Fiscal Origins of Monetary Paradoxes

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Abstract

We revisit the monetary paradoxes of standard monetary models in a liquidity trap and study the channels through which they occur. We focus on two paradoxes: the Forward Guidance Puzzle and the Paradox of Flexibility. First, we propose a decomposition of consumption into *substitution* and *wealth effects*, both of which take into account the general equilibrium effects on output and inflation, and we show that the substitution effect cannot account for the puzzles. Instead, monetary paradoxes are the result of strong wealth effects which, generically, are solely determined by the expected fiscal response to the monetary shocks. We estimate the fiscal response to monetary policy shocks with US data and find responses with the opposite sign to the ones implied by the standard equilibrium. Finally, we introduce the estimated fiscal responses into a medium-size DSGE model. We find that the impulse-response of consumption and inflation do not match the data, suggesting that wealth effects induced by fiscal policy may be important even outside of the liquidity trap. We show that models with constrained agents can produce strong wealth effects if gross private debt is different than zero.

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1 Introduction

In the aftermath of the financial crisis that started in 2008, many central banks reached the effective lower bound on nominal interest rates, leading them to search for alternative, unconventional, instruments. An alternative that has received considerable attention is ”forward guidance”, i.e., promises of future interest rate changes in an attempt to affect current macroeconomic conditions. However, when central banks and academics turned to the standard model of monetary analysis to evaluate the impact of such policies, it was found that forward guidance produces counterfactual responses, to the point many of the results of New Keynesian models in a liquidity trap are deemed as “paradoxes” or “puzzles”.[1] In particular, the prediction that changes in interest rates in the far distant future have arbitrarily large effects on current output is called the “The Forward Guidance Puzzle”.[2]

The extreme sensitivity of current conditions to events in a distant future led to a surge in work related to attenuating the forward looking nature of the New Keynesian model. A burgeoning literature has explored several mechanisms to dampen these forward looking effects, and intertemporal substitution effects in particular, through OLG models (Del Negro et al. (2015)), heterogeneity and incomplete markets (McKay et al. (2016)), deviations from common knowledge (Angeletos and Lian (2016)), behavioral

[1]Despite the prominance of the effects of forward guidance, several other puzzles have been identified, such as the “Paradox of Flexibility” (Eggertsson and Krugman (2012), Werning (2012)), the backloading of fiscal multipliers (Farhi and Werning (2016)), or the ”Paradox of Toil” (Eggertsson (2010), Wieland (2014)).

[2]The term was coined by Del Negro et al. (2015) who also found that the model predictions were at odds with estimates of the effect of forward guidance even for changes in a relatively short horizon.

agents (Gabaix (2016), Farhi and Werning (2017)), or adaptive expectations (Gertler (2017)), to cite a few recent exemples. Recognizing the general relevance of these mechanisms to analyze a range of macroeconomic questions, we propose an alternative diagnosis to the paradoxical results found in a liquidity trap scenario: powerful wealth effects typically caused by strong fiscal responses. Our analysis suggests that weakening intertemporal substitution effects is not necessary, and in many occasions not sufficient, to eliminate the counterfactual predictions of the model.

We start by showing how to decompose the consumption response of *any equilibrium* of the standard New Keynesian model into *substitution* and *wealth effects*. It is well understood that the Forward Guidance Puzzle is the result of general equillibrium effects.[3] Hence, it is important to go beyond the standard decomposition in partial equilibrium and extend it to a general equilibrium setting, where, for instance, the substitution effect is consistent with the inflationary consequences of substituting consumption intertemporally. Our first main result is that, in the absence of wealth effects, the equilibrium does not present any of the paradoxical results: even after fully taking into account general equilibrium effects on output and inflation, the effect of changes in interest rates is *reduced* with the horizon of the intervention and the equilibrium is continuous in the price flexibility parameter.

The importance of wealth effects remains even if we take into account some of the mechanisms proposed to reduce the role of forward looking aspects of the model. In particular, many of the proposed formulations boil down to a modified version of the New Keynesian model with a *discounted Euler equation*, as in McKay et al. (2017). As long the discounting is not too large, such that the equilibrium remains indeterminate under an interest

[3] See, for instance, Angeletos and Lian (2016) and Kaplan et al. (2017).

rate peg, the puzzles are *attenuated*, but not eliminated.[4] We extend our decomposition to an economy with a discounted Euler equation and show that again, in the absence of wealth effects, the puzzles are eliminated.

Given the relevance of wealth effects in the counterfactual predictions of the model, it is important to understand how wealth effects are determined. In the knife-edge case where government debt and proportional taxation are equal to zero, such that monetary actions have no fiscal consequence, wealth effects are purely self-fulfilling. Expectations for a path of output consistent with a higher value for (physical and human) wealth will increase demand, increasing output, and confirming the initial expectation. In the case where either government debt or proportional taxes are different from zero, however small, the response will depend on the expectation of the fiscal response.

Generically, wealth effects are determined by a mechanism we call the *Intermporal Keynesian Cross*, given that its logic is reminiscent of undergraduate textbook analysis. The effect of autonomous movements in wealth, components that are not directly a function of the expected path of output, as expected fiscal transfers or the revaluation of bonds, is amplified in a similar way as autonomous variations in income are amplified in the standard Keynesian cross. We find that the powerful wealth effects required by the liquidity trap equilibrium cannot be generated by the revaluations of bonds, but it is the result of expected fiscal transfers.

Our results are consistent with the findings of Cochrane (2017) and **?** who argue that the puzzles are eliminated by introducing the Fiscal Theory of the Price Level. In his case, the only wealth effects are the ones generated by bond revaluations, which are not enough to create the puzzling

[4] Diba and Loisel (2017) also stress this point.

outcomes. In contrast, we emphasize that whether we are in a monetary or fiscal dominance regime is immaterial, as it is the expected equilibrium behavior of the fiscal authority that matters, not the off-equilibrium interactions of the fiscal and monetary authority. Hence, we view our contribution as complementary to Cochrane's work.

Finally, we analyze whether the channels emphasized in our analysis are a feature of the zero lower bound or they are relevant to understand the monetary policy transmission mechanism in normal times as well. In particular, we study the role of wealth effects and fiscal responses to monetary shocks in a medium-scale DSGE model, as in Smets and Wouters (2007). First, we show that the implicit transfers necessary to sustain the standard equilibrium play an important role in the quantitative predictions of the model. Absent the transfers, the impulse response functions for output and inflation tend to display the opposite signs than with transfers. Next, we ask what is the actual fiscal response to monetary shocks in the data. We use the high-frequency identification approach adopted in Gertler and Karadi (2015), augmented to account for fiscal variables. We find that not only the size, but the sign of the estimated fiscal response go against the ones implied by the standard equilibrium. As a final step, we feed the medium-scale DSGE model with the estimated monetary and fiscal impulse response functions, and compute the resulting dynamics of output and inflation. It is worth emphasizing that this exercise is a test of the New Keynesian model that does not rely on policy rules but instead imposes the observed path of monetary and fiscal variables as restrictions, and hence is independent of the debate about fiscal and monetary dominance. We find that by adding the additional constraint on fiscal policy, the model has difficulty in generating reasonable impulse responses of consumption and inflation to monetary shocks. This finding differs from the result in Kaplan et al. (2017) that the main transmission mechanism of monetary policy in the representative agent New Keynesian model is the intertemporal Euler equation.[5] We conclude that it may be useful to explore models with a richer portfolio structure in order to generate richer wealth effects in the absence of strong fiscal response. In particular, we show that a medium-scale DSGE model augmented by a heterogeneous agents model that produces indebted hand-to-mouth improves the quantitative predictions of the New Keynesian model.

The rest of the paper is organized as follows. Section 2 presents the monetary paradoxes in the context of a continuous-time New Keynesian model. Section 3 presents the equilibrium decomposition into substitution and wealth effects, first with rigid prices, then with sticky prices. Section 4 discuss the determination of wealth effects and the fiscal origins of the monetary paradoxes. Section 5 presents the analysis of these channels outside of the zero lower bound. Section 6 concludes.

2 The Standard Liquidity Trap Equilibrium

We develop a simple New Keynesian model in continuous time in the spirit of Werning (2012) and Cochrane (2017), augmented to incorporate fiscal variables and explicitly account for the households' budget constraint. The objective of this section is to present the monetary paradoxes that characterize the standard Liquidity Trap equilibrium, i.e., the Forward Guidance Puzzle and the Paradox of Flexibility, and show how fiscal variables, in particular lump-sum transfers, adjust in the background.

[5] In footnote 11, Kaplan et al. (2017) quote John Cochrane and suggest that the standard New Keynesian model could be renamed as "sticky-price intertemporal substitution model".

Time is continuous and denoted by $t \in \mathbb{R}_+$. There are two types of agents in the economy: a large number of identical, infinitely-lived households, and an infinitely-lived government. There is also a continuum of mass one of firms that produce a differentiated good. Households' preferences are such that the final consumption is a CES aggregator of the purchases of each of the differentiated goods. In the benchmark model, we assume that the government does not consume, but it raises proportional sales taxes, issues short-term nominal debt (which is in positive net supply in steady-state) and distributes lump-sum transfers (which can be negative).

The main focus of this paper is to understand the sources of the monetary paradoxes when the economy is in a Liquidity Trap. As is standard in the literature, we log-linearize the model around its steady-state equilibrium in order to study the first-order approximation of the equilibrium response of the economy to exogenous shocks. Since the model is linear, we do not need to introduce the shock that takes the economy to a Liquidity Trap. We then study the equilibrium of the economy as if there was an interest rate peg.[6] The joint dynamics of a shock that takes the economy to a Liquidity Trap and our exercises would simply be the sum of the two.[7]

The log-linearized solution to the model can be characterized by four equations: an intertemporal Euler equation

$$\dot{c}_t = \sigma^{-1}(i_t - \pi_t - \rho), \tag{1}$$

[6] See Angeletos and Lian (2016) for a similar strategy.

[7] For a detailed exposition of the dynamics of the economy after a shock that leads to a Liquidity Trap, see Werning (2012) and Cochrane (2017).

a New Keynesian Phillips Curve

$$\dot{\pi}_t = \rho \pi_t - \kappa c_t, \tag{2}$$

the household's intertemporal budget constraint

$$\int_0^\infty e^{-\rho t} c_t dt = \int_0^\infty e^{-\rho t} \left[(1-\overline{\tau}) y_t + \overline{b}(i_t - \pi_t - \rho) + T_t \right] dt, \tag{3}$$

and a resource constraint

$$c_t = y_t, \tag{4}$$

plus a policy rule for monetary and fiscal policies. Here, c_t denotes the percentage difference between actual consumption and the level of consumption in a steady state that features a constant path for the policy variables and zero inflation; π_t denotes inflation; i_t denotes the nominal, short-term, risk-free interest rate; $\overline{\tau}$ is the steady state level of proportional sales taxes; $\overline{b}$ is the steady state level of short-term government debt; T_t is a lump-sum transfer expressed as a fraction of output; σ denotes the inverse of the intertemporal elasticity of substitution; ρ denotes the subjective discount factor of the households; and κ is the slope of the Phillips curve.

Because our analysis emphasizes the role of the household's budget constraint in the dynamic behavior of consumption, it is useful to briefly describe its components. The left-hand side of the household's budget constraint is the present value of consumption. The right-hand side are the sources of income: the after-tax wage and profits, the interest from financial assets, and government's lump-sum transfers. In the standard analysis, equation (3) can be dropped when finding an equilibrium because transfers $\{T_t\}_{t=0}^\infty$ are assumed to automatically adjust so that the government's budget constraint is always satisfied for any path of the endogenous and exogenous variables (the so-called Ricardian fiscal policy).[8] Since lump-sum transfers do not affect any of the other equilibrium equations, they provide a free variable that guarantees that any solution to the system given by (1) and (2) can be an equilibrium of the economy. Given the focus of our analysis, we will explicitly account for the presence of the budget constraint and explore the role of each component of income in the dynamic behavior of consumption.

The system of differential equations (1)-(2) can be written as

$$\begin{bmatrix} \dot{c}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} 0 & -\sigma^{-1} \\ -\kappa & \rho \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1}(i_t - \rho) \\ 0 \end{bmatrix}.$$

The eigenvalues of the system above are given by

$$\overline{\omega} = \frac{\rho + \sqrt{\rho^2 + 4\kappa\sigma^{-1}}}{2}, \quad \underline{\omega} = \frac{\rho - \sqrt{\rho^2 + 4\kappa\sigma^{-1}}}{2}.$$

Notice that the system has a positive and a negative eigenvalue. Focusing on bounded solutions, we need one additional condition to determine equilibrium. The liquidity trap literature has used that the economy arrives to its steady state at some finite time T^*, potentially in the far future, as such condition.[9] That is, there exists T^* such that

$$i_t = \rho \quad \text{and} \quad c_t = \pi_t = 0 \quad \forall t \geq T^*.$$

In this section, we stick to this selection. The next Lemma characterizes the

[8] Ricardian equivalence holds in this model, so only the present value of transfers, $\int_{t=0}^{\infty} e^{-\rho t} T_t dt$, rather than the whole path, $\{T_t\}_{t=0}^{\infty}$, matters for the equilibrium.

[9] See, e.g., Werning (2012) and McKay et al. (2016).

solution and shows some of its properties.

Lemma 1 (Standard Liquidity Trap Equilibrium)**.** *Consider the New Keynesian model with the standard Liquidity Trap equilibrium selection. The response of initial consumption to a monetary shock is given by*

$$c_0^{NK} = -\frac{\kappa\sigma^{-1}}{\sigma(\overline{\omega} - \underline{\omega})} \int_0^{T^*} \left(\frac{e^{-\overline{\omega}t}}{\overline{\omega}} - \frac{e^{-\underline{\omega}t}}{\underline{\omega}} \right) (i_t - \rho) dt, \tag{5}$$

while the response of initial inflation is given by

$$\pi_0^{NK} = -\frac{\kappa\sigma^{-1}}{\overline{\omega} - \underline{\omega}} \int_0^{T^*} (e^{-\underline{\omega}t} - e^{-\overline{\omega}t})(i_t - \rho) dt.$$

Moreover, initial consumption and inflation are decreasing in the nominal interest rate at all horizons,

$$\frac{\partial c_0^{NK}}{\partial i_t} < 0, \qquad \frac{\partial \pi_0^{NK}}{\partial i_t} < 0, \qquad \forall t > 0.$$

Two characteristics of the solution are worth mentioning. First, in the standard equilibrium, a monetary shock that increases the nominal interest rate has a contractionary effect in the economy, reducing initial consumption and inflation. Second, fiscal policy, given by $\overline{\tau}$, $\overline{b}$ and $\{T_t\}_{t=0}^{\infty}$, is irrelevant for the dynamic response to monetary shocks in the standard Liquidity Trap equilibrium as long as the government's intertemporal budget constraint is satisfied. However, as we show below, fiscal policy plays a role in determining the channels through which monetary policy operates.

Next, we present the monetary puzzles and paradoxes the literature has identified and formally show that they are present in the standard Liquidity Trap equilibrium. Suppose that the central bank reduces the short-term nominal interest rate in some period $t < T^*$ (or credibly announces that it will reduce it). Lemma 1 states that consumption and inflation increase at the time of the news. How does the response of consumption depend on the time of the intervention and the degree of price rigidity in the economy? The Forward Guidance Puzzle refers to the theoretical result that the promise to reduce the interest rates in the future becomes more powerful the further in the future the actual intervention takes place. In the extreme, the response of consumption becomes unboundedly large as the horizon of the policy intervention goes to infinity. Another counter-intuitive result attributed to the New Keynesian model is that the effect of monetary policy shocks become stronger as price flexibility increases. As we approach the flexible price limit, monetary policy becomes unboundedly strong. Since shocks to the nominal interest rate have no impact on real variables in a flexible price economy, this result is known as the Paradox of Flexibility. Proposition 1 formally shows that the Forward Guidance Puzzle and the Paradox of Flexibility are present in the standard Liquidity Trap equilibrium.

