

Reallocation Effects of Monetary Policy*

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Abstract

Central banks across the globe are paying increasing attention to the distributional aspects of monetary policy. In this study, we focus on reallocation among heterogeneous firms triggered by nominal growth. Japanese firm-level data show that large firms tend to grow faster than small firms under higher inflation. We then construct a model that introduces nominal rigidity into endogenous growth with heterogeneous firms. The model shows that, under a high nominal growth rate, firms of inferior quality bear a heavier burden of menu cost payments than do firms of superior quality. This outcome increases the market share of superior firms, while some inferior firms exit the market. This reallocation effect, if strong, yields a positive effect of monetary expansion on both real growth and welfare. The optimal nominal growth can be strictly positive even under nominal rigidity, whereas standard New Keynesian models often conclude that zero nominal growth is optimal. Moreover, the presence of menu costs can improve welfare.

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1 Introduction

The literature on reallocation or firm dynamics has emphasized that sizable heterogeneity exists among firms, plants, and establishments even within narrowly defined industries (e.g., Baily et al. (1992); Bartelsman and Doms (2000); Bartelsman et al. (2004)). Firm dynamics has a significant implication on macroeconomic performance (e.g., Caballero and Hammour (1994); Hsieh and Klenow (2009); Hsieh and Klenow (2018); Restuccia and Rogerson (2013)). However, these previous studies do not pay attention to the nominal side of the economy, although it is extremely important to consider how monetary policy influences firm dynamics in deriving the optimal monetary policy or the optimal inflation target. Indeed, Yellen (2016), former Chair of the Federal Reserve, states that economists should place greater emphasis on heterogeneity among firms and households, as ignoring it could overlook important transmission channels of monetary policies. Kuroda (2017), Governor of the Bank of Japan, states that central banks should pay more attention to the distributional effects of monetary policy.

This study investigates the reallocation effects of monetary policy in a framework that introduces nominal rigidity into endogenous growth with heterogeneous firms. In the model, nominal rigidity brings about changes in firm dynamics in response to inflation. Such a relationship between inflation and firm dynamics has empirical relevance. Using Japanese firm-level data, we find that firm-size distribution and firm-level growth are associated with, and also likely to be influenced by, inflation rates. Specifically, under a higher inflation rate, large firms tend to grow faster than small firms in real terms as measured by sales and employment. This result is consistent with our model prediction.

The model has three features: Schumpeterian creative destruction, multi-product firms, and menu costs. The Schumpeterian creative-destruction model developed by Grossman and Helpman (1991) and Aghion and Howitt (1992) enables analysis of firm entry and exit as well as endogenous growth. In the basic model, however, each firm is assumed to produce only one product. For all firms, the amount of sales is the

same and, thus, the firm-size distribution is degenerate. The second feature, multi-product firms, enables analysis of firm dynamics. Klette and Kortum (2004) build a model in which incumbent firms can produce more than one product by continuing R&D investment and taking the markets from other firms. This leads to resource reallocation from inferior to superior firms and ex post heterogeneity in firm size. Lentz and Mortensen (2005, 2008) introduce ex ante heterogeneity in innovativeness of firms as well as selection after entry to investigate the effect of resource reallocation.

The third feature, menu costs, is our theoretical innovation. We shed new light on reallocation stemming from the nominal side of the economy, such as monetary policy, by extending the Lentz and Mortensen (2005) model to introduce menu costs as nominal rigidity (e.g., Sheshinski and Weiss (1977); Golosov and Lucas (2007); Midrigan (2011)). We adopt menu costs for nominal rigidity because they are a fundamental setup in the New Keynesian literature and we believe they are more realistic than Calvo- or Rotemberg-type pricing models. One of the contributions of this study is that it provides a tractable framework to consider the reallocation effects of monetary policy.

The main results from our model are as follows. First, nominal growth can enhance real growth. Second, the optimal nominal growth (and inflation) rate can be positive. These results are in a sharp contrast to the results of standard New Keynesian models in which the economy is the most efficient without nominal growth.¹ Although nominal rigidity incurs a cost for all firms, the burden is relatively larger for lower-quality firms and, thus, higher-quality firms increase their market shares as well as relative R&D expenditures. As a result, the average quality of firms increases under positive nominal growth. If this reallocation effect dominates inefficiency stemming from nominal rigidity, the optimal nominal growth (and inflation) rate can be positive. A key to the welfare implication is heterogeneity in negative externality of R&D, or the business-stealing effect, in Schumpeterian growth models. A heavier menu cost burden crowds out low-quality firms whose innovations provide only small social benefits relative to private benefits. Put differently, menu costs serve as a quality-dependent entry barrier. For this reason, nominal rigidity is not necessarily a source of inefficiency but a benefit to the economy. Low menu costs are better for

¹The optimal inflation target is often zero if the real growth rate is zero and unless other frictions, such as the zero lower bound of a nominal interest rate and downward wage rigidity, are present. See Goodfriend and King (1997), Khan et al. (2003), Burstein and Hellwig (2008), Schmitt-Grohé and Uribe (2010), and Coibion et al. (2012).

welfare. As Keynes mentions in *The General Theory* (Keynes (1936)), an economy is fortunate to have nominal rigidity.

A model calibrated to Denmark based on Lentz and Mortensen (2008) reveals that the optimal nominal growth is indeed positive. Under higher nominal growth, the ratio of large firm's sales to those of small firms increases, which is consistent with our empirical finding. Furthermore, both the average firm size and the average quality of firms increase.

Sensitivity analysis shows that the optimal nominal growth (inflation) rate depends on circumstances. Particularly important factors are the shape of the distribution of innovation ability and a measure of potential entrants by influencing the extent of the reallocation effect. The fatter is the tail of the distribution of innovation ability, the larger is the positive reallocation effect and the higher is the optimal nominal growth (inflation) rate. On the other hand, if the measure of potential entrants is large, the reallocation effect among incumbents weakens and, thus, the optimal nominal growth (inflation) rate declines. This is actually the case for the model calibrated to Japan.

Some existing studies argue that nominal factors, such as inflation and nominal interest rates, affect both real growth and welfare in endogenous growth models (e.g., Bilbiie et al. (2014); Chu and Cozzi (2014); Chu et al. (2017); Arawatari et al. (2018); Oikawa and Ueda (2018)). Chu and Cozzi (2014) show that the optimal nominal interest rate is strictly positive if R&D is overinvested in a Schumpeterian model with cash-in-advance in R&D. Recently, Adam and Weber (2017) derive a positive optimal inflation rate in a New Keynesian model with growth and firm heterogeneity, but both productivity growth and entry in their model are exogenous and, thus, monetary policy does not have distributional impacts. Compared with these studies, ours is the first work to combine a New Keynesian model with an endogenous growth model embedding reallocation and firm dynamics. Thus, we can connect our model to the vast literature on monetary policy based on New Keynesian models in which money is absent. Moreover, our model does not need a case for overinvestment in R&D, which rarely occurs under standard parameter values, to have positive optimal nominal growth if there are sufficient reallocation effects.²

²Oikawa and Ueda (2018) is a homogeneous-firm version of the current model (thus, there is no reallocation). They calibrate the model to the US economy and conclude that there is underinvestment in R&D and that the optimal nominal growth rate is zero.

Acemoglu et al. (2017) is related to our study, although they do not consider the nominal aspect, as they analyze several industry policies (e.g., R&D subsidy) by extending the model of Lentz and Mortensen (2008). One of the main results of their study is that tax should be imposed only on inferior incumbents to achieve the social optimum. However, as the authors mention, governments rarely conduct such a selective policy in reality. By contrast, our model suggests that monetary policy that is not selective has different impacts across firms: inferior firms incur a greater burden of menu cost payments under high nominal growth than superior firms do. In other words, monetary policy automatically works as an entry barrier only for low quality firms.

To study the reallocation effects of monetary policy, other types of models may be useful (e.g., Hopenhayn (1992); Bernard et al. (2003); Melitz (2003); Luttmer (2007); Restuccia and Rogerson (2013); Lucas and Moll (2014)). Compared with these models, our model explicitly incorporates firms' R&D investment decision by both entrants and incumbents. In particular, incumbents' R&D is considered important for growth and reallocation. As documented in Malerba and Orsenigo (1999), the majority of newly granted patents are held by existing patent holders. Argente et al. (2018) report that firms grow by continuously adding products and that incumbents are the main agents of product reallocation. In our model, R&D by incumbents plays the key role in firm growth as well as endogenous economic growth.

Our study complements the growing literature on the effects of monetary policy on heterogeneous agents. For example, Iacoviello (2005) and Gornemann et al. (2016) investigate monetary policy effects on different types of households. A notable difference from these studies is that the focus of our study is not on household heterogeneity but on firm heterogeneity.

The rest of the paper is organized as follows. After Section 2 provides empirical findings, Section 3 builds a theoretical model and Section 4 defines stationary equilibrium. Section 5 investigates the reallocation impacts of nominal growth. Section 6 shows the results of numerical simulations. Section 7 concludes.

2 Relationship between Inflation and Firm-size Distributions: Empirics

Here, we provide some empirical evidence on the relationship between inflation and firm-size distributions. We first briefly review the existing literature and then present observations from Japanese firm-level data.

Existing Literature

Although empirical studies using firm- or plant-level data have been growing rapidly, studies on the relationship between the nominal side of the economy and firm-size distributions are scarce, as there is almost no theoretical analysis of this issue. However, some indirect evidence seems to suggest the existence of a relationship between firm distribution and inflation. For example, Alfaro et al. (2008) report that less developed countries, notably India, tend to have a greater mean establishment size than advanced countries do. Similar findings are reported by Bartelsman et al. (2004) and Poschke (2018). Considering that less developed countries tend to have relatively high inflation rates, this fact seems to suggest that there is some association between inflation and the growth of large firms relative to small ones in a cross-country dimension.

It should be noted, however, that there is considerable cause for concerns about how to accurately measure firm distributions or what drives different firm distributions.³ To alleviate these concerns, we next analyze the relationship between inflation

³There are considerable measurement errors for firm or plant distributions owing to the large cost of collecting data and the presence of informal small establishments. The latter problem is particularly serious in less developed countries, where there are a large mass of informal small establishments. Thus, the more we include informal small establishments as *firms*, the lower is the share of big firms, which may lead to the opposite result: that less developed countries have smaller mean firm size than advanced countries. This issue is related to the so-called “missing middle.” Tybout (2014) calculates the deviation of employment share from the estimated Pareto distribution by country and plant size category and finds that, for India and Indonesia, the smallest and largest size categories are more populated than predicted by the Pareto distribution, whereas for the United States, the largest size category is less populated. In fact, the issue of the missing middle is consistent with our study. In our model, a firm is an agent that makes an R&D investment to invent new ideas and employs people for both production and R&D. This type of firm is not like an informal small establishment. For this reason, we exclude informal small establishments and pay attention to the large and medium-sized category in Tybout (2014). This fact suggests that, in less developed countries, which are typically more inflationary countries, large firms are relatively more common than medium-sized firms, which is consistent with our model’s prediction. Another important note is that cross-country differences in firm distributions are caused by many other factors as well as

and firm-size distributions using Japanese firm-level data. Policies and institutions are considered less heterogeneous within a country than between countries. We control industry differences and time-series changes in policies, institutions, and measurement errors by using industry and year dummies. Furthermore, by using instrumental variables, we aim to study if there is any causality from inflation to firm-size distributions.

Japanese Firm-level Data

As firm-level data, we use the Basic Survey of Japanese Business Structure and Activities surveyed by Ministry of Economy, Trade and Industry in Japan. The survey covers all firms with no less than 50 employees and no less than 30 million Japanese yen in capital in Japan for 1991 and 1994–2015. We focus on manufacturing firms and calculate percentile points of nominal sales within each major division of industries⁴ to observe changes in firm-size distribution. For inflation rates, we use industry-level input price indexes from the producer price index (PPI) reported by the Bank of Japan. We consider the average input inflation during the past 2 years as the main explanatory variable, for consistency with the theoretical model in the next section, in which we analyze a balanced growth path with trend inflation and firms’ pricing decision depends on inflation in production cost. Appendix A shows the time-series of input-PPI inflation in the manufacturing sector in Japan.

