Structural Change and Deindustrialization*

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Abstract

We confirm two of Rodrik's (2016) premature deindustrialization facts: Compared to the advanced economies, countries that have industrialized more recently have had a lower industry share of employment and lower income per capita at their industrial peak. To understand these facts better, we develop a dynamic, multi-sector, multi-country model of structural change that embodies several channels by which structural change can affect industrial employment. We calibrate the model, and back out the "wedges" that account for the evolution of the model's endogenous variables. We then feed the wedges into the model to assess the relative importance of each in accounting for the deindustrialization in recent years.

JEL Classifications: F11, F43, O41, O11

Keywords: Structural change; Deindustrialization; International trade; Wedge-based accounting

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1 Introduction

One of the most important facts about structural change is the reallocation of resources and output away from agriculture towards industry and services. The reallocation involving industry is non-monotonic for most countries: the industry share of employment and/or output first rises, and then declines – it follows a "hump" pattern. Of course, this broad pattern masks a great deal of heterogeneity across countries. Recently, Rodrik (2016) has presented evidence for an intertemporal heterogeneity: compared to countries that industrialized earlier, i.e., the current advanced economies, the more recent industrializing countries have tended to have a lower peak industrial share of employment, and a lower income per capita at that peak. Rodrik has called these and related facts "premature de-industrialization".

The main goal of our paper is to systematically account for premature deindustrialization from the lens of a dynamic, multi-sector, multi-country model of structural change. Our model includes several exogenous forces, or "wedges", and several endogenous mechanisms. The key exogenous forces are sectoral productivity shocks, trade costs, and preference shocks. In addition, we allow for labor market "wedges", as well as shocks to value-added and input-output coefficients, investment rates and investment shares, as well as trade imbalances. We have enough exogenous forces to explain all the observable endogenous variables in our model. The main endogenous mechanisms are non-homothetic preferences, comparative advantage-based international trade, and input-output linkages. Each of the exogenous forces, mediated through the model's mechanisms, will have implications for sectoral output and factor demand, which, in turn, affects sectoral outputs and the sectoral alllocation of factors of production.

For example, a decline in trade costs will affect sectoral employment shares through at least three channels. First, the decline in these costs will lead to more specialization, which, will directly affect the composition of sectoral production, and correspondingly, sectoral factor demands. The extent to which factor demands respond to changes in sectoral production depends on the input-output linkages within and across sectors. Second, to the extent the specialization leads to a more efficient allocation of resources, real income will increase, which, owing to non-homothetic preferences, will engender differential changes in sectoral output demand with corresponding effects on sectoral factors of production (again, with input-output linkages playing a role). Third, to the extent that trade costs decline faster in industry compared to the other sectors effectively amplifies global productivity industry, inducing a decline in the relative price of industry's output thereby shifting final expenditure away from industry and into services.

Our calibrated model includes 28 countries plus a composite rest-of-world, and covers the period 1970 to 2011. The parameters of the model are set to be consistent with existing research. We then use the calibrated model to solve for the seven exogenous variables and time varying parameters that enable the model to fit the sectoral trade, GDP, sectoral prices, employment, expenditure, and other data as closely as possible. We call the exogenous variables and time-varying parameters collectively "wedges". We then conduct a structural accounting decomposition by feeding subsets of the wedges back into the model, which yields implications for structural change. This wedge accounting methodology has its roots in business cycle accounting (e.g. Chari, Kehoe, and McGrattan, 2007).

For our structural accounting decomposition, we ask: "If a particular country's wedge is held constant at its 1970 value, instead of evolving as it did, and if all other wedges evolve as they actually did in the data, what would be the implications for the industry share of employment?" We focus on understanding the industry employment share. We have conducted this exercise for trade costs and we find that the implied peak industry share of employment would vary positively with the year of the peak – instead of negatively, as in the data – and income per capita would be negatively associated with the peak employment share, instead of positively, as in the data. In other words, changes in trade costs alone appear to have qualitatively meaningful effects on premature deindustrialization.

Our paper is most closely related to Święcki (2014) and to Sposi, Yi, and Zhang (2018). Święcki (2014) employs a wedge accounting exercise using a multi-country model of structural change. Relative to his paper, our model includes investment, input-output linkages, and trade imbalances. Furthermore, we focus our attention to a specific subset of middle-income countries to better understand the dynamics of industrial employment around the peak of the employment share. Our framework is essentially the same as that in Sposi, Yi, and Zhang (2018), but we are applying it to a different question. In addition, it is related to other multi-sector, multi-country models that have been employed to investigate, for example, the effects of NAFTA, the evolution of comparative advantage over time, and the changing nature of structural change – Caliendo and Parro (2015), Levchenko and Zhang (2016), and Sposi (2016).

The paper is organized as follows. Section 2 presents the established and new stylized facts about structural change. Section 3 lays out our model. Section 4 describes how we calibrate the parameters of our model. The next section includes preliminary results from our structural accounting decomposition.

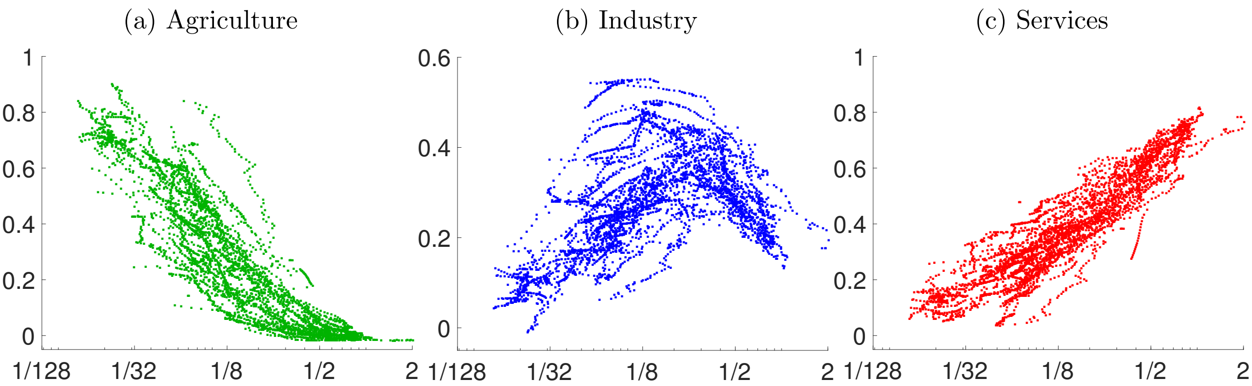
2 Structural change facts

We first present the evolution of broad sectoral shares as per capita income increases. We then present our evidence on premature deindustrialization.

2.1 Three broad structural change facts

Figure 1 plots the employment share across levels of income per capita (relative to the United States in 2011) for each sector. These data is based on an unbalanced panel covering 41 countries and up to 110 years for some countries. For all countries, 2011 is the final year.[1] The figure shows the well known fact that as countries develop the agriculture share of total employment declines, the services share of total employment increases, and, for most countries, the industry (manufacturing+mining+utilities+construction) share of total employment follows a "hump" pattern. Similar patterns hold for the sectoral value added shares over income per capita.