Proposition 1 (The Standard Liquidity Trap Equilibrium and its Paradoxes)**.** *Consider the standard Liquidity Trap equilibrium and let $t < T^*$. The economy presents the following dynamics:*

i) Forward Guidance Puzzle

$$\frac{\partial^2 c_0^{NK}}{\partial t \partial i_t} < 0, \qquad \lim_{t\to\infty} \lim_{T^*\to\infty} \frac{\partial c_0^{NK}}{\partial i_t} = -\infty,$$
$$\frac{\partial^2 \pi_0^{NK}}{\partial t \partial i_t} < 0, \qquad \lim_{t\to\infty} \lim_{T^*\to\infty} \frac{\partial \pi_0^{NK}}{\partial i_t} = -\infty,$$

ii) Paradox of Flexibility

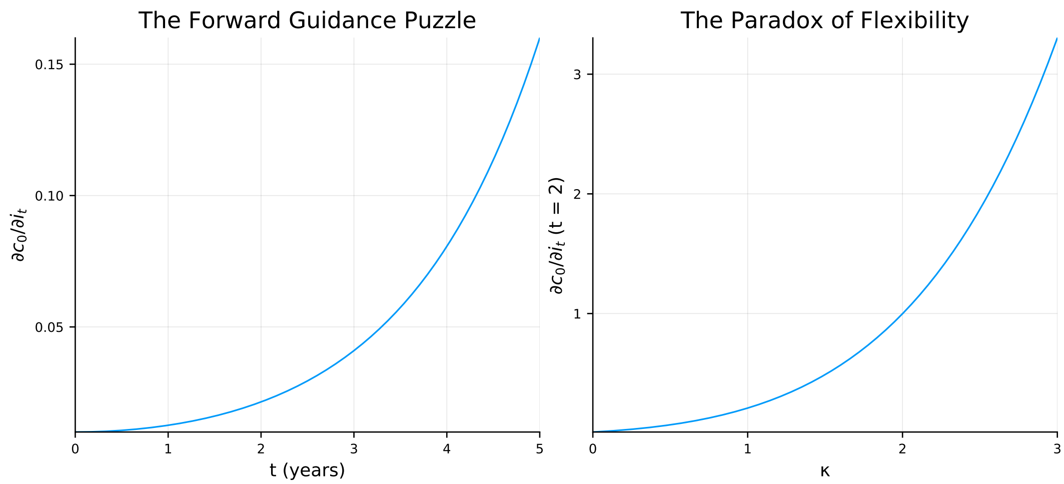

FIGURE 1: The Forward Guidance Puzzle and The Paradox of Flexibility

$$\frac{\partial^2 c_0^{NK}}{\partial \kappa \partial i_t} < 0, \quad \lim_{\kappa \to \infty} \frac{\partial c_0^{NK}}{\partial i_t} = -\infty$$
$$\frac{\partial^2 \pi_0^{NK}}{\partial \kappa \partial i_t} < 0, \quad \lim_{\kappa \to \infty} \frac{\partial \pi_0^{NK}}{\partial i_t} = -\infty.$$

Figure 1 shows graphically these results. The standard narrative is as follows. A reduction of the nominal interest rate translates into a reduction of the real interest rate due to nominal rigidities. This reduction in the real rate generates a boom in consumption and an increase in inflation in the period of the intervention. Because both the Euler equation and the New Keynesian Philips curve are forward looking, the future boom translates into a contemporaneous boom. Moreover, since the effect of inflation accumulates over time, the furthest in the future the policy intervention is, the more inflation it generates in the preceding periods. And because nominal interest rates in previous periods are kept fixed due to the liquidity trap (or the interest rate peg), this implies a larger reduction in real rates and a larger boom in consumption today, i.e., the Forward Guidance Puzzle. Fur-

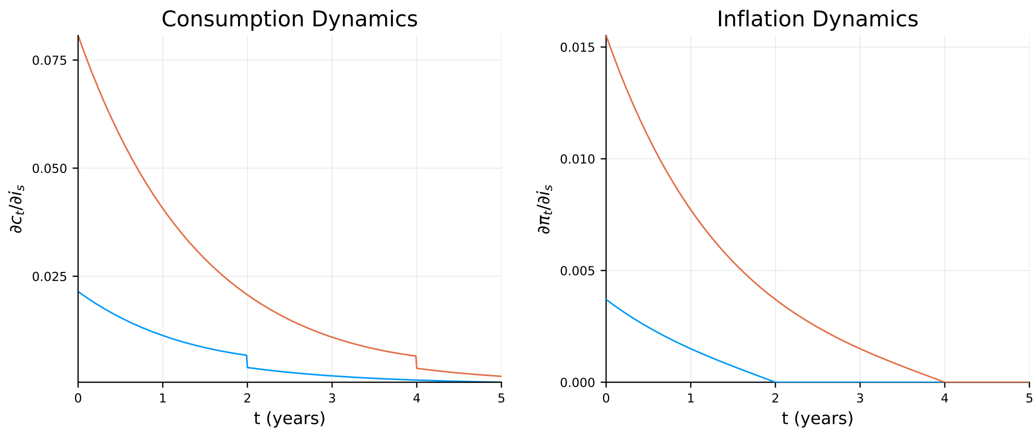

FIGURE 2: Consumption and Inflation Dynamics after a Monetary Shock

thermore, since nominal rigidities are preventing some prices to increase, the more flexible prices are, the more inflation the monetary shocks generates, and hence the more powerful the effect on consumption is. Figure 2 shows the dynamics of consumption and inflation for and expansionary monetary shock in years 2 and 4.

Given this logic, it is natural that the literature has focused on solutions to these puzzles that reduce the forward looking nature of consumption and inflation (e.g., the so-called ”discounting Euler equation”). However, as we show in the next section, most of the solutions can at most attenuate the effects, but do not eliminate them. In contrast, we show that the nature of the puzzles is associated with strong wealth effects which are generically the result of strong fiscal policy responses.

Next, we present a decomposition of households’ behavior which allows us to identify the sources of the paradoxes.

3 Consumption Decomposition: Substitution and Wealth Effects

In this section, we take the point of view of the households and decompose their behavior adapting the tools from consumer theory to general equilibrium analysis. In particular, we decompose c_t into two pieces: a substitution effect and an income/wealth effect. By doing so, we are able to formally identify what components of the households' problem is generating the paradoxical results from Proposition 1.

To this end, we present two objects that will be important in this characterization that follows. First, for a given path of the nominal interest rate and inflation, $\{i_t, \pi_t\}_{t=0}^{\infty}$, we define c_t^H as the household's *Hicksian demand*, which is given by

$$c_t^H \equiv \sigma^{-1} \int_0^t (i_s - \pi_s - \rho) ds - \sigma^{-1} \int_0^{\infty} e^{-\rho s} (i_s - \pi_s - \rho) ds. \tag{6}$$

Equation (6) is the log-linear approximation of the solution to the minimization of the household's expenditures subject to achieving a reservation utility. In this setting, the different goods are the consumption at different dates, and the price of one unit of consumption at date t is $e^{-\int_0^t (i_s - \pi_s - \rho) ds}$. An important property of the Hicksian demand is that the total cost of $\{c_t^H\}_{t=0}^{\infty}$ evaluated at the steady state prices is zero, that is

$$\int_0^{\infty} e^{-\rho t} c_t^H dt = 0. \tag{7}$$

The Hicksian demand will be tightly connected to the substitution effect in general equilibrium.

The second object we define here is the average consumption, C,

$$C \equiv \rho \int_0^\infty e^{-\rho t} \left[(1-\overline{\tau}) y_t + \overline{b}(i_t - \pi_t - \rho) + T_t \right] dt. \tag{8}$$

Average consumption is the consumption path that would prevail if the households were forced to consume the same amount every period while still satisfying their budget constraint. Average consumption will be related to the wealth effect.

The rest of this section analyzes the role of the substitution and wealth effects in generating the dynamics of consumption in the New Keynesian model. We do this in two steps. First, we study an economy with fixed prices. This economy provides a useful benchmark by shutting down an important general equilibrium feedback effect that works through inflation. Next, we allow for prices to move and show that the insights from the fixed prices case are amplified in general equilibrium.

3.1 Rigid Prices

Suppose prices are fixed, i.e., $\kappa = 0$. Solving the Euler equation forward, and imposing $\lim_{t\to\infty} c_t = 0$, we get that in the standard Liquidity Trap equilibrium

$$c_t = -\sigma^{-1} \int_t^\infty (i_s - \rho) ds. \tag{9}$$

Consider the effect of a one time reduction of the interest rate at time $s > 0$. Equation (9) implies that consumption increases by $\sigma^{-1}\Delta i_s$ in all periods $t \leq s$, and goes back to zero afterwards. Moreover, the effect on initial

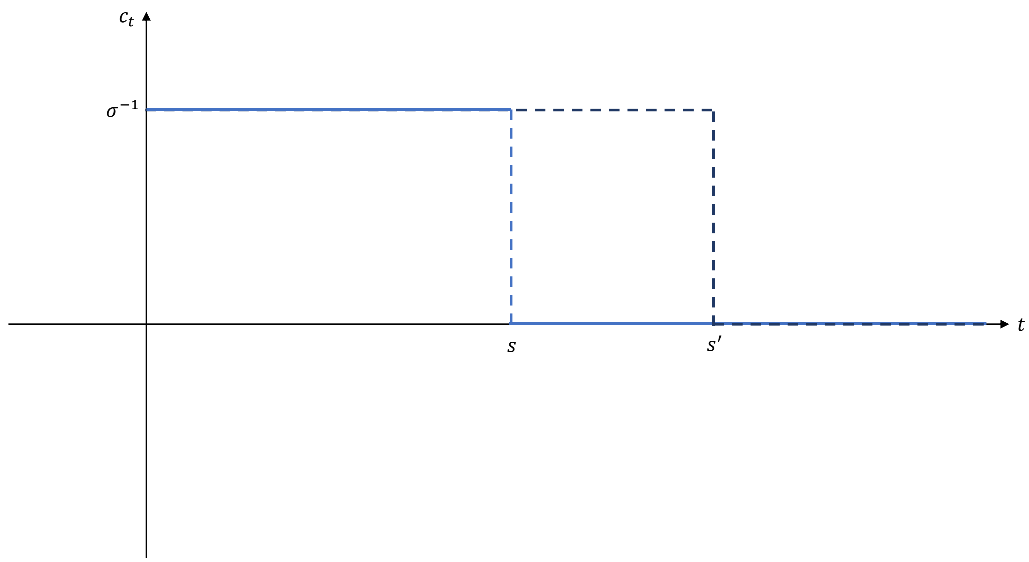

FIGURE 3: Consumption dynamics to a reduction in the future interest rate when prices are rigid.

consumption is independent of the time of the shock,

$$\frac{\partial c_0^{NK}}{\partial i_s} = -\sigma^{-1} \qquad \forall s. \tag{10}$$

Thus, the fixed price case has an attenuated form of the forward guidance puzzle: the time of the intervention is irrelevant for its effect on initial consumption. Note that this does not mean that the time of intervention is irrelevant for the whole *path* of consumption. In fact, the further in the future the intervention takes place, the longer the time that consumption remains above its steady state level. Figure 3 depicts this case.

Despite the independence of the effect of the time of the monetary shock on initial consumption, the channels through which monetary policy affects consumption do vary with the horizon of the intervention. To see this, we decompose consumption into substitution and wealth effects. Under fixed prices, we define the substitution effect as the Hicksian demand evaluated at $\pi_t = 0 \; \forall t$, while the wealth effect is the average consumption, C. It holds that equilibrium consumption is equal to the sum of the substitution and wealth effects. The next Lemma formalizes this result.

Lemma 2 (Substitution and Wealth Effects with Fixed Prices)**.** *Suppose* $\kappa = 0$. *In the standard Liquidity Trap equilibrium, consumption can be decomposed as*

$$c_t^{NK} = \underbrace{\sigma^{-1}\int_0^t (i_s - \rho)ds - \sigma^{-1}\int_0^\infty e^{-\rho s}(i_s - \rho)ds}_{\text{substitution effect}} + \underbrace{\sigma^{-1}\int_0^\infty (e^{-\rho s} - 1)(i_s - \rho)ds}_{\text{wealth effect}}, \quad (11)$$

where the substitution effect equals the Hicksian demand defined in (6) *evaluated at* $\pi_t = 0 \; \forall t$, *and the wealth effect equals the average consumption defined in* (8).

We use this decomposition to study the channels through which monetary policy affects consumption. Consider an increase of the interest rate in period s. We know from (10) that the effect on initial consumption is independent of s. Let's decompose the total effect into substitution and wealth effects. The substitution effect is given by

$$\frac{\partial c_t^H}{\partial i_s} = \begin{cases} -\sigma^{-1}e^{-\rho s} < 0 & \text{if } t < s, \\ \sigma^{-1}(1 - e^{-\rho s}) > 0 & \text{if } t \geq s. \end{cases}$$

These formula shows two things. First, the substitution effect of an increase in the interest rate in s is negative for $t < s$ and positive afterwards. This is the standard result from consumer theory: the substitution effect always is negative This effect is depicted in Figure 4(a) for a negative interest rate shock in $s = 2$. Second, the substitution effect of initial consumption *decreases* with the horizon of the intervention, and vanishes in the limit. Formally,

$$\frac{\partial^2 c_0^H}{\partial s \partial i_s} = \rho\sigma^{-1}e^{-\rho s} > 0, \qquad \lim_{s\to\infty}\lim_{T^*\to\infty}\frac{\partial c_0^H}{\partial i_s} = 0.$$

The intuition for this result is as follows. While the size of the change of total consumption at date s depends only on the elasticity of intertemporal substitution, how much of the adjustment will fall on current versus future consumption depends on the marginal rate of substitution (MRS) between consumption in these two dates. Since the marginal utility of future consumption declines according to $e^{-\rho t}$, the indifference curve between consumption in $t = 0$ and consumption in the future gets flatter over time. Hence, a smaller change in initial consumption will be necessary to keep the same utility level.[10]

That is, the *intertemporal substitution* channel gets smaller as the time of the intervention increases. Thus, absent wealth effects, this model predicts a negative relation between the time of the intervention and the size of the effect on initial consumption and, therefore, does not feature the Forward Guidance Puzzle.

However, we know that the response of initial consumption to changes in the interest rate does not depend on the time of the intervention, so it must be that the wealth effect is picking up the slack. Figure 4(b) shows a graphical representation of the channel. The reduction of the interest rate in period $s = 2$ generates a positive substitution effect in all periods before s and a negative substitution effect afterwards. Since the standard Liquidity

[10]This can be easily seen in a two good example. If σ^{-1} is the elasticity of substitution between goods x and y, and we start at a point where $x = y$, then $\frac{\partial \log x}{\partial p_y/p_x} = \frac{MRS}{1+MRS}\sigma^{-1}$.

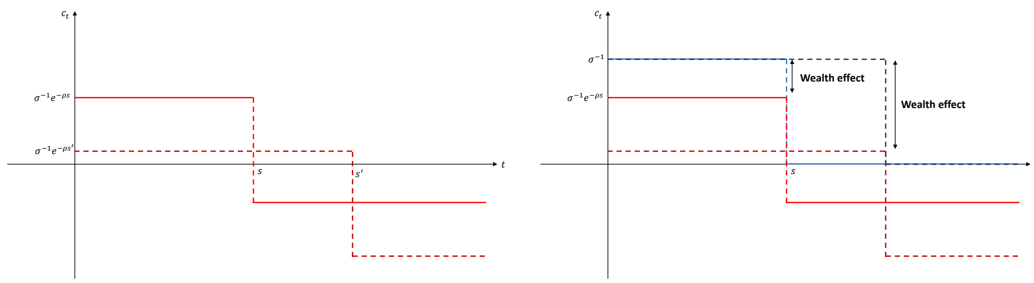

FIGURE 4: Substitution and Wealth Effects with Fixed Prices

Trap equilibrium implies that consumption has to be at its steady state level at s (since consumption is constant afterwards), the wealth effect needs to be exactly the negative of the substitution effect after s. As the time of the intervention s moves further into the future, the substitution effect in 0 decreases, but the substitution effect after s increases in absolute value. Therefore, the wealth effect has to be larger to compensate for the negative substitution effect and allow consumption to go back to the steady state level, moving the whole consumption path upwards.

To fix ideas, consider the following example.[11] Suppose there is an unexpected monetary shock in period 0 that takes the following form:

$$i_t = \begin{cases} \rho & \text{for } t < z, \\ \rho + e^{-\eta(t-z)}(r - \rho) & \text{for } t \geq z, \end{cases}$$

where $r > \rho$, and z is known to the households. Note that $z = 0$ corresponds to a contemporaneous monetary shock with persistence governed by η. For $z > 0$ it becomes a forward guidance shock. Replacing this path for the

[11]This is an extension of an example in Kaplan et al. (2017).

interest rate in (11) at $t = 0$, we get

$$c_0^{NK} = -\left[\underbrace{\frac{\sigma^{-1}}{\eta}\frac{\eta e^{-\rho z}}{\rho+\eta}}_{\text{substitution effect}} + \underbrace{\frac{\sigma^{-1}}{\eta}\frac{\rho+\eta(1-e^{-\rho z})}{\rho+\eta}}_{\text{wealth effect}}\right](r-\rho).$$

For $z = 0$, the relative importance of the substitution and the wealth effect is governed by the relative magnitudes of η and ρ. Kaplan et al. (2017) calibrate this case and argue that for reasonable values of η and ρ, contemporaneous shocks are mostly driven by the substitution effect. However, we can see that as z increases, the relative importance of the substitution effect decreases. In particular, as $z \to \infty$, the effect of a forward guidance shock is exclusively explained by the wealth effect.

