rho_\psi}\right)-\left(\frac{\overline{D}}{\overline{P}_H\overline{Y}}\right)^{-1}\left[\varsigma_g(\varphi(1+\phi)-1)\kappa_y+\left(\overline{\tau}^v+\overline{\tau}^l\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\right)\kappa_y+(1-\varsigma_g)\kappa_\tau\right]\frac{\kappa_\theta}{\rho+\rho_\psi}\right] \tag{117}$$

Hence, government debt is increasing if and only if

$$\frac{\overline{D}}{\overline{P}_H\overline{Y}} > \frac{\left[\varsigma_g(\varphi(1+\phi)-1)\kappa_y+\left(\overline{\tau}^v+\overline{\tau}^l\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\right)\kappa_y+(1-\varsigma_g)\kappa_\tau\right]\frac{\kappa_\theta}{\rho+\rho_\psi}}{1-(\varphi(1+\phi)-1)\frac{\rho\kappa_y\kappa_\theta}{\rho+\rho_\psi}} \tag{118}$$

In the case of passive fiscal policy, the expression for the debt is given by

$$\hat{D}_t^g = \frac{1-e^{-\rho_\psi t}}{\rho_\psi}\psi_0\left(\frac{\overline{D}}{\overline{P}_H\overline{Y}}\right)^{-1}\left[\frac{\overline{D}}{\overline{P}_H\overline{Y}}\left(1-(\varphi(1+\phi)-1)\frac{\rho\kappa_y^P}{\rho+\rho_\psi}\right)-\left(\overline{\tau}^v+\overline{\tau}^l\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\right)\frac{\kappa_y^P}{\rho+\rho_\psi}\right] \tag{119}$$

iii. **Fiscal consolidation**: For γ sufficiently large, $\kappa_y\kappa_\theta$ converges to κ_y^P. This will guarantee the government debt is larger under the passive policy. □

C Derivations for section 5

C.1 Proof of propositions 8

i. **Optimal allocation**: The optimal policy can be obtained by solving the linear quadratic problem:

$$\min_{[\theta_t,g_t,y_t]}\left\{\frac{1}{2}\int_0^\infty e^{-\rho t}\left[\varrho_c\theta_t^2+\varrho_g g_t^2+\varrho_y y_t^2-2\varrho_\Psi\theta_t\hat{\Psi}_t\right]dt\right\} \tag{120}$$

subject to

$$y_t = \varsigma_c\theta_t+\varsigma_g g_t \tag{121}$$

$$0 = \int_0^\infty e^{-\rho t}\theta_t dt \tag{122}$$

The first-order conditions are

$$\begin{aligned} g_t &= \lambda_t \\ -(\varphi(1+\phi)-1)y_t &= \lambda_t \\ \varrho_c\theta_t-\varrho_\Psi\hat{\Psi}_t &= \varsigma_c\lambda_t+\Gamma \end{aligned}$$

This imply the usual condition for government spending:

$$g_t = -(\varphi(1+\phi)-1)y_t \tag{123}$$

Output is then given by

$$y_t = \frac{\varsigma_c}{1+(\varphi(1+\phi)-1)\varsigma_g}\theta_t \tag{124}$$

The Lagrange multiplier is given by

$$\lambda_t = -\frac{(\varphi(1+\phi)-1)\varsigma_c}{1+(\varphi(1+\phi)-1)\varsigma_g}\theta_t \tag{125}$$

Differentiating the condition for θ_t, we get

$$\dot{\theta}_t = \frac{\alpha(\gamma+1-\alpha)}{\gamma+(\gamma-\alpha)(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}\frac{\psi_t}{\sigma} \equiv \frac{\kappa_\theta^R}{\sigma}\psi_t \tag{126}$$

where $\kappa_\theta^R > 0$ and

$$1-\kappa_\theta^R = \frac{(1-\alpha)(\gamma-\alpha)+(\gamma-\alpha)(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}{\gamma+(\gamma-\alpha)(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} > 0 \tag{127}$$

The evolution of the consumption tax is given by

$$\dot{\theta}_t = \frac{1}{\sigma}\left[\psi_t - \dot{\hat{\tau}}_t^c\right] \Rightarrow \dot{\hat{\tau}}_t^c = \left(1-\kappa_\theta^R\right)\psi_t \tag{128}$$

Plugging into the external solvency condition,

$$\theta_0 = -\frac{\kappa_\theta^R}{\sigma}\int_0^\infty e^{-\rho t}\psi_t dt \tag{129}$$

Therefore, θ_t is given by

$$\theta_t = \frac{\kappa_\theta^R}{\sigma}\left[\int_0^t \psi_s ds - \int_0^\infty e^{-\rho s}\psi_s ds\right] \tag{130}$$

The labor wedge is given by

$$\hat{\omega}_t^l = -\sigma\theta_t - (\varphi(1+\phi)-1)y_t = -\sigma\left[1+(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]\theta_t \tag{131}$$

ii. **Optimal versus passive policy**: Under passive fiscal policy, θ_t is given by

$$\theta_t = \frac{1}{\sigma}\left[\int_0^t \psi_s ds - \int_0^\infty e^{-\rho s}\psi_s ds\right] \tag{132}$$

Output is given by $y_t = \varsigma_c\theta_t$, consumption by $c_t = \theta_t$, and net exports by $nx_t = -\varsigma_x\theta_t$. The labor wedge is given by

$$\hat{\omega}_t^{L,P} = -\sigma\theta_t - (\varphi(1+\phi)-1)y_t = -\sigma\left[1+(\xi-\alpha)(\varphi(1+\phi)-1)(1-\varsigma_g)\right]\theta_t^P \tag{133}$$

Since $0 < \kappa_\theta^R < 1$ and $\frac{1}{1+(\varphi(1+\phi)-1)\varsigma_g} < 1$, then

$$\frac{\theta_t}{\theta_t^P} < 1; \qquad \frac{y_t}{y_t^P} < 1; \qquad \frac{c_t}{c_t^P} < 1; \qquad \frac{nx_t}{nx_t^P} < 1; \qquad \frac{\hat{\omega}_t^L}{\hat{\omega}_t^{L,P}} < 1 \tag{134}$$

iii. **Role of openness and international demand**: The coefficient κ_θ^R can be expressed as

$$\kappa_\theta^R = \frac{\alpha}{\frac{\gamma}{\gamma+1-\alpha} + \frac{\gamma-\alpha}{\gamma+1-\alpha}(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} \tag{135}$$

Clearly, $\kappa_\theta^R = 0$ for $\alpha = 0$ and $\kappa_\theta^R = 1$ for $\alpha = 1$. Since the denominator is increasing in γ, then $\frac{\partial \kappa_\theta^R}{\partial \gamma} < 0$. Taking the limit as $\gamma \to \infty$, we obtain

$$\lim_{\gamma\to\infty} \kappa_\theta^R = \frac{\alpha}{1 + (1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} > 0 \tag{136}$$

C.2 Sticky prices

Derivation of the loss function

Taking a second-order approximation of the expression for the domestic price index

$$P_{H,t}^{1-\epsilon} = \int_0^1 P_{H,t}(j)^{1-\epsilon} dj \Rightarrow \int_0^1 (p_{h,t}(j) - p_{H,t})dj = \frac{\epsilon-1}{2}\int_0^1 (p_{h,t}(j) - p_{H,t})^2 dj \tag{137}$$

Approximating the expression for the price dispersion (10) up to second-order, we obtain

$$\begin{aligned}\hat{\Delta}_t &= -\epsilon\varphi \int_0^1 (p_{H,t}(j) - p_{H,t})dj + \frac{(\epsilon\varphi)^2}{2}\int_0^1 (p_{H,t}(j) - p_{H,t})^2 dj \\ &= \frac{\epsilon\varphi((\varphi-1)\epsilon+1)}{2}\int_0^1 (p_{H,t}(j) - p_{H,t})^2 dj\end{aligned}$$

Hence, up to first-order $\hat{\Delta}_t = 0$, so $n_t = \varphi(y_t - a_t)$. Applying an exact law of large numbers:

$$\hat{\Delta}_t = \frac{\epsilon\varphi((\varphi-1)\epsilon+1)}{2}\rho_\delta \int_{-\infty}^t e^{-\rho_\delta(t-s)}(\overline{p}_s - p_{H,s})^2 ds \Rightarrow \dot{\hat{\Delta}}_t = \frac{\epsilon\varphi((\varphi-1)\epsilon+1)}{2}\varrho_\delta(\overline{p}_t - p_{H,t})^2 - \rho_\delta\hat{\Delta}_t \tag{138}$$

Subtracting $\rho\hat{\Delta}_t$ from both sides, multiplying by $e^{-\rho t}$, and integrating the expression above, we obtain

$$\int_0^\infty e^{-\rho t}\hat{\Delta}_t dt = \frac{\epsilon\varphi}{2\kappa}\int_0^\infty e^{-\rho t}\pi_{H,t}^2 dt \tag{139}$$

where $\kappa \equiv \frac{\rho_\delta(\rho+\rho_\delta)}{(\varphi-1)\epsilon+1}$ and using the fact that $\hat{\Delta}_0 = 0$ and $\pi_{H,t} = \rho_\delta(\overline{p}_t - p_{H,t})$.

Taking a second-order approximation of the labor disutility gives

$$\frac{N_t^{1+\phi}}{1+\phi} = \frac{\overline{N}^{1+\phi}}{1+\phi} e^{(1+\phi)\hat{\Delta}_t + \varphi(1+\phi)(y_t - a_{H,t})} \approx \frac{\overline{N}^{1+\phi}}{1+\phi} + \varphi \overline{N}^{1+\phi} \left[\frac{\hat{\Delta}_t}{\varphi} + (y_t - a_{H,t}) + \frac{\varphi(1+\phi)}{2} (y_t - a_{H,t})^2 \right] \tag{140}$$

Hence, the (per period) loss function under sticky prices corresponds to the one under flexible prices plus the term $\frac{\epsilon}{\kappa}\pi_t^2$.