Figure 1: Sectoral employment shares: 1970-2011



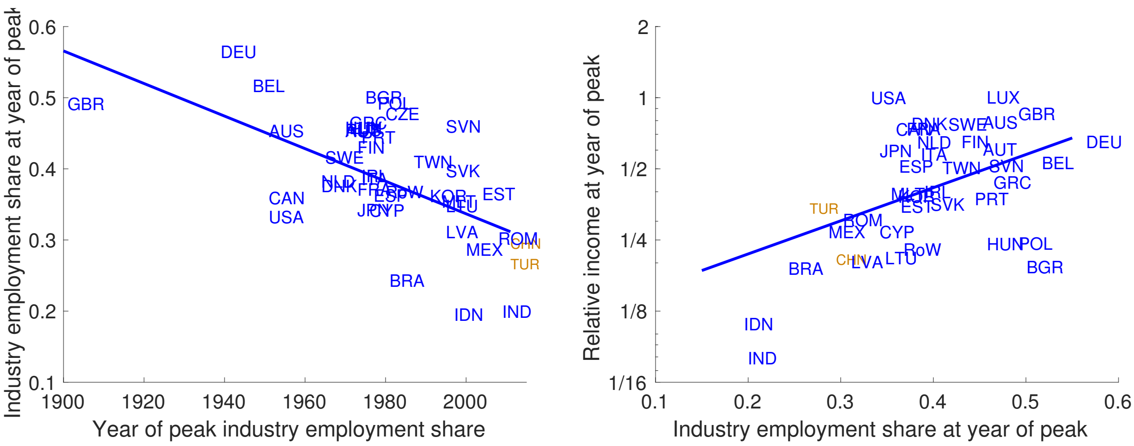
Notes: Horizontal axes - Real income per capita at PPP, relative to United States in 2011. The data comes from an unbalanced panel covering 40 countries from 1900-2011 and is HP filtered trends.

The second and third facts are about the behavior of the industry employment share at its peak. The left panel of Figure 2 plots the peak level of the industry employment share versus the year that the peak share was attained across our sample countries. The right panel plots the relative income level, vis-à-vis the United States, in the year that the peak occured versus the peak share attained. In both graphs China and Turkey are marked in a different

[1] See Appendix for list of countries and details on our data sources.

color indicating that they did not reach a peak by 2011; these countries are excluded from the estimated trend lines. Both graphs show a clear negative relation. Countries that peak later tend to have a lower share of employment in industry at that peak. And those countries that peak later tend to have a lower per-capita income relative to U.S. in that year. These two facts are consistent with what Rodrik (2016) calls "premature deindustrialization".

Figure 2: Patterns of peak industry employment shares



Notes: The data is an unbalanced panel covering 40 countries from 1900-2011.

3 Model

We now describe the model for our structural accounting exercises. We employ a three-sector, multi-country, Ricardian model of trade along the lines of Uy, Yi, and Zhang (2013), Święcki (2014), and Sposi (2016). As noted above, the model follows closely the one in Sposi, Yi, and Zhang (2018).[2]

There are N countries indexed by $(i, j) = 1, \ldots, I$ and three sectors: agriculture, industry, and services, denoted by $(k, \ell) \in \{a, m, s\}$ respectively. Time is discrete and runs from $t = 1, \ldots, T$.

3.1 Endowments

Each country is inhabited by a representative household. The representative household consists of a labor force of size L_{it} and its owns its country's stock of capital, K_t. The initial

[2]In future versions of this paper, we will adopt a somewhat different framework.

capital stock, K_{i1} is exogenous to the model, but the capital stock at the beginning of each subsequent period is predetermined based on preceding investment outlays. In each period, both factors are supplied inelastically to domestic firms.

3.2 Technology

There is a unit interval of varieties in each sector. Each variety within each sector is tradable and is indexed by $x^k \in [0,1]$ for $k \in \{a, m, s\}$.

Composite goods Within each sector, all of the varieties are combined with constant elasticity in order to construct a sectoral composite good according to

$$Q_{it}^k = \left[\int q_{it}^k(x^k)^{(\eta-1)/\eta} dx^k \right]^{\eta/(\eta-1)},$$

where η is the elasticity of substitution between any two varieties and is constant across countries, across sectors, and over time.[3] The term $q_{it}^k(x^k)$ is the quantity of variety x^k used by country i at time t to construct the sector k composite good. The resulting composite good, Q_{it}^k, is the quantity of the sector k composite good available in country i to use as an intermediate input or for final consumption or investment at time t.

Individual varieties Each individual variety is produced using capital, labor and intermediate (composite) goods from each sector. The technologies for producing each variety in each sector are given by

$$Y_{it}^k(x) = z(x) \left(A_{it}^k K_{it}^k(x)^\alpha L_{it}^k(x)^{1-\alpha} \right)^{\nu_{it}^k} \left(\prod_{\ell \in \{a,m,s\}} M_{it}^{k\ell}(x)^{\mu_{it}^{k\ell}} \right)^{1-\nu_{it}^k}.$$

The term $M_{it}^{k\ell}(x)$, for $(k,\ell) \in \{a, m, s\}$, denotes the quantity of the composite good of type ℓ used by country i as an input to produce sector k, variety x at time t. Similarly, $K_{it}^k(x)$ and $L_{it}^k(x)$ denote the quantities of capital and labor employed. Capital's share in value added, α, is constant across countries and over time. The parameter $\nu_{it}^k \in [0,1]$, for $k \in \{a, m, s\}$, denotes the share of value added in total output in sector k, while $\mu_{it}^{k\ell} \in [0,1]$ denotes the share of composite good ℓ in total spending on intermediates by producers in sector k, with $\sum_l \mu_{it}^{k\ell} = 1$ at all t. Each of these coefficients is country-specific and varies over time.

[3]The value η plays no quantitative role other than ensuring convergence of the integrals.

Value-added productivity is given by A^k_{it}, specific to a country-year-sector. The term $z(x)$ denotes country i's idiosyncratic productivity for producing variety x. Following Eaton and Kortum (2002), the idiosyncratic draw comes from independent Fréchet distributions with shape parameters θ^k for $k \in \{a, m, s\}$, with c.d.f. given by $F^k_{it}(z) = \exp(-z^{-\theta^k})$.

3.3 Labor market wedges

Workers are homogenous within a country and take home a common wage rate, w_{it} regardless of the sector that they work in. All firms hire from the same domestic market but face sector-specific wedges. Specifically, firms in sector k pay $(1 + \tau^k_{it})w_{it}$. For example, labor-embodied productivity, such as human capital, may differ across sectors and workers must be compensated accordingly to remain indifferent between working in each sector. As a normalization, we assume that there is no distortion in industry; $\tau^m_{it} = 0$. Compensating differentials are returned as a lump sum transfer to the household.

3.4 Trade

All international trade is subject to barriers that take the form of iceberg costs. Country i must purchase $d^k_{ijt} \geq 1$ units of any individual variety of sector k from country j in order for one unit to arrive at time t; $d^k_{ijt} - 1$ units *melt* away in transit. The trade costs vary across sectors and over time. As a normalization we assume that $d^k_{iit} = 1$ for all (k, i).