Table 1(a) shows the panel estimation results on the relationship between input inflation, $\bar{\pi}_{it}^{\text{input}}$, and sales dispersions, which are measured by the ratios of the 90th percentile to the median (Top/Middle) and the 90th percentile to the 10th percentile (Top/Bottom), using such equations as

$$\text{Dispersion measure}_{it} = \beta_0 + \beta_1 \bar{\pi}_{it}^{\text{input}} + \mathbf{Z}'_{it} \boldsymbol{\zeta} + \zeta_t^T + \zeta_i^I + \epsilon_{it}, \quad (1)$$

where ζ_t^T and ζ_i^I are year and industry fixed effects, respectively. As the basic OLS regression shows, in columns (1) and (4) in Table 1(a), the coefficients on inflation are positive and significant. Sales dispersion is increased under input inflation. For robustness, we introduce control variables, \mathbf{Z}_{it} , such as industry-level financing po-

differences in the nominal side of the economy. Clearly, policies and institutions differ by country.

⁴We follow the industry classification used in the producer price index. Because the classification has been revised a few times during the sample periods, we use the 2005 industry classification to obtain consistent price index sequences. The list of the 14 we use is summarized in Appendix A.

sition index, the Diffusion Index (DI) from *Tankan*,⁵ and industry-level real sales, industry RS, as shown in columns (2) and (5).⁶ The DI is a proxy for the degree of financial constraint (the higher the DI is, the looser is the constraint). We control the degree of financial constraint, because if large firms have more access to finance, it affects the size distribution. In the table, “DI” is the industry-level index and “DI gap” represents the gaps in DI between large and medium-sized firms (T/M) and between large and small firms (T/B). We include industry-level real sales to control aggregate real factors. The table shows that the coefficients on inflation are positive.

These impacts on firm-size distribution from inflation may be subject to an endogeneity problem, because high demand for inputs from large growing firms could drive up input prices in an industry. In columns (3) and (6) of Table 1(a), we perform second-stage least squares (2SLS) using the information on international primary commodity prices as the instrumental variable, which is reported by the IMF (converted to Japanese yen by the average nominal exchange rate in each year). Because the commodity price is an aggregate variable, we set our instrumental variables as the cross-terms of industry dummies and inflation in primary commodity prices during the past 2 years. The positive association between inflation and sales dispersion is significant again. We also observe that the industry-level DI is negatively correlated with sales dispersion, implying that less financial constraint shrinks the size gap among firms. The coefficients in the 2SLS regressions indicate that a 1% increase in inflation leads to about a 2-unit increase in the T/M ratio and about a 6-unit increase in the T/B ratio.⁷

Table 1(b) shows the estimation result when we measure firm size by total employment. The result is similar to that for sales dispersion. Employment dispersion is also positively associated with input inflation, and is robust when we consider other control variables and endogeneity.

To understand the source of the rise in these observed dispersions, we examine the relationship between firm-level growth and inflation, in particular its size dependency. We divide firms into 10 size groups according to firm-size distribution in terms of

⁵The financing position index is one of the DIs from *Tankan*, surveyed by the Bank of Japan. It indicates the difference in the shares of firms that answer whether their financial positions are “easy” and “tight.” Data on industry and firm size (large, medium, or small) are available.

⁶Real sales are calculated by dividing nominal sales by the industry-level output price index from the PPI.

⁷The means of the T/M and T/B ratios are about 11 and 43 in sales, respectively, and about 5 and 10 in employment, respectively.

sales or employment. Then, we calculate the average firm growth rates from $t - 1$ to t within size groups, based on size distributions at $t - 1$. The size group index comprises integers from 1 to 10, where groups 1 and 10 represent firms smaller than the 10th percentile point and those larger than the 90th percentile point, respectively. The equation regressed is

$$\text{Firm growth}_{ist} = \beta_0 + \beta_1 \bar{\pi}_{it}^{\text{input}} + \beta_2 \cdot \text{Size group} + \beta_3 \cdot \text{Size group} \times \bar{\pi}_{it}^{\text{input}} + \mathbf{Z}'_{ist} \cdot \boldsymbol{\zeta} + \zeta_t^T + \zeta_i^I + \epsilon_{it}, \quad (2)$$

where the dependent variable is the average growth rate of real sales or employment at the firm level in industry i , size group s , and year t . The control \mathbf{Z}_{ist} contains the average financial DI within size groups and industry-level real sales. Table 2 shows that input inflation has a negative impact on firm growth and that a larger firm size leads to lower growth.⁸ A more important result is observed in the cross-term of inflation and size group. In all regressions, the cross-term is positive and significant, implying that the negative impact of input inflation on growth is partly offset when a firm is large. This result is consistent with the increased firm-size dispersion under inflation, which is observed in Table 1. Columns (3) and (6) of Table 2 present the 2SLS results with the same instrumental variable strategy as in Table 1. We confirm the robustness of the complementarity between firm size and inflation.⁹

In summary, the results from the Japanese firm-level data suggest that there is a significant response in firm distributions to inflation. In the next section, we build a theoretical model that is consistent with the fact and derive growth and welfare implications of monetary policy through reallocation.

3 Model

The model is based on Lentz and Mortensen (2005) and Oikawa and Ueda (2018). A representative household consumes and supplies a fixed amount of labor. Firms, both potential entrants and incumbents, make R&D investments to develop new products with superior quality and take over the markets of other firms. Firms must pay menu

⁸There is vast literature on Gibrat's law on firm growth, which states that the firm growth rate is independent of firm size. Empirically, many papers report that size has a negative impact on growth at the firm level. See Santarelli et al. (2006) for an empirical literature survey.

⁹Appendix A runs the same regressions in this section using dispersion and firm growth in domestic sales for a robustness check.

costs when they revise their output price as well as when they enter the market and set an initial price. Because of the menu cost payments, some firms decide not to enter the market even if they develop a new product when their product quality is not sufficiently high. Firms are ex post heterogeneous in their product quality and, in turn, firm size.

We focus on a balanced growth path. We assume for simplicity that a central bank exogenously sets the growth rate of nominal aggregate output, E_t , at n .¹⁰ Denoting the quality-adjusted aggregate price index and its change (inflation rate) by P_t and π , respectively, we can write the real growth rate, g , as $g = n - \pi$. For convenience, we denote the initial values in period $t = 0$ without time subscripts, for example, $E_0 = E$. In the following analysis, we consider the case with $n \geq 0$, because trend nominal growth rates are usually positive. Even in the lost decades in Japan with long-term stagnation periods, the average per capita nominal GDP growth rate is almost zero.

In the current model setting, we focus on an extensive margin to explain firm growth. By an extensive margin, we mean a change in the number of product lines that firms produce. Because our model assumes unit elasticity of substitution between different product lines and infinite elasticity of substitution between products in the same product line, as in Grossman and Helpman (1991) and Aghion and Howitt (1992), firms' sales relative to nominal aggregate expenditure never grow as long as firms engage in production of a single product. Changes in the intensive margin, that is, an improvement of productivity or a quality update in the same product line, cannot increase firms' sales. Incumbent firms can grow their sales size only by changing the extensive margin, that is, taking the market share of other firms and producing more than one product line. For this reason, we need multi-product firms to obtain a dispersed firm distribution.¹¹ This model strategy is consistent with the finding by Cao et al. (2018) that firms grow by adding new establishments (extensive margin) rather than by increasing employment per establishment (intensive margin). Certainly, we do not intend to argue that the intensive margin is unimportant. For example, the models in Luttmer (2007) and Lucas and Moll (2014) generate firm-

¹⁰Our results are essentially the same even if we assume that a central bank sets a certain inflation target instead of controlling n . See Oikawa and Ueda (2018).

¹¹The extensive margin also plays an important role in reallocation in the model of Melitz (2003), in which firms expand their sales via exports. Using product- and firm-level data, Argente et al. (2018) show that firms grow only by adding new products.

size distribution by the intensive margin, often through heterogeneous and exogenous changes in productivity.¹²

3.1 Household

A representative household has the following preferences over all versions of $a \in \{0, 1, \dots, A_t(j)\}$ of each product line $j \in [0, 1]$:

$$U_t = \int_t^\infty e^{-\rho(t'-t)} \log C_{t'} dt', \quad (3)$$

$$\log C_{t'} = \int_0^1 \log \left[\sum_{a=0}^{A_{t'}(j)} Q(j, a) x_{t'}(j, a) \right] dj, \quad (4)$$

where ρ represents the subjective discount rate, C_t is aggregate consumption, and $x_t(j, a)$ and $Q(j, a)$ are consumption and quality of version a of product j , respectively.¹³ The quality evolves as

$$Q(j, a) = \prod_{a'=0}^a q(j, a'), \quad q(j, a') > 1 \quad \forall a', j,$$

where $q(j, a')$ is the quality update created by each innovation.

3.2 Firms' Decision about Price Revisions

This part is based on Oikawa and Ueda (2018). The only difference is that a firm has to pay a menu cost when it enters the market.¹⁴ Price setting requires menu cost,

$$\kappa E_t / P_t, \quad (5)$$

at time t in real terms, which is paid by consumption goods.

¹²Our assumption about the elasticity of substitution is for analytical simplicity. Garcia-Macia et al. (2016) argue that one of the main sources of growth is incumbents' quality updates on their own product lines.

¹³Extending the form of aggregate consumption to the constant elasticity of substitution form does not change our results as long as the elasticity of substitution between products in two different product lines is not too large. See Lentz and Mortensen (2008).

¹⁴If menu cost payment is not required at entry for each product line, firms with very low q supply goods until the first timing of price revision and exit the market without revising the initial price. Our assumption is to avoid this complication.

Suppose that nominal prices are reset at t_i for $i = -1, 0, 1, \dots$. If nominal price $p_{t_{-1}}$ posted at time t_{-1} is not revised thereafter, its real price, say ξ , decreases at the growth rate of nominal production costs (wage, W_t), n . Let $\xi_{t_{-1}, \tau} \equiv p_{t_{-1}} e^{-n(t_{-1} + \tau)} \equiv \xi_{-1} e^{-n\tau}$, where τ is the elapsed time from the previous price revision and ξ_i is the revised real price at t_i (i.e., $\xi_{t_i, 0} \equiv \xi_i$).¹⁵

Then, when the price is reset at t , the real value of each product line is expressed as

$$V_t = \max_{\{t_i, \xi_i\}_{i=1}^{\infty}} \sum_{i=0}^{\infty} \left(\int_{t_i}^{t_{i+1}} \Pi(\xi_{t_i, t' - t_i} e^{-n(t' - t_i)}) \frac{E_{t'}}{P_{t'}} e^{-(\rho + g + \delta)(t' - t)} dt' - \frac{\kappa E_{t_i}}{P_{t_i}} e^{-(\rho + g + \delta)(t_i - t)} \right), \quad (6)$$

where δ is the rate of creative destruction, $\Pi(\cdot) E_t / P_t$ represents the real period profit, and $t_0 = t$. The second term represents menu cost payments at t_i . Because of the recursive structure of the price adjustment problem, this maximization problem can be reduced to the following problem by controlling the interval of price revision, Δ , and the reset level of real price, ξ_0 , where we drop the absolute time subscript from $\xi_{t_i, \tau}$. Let $\nu_{\tau}(q|\delta, n)$ be the normalized value of product line, $P_t V_t / E_t$, with elapsed time $\tau = t - t_i \in [0, \Delta]$ for some i . The problem (6) can be reduced to

$$\nu_0(q|\delta, n) = \max \{ \tilde{\nu}_0(q|\delta, n), 0 \}, \quad (7)$$

$$\tilde{\nu}_0(q|\delta, n) = \max_{\Delta, \xi_0} \sum_{i=0}^{\infty} e^{-(\rho + \delta)i\Delta} \left(\int_0^{\Delta} \Pi(\xi_0 e^{-n\tau}) e^{-(\rho + \delta)\tau} d\tau - \kappa \right), \quad (8)$$

where $\tilde{\nu}_0$ is the value of a product line just before setting the price, which can be negative because of menu cost payment. If it is negative, the firm decides not to produce or not to enter the market. Firms choose the optimal interval of price revision Δ and real reset price ξ_0 as $\Delta(q|\delta, n)$ and $\xi_0(q|\delta, n)$, respectively. They both depend on their quality step q so that price revision behaviors are also heterogeneous among firms. We assume that each good is produced by linear technology with productivity of 1. Then, $\Pi(\xi) = (\xi - W)/\xi$.