Interestingly, the results in this subsection imply that even though the response of initial consumption to a monetary policy is independent of the timing of the intervention, the channel through which this happens varies with it. In particular, the substitution effect is more relevant for contemporaneous shocks while the wealth effect is the most important channel for forward guidance shocks. Moreover, we showed that it is the wealth effect the one that explains the non-decreasing response of initial consumption to the horizon of the policy intervention, and not a strong intertemporal substitution effect. Since average consumption is independent of preferences, we claim that, when prices are rigid, these results imply that the Forward Guidance Puzzle is not related to the Euler equation.

Next, we present the general case with $\kappa > 0$ and show that an even stronger result arises when prices are allowed to move.

3.2 Sticky Prices

Decomposing consumption into substitution and wealth effects is relatively easy when prices are fixed. Because there are no general equilibrium feedback effects through inflation, the substitution effect is simply the Hicksian demand evaluated at the new path for the nominal interest rate. However, in general equilibrium, the ”right” decomposition is less obvious. In particular, we need to take a stand on what inflation rate should the Hicksian demand be evaluated at. A natural answer is to evaluate it at the equilibrium inflation rate. However, the next example shows some conceptual difficulties of this approach, and justifies the choices we make for the rest of the paper.

Consider an economy that is at the zero inflation steady state. Suppose that households receive an unexpected positive endowment in period zero, $e_0 > 0$. This represents a positive *wealth shock*. As a consequence, the households want to increase their consumption in all periods. This behavioral response generates some inflationary pressures if nominal interest rates remain fixed. Now, let's decompose the response of consumption into substitution and wealth effects. If we were to use the equilibrium inflation to calculate the substitution effect, we would end up attributing a positive fraction of the response to it. However, we know, by construction, that the shock was a purely wealth effect. Since we want our decomposition to avoid this type of results, we propose a different direction. We define the substitution effect as the Hicksian demand for the observed path of nominal rates, $\{i_t\}_{t=0}^{\infty}$, and the inflation *induced* by the substitution effect, $\{\pi_t^H\}_{t=0}^{\infty}$. That is, $\{c_t^H, \pi_t^H\}_{t=0}^{\infty}$ is the solution to the following system of equations

$$c_t^H = \sigma^{-1} \int_0^t (i_s - \pi_s^H - \rho) ds - \sigma^{-1} \int_0^\infty e^{-\rho s} (i_s - \pi_s^H - \rho) ds, \tag{12}$$

$$\pi_t^H = \kappa \int_t^\infty e^{-\rho(s-t)} c_s^S ds. \tag{13}$$

where (13) is the New Keynesian Philips curve integrated forward. In the example above, since $i_t = \rho \ \forall t$, the only bounded solution to the system above has $c_t^H = 0 \ \forall t$.

It turns out that because the consumption bundle prescribed by the Hicksian demand has zero cost (relative to the steady state bundle; see equation (7)), our choices for the substitution effect can be motivated as being the solution to the system of equations (1)-(2) imposing no change in the households' wealth.

Lemma 3. *The solution to* (12) *and* (13) *is the solution to the system* (1)-(2) *and*

$$\int_0^\infty e^{-\rho t} c_t dt = 0.$$

Thus, we can justify our choice as being the solution to the basic system of differential equations that keeps the household's wealth fixed.

Next, we show that this decomposition is useful to understand the equilibrium dynamics of consumption and inflation in the basic New Keynesian model. In particular, we show how to decompose equilibrium consumption and inflation into substitution and wealth effects which resembles the standard decomposition in consumer theory, augmented to account for general equilibrium effects.

Proposition 2 (Substitution and Wealth Effects with Sticky Prices)**.** *Suppose*

$\kappa > 0$. *Consumption can be decomposed as*

$$c_t = \underbrace{c_t^H}_{\text{substitution effect}} + \underbrace{\underbrace{\frac{\overline{\omega}}{\rho} e^{\underline{\omega} t}}_{\text{GE multiplier}} \times \underbrace{C}_{\text{average consumption}}}_{\text{wealth effect}},$$

The GE multiplier is always greater than 1 *in* $t = 0$.

As it was evident in the example with the endowment shock, the wealth effect affects the economy through two different channels. The first channel is the standard partial equilibrium logic: when their wealth increases, households respond by increasing their consumption in all periods. But now there is a second, general equilibrium, channel, which takes into account that the direct effect has an impact on prices which indirectly affects consumption. In particular, a positive wealth shock generates inflation which translates into a reduction in the real interest rate when nominal rates are fixed. This is an extra positive force on initial consumption.

It turns out that the decomposition in Proposition 2 is not limited to the standard Liquidity Trap equilibrium, but it characterizes *all* the solutions to the system (1)-(2) for a given path of the nominal interest rate. This result is important because it provides a new perspective about the multiplicity of equilibria of the New Keynesian model under an interest rate peg. When we defined the substitution effect, we showed that, given a path for interest rates $\{i_t\}_{t=0}^{\infty}$, the solution to (12)-(13) is unique. This implies that the substitution effect is *common* to all equilibria. Moreover, the GE multiplier depends only on fundamental parameters of the economy, so it cannot vary across equilibria. This means that *all* equilibria of the New Keynesian model under an interest rate peg can be indexed simply by their effect on average consumption. In this sense, the standard Liquidity Trap equilibrium selects a particular wealth effect.

Lemma 4. *In the bounded solutions to the system* (1)-(3)*, consumption is given by*

$$c_t = c_t^H + \frac{\overline{\omega}}{\rho} e^{\underline{\omega} t} C,$$

and inflation is given by

$$\pi_t = \pi_t^H + \frac{\kappa}{\rho} e^{\underline{\omega} t} C \tag{14}$$

where $\{c_t^H, \pi_t^H\}_{t=0}^{\infty}$ *is the solution to* (12)-(13)*, and C is the average consumption as defined in* (8)*.*

For $t = 0$*, we have*

$$\pi_0 = \frac{\kappa}{\rho} C.$$

Moreover, all equilibria satisfy monetary neutrality in the long run.

Interestingly, inflation features a similar decomposition. This result helps understand the channels through which monetary policy affects inflation. Proposition 2 shows that initial inflation is simply determined by average consumption, since the inflation generated by the substitution effect in $t = 0$ is zero, $\pi_0^H = 0$. Thus, inflation in period 0 only depends only on whether agents are richer or poorer after the monetary policy shock, but independent of any consumption switching due to intertemporal substitution.

With this decomposition, we now analyze which of the components of consumption and inflation are responsible for the monetary paradoxes of Proposition 1. Since we know that $\frac{\partial^2 c_0}{\partial t \partial i_t} < 0$ and $\frac{\partial^2 \pi_0}{\partial t \partial i_t} < 0$, it has to be true that either the substitution effect or average consumption (or both) feature the monetary paradoxes as well. The main result of this section is that the substitution effect is a force *against* the paradoxes.

Proposition 3 (Substitution Effect with Sticky Prices)**.** *Suppose $\kappa > 0$ and let $t < T^*$. The substitution effect satisfies*

$$\frac{\partial c_0^H}{\partial i_t} < 0, \quad \frac{\partial^2 c_0^H}{\partial t \partial i_t} > 0, \quad \lim_{t\to\infty} \lim_{T^*\to\infty} \frac{\partial c_0^H}{\partial i_t} = 0, \quad \lim_{\kappa\to\infty} \frac{\partial c_0^H}{\partial i_t} = 0.$$

Proposition 3 implies that the paradoxes are not a result of general equilibrium amplification of intertemporal substitution. In the absence of wealth effects, an increase in interest rates shifts consumption from the present to the future. However, the effect becomes weaker as the date of the shock moves further into the future. Moreover, the substitution effect is continuous in the price flexibility parameter κ, in sharp contrast with the standard Liquidity Trap equilibrium.

Thus, both the fixed and sticky prices exercise identify the wealth effect, through the average consumption, as the channel that generates the monetary paradoxes. Taking the Forward Guidance Puzzle, we found that

$$\underbrace{\frac{\partial^2 c_0}{\partial t \partial i_t}}_{<0} = \underbrace{\frac{\partial^2 c_0^H}{\partial t \partial i_t}}_{>0} + \underbrace{\frac{\overline{\omega}}{\rho}}_{>0} \times \underbrace{\frac{\partial^2 C}{\partial t \partial i_t}}_{\Rightarrow <0}.$$

In the next section, we dig deeper into the average consumption and study the dynamics of each one of its components, with a special interest in the fiscal variables. But before that, we take a short detour to compare our findings to the literature. By identifying average consumption as the responsible for the monetary paradoxes, and since average consumption does not depend on preferences, this result casts doubts about whether the ”discounted Euler equation” principle can resolve the monetary paradoxes of the New Keynesian model. Next, we show that, in fact, the solutions proposed in the literature that are based on the ”discounted Euler equation” principle cannot successfully get rid of the paradoxes but they attenuate them.

3.3 The Discounted Euler Equation

A range of different economic environments have been proposed attempting to solve the Forward Guidance Puzzle and related paradoxes. For instance, McKay et al. (2017) propose a heterogenous agent model with incomplete markets, Angeletos and Lian (2016) relax the assumption of common knowledge, Gabaix (2016) introduces (behavioral) inattention, and Gertler (2017) adaptive expectations. Perhaps surprisingly, despite the vastly different microfoundations, all these proposed solutions essentially boil down to changes in the aggregate Euler equation, where the direct impact of future interest rate changes is attenuated.

In order to illustrate the mechanism, we consider a version of the heterogeneous agent model in McKay et al. (2017), henceforth MNS, but our results apply to any environment that generates an aggregate discounted Euler equation. Households now face uninsurable idiosyncratic risk and they cannot borrow. Low-productivity households cannot produce, but they receive a government transfer. All the labor supply is provided by the high-productivity workers. A household of type $j \in \{H, L\}$ switches type with Poisson intensity $\lambda_j \geq 0$. There is no liquidity in this economy, i.e., government bonds are equal to zero at all dates, $B_t = 0\ \forall t$. In equilibrium, the high-productivity households are unconstrained and their Euler equation is given by

$$\frac{\dot{C}_{H,t}}{C_{H,t}} = \frac{r_t - \rho}{\sigma} + \frac{\lambda_H}{\sigma}\left(\frac{C_{L,t}^{-\sigma} - C_{H,t}^{-\sigma}}{C_{H,t}^{-\sigma}}\right)$$

The second term in the expression captures the *self-insurance* motive, and it will imply that consumption reacts less strongly to future real interest rate changes. By linearizing the Euler equation around a symmetric steady state, and assuming that the transfer to low-productivity households stays at the steady state level as in MNS[12], we can derive the *discounted Euler equation*:

$$\dot{c}_t = \delta c_t + \zeta\sigma^{-1}(r_t - \rho) \tag{15}$$

where $\zeta = \frac{\lambda_L}{\lambda_L + \lambda_H}$, $\delta = \zeta\lambda_H$, and c_t denotes deviations of aggregate consumption from its steady state level.

The parameter δ controls the amount of discounting, and it is zero in the special case where $\lambda_H = 0$, when we recover the representative agent model. Integrating the discounted Euler equation forward, we obtain

$$c_t = -\zeta\sigma^{-1}\int_t^\infty e^{-\delta(s-t)}(i_s - \pi_s - \rho)ds.$$

Despite the extra discounting, moderate values of δ are not enough to get rid of the puzzles, even though it attenuates its impact. The following proposition shows that the Forward Guidance Puzzle and the Paradox of Flexibility are still present even with a discounted Euler equation.

Proposition 4. *Suppose* $0 < \delta < \frac{\kappa\zeta\sigma^{-1}}{\rho}$.[13] *Let* c_t^{DE} *denote consumption in an*

[12] Bilbiie (2017a) shows that to obtain a discounted Euler equation the consumption of low-productivity households must react less than one-to-one to aggregate income. See also Werning (2015) for the role of the cyclicality of income on the aggregate Euler equation.

[13] The upper bound on δ guarantees that there is multiplicity of equilibrium under an interest rate peg, as in the standard New Keynesian model. For a standard calibration, the

equilibrium with the discounted Euler equation (15)*, and apply the standard equilibrium selection. Then,*

$$\frac{\partial c_0^{DE}}{\partial i_t} < 0, \qquad \lim_{t\to\infty} \frac{\partial c_0^{DE}}{\partial i_t} = -\infty, \qquad \lim_{\kappa\to\infty} \frac{\partial c_0^{DE}}{\partial i_t} = -\infty, \qquad \lim_{t\to\infty} \frac{\frac{\partial c_0^{DE}}{\partial i_t}}{\frac{\partial c_0^{NK}}{\partial i_t}} = 0$$

Changes in the nominal interest rates in the far distant future still have arbitrarily large effects on consumption, but this effect grows at a smaller rate than in the case with the standard Euler equation. Decomposing the consumption allocation between a wealth effect $C^{DE} = \rho \int_0^\infty e^{-\rho t} c_t^{DE} dt$ and a substitution effect may help provide some intuition for this result.

Proposition 5. *Suppose* $0 < \delta < \frac{\kappa \zeta \sigma^{-1}}{\rho}$*. Let* c_t^{DE} *denote consumption in an equilibrium with the discounted Euler equation* (15) *and arbitrary equilibrium selection. Then,*

1. *Consumption decomposition:*

$$c_t^{DE} = c_t^{DE,H} + \frac{\overline{\omega}_d - \delta}{\rho} e^{\underline{\omega}_d t} C^{DE} \tag{16}$$

where $\overline{\omega}_d > \delta$ *and* $\underline{\omega}_d < 0$ *are parameters defined in the appendix.*

2. *Substitution effect:*

$$\frac{\partial c_0^{DE,H}}{\partial i_t} < 0, \quad \lim_{t\to\infty} \frac{\partial c_0^{DE,H}}{\partial i_t} = 0, \quad \lim_{\kappa\to\infty} \frac{\partial c_0^{DE,H}}{\partial i_t} = 0, \quad \int_0^\infty e^{-\rho t} c_t^{DE,H} dt = 0$$

As before, any equilibrium in the economy with a discounted Euler equation can be decomposed into a substitution effect $c_t^{DE,H}$ and a wealth effect C^{DE}. Also as before, the substitution effect can be interpreted as an

value of δ is more than one order of magnitude smaller than the bound.

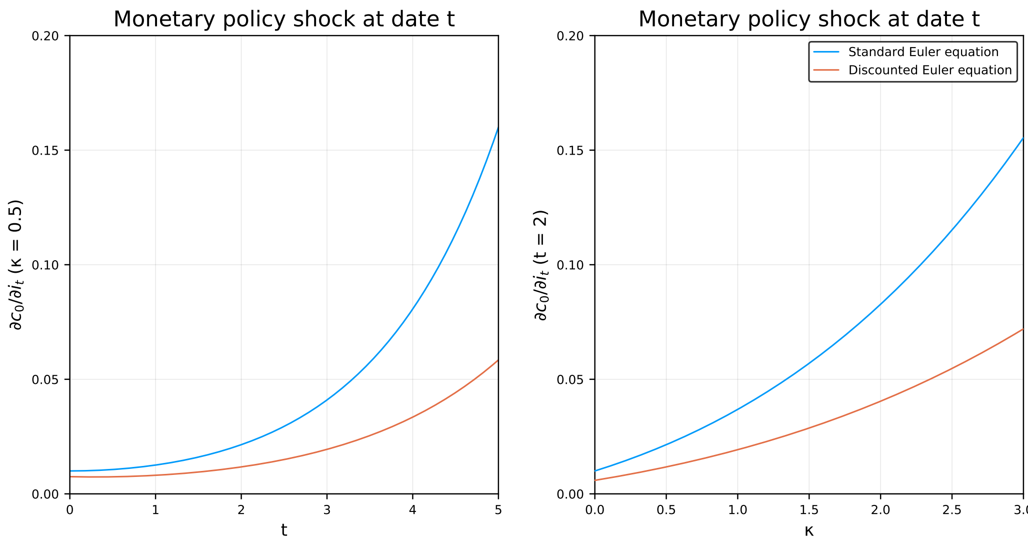

FIGURE 5: The Forward Guidance Puzzle with a Discounted Euler Equation

equilibrium of the model when average consumption is equal to zero: intertemporal substitution only reallocates consumption over time. As a consequence, even with a discounted Euler equation, monetary paradoxes are the result of strong wealth effects. Still, the model manages to dampen the quantitative effects of the Forward Guidance Puzzle. Figure 5 shows this result quantitatively for our calibration.

Hence, wealth effects are important to understand the Forward Guidance Puzzle and the Paradox of Flexibility even after allowing for a discounted Euler equation. In the next section, we show how fiscal policy is an important determinant of the wealth effect in the standard New Keynesian model and, generically, the sole responsible for the monetary paradoxes.

4 The Fiscal Origins of Monetary Paradoxes

Recall that average consumption is given by

$$C = \rho \int_0^\infty e^{-\rho t} \left[(1-\overline{\tau}) y_t + \overline{b}(i_t - \pi_t - \rho) + T_t \right] dt.$$

Since we know that the paradoxes are coming from C, the next step is to study which components of C are generating the results.