3.5 Preferences

Aggregate consumption is a Cobb-Douglas aggregate of agriculture and non-agriculture, with Stone-Geary subsistence requirement for agriculture. Consumption of the non-agricultural composite is implicitly defined as a generalized, non-homothetic CES aggregate over industry and services, along the lines of Comin, Lashkari, and Mestieri (2015).

$$C_{it} = (C^a_{it} - L_{it}\bar{c}^a)^{\omega^{Ca}_{it}} \left(C^N_{it}\right)^{1-\omega^{Ca}_{it}}$$

$$\sum_{k\in\{m,s\}} \omega^{Ck}_{it} \left(\frac{C^N_{it}}{L_{it}}\right)^{\frac{\varepsilon^k-\sigma}{\sigma}} \left(\frac{C^k_{it}}{L_{it}}\right)^{\frac{\sigma-1}{\sigma}} = 1.$$

Aggregate *discretionary* (non-subsistence) consumption is denoted by C_{it}. Total consumption is given by $C_{it} + L_{it}\bar{c}^a$, where $\bar{c^a}$ is the per-capita subsistence requirement of agriculture.

Total agricultural consumption is denoted by C_{it}^{a}, and C_{it}^{N} is the non-agricultural consumption composite, which bundles consumption of both industry and services, C_{it}^{m} and C_{it}^{s}, respectively. The parameter $\omega_{it}^{Ca} \in [0,1]$ determines the share of consumption expenditures on *discretionary* agricultural consumption in country i at time t, and $\omega_{it}^{CN} (= 1 - \omega_{it}^{Ca})$ is the share of spending on non-agriculture. Within the non-agricultural composite, the term $\sigma > 0$ governs the elasticity of substitution between industry and services, while ε^{m} and ε^{s} denote the income elasticities for each good; each of which is constant across countries and over time.[4] Finally, ω_{it}^{Cm} and $\omega_{it}^{Cs} (= 1 - \omega_{it}^{Cm})$ denote the relative weights of each good within the bundle.

3.6 Saving technologies

National saving is motivated by allowing for investment and non-zero net exports. Both components of saving are introduced in a tractable manner as we describe below, thereby providing the model with additional flexibility to replicate national accounts data.

Investment First, we define aggregate investment, X_{it}, as a Cobb-Douglass aggregate of each sector's composite good:

$$X_{it} = \prod_{k \in \{a,m,s\}} (X_{it}^{k})^{\omega_{it}^{Xk}}.$$

An exogenous share of GDP, ρ_{it}, is allocated to investment spending. GDP equals capital and labor income plus transfers from the labor market distortions: $w_{it}L_{it} + r_{it}K_{it} + T_{it}^{L}$. Denote the ideal price index for a bundle of investment goods by P_{it}^{X}, then

$$P_{it}^{X} X_{it} = \rho_{it}(r_{it}K_{it} + w_{it}L_{it} + T_{it}^{L}),$$

Aggregate investment augments the existing stock of capital subject one-period time to build with linear depreciation:

$$K_{it+1} = (1-\delta)K_{it} + X_{it}.$$

Although the nominal investment rate is exogenous, the real investment rate is endogenous since it depends on relative price of investment.

[4] Only the difference in the income elasticities matters for allocations. Changing the levels, holding the difference fixed, affects only the cardinal properties of the utility function.

International saving Households borrow from (lend to) the rest of the world by running trade deficits (surpluses). Trade imbalances are modeled as proceeds from a global portfolio as in Caliendo, Parro, Rossi-Hansberg, and Sarte (2014). A pre-determined share of GDP, ϕ_{it}, is sent to a global portfolio. The global portfolio disperses a lump sum transfer, $L_{it}T_t^P$, to every country, where T_t^P is the per-capita transfer at time t common to all countries. This implies that country i's net exports, the difference between GDP and final expenditures is $\phi_{it}(w_{it}L_{it} + r_{it}K_{it} + T_{it}^L) - L_{it}T_t^P$.While the share of GDP allocated to the global portfolio is exogenous, the proceeds are endogenous, because they depend of the size of the global portfolio. Therefore, the overall trade imbalance is endogenous, both in terms of absolute level and as shares of local and global GDP.

3.7 Budget constraint

The household spends its income on consumption and investment in the three goods; the period budget constraint is

$$\underbrace{\sum_{k\in\{a,m,s\}} P_{it}^k C_{it}^k}_{P_{it}^C C_{it} + P_{it}^a \bar{c}^a L_{it}} + \underbrace{\sum_{k\in\{a,m,s\}} P_{it}^k X_{it}^k}_{P_{it}^X X_{it}} = (1-\phi_{it})(r_{it}K_{it} + w_{it}L_{it} + T_{it}^L) + L_{it}T_t^P.$$

The ideal price indexes for consumption and investment are P_{it}^C and P_{it}^X, respectively.

3.8 Equilibrium

A competitive equilibrium satisfies the following conditions: 1) the representative household maximizes utility taking prices as given, 2) firms maximize profits taking prices as given, 3) each country purchases each variety from its least-cost supplier, and 4) markets clear. We describe each equilibrium condition in detail below and omit time subscripts to simplify notation. (Table B.1 in Appendix B lists the equations for all of the equilibrium conditions.)

3.8.1 Household optimization

Household take income as given. The income is allocated across net lending to the world, investment in the three sectors, and consumption in the three sectors.

Saving and investment A predetermined share of income, ϕ_{it}, is sent to the global portfolio. Another predetermined share of income, ρ_{it}, is spent on investment. Investment spending is allocated across sectors as

$$P_{it}^k X_{it}^k = \omega_{it}^{Xk} P_{it}^X X_{it}.$$

Consumption Income that is not saved is allocated toward consumption expenditures. The problem of allocating consumption across the three sectors can be broken down into three steps. First, given aggregate consumption spending, consumption of agriculture is given by

$$C_{it}^a = \frac{\omega_{it}^{Ca}\left(P_{it}^C C_{it} - L_{it} P_{it}^a \bar{c}^a\right) + L_{it} P_{it}^a \bar{c}^a}{P_{it}^a} \tag{1}$$

Second, the non-agricultural consumption demand and corresponding ideal price index satisfy the following two simultaneous equations:

$$C_{it}^N = \frac{\omega_{it}^{CN}\left(P_{it}^C C_{it} - L_{it} P_{it}^a \bar{c}^a\right)}{P_{it}^N}$$

$$P_{it}^N = \left(\sum_{k\in\{m,s\}} (\omega_{it}^{Ck})^\sigma \left(\frac{C_{it}^N}{L_{it}}\right)^{\varepsilon_k - 1} (P_{it}^k)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

Third, taking non-agricultural consumption and the corresponding ideal price index as given, the sectoral demand for consumption of industry and services satisfies

$$C_{it}^k = L_{it}(\omega_{it}^{Ck})^\sigma \left(\frac{P_{it}^k}{P_{it}^N}\right)^{-\sigma} \left(\frac{C_{it}^N}{L_{it}}\right)^{\varepsilon^k}.$$