New Keynesian Phillips Curve

Up to first-order, we have $p_{H,t} = \int_0^1 p_{H,t}(j)dj$ and, applying an exact law of large numbers, we obtain

$$p_{H,t} = \rho_\delta \int_{-\infty}^{t} e^{-\rho_\delta (t-s)} \overline{p}_s ds \Rightarrow \pi_{H,t} = \rho_\delta (\overline{p}_t - p_{H,t}) \tag{141}$$

The optimal price setting condition is given by

$$\int_0^\infty e^{-(\rho+\rho_\delta)s} \frac{C_{t+s}^{-\sigma}}{P_{t+s}(1+\tau_{t+s}^c)} \left[(1-\tau_{t+s}^v) \frac{\overline{P}_t(j)^{-\epsilon}}{P_{H,t+s}^{-\epsilon}} Y_{t+s} - \frac{\varphi\epsilon}{\epsilon - 1} W_{t+s} \frac{\overline{P}_t(j)^{-\varphi\epsilon - 1}}{P_{H,t+s}^{-\varphi\epsilon}} \left(\frac{Y_{t+s}}{A_{t+s}} \right)^\varphi \right] ds = 0 \tag{142}$$

Linearizing the expression above

$$\int_t^\infty e^{-(\rho+\rho_\delta)(s-t)} \left[-\hat{\tau}_s^v - \epsilon(\overline{p}_t - p_{H,s}) + y_t - (\hat{\tau}_s^c + \hat{\tau}_s^l + \sigma c_s + \alpha s_t + \varphi(1+\phi)(y_s - a_s)) + (\varphi\epsilon + 1)(\overline{p}_t - p_{H,s}) \right] ds = 0 \tag{143}$$

rearranging

$$\overline{p}_t = \frac{(\rho + \rho_\delta)}{(\varphi - 1)\epsilon + 1} \int_t^\infty e^{-(\rho+\rho_\delta)(s-t)} \left[\hat{\tau}_s^v + \hat{\tau}_s^c + \hat{\tau}_s^l + \sigma c_s + \alpha s_s + \varphi(1+\phi)(y_s - a_s) - y_t + ((\varphi - 1)\epsilon + 1) p_{H,s} \right] ds \tag{144}$$

Taking the derivative of the expression above with respect to t and using the fact that $\pi_{H,t} = \rho_\delta(\overline{p}_t - p_{H,t})$, we obtain

$$\dot{\pi}_{H,t} = \rho \pi_{H,t} - \kappa \left[\hat{\tau}_t^v + \hat{\tau}_t^c + \hat{\tau}_t^l + \sigma c_t + \alpha s_t + \varphi(1+\phi)(y_t - a_t) - y_t \right] \tag{145}$$

Optimal policy

The Ramsey problem takes the form of a continuous-time optimal control problem

$$\min_{[\theta_t, s_t, g_t, y_t, c_t, \pi_{H,t}]} \left\{ \frac{1}{2} \int_0^\infty e^{-\rho t} \left[\varrho_c \left(c_t - \varrho_{cs} s_t \right)^2 + \varrho_g g_t^2 + \varrho_y y_t^2 + \varrho_s s_t^2 + 2\varrho_\Psi \left((\gamma - \xi) s_t - \theta_t \right) \hat{\Psi}_t + \frac{\epsilon}{\kappa} \pi_{H,t}^2 \right] dt \right\}$$

subject to

$$\begin{aligned} c_t &= \theta_t + (\xi - \alpha)s_t \\ y_t &= \varsigma_c\theta_t + \varsigma_g g_t + (\varsigma_c\xi + \varsigma_x\gamma)s_t \\ 0 &= \int_0^\infty e^{-\rho t}\left[(\gamma - \xi)s_t - \theta_t\right] dt \\ \dot{s}_t &= -\pi_{H,t} \end{aligned}$$

where $s_0 = 0$.

The first-order conditions are

$$g_t = -\lambda_t$$

$$(\varphi(1+\phi) - 1)\, y_t = \lambda_t$$

$$\varrho_c\left(\theta_t + (\xi - \alpha)\frac{\gamma - \alpha}{\gamma}s_t\right) - \varrho_\Psi\hat{\Psi}_t + \varsigma_c\lambda_t + \Gamma = 0$$

$$\varrho_c(\xi - \alpha)\frac{\gamma - \alpha}{\gamma}\left(\theta_t + (\xi - \alpha)\frac{\gamma - \alpha}{\gamma}s_t\right) + \varrho_s s_t + \varrho_\Psi(\gamma - \xi)\hat{\Psi}_t + \lambda_t(\varsigma_c\xi + \varsigma_x\gamma) - \Gamma(\gamma - \xi) = \rho\mu_{S,t} + \dot{\mu}_{S,t}$$

$$\frac{\epsilon}{\kappa}\pi_{H,t} = \mu_{S,t}$$

Government spending is given by:

$$g_t = -(\varphi(1+\phi) - 1)y_t \tag{146}$$

Plugging the expression above into the aggregate demand condition

$$y_t = \frac{\varsigma_c\theta_t + (\varsigma_c\xi + \varsigma_x\gamma)s_t}{1 + (\varphi(1+\phi) - 1)\,\varsigma_g} \tag{147}$$

Note that λ_t is given by

$$\lambda_t = \frac{(\varphi(1+\phi) - 1)(1 - \varsigma_g)}{1 + (\varphi(1+\phi) - 1)\,\varsigma_g}\left[(1 - \alpha)\theta_t + ((1 - \alpha)\xi + \alpha\gamma)s_t\right] \tag{148}$$

The optimality condition for θ_t can be written as

$$\frac{\gamma(1 - \varsigma_g)\sigma}{\gamma - \alpha}\theta_t + \varsigma_c s_t - \varrho_\Psi\hat{\Psi}_t + \Gamma = -\varsigma_c\lambda_t \tag{149}$$

plugging in the expression for λ_t, we get

$$\omega_{\theta\theta}\theta_t + \omega_{\theta s}s_t = \varrho_\Psi\hat{\Psi}_t - \Gamma \tag{150}$$

where

$$\omega_{\theta\theta} \equiv \frac{\gamma(1-\varsigma_g)\sigma}{\gamma-\alpha}\left[1+\frac{\gamma-\alpha}{\gamma}(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]$$

$$\omega_{\theta s} \equiv (1-\alpha)(1-\varsigma_g)\left[1+((1-\alpha)\xi+\alpha\gamma)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]$$

We can use the expression above to express θ_t in terms of s_t:

$$\theta_t = -\frac{\omega_{\theta s}}{\omega_{\theta\theta}}s_t + \frac{\varrho_\Psi \hat{\Psi}_t - \Gamma}{\omega_{\theta\theta}} \tag{151}$$

The optimality condition for s_t can be written as

$$\omega_{s\theta}\theta_t + \omega_{ss}s_t + (\gamma-\xi)\left(\varrho_\Psi \hat{\Psi}_t - \Gamma\right) = \rho\mu_{S,t} - \dot{\mu}_{S,t} \tag{152}$$

where

$$\omega_{ss} \equiv (1-\varsigma_g)\left[(1-\alpha)\xi + \alpha\left[\gamma+(1-\alpha)\frac{\gamma-\xi}{\gamma-\alpha}\right] + ((1-\alpha)\xi+\alpha\gamma)^2\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]$$

$$\omega_{s\theta} \equiv (1-\alpha)(1-\varsigma_g)\left[1+((1-\alpha)\xi+\alpha\gamma)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]$$

Using the optimality condition for inflation and the dynamic constraint, we get

$$\mu_{S,t} = -\frac{\epsilon}{\kappa}\dot{s}_t; \qquad \dot{\mu}_{S,t} = -\frac{\epsilon}{\kappa}\ddot{s}_t \tag{153}$$

Plugging (151) into the expression for s_t, we get

$$\ddot{s}_t - \rho\dot{s}_t = \frac{\kappa}{\epsilon}\left(\frac{\omega_{ss}\omega_{\theta\theta}-\omega_{s\theta}^2}{\omega_{\theta\theta}}\right)s_t + \frac{\kappa}{\epsilon}\left(\frac{\omega_{s\theta}}{\omega_{\theta\theta}}+(\gamma-\xi)\right)\left(\varrho_\Psi\hat{\Psi}_t-\Gamma\right) \tag{154}$$

where I used the fact that $\omega_{s\theta} = \omega_{\theta s}$.

More compactly, expression above can be written as

$$\ddot{s}_t = \rho\dot{s}_t + \omega_s s_t + u_t \tag{155}$$

where

$$\omega_s = \frac{\kappa}{\epsilon}\left(\frac{\omega_{ss}\omega_{\theta\theta}-\omega_{s\theta}^2}{\omega_{\theta\theta}}\right);$$

$$u_t = \frac{\kappa}{\epsilon}\left(\frac{\omega_{s\theta}}{\omega_{\theta\theta}}+(\gamma-\xi)\right)\left(\varrho_\Psi\hat{\Psi}_t-\Gamma\right);$$

$$\omega_{ss} \equiv (1-\varsigma_g)\left[(1-\alpha)\xi + \alpha\left[\gamma + (1-\alpha)\frac{\gamma-\xi}{\gamma-\alpha}\right] + ((1-\alpha)\xi+\alpha\gamma)^2\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]$$
$$\omega_{s\theta} \equiv (1-\alpha)(1-\varsigma_g)\left[1 + ((1-\alpha)\xi+\alpha\gamma)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]$$

The second order differential equation above can be written as a system of first order ODEs:

$$\begin{bmatrix}\ddot{s}_t \\ \dot{s}_t\end{bmatrix} = \begin{bmatrix}\rho & \omega_s \\ 1 & 0\end{bmatrix}\begin{bmatrix}\dot{s}_t \\ s_t\end{bmatrix} + \begin{bmatrix}u_t \\ 0\end{bmatrix} \tag{156}$$

Define the eigenvalues of the coefficient matrix:

$$\underline{\rho} \equiv \frac{\rho - \sqrt{\rho^2 + 4\omega_s}}{2}; \qquad \overline{\rho} \equiv \frac{\rho + \sqrt{\rho^2 + 4\omega_s}}{2}; \tag{157}$$

The coefficient matrix can be written as

$$\begin{bmatrix}\rho & \omega_s \\ 1 & 0\end{bmatrix} = \frac{\omega_s}{\overline{\rho}-\underline{\rho}}\begin{bmatrix}1 & 1 \\ \overline{\rho}^{-1} & \underline{\rho}^{-1}\end{bmatrix}\begin{bmatrix}\overline{\rho} & 0 \\ 0 & \underline{\rho}\end{bmatrix}\begin{bmatrix}-\underline{\rho}^{-1} & 1 \\ \overline{\rho}^{-1} & -1\end{bmatrix} \tag{158}$$