To simplify the pricing strategy, we assume that knowledge on outdated technologies is automatically in the public domain. With this assumption, firms with secondary technology, if any, are under perfect competition and thus their prices are marginal cost, W . Hence, the limit price of a leading firm is qW . From log utility,

¹⁵In the current model, we do not consider indexation of final goods prices. The model works unless the nominal prices are perfectly linked to nominal growth rate, n .

the optimal (re)set real price is

$$\xi_0(q|\delta, n) = qW. \quad (9)$$

Note that $\Delta(q|\delta, 0) = \infty$ because firms do not need to reset prices when costs do not change. The optimal choice of price revision is expressed by

$$\frac{d}{d\Delta} \tilde{\nu}_0(q|\delta, n) = 0 = -(\rho + \delta)\tilde{\nu}_0 + \Pi(\xi_0 e^{-n\Delta}) + \kappa(\rho + \delta). \quad (10)$$

Substituting this into equation (8), we obtain the maximized value of a product line for type- q firm as

$$\tilde{\nu}_0(q|\delta, n) = \begin{cases} \frac{1}{\rho+\delta} \left(1 - \frac{e^{n\Delta(q|\delta, n)}}{q}\right) & \text{for } n > 0, \\ \frac{1}{\rho+\delta} \left(1 - \frac{1}{q}\right) - \kappa & \text{for } n = 0. \end{cases} \quad (11)$$

Lemma 3 in Appendix D shows that $\tilde{\nu}_0(q|\delta, n)$ is strictly increasing in q and strictly decreasing in δ and n . Moreover, the second-order cross-derivative of $\tilde{\nu}_0(q|\delta, n)$ with respect to q and n is positive.

The intuitions is as follows. Rapid nominal growth (high n) implies frequent price reset (low Δ) and high menu cost payments, leading to lower value of a product line ($\tilde{\nu}_0$). The burden from a rise in menu cost payments, which is triggered by an increase in n , is relatively smaller for firms with greater q , because menu costs are independent of q , while the return from the price reset is increasing in q . Higher- q firms have larger markup and choose larger time interval, Δ . This increases the sensitivity of such firms' profits to a change in n , which causes more frequent price changes when n increases (i.e., $(d^2\Delta)/(dqdn) < 0$).

The product line value for low-quality firms can be negative when there are positive menu costs, $\kappa > 0$. Here, we denote $\underline{q}(\delta, n)$ as the threshold level of q ,

$$\underline{q}(\delta, n) = \max \{q \mid \tilde{\nu}_0(q|\delta, n) \leq 0\}. \quad (12)$$

Proposition 1 shows that $\underline{q}(\delta, n)$ uniquely exists if κ is sufficiently small, and it is increasing in n when $n \geq 0$, implying that faster nominal growth pushes inferior firms out of the market.

Proposition 1 *If $\kappa < \frac{1}{\rho+\delta}$, then $\underline{q}(\delta, n)$ uniquely exists and $\underline{q}(\delta, n)$ is increasing in n*

for $n \geq 0$.

All proofs are in Appendix D. Throughout the study, we assume that the menu cost parameter κ is small enough for Proposition 1 to hold.

3.3 Incumbent Firms' R&D

Here, we consider incumbents' R&D decision, taking the optimal pricing decision as given. This subsection is based on Lentz and Mortensen (2005, 2008). We assume that the size of quality improvement, q , is firm specific and unchanged over time for each incumbent.

Suppose that an incumbent firm is type q and supplies $k \geq 1$ products. The R&D cost is $kW_t c(\gamma)$, where $\gamma \geq 0$ is the R&D intensity. With probability $k\gamma$, the firm comes up with a new idea in a randomly chosen product line. The function $c(\gamma)$ is increasing and strictly convex with $c(0) = 0$. We define the real wage by $w \equiv W/E$.

The Bellman equation for the incumbent's problem is a bit complicated, because profits from a product line depend on the elapsed time from the previous price revision. However, as we see later in this subsection, the resultant value function is similar to that in Lentz and Mortensen (2008). Let $T_k \equiv \{\tau_j\}_{j=1}^k$, the set of elapsed times from the previous price setting. The real value of the firm with q , k , and T_k under the environment of (δ, w, n) , say $v_k(T_k, q|\delta, w, n)$, satisfies the following Bellman equation:

$$\begin{aligned}
\rho v_k(T_k, q|\delta, w, n) = & \max_{\gamma} \sum_{j \notin \Omega} \left[\Pi^0(\xi_0 e^{-n\tau_j}) + \frac{\partial v_k(T'_k, q|\delta, w, n)}{\partial \tau_j} \right] \\
& + \sum_{j \in \Omega} \left[\Pi^0(\xi_0 e^{-n\tau_j}) - \kappa + \frac{\partial v_k(T'_k, q|\delta, w, n)}{\partial \tau_j} \right] \\
& - kwc(\gamma) \\
& + k\gamma [v_{k+1}(\{T'_k, 0\}, q|\delta, w, n) - v_k(T'_k, q|\delta, w, n)] \\
& + k\delta \left[\frac{1}{k} \sum_{j=1}^k v_{k-1}(T'_{k-1, <j>}, q|\delta, w, n) - v_k(T'_k, q|\delta, w, n) \right],
\end{aligned} \tag{13}$$

where $\Omega \equiv \{j|\tau_j = \Delta(q|\delta, n)\}$ represents the set of products whose prices are to be revised, and $T'_k \equiv \{\tau'_j\}_{j=1}^k$ consists of $\tau'_j = 0$ if $j \in \Omega$ and $\tau'_j = \tau_j$ otherwise. $T_{k-1, <j>}$

is the set comprising elapsed time of the firm when it exits from j -th product market. On the right-hand side of equation (13), the first and second lines represent the real period profit when nominal prices are not reset and when nominal prices are reset, respectively. The real period profit decreases as τ lengthens. The third line represents R&D investment costs, which are proportional to the number of product lines k . The fourth and fifth lines represent the changes in real firm value when the firm increases and decreases its product lines by one, respectively.

Appendix B shows that the value function, v_k , is solved as

$$v_k(T_k, q|\delta, w, n) = \sum_{i=1}^k \nu_{\tau_i}(q|\delta, n) + k\psi(q|\delta, w, n), \quad (14)$$

where $k\psi(q|\delta, w, n)$ is the value from R&D in the future, which depends on the current number of product lines, or knowledge level, because it contributes to probability of success in R&D. The appendix also shows that incumbents make the following optimal R&D intensity choice, γ :

$$wc'(\gamma) = \max_{\gamma \in [0, \rho + \delta]} \frac{(\rho + \delta)\nu_0(q|\delta, n) - wc(\gamma)}{\rho + \delta - \gamma}. \quad (15)$$

The next proposition summarizes the properties of the optimal γ . It shows that firms with greater q have a higher R&D intensity and there is a complementarity effect among q and n . Since more rapid nominal growth monotonically decreases R&D intensity for any firm, the decline in R&D intensity under higher n is relatively small for firms with greater q .

Proposition 2 *Fix n and $\delta > 0$. $\gamma(q|\delta, w, n)$ uniquely exists for sufficiently large w . If such γ exists, $\gamma(q|\delta, w, n)$ is increasing in q and decreasing in n , δ , and w . Moreover, for $n \neq 0$,*

$$\frac{\partial^2 \gamma(q|\delta, w, n)}{\partial q \partial n} > 0 \quad \text{and} \quad \frac{\partial^2 \gamma(q|\delta, w, n)}{\partial q \partial w} < 0.$$

The optimal choice $\gamma(q|\delta, w, n)$ is increasing in $\nu_0(q|\delta, n)$. Hence, higher q implies a higher value of a product line and, at the same time, greater R&D investment.

3.4 Firm Entry

The entry stage consists of two steps. First, a measure h of potential entrants make R&D decisions before knowing their own q . The exogenous density of types of potential entrants is $\bar{\phi}(q)$ on $[1, \bar{q}]$, where \bar{q} can be infinite. We assume that $\bar{\phi}(q)$ is continuous on its support. Second, some of the firms that observe $q < \underline{q}(\delta, n)$, defined in equation (12), call off their entry, because menu cost payment makes them worse off. We denote the ex post entry rate by η . Here, for simplicity, we do not allow for the possibility that such firms coordinate with their incumbents to enter the market. While entrants give up their entry, incumbents continue to monopolize the market. We denote the ex post distribution of q for the entrants as

$$\phi(q|\delta, n) = \begin{cases} 0 & \text{for } q < \underline{q}(\delta, n), \\ \frac{\bar{\phi}(q)}{1 - \bar{\Phi}(\underline{q}(\delta, n))} & \text{for } q \geq \underline{q}(\delta, n), \end{cases} \quad (16)$$

where $\bar{\Phi}$ is the cumulative distribution associated with $\bar{\phi}$.

We assume free entry to the market. In equilibrium, we have the following free-entry (FE) condition,

$$\int_1^\infty \bar{\phi}(q) v_1(\{0\}, q|\delta, w, n) dq = wc'(\gamma_\eta(\delta, w, n)), \quad (17)$$

$$\gamma_\eta(\delta, w, n) \equiv \frac{\eta(\delta, w, n)}{1 - \bar{\Phi}(\underline{q}(\delta, n))} \frac{1}{h}, \quad (18)$$

where $v_1(\{0\}, q|\delta, w, n)$ is the firm value of an entrant and $\gamma_\eta(\delta, w, n)$ is entrants' R&D intensity, giving the ex post entry rate of η . The R&D cost function c for potential entrants is the same as that for incumbents.

3.5 Firm Distribution

3.5.1 Price Distribution

Because of the Ss pricing rule due to menu costs, firms are heterogeneous in prices even within firms with the same q . From Oikawa and Ueda (2018), the stationary

distribution of real prices among type- q firms is

$$f(\xi(\tau)|q, n) = \frac{\delta e^{-\delta\tau}}{1 - e^{-\delta\Delta(q|\delta, n)}} \quad \text{for } \tau \in [0, \Delta(q|\delta, n)]. \quad (19)$$

3.5.2 Firm Size and Quality

Let $M_k(q|\delta, w, n)$ be the measure of type- q firms that produce k products under the environment (δ, w, n) . Following Lentz and Mortensen (2005), the stationary distribution satisfies

$$\begin{aligned} \gamma(q|\delta, w, n)(k-1)M_{k-1}(q|\delta, w, n) + \delta(k+1)M_{k+1}(q|\delta, w, n) \\ = (\gamma(q|\delta, w, n) + \delta)kM_k(q|\delta, w, n) \quad \text{for } k \geq 2, \end{aligned} \quad (20)$$

with

$$\phi(q|\delta, n)\eta = \delta M_1(q|\delta, w, n), \quad (21)$$

$$\phi(q|\delta, n)\eta + 2\delta M_2(q|\delta, w, n) = (\gamma(q|\delta, w, n) + \delta)M_1(q|\delta, w, n). \quad (22)$$

Based on Appendix C, we derive the following equation:

$$1 = \eta \int_{\underline{q}(\delta, n)}^{\infty} \frac{\phi(q|\delta, n)}{\delta - \gamma(q|\delta, w, n)} dq. \quad (23)$$

Equation (23) pins down the entry rate in equilibrium, $\eta(\delta, w, n)$. Because the measure of products supplied by type- q firms is represented by

$$K(q|\delta, w, n) \equiv \frac{\eta(\delta, w, n)\phi(q|\delta, n)}{\delta - \gamma(q|\delta, w, n)},$$

condition (23) is equivalent to the condition that the total measure of product lines must be one.

The equilibrium entry rate, $\eta(\delta, w, n)$, has the following properties. First, η is increasing in w , because $\partial\gamma/\partial w < 0$ for any $q > \underline{q}(\delta, n)$ so that the integral in equation (23) decreases with w . Second, the relationship between δ and η is ambiguous. In response to an increase in δ , $\delta - \gamma(q|\delta, w, n)$ increases for any $q > \underline{q}(\delta, n)$ because $\partial\gamma/\partial\delta < 0$, but at the same time, $\phi(q|\delta_1, n)$ stochastically dominates $\phi(q|\delta_2, n)$ if $\delta_1 > \delta_2$. The former effect reduces the integral in equation (23) but the latter distribution

effect enlarges it. Although it is possible that the distribution effect dominates and η is decreasing in δ if we consider ad hoc distribution for $\bar{\phi}$,¹⁶ we assume that the distribution effect does not dominate in the following analyses, meaning, roughly speaking, that we assume that $\bar{\phi}$ is sufficiently smooth.

Substituting $\eta(\delta, w, n)$ into the free-entry condition, equation (17), we obtain Proposition 3, which shows that the free-entry condition is depicted as a downward-sloping curve on the δ - w space unless the distribution effect is too strong.

Proposition 3 *For given δ , the wage w that satisfies the free-entry condition uniquely exists. Moreover, as long as $\eta(\delta, w, n)$ is increasing in δ , the free-entry condition implies a negative association between δ and w .*

Intuitively, a greater creative destruction rate should be at least partly driven by a higher entry rate, although it implies a lower expected firm value after entry, which dampens innovation incentives. Wage reduction is required to fill the gap, as it would increase post-entry firm value and simultaneously lead to greater incumbents' R&D, which implies a lower share of entry to the creative destruction rate.