3.8.2 Firm optimization

Markets are perfectly competitive, so firms set prices equal to marginal costs. Denote the price of variety z, produced in country j and purchased by country i, as $p_{ijt}^k(z)$. Then $p_{ijt}^k(z) = p_{jjt}^k(z) d_{ijt}^k$, where $p_{jj}^k(z)$ is the marginal cost of producing variety z in country j. Since country i purchases each variety from the country that can deliver it at the lowest price, the price in country i is $p_{it}^k(z) = \min_{j=1,\dots,I}[p_{jjt}^k(z) d_{ijt}^k]$. The price of the sector k composite good in country i is then

$$P_{it}^k = \gamma_k \left[\sum_{j=1}^{I} \left((A_{jt}^k)^{-\nu_j^k} u_{jt}^k d_{ijt}^k\right)^{-\theta^k}\right]^{-\frac{1}{\theta^k}}, \tag{2}$$

where the unit cost for a bundle of inputs for producers in sector k in country i is

$$u_{it}^{k} = \left(\frac{r_i}{\alpha\nu_{it}^{k}}\right)^{\alpha\nu_{it}^{k}} \left(\frac{(1+\tau_i^k)w_{it}}{(1-\alpha)\nu_{it}^{k}}\right)^{(1-\alpha)\nu_{it}^{k}} \left[\prod_{\ell\in\{a,m,s\}} \left(\frac{P_{it}^{\ell}}{\mu_{it}^{k\ell}}\right)^{\mu_{it}^{k\ell}}\right]^{1-\nu_{it}^{k}}. \tag{3}$$

Next we define total factor usage in sector k by aggregating the individual varieties.

$$K_{it}^{k} = \int K_{it}^{k}(x)dx, \qquad L_{it}^{k} = \int L_{it}^{k}(x)dx,$$
$$M_{it}^{k\ell} = \int M_{it}^{k\ell}(x)dx, \qquad Y_{it}^{k} = \int Y_{it}^{k}(x)dx.$$

The term $K_{it}^{k}(x)$ denotes the quantity of labor used in the production of variety x. If country i imports variety x, then $K_{it}^{k}(x) = 0$. Hence, K_{it}^{k} is the total capital used in sector k in country i. Similarly, L_{it}^{k} is the total quantity of labor used, $M_{it}^{k\ell}$ denotes the quantity of good ℓ that country i uses as an intermediate input in production in sector k, and Y_{it}^{k} is the quantity of the sector k output produced by country i.

Cost minimization by firms operating under constant returns to scale implies that, within each sector, factor expenses exhaust the value of output:

$$\begin{aligned} r_{it}K_{it}^{k} &= \alpha\nu_{it}^{k}P_{it}^{k}Y_{it}^{k}, \\ (1+\tau_{it}^{k})w_{it}L_{it}^{k} &= (1-\alpha)\nu_{it}^{k}P_{it}^{k}Y_{it}^{k}, \\ P_{it}^{\ell}M_{it}^{k\ell} &= (1-\nu_{it}^{k})\mu_{it}^{k\ell}P_{it}^{k}Y_{it}^{k}. \end{aligned}$$

3.8.3 Trade flows

In sector k the fraction of country i's expenditures allocated to goods produced by country j is given by

$$\pi_{ijt}^{k} = \frac{\left((A_{jt}^{k})^{-\nu_{it}^{k}}u_j^k d_{ijt}^{k}\right)^{-\theta^k}}{\sum_{h=1}^{I}\left((A_{ht}^{k})^{-\nu_{ht}^{k}}u_{ht}^{k}d_{iht}^{k}\right)^{-\theta^k}}. \tag{4}$$

3.8.4 Market clearing conditions

We begin by describing the domestic market clearing conditions:

$$K_{it} = \sum_{k \in \{a,m,s\}} K_{it}^k,$$
$$L_{it} = \sum_{k \in \{a,m,s\}} L_{it}^k$$
$$Q_{it}^k = C_{it}^k + X_{it}^k + \sum_{\ell \in \{a,m,s\}} M_{it}^{\ell k}.$$

The first two conditions impose capital and labor market clearing in country i. The third condition, which applies to each sector $k \in \{a, m, s\}$, requires that the use of composite good k equal its supply. Its use consists of consumption and investment by the representative household and intermediate use by firms in each sector. Its supply is the quantity of the composite good, which consists of both domestically- and foreign-produced varieties.

The next conditions require that the value of output produced by country i is equal to the value that all countries purchase from country i. That is,

$$P_{it}^k Y_{it}^k = \sum_{j=1}^{I} \left(P_{it}^k C_{it}^k + P_{it}^k X_{it}^k + \sum_{\ell \in \{a,m,s\}} P_{it}^k M_{it}^{\ell k} \right) \pi_{ijt}^k. \tag{5}$$

Finally we impose an aggregate resource constraint in each country: the sum of net exports across sectors must equal the value of net lending.

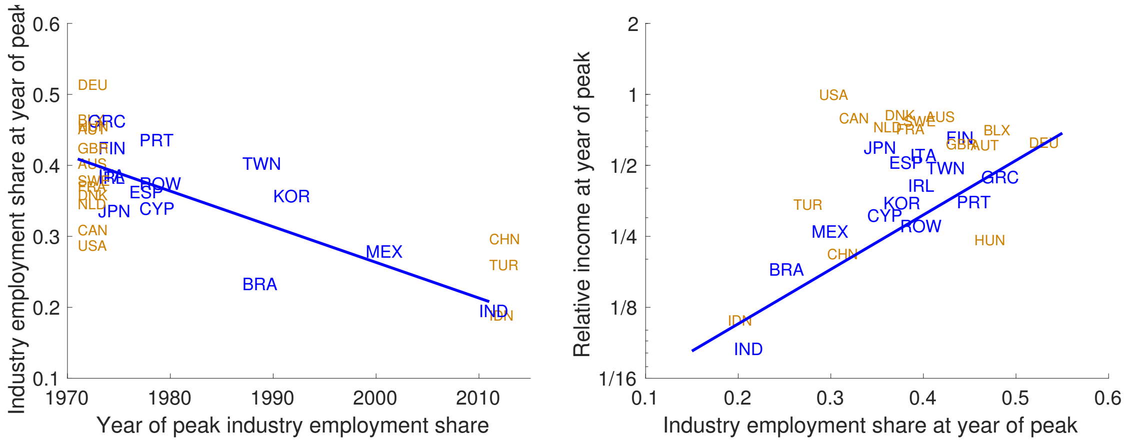
$$\phi_{it}(w_{it} L_{it} + r_{it} K_{it} + T_{it}^L) - L_{it} T_t^P = \sum_{k \in \{a,m,s\}} P_{it}^k Y_{it}^k - P_{it}^k Q_{it}^k. \tag{6}$$

The left-hand side is the difference between the household's income and spending. The right-hand side is the value of gross production minus gross absorption. In equilibrium, both sides must equal net exports.

4 Measurement and calibration of "wedges"

Our quantitative analysis is constrained to 28 countries plus a rest-of-world aggregate from 1970 to 2011, due to data requirements and their limitations. All empirical facts presented in section 2 hold true in this smaller sample. Particularly, for countries that peak earlier tend to peak at high industry employment shares and at high levels of GDP per capita. The left panel of Figure 3 shows that countries with the industry employment share peaked later have lower industry employment shares. The level of income per capita at the time of the peak correlates positively with the level of the peak industry employment share, as shown in the right panel of Figure 3. The countries marked in a beige color either peaked prior to 1970, or have not yet peaked by 2011. Thus, the fitted trend line is based only on countries that have peaked between 1970 and 2011.