Define the following transformed variables:

$$Z_t \equiv \begin{bmatrix}Z_{1,t} \\ Z_{2,t}\end{bmatrix} = \begin{bmatrix}1 & 1 \\ \overline{\rho}^{-1} & \underline{\rho}^{-1}\end{bmatrix}^{-1}\begin{bmatrix}\dot{s}_t \\ s_t\end{bmatrix} = \frac{\omega_s}{\overline{\rho}-\underline{\rho}}\begin{bmatrix}s_t - \underline{\rho}^{-1}\dot{s}_t \\ \overline{\rho}^{-1}\dot{s}_t - s_t\end{bmatrix} \tag{159}$$

The system can be rewritten as

$$\dot{Z}_{1,t} = \overline{\rho}Z_{1,t} - \frac{\omega_s\underline{\rho}^{-1}}{\overline{\rho}-\underline{\rho}}u_t$$
$$\dot{Z}_{2,t} = \underline{\rho}Z_{2,t} + \frac{\omega_s\overline{\rho}^{-1}}{\overline{\rho}-\underline{\rho}}u_t$$

Notice the second-order conditions for the optimization problem guarantee that $\omega_s > 0$. Hence, $\overline{\rho} > 0 > \underline{\rho}$. We can then solve the first equation forward and the second one backwards:

$$Z_{1,t} = \frac{\omega_s\underline{\rho}^{-1}}{\overline{\rho}-\underline{\rho}}\int_t^\infty e^{-\overline{\rho}(s-t)}u_s ds$$
$$Z_{2,t} = e^{\underline{\rho}t}Z_{2,0} + \frac{\omega_s\overline{\rho}^{-1}}{\overline{\rho}-\underline{\rho}}\int_0^t e^{\underline{\rho}(t-s)}u_s ds$$

Evaluating $Z_{1,t}$ at period zero we obtain the remaining boundary condition:

$$-\frac{\omega_s\underline{\rho}^{-1}}{\overline{\rho}-\underline{\rho}}\dot{s}_0 = \frac{\omega_s\underline{\rho}^{-1}}{\overline{\rho}-\underline{\rho}}\int_0^\infty e^{-\overline{\rho}t}u_t dt \Rightarrow \dot{s}_0 = -\int_0^\infty e^{-\overline{\rho}t}u_t dt \tag{160}$$

where I used the fact $s_0 = 0$.

The initial value of $Z_{2,t}$ is given by

$$Z_{2,0} = \frac{\omega_s \overline{\rho}^{-1}}{\overline{\rho} - \underline{\rho}} \dot{s}_0 = -\frac{\omega_s \overline{\rho}^{-1}}{\overline{\rho} - \underline{\rho}} \int_0^\infty e^{-\overline{\rho} t} u_t dt \tag{161}$$

The variable $Z_{2,t}$ can then be written as

$$Z_{2,t} = -e^{\underline{\rho} t} \frac{\omega_s \overline{\rho}^{-1}}{\overline{\rho} - \underline{\rho}} \int_0^\infty e^{-\overline{\rho} s} u_s ds + \frac{\omega_s \overline{\rho}^{-1}}{\overline{\rho} - \underline{\rho}} \int_0^t e^{\underline{\rho}(t-s)} u_s ds \tag{162}$$

Rotating the system back to the original coordinates, we get

$$\begin{bmatrix} \dot{s}_t \\ s_t \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \overline{\rho}^{-1} & \underline{\rho}^{-1} \end{bmatrix} \begin{bmatrix} Z_{1,t} \\ Z_{2,t} \end{bmatrix} = \begin{bmatrix} Z_{1,t} + Z_{2,t} \\ \overline{\rho}^{-1} Z_{1,t} + \underline{\rho}^{-1} Z_{2,t} \end{bmatrix}$$

The terms of trade is then given by

$$s_t = \frac{1}{\overline{\rho} - \underline{\rho}} \left[e^{\underline{\rho} t} \int_0^\infty e^{-\overline{\rho} s} u_s ds - \int_0^t e^{\underline{\rho}(t-s)} u_s ds - \int_t^\infty e^{-\overline{\rho}(s-t)} u_s ds \right] \tag{163}$$

Let's consider now the present discounted value of s_t:

$$\int_0^\infty e^{-\rho t} s_t dt = \frac{1}{\omega_s} \int_0^\infty \left(e^{-\overline{\rho} t} - e^{-\rho t} \right) u_t dt = \frac{\kappa}{\epsilon \omega_s} \left(\frac{\omega_{s\theta}}{\omega_{\theta\theta}} + (\gamma - \xi) \right) \int_0^\infty \left(e^{-\overline{\rho} t} - e^{-\rho t} \right) \left(\varrho_\Psi \hat{\Psi}_t - \Gamma \right) dt \tag{164}$$

The external solvency condition is

$$\int_0^\infty e^{-\rho t} \left[(\gamma - \xi) s_t - \theta_t \right] dt = 0 \iff \int_0^\infty e^{-\rho t} \left[\left((\gamma - \xi) \omega_{\theta\theta} + \omega_{\theta s} \right) s_t - \left(\varrho_\Psi \hat{\Psi}_t - \Gamma \right) \right] dt = 0 \tag{165}$$

The condition above can be written as

$$\kappa \frac{\left((\gamma - \xi) \omega_{\theta\theta} + \omega_{\theta s} \right)^2}{\epsilon \omega_s \omega_{\theta\theta}} \int_0^\infty \left(e^{-\overline{\rho} t} - e^{-\rho t} \right) \left(\varrho_\Psi \hat{\Psi}_t - \Gamma \right) dt = \int_0^\infty e^{-\rho t} \left(\varrho_\Psi \hat{\Psi}_t - \Gamma \right) dt \tag{166}$$

where I used the fact $\omega_{\theta s} = \omega_{s\theta}$.

We can then solve for Γ:

$$\Gamma = \frac{\varrho_\Psi \sigma^{-1}}{1 - \frac{\rho}{\overline{\rho}} \frac{\kappa \left((\gamma - \xi) \omega_{\theta\theta} + \omega_{\theta s} \right)^2}{\kappa \left((\gamma - \xi) \omega_{\theta\theta} + \omega_{\theta s} \right)^2 + \epsilon \omega_s \omega_{\theta\theta}}} \left[\int_0^\infty e^{-\rho t} \psi_t dt - \frac{\rho}{\overline{\rho}} \frac{\kappa \left((\gamma - \xi) \omega_{\theta\theta} + \omega_{\theta s} \right)^2}{\kappa \left((\gamma - \xi) \omega_{\theta\theta} + \omega_{\theta s} \right)^2 + \epsilon \omega_s \omega_{\theta\theta}} \int_0^\infty e^{-\overline{\rho} t} \psi_t dt \right] \tag{167}$$

If ψ_t decays exponentially, $\psi_t = e^{-\rho_\psi t}\psi_0$, then

$$\Gamma = \frac{\varrho_\Psi}{\sigma}\frac{\psi_0}{\rho+\rho_\psi}\frac{1-\frac{\underline{\rho}}{\overline{\rho}}\frac{\kappa((\gamma-\xi)\omega_{\theta\theta}+\omega_{\theta s})^2}{\kappa((\gamma-\xi)\omega_{\theta\theta}+\omega_{\theta s})^2+\epsilon\omega_s\omega_{\theta\theta}}\frac{\rho+\rho_\psi}{\overline{\rho}+\rho_\psi}}{1-\frac{\underline{\rho}}{\overline{\rho}}\frac{\kappa((\gamma-\xi)\omega_{\theta\theta}+\omega_{\theta s})^2}{\kappa((\gamma-\xi)\omega_{\theta\theta}+\omega_{\theta s})^2+\epsilon\omega_s\omega_{\theta\theta}}} \tag{168}$$

where the expression above is positive.

The terms of trade are given by

$$s_t = \frac{\kappa}{\epsilon}\left(\frac{\omega_{s\theta}+(\gamma-\xi)\omega_{\theta\theta}}{\omega_{\theta\theta}}\right)\left[e^{\underline{\rho}t}\int_0^\infty e^{-\overline{\rho}s}\frac{\varrho_\Psi\hat{\Psi}_s}{\overline{\rho}-\underline{\rho}}ds - \int_0^t e^{\underline{\rho}(t-s)}\frac{\varrho_\Psi\hat{\Psi}_s}{\overline{\rho}-\underline{\rho}}ds - \int_t^\infty e^{-\overline{\rho}(s-t)}\frac{\varrho_\Psi\hat{\Psi}_s}{\overline{\rho}-\underline{\rho}}ds + \frac{1-e^{\underline{\rho}t}}{\omega_s}\Gamma\right] \tag{169}$$

Assuming that ψ_t is exponentially decaying, the terms of trade can be written as

$$s_t = \frac{\kappa}{\epsilon}\left(\frac{\omega_{s\theta}+(\gamma-\xi)\omega_{\theta\theta}}{\omega_{\theta\theta}}\right)\left[\frac{\varrho_\Psi\psi_0 e^{\underline{\rho}t}}{\sigma\overline{\rho}(\overline{\rho}+\rho_\psi)(\overline{\rho}-\underline{\rho})} + \varrho_\Psi\psi_0\frac{\underline{\rho}+\rho_\psi-\rho_\psi e^{\underline{\rho}t}-\underline{\rho}e^{-\rho_\psi t}}{\sigma\rho_\psi\underline{\rho}(\rho_\psi+\underline{\rho})(\overline{\rho}-\underline{\rho})} - \varrho_\Psi\psi_0\frac{\overline{\rho}+\rho_\psi-\overline{\rho}e^{-\rho_\psi t}}{\sigma\rho_\psi\overline{\rho}(\overline{\rho}+\rho_\psi)(\overline{\rho}-\underline{\rho})} + \frac{1-e^{\underline{\rho}t}}{\omega_s}\Gamma\right] \tag{170}$$