3.6 Labor Market

Labor consists of three groups of workers: production workers L_X , R&D workers hired by incumbents $L_{R,inc}$, and R&D workers hired by entrants $L_{R,ent}$. There is no skill difference across sectors.

Production workers are represented by

$$L_X(\delta, w, n) = \int_{\underline{q}(\delta, n)}^{\infty} K(q|\delta, w, n)L_{X,q}(q|\delta, w, n)dq, \quad (24)$$

where $L_{X,q}(q|\delta, w, n)$ is production labor demanded by type- q firms, which is derived

¹⁶This situation can arise when the increase in $\underline{q}(\delta, n)$ in response to an increase in δ eliminates a large mass of firms, as indicated by the following example. Suppose that $\bar{\phi}$ is defined on a discrete space and has two mass points with equal probabilities at $\{q_1, q_2\}$ and there are negligible probabilities on the other q . Suppose \underline{q} is very close to q_1 so that $\gamma(q_1)$ is almost zero. In addition, suppose $\gamma(q_2) = \frac{\delta}{2}$. Now consider the impact of an increase in δ to δ' , where the increase is tiny but enough to eliminate q_1 -type firms from the market. Before the change, the integral in equation (23), the expected value of $(\delta - \gamma(q))^{-1}$, is approximately $\frac{1.5}{\delta}$. After the change, it becomes approximately $\frac{2}{\delta'}$. If δ'/δ is sufficiently small, then η is decreasing in δ around the current δ .

from the price density function (19), such as

$$L_{X,q}(q|\delta, w, n) = \int_0^{\Delta(q|\delta, n)} f(\xi(\tau)) \frac{1}{\xi(\tau)} d\tau = \frac{\delta}{\delta - n} \frac{1}{Ewq} \frac{1 - e^{-(\delta-n)\Delta(q|\delta, n)}}{1 - e^{-\delta\Delta(q|\delta, n)}}. \quad (25)$$

The two types of R&D labor demands are written as

$$L_{R,inc}(\delta, w, n) = \int_{\underline{q}(\delta, n)}^{\infty} K(q|\delta, w, n) c(\gamma(q|\delta, w, n)) dq. \quad (26)$$

$$L_{R,ent}(\delta, w, n) = hc(\gamma_{\eta}(\delta, w, n)), \quad (27)$$

The labor market clearing (LMC) condition is defined as

$$L = L_X(\delta, w, n) + L_{R,inc}(\delta, w, n) + L_{R,ent}(\delta, w, n) \equiv L_D(\delta, w, n). \quad (28)$$

As discussed in Appendix E, we consider it natural to have

$$\frac{\partial L_D}{\partial \delta} > 0 \quad \text{and} \quad \frac{\partial L_D}{\partial w} < 0.$$

This yields a positive association between δ and w for the LMC condition as depicted in Figure 1.

4 Stationary Equilibrium

4.1 Equilibrium

Given exogenous $\{n, \bar{\phi}(q)\}$, a stationary equilibrium consists of constant endogenous variables $\{\nu_0(q|\delta, n), \Delta(q|\delta, n), \xi_0(q, \delta, w, n), \gamma(q|\delta, w, n), w, \delta, \eta\}$ and they satisfy producers' optimal Ss-pricing (9) and (10), firm value (11), incumbents' optimal R&D intensity choice (15), the free-entry condition (17), the entry/exit rate equation (23), and the labor market clearing condition (28) as well as stationary distributions $\{M_k(q|\delta, w, n), \phi(q|\delta, n)\}$ that satisfy equations (16) and (20)-(22).

Figure 1 illustrates a typical loci of FE and LMC curves with the curve of $w(\delta, n)$, defined in Appendix C, above which a nondegenerated quality distribution exists in the stationary state. The left edge of the LMC curve should coincide with a point on

the curve of $\underline{w}(\delta, n)$. This is because $L_D(\delta, \underline{w}(\delta, n), n)$ is continuous in δ and

$$\lim_{\delta \rightarrow 0} L_D(\delta, \underline{w}(\delta, n), n) = 0 < L < \lim_{\delta \rightarrow \frac{1}{\kappa} + \rho} L_D(\delta, \underline{w}(\delta, n), n),$$

where the upper bound of δ is defined to have a finite \underline{q} .¹⁷

Since the locus of FE is always above $\underline{w}(\delta, n)$, we can show the existence of a stationary equilibrium as in the next proposition.

Proposition 4 *For given $n \geq 0$, a stationary equilibrium exists and the entry rate in a stationary state is strictly positive.*

Because uniqueness of the equilibrium depends on monotonicity of the labor demand function, multiple equilibria may occur under some kind of potential distribution, $\bar{\phi}$. However, we do no longer debate this possibility anymore because, according to our simulations with smooth potential distributions, this nonmonotonicity is not relevant.

4.2 Welfare

Using the endogenous variables determined in a stationary equilibrium, we can pin down other endogenous variables as well: $\{C, P, \pi, g, U\}$.

The goods market is cleared as

$$\begin{aligned} E &= PC + \kappa E \int dq \sum_{k=1}^{\infty} M_k(q|\delta, w, n) k f(\xi_0(q, n)) \\ \Rightarrow C &= \frac{E}{P} \left(1 - \kappa \delta \int \frac{K(q|\delta, w, n)}{1 - e^{-\delta \Delta(q|\delta, n)}} dq \right), \end{aligned} \quad (29)$$

where the second term in the large parenthesis is the total menu cost payments.

Appealing to Oikawa and Ueda (2018), the aggregate price level and the inflation rate are given by

$$\log P = \int K(q|\delta, w, n) \left\{ \log \xi_0(q, n) + n \Delta \frac{e^{-\delta \Delta(q|\delta, n)}}{1 - e^{-\delta \Delta(q|\delta, n)}} - \frac{n}{\delta} \right\} dq, \quad (30)$$

$$\pi = n - \delta \int K(q|\delta, w, n) \log q dq. \quad (31)$$

¹⁷Note that $\lim_{\delta \rightarrow 0} \underline{w}(\delta) = \infty$ and $\lim_{\delta \rightarrow \frac{1}{\kappa} + \rho} L_{R,ent} = \infty$ because $\underline{q}(\frac{1}{\kappa} + \rho, n) = \infty$.

Then, the real growth rate g is

$$g = n - \pi = \delta \int_{\underline{q}(\delta, n)}^{\infty} K(q|\delta, w, n) \log q dq. \quad (32)$$

Welfare at $t = 0$ is given by

$$U = \int_0^{\infty} e^{-\rho t} \log C_t dt = \frac{g}{\rho^2} + \frac{\log C}{\rho}. \quad (33)$$

5 Impacts of Nominal Growth

In the current model, monetary policy is described as a change in the nominal growth rate, n . We simply assume that the central bank chooses n and compare the stationary states across different n .¹⁸

5.1 Reallocation Effects on the Real Growth Rate

Nonzero nominal growth has two kinds of reallocation effects. First, it pushes low-quality firms out of the market because of greater burden of menu cost payments ($\partial q/\partial n > 0$). Second, it causes higher-quality firms to hold more product lines, because $\partial^2 \gamma/\partial n \partial q > 0$: the decline in R&D intensity caused by an upward shift of n is relatively small for firms with greater q . Moreover, the increase in the product line share owned by high- q firms benefits efficiency in R&D investment, because the probability of success is $k\gamma$. These reallocation effects are interpreted as the “cleansing effect” named by Caballero and Hammour (1994). The reallocation effects are positive for the real growth rate, g .

At the same time, there is a negative impact on g from faster nominal growth, which is examined in the Oikawa and Ueda (2018) model that abstracts ex ante firm heterogeneity. When the economy grows faster in nominal terms, firms revise their prices more frequently and pay more menu costs, leading to lower firm values and lower innovation incentives.

The overall impact of faster nominal growth on real growth is determined by the balance between these effects. As shown in equation (32), the real growth rate is

¹⁸The current model conforms to Lentz and Mortensen (2008) (although distributions are continuous here) if there is no menu cost, or $\kappa = 0$. When $n = 0$, the current model becomes Lentz and Mortensen (2008) with fixed cost of entry to product line.

determined by the creative destruction rate, δ , multiplied by the weighted average of quality gaps, $\log q$. Even though the creative destruction rate decreases with n , when the reallocation effect is sufficiently large, the increase in the weighted average of quality gap, $\int K(q|\delta, w, n) \log q dq$, dominates the negative effect on δ , and then the real growth rate, g , becomes increasing in the nominal growth rate, n .

5.2 Reallocation Effects on Welfare

Here, we compare welfare in two balanced growth paths that are distinct only in the nominal growth rate. The impacts on welfare consist of several factors. As equations (29) and (33) show, the impacts should be divided into the real growth channel, menu cost channel, and initial price channel. The real growth channel consists of two elements. The one is simply from the abovementioned real growth effect. Since the market equilibrium is suboptimal, more rapid growth improves welfare if positive externality of R&D is sufficiently strong. The other element stems from reduction in negative externality in R&D, or the business-stealing effect (Aghion and Howitt (1992)). This effect is derived from the real product line value “stolen” from the former leading firm. Thus, innovations by high- q firms tend to be accompanied by lower business-stealing effects than the social benefits of their innovations. In other words, menu costs can improve welfare by automatically hindering entry of low-quality firms. Thus, nominal rigidity improves welfare through reallocation.

The menu cost channel basically has an ambiguous effect on welfare. An increase in n imposes more frequent menu cost payments for all firms. However, at the same time, an increase in n tends to bring less creative destruction (and less entry), which reduces the total menu cost. Moreover, a greater n makes $K(q|\delta, w, n)$ be denser with higher q . Because the increase in frequency of price revisions caused by an increase in n is relatively small for high- q firms, the total menu cost payments may decrease under rapid nominal growth.

The price channel is two-fold and both paths have ambiguous effects. The first one comes from a change in markups. The markup rate just after price revision is high among the products supplied by high- q firms. Because distribution $K(q|\delta, w, n)$ becomes denser with higher q when we have a higher n , the real prices tend to be high. However, this impact is cancelled out partly or fully by a lower rate of entry and a rapid decline in real prices, caused by an increase in n . The second path is

derived from a change in wage, w . The price level P is monotonically increasing in w but the equilibrium value of w can be decreasing and increasing in n , according to the labor market conditions. If a smaller w is realized by an increase in n , it positively affects welfare.

6 Simulations

In this section, we present the results of numerical simulations.

6.1 Parameter Setting

We set the total number of workers, L , to one. For the parameter of the menu cost, we use $\kappa = 0.022$, based on the work of Midrigan (2011). For the other parameters, we follow the work of Lentz and Mortensen (2008), whose model is equivalent to our model without nominal rigidity, that is, $\kappa = 0$ or roughly speaking, $n = 0$. We calibrate the model to the economy of Denmark. The discount rate ρ is set to 0.0361, which equals the interest rate 0.05 minus the real growth rate $g = 0.0139$ in Lentz and Mortensen (2008). The R&D cost function is given by $c(\gamma) = c_0 \times \gamma^{c_1}$, where $c_1 = 3.728$. As for c_0 and the measure of potential entrants, h , we calibrate the values so that the rate of creative destruction, δ , equals 0.071 and the rate of entry, η , equals 0.045 in the case of $n = 0$, in which menu costs do not affect firms' price setting in our model.¹⁹ We then obtain $c_0 = 2.209 \cdot 10^4$ and $h = 2.699$.

We assume that the distribution of innovation ability, $\bar{\phi}(q)$, obeys the finite beta distribution, $B(a, b)$, where the two coefficients a and b are calibrated to be consistent with Lentz and Mortensen (2008). Lentz and Mortensen (2008) assume a simple discrete distribution for q , which takes only three values, that is, $q = 1$ (no quality growth), $q_L > 1$, and $q_H > 1$ and then estimate the probability for each q . Here, it is important to note that firms with $q = 1$ never enter the market because of menu cost payments in our model, whereas 85% of firms have the quality of $q = 1$ in the estimated model of Lentz and Mortensen (2008). Thus, we calibrate the beta coefficients by targeting the mean and the variance of q in Lentz and Mortensen (2008) by setting the lower and upper bounds of q to 1 and 1.2, respectively, where

¹⁹The results presented below in this subsection hardly change when we calibrate parameter values by assuming $n = 0.031$ which corresponds to the average rate of nominal GDP per capita from 1990 to 2016 in Denmark.

the upper bound is chosen to be as large as possible under the condition that we can solve for a and b . We then obtain $a = 0.196$ and $b = 1.724$, respectively. In what follows, we discuss the robustness of our results to changes in the shape of the quality distribution, $\bar{\phi}(q)$.²⁰

6.2 Simulation Results

Sales and Employment Dispersion Figure 2 shows simulation results that are analogous to our empirical findings reported in Section 2. In the left (right) panel, we calculate the ratio of sales (employment) per firm for the top 0.1%, 1%, and 10% firms to those for a median firm for different n .²¹ The lines have positive slopes with respect to n , suggesting that larger firms tend to grow more rapidly than do smaller firms when the nominal growth rate, n , increases.