Figure 3: Patterns of peak industry employment shares in restricted sample



Notes: The blue points are for countries that have the industry employment share peaked between 1970–2011. The beige points are for countries that have the industry employment share either peaked prior to 1970, or have not yet peaked by 2011.

In this section, we first describe how we set the parameters that are common across sectors or countries. Then, we present how we determine each of the seven sets of country and/or sector-specific, as well as time-varying parameters and exogenous shocks using the mthodology in Sposi, Yi, and Zhang (2018). We will call the time-varying parameters and exogenous shocks "wedges".

4.1 Common parameters

Table 1 reports the parameters that are common across countries and constant over time. Starting with the preferences parameters, we choose the substance level, $\bar{c}^a$, so that the the maximum ratio of subsistence to agricultural consumption is 0.75. From Lewis, Monarch, Sposi, and Zhang (2018), we take $\sigma = 0.39$, $\varepsilon^m = 1$ (normalization), and $\varepsilon^s = 1.40$. They estimate these parameters in a model with preferences defined over goods and services using a similar set of countries over a similar time period. In their model, goods include both agriculture and manufacturing.

Table 1: Common parameters

$\bar{c}^a$	Per-capita subsistence level of agriculture consumption	1.55×10^{-6}
σ	Elasticity of substitution between industry and services consumption	0.39
ε^m	Income elasticity of demand for industry consumption	1
ε^s	Income elasticity of demand for services consumption	1.40
α	Capital's share in value added	0.33
δ	Annual depreciation rate for stock of capital	0.06
θ^k	Trade elasticity	4
η	Elasticity of substitution between varieties	2

Notes: The per-capita subsistence, $\bar{c}^a$, implies that the subsistence share in U.S. agriculture consumption is 0.15 in 2011. The model's units are normalized so that world labor compensation equals 1 in every period: $\sum_{i=1}^{I} w_{it} L_{it} = 1, \forall t$.

Next we describe the production parameters. Capital's share in value added is set to $\alpha = 0.33$ as in Gollin (2002). The depreciation rate is set to $\delta = 0.06$, as is standard in macro models using annual data. Simonovska and Waugh (2014) estimate the trade elasticity for manufacturing to be 4. We apply this estimate to all sectors and set $\theta^k = 4$ in each sector. The elasticity of substitution between individual goods within the composite good plays no quantitative role in the model other than satisfying a technical condition: $1 + \frac{1}{\theta_b}(1 - \eta) > 0$. Following the literature we set $\eta = 2$.

4.2 Country-specific and time-varying exogenous variables and parameters

As a reminder, the exogenous variables and time varying parameters, i.e., the wedges, that we back out from the data through the lens of the model are:

1. Total factor (fundamental) productivity: A_{it}^k
2. Trade costs: d_{ijt}^k
3. Preference shocks: ω_{it}^{Ck}
4. Labor market wedges: τ_{it}^k

5. Value-added and input-output coefficients: ν_{it}^{k} and $\mu_{it}^{k\ell}$

6. Investment rates and sectoral investment shares: ρ_{it} and ω_{it}^{Xk}

7. Trade imbalance shares: ϕ_{it}

These wedges are "chosen" by the model to match the following data moments: sectoral value added, sectoral bilateral trade shares, sectoral consumption expenditures, sectoral employment, sectoral gross production and intermediate-input usage, sectoral investment expenditures, and aggregate trade imbalances. In what follows below, "hats" on variables refer to data. We also make use of data on aggregate employment in each country and the initial stock of capital in each country.

Fundamental productivity Sectoral productivity can be recovered from from the sectoral prices. Using equations (2) and (4), the price of the sector k composite good can be expressed as $P_{it}^{k} = \frac{u_{it}^{k}}{Z_{it}^{k}}$, where u^{k} denotes the unit cost for a bundle of inputs in sector k and Z^{k} denotes the measured productivity. Expanding out for the unit costs and rearranging, measured productivity can be backed out as

$$Z_{it}^{k} = \left(\frac{1}{\widehat{P}_{it}^{k}}\right)\left(\frac{r_{it}}{\alpha\nu_{it}^{k}}\right)^{\alpha\nu_{it}^{k}}\left(\frac{(1+\tau_{it}^{k})\widehat{w}_{it}}{(1-\alpha)\nu_{it}^{k}}\right)^{(1-\alpha)\nu_{it}^{k}}\left[\prod_{\ell\in\{a,m,s\}}\left(\frac{\widehat{P}_{it}^{\ell}}{\mu_{it}^{k\ell}(1-\nu_{it}^{k})}\right)^{\mu_{it}^{k\ell}}\right]^{1-\nu_{kit}}. \quad (7)$$

Sector prices, $\widehat{P}^{k}$, are taken directly from the data. The aggregate wage is taken from the data as the aggregate value added per worker, $\widehat{w_{it}} = \frac{\widehat{VA}_{it}}{\widehat{L}_{it}}$. The labor wedges, $\widehat{\tau^{k}}$, are computed below using equation (14). The production elasticities, $\nu^{k}it$ and $\mu_{it}^{k\ell}$ are computed below using equations (15) and (16). This leaves the unobserved rental rate for capital, r_{it}, which we impute using the models structure and data on aggregate capital stocks.

In the model, cost minimization by firms implies that $\frac{(1+\tau_{it}^{k})w_{it}L_{it}^{k}}{1-\alpha} = \frac{r_{it}K_{it}^{k}}{\alpha}$ in each sector. Aggregating across sectors and imposing market clearing for capital—$K_{it} = \sum_{k\in\{a,m,s\}} K_{it}^{k}$— implies that the rental rate can be imputed as

$$r_{it} = \left(\frac{\widehat{w}_{it}}{\widehat{K}_{it}}\right)\left(\frac{1-\alpha}{\alpha}\right)\sum_{k\in\{a,m,s\}}(1+\tau_{it}^{k})\widehat{L}_{it}^{k}.$$

The aggregate capital stock, $\widehat{K}_{it}$ is taken directly from the data.

Given the measured productivity, Z_{it}^k, the fundamental productivity is recovered using data on sectoral home trade shares:

$$A_{it}^k = \left(\gamma^k Z_{it}^k \left(\widehat{\pi}_{iit}^k\right)^{\frac{1}{\theta^k}}\right)^{\frac{1}{\nu_{it}^k}} \tag{8}$$

Bilateral trade costs Through the lens of the model, the bilateral trade barrier between two countries appears as a wedge that reconciles the pattern of trade between them, taking the prices in both countries as given.

$$d_{ijt}^k = \left(\frac{\widehat{\pi}_{ijt}^k}{\widehat{\pi}_{jjt}^k}\right)^{-\frac{1}{\theta_k}} \left(\frac{\widehat{P}_{it}^k}{\widehat{P}_{jt}^k}\right) \tag{9}$$

For each sector $k \in \{a, m, s\}$, we make use of the data on bilateral trade shares, $\widehat{\pi}_{ijt}^k$, and prices, $\widehat{P}_{it}^k$, to compute the bilateral trade costs directly. In cases where $\widehat{\pi}_{ijt}^k = 0$ in the data, we set $d_{ij}^k = 10^8$ (this is arbitrarily large enough to ensure that $\pi_{ijt}^k \approx 0$ in the model). In cases where the computed barrier is less than 1, we set $d_{ijt}^k = 1$.