In more compact notation, we can write

$$s_t = \kappa_{s\underline{\rho}}\left(1-e^{\overline{\rho}t}\right) + \kappa_{s\rho_\psi}\left(1-e^{-\rho t}\right) \tag{171}$$

where

$$\kappa_{s\underline{\rho}} \equiv -\frac{\kappa}{\omega_s\epsilon}\left(\frac{\omega_{s\theta}+(\gamma-\xi)\omega_{\theta\theta}}{\omega_{\theta\theta}}\right)\frac{\varrho_\Psi}{\sigma}\frac{\psi_0}{\rho+\rho_\psi}\left[\frac{(\rho+\rho_\psi)^2}{(\overline{\rho}+\rho_\psi)(\underline{\rho}+\rho_\psi)} - \frac{1-\frac{\underline{\rho}}{\overline{\rho}}\frac{\kappa((\gamma-\psi)\omega_{\theta\theta}+\omega_{\theta s})^2}{\kappa((\gamma-\psi)\omega_{\theta\theta}+\omega_{\theta s})^2+\epsilon\omega_s\omega_{\theta\theta}}\frac{\rho+\rho_\psi}{\overline{\rho}+\rho_\psi}}{1-\frac{\underline{\rho}}{\overline{\rho}}\frac{\kappa((\gamma-\psi)\omega_{\theta\theta}+\omega_{\theta s})^2}{\kappa((\gamma-\psi)\omega_{\theta\theta}+\omega_{\theta s})^2+\epsilon\omega_s\omega_{\theta\theta}}}\right]$$

$$\kappa_{s\rho_\psi} \equiv \frac{\kappa}{\epsilon}\left(\frac{\omega_{s\theta}+(\gamma-\xi)\omega_{\theta\theta}}{\omega_{\theta\theta}}\right)\frac{\varrho_\Psi}{\sigma}\frac{\psi_0}{\rho+\rho_\psi}\frac{(\rho+\rho_\psi)}{(\overline{\rho}+\rho_\psi)(\underline{\rho}+\rho_\psi)}\frac{1}{\rho_\psi}$$

where $\kappa_{s\underline{\rho}}$ is negative for $\rho_\psi+\underline{\rho}>0$, positive if $\rho_\psi+\underline{\rho}<0$, and $\kappa_{s\rho_\psi}$ has the opposite sign of $\kappa_{s\underline{\rho}}$.

C.3 Sticky prices: passive policy

The equilibrium conditions under passive policy are given by

$$\begin{aligned}
c_t &= \theta_t + (\xi-\alpha)s_t \\
y_t &= \varsigma_c\theta_t + (\varsigma_c\xi+\varsigma_x\gamma)s_t \\
0 &= \int_0^\infty e^{-\rho t}\left[(\gamma-\xi)s_t-\theta_t\right]dt \\
\dot{s}_t &= -\pi_{H,t} \\
\dot{\pi}_{H,t} &= \rho\pi_{H,t} - \kappa\left[(\varphi(\phi+1)-1)y_t + \sigma\theta_t + s_t\right] \\
\dot{\theta}_t &= \frac{\psi_t}{\sigma}
\end{aligned}$$

Given the path of θ_t, we can obtain a system of differential equations in $(s_t, \dot{s}_t)$:

$$\begin{bmatrix} \ddot{s}_t \\ \dot{s}_t \end{bmatrix} = \begin{bmatrix} \rho & \tilde{\omega}_s \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{s}_t \\ s_t \end{bmatrix} + \begin{bmatrix} \tilde{u}_t \\ 0 \end{bmatrix} \tag{172}$$

where

$$\begin{aligned} \tilde{\omega}_s &= \kappa \left[(\varphi(\phi+1)-1)(1-\varsigma_g)((1-\alpha)\xi + \alpha\gamma) + 1 \right] \\ \tilde{u}_t &= \kappa\sigma \left[(\varphi(\phi+1)-1)(1-\varsigma_g)(\xi-\alpha) + 1 \right] \theta_t \end{aligned}$$

Define the eigenvalues

$$\underline{\rho}_P \equiv \frac{\rho - \sqrt{\rho^2 + 4\tilde{\omega}_s}}{2}; \qquad \overline{\rho}_P \equiv \frac{\rho + \sqrt{\rho^2 + 4\tilde{\omega}_s}}{2} \tag{173}$$

The solution is analogous to the one under optimal policy, then

$$s_t = \frac{1}{\overline{\rho}_P - \underline{\rho}_P} \left[e^{\underline{\rho}_P t} \int_0^\infty e^{-\overline{\rho}_P s} \tilde{u}_s ds - \int_0^t e^{\underline{\rho}_P (t-s)} \tilde{u}_s ds - \int_t^\infty e^{-\overline{\rho}_P (s-t)} \tilde{u}_s ds \right] \tag{174}$$

The present discounted value of s_t is then given by

$$\int_0^\infty e^{-\rho t} s_t dt = \frac{1}{\tilde{\omega}_s} \int_0^\infty \left(e^{-\overline{\rho}_P t} - e^{-\rho t} \right) \tilde{u}_t dt \tag{175}$$

From the external solvency constraint, we have

$$\int_0^\infty e^{-\rho t} (\gamma - \xi) s_t dt = \int_0^\infty e^{-\rho t} \theta_t dt \tag{176}$$

Combining the previous two expressions and rearranging

$$\int_0^\infty e^{-\rho t} \theta_t dt = \frac{\sigma \frac{(\gamma-\xi)\left[(\varphi(\phi+1)-1)(1-\varsigma_g)(\xi-\alpha)+1\right]}{(\varphi(\phi+1)-1)(1-\varsigma_g)((1-\alpha)\xi+\alpha\gamma)+1}}{1 + \sigma \frac{(\gamma-\xi)\left[(\varphi(\phi+1)-1)(1-\varsigma_g)(\xi-\alpha)+1\right]}{(\varphi(\phi+1)-1)(1-\varsigma_g)((1-\alpha)\xi+\alpha\gamma)+1}} \int_0^\infty e^{-\overline{\rho}_P t} \theta_t dt \tag{177}$$

Solving the integrals

$$\frac{1}{\rho} \left[\theta_0 + \frac{1}{\sigma} \frac{\psi_0}{\rho + \rho_\psi} \right] = \frac{\sigma \frac{(\gamma-\xi)\left[(\varphi(\phi+1)-1)(1-\varsigma_g)(\xi-\alpha)+1\right]}{(\varphi(\phi+1)-1)(1-\varsigma_g)((1-\alpha)\xi+\alpha\gamma)+1}}{1 + \sigma \frac{(\gamma-\xi)\left[(\varphi(\phi+1)-1)(1-\varsigma_g)(\xi-\alpha)+1\right]}{(\varphi(\phi+1)-1)(1-\varsigma_g)((1-\alpha)\xi+\alpha\gamma)+1}} \frac{1}{\overline{\rho}_P} \left[\theta_0 + \frac{1}{\sigma} \frac{\psi_0}{\overline{\rho}_P + \rho_\psi} \right] \tag{178}$$

The value of θ_0 is then given by

$$\theta_0 = -\frac{1}{\sigma} \frac{1 + \sigma \frac{(\gamma-\xi)\left[(\varphi(\phi+1)-1)(1-\varsigma_g)(\xi-\alpha)+1\right]}{(\varphi(\phi+1)-1)(1-\varsigma_g)((1-\alpha)\xi+\alpha\gamma)+1} \left(1 - \frac{\rho}{\overline{\rho}_P} \frac{\rho+\rho_\psi}{\overline{\rho}_P+\rho_\psi}\right)}{1 + \sigma \frac{(\gamma-\xi)\left[(\varphi(\phi+1)-1)(1-\varsigma_g)(\xi-\alpha)+1\right]}{(\varphi(\phi+1)-1)(1-\varsigma_g)((1-\alpha)\xi+\alpha\gamma)+1} \left(1 - \frac{\rho}{\overline{\rho}_P}\right)} \frac{\psi_0}{\rho + \rho_\psi} \tag{179}$$

Notice that the present discounted value of θ_t is given by

$$\int_0^\infty e^{-\rho t}\theta_t dt = -\frac{1}{\rho}\frac{\frac{(\gamma-\xi)\left[(\varphi(\phi+1)-1)(1-\varsigma_g)(\xi-\alpha)+1\right]}{(\varphi(\phi+1)-1)(1-\varsigma_g)((1-\alpha)\xi+\alpha\gamma)+1}\frac{\rho}{\bar{\rho}_P}\left(1-\frac{\rho+\rho_\psi}{\bar{\rho}_P+\rho_\psi}\right)}{1+\sigma\frac{(\gamma-\xi)\left[(\varphi(\phi+1)-1)(1-\varsigma_g)(\xi-\alpha)+1\right]}{(\varphi(\phi+1)-1)(1-\varsigma_g)((1-\alpha)\xi+\alpha\gamma)+1}\left(1-\frac{\rho}{\bar{\rho}_P}\right)}\frac{\psi_0}{\rho+\rho_\psi} \tag{180}$$

The terms of trade can then be written

$$s_t = \kappa\sigma\left[(\varphi(\phi+1)-1)(1-\varsigma_g)(\xi-\alpha)+1\right]\left[e^{\underline{\rho}_P t}\int_0^\infty e^{-\bar{\rho}_P s}\frac{\hat{\Upsilon}_s}{\bar{\rho}_P-\underline{\rho}_P}ds - \int_0^t e^{\underline{\rho}_P(t-s)}\frac{\hat{\Upsilon}_s}{\bar{\rho}_P-\underline{\rho}_P}ds - \int_t^\infty e^{-\bar{\rho}_P(s-t)}\frac{\hat{\Upsilon}_s}{\bar{\rho}_P-\underline{\rho}_P}ds - \frac{1-e^{\underline{\rho}_P t}}{\tilde{\omega}_s}\theta_0\right] \tag{181}$$