This simulation result is consistent with that from the Japanese firm-level data in Section 2: the growth difference between large and small firms becomes larger as the rate of changes in firms' input price increases. Note that the rate of changes in the input price is analogous to the rate of changes in firms' production costs n , or more precisely, $\pi = n - g$.

Furthermore, this simulation result confirms Proposition 2. In the proposition, we show that the ratio of R&D investment by high- q firms to that by low- q firms widens as n increases. This results in the widening of the ratio of sales for large firms to those for small firms.

Aggregate Variables To understand the mechanism for how changes in the nominal growth rate n influence the economy, Figure 3 shows that the real growth rate, g , is increasing with n . Two opposing effects lie behind this, as discussed in Section 5.

A negative effect of an increase in n is regarded as a decrease in the creative

²⁰Previous studies, such as Lucas (1978), Melitz (2003), and Luttmer (2007), often assume Pareto distribution. We assume beta distribution, because q needs to be finite to conduct numerical simulations. As nominal growth increases, the importance of high- q firms in the right tail increases, which worsens computational error if we approximate Pareto distribution by using a finite bound for q . Thus, we assume beta distribution, which is bounded and has much more flexibility than does the uniform distribution.

²¹In our calibrated model, the bottom half of firms produce only one product ($k = 1$). Thus, for example, the ratio of sales for the top 1% of firms to the bottom 1% of firms is almost equal to the ratio of sales for the top 1% of firms to a median firm.

destruction rate, δ . Because a higher n implies greater burden of menu cost payments and, thus, aggregate menu cost payments increase, incentives to innovate decrease (innovation incentive effect). As a result, the entry rate, η , as well as R&D investment, $\gamma(q)$, for low- q firms decreases with n .

However, there is a positive reallocation effect. The average $\log q$ increases, because, under a greater n , $K(q|\delta, w, n)$ has more weights for higher q , while firms with relatively small q cannot survive. Indeed, the lower threshold of q for the firm entry, \underline{q} , increases with n . Thus, the average firm size, k , increases with n , while the gap of $k(q)$ widens for firms with different q . Despite greater menu cost payments, the R&D investment, $\gamma(q)$, for high- q firms does not decrease with n . As a result, the average $\log q$ increases, while the average markup also increases. This markup increase decreases the demand for production workers. Combined with a decrease in the demand for R&D workers for possible entrants, $L_r(ent)$ in Figure 3, this results in an increase in demand for R&D workers for incumbents, $L_r(inc)$ in the figure, because of the labor market clearing condition. In this simulation, we find that the latter reallocation effect dominates the former innovation incentive effect. Thus, the real growth rate increases with the nominal growth rate.

Furthermore, Figure 3 shows that welfare, U , peaks at positive n . This result is in sharp contrast to the model without firm reallocation. Oikawa and Ueda (2018) show that real growth is maximized at $n = 0$ and, thus, the growth-maximizing inflation rate is negative. By contrast, in the model with firm reallocation, the welfare, U , increases under positive n , mainly because the real growth rate, g , increases with n . However, too high n decreases welfare because of increased menu cost payments and the negative business-stealing effect. Welfare is the highest at $n = 0.13$. Note that this value is unimportant, because it is sensitive to economic circumstances, as we show later in this section.

Firm Distributions In this subsection, we discuss the reallocation effect by showing how distributions change depending on the value of the nominal growth rate, n . Figure 4 shows changes in the distribution of realized q , $\phi(q)$ (note it is not the exogenous $\bar{\phi}(q)$), changes. As n increases, \underline{q} increases, which causes firms with low q to exit the market. Thus, the average quality improves with n .

Next, Figure 5 shows the cumulative distribution for sales and employment per firm. The sales distribution in the model coincides with the distribution of the num-

ber of products a firm produces, k . Employment includes that for both production and R&D investment. Following the convention of empirical studies, we depict the distribution on the log-log scale and calculate the tail distribution (1 minus cumulative distribution) of sales or employment. We find from the figure that both sales and employment per firm have heavier tails under positive nominal growth and the slopes of the tails are around one, although the tails hardly increase as nominal growth increases when $n > 0.05$. Empirically, the slope of the tail is shown to be close to one on the log-log scale by, for example, Luttmer (2007). Our simulation results show that introducing nominal rigidity significantly improves the fit of the model compared with the models of Klette and Kortum (2004) and Lentz and Mortensen (2005, 2008).

6.3 Does Menu Cost Improve Welfare?

It is sometimes said that it would be illogical for labour to resist a reduction of money-wages but not to resist a reduction of real wages. For reasons given below, this might not be so illogical as it appears at first; and, as we shall see later, **fortunately so** [emphasis added].

— Keynes (1936)

Because of the reallocation effect, price stickiness may improve welfare. To observe this, in Figure 6, we calculate the effects of nominal growth on welfare by varying menu cost parameter values, κ , from 0.022 to 0, 0.002, or 0.012.²² The figure shows that price stickiness indeed improves welfare. Even with a slight menu cost value ($\kappa > 0$), welfare improves relative to the case with no menu cost ($\kappa = 0$) for $n > 0$. As Keynes says, it is fortunate for an economy to have nominal rigidity.²³

6.4 Growth Decomposition

In this subsection, we decompose the impact of nominal growth on the real growth rate, that is, $g(n) - g(0)$ into four components. First, following Lentz and Mortensen (2008), we examine the composition of the real growth rate at $n = 0$, as the reference

²²The welfare levels for each κ are slightly different at $n = 0$, because we assume that firms pay menu costs at the first price setting.

²³We thank Prof. Katsuhito Iwai for pointing out the last two words in the quote.

point, into the entry/exit effect, the selection effect, and the within (residual) effect:

$$\begin{aligned}
g(0) &= \int_{\underline{q}(0)}^{\infty} \eta(0)\phi(q|0) \log q \, dq \\
&+ \int_{\underline{q}(0)}^{\infty} [K(q|0) - \phi(q|0)] \gamma(q|0) \log q \, dq \\
&+ \int_{\underline{q}(0)}^{\infty} \phi(q|0)\gamma(q|0) \log q \, dq.
\end{aligned} \tag{34}$$

The first term represents the *net entry effect* by entrants. The second term represents the *selection effect*, which indicates the contribution to real growth through heterogeneous firm growth, which depends on quality. The last term represents the *within effect*, which Lentz and Mortensen (2008) call the residual effect: the contribution to real growth under the assumption that the share of products by firms is the same as that at entry.

In our calibrated model, the entry/exit effect, the selection effect, and the within effect account for 43.2%, 15.4%, and 41.4% of the real growth at $n = 0$, respectively. The contribution of the selection effect is the smallest, whereas that of the entry/exit effect is the largest. This result is in sharp contrast to that in Lentz and Mortensen (2008), where the selection effect accounts for the largest share (52.8%), followed by the within effect (26.8%) and the entry/exit effect (21.1%). This difference arises from the fact that our model incorporates menu costs. Even under no nominal growth, our model has $\underline{q}(0) > 1$, that is, entrants need to pay menu costs, which serves as an entry barrier. The inclusion of menu costs generates another selection effect at the time of entry: only firms with sufficiently high quality decide to enter the market, which instead weakens the selection effect that works through the second term in equation (34).

Having positive nominal growth, $n > 0$, we can write down the change in the real

growth as follows:

$$\begin{aligned}
g(n) - g(0) &= -\delta(0) \int_{\underline{q}(0)}^{\underline{q}(n)} K(q|0) \log q \, dq \\
&+ \int_{\underline{q}(n)}^{\infty} \{\eta(n)\phi(q|n) - \eta(0)\phi(q|0)\} \log q \, dq \\
&+ \int_{\underline{q}(n)}^{\infty} \{[K(q|n) - \phi(q|n)]\gamma(q|n) - [K(q|0) - \phi(q|0)]\gamma(q|0)\} \log q \, dq \\
&+ \int_{\underline{q}(n)}^{\infty} \{\phi(q|n)\gamma(q|n) - \phi(q|0)\gamma(q|0)\} \log q \, dq. \tag{35}
\end{aligned}$$

Equation (35) shows the growth decomposition of the real growth rate at n minus that at $n = 0$, that is, $g(n) - g(0)$. A new component appears in the first term on the right-hand side of the equation, which we call the *entry barrier effect*. This effect is one of the two reallocation effects in our model. An increase in n causes low-quality firms to exit the market owing to greater burden of menu cost payments ($\partial \underline{q} / \partial n > 0$). Because these firms would contribute to economic growth if they were in the market, the sign of the entry barrier effect becomes negative. The second to fourth effects are labelled in the same way as before: the entry/exit effect, the selection effect, and the within effect, respectively. However, they differ in that the effects are measured relative to those under no nominal growth, $n = 0$.

Consider how these effects change when the nominal growth rate, n , increases. The selection effect increases real growth, because the ex post distribution has more density for greater q than does the ex ante distribution. The within effect on real growth is positive, because the average q as well as the gap of R&D intensity between high- and low- q firms increase with n . The entry/exit effect on real growth is ambiguous, because the entry rate η decreases with n , while the average q rises.

Figure 7 shows the growth decomposition of $g(n) - g(0)$. The selection effect and within effect make the largest contributions to the change in real growth. Both the entry/exit effect and the entry barrier effect negatively influence the real growth rate. When considering the effect of monetary policy on real growth, the selection effect as well as the within effect are important, whereas the within effect and the entry/exit effect are important when nominal rigidity is absent at $n = 0$. Adam and Weber (2017) argue that the optimal inflation rate is around 1–2% in the model in which firms are heterogeneous. Compared with their model, our model highlights additional effects of monetary policy that work through reallocations. In particular,

both the selection effect, which arises because firms' heterogeneity is endogenous, and the within effect, which arises because firms' R&D is endogenous, suggest that the optimal inflation rate is even higher.

6.5 Further Analyses

Sensitivity to the Shape of Distributions In the baseline simulation, we assume the beta distribution for the shape of the distribution of innovation ability, $\bar{\phi}(q)$. To observe the robustness of the result, in Figure 8, we show the effects of nominal growth n on real growth g and welfare U , by assuming Pareto and uniform distributions. In the case of Pareto distribution, we calibrate the Pareto coefficient by targeting the variance of q in Lentz and Mortensen (2008), which leads to 17.50. For numerical computations, we have to set the upper bound of q . We set it to 1.5, which is higher than 1.2 in the case of beta distribution. In the case of uniform distribution, the upper bound of q is selected so that the variance of q coincides with that of q in Lentz and Mortensen (2008).

Figure 8 shows that real growth and welfare implications depend on the shape of the distribution. Both the real growth rate and welfare are increasing with n in the case of Pareto distribution, but they are decreasing with n in the case of uniform distribution. Such differences can be understood by the difference in the degree of the reallocation effect of nominal growth. This effect is strong in the case of Pareto distribution, because infinitely high- q firms exist and they can increase their share as n increases. On the other hand, the reallocation effect is weak under uniform distribution, because there is an upper bound of q and because there are fewer low- q firms that exit the market with high n than under beta and Pareto distributions.

Comparison of Denmark and Japan Our baseline simulation is based on parameters calibrated to the Danish economy by Lentz and Mortensen (2008). For comparison, we conduct another simulation using parameters calibrated to the Japanese economy using the work of Murao and Nirei (2011), who apply a model based on Lentz and Mortensen (2008) to Japan. Our calibration approach is the same. The discount rate ρ is set to 0.0385, which equals the interest rate 0.05 minus the real growth rate $g = 0.0115$ in Lentz and Mortensen (2008). The R&D cost function is given by $c(\gamma) = c_0 \times \gamma^{c_1}$, where $c_1 = 1.923$. As for c_0 and the measure of potential entrants, h , we calibrate the values so that the rate of creative destruction, δ , equals

0.085 and the rate of entry, η , equals 0.079 in the case of $n = 0$. We then obtain $c_0 = 3.046 \cdot 10^3$ and $h = 13.250$. We assume the distribution of innovation ability, $\bar{\phi}(q)$, obeys beta distribution and sets the two parameters by targeting the mean and variance of q .