Preference weights Calibration of the preference weights makes use of data on sectoral consumption expenditures and prices. Denote the consumption expenditures in sector k as $E_{it}^{Ck} = P_{it}^k C_{it}^k$. Noting that $\omega_{it}^{CN} = 1 - \omega_{it}^{Ca}$, the preference weight for agriculture is computed from

$$\frac{\omega_{it}^{Ca}}{\omega_{it}^{CN}} = \frac{\widehat{E}_{it}^{Ca} - \widehat{L}_{it}\widehat{P}_{it}^a\bar{c}^a}{\widehat{E}_{it}^{Cm} + \widehat{E}_{it}^{Cs}}. \tag{10}$$

The preference weights for industry and services can be identified off of data on relative expenditures, relative prices, and non-agricultural consumption levels. Noting that $\omega_{it}^{Cs} = 1 - \omega_{it}^{Cs}$, the preference weight for industry is computed from

$$\frac{\widehat{E}_{it}^{Cm}}{\widehat{E}_{it}^{Cs}} = \left(\frac{\omega_{it}^{Cm}}{\omega_{it}^{Cs}}\right)^{\sigma} \left(\frac{\widehat{P}_{it}^{cm}}{\widehat{P}_{it}^{cs}}\right)^{1-\sigma} \left(\frac{C_{it}^N}{\widehat{L}_{it}}\right)^{\varepsilon^m - \varepsilon^s}. \tag{11}$$

However, the index for consumption of non-agriculture is not observed. We impute the level of non-agricultural consumption bundle using the observed prices and levels of consumption of industry and services in addition to the level of non-agricultural consumption expenditures as dictated by the model using

$$\frac{\widehat{E}_{it}^{Cm}+\widehat{E}_{it}^{Cs}}{\widehat{L}_{it}}=\left(\sum_{k\in\{m,s\}}(\omega_{it}^{Ck})^{\sigma}\left(\frac{C_{it}^{N}}{\widehat{L}_{it}}\right)^{\varepsilon^k-\sigma}(\widehat{P}_{it}^{k})^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{12}$$

Solving for equations (11) and (12) jointly given the industry preference weights and the level of non-agricultural consumption. The price level for nonagricultural consumption is defined as the ratio of nominal expenditures to the level of consumption:

$$P_{it}^{N}=\frac{\widehat{E}_{it}^{Cm}+\widehat{E}_{it}^{Cs}}{C_{it}^{N}}. \tag{13}$$

Labor endowments and labor market wedges L_{it} are computed directly from data as the numbers of persons engaged across the three broad sectors.

The labor market wedges are defined to reconcile the disparity between sectoral employment shares and sectoral value added shares. In the model, sector k value added is $(1+\tau_{it}^{k})w_{it}L_{it}^{k}+r_{it}K_{it}^{k}$. Moreover, in each sector the capital-labor ratio is given by $\frac{K_{it}^{k}}{L_{it}^{k}}=\frac{\alpha}{1-\alpha}\frac{(1+\tau_{it}^{k})w_{it}}{r_{it}}$. This implies that sector k value added equals $\frac{(1+\tau_{it}^{k})w_{it}L_{it}^{k}}{1-\alpha}$. By normalizing the industry labor wedge, $\tau_{it}^{m}=0$, the wedge in sector k is computed as

$$1+\tau_{it}^{k}=\frac{\widehat{VA}_{it}^{k}/\widehat{VA}_{it}^{m}}{\widehat{L}_{it}^{k}/\widehat{L}_{it}^{m}}. \tag{14}$$

Value-added and Input-output coefficients In the model, ν_{it}^{k} is the share of gross output that compensates capital and labor: $\frac{r_{it}K_{it}^{k}+(1+\tau_{it}^{k})w_{it}}{P_{it}^{k}Y_{it}^{k}}$. We compute it directly as the ratio of value added, VA, to gross output, GO, in each sector-country-year:

$$\nu_{it}^{k}=\frac{\widehat{VA}_{it}^{k}}{\widehat{GO}_{it}^{k}}. \tag{15}$$

The intermediate input share $\mu_{it}^{k\ell}$ is the share of sector ℓ in intermediates in sector k: $\frac{P_{it}^{\ell}M_{it}^{k\ell}}{\sum_{b\in\{a,m,s\}}P_{it}^{b}M_{it}^{kb}}$. We compute is as the ratio of sector k's spending on inputs from sector ℓ, $IO^{k\ell}$, to sector k's total spending on intermediate inputs (gross output net of value added).

$$\mu_{it}^{k\ell}=\frac{\widehat{IO}_{it}^{k\ell}}{\widehat{GO}_{it}^{k}-\widehat{VA}_{it}^{k}}. \tag{16}$$

Note that $\sum_{\ell \in \{a,m,s\}} \mu_{it}^{k\ell} = 1$, for all (k, i, t), since $\widehat{GO}_{it}^{k} = \widehat{VA}_{it}^{k} + \sum\limits_{\ell \in \{a,m,s\}} \widehat{IO}_{it}^{k\ell}$.

Investment rates The nominal investment rate in the model is the ratio of gross fixed capital formation to GDP: $\rho_{it} = \frac{P_{it}^{X} X_{it}}{w_{it} L_{it} + r_{it} K_{it} + R_{it}^{G}}$. We compute it using these data directly.

$$\rho_{it} = \frac{\widehat{GFCF}_{it}}{\widehat{GDP}_{it}}. \tag{17}$$

Each sector's share in aggregate investment is given by $\omega_{it}^{Xk} = \frac{P_{it}^{k} X_{it}^{k}}{P_{it}^{X} X_{it}}$. We compute it as the ratio of investment expenditures on goods from sector k, $GFCF^{k}$ to aggregate investment expenditures:

$$\omega_{it}^{Xk} = \frac{\widehat{GFCF}_{it}^{k}}{\widehat{GFCF}_{it}}. \tag{18}$$

Trade Imbalances In the baseline model we assume that transfers from the global portfolio, R^{G} are equal to 0 in every period, so that the fraction of income sent to the global portfolio equals the ratio of net exports to GDP: $\phi_{it} = \frac{r_{it} K_{it} + w_{it} L_{it} + T_{it}^{L} - P_{it}^{C} C_{it} - P_{it}^{X} X_{it}}{r_{it} K_{it} + w_{it} L_{it} + T_{it}^{L}}$ (the numerator is equal to net exports, NX). Thus, we have

$$\phi_{it} = \frac{\widehat{NX}_{it}}{\widehat{GDP}_{it}}. \tag{19}$$

In the data we ensure that $\sum_{i=1}^{I} \widehat{NX}_{it} = 0$ so the global portfolio in balanced in every year.