Assuming exponentially decaying risk premium shocks, we obtain

$$s_t = \tilde{\kappa}_{s\underline{\rho}}\left(1-e^{\underline{\rho}_P t}\right) + \tilde{\kappa}_{s\rho_\psi}\left(1-e^{-\rho_\psi t}\right) \tag{182}$$

where

$$\tilde{\kappa}_{s\underline{\rho}} \equiv -\frac{\kappa}{\tilde{\omega}_s}\left[(\varphi(\phi+1)-1)(1-\varsigma_g)(\xi-\alpha)+1\right]\frac{\psi_0}{\rho+\rho_\psi}\left[\frac{(\rho+\rho_\psi)^2}{(\bar{\rho}_P+\rho_\psi)(\underline{\rho}_P+\rho_\psi)} - \frac{1+\sigma\frac{(\gamma-\xi)\left[(\varphi(\phi+1)-1)(1-\varsigma_g)(\xi-\alpha)+1\right]}{(\varphi(\phi+1)-1)(1-\varsigma_g)((1-\alpha)\xi+\alpha\gamma)+1}\left(1-\frac{\rho}{\bar{\rho}_P}\frac{\rho+\rho_\psi}{\bar{\rho}_P+\rho_\psi}\right)}{1+\sigma\frac{(\gamma-\xi)\left[(\varphi(\phi+1)-1)(1-\varsigma_g)(\xi-\alpha)+1\right]}{(\varphi(\phi+1)-1)(1-\varsigma_g)((1-\alpha)\xi+\alpha\gamma)+1}\left(1-\frac{\rho}{\bar{\rho}_P}\right)}\right]$$

$$\tilde{\kappa}_{s\rho_\psi} \equiv \kappa\left[(\varphi(\phi+1)-1)(1-\varsigma_g)(\xi-\alpha)+1\right]\frac{\psi_0}{\rho+\rho_\psi}\frac{(\rho+\rho_\psi)}{(\bar{\rho}+\rho_\psi)(\underline{\rho}+\rho_\psi)}\frac{1}{\rho_\psi}$$

$$s_t = \frac{\kappa}{\epsilon}\left(\frac{\omega_{s\theta}+(\gamma-\xi)\omega_{\theta\theta}}{\omega_{\theta\theta}}\right)\left[\frac{\varrho_\Psi\psi_0 e^{\underline{\rho}t}}{\sigma\bar{\rho}(\bar{\rho}+\rho_\psi)(\bar{\rho}-\underline{\rho})} + \varrho_\Psi\psi_0\frac{\underline{\rho}+\rho_\psi-\rho_\psi e^{\underline{\rho}t}-\underline{\rho}e^{-\rho_\psi t}}{\sigma\rho_\psi\underline{\rho}(\rho_\psi+\underline{\rho})(\bar{\rho}-\underline{\rho})} - \varrho_\Psi\psi_0\frac{\bar{\rho}+\rho_\psi-\bar{\rho}e^{-\rho_\psi t}}{\sigma\rho_\psi\bar{\rho}(\bar{\rho}+\rho_\psi)(\bar{\rho}-\underline{\rho})} + \frac{1-e^{\underline{\rho}t}}{\omega_s}\Gamma\right] \tag{183}$$

Given the path of θ_t and s_t, we can solve for the remaining variables:

$$\begin{aligned} c_t &= \theta + (\xi-\alpha)s_t \\ y_t &= \varsigma_c\theta_t + (\varsigma_c\xi + \varsigma_x\gamma)s_t \\ nx_t &= \varsigma_x((\gamma-\xi)s_t - \theta_t) \end{aligned}$$

C.4 Proof of proposition 9

i. **Effect on impact and dynamics**: The value of θ_t is given by the equation:

$$\theta_t = -\frac{\omega_{\theta s}}{\omega_{\theta\theta}}s_t + \frac{1}{\omega_{\theta\theta}}\left[\frac{\varrho_\Psi}{\sigma}\int_0^t \psi_s ds - \Gamma\right] \tag{184}$$

Evaluating the expression at $t=0$, we obtain

$$\theta_0 = -\frac{\Gamma}{\omega_{\theta\theta}} < 0 \tag{185}$$

or more explicitly

$$\theta_0 = -\frac{1}{\sigma}\frac{\psi_0}{\rho+\rho_\psi}\frac{\alpha+(1-\alpha)\frac{\alpha}{\gamma}}{1+\frac{\gamma-\alpha}{\gamma}(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}\frac{1-\frac{\rho}{\bar{\rho}}\frac{\kappa((\gamma-\xi)\omega_{\theta\theta}+\omega_{\theta s})^2}{\kappa((\gamma-\xi)\omega_{\theta\theta}+\omega_{\theta s})^2+\epsilon\omega_s\omega_{\theta\theta}}\frac{\rho+\rho_\psi}{\bar{\rho}+\rho_\psi}}{1-\frac{\rho}{\bar{\rho}}\frac{\kappa((\gamma-\xi)\omega_{\theta\theta}+\omega_{\theta s})^2}{\kappa((\gamma-\xi)\omega_{\theta\theta}+\omega_{\theta s})^2+\epsilon\omega_s\omega_{\theta\theta}}} \tag{186}$$

Hence, we can write $\theta_t = \theta_0 - \kappa_{\theta,s}s_t + \kappa_{\theta,\Psi}\hat{\Psi}_t$, where

$$\kappa_{\theta,s} \equiv \frac{\omega_{\theta s}}{\omega_{\theta\theta}} = (\xi-\alpha)\frac{\gamma-\alpha}{\gamma}\frac{1+((1-\alpha)\xi+\alpha\gamma)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}{1+\frac{\gamma-\alpha}{\gamma}(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} > 0$$

$$\kappa_{\theta,\Psi} \equiv \frac{\varrho_\Psi}{\omega_{\theta\theta}} = \frac{1}{\gamma}\frac{\alpha(\gamma+1-\alpha)}{1+\frac{\gamma-\alpha}{\gamma}(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} > 0$$

Output is given by

$$y_t = \frac{\varsigma_c\theta_t + (\varsigma_c\xi+\varsigma_x\gamma)s_t}{1+(\varphi(1+\phi)-1)\,\varsigma_g} \tag{187}$$

Evaluating at $t=0$, we obtain

$$y_0 = \frac{\varsigma_c\theta_0}{1+(\varphi(1+\phi)-1)} < 0 \tag{188}$$

Output can then be written as $y_t = y_0 + \kappa_{y,s}s_t + \kappa_{s,\Psi}\hat{\Psi}_t$, where

$$\kappa_{y,s} = (1-\varsigma_g)\frac{-(1-\alpha)\frac{\omega_{\theta s}}{\omega_{\theta\theta}} + ((1-\alpha)\xi+\alpha\gamma)}{1+(\varphi(1+\phi)-1)\,\varsigma_g} \tag{189}$$

$$\kappa_{y,\Psi} = \frac{(1-\varsigma_g)(1-\alpha)\frac{\varrho_\Psi}{\omega_{\theta\theta}}}{1+(\varphi(1+\phi)-1)\varsigma_g} \tag{190}$$

The coefficient $\kappa_{y,\Psi}$ is clearly positive. We can write $\kappa_{y,s}$ as follows

$$\kappa_{y,s} = \frac{1-\varsigma_g}{1+(\varphi(1+\phi)-1)\varsigma_g}\frac{(1-\alpha)\xi+\alpha\gamma-(1-\alpha)(\xi-\alpha)\frac{\gamma-\alpha}{\gamma}}{1+\frac{\gamma-\alpha}{\gamma}(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} > 0 \tag{191}$$

Net exports are given by

$$nx_t = \varsigma_x\left[(\gamma-\xi)s_t - \theta_t\right] \tag{192}$$

where $nx_0 = -\varsigma_x\theta_0 > 0$.

Net exports can be written as $nx_t = nx_0 + \kappa_{nx,s}s_t - \kappa_{nx,\Psi}\hat{\Psi}_t$, where

$$\kappa_{nx,s} = \varsigma_x \left[\gamma - \xi + (\xi - \alpha)\frac{\gamma - \alpha}{\gamma}\frac{1 + ((1-\alpha)\xi + \alpha\gamma)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}{1 + \frac{\gamma-\alpha}{\gamma}(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}\right] > 0 \tag{193}$$

$$\kappa_{nx,\Psi} = \frac{1}{\gamma}\frac{\alpha(\gamma + 1 - \alpha)}{1 + \frac{\gamma-\alpha}{\gamma}(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} > 0 \tag{194}$$

$$\varrho_\Psi \equiv \frac{\gamma + 1 - \alpha}{\gamma - \alpha}\frac{\alpha\varsigma_c}{\xi - \alpha} \tag{195}$$

Consumption is given by

$$c_t = \theta_t + (\xi - \alpha)s_t \tag{196}$$

where $c_0 = \theta_0 < 0$.

Consumption can be written as $c_t = c_0 + \kappa_{c,s}s_t + \kappa_{c,\Psi}\hat{\Psi}_t$, where $\kappa_{c,\Psi} = \kappa_{\theta,\Psi}$ and

$$\kappa_{c,s} = (\xi - \alpha)\left[1 - \frac{\gamma - \alpha}{\gamma}\frac{1 + ((1-\alpha)\xi + \alpha\gamma)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}{1 + \frac{\gamma-\alpha}{\gamma}(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}\right] \tag{197}$$

If $\gamma = \alpha$, then $\kappa_{c,s} > 0$ and $\lim_{\gamma\to\infty}\kappa_{c,s} < 0$. Hence, the sign of $\kappa_{c,s}$ is ambiguous.

The labor wedge is given by $\hat{\omega}_t^L = -(\sigma\theta_t + s_t + (\varphi(1+\phi)-1)y_t)$. Given that $\theta_0 < 0$, $s_0 = 0$, and $y_0 < 0$, then $\hat{\omega}_0^L > 0$.

ii. **Average consumption:** The present discounted value of θ_t is given by

$$\begin{aligned}
\int_0^\infty e^{-\rho t}\theta_t dt &= (\gamma - \xi)\int_0^\infty e^{-\rho t}s_t dt \\
&= \frac{(\gamma-\xi)\omega_{\theta\theta}}{(\gamma-\xi)\omega_{\theta\theta} + \omega_{\theta s}}\int_0^\infty e^{-\rho t}\left(\frac{\varrho_\Psi}{\omega_{\theta\theta}}\hat{\Psi}_t - \frac{\Gamma}{\omega_{\theta\theta}}\right)dt \\
&= \frac{(\gamma-\xi)\omega_{\theta\theta}}{(\gamma-\xi)\omega_{\theta\theta} + \omega_{\theta s}}\frac{\varrho_\Psi}{\omega_{\theta\theta}}\frac{\psi_0}{\rho + \rho_\psi}\frac{1}{\sigma\rho}\left[1 - \frac{1 - \frac{\rho}{\bar{\rho}}\frac{\kappa((\gamma-\xi)\omega_{\theta\theta}+\omega_{\theta s})^2}{\kappa((\gamma-\xi)\omega_{\theta\theta}+\omega_{\theta s})^2 + \epsilon\omega_s\omega_{\theta\theta}}\frac{\rho+\rho_\psi}{\bar{\rho}+\rho_\psi}}{1 - \frac{\rho}{\bar{\rho}}\frac{\kappa((\gamma-\xi)\omega_{\theta\theta}+\omega_{\theta s})^2}{\kappa((\gamma-\xi)\omega_{\theta\theta}+\omega_{\theta s})^2 + \epsilon\omega_s\omega_{\theta\theta}}}\right] < 0
\end{aligned}$$