There are two main forces that determine the equilibrium average consumption: the *spending-income spiral* and the *spending-inflation spiral*, given, respectively, by

$$\rho \int_0^\infty e^{-\rho t} y_t dt = C, \qquad \pi_t = \pi_t^H + \frac{\kappa}{\rho} e^{\underline{\omega} t} C.$$

The spending-income spiral states that average income equals average consumption, and higher income leads to higher consumption. The spending-inflation spiral states that, given a path for the nominal interest rate, the inflation rate increases with average consumption. Thus, plugging in these two relations into average consumption, we get

$$C = [1 - (\overline{\tau} - \sigma \underline{\omega} \overline{b})] C + \rho \int_0^\infty e^{-\rho t} [\overline{b}(i_t - \pi_t^H - \rho) + T_t] dt. \tag{17}$$

Interestingly, equation (17) shows that average consumption is determined according to an *Intertemporal Keyensian Cross*, in the spirit of the old Keynesian logic found in many introductory textbooks. On the one hand, one could interpret $1 - (\overline{\tau} - \sigma \underline{\omega} \overline{b})$ as the analogous to the marginal propensity to consume (MPC), and $\rho \int_0^\infty e^{-\rho t}[\overline{b}(i_t - \pi_t^H - \rho) + T_t]dt$ as the autonomous spending. For future reference, let

$$\begin{aligned} A \equiv& \rho \int_0^\infty e^{-\rho t}[\overline{b}(i_t - \pi_t^H - \rho) + T_t]dt, \\ C^F \equiv& \rho \int_0^\infty e^{-\rho t}\overline{b}(i_t - \pi_t^H - \rho)dt, \\ T \equiv& \rho \int_0^\infty e^{-\rho t}T_t dt, \end{aligned}$$

where A is the autonomous spending, C^F is the *Hicksian financial wealth* (since it is calculated using the inflation rate from the substitution effect), and T is the present of taxes. Trivially, $A = C^F + T$.

In order to determine the equilibrium value of C, we need to consider two separate cases: i) monetary policy has no fiscal consequence, $\overline{\tau} = \overline{b} = 0$; ii) monetary policy has fiscal consequences, either $\overline{\tau} > 0$ or $\overline{b} > 0$ (or both). The equilibrium properties of model are very different in these two cases.

Let's start with $\overline{\tau} = \overline{b} = 0$. Note that this case is knife-edge (in the sense of being non-generic) and not the empirically relevant one. Still, it is useful to understand this case from a theoretical point of view. Evaluating equation (17) at $\overline{\tau} = \overline{b} = 0$, we get

$$C = C + A \Rightarrow A = T = 0,$$

that is, the only restriction that this equation imposes on equilibrium is that the present value of transfers has to be zero. But this implies that the budget constraint of the household imposes no restriction on what average consumption is. In particular, the level of average consumption, and hence the wealth effect, has a self-fulfilling nature (using the old-Keynesian analogy, the MPC is equal to one in this case). If agents expect to receive higher income, $\int_0^\infty e^{-\rho t} y_t dt$, they increase their consumption accordingly, and since output is demand determined in this model, output increases to satisfy that demand. But since the household's income equals the present value of output, the increase in consumption becomes self-fulfilling. In the standard equilibrium selection, the Taylor rule pins down C by imposing that only a specific path of inflation is consistent with a bounded equilibrium. However, this presents a challenge to testing this theory, since, given observables, a continuum of paths for consumption and inflation (indexed by C) are consistent with the system of equations governing the equilibrium. That is, unless one knows the monetary policy rule the central bank is following, the researcher has many degrees of freedom (a continuum) to fit the model to the data.

However, this indeterminacy is limited to the knife-edge case. As we move away from $\overline{\tau} = \overline{b} = 0$, average consumption is determined by the *observed paths* of policy variables. It is important to highlight that this decomposition has no implication about the discussion of passive-active fiscal policy. Our analysis is consistent with the so-called Ricardian fiscal policy as well as with the Fiscal Theory of the Price Level. The decomposition simply brings to the forefront the role that policy variables have on equilibrium paths, and the channels through which they operate.

Suppose $\overline{\tau} > 0$ and $\overline{b} > 0$. In this case, we can isolate C from equation (17) and obtain

$$C = \frac{A}{\overline{\tau} - \sigma \underline{\omega} \overline{\overline{b}}} = \frac{C^F + T}{\overline{\tau} - \sigma \underline{\omega} \overline{\overline{b}}}. \tag{18}$$

That is, given a path for the nominal interest rate and government transfers, average consumption is completely determined by equation (18). To understand the importance of this result, recall that equilibrium consumption is given by

$$c_t = c_t^H + \frac{\overline{\omega}}{\rho} e^{\underline{\omega} t} C.$$

We already know that c_t^H is determined by the path of nominal interest rates, without resorting to any monetary or fiscal rule. Moreover, the GE multiplier is given by the value of fundamental parameters in the economy, so it does not change with policy rules. Equation (18) is saying that average consumption, the last piece needed to determine equilibrium consumption, is also determined by the observed paths of monetary and fiscal policy. Hence, when monetary policy has fiscal consequences, the model produces a unique path for consumption and inflation conditional on $\{i_t, T_t\}_{t=0}^{\infty}$, without needing to resort to the policy rule that led to those paths.

To sum up, we had identified C as the responsible for the monetary paradoxes, and now we found that, generically, C is proportional to $C^F + T$. This reduces the candidates of generating the monetary paradoxes to two: the Hicksian financial wealth and government transfers. The next proposition states that it is not the Hicksian financial wealth.

Proposition 6. *Suppose $\overline{b} > 0$. For a given sequence of $\{i_t\}_{t=0}^{\infty}$*

$$\frac{\partial C^F}{\partial i_t} > 0, \quad \frac{\partial^2 C^F}{\partial t \partial i_t} < 0, \quad \lim_{t \to \infty} \frac{\partial C^F}{\partial i_t} = 0, \quad \lim_{\kappa \to \infty} \frac{\partial C^F}{\partial i_t} = 0.$$

Thus, the paradoxes need to be coming from the fiscal policy.

Proposition 7. *Suppose $\overline{\tau} > 0$ and $\overline{b} > 0$. In the standard Liquidity Trap equilibrium, the Forward Guidance Puzzle and the Paradox of Flexibility are the consequence of the fiscal response to monetary shocks. In particular,*

$$\lim_{t \to \infty} \frac{\partial c_0}{\partial i_t} = -\infty \iff \lim_{t \to \infty} \frac{\partial T}{\partial i_t} = -\infty$$

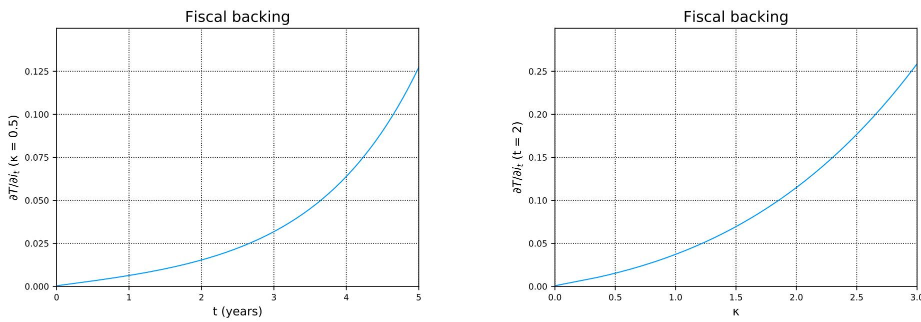

FIGURE 6: Fiscal Backing to Shocks in the Standard Liquidity Trap Equilibrium

where $T = \rho \int_0^\infty e^{-\rho t} T_t dt$ *and*

$$\lim_{\kappa \to \infty} \frac{\partial c_0}{\partial i_t} = 0 \iff \lim_{\kappa \to \infty} \frac{\partial T}{\partial i_t} = 0.$$

This is a powerful result. It states that if government transfers don't grow too fast as we postpone the time of the monetary shock or as we increase price flexibility, the New Keynesian model does not exhibit any of the monetary paradoxes emphasized in the literature. Thus, monetary paradoxes are not the result of a fiscal response to monetary shocks but to a sufficiently large one. Proposition 7 shows that the Forward Guidance Puzzle and the Paradox of Flexibility imply a fiscal response with transfers that increases unboundedly (in absolute terms) as the horizon and the price flexibility go to infinity. Figure 6 shows the magnitude of the fiscal transfers necessary to sustain the standard Liquidity Trap equilibrium in our baseline calibration.

To sum up, we decomposed equilibrium consumption in order to identify the source of the paradoxical responses of the economy to monetary policy shocks. We concluded that, generically, the monetary paradoxes are the result of extreme fiscal policy reaction implied in the standard equilibrium selection. Next, we reinforce the importance of this result by using the tools developed in this paper to study the predictions of the model in normal times. This exercise is useful for two reasons. First, it will allow us to argue that the reliance of the model to strong fiscal responses to monetary shocks is not only a feature of the Liquidity Trap or interest rate peg. That is, the results presented in the previous sections are driven by anomalies of the New Keynesian model in extreme situations, but they are at the core of the standard model. Second, it will allow us to test the model quantitatively. Unfortunately, data of economies in a Liquidity Trap is limited. Thus, testing the model in normal times can shed light about the plausibility of the fiscal responses that lead to the monetary paradoxes.

5 Outside the ZLB: Monetary Policy in Normal Times

The objective of this section is twofold. First, we explore the role that fiscal policy plays in the quantitative success of the New Keynesian model emphasized in the literature (e.g., Christiano et al. (2005)). Our results show that fiscal policy is crucial in the model, suggesting that our findings in the previous sections about the role of fiscal variables in the standard equilibrium is not limited to the Liquidity Trap. Second, we design a test of the model to assess its quantitative performance when fiscal variables are taken into account. We conclude that the RANK model does not produce impulse response functions consistent with the data.

The rest of this section is organized as follows. First, we briefly describe the model we use in our quantitative exercise. Next, we use the strategy in Gertler and Karadi (2015) to estimate the fiscal response to a monetary shock in the US data. We then feed the model with the impulse response functions for monetary and fiscal variables we estimated from the data, and compare the predicted impulse response functions for output and inflation with the ones obtained in the data. Finally, we build a simple HANK model and argue that it produces the forces necessary to fit the data, but it is crucial that it incorporates information from the fiscal policy side.

5.1 The Model

The model is the natural extension of Smets and Wouters (2007) to account for fiscal variables. Time is discrete and denoted by $t = 0, 1, 2, \ldots, \infty$. The economy is populated by a continuum of mass one of infinitely-lived households. Households derive utility from the consumption of a final good and leisure. Their preference for consumption exhibits an external habit variable. Labor supply is differentiated across households. It is assumed that the wages of each type of labor is negotiated by a union, which chooses the wage but is subject to nominal rigidities à la Calvo. Households are the owners of the capital of the economy. They rent capital services to the firms, which is a function of the capital stock they hold and the utilization level they choose, which comes at the cost of higher depreciation. Households also decide how much capital to accumulate given the adjustment costs they face.

There are two types of firms in the economy. There is a continuum of intermediate goods producer firms, which transform labor and capital services into a differentiated good and set prices subject to the Calvo friction. Those wages and prices that cannot be re-optimized in a given period are partially indexed to past inflation. The second type of firm is a representative firm that produces the final consumption good using the intermediate goods as inputs and sells the output in competitive markets. Finally, there is a government that chooses a path for the nominal interest rate, government spending, proportional sales taxes, lump-sum transfers and debt.

The model follows very closely Smets and Wouters (2007), with two differences. First, because we introduce sales taxes, the NK Phillips curve has an extra term and the coefficients depend on the taxes. This change has no relevant quantitative impact on the model. Second, we explicitly introduce the household's budget constraint. The budget constraint of the household is now given by

$$c_y c_t + x_y x_t + q^L b_y (b_t^L + q_t^L) = (1-\overline{\tau})(y_t - \tau_t) - z_y z_t + \frac{\rho_L q^L b_y}{1+\overline{\pi}} q_t^L - \frac{(1+\rho_L q^L) b_y}{1+\overline{\pi}} \pi_t + \frac{(1+\rho_L q^L) b_y}{1+\overline{\pi}} b_{t-1}^L + T_t, \quad (19)$$

where q_L is the price of long-term government bonds

$$q_t^L = \frac{\rho_L}{1+\bar{i}}\, q_{t+1}^L - i_t, \quad (20)$$

y_t is output, c_t is consumption, x_t is investment, g_t is government spending, z_t is the capital utilization rate, l_t is hours worked, i_t is the nominal interest rate (set by the monetary authority), π_t is the inflation rate, q_t is the Tobin's Q, r_t^k is the rental rate of capital services, k_t^s is capital services, k_t is the stock of capital, w_t is the real wage, μ_t^p is the price mark-up, μ_t^w is the wage mark-up, τ_t is the proportional sales tax, b_t^L is government bonds, q_t^L is the price of the long-term bond, and T_t are government lump-sum transfers. The rest of the variables are positive constants defined in the appendix. As we emphasized in the previous section, the benefit of explicitly accounting for the households' budget constraint is that it allows us to study the interaction between the monetary shock and the fiscal policy. By adding proportional taxes and government bonds, monetary shocks have fiscal consequences. Since these are instruments widely used by governments, they provide a testable implication of the model.

The reader can refer to the appendix for a detailed derivation of the rest of the model.

Now, we use the constructed medium-scale New Keynesian model to uncover the importance of the implicit fiscal adjustment of the economy to monetary shocks. That is, we ask the following question: in the standard equilibrium selection (i.e., with a Taylor rule for monetary policy and passive fiscal policy), how large are the implicit transfers after a monetary shock? We answer this question by simulating the model above and calculate the implied transfers from the budget constraint.

Figure 7 shows the results. The solid line depicts the impulse response functions of inflation and GDP to a monetary shock, as well as the transfers that come out from the budget constraint. The transfers are negative and of a similar order of magnitude of the change in output. The dashed line shows how inflation and output react to the monetary shock if transfers did not change. This exercise is not meant to convey the idea that if transfers had not reacted as the solid line shows then *equilibrium* inflation and output would have behaved as the dashed line shows. On the contrary, it should be understood as a decomposition exercise. In particular, it shows how much of the dynamics comes from the change in the interest rates, and the gap is the dynamics generated by the transfers. The impulse response functions of inflation and GDP are dramatically different. First, the mone-

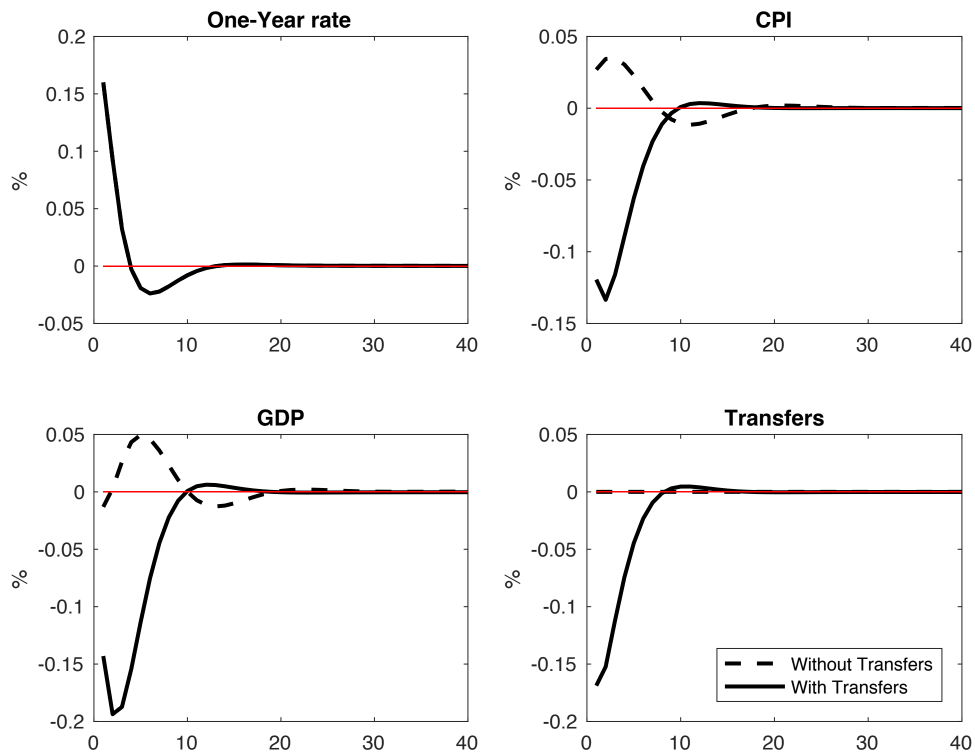

FIGURE 7: SW with and without transfers

tary shock generates inflation on impact, rather than deflation. Second, the shock generates a small recession on impact followed by a boom. How can this difference be explained? The answer is the decomposition from Section 4. The large effect of transfers is not coming from a ”permanent income” logic, which is rather small, but from the interaction of transfers and GE effects through inflation. As Figure 7 shows, the transfers have a large impact on inflation, and the effect feeds to itself. Thus, transfers play an important role even in a medium-sized New Keynesian model outside of a Liquidity Trap.