4.3 Calibrated wedges

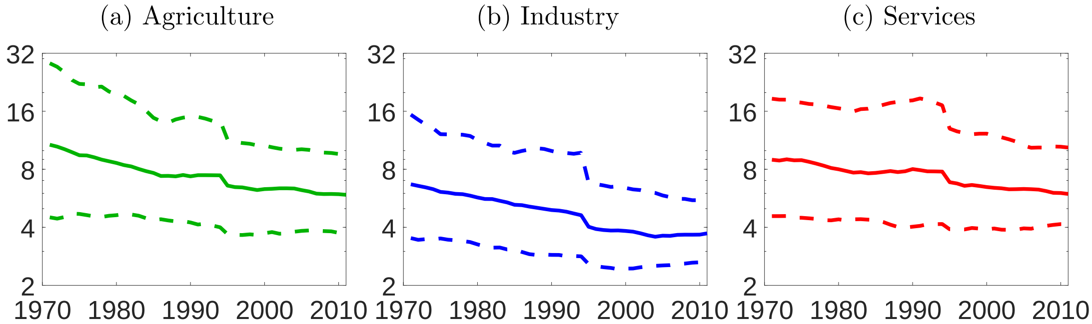
The previous sub-section described how we back out the seven sets of wedges, with each wedge corresponding to a mechanism for structural change, using data and the model.[5] We now show how the world distribution of calibrated trade costs varied over time.

Through the lens of the model these costs are wedges that are needed to reconcile the observed pattern of bilateral trade given the levels of technology. They therefore capture both policy and non-policy impediments to trade. Since the bilateral trade costs are asymmetric across countries, we plot, for each year, the 25^{th}, the 50^{th}, and the 75^{th} percentiles for each sector's trade cost in Figure 4. Trade costs in every sector have declined over our sample

[5] As a reminder, our data are HP-filtered to remove the business cycle frequencies; the HP-filtered data are used to back out the wedges. Hence, the wedges capture only secular trends over time.

period, particulalry the 75^{th} percentile, reflecting low-income countries undergoing large trade liberalizations and partaking in the global economy. Not surprisingly trade costs in industry, in general, are lower than in the other sectors and have declined relatively faster at all levels of the distribution.

Figure 4: Sectoral Bilateral Trade Costs: 1970-2011



Notes: 25^{th}, 50^{th}, and 75^{th} percentiles for the world distribution of bilateral trade costs in each sector. A value of 1 corresponds to frictionless trade.

5 Structural accounting decompositions

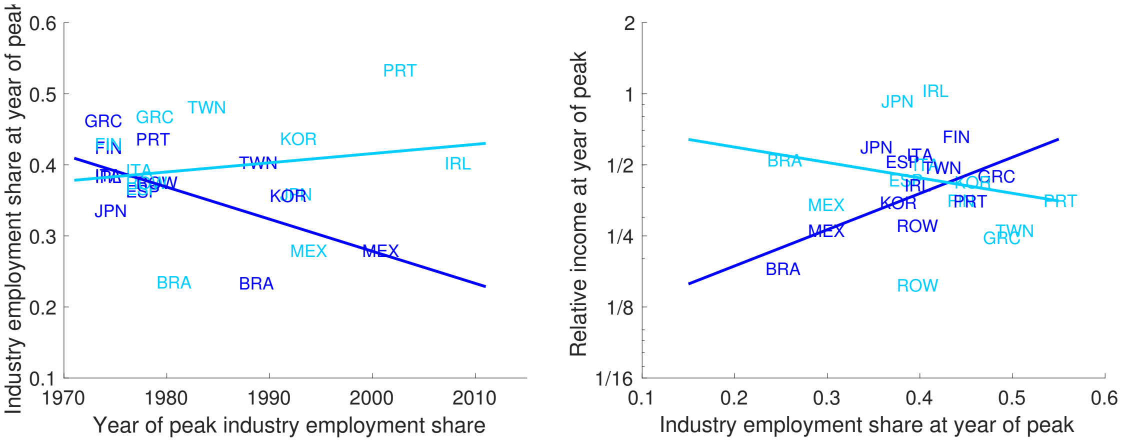
The wedge analysis in the previous section is informative about the size and the dynamics of these wedges. They provide suggestive evidence on their relative importance in affecting the key variables like the sectoral employment shares. However, they do not provide a definitive assessment of how important each set of wedges is in accounting for the industry share of employment, for example. In order to measure this importance, we need to conduct counterfactual simulations, which we call structural accounting decompositions, with our model. For each set of wedges, we examine the effect of holding the wedges constant at their 1970 levels. We focus on accounting for the evolution of the industry employment share.

5.1 Hold country-level trade costs constant

In this counterfactual, we examine how changing trade costs contributed to the facts on premature deindustrialization. To do this, we hold every country's trade costs in all sectors constant at 1970 levels, $d^k_{ijt} = d^k_{i\ell 1970}, \forall (i, j, k, t)$, but allow all other wedges to vary over time.

Figure 5 shows facts 2 and 3 in both the baseline (data) and in the counterfactual. Countries that do not reach a peak strictly between 1970 and 2011 are removed from the plot. The dark blue refers to the baseline and the light blue refers to the counterfactual. Clearly, the elimination of time-variation in trade costs implies a complete disappearance of both facts 2 and 3. That is, the implied peak industry share of employment would vary positively with the year of the peak – instead of negatively, as in the data – and income per capita would be negatively associated with the peak employment share, instead of positively, as in the data. This finding suggests that changes in trade costs over time, or trade integration over time, have contributes to the premature deindustrialization.

Figure 5: Patterns of peak industry employment shares with constant trade costs



6 Conclusion

Our paper develops a structural accounting decomposition framework for studying the evolution of structural change across countries. The framework is a dynamic multi-country, multi-sector model that embodies three main mechanisms of structural change – non-homothetic preferences, asymmetric sectoral productivity growth, and international trade, as well as four additional mechanisms, including intermediate goods and input-output linkages. We then parameterize and calibrate our model to 28 countries and a composite rest-of-world aggregate covering 1970 to 2011. We use the model to recover the exogenous variables and time-varying parameters (the wedges) that can fully account for the GDP, employment, expenditure, trade and other data.

We use the calibrated model to structurally systematically measure the effect of changes in trade costs on premature deindustrialization. We find that the implied peak industry share of employment would vary positively with the year of the peak – instead of negatively, as in the data – and income per capita would be negatively associated with the peak employment share, instead of positively, as in the data. In other words, changes in trade costs alone appear to have qualitatively meaningful effects on premature deindustrialization.

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A Data Sources

Our data draw from numerous sources including the WIOD, EU-KLEMS, Penn World Tables, GGDC 10-sector Database, and many other databases.

The list of countries in Figures 1 and 2 are: Australia (AUS), Austria (AUT), Belgium-Luxembourg (BLX), Brazil, (BRA), Bulgaria (BGR), Canada (CAN), China (CHN), Cyprus (CYP), Germany (DEU), Denmark (DNK), Spain (ESP), Finland (FIN), France (FRA), United Kingdom (GBR), Greece (GRC), Hungary (HUN), Indonesia (IDN), India (IND), Ireland (IRL), Italy (ITA), Japan (JPN), South Korea (KOR), Mexico (MEX), Netherlands (NLD), Poland (POL), Portugal (PRT), Sweden (SWE), Switzerland (CHE), Turkey (TUR), Taiwan (TWN), United States (USA), Argentina (ARG), Malaysia (MYS), Mauritius (MUS), Morocco (MAR), Philippines (PHL), Singapore (SGP), South Africa (ZAF), Thailand (THA), and the Rest-of-world (ROW).