Since the present discounted value of θ_t and s_t fall with the shock, then the present discounted value of c_t and y_t also fall, since $c_t = \theta + (\xi - \alpha)s_t$ and $y_t = \frac{\varsigma_c\theta_t + (\varsigma_c\psi + \varsigma_x\gamma)s_t}{1+(\varphi(1+\phi)-1)\varsigma_g}$. Similarly, the average value of the labor wedge is positive. Moreover, notice that the expression above converges to zero if $\kappa \to 0$ and $\kappa \to \infty$.

iii. **Terms of trade**: The derivative of the terms of trade at zero is given by

$$\begin{aligned}\dot{s}_0 &= -\frac{\kappa}{\epsilon}\left(\frac{\omega_{s\theta}}{\omega_{\theta\theta}}+(\gamma-\xi)\right)\int_0^\infty e^{-\bar{\rho}t}\left(\varrho_\Psi\hat{\Psi}_t-\Gamma\right)dt \\ &= \frac{\kappa}{\epsilon}\left(\frac{\omega_{s\theta}}{\omega_{\theta\theta}}+(\gamma-\xi)\right)\frac{1}{\bar{\rho}}\left[\Gamma-\frac{\varrho_\Psi}{\sigma}\frac{\psi_0}{\bar{\rho}+\rho_\psi}\right] \\ &= \frac{\kappa}{\epsilon}\left(\frac{\omega_{s\theta}}{\omega_{\theta\theta}}+(\gamma-\xi)\right)\frac{1}{\bar{\rho}}\frac{\varrho_\Psi}{\sigma}\frac{\frac{\psi_0}{\rho+\rho_\psi}-\frac{\psi_0}{\bar{\rho}+\rho_\psi}}{1-\frac{\rho}{\bar{\rho}}\frac{\kappa((\gamma-\psi)\omega_{\theta\theta}+\omega_{\theta s})^2}{\kappa((\gamma-\psi)\omega_{\theta\theta}+\omega_{\theta s})^2+\epsilon\omega_s\omega_{\theta\theta}}}>0\end{aligned} \tag{198}$$

Hence, inflation is negative on impact. Taking the limit as $t\to\infty$ of s_t

$$\begin{aligned}\lim_{t\to\infty}s_t &= \frac{\kappa}{\epsilon}\left(\frac{\omega_{s\theta}+(\gamma-\xi)\omega_{\theta\theta}}{\omega_{\theta\theta}}\right)\left[-\frac{\varrho_\Psi\psi_0}{\sigma\rho_\psi\omega_s}+\frac{1}{\omega_s}\Gamma\right] \\ &= -\frac{\kappa}{\epsilon}\left(\frac{\omega_{s\theta}+(\gamma-\xi)\omega_{\theta\theta}}{\omega_{\theta\theta}}\right)\frac{\varrho_\Psi\psi_0}{\sigma\rho_\psi\omega_s}\left[\frac{\frac{\rho}{\rho+\rho_\psi}-\frac{\rho}{\rho+\rho_\psi}\frac{\kappa((\gamma-\psi)\omega_{\theta\theta}+\omega_{\theta s})^2}{\kappa((\gamma-\psi)\omega_{\theta\theta}+\omega_{\theta s})^2+\epsilon\omega_s\omega_{\theta\theta}}\frac{\rho+\rho_\psi}{\bar{\rho}+\rho_\psi}}{1-\frac{\rho}{\bar{\rho}}\frac{\kappa((\gamma-\psi)\omega_{\theta\theta}+\omega_{\theta s})^2}{\kappa((\gamma-\psi)\omega_{\theta\theta}+\omega_{\theta s})^2+\epsilon\omega_s\omega_{\theta\theta}}}\right]<0\end{aligned}$$

Hence, the terms of trade starts at zero, it is initially increasing, and it is eventually negative.

C.5 Proof of proposition 10

i. **Consumption taxes**: The time derivative of θ_t is given by

$$\dot{\theta}_t = -\frac{\omega_{\theta s}}{\omega_{\theta\theta}}\dot{s}_t+\frac{\varrho_\Psi}{\omega_{\theta\theta}}\frac{\psi_t}{\sigma} \tag{199}$$

Consumption taxes are given by

$$\dot{\hat{\tau}}^c_t = \psi_t-\sigma\dot{\theta}_t = \sigma\frac{\omega_{\theta s}}{\omega_{\theta\theta}}\dot{s}_t+\left(1-\frac{\varrho_\Psi}{\omega_{\theta\theta}}\right)\psi_t \tag{200}$$

Using the fact that $\dot{s}_t=-\pi_{H,t}$, we can write

$$\dot{\hat{\tau}}^c_t = -\kappa_{\tau^c,\pi}\pi_{H,t}+\kappa_{\tau^c,\psi}\psi_t \tag{201}$$

where

$$\begin{aligned}\kappa_{\tau^c,\pi} &\equiv \frac{(1-\alpha)+(1-\alpha)((1-\alpha)\xi+\alpha\gamma)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}{\frac{\gamma}{\gamma-\alpha}+(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}} \\ \kappa_{\tau^c,\psi} &\equiv 1-\frac{\alpha+(1-\alpha)\frac{\alpha}{\gamma}}{1+\frac{\gamma-\alpha}{\gamma}(1-\alpha)(\xi-\alpha)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}}\end{aligned}$$

and $\kappa_{\tau^c,\psi}\in(0,1)$.

ii. **Sum of taxes**: The differential equation for s_t can be expressed in terms of $\pi_{H,t}$:

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - [\omega_s s_t + u_t] \tag{202}$$

The New Keynesian Philipps curve is given by

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \kappa\left[(\varphi(\phi+1)-1)y_t + \sigma c_t + \alpha s_t + \hat{\tau}_t^v + \hat{\tau}_t^c\right] \tag{203}$$

more compactly

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \left[\tilde{\omega}_s s_t + \tilde{\omega}_u u_t + \kappa\left(\hat{\tau}_t^v + \hat{\tau}_t^c\right)\right] \tag{204}$$

where

$$\begin{aligned}
\tilde{\omega}_s &= \kappa\left[\frac{(\varphi(1+\phi)-1)(\varsigma_c\xi+\varsigma_x\gamma)}{1+(\varphi(1+\phi)-1)\varsigma_g} + 1 - \frac{\omega_{\theta s}}{\omega_{\theta\theta}}\left[\frac{(\varphi(1+\phi)-1)\varsigma_c}{1+(\varphi(1+\phi)-1)\varsigma_g}+\sigma\right]\right] \\
\tilde{\omega}_u &= \left[\frac{(\varphi(1+\phi)-1)\varsigma_c}{1+(\varphi(1+\phi)-1)\varsigma_g}+\sigma\right]\frac{\epsilon}{\omega_{s\theta}+(\gamma-\xi)\omega_{\theta\theta}}
\end{aligned}$$

Combining the expression above with the dynamics for s_t, we get

$$\hat{\tau}_t^v + \hat{\tau}_t^c = \frac{1}{\kappa}\left[\left(\omega_s - \tilde{\omega}_s\right)s_t + \left(1-\tilde{\omega}_u\right)u_t\right] \tag{205}$$

Hence, the initial level of taxes is given by

$$\begin{aligned}
\hat{\tau}_0^v + \hat{\tau}_0^c &= -\left[\frac{\omega_{s\theta}+(\gamma-\xi)\omega_{\theta\theta}}{\epsilon} - \left(\frac{(\varphi(1+\phi)-1)\varsigma_c}{1+(\varphi(1+\phi)-1)\varsigma_g}+\sigma\right)\right]\frac{\Gamma}{\omega_{\theta\theta}} \\
&= \sigma(1-\varsigma_g)\left[\left(\frac{\gamma(\gamma-\xi)}{\gamma-\alpha}+\xi-\alpha-\frac{\epsilon}{1-\varsigma_g}\right)+(\xi-\alpha)\left(\gamma-\frac{\epsilon}{1-\varsigma_g}\right)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]\frac{\theta_0}{\epsilon}
\end{aligned}$$

Taxes can then be written as

$$\hat{\tau}_t^v + \hat{\tau}_t^c = \hat{\tau}_0^v + \hat{\tau}_0^c + \kappa_{\tau,s}s_t + \kappa_{\tau,\Psi}\hat{\Psi}_t \tag{206}$$

where

$$\begin{aligned}
\kappa_{\tau,\Psi} &= -\frac{\sigma}{\epsilon}\left[\epsilon-\gamma(1-\varsigma_g)+\varsigma_x\frac{\xi-\alpha}{\gamma-\alpha}+(\xi-\alpha)\left(\epsilon-\gamma(1-\varsigma_g)\right)\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right] \\
\kappa_{\tau,s} &= \frac{\omega_s-\tilde{\omega}_s}{\kappa}
\end{aligned}$$

and $\kappa_{\tau,\Psi} < 0$ if $\gamma < \frac{\epsilon}{1-\varsigma_g}$, $\kappa_{\tau,\Psi} > 0$ if $\gamma >> \frac{\epsilon}{1-\varsigma_g}$.

iii. **No home-bias limit**: Taking the limit $\alpha \to 1$, we obtain

$$\dot{\tau}_t^c = 0 \tag{207}$$

The coefficient $\kappa_{\tau,\Psi}$ is given by

$$\kappa_{\tau,\Psi} = -\frac{\sigma}{\epsilon}\left(\epsilon - \gamma(1-\varsigma_g)\right) \tag{208}$$

The following reduced-form coefficients are given by

$$\omega_s = \frac{\kappa}{\epsilon}(1-\varsigma_g)\gamma\left[1+\gamma\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]$$
$$\tilde{\omega}_s = \kappa\left[1+\gamma\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right]$$

The coefficients $\kappa_{\tau,s}$ is then given by

$$\kappa_{\tau,s} = \frac{1}{\epsilon}\left(\epsilon - \gamma(1-\varsigma_g)\right)\left[1+\gamma\frac{(\varphi(1+\phi)-1)(1-\varsigma_g)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right] \tag{209}$$

C.6 Government debt derivations

i. **Fiscal needs of the government**: The government solvency constraint is given by

$$0 = \int_0^\infty e^{-\rho t}\left[\left(-\sigma(c_t - c_0) - \alpha s_t - \int_0^t \dot{\hat{\tau}}^c_s ds\right)\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} + (1-\overline{\tau}^v)\hat{\tau}^v_t + \overline{\tau}^v y_t + (1-\varsigma_g)\hat{\tau}^c_t - \varsigma_g g_t \right.$$
$$\left. + \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\left(\hat{\tau}^c_t + \sigma c_t + \alpha s_t + \varphi(1+\phi)y_t\right) + \hat{T}\right]dt$$

Under the optimal policy, the expression above can be written as

$$\frac{\psi_0}{\rho+\rho_\psi}\frac{\overline{D}}{\overline{P}_H\overline{Y}} = \int_0^\infty e^{-\rho t}\left[-s_t\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} + \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\left(\sigma\theta_0 + s_t + \sigma\hat{\Psi}_t\right) + (1-\overline{\tau}^v)(\hat{\tau}^v_t + \hat{\tau}^c_t) + \left[\overline{\tau}^v - \varsigma_g\right]\hat{\tau}^c_t \right.$$
$$\left. + \left[\overline{\tau}^v + (\varphi(1+\phi)-1)\varsigma_g + \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\varphi(1+\phi)\right]y_t + \hat{T}\right]dt$$

using $-\sigma(c_t - c_0) - \alpha s_t - \int_0^t \dot{\hat{\tau}}^c_s ds = -s_t - \sigma\hat{\Psi}_t$.