Next, we go to the data and estimate the response of fiscal variables to monetary shocks. We want to evaluate the empirical plausibility of the transfers derived from the theory. In particular, we check whether the predicted fiscal response in the model is supported by the data.

5.2 Empirical Evidence of Fiscal Response to Monetary Shocks

There is an extensive literature that studies the response of the main macroeconomic variables (e.g., output, consumption, investment, inflation) to monetary policy shocks. However, there is still not a consensus with respect to the best set of assumptions that identify exogenous monetary innovations. Our objective in this section is not to contribute to this debate, but to extend the results in the literature to incorporate the dynamics of fiscal variables.

To this end, we extend Gertler and Karadi (2015) to account for the dynamics of fiscal policy after an exogenous monetary shock. The exercise uses a combination of a VAR estimation and an external instruments approach. First, we estimate a VAR in seven variables. The first four variables are the ones in Gertler and Karadi (2015): the one-year government bond rate, log industrial production, log consumer price index and the Gilchrist and Zakrajšek (2012) excess bond premium as a measure of credit spread. We also include three fiscal variables: government spending, total revenues over GDP and government transfers. From this VAR, we obtain the reduced form shocks, which are a linear combination of the structural shocks, in particular, the monetary policy shock.

The second stage estimates the sensitivity of the VAR variables to a monetary policy shock. A standard approach consists of putting the so-called "timing restrictions" on the relationship between reduced form and structural shocks.[14] Instead, Gertler and Karadi (2015) use an external in-

[14] For a detailed explanation, see Christiano et al. (1999).

strument approach. They regress the estimated reduced-form residuals to the change of the three-months ahead fed-funds rate futures in a 30-minute window around FOMC announcements, where the change in fed-funds futures acts as an instrument of the actual monetary shocks.

We use quarterly data over the period 1979:3 to 2012:2. This is a substantial reduction on the number of observation from the original paper, which uses monthly data. Unfortunately, fiscal variables are only available in a quarterly basis. However, our results suggest that there are no important differences in the estimates of the impulse response functions, though we get wider confidence intervals.

Figure 8 shows the results. As in Gertler and Karadi (2015), we find that a positive monetary shock reduces output and prices, though only the effect on output is significant at 90% confidence level. With respect to the dynamics of fiscal variables, a monetary shock has a statistically zero effect on government purchases, while revenues over output (a proxy for proportional taxes) decrease, and transfers increase. The effect on proportional taxes and transfers is likely to come from the automatic stabilizer mechanisms embedded in the government accounts. Because a monetary shock is contractionary, households' income and employment decrease. This has two effects. First, because income taxes are progressive, the average income tax in the economy decreases. Second, because a large fraction of government transfers are unemployment benefits, it is natural that they increase in recessions.

These results show that a monetary policy shock does not trigger the fiscal response implied by the standard New Keynesian equilibrium selection. Compare the impulse responses in Figure 7 and Figure 8. While the model requires a *contractionary* fiscal policy, the data suggests that fiscal policy is

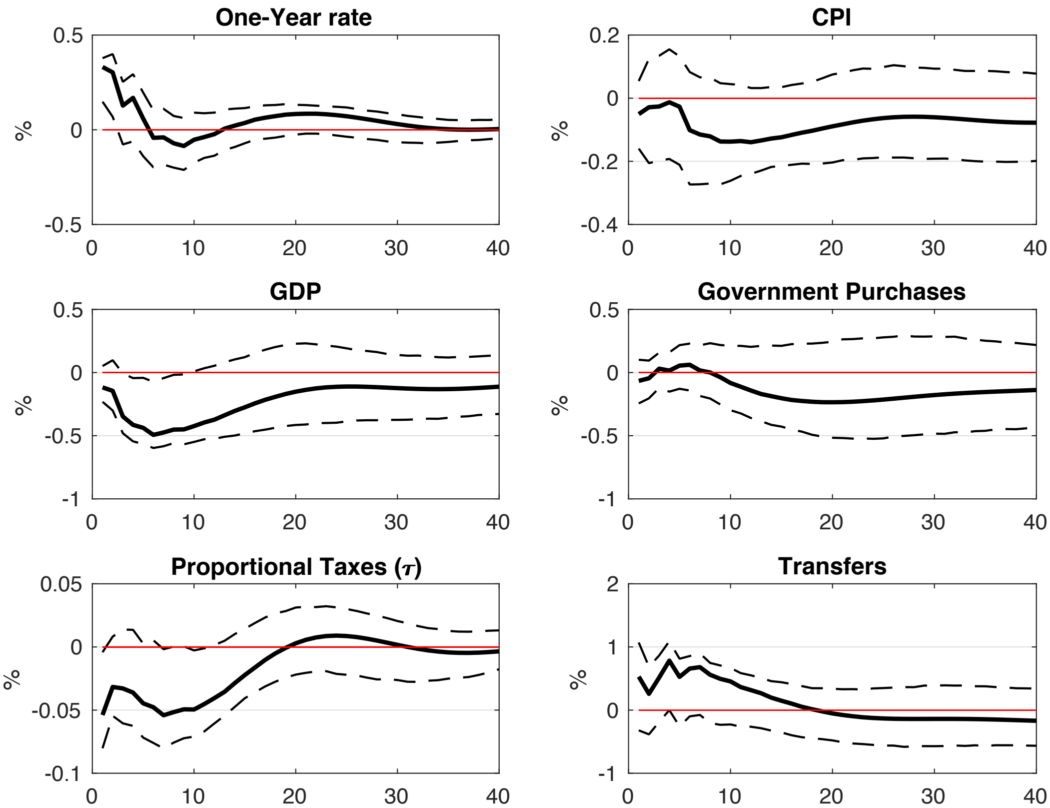

FIGURE 8: Impulse Response Function to a monetary shock

expansionary as a response to a monetary shock. Both government transfers and proportional taxes increase rather than decrease. The point estimate of government spending is negative, which contributes to the standard equilibrium mechanism, but statistically zero. Thus, the evidence does not seem to support the standard equilibrium selection.

Next, we feed the monetary and fiscal policies impulse response functions in the medium-size New Keynesian model to evaluate the quantitative success of the model when we force both monetary and fiscal policies to follow the observed paths in the data.

5.3 Testing the New Keynesian model

Next, we present the main exercise of this section. The objective is to design a test of the model that incorporates information of both monetary and fiscal variables. We do this by feeding the model with the impulse responses for the interest rate, government purchases, proportional taxes and transfers estimated from the data, and compare the resulting impulse response functions for inflation and output with the ones obtained from the data. To do this, we need to drop the model's Taylor rule for monetary policy. As in the simple model, conditional on the observed path for interest rates, government spending and taxes, the New Keynesian model predicts a unique path for output, inflation, consumption and investment. But as we emphasized before, this does not imply that the economy is inconsistent with policy rules. In fact, one can ex-post construct a rule consistent with the observed path of interest rates, inflation and output.[15] Importantly, this rule can be of the active-monetary or active-fiscal type, without affecting the interpretation of our results. The next Lemma formalizes this idea.

Lemma 5. *Suppose* $\Xi^* \equiv (y_t^*, c_t^*, l_t^*, i_t^*, z_t*, q_t*, r_t^{k*}, \pi_t^*, k_t^*, k_t^{s*}, \mu_t^{p*}, \mu_t^{w*}, w_t^*, b_t^*)$ *satisfies the system of equations given by* (22)-(36) *given a sequence of monetary and fiscal variables* (r_t, g_t, τ_t, T_t)*. Then, there exists a Taylor rule that implements* Ξ^* *as an equilibrium of an economy characterized by equations* (22)-(34)*, given a sequence of fiscal variables* (g_t, τ_t)*. Moreover, suppose* Ξ^* *satisfies the system of equations given by* (22)-(34) *and a Taylor rule, given a sequence of fiscal variables* (g_t, τ_t)*. Then, there exists a sequence* $(\tilde{r}_t, \tilde{T}_t)$ *that implements* Ξ^* *as a solution to* $(22) - (36)$.

[15] A similar result is emphasized in Werning (2012) and Cochrane (2017) in a different context.

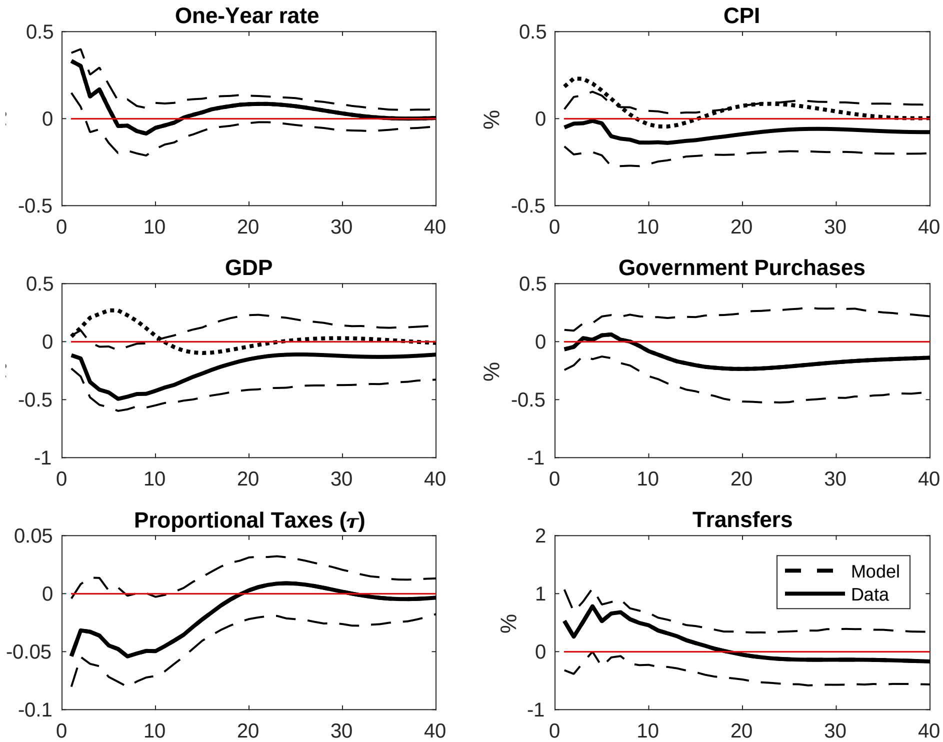

FIGURE 9: Model impulse response functions to a monetary shock. Interest rates and fiscal variables match the data.

Lemma 5 implies that we can test the predictions of the model independently of the policy regime that led to those paths for policy variables. If by doing this we reject the model, then assuming monetary or fiscal dominance will not change the results. In this sense, our exercise is about in-sample evaluation of the model, which does not require a stand on policy rules.

Figure 9 depicts the results. The solid line is the impulse response estimated from the data (as in Figure 8), with the dashed lines being the 90% confidence intervals. The pointed line is the impulse response of the model. We can see that the model predicts higher inflation than the data, though mostly inside the confidence bands. However, the data rejects the impulse

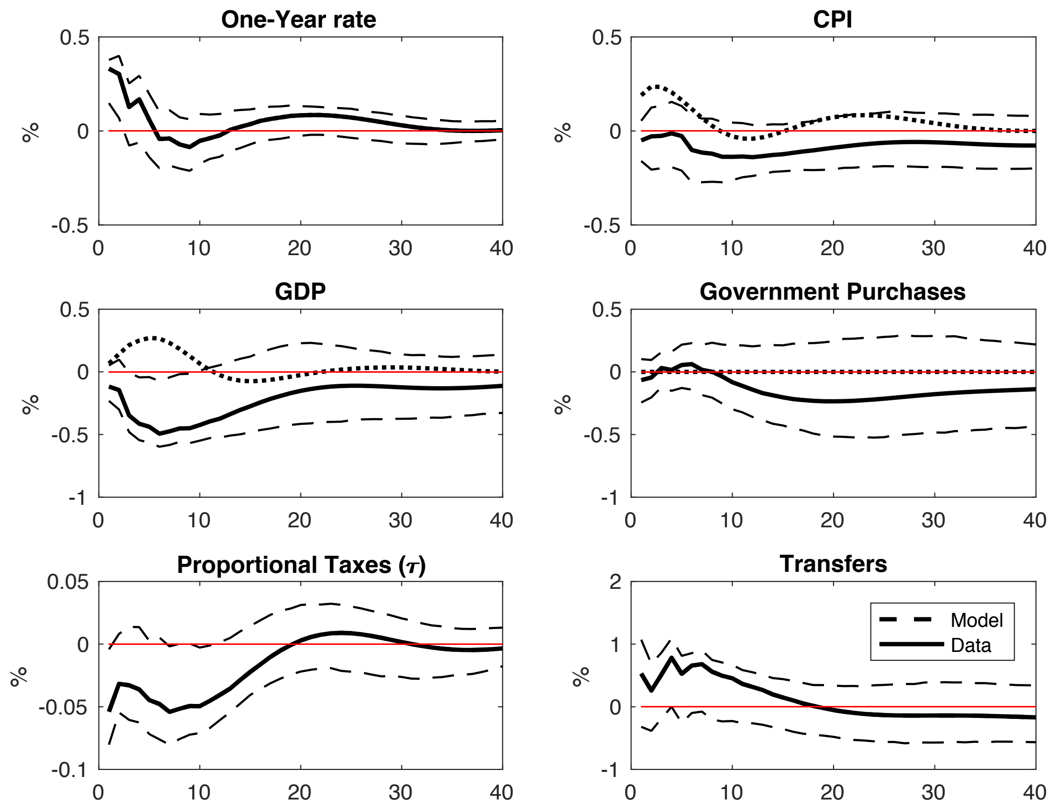

FIGURE 10: Model impulse response functions to a monetary shock. Government spending is set to zero.

response for output. While the data implies that a positive monetary shock generates a recession in the short-run, the model predicts a boom. Figure 10 performs a similar exercise but imposing that government spending does not react to a monetary shock, motivated by the fact that the data cannot reject that the changes are zero. The difference with the data exacerbates. Since the point estimate of the impulse response of government purchases is negative, their contribution in the model is a force towards lower inflation and output. When we set government spending to zero, the output boom exacerbates, and now we have statistically significant increase in inflation.

Thus, we conclude that the standard New Keynesian model cannot produce impulse response functions on output and inflation that resemble the ones obtained in the data. This result contrasts with some of the findings in the literature, for example Christiano et al. (2005). The difference relies on our use of fiscal policy as an extra restriction in the model. When fiscal transfers are set to match their empirical counterpart, the standard New Keynesian model is dominated by substitution effects that are too weak to match the data. The way the model produces stronger wealth effects is by increasing government transfers. But once government transfers are constrained to match the data, the model's performance worsens. Importantly, we want to emphasize that this result does not depend on equilibrium selection. Thus, the test in this section can be interpreted as being the answer to the following question: is there *any* equilibrium in the standard New Keynesian model that can produce impulse response functions similar to the ones in the data, *once we impose observed paths for monetary and fiscal variables after a monetary shock*? The answer is no.

In light of these results, next we briefly explore an extension of the standard model that relies on endogenous strong *private* wealth effects in response to monetary shocks. We show that when forced to match the monetary and fiscal dynamics of the data, this model produces dynamics for inflation and output similar to the data.

5.4 Private Wealth Effects: Indebted Hand-to-Mouth

The previous analysis showed two things: first, implied government transfers are important for the quantitative success of the standard New Keynesian model; second, once transfers are disciplined by the data, the standard New Keynesian model cannot produce wealth effects strong enough to match the impulse response functions of output and inflation in the data. With this in mind, we study a very simple extension of the New Keynesian model to account for private wealth effects, and show that it produces dynamics closer to the ones found in the data. The model is based on Eggertsson and Krugman (2012) and the new literature on HANK models.[16]

The model is a standard borrower-saver economy. There are two types of agents. The main difference between these two groups is their discount factor: borrowers are more impatient than savers. Agents in this economy are subject to a borrowing constraint of the form:

$$\frac{B_t}{P_t} \leq \overline{D},$$

where B_t is the level of the agent's nominal debt, P_t is the price level, and $\overline{D}$ is a real borrowing limit. The rest of the economy follows Smets and Wouters (2007).

In the steady state of the economy, the borrowers are against their borrowing constraint due to their impatience. Therefore, their consumption in period t is given by

$$C_t^b = w_t l_t^b - \frac{r_t}{1+r_t}\overline{D} + T_t,$$

where C_t^b is consumption of borrowers, w_t is the real wage, l_t^b is labor of the borrowers, r_t is the real interest rate, and T_t is the lump-sum transfers.