For the regression analysis, the countries are grouped as follows: Eastern Europe (10): Bulgaria, Czech Republic, Estonia, Hungary, Lithuania, Latvia, Poland, Romania, Solvakia, and Slovenia. Southern Europe (6): Cyprus, Spain, Greece, Italy, Malta, and Portugal. Northern Europe (11): Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Luxembourg, Netherlands, Sweden, and United Kingdom. Other advanced (6): Australia, Canada, Japan, South Korea, Taiwan, United States Emerging (7): Brazil, China, Indonesia, India, Mexico, Russia, Turkey

For the quantitative analysis (Sections 4 and 5), we use 28 countries/regions: Australia (AUS), Austria (AUT), Belgium-Luxembourg (BLX), Brazil, (BRA), Canada (CAN), China (CHN), Cyprus (CYP), Germany (DEU), Denmark (DNK), Spain (ESP), Finland (FIN), France (FRA), United Kingdom (GBR), Greece (GRC), Hungary (HUN), Indonesia (IDN), India (IND), Ireland (IRL), Italy (ITA), Japan (JPN), South Korea (KOR), Mexico (MEX), Malta (MLT), Netherlands (NLD), Portugal (PRT), Sweden (SWE), Turkey (TUR), Taiwan (TWN), United States (USA), and the Rest-of-world (ROW).

B Solving the equilibrium

Table B.1 provides a set of equations that characterize the equilibrium of the economy.

Table B.1: Equilibrium conditions

(S1)	$r_{it}K_{it}^k = \alpha\nu_{it}^k P_{it}^k Y_{it}^k$	$\forall(k,i,t)$
(S2)	$(1+\tau_{it}^k)w_{it}L_{it} = (1-\alpha)\nu_{it}^k P_{it}^k Y_{it}^k$	$\forall(k,i,t)$
(S3)	$P_{it}^{\ell}M_{it}^{k\ell} = (1-\nu_{it}^k)\mu_{it}^{k\ell}P_{it}^k Y_{it}^k$	$\forall(k,\ell,i,t)$
(S4)	$P_{it}^k = \gamma^k\left(\sum_{j=1}^{I}\left((A_{jt}^k)^{-\nu_{jt}^k}u_{jt}^k d_{ijt}^k\right)^{-\theta^k}\right)^{-\frac{1}{\theta^k}}$	$\forall(k,i,t)$
(S5)	$\pi_{ijt}^k = \frac{\left((A_{jt}^k)^{-\nu_{jt}^k}u_{jt}^k d_{ijt}^k\right)^{-\theta^k}}{\sum_{h=1}^{I}\left((A_{ht}^k)^{-\nu_{ht}^k}u_{ht}^k d_{iht}^k\right)^{-\theta^k}}$	$\forall(k,i,t)$
(D1)	$P_{it}^a C_{it}^a = \omega_{it}^{Ca}(P_{it}^C C_{it} - L_{it}P_{it}^a\bar{c}^a) + L_{it}P_{it}^a\bar{c}^a$	$\forall(i,t)$
(D2)	$P_{it}^k C_{it}^k = \omega_{it}^{Ck}\left(\frac{P_{it}^k}{P_{it}^N}\right)^{1-\sigma}\left(\frac{C_{it}^N}{L_{it}}\right)^{\varepsilon^k-1}$	$\forall(i,t)$ & $k\in\{m,s\}$
(D3)	$P_{it}^N C_{it}^N = (1-\omega_{it}^{Ca})(P_{it}^C C_{it} - L_{it}P_{it}^a\bar{c}^a)$	$\forall(i,t)$
(D4)	$P_{it}^N = \left(\sum_{k\in\{a,m,s\}}(\omega_{it}^{Ck})^{\sigma}\left(\frac{C_{it}^N}{L_{it}}\right)^{\varepsilon^k-1}(P_{it}^k)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$	$\forall(i,t)$
(D5)	$P_{it}^C = \left(\frac{P_{it}^a}{\omega_{it}^{Ca}}\right)^{\omega_{it}^{Ca}}\left(\frac{P_{it}^N}{1-\omega_{it}^{Ca}}\right)^{1-\omega_{it}^{Ca}}$	$\forall(i,t)$
(D6)	$P_{it}^k X_{it}^k = \omega_{it}^{Xk}P_{it}^X X_{it}$	$\forall(i,t)$
(D7)	$X_{it} = \prod_{k\in\{a,m,s\}}\left(X_{it}^k\right)^{\omega_{it}^{Xa}}$	$\forall(i,t)$
(D8)	$P_{it}^X = \prod_{k\in\{a,m,s\}}\left(\frac{P_{it}^k}{\omega_{it}^{Xk}}\right)^{\omega_{it}^{Xa}}$	$\forall(i,t)$
(D9)	$P_{it}^X X_{it} = \rho_{it}(r_{it}K_{it} + w_{it}L_{it} + T_{it}^L)$	$\forall(i,t)$
(D10)	$K_{it+1} = (1-\delta)K_{it} + X_{it}$	$\forall(i,t)$
(D11)	$P_{it}^C C_{it} + L_{it}P_{it}^a\bar{c}^a + P_{it}^X X_{it} = (1-\phi_{it})(r_{it}K_{it} + w_{it}L_{it} + T_{it}^L) + T_t^P L_{it}$	$\forall(i,t)$
(M1)	$K_{it} = \sum_{k\in\{a,m,s\}}K_{it}^k$	$\forall(i,t)$
(M2)	$L_{it} = \sum_{k\in\{a,m,s\}}L_{it}^k$	$\forall(i,t)$
(M3)	$Q_{it}^k = C_{it}^k + X_{it}^k + \sum_{\ell\in\{a,m,s\}}M_{it}^{k\ell}$	$\forall(k,i,t)$
(M4)	$T_{it}^L = w_{it}\sum_{k\in\{a,m,s\}}\tau_{it}^k L_{it}^k$	$\forall(i,t)$
(M5)	$\sum_{i=1}^{I}L_{it}T_t^P = \sum_{i=1}^{I}\phi_{it}(w_{it}L_{it} + r_{it}K_{it} + T_{it}^L)$	$\forall(t)$

Notes: $u_{jt}^k = \left(\frac{r_{jt}}{\alpha\nu_{jt}^k}\right)^{\alpha\nu_{jt}^k}\left(\frac{(1+\tau_{jt}^k)w_{jt}}{(1-\alpha)\nu_{jt}^k}\right)^{(1-\alpha)\nu_{jt}^k}\left(\prod_{\ell\in\{a,m,s\}}\left(\frac{P_{jt}^{\ell}}{\mu_{jt}^{k\ell}}\right)^{\mu_{jt}^{k\ell}}\right)^{1-\nu_{jt}^k}$.