Figure 9 shows the simulation results for Japan. The positive effects of nominal growth on real growth and welfare are greatly decreased. The real growth rate is the highest around $n = 0.08$ and welfare is the highest around $n = 0.01$. While there are many differences in parameter values, the most important difference is the measure of potential entrants, h (13.250 for Japan and $h = 2.699$ for Denmark). This difference stems from the difference in the entry rate (η : 0.079 for Japan and 0.045 for Denmark), whereas the rate of creative destruction, δ , is not much different. In other words, entrants are more important in Japan than in Denmark, while incumbents are more important in Denmark than in Japan. This difference weakens the reallocation effects for Japan, which, if they were strong, should generate positive impacts of nominal growth on real growth and welfare. Figure 10 shows a simulation of the Danish economy with only the potential entrants, h , replaced with that in the Japanese economy. The strong reallocation effect vanishes with higher h .

This simulation exercise does not necessarily mean that inflation is good for Denmark but bad for Japan. It is important to emphasize that the measurement of entry and exit rates are subject to errors and, thus, the parameters we use are indecisive. An important implication is that the effects of nominal growth depend on circumstances. In particular, in an economy with a great mass of potential entrants, low inflation is desirable to ensure frequent turnover and minimize menu cost payments. By contrast, in an economy in which incumbents play a big role in innovations, high inflation is desirable to maximize reallocation effects.

7 Concluding Remarks

In this study, we built an endogenous growth model with firm heterogeneity and nominal rigidity to analyze the impacts of monetary policy on long-run economic growth and welfare through reallocation. Our results show that nominal growth can enhance growth and welfare if the reallocation effect is sufficiently strong. Beyond theoretical possibility, we found that this is indeed the case in the model calibrated to Denmark. According to the model, the optimal nominal growth rate is strictly

positive, whereas it is zero in standard New Keynesian models. Menu cost burdens suppress entry of least innovative firms and, moreover, reallocate R&D resources from low-quality to high-quality firms among survivors.

It is worth noting that this reallocation effect works selectively, while nominal growth or inflation through monetary policy is a purely aggregate phenomenon. Selection occurs because menu costs at each price revision are independent of firm quality, whereas the return from the price reset is increasing in firm quality.

Our model postulates nondirected R&D and thus, incumbents do not invest in quality improvement of the products they currently produce. As Garcia-Macia et al. (2016) point out, a significant share of innovations is attributed to incumbents' quality updates of their own products. We consider that the reallocation effect in the current model holds at least partly even after introducing incumbents' quality updates in their own product lines, because such an extension does not directly change the impact of nominal growth on product or firm values. In the context of data fitting, another deficiency of our model is the unit elasticity of substitution among products. This assumption significantly simplifies the price-setting rule and its influence on the firm value for the model to be tractable. Rigorous quantitative analyses with those extensions are our future research topics.

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A Data Summary and Regression Results with Domestic Sales

Summary Statistics Table 3 summarizes input inflation and firm-size dispersion measures by 14 manufacturing industries, examined in Section 2. Figure 11 shows the dynamics of the 2-year average inflation rate of PPI (input) in the Japanese manufacturing sector. There is no trend in input inflation.

Regression Results with Domestic Sales When we consider the relationship between firm-size dispersion and input inflation, one concern is that some part of the inflation volatility arises from the changes in the exchange rate. The depreciation of the Japanese yen increases the cost of imported inputs and simultaneously raises exports and total sales. Because large firms tend to be exporters, such a depreciation might generate spurious positive correlation between input PPI and the firm-size gap. To avoid this problem, we subtract export revenues from total sales and run the same regressions as in Section 2. Table 4 shows the results. When we focus on domestic sales, we again observe significantly positive impacts on sales dispersion from input inflation. However, as shown in columns (1)–(3) in Table 4(b), the impact on domestic real sales growth from inflation is insignificant, because growth in domestic sales is affected by many other factors. For example, an increase in exports could be associated with a decrease in sales to the domestic market especially under a capacity constraint. Adjustment between domestic and foreign outlets results in noise for our estimation. To avoid complicated interrelations with growth rates, we simply count firms with positive growth rates in real sales or employment and set the frequency within each size group as the dependent variable in columns (4)–(6) of Table 4(b). The result shows that there are fewer growing firms under inflation but the negative impact is partly offset for large firms, which is consistent with the result in Section 2.²⁴

²⁴We obtain the same result with total sales and employment when we use the share of growing firms as the dependent variable.

B Derivation of Equation (15)

Now suppose that the firm value comprises profit flows from each product plus the return from R&D that depends on k . Let the present-discount value of the sum of profit flows for a product with τ_i be $\nu_{\tau_i}(q|\delta, n)$. Thus, our guess for the value function is

$$v_k(T_k, q|\delta, w, n) = \sum_{i=1}^k \nu_{\tau_i}(q|\delta, n) + k\psi(q|\delta, w, n). \quad (36)$$

The guess of equation (36) yields

$$\frac{1}{k} \sum_{i=1}^k v_{k-1}(T'_{k-1, <i>, q|\delta, w, n) = \left(1 - \frac{1}{k}\right) \sum_{i=1}^k \nu_{\tau'_i}(q|\delta, n) + (k-1)\psi(q|\delta, w, n) \quad (37)$$

and, thus,

$$\frac{1}{k} \sum_{i=1}^k v_{k-1}(T'_{k-1, <i>, q|\delta, w, n) - v_k(T'_k, q|\delta, w, n) = -\frac{1}{k} \sum_{i=1}^k \nu_{\tau'_i}(q|\delta, n) - \psi(q|\delta, w, n). \quad (38)$$

Equation (13) is then written as

$$\begin{aligned} & \rho \sum_{i=1}^k \nu_{\tau_i}(q|\delta, n) + \rho k \psi(q|\delta, w, n) \\ &= \sum_{i=1}^k \left[\Pi^0(\xi_0 e^{-n\tau_i}) - I\{\tau_i = \Delta(q|\delta, n)\} \kappa + \frac{\partial \nu_{\tau_i}(q|\delta, n)}{\partial \tau} \right] \\ & \quad - \delta \sum_{i=1}^k \nu_{\tau'_i}(q|\delta, n) - k \delta \psi(q|\delta, w, n) \\ & \quad + k \max_{\gamma} \{ \gamma [v_{k+1}(\{T'_k, 0\}, q|\delta, w, n) - v_k(T'_k, q|\delta, w, n)] - wc(\gamma) \}. \end{aligned} \quad (39)$$

Noticing that

$$\rho \nu_{\tau_i}(q|\delta, n) = \begin{cases} \Pi^0(\xi_0 e^{-n\tau_i}) + \frac{\partial \nu_{\tau_i}(q|\delta, n)}{\partial \tau} - \delta \nu_{\tau_i}(q|\delta, n) & \text{for } \tau_i \in [0, \Delta(q|\delta, n)), \\ \Pi^0(\xi_0 e^{-n\tau_i}) - \kappa + \frac{\partial \nu_0(q|\delta, n)}{\partial \tau} - \delta \nu_0(q|\delta, n) & \text{for } \tau_i = \Delta(q|\delta, n), \end{cases} \quad (40)$$

we have

$$(\rho + \delta) \psi(q|\delta, w, n) = \max_{\gamma} \{ \gamma [\nu_0(q|\delta, n) + \psi(q|\delta, w, n)] - wc(\gamma) \}. \quad (41)$$

The R&D intensity is determined at

$$\nu_0(q|\delta, n) + \psi(q|\delta, w, n) = wc'(\gamma). \quad (42)$$

Using equation (41) to eliminate ψ , this first-order condition can be rewritten as

$$wc'(\gamma) = \max_{\gamma \in [0, \rho + \delta]} \frac{(\rho + \delta)\nu_0(q|\delta, n) - wc(\gamma)}{\rho + \delta - \gamma}, \quad (43)$$

where the constraint $\gamma < \rho + \delta$ should hold because ψ is not well defined otherwise.

C Firm Size and Quality Distribution

Equation (20) leads to

$$M_k(q|\delta, w, n) = \frac{\phi(q|\delta, n)\eta}{\delta k} \left(\frac{\gamma(q|\delta, w, n)}{\delta} \right)^{k-1}. \quad (44)$$

The mass of type- q firms in the stationary state, $M(q|\delta, w, n)$, is $\sum_{k=1}^{\infty} M_k(q|\delta, w, n)$. If $\gamma(q|\delta, w, n) < \delta$ for almost all q , then $M(q|\delta, w, n)$ is well defined, such as

$$M(q|\delta, w, n) = \frac{\eta}{\delta} \left[\log \left(\frac{\delta}{\delta - \gamma(q|\delta, w, n)} \right) \right] \frac{\delta\phi(q|\delta, n)}{\gamma(q|\delta, w, n)}. \quad (45)$$

The condition $\sup_q \gamma(q|\delta, w, n) < \delta$ is supported when w is sufficiently large. The threshold level of w is determined by the first-order condition of the maximization in equation (15). Let $\underline{w}(q|\delta, n)$ be the individual threshold wage level such that $\gamma(q|\delta, w, n) < \delta$ for $w > \underline{w}(q|\delta, n)$. The threshold wage in the whole economy is $\underline{w}(\delta, n) \equiv \sup_q \underline{w}(q|\delta, n)$. Since $\nu(q|\delta, n)$ is monotonically increasing in q and $\lim_{q \rightarrow \infty} \nu(q|\delta, n) = \frac{1}{\rho + \delta} - \kappa$, we have

$$\underline{w}(\delta, n) = \begin{cases} \frac{1 - (\rho + \delta)\kappa}{c(\delta) + \rho c'(\delta)} & \text{if } \bar{q} \rightarrow \infty, \\ \frac{(\rho + \delta)\nu(\bar{q}|\delta, n)}{c(\delta) + \rho c'(\delta)} & \text{if } \bar{q} \text{ is finite.} \end{cases} \quad (46)$$

Two remarks are in order. First, one can easily show that in both cases, \underline{w} is decreasing in the creative destruction rate, δ . This is because a decline in the creative destruction rate and a decline in the wage both stimulate incumbents' R&D. Second,

\underline{w} is independent of n if the support of $\bar{\phi}$ is not bounded, whereas greater nominal growth enlarges the admissible set otherwise.

Any equilibrium has $w > \underline{w}(\delta, n)$. Because the total mass of products is one, the creative destruction rate must equal the sum of the entry rate and the creation rates of all the incumbents. As long as $w > \underline{w}(\delta, n)$, we have

$$\begin{aligned}\delta &= \eta + \int_{\underline{q}(\delta, n)}^{\infty} dq \sum_{k=1}^{\infty} k M_k(q|\delta, w, n) \gamma(q|\delta, w, n) \\ &= \eta \int_{\underline{q}(\delta, n)}^{\infty} \frac{\delta \phi(q|\delta, n)}{\delta - \gamma(q|\delta, w, n)} dq,\end{aligned}$$

which leads to the following equation:

$$1 = \eta \int_{\underline{q}(\delta, n)}^{\infty} \frac{\phi(q|\delta, n)}{\delta - \gamma(q|\delta, w, n)} dq. \quad (47)$$

D Proofs

Lemma 1 *Suppose $n > 0$. $\Delta(q|\delta, n)$ is increasing in q and decreasing in $|n|$. $|n|\Delta$ is increasing in $|n|$. Moreover, $\Delta(q|\delta, n)$ is increasing in δ .*

Proof of Lemma 1 From equations (10) and (11), we obtain

$$q\kappa = \frac{e^{n\Delta} - e^{-(\rho+\delta-n)\Delta}}{\rho + \delta} - \frac{1 - e^{-(\rho+\delta-n)\Delta}}{\rho + \delta - n}. \quad (48)$$