It is immediate to see that this model has the potential of producing strong wealth effects from monetary policy shocks. The borrowers are effectively hand-to-mouth consumers: every period they consume all their income net of interest payments. If the interest rate goes up, borrowers

[16] E.g., Debortoli and Galí (2017); see Kaplan et al. (2017) for a continuum of types version.

will not be able to smooth consumption due to the binding borrowing constraint. As a consequence, their consumption level adjusts one-to-one to the higher interest payments, introducing a channel that, as we show below, greatly amplifies the effect of monetary shocks.

To understand the importance of having positive debt rather than standard hand-to-mouth agents, the next Lemma shows that if $\overline{D} = 0$, aggregate variables behave as if they were generated by a representative agent model.

Lemma 6. *Suppose $\overline{D} = 0$. Then, aggregate variables in the heterogenous agents economy behave as if they were the result of a representative agent model.*

This Lemma implies that the results we obtain below are not simply coming from having hand-to-mouth agents, but from the fact that these agents are indebted and, as a consequence, their reaction to interest rate changes has a direct effect on their consumption decisions.

Suppose that $\overline{D} > 0$. We evaluate the quantitative performance this Smets and Wouters (2007) model augmented by fiscal variables and heterogeneous households. The only difference with respect to the representative agent model is that now aggregate consumption is given by the sum of the consumption of borrowers and savers. In the calibration, there are two important new parameters: the fraction of borrowers and the fraction of private debt-to-GDP. As a benchmark, we set the fraction of borrowers to $1/3$, in line with the findings of Kaplan et al. (2014), and private debt-to-GDP to 50%.[17] It is important to emphasize that this exercise is a proof-of-concept and not a serious quantitative evaluation of a model with heterogeneous agents. The objective of this subsection is to show that this direction of research is promising in producing dynamics that match the data. A serious

[17] As a reference, total private debt in the US is close to one GDP.

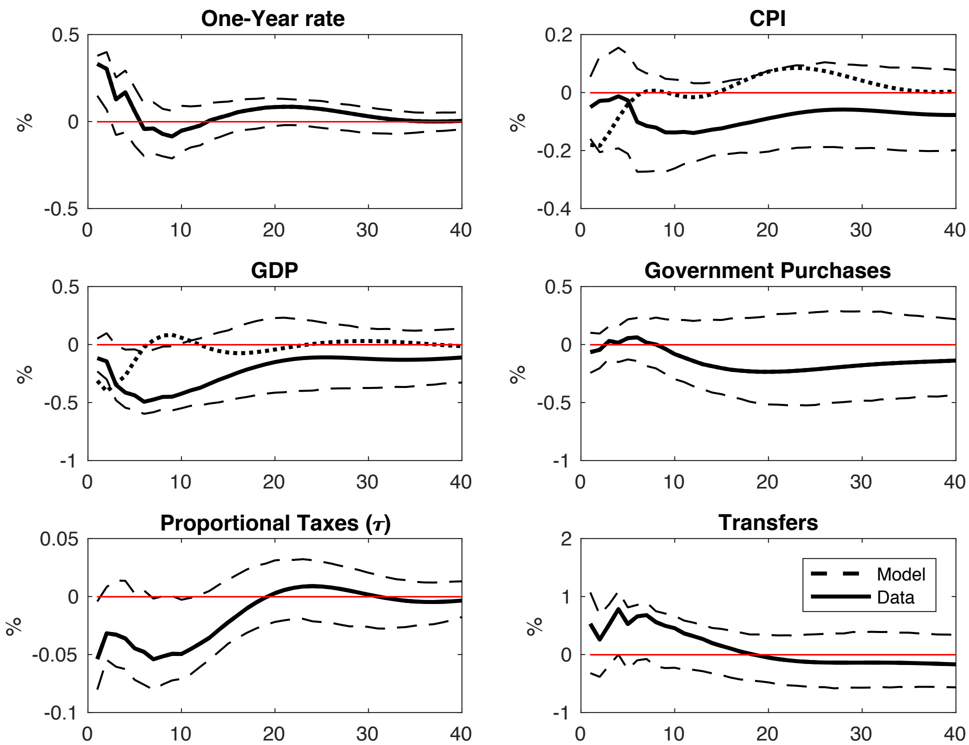

FIGURE 11: Heterogeneous Agent Model impulse response functions to a monetary shock.

evaluation would consider a richer model and a more careful calibration.

Figure 11 shows the results. The fit of the model improves considerably with respect to the standard RANK. The impulse response functions for output and inflation match the effect of a monetary shock on impact, though there are some discrepancies on the effects after one year. The change in the interest rate has a strong effect on the borrowers' consumption, which cannot be smoothed because they are against their constraint. This partially offsets the absence of wealth effects coming from government transfers, improving the quantitative performance of the model when fiscal variables are set to match the data. These results show that, in order to understand the monetary policy transmission mechanism, a promising avenue of research is to better understand the determinants of wealth effects and its interactions with monetary policy.

6 Conclusion

In this paper we revisited the monetary paradoxes in the standard New Keynesian model and study the channels through which they occur. We focus on two paradoxes: the Forward Guidance Puzzle and the Paradox of Flexibility. Our main finding is that monetary paradoxes are mainly driven by strong wealth effects embedded in the standard equilibrium selection, rather than on the intertemporal substitution effect. In particular, we show that, generically, the paradoxes arise exclusively due to a counterfactual fiscal response to monetary shocks.

To do this, we propose a decomposition of consumption into substitution and wealth effects, both of which take into account the general equilibrium effects of output and inflation. Our first main result is that, in the absence of wealth effects, the equilibrium does not present any of the paradoxical results: even after fully taking into account general equilibrium effects on output and inflation, the effect of changes in interest rates is reduced with the horizon of the intervention and the equilibrium is continuous in the price flexibility parameter. Instead, monetary paradoxes are the result of strong wealth effects which, in the presence of fiscal consequences of monetary shocks, are solely determined by the expected fiscal response to the shock. Moreover, we show that this result holds in models that boil down to a discounted Euler equation, in which case the paradoxes are attenuated but do not disappear.

Moreover, we find that the prescribed fiscal response in the standard equilibrium does not hold in the data. We estimate the fiscal response to monetary shocks using the high-frequency data approach in Gertler and Karadi (2015), which combines a VAR estimation and an external instruments approach, where the instruments are the changes of the three-month ahead fed funds rate in a 30-minute window around FOMC announcements. We find empirical fiscal responses to monetary shocks with the opposite sign to the ones implied by the standard equilibrium.

Finally, we introduce the estimated fiscal responses into a medium-size DSGE model. We find that the impulse-response of consumption and inflation do not match the data, suggesting that wealth effects induced by fiscal policy may be important even outside of the liquidity trap. This result suggests that the New Keynesian model should be augmented with mechanisms that generate stronger wealth effects in order to get impulse responses of output and inflation consistent with the data. In particular, we show that introducing an indebted hand-to-mouth improves the quantitative performance of the model.

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APPENDIX

Proof of Lemma 1. Define the following rotation of the system:

$$Z_t = \begin{bmatrix} Z_{1t} \\ Z_{2t} \end{bmatrix} = -\frac{\kappa\omega_c}{\overline{\omega} - \underline{\omega}} \begin{bmatrix} 1 & (\sigma\underline{\omega})^{-1} \\ -1 & -(\sigma\overline{\omega})^{-1} \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \end{bmatrix}$$

$$\eta_t = \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix} = \begin{bmatrix} -\frac{\kappa\omega_c}{\overline{\omega}-\underline{\omega}} m_t - \frac{\overline{\omega}}{\overline{\omega}-\underline{\omega}} f_t \\ \frac{\kappa\omega_c}{\overline{\omega}-\underline{\omega}} m_t + \frac{\underline{\omega}}{\overline{\omega}-\underline{\omega}} f_t \end{bmatrix}$$

where $f_t \equiv \kappa\left(\omega_g g_t + \tau_t\right)$ and $m_t \equiv \sigma^{-1}(i_t - \rho)$.

The system in the new coordinates can be written as

$$\begin{bmatrix} \dot{Z}_{1t} \\ \dot{Z}_{2t} \end{bmatrix} = \begin{bmatrix} \overline{\omega} & 0 \\ 0 & \underline{\omega} \end{bmatrix} \begin{bmatrix} Z_{1t} \\ Z_{2t} \end{bmatrix} + \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix}$$

Integrating the system above:

$$e^{-\overline{\omega}T} Z_{1T} - e^{-\overline{\omega}t} Z_{1t} = \int_t^T e^{-\overline{\omega}s} \eta_{1s} ds$$
$$e^{-\underline{\omega}T} Z_{2T} - e^{-\underline{\omega}t} Z_{2t} = \int_t^T e^{-\underline{\omega}s} \eta_{2s} ds$$

Solving the first equation forward and the second backwards, we obtain

$$Z_{1t} = -\int_t^\infty e^{-\overline{\omega}(s-t)} \eta_{1s} ds$$
$$Z_{2t} = e^{\underline{\omega}t} Z_{20} + \int_0^t e^{\underline{\omega}(t-s)} \eta_{2s} ds$$

Rotating the system back to the original coordinates, we obtain

$$c_t = -(\sigma\underline{\omega})^{-1} e^{\underline{\omega}t} Z_{20} - (\sigma\underline{\omega})^{-1} \int_0^t e^{\underline{\omega}(t-s)} \eta_{2s} ds + (\sigma\overline{\omega})^{-1} \int_t^\infty e^{-\overline{\omega}(s-t)} \eta_{1s} ds$$
$$\pi_t = e^{\underline{\omega}t} Z_{20} + \int_0^t e^{\underline{\omega}(t-s)} \eta_{2s} ds - \int_t^\infty e^{-\overline{\omega}(s-t)} \eta_{1s} ds.$$

Evaluating these expressions at $t=0$, we get

$$
\begin{aligned}
c_0 &= -\left(\sigma\underline{\omega}\right)^{-1} Z_{20} + \left(\sigma\overline{\omega}\right)^{-1}\int_0^{\infty} e^{-\overline{\omega}s}\eta_{1s}ds \\
\pi_0 &= Z_{20} - \int_0^{\infty} e^{-\overline{\omega}s}\eta_{1s}ds.
\end{aligned}
$$

Thus, the relationship between c_0 and π_0 is given by

$$
\pi_0 = \sigma\underline{\omega}\left[\frac{\overline{\omega}-\underline{\omega}}{\sigma\hat{\kappa}}\int_0^{\infty} e^{-\overline{\omega}s}\eta_{1s}ds - c_0\right]
$$

Replacing back in c_t and π_t, we get

$$
\begin{aligned}
c_t &= e^{\underline{\omega}t}c_0 - \sigma^{-1}e^{\underline{\omega}t}\int_0^t\left(\frac{e^{-\overline{\omega}s}}{\overline{\omega}}\eta_{1s} + \frac{e^{-\underline{\omega}s}}{\underline{\omega}}\eta_{2s}\right)ds + \sigma^{-1}\frac{e^{\overline{\omega}t}-e^{\underline{\omega}t}}{\overline{\omega}}\int_t^{\infty} e^{-\overline{\omega}s}\eta_{1s}ds \\
\pi_t &= e^{\underline{\omega}t}\pi_0 + e^{\underline{\omega}t}\int_0^t\left(e^{-\overline{\omega}s}\eta_{1s} + e^{-\underline{\omega}s}\eta_{2s}\right)ds - \left(e^{\overline{\omega}t}-e^{\underline{\omega}t}\right)\int_t^{\infty} e^{-\overline{\omega}s}\eta_{1s}ds
\end{aligned}
$$

Finally, using the expression for η_t we get

$$
\begin{aligned}
c_t &= e^{\underline{\omega}t}c_0 + c_t^m + c_t^f \\
\pi_t &= e^{\underline{\omega}t}\pi_0 + \pi_t^m + \pi_t^f
\end{aligned}
$$

where

$$
\begin{aligned}
c_t^m &\equiv \frac{\hat{\kappa}\sigma^{-1}}{\overline{\omega}-\underline{\omega}}\left[e^{\underline{\omega}t}\int_0^t\left(\frac{e^{-\overline{\omega}s}}{\overline{\omega}} - \frac{e^{-\underline{\omega}s}}{\underline{\omega}}\right)(i_s-\rho)ds - \frac{e^{\overline{\omega}t}-e^{\underline{\omega}t}}{\overline{\omega}}\int_t^{\infty} e^{-\overline{\omega}s}(i_s-\rho)ds\right] \\
c_t^f &\equiv \frac{\kappa\sigma^{-1}}{\overline{\omega}-\underline{\omega}}\left[e^{\underline{\omega}t}\int_0^t\left(e^{-\overline{\omega}s} - e^{-\underline{\omega}s}\right)(\omega_g g_s+\tau_s)ds - \left(e^{\overline{\omega}t}-e^{\underline{\omega}t}\right)\int_t^{\infty} e^{-\overline{\omega}s}(\omega_g g_s+\tau_s)ds\right] \\
\pi_t^m &\equiv \frac{\hat{\kappa}}{\overline{\omega}-\underline{\omega}}\left[-e^{\underline{\omega}t}\int_0^t\left(e^{-\overline{\omega}s} - e^{-\underline{\omega}s}\right)(i_s-\rho)ds + \left(e^{\overline{\omega}t}-e^{\underline{\omega}t}\right)\int_t^{\infty} e^{-\overline{\omega}s}(i_s-\rho)ds\right] \\
\pi_t^f &\equiv \frac{\kappa}{\overline{\omega}-\underline{\omega}}\left[-e^{\underline{\omega}t}\int_0^t\left(\overline{\omega}e^{-\overline{\omega}s} - \underline{\omega}e^{-\underline{\omega}s}\right)(\omega_g g_s+\tau_s)ds + \overline{\omega}\left(e^{\overline{\omega}t}-e^{\underline{\omega}t}\right)\int_t^{\infty} e^{-\overline{\omega}s}(\omega_g g_s+\tau_s)ds\right]
\end{aligned}
$$

The standard liquidity trap equilibrium selection sets $c_T = 0$ and assumes no shocks after T. Thus, initial consumption is given by

$$c_0^{NK} = -e^{-\underline{\omega}T}(c_T^m + c_T^f),$$
$$= -\frac{\kappa}{\sigma(\overline{\omega} - \underline{\omega})}\left[\sigma^{-1}\omega_c \int_0^T \left(\frac{e^{-\overline{\omega}s}}{\overline{\omega}} - \frac{e^{-\underline{\omega}s}}{\underline{\omega}}\right)(i_s - \rho)ds + \int_0^T \left(e^{-\overline{\omega}s} - e^{-\underline{\omega}s}\right)(\omega_g g_s + \tau_s)ds\right]$$

and initial inflation is given by

$$\pi_0^{NK} = \sigma\underline{\omega}\left[\int_0^T e^{-\overline{\omega}s}\left[-\sigma^{-1}(i_s - \rho) + \frac{\kappa}{\sigma\underline{\omega}}(\omega_g g_s + \tau_s)ds\right] - c_0^{NK}\right].$$

Finally, the comparative statics for consumption are

$$\frac{\partial c_0^{NK}}{\partial i_s} = -\frac{\hat{\kappa}}{\sigma(\overline{\omega} - \underline{\omega})}\left(\frac{e^{-\overline{\omega}s}}{\overline{\omega}} - \frac{e^{-\underline{\omega}s}}{\underline{\omega}}\right) < 0,$$
$$\frac{\partial c_0^{NK}}{\partial g_s} = -\frac{\kappa\omega_g}{\sigma(\overline{\omega} - \underline{\omega})}\left(e^{-\overline{\omega}s} - e^{-\underline{\omega}s}\right) > 0,$$
$$\frac{\partial c_0^{NK}}{\partial \tau_s} = -\frac{\kappa}{\sigma(\overline{\omega} - \underline{\omega})}\left(e^{-\overline{\omega}s} - e^{-\underline{\omega}s}\right) > 0,$$

and for inflation

$$\frac{\partial \pi_0^{NK}}{\partial i_s} = \sigma\underline{\omega}\left(-\sigma^{-1}e^{-\overline{\omega}s} - \frac{\partial c_0^{NK}}{\partial i_s}\right) = \frac{\hat{\kappa}}{\overline{\omega} - \underline{\omega}}\left(e^{-\overline{\omega}s} - e^{-\underline{\omega}s}\right) < 0,$$
$$\frac{\partial \pi_0^{NK}}{\partial g_s} = \sigma\underline{\omega}\left(\frac{\kappa}{\sigma\underline{\omega}}\omega_g e^{-\overline{\omega}s} - \frac{\partial c_0^{NK}}{\partial g_s}\right) = \frac{\kappa\omega_g}{\overline{\omega} - \underline{\omega}}\left(\overline{\omega}e^{-\overline{\omega}s} - \underline{\omega}e^{-\underline{\omega}s}\right) > 0,$$
$$\frac{\partial \pi_0^{NK}}{\partial \tau_s} = \sigma\underline{\omega}\left(\frac{\kappa}{\sigma\underline{\omega}}e^{-\overline{\omega}s} - \frac{\partial c_0^{NK}}{\partial \tau_s}\right) = \frac{\kappa}{\overline{\omega} - \underline{\omega}}\left(\overline{\omega}e^{-\overline{\omega}s} - \underline{\omega}e^{-\underline{\omega}s}\right) > 0,,$$