Using the expression for y_t and the fact that $\int_0^\infty e^{-\rho t}\theta_t dt = (\gamma - \xi)\int_0^\infty e^{-\rho t}s_t dt$, we obtain

$$\hat{T} = \frac{\psi_0}{\rho+\rho_\psi}\left[\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} - \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l - (1-\overline{\tau}^v)\frac{\kappa_{\tau,\Psi}}{\sigma} - \left(\overline{\tau}^v - \varsigma_g\right)\frac{\kappa_{\tau^c,\Psi}}{\sigma}\right] - \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\sigma\theta_0 - (1-\overline{\tau}^v)(\hat{\tau}^v_0 + \hat{\tau}^c_0) - \rho\kappa_{T,s}\int_0^\infty e^{-\rho t}s_t dt$$

where

$$\kappa_{T,s} = -\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} + \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l + (1-\overline{\tau}^v)\kappa_{\tau,s} + \left[\overline{\tau}^v - \varsigma_g\right]\sigma\frac{\omega_{\theta s}}{\omega_{\theta\theta}} + (1-\varsigma_g)\gamma\left(\frac{\overline{\tau}^v + (\varphi(1+\phi)-1)\varsigma_g + \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\varphi(1+\phi)}{1+(\varphi(1+\phi)-1)\varsigma_g}\right)$$

Similarly, for the case of passive policy we have

$$\hat{T} = \frac{\psi_0}{\rho+\rho_\psi}\left[\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} - \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\right] - \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\sigma\theta_0 - \rho\kappa^P_{T,s}\int_0^\infty e^{-\rho t}s_t dt$$

where

$$\kappa_{T,s}^{P} = -\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} + \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l + (1-\varsigma_g)\gamma\left(\overline{\tau}^v + \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\varphi(1+\phi)\right)$$

ii. **Debt dynamics**: Debt at period t can satisfy the condition

$$\begin{aligned}\left[-\sigma\theta_{t_0} + \hat{D}^g_{t_0}\right]\frac{\overline{D}}{\overline{P}_H\overline{Y}} = \int_{t_0}^{\infty} e^{-\rho(t-t_0)}\Bigg[&\left(-\sigma\theta_t - s_t - (\hat{\tau}^c_t - \hat{\tau}^c_{t_0})\right)\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} + (1-\overline{\tau}^v)(\hat{\tau}^v_t + \tau^c_t) + (\overline{\tau}^v - \varsigma_g)\hat{\tau}^c_t \\ &+\overline{\tau}^v y_t + \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\left(\hat{\tau}^c_t + \sigma\theta_t + s_t + \varphi(1+\phi)y_t\right) - \varsigma_g g_t + \hat{T}\Bigg]dt\end{aligned}$$

Using $\sigma\theta_t + \hat{\tau}^c_t = \sigma\theta_0 + \sigma\hat{\Psi}_t$, we obtain

$$\begin{aligned}\left[-\sigma(\theta_{t_0} - \theta_0) - \hat{\tau}^c_{t_0} + \hat{D}^g_{t_0}\right]\frac{\overline{D}}{\overline{P}_H\overline{Y}} = \int_{t_0}^{\infty} e^{-\rho(t-t_0)}\Bigg[&\left(-\sigma\hat{\Psi}_t - s_t\right)\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} + (1-\overline{\tau}^v)(\hat{\tau}^v_t + \tau^c_t) + (\overline{\tau}^v - \varsigma_g)\hat{\tau}^c_t \\ &+\overline{\tau}^v y_t + \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\left(\sigma\theta_0 + \sigma\hat{\Psi}_t + s_t + \varphi(1+\phi)y_t\right) - \varsigma_g g_t + \hat{T}\Bigg]dt\end{aligned}$$

The expression above can be written as

$$\left[-\sigma(\theta_{t_0} - \theta_0) - \hat{\tau}^c_{t_0} + \hat{D}^g_{t_0}\right]\frac{\overline{D}}{\overline{P}_H\overline{Y}} = \kappa_{D,0} + \int_{t_0}^{\infty} e^{-\rho(t-t_0)}\left[\kappa_{D,s}s_t + \kappa_{D,\Psi}\hat{\Psi}_t\right]dt$$

where

$$\begin{aligned}\kappa_{D,\theta} &= (1-\alpha)(1-\varsigma_g)\frac{\overline{\tau}^v + \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\varphi(1+\phi) + (\varphi(1+\phi)-1)\varsigma_g}{1+(\varphi(1+\phi)-1)\varsigma_g} \\ \kappa_{D,s} &= -\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} + ((1-\alpha)\xi + \alpha\gamma)(1-\varsigma_g)\frac{\overline{\tau}^v + \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\varphi(1+\phi) + (\varphi(1+\phi)-1)\varsigma_g}{1+(\varphi(1+\phi)-1)\varsigma_g} + \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l \\ &\quad + (1-\overline{\tau}^v)\kappa_{\tau,s} + \left(\overline{\tau}^v - \varsigma_g\right)\kappa_{\tau^c,s} + \kappa_{D,\theta}\kappa_{\theta,s} \\ \kappa_{D,0} &= -\left(\frac{\overline{D}}{\overline{P}_H\overline{Y}} - \frac{1}{\rho}\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\right)\frac{\psi_0}{\rho+\rho_\psi} + \frac{\sigma\theta_0}{\rho}\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l + \frac{(1-\overline{\tau}^v)(\hat{\tau}^v_0 + \tau^c_0) + \hat{T}}{\rho} + \frac{\kappa_{D,\theta}}{\rho}\theta_0 \\ \kappa_{D,\Psi} &= (1-\overline{\tau}^v)\kappa_{\tau,\Psi} + \left(\overline{\tau}^v - \varsigma_g\right)\kappa_{\tau^c,\Psi} + \kappa_{D,\theta}\kappa_{\theta,\Psi}\end{aligned}$$

Solving the integrals, we obtain

$$\left[-\sigma(\theta_{t_0} - \theta_0) - \hat{\tau}^c_{t_0} + \hat{D}^g_{t_0}\right]\frac{\overline{D}}{\overline{P}_H\overline{Y}} = \kappa_{D,0} + \frac{\kappa_{D,s}s_t + \kappa_{D,\Psi}\hat{\Psi}_t}{\rho} - \kappa_{D,s}\kappa_{s,\underline{\rho}}\frac{\rho}{\overline{\underline{\rho}}}\frac{e^{\underline{\rho}t}}{\rho} + \left(\kappa_{D,s}\kappa_{s,\rho_\psi}\frac{\rho_\psi}{\rho} + \kappa_{D,\Psi}\frac{\psi_0}{\rho\sigma}\right)\frac{e^{-\rho_\psi t}}{\rho+\rho_\psi}$$

rearranging

$$\hat{D}^g_t = \tilde{\kappa}_{D,0} + \tilde{\kappa}_{D,\underline{\rho}}e^{\underline{\rho}t} + \tilde{\kappa}_{D,\psi}e^{-\rho_\psi t} \tag{210}$$

where

$$\tilde{\kappa}_{D,0} = \left(\frac{\overline{D}}{\overline{P}_H\overline{Y}}\right)^{-1}\left[\left((\sigma\kappa_{\theta,s}+\kappa_{\tau^c,s})\frac{\overline{D}}{\overline{P}_H\overline{Y}}+\frac{\kappa_{D,s}}{\rho}\right)\left(\kappa_{s,\underline{\rho}}+\kappa_{s,\rho_\psi}\right)+\left(\frac{\kappa_{\theta,\Psi}}{\rho_\psi}+\frac{\kappa_{\tau^c,\Psi}}{\rho_\psi\sigma}\right)\frac{\psi_0\overline{D}}{\overline{P}_H\overline{Y}}+\frac{\psi_0\kappa_{D,\Psi}}{\rho\sigma\rho_\psi}+\kappa_{D,0}\right]$$

$$\tilde{\kappa}_{D,\underline{\rho}} = \left(\frac{\overline{D}}{\overline{P}_H\overline{Y}}\right)^{-1}\left[-\left((\sigma\kappa_{\theta,s}+\kappa_{\tau^c,s})\frac{\overline{D}}{\overline{P}_H\overline{Y}}+\frac{\kappa_{D,s}}{\rho}\right)-\frac{\kappa_{D,s}}{\rho}\frac{\underline{\rho}}{\overline{\overline{\rho}}}\right]\kappa_{s,\underline{\rho}}$$

$$\tilde{\kappa}_{D,\rho_\psi} = \left(\frac{\overline{D}}{\overline{P}_H\overline{Y}}\right)^{-1}\left[-\left((\sigma\kappa_{\theta,s}+\kappa_{\tau^c,s})\frac{\overline{D}}{\overline{P}_H\overline{Y}}+\frac{\kappa_{D,s}}{\rho}\right)\kappa_{s,\rho_\psi}-\left(\frac{\kappa_{\theta,\Psi}}{\rho_\psi}+\frac{\kappa_{\tau^c,\Psi}}{\rho_\psi\sigma}\right)\frac{\psi_0\overline{D}}{\overline{P}_H\overline{Y}}-\frac{\psi_0\kappa_{D,\Psi}}{\rho\sigma\rho_\psi}+\left(\frac{\kappa_{D,s}\kappa_{s,\rho_\psi}\frac{\rho_\psi}{\rho}+\kappa_{D,\Psi}\frac{\psi_0}{\sigma\rho}}{\rho+\rho_\psi}\right)\right]$$