The total differentiation of equation (48) is

$$\begin{aligned}
& \kappa dq - \frac{ne^{n\Delta}(1 - e^{-(\rho+\delta)\Delta})}{\rho + \delta} d\Delta \\
&= \left[e^{n\Delta} \left(\frac{\Delta e^{-(\rho+\delta)\Delta}}{\rho + \delta} - \frac{1 - e^{-(\rho+\delta)\Delta}}{(\rho + \delta)^2} \right) - \left(\frac{\Delta e^{-(\rho+\delta-n)\Delta}}{\rho + \delta - n} - \frac{1 - e^{-(\rho+\delta-n)\Delta}}{(\rho + \delta - n)^2} \right) \right] d\delta \\
&+ \left[\frac{\Delta e^{n\Delta}(1 - e^{-(\rho+\delta)\Delta})}{\rho + \delta} + \frac{\Delta e^{-(\rho+\delta-n)\Delta}}{\rho + \delta - n} - \frac{1 - e^{-(\rho+\delta-n)\Delta}}{(\rho + \delta - n)^2} \right] dn \\
&= e^{-(\rho+\delta-n)\Delta} \left[\left(\frac{\Delta}{\rho + \delta} - \frac{e^{(\rho+\delta)\Delta} - 1}{(\rho + \delta)^2} \right) - \left(\frac{\Delta}{\rho + \delta - n} - \frac{e^{(\rho+\delta-n)\Delta} - 1}{(\rho + \delta - n)^2} \right) \right] d\delta \\
&+ \left[q\kappa\Delta + \left(\frac{\Delta}{\rho + \delta - n} - \frac{1 - e^{-(\rho+\delta-n)\Delta}}{(\rho + \delta - n)^2} \right) \right] dn \tag{49} \\
&= e^{-(\rho+\delta-n)\Delta} [h_2(\rho + \delta, \Delta) - h_2(\rho + \delta - n, \Delta)] d\delta + [q\kappa\Delta + h_1(\rho + \delta - n, \Delta)] dn,
\end{aligned}$$

where we substitute equation (48) in the second equality. Functions h_1 and h_2 are defined in Lemma 2 below. Owing to Lemma 2, the signs of the coefficients of $d\delta$ and dn are determined uniquely and we have $d\Delta/dq > 0$, $d\Delta/d\delta > 0$, and $d\Delta/dn < 0$. ■

Lemma 2 Define the following functions over $x \neq 0$ with $y \geq 0$ by

$$\begin{aligned}
h_1(x, y) &= \frac{y}{x} - \frac{1 - e^{-xy}}{x^2}, \\
h_2(x, y) &= \frac{y}{x} - \frac{e^{xy} - 1}{x^2}.
\end{aligned}$$

Then, for any $x \neq 0$ and $y \geq 0$, the following relationships hold:

$$\begin{aligned}
h_1(x, y) &\geq 0, & h_2(x, y) &\leq 0, \\
\frac{\partial h_1(x, y)}{\partial x} &\leq 0, & \frac{\partial h_2(x, y)}{\partial x} &\leq 0,
\end{aligned}$$

with equalities only when $y = 0$.

Proving the signs of the functions is straightforward:

$$\begin{aligned}
h_1(x, y) &= \frac{xy - (1 - e^{-xy})}{x^2} \geq 0, \\
h_2(x, y) &= \frac{xy - (e^{xy} - 1)}{x^2} \leq 0.
\end{aligned}$$

For the derivative of h_1 , we have

$$\frac{\partial h_1(x, y)}{\partial x} = -\frac{2(1 + e^{-xy})}{x^3} \left[\frac{xy}{2} - \frac{1 - e^{-xy}}{1 + e^{-xy}} \right].$$

Since $y \geq 0$ and

$$\frac{xy}{2} \geq \frac{1 - e^{-xy}}{1 + e^{-xy}} \quad \text{if and only if } xy \geq 0,$$

we have $\frac{\partial h_1(x, y)}{\partial x} \leq 0$.

Last, the partial derivative of h_2 is expressed as

$$\frac{\partial h_2(x, y)}{\partial x} = -\frac{2(1 + e^{xy})}{x^3} \left[\frac{xy}{2} - \frac{1 - e^{-xy}}{1 + e^{-xy}} \right].$$

Thus, the sign of $\partial h_2(x, y)/\partial x$ is equivalent to that of $\partial h_1(x, y)/\partial x$. ■

Lemma 3 $\tilde{v}_0(q|\delta, n)$ is strictly increasing in q and strictly decreasing in δ . For $n > 0$,

$$\frac{\partial \tilde{v}_0(q|\delta, n)}{\partial n} < 0, \quad \frac{\partial^2 \tilde{v}_0(q|\delta, n)}{\partial q \partial n} > 0.$$

Proof of Lemma 3 It is obvious to observe the former part of the proposition: $\partial \tilde{v}_0(q|\delta, n)/\partial q > 0$ and $\partial \tilde{v}_0(q|\delta, n)/\partial \delta < 0$. As for the latter part of the proposition,

$$\begin{aligned} \frac{\partial^2 \tilde{v}_0(q|\delta, n)}{\partial q \partial n} &= \frac{\partial(n\Delta)}{\partial(n)} \frac{\partial^2 \tilde{v}_0(q|\delta, n)}{\partial q \partial(n\Delta)} = -\frac{1}{\rho + \delta} \frac{\partial(n\Delta)}{\partial(n)} \frac{e^{n\Delta}}{q} \left(n \frac{\partial \Delta}{\partial q} - \frac{1}{q} \right) \\ &= -\frac{1}{\rho + \delta} \frac{\partial(n\Delta)}{\partial(n)} \frac{e^{n\Delta}}{q} \frac{(\rho + \delta)q\kappa - (e^{n\Delta} - e^{-(\rho + \delta - n)\Delta})}{q(e^{n\Delta} - e^{-(\rho + \delta - n)\Delta})} \\ &= \frac{1}{\rho + \delta} \frac{\partial(n\Delta)}{\partial(n)} \frac{e^{n\Delta}}{q} \frac{1}{q(e^{n\Delta} - e^{-(\rho + \delta - n)\Delta})} \frac{\rho + \delta}{\rho + \delta - n} (1 - e^{-(\rho + \delta - n)\Delta}) \\ &> 0. \end{aligned}$$

For the derivation of the last line, we use Lemma 1. ■

Proof of Proposition 1 From equation (11), $\kappa \geq \frac{1}{\rho + \delta}$ implies that $\underline{q}(\delta, n) \rightarrow \infty$ and no firm can yield profits. Thus, we assume $\kappa < \frac{1}{\rho + \delta}$ below.

When $n = 0$, then $\underline{q}(\delta, 0) = \frac{1}{1 - (\rho + \delta)\kappa}$.

When $n > 0$, $\tilde{\nu}_0(q|\delta, n) < 0$ implies that $\Delta(q|\delta, n) > \frac{\log q}{n} \equiv \bar{\Delta}$. From the first-order condition when choosing Δ :

$$q\kappa = \frac{e^{n\Delta} - e^{-(\rho+\delta-n)\Delta}}{\rho + \delta} - \frac{1 - e^{-(\rho+\delta-n)\Delta}}{\rho + \delta - n},$$

$\underline{q}(\delta, n)$ is q satisfying

$$\kappa = \frac{1 - e^{-(\rho+\delta)\bar{\Delta}}}{\rho + \delta} - \frac{1/q - e^{-(\rho+\delta)\bar{\Delta}}}{\rho + \delta - n} = \frac{1}{\rho + \delta} - \frac{q^{-1} - \frac{n}{\rho+\delta}q^{-\frac{\rho+\delta}{n}}}{\rho + \delta - n}. \quad (50)$$

Let $A_1(q, n)$ be the right-hand side of equation (50). Note that $A_1(1, n) = 0$ and $\lim_{q \rightarrow \infty} A_1(q, n) = \frac{1}{\rho+\delta}$.

Suppose $n \neq \rho + \delta$. Since

$$\frac{\partial A_1(q, n)}{\partial q} = \frac{\rho + \delta}{q^2} \frac{1 - q^{1-\frac{\rho+\delta}{n}}}{\rho + \delta - n} > 0,$$

$\underline{q}(\delta, n)$ uniquely exists. Next, $\partial \underline{q} / \partial n < 0$ comes from

$$\frac{\partial A_1(q, n)}{\partial n} = \frac{q^{-\frac{\rho+\delta}{n}}}{(\rho + \delta - n)^2} \left[1 + \log q \frac{\rho+\delta}{n} - 1 - q^{\frac{\rho+\delta}{n}-1} \right] < 0 \quad \text{for } q > 1.$$

Moreover, since $A_1(q, n)$ is continuous as $n \downarrow 0$, $\underline{q}(\delta, n) > 1/[1 - (\rho + \delta)\kappa]$ for $n > 0$. Thus, $\partial \underline{q}(\delta, n) / \partial n > 0$.

For the case of $n = \rho + \delta$, $A_1(q, n) = \frac{1-1/q}{\rho+\delta}$, which implies $A_1(q, n)$ is strictly increasing in q . Further, we have

$$\lim_{n \rightarrow \rho+\delta} \frac{\partial A_1(q, n)}{\partial n} = \lim_{n \rightarrow \rho+\delta} -\frac{1}{2} \frac{\rho + \delta}{n^2} q^{-\frac{\rho+\delta}{n}} (\log q)^2 = -\frac{(\log q)^2}{2q(\rho + \delta)} < 0.$$

Thus, $\underline{q}(\delta, n)$ uniquely exists and $\partial \underline{q}(\delta, n) / \partial n > 0$.

■

Proof of Proposition 2 From equation (15), the optimal γ satisfies

$$\frac{(\rho + \delta)\nu_0(q|\delta, n)}{w} = c(\gamma) + (\rho + \delta - \gamma)c'(\gamma), \quad \gamma \in [0, \rho + \delta].$$

From the strict convexity of $c(\gamma)$ and the assumption of $c(0) = 0$, γ has a unique interior solution if w is sufficiently large. $\partial\gamma/\partial|n| < 0$ and $\partial\gamma/\partial w < 0$ are obvious from the first-order condition. For the impact of δ , $(\rho + \delta)\nu_0(q|\delta, n)$ is strictly decreasing in δ for $n \geq 0$ from Lemma 1. Thus, $\partial\gamma/\partial\delta < 0$. In addition,

$$\frac{\partial\gamma}{\partial q} = \frac{1}{wc''(\gamma)} \frac{\rho + \delta}{\rho + \delta - \gamma} \frac{\partial\nu_0(q|\delta, n)}{\partial q} > 0, \quad (51)$$

$$\frac{\partial^2\gamma}{\partial q \partial n} = \frac{1}{wc''(\gamma)} \frac{\rho + \delta}{\rho + \delta - \gamma} \frac{\partial^2\nu_0(q|\delta, n)}{\partial q \partial n} \begin{cases} > 0 & \text{for } n > 0 \\ < 0 & \text{for } n < 0, \end{cases} \quad (52)$$

$$\frac{\partial^2\gamma}{\partial q \partial w} = -\frac{1}{w^2c''(\gamma)} \frac{\rho + \delta}{\rho + \delta - \gamma} \frac{\partial\nu_0(q|\delta, n)}{\partial q} < 0. \quad (53)$$

■

Proof of Proposition 3 The free-entry condition, equation (17), can be rewritten as

$$\int_{\underline{q}(\delta, n)}^{\infty} \bar{\phi}(q) c'(\gamma(q|\delta, w, n)) dq = c'(\gamma_\eta(\delta, w, n)), \quad (54)$$

by substituting ψ from equation (15).

Fix $\delta > 0$ arbitrarily. The right-hand side of equation (54) is increasing in w because $\eta(\delta, w, n)$ is increasing in w and $c'(\gamma)$ is positive. On the other hand, the left-hand side of the equation is decreasing in w because $\gamma(q|\delta, w, n)$ is decreasing in w for any $q > \underline{q}(\delta, n)$ in the admissible set. Hence, if w satisfies the free-entry condition for a given δ , then w is unique. Its existence is guaranteed if we can prove that the right-hand side is zero at $\underline{w}(\delta, n)$, because the left-hand side is strictly positive at $\underline{w}(\delta, n)$ and both the left- and right-hand sides are continuous for $w \geq \underline{w}(\delta, n)$. The right-hand side is shown to be zero at $\underline{w}(\delta, n)$ because equation (44) suggests that, in the limit of $\delta = \gamma$, η has to converge to zero to make the total measure of firms finite.