■

Proof of Proposition 1. For the Forward Guidance Puzzle we have

$$\frac{\partial^2 c_0^{NK}}{\partial t \partial i_s} = -\frac{\hat{\kappa}}{\sigma(\overline{\omega}-\underline{\omega})}\left(-e^{-\overline{\omega}s}+e^{-\underline{\omega}s}\right) < 0,$$
$$\frac{\partial^2 \pi_0^{NK}}{\partial t \partial i_s} = \frac{\hat{\kappa}}{\overline{\omega}-\underline{\omega}}\left(-\overline{\omega}e^{-\overline{\omega}s}+\underline{\omega}e^{-\underline{\omega}s}\right) < 0,$$

and the limits are straightforward from Lemma 1. For the Paradox of Flexibility note that

$$\lim_{\kappa\to\infty}\hat{\kappa}=\infty, \quad \lim_{\kappa\to\infty}\overline{\omega}=\infty, \quad \lim_{\kappa\to\infty}\underline{\omega}=-\infty, \quad \lim_{\kappa\to\infty}\frac{\hat{\kappa}}{\overline{\omega}-\underline{\omega}}=\infty.$$

Then, the result is straightforward after some manipulation of the expressions. ■

Proof of Proposition 7. From the households' budget constraint we get that

$$c_0 = \int_0^\infty \left[\chi_{i,t}^c(i_t-\rho)+\chi_{g,t}^c g_t+\chi_{\tau,t}^c\hat{\tau}_t\right]dt+\chi_T^c T,$$

where $T \equiv \int_0^\infty e^{-\rho t}T_t dt$ is the present value of government transfers, and

$$\chi_{i,t}^c \equiv -\sigma^{-1}\frac{\overline{\tau}\varsigma_c-\sigma\rho\varsigma_d}{\overline{\tau}\varsigma_c-\sigma\underline{\omega}\varsigma_d}e^{-\overline{\omega}t},$$
$$\chi_{g,t}^c \equiv \overline{\omega}\frac{\overline{\tau}\varsigma_c-\sigma\rho\varsigma_d}{\overline{\tau}\varsigma_c-\sigma\underline{\omega}\varsigma_d}\frac{\omega_g}{\omega_c}(e^{-\rho t}-e^{-\overline{\omega}t})+\overline{\omega}\frac{1-\overline{\tau}}{\overline{\tau}\varsigma_c-\sigma\underline{\omega}\varsigma_d}\varsigma_g e^{-\rho t},$$
$$\chi_{\tau,t}^c \equiv \overline{\omega}\frac{\overline{\tau}\varsigma_c-\sigma\rho\varsigma_d}{\overline{\tau}\varsigma_c-\sigma\underline{\omega}\varsigma_d}\frac{1}{\omega_c}(e^{-\rho t}-e^{-\overline{\omega}t})-\overline{\omega}\frac{1-\overline{\tau}}{\overline{\tau}\varsigma_c-\sigma\underline{\omega}\varsigma_d}e^{-\rho t},$$
$$\chi_T^c \equiv \frac{\overline{\omega}}{\overline{\tau}\varsigma_c-\sigma\underline{\omega}\varsigma_d}.$$

On the other hand, we know that in the standard equilibrium selection, initial consumption is given by

$$c_0^{NK} = -\frac{\kappa}{\sigma(\overline{\omega}-\underline{\omega})}\left[\sigma^{-1}\omega_c\int_0^T\left(\frac{e^{-\overline{\omega}s}}{\overline{\omega}}-\frac{e^{-\underline{\omega}s}}{\underline{\omega}}\right)(i_s-\rho)ds+\int_0^T\left(e^{-\overline{\omega}s}-e^{-\underline{\omega}s}\right)(\omega_g g_s+\tau_s)ds\right].$$

Equalizing $c_0^{NK} = c_0$, we can isolate T and get

$$T^{NK} = \int_0^T \left[\chi^T_{i,t}(i_t - \rho) + \chi^T_{g,t} g_t + \chi^T_{\tau,t}\tau_t \right] dt,$$

where

$$\chi^T_{i,t} = -\frac{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d}{\overline{\omega}} \left[\frac{\hat{\kappa}}{\sigma(\overline{\omega} - \underline{\omega})} \left(\frac{e^{-\overline{\omega}t}}{\overline{\omega}} - \frac{e^{-\underline{\omega}t}}{\underline{\omega}} \right) - \sigma^{-1} \frac{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d}{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d} e^{-\overline{\omega}t} \right],$$

$$\chi^T_{g,t} = -\frac{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d}{\overline{\omega}} \left[\frac{\kappa\omega_g}{\sigma(\overline{\omega} - \underline{\omega})} (e^{-\overline{\omega}t} - e^{-\underline{\omega}t}) + \overline{\omega} \frac{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d}{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d} \frac{\omega_g}{\omega_c} (e^{-\rho t} - e^{-\overline{\omega}t}) + \overline{\omega} \frac{1 - \overline{\tau}}{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d} \varsigma\right.$$

$$\chi^T_{\tau,t} = -\frac{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d}{\overline{\omega}} \left[\frac{\kappa}{\sigma(\overline{\omega} - \underline{\omega})} (e^{-\overline{\omega}t} - e^{-\underline{\omega}t}) + \overline{\omega} \frac{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d}{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d} \frac{1}{\omega_c} (e^{-\rho t} - e^{-\overline{\omega}t}) - \overline{\omega} \frac{1 - \overline{\tau}}{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d} e\right.$$

Note that $\chi^T_{i,0} = -\frac{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d}{\overline{\omega}} \left[\frac{\hat{\kappa}}{\sigma(\overline{\omega} - \underline{\omega})} \frac{\underline{\omega} - \overline{\omega}}{\overline{\omega}\underline{\omega}} - \sigma^{-1} \frac{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d}{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d} e^{-\overline{\omega}t} \right]$ and since $\overline{\omega}\underline{\omega} = -\hat{\kappa}$, and $\frac{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d}{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d} > 1$, then $\chi^T_{i,0} < 0$. Moreover,

$$\frac{\partial \chi^T_{i,t}}{\partial t} < 0.$$

Hence

$$\frac{\partial T}{\partial i_t} < 0.$$

Taking the partial derivative with respect to time, we get

$$\frac{\partial^2 T}{\partial t \partial i_t} = -\frac{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d}{\overline{\omega}} \left[\frac{\hat{\kappa}}{\sigma(\overline{\omega} - \underline{\omega})} \left(-e^{-\overline{\omega}t} + e^{-\underline{\omega}t} \right) + \overline{\omega}\sigma^{-1} \frac{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d}{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d} e^{-\overline{\omega}t} \right] < 0.$$

It is straightforward to see that as $t \to \infty$, $\chi^T_{i,t} \to -\infty$. ■

Proof of Proposition ??. The effect of government spending on initial consumption is given by

$$\frac{\partial c_0^{NK}}{\partial g_t} = -\frac{\kappa\omega_g}{\sigma(\overline{\omega} - \underline{\omega})} (e^{-\overline{\omega}s} - e^{-\underline{\omega}s}) > 0,$$

$$\frac{\partial^2 c_0^{NK}}{\partial t \partial g_t} = -\frac{\kappa\omega_g}{\sigma(\overline{\omega} - \underline{\omega})} (-\overline{\omega}e^{-\overline{\omega}s} + \underline{\omega}e^{-\underline{\omega}s}) > 0,$$

$$\lim_{t\to\infty}\frac{\partial c_0^{NK}}{\partial g_t} = \lim_{t\to\infty} -\frac{\kappa\omega_g}{\sigma(\overline{\omega}-\underline{\omega})}(\underbrace{e^{-\overline{\omega}s}}_{\to 0} - \underbrace{e^{-\underline{\omega}s}}_{\to\infty}) = \infty$$

$$\lim_{\kappa\to\infty}\frac{\partial c_0^{NK}}{\partial g_t} = \lim_{\kappa\to\infty} -\frac{\kappa\omega_g}{\underbrace{\sigma(\overline{\omega}-\underline{\omega})}_{\to\infty}}(\underbrace{e^{-\overline{\omega}s}}_{\to 0} - \underbrace{e^{-\underline{\omega}s}}_{\to\infty}) = \infty,$$

and on inflation, by

$$\frac{\partial \pi_0^{NK}}{\partial g_t} = \frac{\kappa\omega_g}{\overline{\omega}-\underline{\omega}}\left(\overline{\omega}e^{-\overline{\omega}s} - \underline{\omega}e^{-\underline{\omega}s}\right) > 0,$$

$$\lim_{t\to\infty}\frac{\partial \pi_0^{NK}}{\partial g_t} = \lim_{t\to\infty}\frac{\kappa\omega_g}{\overline{\omega}-\underline{\omega}}\left(\underbrace{\overline{\omega}e^{-\overline{\omega}s}}_{\to 0} - \underbrace{\underline{\omega}e^{-\underline{\omega}s}}_{\to -\infty}\right) = \infty,$$

$$\lim_{\kappa\to\infty}\frac{\partial \pi_0^{NK}}{\partial g_t} = \lim_{\kappa\to\infty}\frac{\kappa\omega_g}{\underbrace{\overline{\omega}-\underline{\omega}}_{\to\infty}}\left(\underbrace{\overline{\omega}e^{-\overline{\omega}s}}_{\to 0} - \underbrace{\underline{\omega}e^{-\underline{\omega}s}}_{\to -\infty}\right) = \infty.$$

Since $\frac{\partial c_0^N K}{\partial g_t} = \omega_g \frac{\partial c_0^{NK}}{\partial \tau_t}$, all the results go through for τ_t as well. ■

Proof of Lemma ??. From the proof of Lemma 1 we know that any equilibrium of the New Keynesian model can be written as

$$c_t = e^{\underline{\omega}t}c_0 + c_t^m + c_t^f$$
$$\pi_t = e^{\underline{\omega}t}\pi_0 + \pi_t^m + \pi_t^f$$

for some $c_t^m, c_t^f, \pi_t^m, \pi_t^f$, functions of $\{i_t, g_t, \tau_t\}_{t=0}^{\infty}$,

$$\pi_0 = \sigma\underline{\omega}\left[\frac{\overline{\omega}-\underline{\omega}}{\sigma\hat{\kappa}}\int_0^\infty e^{-\overline{\omega}s}\eta_{1s}ds - c_0\right],$$

and

$$c_0 = \int_0^\infty \left[\chi_{i,t}^c(i_t-\rho) + \chi_{g,t}^c g_t + \chi_{\tau,t}^c\hat{\tau}_t\right]dt + \chi_T^c T,$$

for some $\chi_{i,t}^c, \chi_{g,t}^c, \chi_{\tau,t}^c, \chi_{T,t}^c$, independent of the shocks. Thus, we can index all equilibria of the New Keynesian model by the level of lump-sum transfers, T. By choosing T, the equations above completely characterize the equilibrium path of consumption and inflation. In particular, we have

$$\tilde{c}_0 \equiv \int_0^\infty \left[\chi^c_{i,t}(i_t - \rho) + \chi^c_{g,t} g_t + \chi^c_{\tau,t}\hat{\tau}_t\right] dt,$$

$$\tilde{\pi}_0 = \sigma\underline{\omega}\left[\frac{\overline{\omega} - \underline{\omega}}{\sigma\hat{\kappa}}\int_0^\infty e^{-\overline{\omega}s}\eta_{1s} ds - \tilde{c}_0\right],$$

and

$$\tilde{c}_t = e^{\underline{\omega}t}\tilde{c}_0 + c^m_t + c^f_t$$
$$\tilde{\pi}_t = e^{\underline{\omega}t}\tilde{\pi}_0 + \pi^m_t + \pi^f_t.$$

It is straightforward to see that

$$c_t = \tilde{c}_t + \frac{\overline{\omega}}{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d} e^{\underline{\omega}t} T,$$

and

$$\pi_t = \tilde{\pi}_t + \frac{\kappa\omega_c}{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d} e^{\underline{\omega}t} T.$$

■

Proof of Proposition ??. We have

$$\frac{\partial \tilde{c}_0}{\partial i_t} = \chi^c_{i,t} = -\sigma^{-1}\frac{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d}{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d} e^{-\overline{\omega}t} < 0,$$

$$\frac{\partial^2 \tilde{c}_0}{\partial t \partial i_t} = \sigma^{-1}\frac{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d}{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d}\overline{\omega} e^{-\overline{\omega}t} > 0,$$

$$\lim_{t\to\infty}\frac{\partial \tilde{c}_0}{\partial i_t} = -\sigma^{-1}\frac{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d}{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d}\lim_{t\to\infty} e^{-\overline{\omega}t} = 0,$$

$$\lim_{\kappa\to\infty}\frac{\partial \tilde{c}_0}{\partial i_t} = -\lim_{\kappa\to\infty}\sigma^{-1}\underbrace{\frac{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d}{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d}}_{\to 0}\underbrace{e^{-\overline{\omega}t}}_{\to 0} = 0.$$

■

Proof Lemma ??. First, let's compute the substitution effect. The Hicksian demand of the non-linear model is obtained as the solution to the following problem

$$\min_{\{C_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\int_0^t (i_s - \pi_s) ds} C_t dt$$

$$st \quad \int_0^{\infty} e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} dt \geq \overline{U},$$

for some $\overline{U} \in \mathbb{R}$. The FOCs of this problem are given by

$$e^{-\int_0^t (i_s - \pi_s) ds} = \lambda e^{-\rho t} C_t^{-\sigma} \quad \forall t,$$

where λ is the Lagrange multiplier associated to the constraint. This implies that

$$C_t = e^{\frac{1}{\sigma} \int_0^t (i_s - \pi_s) ds} \lambda^{\frac{1}{\sigma}} e^{-\frac{\rho}{\sigma} t} \implies e^{-\rho t} C_t^{1-\sigma} = e^{-\frac{\rho}{\sigma} t} e^{-\frac{\sigma - 1}{\sigma} \int_0^t (i_s - \pi_s) ds} \lambda^{\frac{1-\sigma}{\sigma}},$$

and hence

$$\lambda = \left[\frac{(1-\sigma)\overline{U}}{\int_0^{\infty} e^{-\frac{\rho}{\sigma} t} e^{-\frac{\sigma-1}{\sigma} \int_0^t (i_s - \pi_s) ds} dt} \right]^{\frac{\sigma}{1-\sigma}}.$$

Replacing in the FOC for C_t, we get

$$C_t = \frac{e^{-\frac{\rho}{\sigma} t} e^{\frac{1}{\sigma} \int_0^t (i_s - \pi_s) ds}}{\left[\int_0^{\infty} e^{-\frac{\rho}{\sigma} t} e^{-\frac{\sigma-1}{\sigma} \int_0^t (i_s - \pi_s) ds} dt \right]^{\frac{1}{1-\sigma}}} \left[(1-\sigma)\overline{U} \right]^{\frac{1}{1-\sigma}}.$$

Log-linearizing around the zero inflation steady state we get,

$$c_t^S = \frac{1}{\sigma} \int_0^t (i_s - \pi_s - \rho) ds - \frac{1}{\sigma} \int_0^{\infty} e^{-\rho t} (i_s - \pi_s - \rho) dt.$$

The present discounted value of the substitution effect is given by

$$\begin{aligned}\int_0^\infty e^{-\rho t} c_t^S dt &= \frac{1}{\sigma}\int_0^\infty e^{-\rho t}\int_0^t (i_s - \pi_s - \rho) ds dt - \frac{1}{\sigma}\int_0^\infty e^{-\rho t}\int_0^\infty e^{-\rho s}(i_s - \pi_s - \rho) ds, \\ &= \frac{1}{\rho\sigma}\int_0^\infty e^{-\rho t}(i_s - \pi_s - \rho) ds - \frac{1}{\rho\sigma}\int_0^\infty e^{-\rho s}(i_s - \pi_s - \rho) ds, \\ &= 0.\end{aligned}$$

Now, let's calculate the wealth effect. From the Euler equation we have

$$c_t = c_0 + \frac{1}{\sigma}\int_0^t e^{-\rho s}(i_s - \pi_s - \rho) ds,$$

hence

$$c_t^W = c_t - c_t^S = c_0 + \frac{1}{\sigma}\int_0^\infty e^{-\rho s}(i_s - \pi_s - \rho) ds,$$

which is constant over time. Let $C \equiv c_t^W$. Then net present value of consumption is given by

$$\int_0^\infty e^{-\rho t} \varsigma_c c_t dt = \int_0^\infty e^{-\rho t} \varsigma_c (c_t^S + C) dt = \frac{\varsigma_c}{\rho} C,$$

where the last equality follows from the fact that the present value of the substitution effect is zero. Introducing this in the budget constraint, we get

$$c_t^W = C = \frac{\rho}{\varsigma_c}\int_0^\infty e^{-\rho t}[(1-\overline{\tau})(y_t - \tau_t) + \varsigma_d(i_t - \pi_t - \rho)] dt.$$