A similar expression holds for the case of passive policy with the coefficients

$$\kappa^P_{D,\theta} = (1-\alpha)(1-\varsigma_g)\left(\overline{\tau}^v+\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\varphi(1+\phi)\right)$$

$$\kappa^P_{D,s} = -\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}}+((1-\alpha)\xi+\alpha\gamma)(1-\varsigma_g)\left(\overline{\tau}^v+\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\varphi(1+\phi)\right)+\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l$$

$$\kappa^P_{D,0} = -\left(\frac{\overline{D}}{\overline{P}_H\overline{Y}}-\frac{1}{\rho}\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l\right)\frac{\psi_0}{\rho+\rho_\psi}+\frac{\sigma\theta_0}{\rho}\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\overline{\tau}^l+\frac{\hat{T}}{\rho}+\frac{\kappa_{D,\theta}}{\rho}\theta_0$$

$$\kappa^P_{D,\Psi} = \kappa_{D,\theta}\kappa_{\theta,\Psi}$$

and

$$\tilde{\kappa}^P_{D,0} = \left(\frac{\overline{D}}{\overline{P}_H\overline{Y}}\right)^{-1}\left[\frac{\kappa_{D,s}}{\rho}\left(\kappa_{s,\underline{\rho}}+\kappa_{s,\rho_\psi}\right)+\frac{\psi_0}{\rho_\psi}\frac{\overline{D}}{\overline{P}_H\overline{Y}}+\frac{\psi_0\kappa_{D,\Psi}}{\rho\sigma\rho_\psi}+\kappa_{D,0}\right]$$

$$\tilde{\kappa}^P_{D,\underline{\rho}} = \left(\frac{\overline{D}}{\overline{P}_H\overline{Y}}\right)^{-1}\left[-\frac{\kappa_{D,s}}{\rho}-\frac{\kappa_{D,s}}{\rho}\frac{\underline{\rho}}{\overline{\overline{\rho}}}\right]\kappa_{s,\underline{\rho}}$$

$$\tilde{\kappa}^P_{D,\rho_\psi} = \left(\frac{\overline{D}}{\overline{P}_H\overline{Y}}\right)^{-1}\left[-\frac{\kappa_{D,s}}{\rho}\kappa_{s,\rho_\psi}-\frac{\kappa_{\theta,\Psi}}{\rho_\psi}\frac{\psi_0\overline{D}}{\overline{P}_H\overline{Y}}-\frac{\psi_0\kappa_{D,\Psi}}{\rho\sigma\rho_\psi}+\left(\frac{\kappa_{D,s}\kappa_{s,\rho_\psi}\frac{\rho_\psi}{\rho}+\kappa_{D,\Psi}\frac{\psi_0}{\sigma\rho}}{\rho+\rho_\psi}\right)\right]$$

D Downward Nominal Wage Rigidities

The wage and pricing equations are given by

$$w_t - p_t = \sigma c_t + \phi n_t + \hat{\tau}^c_t + \hat{\tau}^l_t \tag{211}$$

$$p_{H,t} = (\varphi-1)y_t + w_t + \hat{\tau}^v_t \tag{212}$$

Suppose now that wages are downward rigid $w_t \geq 0$, i.e., wages are allowed to go up, but not to go down. If the constraint is binding, then there will be unemployment, i.e., labor supply exceeds labor demand. We can rewrite the equations above as a labor demand and labor supply equations:

$$
\begin{aligned}
n_t^s &= \frac{1}{\phi}\left[w_t + (1-\alpha)s_t - \sigma c_t - \hat{\tau}_t^c - \hat{\tau}_t^l\right] \\
n_t^d &= \frac{\varphi}{\varphi - 1}\left[-w_t - s_t - \hat{\tau}_t^v\right]
\end{aligned}
$$

using $p_t = -(1-\alpha)s_t$.

Unemployment is then given by

$$
u_t = n_t^s - n_t^d = \frac{1}{\phi}\left[-\sigma\theta_t - \hat{\tau}_t^c - \hat{\tau}_t^l\right] - \frac{\varphi}{\varphi - 1}\left[-s_t - \hat{\tau}_t^v\right] \tag{213}
$$

where I used the fact that $w_t = 0$.

Passive Policy

Consider first the equilibrium under passive policy. Let's conjecture the economy starts with positive unemployment given a positive risk premium shock. Equilibrium is then determined by

$$
\begin{aligned}
y_t &= \varsigma_c\theta_t + (\varsigma_c\xi + \varsigma_x\gamma)s_t \\
-s_t &= (\varphi - 1)y_t
\end{aligned}
$$

solving the system

$$
y_t = \frac{\varsigma_c\theta_t}{1 + (\varsigma_c\xi + \varsigma_x\gamma)(\varphi - 1)}; \qquad s_t = -\frac{\varsigma_c(\varphi - 1)\theta_t}{1 + (\varsigma_c\xi + \varsigma_x\gamma)(\varphi - 1)} \tag{214}
$$

Unemployment is given by

$$
u_t = -\left[\frac{\sigma}{\phi} + \frac{\varphi\varsigma_c}{1 + (\varsigma_c\xi + \varsigma_x\gamma)(\varphi - 1)}\right]\theta_t \tag{215}
$$

For simplicity, assume $\xi = \gamma$. This imply that, regardless of the length of the unemployment period, θ_t is given by

$$
\theta_t = \frac{1}{\sigma}\left[\int_0^t \psi_s ds - \int_0^\infty e^{-\rho s}\psi_s ds\right] \tag{216}
$$

Hence, on impact output will decrease, unemployment will increase, and, in the case of decreasing returns to scale, the terms of trade will depreciate.

Define T_0 as the period where $\theta_{T_0} = 0$. For $t \geq T_0$, equilibrium will coincide with the flexible price allocation:

$$
y_t = -\frac{\varsigma_x\sigma}{1 + (\varphi(1+\phi) - 1)(1 - \varsigma_g)\gamma}\theta_t \tag{217}
$$

$$
s_t = -\frac{\sigma + (\varphi(1+\phi) - 1)\varsigma_c}{1 + (\varphi(1+\phi) - 1)(1 - \varsigma_g)\gamma}\theta_t \tag{218}
$$

The nominal wage is given by

$$w_t = \sigma\theta_t + \varphi\phi y_t = \sigma\left[1 - \frac{(1-\varsigma_g)\alpha\varphi\phi}{1+(\varphi(1+\phi)-1)(1-\varsigma_g)\gamma}\right]\theta_t \tag{219}$$

The coefficient above is positive, since $\gamma > \alpha$. Hence, the wage is positive and increasing for $t > T_0$. Notice also that for $t < T_0$, the wage would be below the steady state level, contradicting the downward wage rigidity.

Optimal policy

Consider now the optimal policy. Given enough instruments, the downward nominal wage rigidity does not impose any additional restriction to the problem under flexible prices. Hence, the solution will be

$$y_t = 0; \qquad s_t = -\frac{1-\alpha}{\gamma}\theta_t \tag{220}$$

The implementation, however, is different. Given s_t and y_t, labor supply is given by

$$n_t^s = \frac{1}{\phi}\left[-\sigma\frac{\alpha}{\gamma}\theta_t - \left(\hat{\tau}_t^c + \hat{\tau}_t^l + \hat{\tau}_t^v\right)\right] \tag{221}$$

The decline in consumption will increase the labor supply and create unemployment. The following condition guarantees labor supply equals labor demand

$$\hat{\tau}_t^c + \hat{\tau}_t^v + \hat{\tau}_t^l = -\frac{\sigma\alpha}{\gamma}\theta_t \tag{222}$$

The nominal wage is given by

$$w_t = \sigma\theta_t + \hat{\tau}_t^c + \hat{\tau}_t^l = \sigma\frac{\gamma-\alpha}{\gamma}\theta_t - \hat{\tau}_t^v \geq 0 \tag{223}$$

This requires the VAT to satisfy the condition

$$\hat{\tau}_t^v = \sigma\frac{\gamma-\alpha}{\gamma}\theta_t \tag{224}$$

This imply that the labor income tax is

$$\hat{\tau}_t^l = -\left(\sigma\theta_t + \hat{\tau}_t^c\right) = -\int_0^t \psi_s ds - \left(\sigma\theta_0 + \hat{\tau}_0^c\right) \tag{225}$$

If we set $\hat{\tau}_0^c = 0$, then the VAT declines and the labor tax increases on impact.

Fiscal needs of the government

Consider now the government solvency constraint:

$$-\sigma\theta_0\frac{\overline{D}}{\overline{P}_H\overline{Y}} = \int_0^\infty e^{-\rho t}\left[\left((\varphi(1+\phi)-1)\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} + \overline{\tau}^v + \overline{\tau}^l\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\right)y_t + (1-\varsigma_g)(\hat{\tau}_t^v + \hat{\tau}_t^c + \hat{\tau}_t^l) - \left((1-\overline{\tau}^v) - \frac{\overline{WN}}{\overline{P}_H\overline{Y}}\right)\hat{\tau}_t^l - \varsigma_g g_t - \frac{\overline{T}}{\overline{Y}}\hat{T}_t\right]dt$$

using $\hat{\tau}_0^c = 0$.

Under the optimal policy, the present discounted value of output, government spending, and the sum of taxes are all equal to zero. If $\alpha = 1$, then the present discounted value of $\hat{\tau}_t^l$ is also zero, since $\sigma\theta_0 = -\int_0^\infty e^{-\rho t}\psi_t dt$. The required lump-sum tax is given by

$$\tilde{T}_t = -\sigma\theta_0 \frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} \tag{226}$$

Under the passive policy, it is given by

$$\tilde{T}_t = -\sigma\theta_0 \frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} - \rho\int_0^\infty e^{-\rho t}\left((\varphi(1+\phi)-1)\frac{\rho\overline{D}}{\overline{P}_H\overline{Y}} + \overline{\tau}^v + \overline{\tau}^l\frac{\overline{WN}}{\overline{P}_H\overline{Y}}\right) y_t dt \tag{227}$$

Since $y_t^P \leq 0$ for all $t \geq 0$ and θ_0 is the same under passive and optimal policy, then the fiscal needs of the government are larger under the passive policy.