■

Proof of Lemma ??. Immediate from the expression for c_t^S. ■

Proof of Proposition ??. The substitution effect is given by

$$c_t^S = \sigma^{-1}\int_0^t (i_s - \pi_s - \rho) ds - \sigma^{-1}\int_0^\infty e^{-\rho s}(i_s - \pi_s - \rho) ds.$$

Consider a monetary shock at date s. The substitution effect of consumption at dates $t < s$ is given by

$$-\sigma^{-1} e^{-\rho s} + \sigma^{-1} \int_0^\infty e^{-\rho z} \frac{\partial \pi_z}{\partial i_s} dz.$$

Moreover,

$$\frac{\partial \pi_z}{\partial i_s} = e^{\underline{\omega} z} \frac{\partial \pi_0}{\partial i_s} + \frac{\partial \pi_z^m}{\partial i_s},$$

where

$$\frac{\partial \pi_0}{\partial i_s} = -\underline{\omega} \frac{\sigma \overline{\omega} \varsigma_d}{\overline{\tau} \varsigma_c - \sigma \underline{\omega} \varsigma_d} e^{-\overline{\omega} s}$$

and

$$\frac{\partial \pi_z^m}{\partial i_s} = \begin{cases} \frac{\hat{\kappa}}{\overline{\omega} - \underline{\omega}} (e^{\overline{\omega} z} - e^{\underline{\omega} z}) e^{-\overline{\omega} s}, & \text{if } z < s \\ -\frac{\hat{\kappa}}{\overline{\omega} - \underline{\omega}} e^{\underline{\omega} z} (e^{-\overline{\omega} s} - e^{-\underline{\omega} s}), & \text{if } z > s \end{cases}$$

Putting all the pieces together, and after some algebra, we get

$$\left. \frac{\partial c_t^S}{\partial i_s} \right|_{t<s} = -\sigma^{-1} \frac{\overline{\tau} \varsigma_c}{\overline{\tau} \varsigma_c - \sigma \rho \varsigma_d} e^{-\overline{\omega} s} < 0.$$

It is straightforward that $\lim_{s \to \infty} \frac{\partial c_0^S}{\partial i_s} = 0$. Moreover,

$$\lim_{\kappa \to \infty} \frac{\partial c_0^S}{\partial i_s} = \left(\frac{1}{\omega_c} - \sigma^{-1} \right) e^{-\rho s}.$$

Now consider the substitution effect at dates $t > s$

$$\sigma^{-1}(1 - e^{-\rho s}) - \sigma^{-1} \int_0^t \frac{\partial \pi_z}{\partial i_s} dz + \sigma^{-1} \int_0^\infty e^{-\rho z} \frac{\partial \pi_z}{\partial i_s} dz$$

Consider the third term

$$\int_0^\infty e^{-\rho z} \left(e^{\underline{\omega} z} \frac{\partial \pi_0}{\partial i_s} + \frac{\partial \pi_z^m}{\partial i_s} \right) dz = \frac{1}{\overline{\omega}} \frac{\partial \pi_0}{\partial i_s} + \int_0^\infty e^{-\rho z} \frac{\partial \pi_z^m}{\partial i_s} dz$$

where

$$\frac{\partial \pi_0}{\partial i_s} = -\underline{\omega} \frac{\sigma \overline{\omega} \varsigma_d}{\overline{\tau} \varsigma_c - \sigma \underline{\omega} \varsigma_d} e^{-\overline{\omega} s}$$

$$\frac{\hat{\kappa}}{\overline{\omega}-\underline{\omega}}\int_0^s e^{-\rho z}\left(e^{\overline{\omega}z}-e^{\underline{\omega}z}\right)e^{-\overline{\omega}s}dz=\frac{\hat{\kappa}}{\overline{\omega}-\underline{\omega}}\left(-\frac{e^{-\rho s}}{\underline{\omega}}+\frac{e^{-\overline{\omega}s}}{\underline{\omega}}+\frac{e^{-2\overline{\omega}s}}{\overline{\omega}}-\frac{e^{-\overline{\omega}s}}{\overline{\omega}}\right)$$

$$-\frac{\hat{\kappa}}{\overline{\omega}-\underline{\omega}}\int_s^\infty e^{-\rho z}e^{\underline{\omega}z}\left(e^{-\overline{\omega}s}-e^{-\underline{\omega}s}\right)dz=-\frac{\hat{\kappa}}{\overline{\omega}-\underline{\omega}}\left(\frac{e^{-2\overline{\omega}s}}{\overline{\omega}}-\frac{e^{-\rho s}}{\overline{\omega}}\right)$$

$$\int_0^\infty e^{-\rho z}\frac{\partial\pi_z^m}{\partial i_s}dz=e^{-\rho s}-e^{-\overline{\omega}s}$$

Hence

$$\sigma^{-1}\int_0^\infty e^{-\rho z}\frac{\partial\pi_z}{\partial i_s}dz=-\sigma^{-1}\frac{\sigma\underline{\omega}\varsigma_d}{\overline{\tau}\varsigma_c-\sigma\underline{\omega}\varsigma_d}e^{-\overline{\omega}s}+\sigma^{-1}\left(e^{-\rho s}-e^{-\overline{\omega}s}\right).$$

Now consider the first term

$$\int_0^t\left(e^{\underline{\omega}z}\frac{\partial\pi_0}{\partial i_s}+\frac{\partial\pi_z^m}{\partial i_s}\right)dz=\frac{e^{\underline{\omega}t}-1}{\underline{\omega}}\frac{\partial\pi_0}{\partial i_s}+\int_0^t\frac{\partial\pi_z^m}{\partial i_s}dz$$

where

$$\frac{\hat{\kappa}}{\overline{\omega}-\underline{\omega}}\int_0^s\left(e^{\overline{\omega}z}-e^{\underline{\omega}z}\right)e^{-\overline{\omega}s}dz=\frac{\hat{\kappa}}{\overline{\omega}-\underline{\omega}}\left(\frac{1-e^{-\overline{\omega}s}}{\overline{\omega}}-\frac{e^{(\underline{\omega}-\overline{\omega})s}-e^{-\overline{\omega}s}}{\underline{\omega}}\right)$$

$$-\frac{\hat{\kappa}}{\overline{\omega}-\underline{\omega}}\int_s^t e^{\underline{\omega}z}\left(e^{-\overline{\omega}s}-e^{-\underline{\omega}s}\right)dz=-\frac{\hat{\kappa}}{\overline{\omega}-\underline{\omega}}\frac{e^{\underline{\omega}t}\left(e^{-\overline{\omega}s}-e^{-\underline{\omega}s}\right)-e^{(\underline{\omega}-\overline{\omega})s}+1}{\underline{\omega}}$$

$$\int_0^t\frac{\partial\pi_z^m}{\partial i_s}dz=\left(1-e^{-\overline{\omega}s}\right)-\frac{\hat{\kappa}}{\overline{\omega}-\underline{\omega}}\frac{e^{\underline{\omega}t}\left(e^{-\overline{\omega}s}-e^{-\underline{\omega}s}\right)}{\underline{\omega}}$$

Hence

$$-\sigma^{-1}\int_0^t\frac{\partial\pi_z^m}{\partial i_s}dz=-\sigma^{-1}\left(1-e^{-\overline{\omega}s}\right)+\frac{\sigma^{-1}\hat{\kappa}}{\overline{\omega}-\underline{\omega}}\frac{e^{\underline{\omega}t}\left(e^{-\overline{\omega}s}-e^{-\underline{\omega}s}\right)}{\underline{\omega}}$$

Therefore

$$\sigma^{-1}\frac{\sigma\overline{\omega}\varsigma_d}{\overline{\tau}\varsigma_c-\sigma\underline{\omega}\varsigma_d}e^{-\overline{\omega}s}e^{\underline{\omega}t}-\sigma^{-1}\frac{\sigma\rho\varsigma_d}{\overline{\tau}\varsigma_c-\sigma\underline{\omega}\varsigma_d}e^{-\overline{\omega}s}-\sigma^{-1}\frac{\overline{\omega}}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\left(e^{-\overline{\omega}s}-e^{-\underline{\omega}s}\right)$$

■

Proof of Proposition ??. Then

$$\rho\int_0^\infty e^{-\rho s}(1-\overline{\tau})(\tilde{y}_s-\tau_s)ds = \underbrace{\rho\int_0^\infty e^{-\rho s}(1-\overline{\tau})\tilde{c}_s ds}_{=(1-\overline{\tau})C} + \rho\int_0^\infty e^{-\rho s}(1-\overline{\tau})(g_s-\tau_s)ds$$
$$= (1-\overline{\tau})C + \rho\int_0^\infty e^{-\rho s}(1-\overline{\tau})(g_s-\tau_s)ds,$$

and

$$\rho\overline{b}\int_0^\infty e^{-\rho s}(i_s-\tilde{\pi}_s-\rho)ds = \rho\overline{b}\int_0^\infty e^{-\rho s}(i_s-\pi_t^*+\sigma\underline{\omega}\tilde{c}_t-\rho)ds$$
$$= \rho\overline{b}\int_0^\infty e^{-\rho s}(i_s-\pi_t^*-\rho)ds + \sigma\underline{\omega}\overline{b}C.$$

Introducing these two results into (**??**), we get

$$C = [1-(\overline{\tau}-\sigma\underline{\omega}\overline{b})]C + A, \tag{21}$$

or

$$C = \frac{A}{\overline{\tau}-\sigma\underline{\omega}\overline{b}},$$

where $A \equiv \rho\overline{b}\int_0^\infty e^{-\rho s}(i_s-\pi_t^*-\rho)ds + \rho\int_0^\infty e^{-\rho s}(1-\overline{\tau})(g_s-\tau_s)ds$ is the autonomous component of consumption. ■

Proof of Lemma ??. Consider firm's j problem in a flexible price economy

$$\max_{P_t(j)}(1-\tau_t)P_t(j)Y_t(j) - W_t\left(\frac{Y_t(j)}{A_t}\right)^{\frac{1}{\alpha}}$$

$$s.t. \quad Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}Y_t.$$

Plugging in the demand in the objective function and taking firs order conditions gives

$$(\epsilon-1)(1-\tau_t)\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}Y_t = \frac{\epsilon}{\alpha}\frac{W_t}{P_t(j)}\left(\frac{P_t(j)}{P_t}\right)^{-\frac{\epsilon}{\alpha}}\left(\frac{Y_t}{A_t}\right)^{\frac{1}{\alpha}}.$$

Rearranging,

$$P_t = \frac{\epsilon}{\epsilon - 1}\frac{1}{1-\tau_t}\frac{W_t}{\alpha}\frac{Y_t^{\frac{1-\alpha}{\alpha}}}{A_t^{\frac{1}{\alpha}}} = \frac{\epsilon}{\epsilon - 1}\frac{1}{1-\tau_t}\frac{W_t}{\alpha}\frac{N_t^{1-\alpha}}{A_t} \implies w_t - p_t = -\tau_t - (1-\alpha)n_t.$$

Labor supply is given by

$$\sigma c_t + \phi n_t = w_t - p_t.$$

The resource constraint is given by

$$y_t = c_t + g_t.$$

The production function gives

$$y_t = \alpha n_t.$$

Combining labor demand and labor supply, we obtain

$$n_t = -\frac{\sigma c_t + \tau_t}{\phi + 1 - \alpha}.$$

Plugging the production function and the expression for labor into the resource constraint

$$-\frac{\alpha\sigma}{\phi+1-\alpha}c_t - \frac{\tau_t}{\phi+1-\alpha} = c_t + g_t \implies c_t = -\underbrace{\frac{\phi+1-\alpha}{\phi+1-\alpha+\alpha\sigma}}_{\omega_g} g_t - \underbrace{\frac{1}{\phi+1-\alpha+\alpha\sigma}}_{\omega_\tau}\tau_t.$$

Hence, the present discounted value of consumption is given by

$$C^{fp} = -\rho\int_0^\infty e^{-\rho t}[\omega_g g_t + \omega_\tau \tau_t]dt,$$

which gives us the derivatives $\frac{\partial C^{fp}}{\partial g_t} = -\rho e^{-\rho t}\omega_g$ and $\frac{\partial C^{fb}}{\partial \tau_t} = -\rho e^{-\rho t}\omega_\tau$.

Consider the limit involving T

$$\lim_{\kappa\to\infty} \frac{T}{\overline{\tau} - \underline{\omega}\sigma\varsigma_d} = \lim_{\kappa\to\infty} \frac{\sqrt{\kappa}}{\overline{\tau} - \underline{\omega}\sigma\varsigma_d} \frac{T}{\sqrt{\kappa}} = \lim_{\kappa\to\infty} \sqrt{-\sigma\frac{\overline{\omega}}{\underline{\omega}}} \frac{-\underline{\omega}}{\overline{\tau} - \underline{\omega}\sigma\varsigma_d} \frac{T}{\sqrt{\kappa}} \propto \lim_{\kappa\to\infty} \frac{T}{\sqrt{\kappa}},$$

since $\lim_{\kappa\to\infty} \sqrt{-\sigma\frac{\overline{\omega}}{\underline{\omega}}} \frac{-\underline{\omega}}{\overline{\tau}-\underline{\omega}\sigma\varsigma_d} = \frac{1}{\sqrt{\sigma\varsigma_d}}$. Hence, the wealth effect converges to its value in the flexible price equilibrium if and only if $\lim_{\kappa\to\infty} \frac{T}{\sqrt{\kappa}} = 0$. ■

.1 Smets & Wouters (2007) Revisited

In order to keep the exposition short, we present the log-linearized version of the model, followed by a description of the differences that incorporating the fiscal variables introduce to the model.

The equilibrium of the economy is characterized by the following system of linear rational expectations equations:

- the aggregate resource constraint

$$y_t = c_y c_t + i_y i_t + g_y g_t + z_y z_t, \tag{22}$$

- the consumption Euler equation

$$c_t = c_1 c_{t-1} + (1 - c_1)E_t c_{t+1} + c_2(l_t - E_t l_{t+1}) - c_3(r_t - E_t \pi_{t+1}), \tag{23}$$

- the investment Euler equation

$$i_t = i_1 i_{t-1} + (1 - i_1)E_t i_{t+1} + i_2 q_t, \tag{24}$$

- the Tobin's Q

$$q_t = q_1 E_t q_{t+1} + (1 - q_1)E_t r^k_{t+1} - (r_t - E_t \pi_{t+1}), \tag{25}$$

- the production function

$$y_t = \phi_p \left(\alpha k_t^s + (1-\alpha) l_t \right), \tag{26}$$

- the capital services equation

$$k_t^s = k_{t-1} + z_t, \tag{27}$$

- the capital utilization rate

$$z_t = z_1 r_t^k, \tag{28}$$

- the capital accumulation equation

$$k_t = k_1 k_{t-1} + (1-k_1) i_t, \tag{29}$$

- price mark-up

$$\mu_t^p = \alpha (k_t^s - l_t) - w_t, \tag{30}$$

- the NK Phillips curve

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \pi_3 (1-\tau^*) \tau_t, \tag{31}$$

- firms' cost minimization equation

$$r_t^k = -(k_t^s - l_t) + w_t, \tag{32}$$

- wage mark-up

$$\mu_t^w = w_t - \left[\sigma_l l_t + \frac{1}{1-\frac{h}{\gamma}} \left(c_t - \frac{h}{\gamma} c_{t-1} \right) \right], \tag{33}$$

- the aggregate wage index

$$w_t = w_1 w_{t-1} + (1 - w_1)(E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w, \tag{34}$$

- long-term bond price

$$q_t^L = \frac{\rho_L}{1 + \bar{r}} * q_{t+1}^L - r_t; \tag{35}$$

- the household's budget constraint

$$c_y c_t + i_y i_t + q^L b_y (b_t^L + q_t^L) = (1 - \tau^*)(y_t - \tau_t) - z_y z_t + \frac{\rho_L q^L b_y}{1 + \pi^*} q_t^L - \frac{(1 + \rho_L q^L) b_y}{1 + \pi^*} \pi_t + \frac{(1 + \rho_L q^L) b_y}{1 + \pi^*} b_{t-1}^L + T_t, \tag{36}$$

where y_t is output, c_t is consumption, i_t is investment, g_t is government spending, z_t is the capital utilization rate, l_t is hours worked, r_t is the nominal interest rate (set by the monetary authority), π_t is the inflation rate, q_t is the Tobin's Q, r_t^k is the rental rate of capital services, k_t^s is capital services, k_t is the stock of capital, w_t is the real wage, μ_t^p is the price mark-up, μ_t^w is the wage mark-up, τ_t is the proportional sales tax, b_t^L is government bonds, q_t^L is the price of the long-term bond, and T_t are government lump-sum transfers.