As long as η is positively correlated with δ in condition (23), an increase in δ drives up $\gamma_\eta(\delta, w, n)$ for given w and n while it reduces the left-hand side of equation (54) by an increase in $\underline{q}(\delta, n)$ and declines in $\gamma(q|\delta, w, n)$ for any $q > \underline{q}(\delta, n)$. Thus, when δ increases, it is necessary to have a smaller w to satisfy the FE, because γ is decreasing in w and γ_η is increasing in w . ■

Proof of Proposition 4 Define $\delta_{FE}(w, n)$ as δ that satisfies equation (23) and the FE condition, (54). Consider $\eta(\delta_{FE}(w, n), w, n)$ by substituting $\delta_{FE}(w, n)$ into equation (23). As $w \rightarrow \infty$, $\underline{w}^{-1}(w, n)$ and the left-hand side of equation (54) go to zero. η (and δ) must be zero. Then, $\lim_{w \rightarrow \infty} \eta(\delta_{FE}(w, n), w, n) = 0$. On the other hand, when $w \rightarrow 0$, $\delta_{FE}(w, n) \rightarrow \infty$ because $\underline{w}^{-1}(w, n) \rightarrow \infty$. Thus, $\lim_{w \rightarrow 0} \eta(\delta_{FE}(w, n), w, n) = \infty$. In terms of employment, when $w \rightarrow \infty$, we observe

$$\lim_{w \rightarrow \infty} L_{R,ent}(\delta_{FE}(w, n), w, n) = 0, \quad \lim_{w \rightarrow \infty} L_{R,inc} = \lim_{w \rightarrow \infty} L_X = 0,$$

which leads to $\lim_{w \rightarrow \infty} L_D(\delta_{FE}(w, n), w, n) = 0$ for any $n \geq 0$. When $w \rightarrow 0$, $\lim_{w \rightarrow 0} L_{R,ent}(\delta_{FE}(w, n), w, n) = \infty$ for any $n \geq 0$, which implies $\lim_{w \rightarrow 0} L_D(\delta_{FE}(w, n), w, n) = \infty$.

Because L_D is continuous in w , at least one w that satisfies $L_D(\delta_{FE}(w, n), w, n) = L$ exists for any $n \geq 0$.

Since the FE curve lies above the curve of \underline{w} , any stationary state has a nondegenerate firm-size distribution. Such a distribution cannot be compatible with no entry, because $\eta = 0$ implies that \underline{q} should be the upper bound of the support of q (or infinity) to satisfy the FE condition, (54). ■

E Labor Demand

Lemma 4 Fix $\delta > 0$ and $n \geq 0$. The cumulative distribution of $K(q|\delta, w_1, n)$ stochastically dominates that of $K(q|\delta, w_2, n)$ if $w_1 < w_2$.

Proof of Lemma 4 From equation (23) and the definition of K , $K(q|\delta, w, n)$ is increasing in w if and only if

$$\int_{\underline{q}(\delta, n)}^{\infty} -\frac{\partial \gamma(q'|\delta, w, n)}{\partial w} \frac{K(q'|\delta, w, n)}{\delta - \gamma(q'|\delta, w, n)} dq' > -\frac{\partial \gamma(q|\delta, w, n)}{\partial w} \frac{1}{\delta - \gamma(q|\delta, w, n)}.$$

The left-hand side is a positive constant for any q , while the right-hand side is zero at $q = \underline{q}(\delta, n)$ and monotonically increasing in q from Proposition 2. To keep $\int_{\underline{q}}^{\infty} K(q|\delta, w, n) dq = 1$, $\hat{q} > \underline{q}$ exists such that $\partial K/\partial w > 0$ if and only if $q < \hat{q}$.

Since $\frac{\partial}{\partial w} \int_{\underline{q}}^{\infty} K(q'|\delta, w, n) dq'$ must be zero by definition,

$$0 = \frac{\partial}{\partial w} \int_{\underline{q}}^{\hat{q}} K(q'|\delta, w, n) dq' + \frac{\partial}{\partial w} \int_{\hat{q}}^{\infty} K(q'|\delta, w, n) dq'.$$

Hence, for any $q \in (\hat{q}, \infty)$,

$$\frac{\partial}{\partial w} \int_{\underline{q}}^{\hat{q}} K(q'|\delta, w, n) dq' > -\frac{\partial}{\partial w} \int_{\hat{q}}^q K(q'|\delta, w, n) dq'.$$

Therefore, $\frac{\partial}{\partial w} \int_{\underline{q}}^q K(q'|\delta, w, n) dq' > 0$ for any finite q , which implies the stated stochastic dominance. ■

We assume that this effect does not dominate.

Proposition 5 $L_{R,ent}$ is increasing in w . $L_{R,inc}$ is decreasing in w . L_X is decreasing in w if the distribution effect is sufficiently weak.

Proof of Proposition 5 Fix $\delta > 0$ and n arbitrarily. $L_{R,ent}$ is increasing in w because

$$\frac{\partial \gamma_n(\delta, w, n)}{\partial w} \propto \frac{\partial \eta(\delta, w, n)}{\partial w} > 0.$$

Next, the response of $L_{R,inc}$ to an increase in w is

$$\begin{aligned} \frac{\partial L_{R,inc}}{\partial w} &= \int_{\underline{q}(\delta, n)}^{\infty} \frac{\partial K(q|\delta, w, n)}{\partial w} c(\gamma(q|\delta, w, n)) dq \\ &\quad + \int_{\underline{q}(\delta, n)}^{\infty} K(q|\delta, w, n) c'(\gamma(q|\delta, w, n)) \frac{\partial \gamma(q|\delta, w, n)}{\partial w} dq \end{aligned} \quad (55)$$

The last term is negative from Proposition 2. The first term on the right-hand side depends on $\partial K/\partial w$. Because $c(\gamma(q))$ is an increasing function of q , Lemma 4 implies that the first term on the right-hand side of equation (55) is negative. Hence, R&D labor demand from incumbents is decreasing in w .

The impact of an increase in w on L_X is

$$\frac{\partial L_X}{\partial w} = -\frac{L_X}{w} + \int_{\underline{q}(\delta, n)}^{\infty} \frac{\partial K(q|\delta, w, n)}{\partial w} L_{X,q}(q|\delta, w, n) dq \quad (56)$$

The last term is positive because $L_{X,q}(q|\delta, w, n)$ is a decreasing function of q and K

is stochastically dominated when w increases, as shown in Lemma 4.²⁵ Thus, to have L_X decreasing in w , the distribution effect is sufficiently small. ■

The LMC condition does not necessarily imply a monotonic relationship between δ and w . Suppose that $w > \underline{w}(\delta, n)$ and $\eta(\delta, w, n)$ is increasing in δ in condition (23). Then, we have

$$\frac{\partial L_D}{\partial \delta} = \underbrace{\frac{\partial L_X}{\partial \delta}}_{(+)} + \underbrace{\frac{\partial L_{R,inc}}{\partial \delta}}_{-} + \underbrace{\frac{\partial L_{R,ent}}{\partial \delta}}_{+} \quad (57a)$$

$$\frac{\partial L_D}{\partial w} = \underbrace{\frac{\partial L_X}{\partial w}}_{(-)} + \underbrace{\frac{\partial L_{R,inc}}{\partial w}}_{-} + \underbrace{\frac{\partial L_{R,ent}}{\partial w}}_{+} \quad (57b)$$

The signs of responses of L_X in equations (57) are in parentheses because the signs do not hold in general. In equation (57a), with a rise in the creative destruction rate, firms extend the interval between price resets, leading to lower markups under positive nominal growth. This is the primary factor that increases production employment. However, a rise in δ also brings more takeover of product lines with the highest markup at takeover. Furthermore, there is a reallocation effect: $K(q|\delta, w, n)$ becomes less concentrated with higher q , because firm growth tends to be low under high frequency of creative destruction²⁶ but we have higher $\underline{q}(\delta, n)$.

In equation (57b), $\partial L_X/\partial w$ could be positive when the reallocation effect is too strong. Lemma 4 implies that higher w leads to a reduction in average markup and thereby an increase in production.

²⁵ $L_{X,q}(q|\delta, w, n)$ is decreasing in q because the value of the product line obtained from price revision increases with q , and thus, a firm with greater q does not wait for a decline in real price lower than the optimal lower bound chosen by firms with smaller q . Hence, the lower bound of real price, $e^{n\Delta(q|\delta, n)}$, is increasing in q . The average real price is definitely higher for high-quality firms.

²⁶If δ is extremely high, $K(q|\delta, w, n)$ is close to $\phi(q|\delta, n)$.

F Other Properties

F.1 Firm Size and Quality

Relation to Market Concentration Firm size and firm quality are positively correlated. The expected number of product lines supplied by a type- q firm is

$$E[k|q] = \frac{K(q, \delta, w, n)}{M(q, \delta, w, n)} = \frac{\gamma(q|\delta, w, n)}{\delta - \gamma(q|\delta, w, n)} \frac{1}{\log\left(\frac{\delta}{\delta - \gamma(q|\delta, w, n)}\right)}, \quad (58)$$

which is strictly increasing in q from Proposition 2. This relationship also implies that firm size and average markup are positively correlated, because q determines the maximum markup rate.

Another important implication from equation (58) concerns concentration in the market. If there are more high-quality firms, more product lines are occupied by those firms and the degree of concentration increases.

Nominal Sales Distribution Because of log utility, nominal sales in each product line equal nominal income, E_t , independent of prices. Hence, the sales distribution across firms is the same as the distribution of the number of products, k . This is why we examined the relationship between sales distribution and inflation in Section 2.

Let $R(k|\delta, w, n)$ be the density around the sales of kE_t ,

$$\begin{aligned} R(k|\delta, w, n) &\equiv \int_{\underline{q}(\delta, n)}^{\infty} \phi(q|\delta, n) \frac{M_k(q|\delta, w, n)}{M(q|\delta, w, n)} dq \\ &= \frac{1}{k} \int_{\underline{q}(\delta, n)}^{\infty} \phi(q|\delta, n) \left(\frac{\gamma(q|\delta, w, n)}{\delta} \right)^k \left[\log \frac{\delta}{\delta - \gamma(q|\delta, w, n)} \right]^{-1} dq. \end{aligned} \quad (59)$$

Table 1: Inflation and Reallocation
(a) Sales distribution:

	Top/Middle ratio			Top/Bottom ratio		
	(1) OLS	(2) OLS	(3) 2SLS	(4) OLS	(5) OLS	(6) 2SLS
$\bar{\pi}^{\text{input}}$	100.6*** (20.16)	102.7*** (20.58)	194.6*** (27.73)	282.7*** (63.48)	289.0*** (64.53)	620.5*** (87.52)
D.I. gap (T/M or T/B)		-0.0454 (0.18)	0.135 (0.18)		-0.134 (0.49)	0.365 (0.53)
D.I.		-0.358 (0.22)	-0.547** (0.23)		-1.220* (0.70)	-1.876** (0.73)
Industry RS		2.168 (5.69)	3.437 (6.08)		-6.377 (17.85)	-5.679 (19.26)
Constant	4.591 (5.40)	-20.99 (72.16)	-33.84 (77.19)	17.98 (17.00)	104.8 (226.40)	106.2 (244.40)
Year/Industry FE	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes
Obs.	322	316	302	322	316	302
\bar{R}^2	0.509	0.507	0.483	0.679	0.679	0.653
Underidentification			164.1			165.4
Weak identification			22.97			23.37

(b) Employment distribution:

	Top/Middle ratio			Top/Bottom ratio		
	(1) OLS	(2) OLS	(3) 2SLS	(4) OLS	(5) OLS	(6) 2SLS
$\bar{\pi}^{\text{input}}$	3.484*** (1.027)	3.343*** (0.992)	5.903*** (1.309)	11.24*** (2.734)	11.80*** (2.674)	16.73*** (3.497)
D.I. gap (T/M or T/B)		-0.0215*** (0.008)	-0.0116 (0.009)		0.0242 (0.020)	0.0436** (0.021)
D.I.		-0.0245** (0.011)	-0.0279*** (0.011)		-0.0529* (0.029)	-0.0588** (0.029)
Industry RS		1.097*** (0.273)	1.290*** (0.284)		2.735*** (0.737)	3.172*** (0.764)
Constant	4.510*** (0.275)	-9.090*** (3.455)	-10.95*** (3.608)	11.21*** (0.732)	-23.19** (9.337)	-28.16*** (9.661)
Year/Industry FE	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes
Obs.	322	316	302	322	316	302
\bar{R}^2	0.7	0.729	0.723	0.776	0.791	0.786
Underidentification			163.7			165.1
Weak identification			22.84			23.27

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Inflation and firm growth

	Sales growth			Employment growth		
	(1)	(2)	(3)	(4)	(5)	(6)
	OLS	OLS	2SLS	OLS	OLS	2SLS
$\bar{\pi}^{\text{input}}$	-0.592*** (0.0792)	-0.628*** (0.0797)	-0.623*** (0.1500)	-0.0464** (0.0191)	-0.0611*** (0.0187)	-0.0408 (0.0354)
Size group	-0.00687*** (0.0007)	-0.00838*** (0.0008)	-0.00837*** (0.0009)	-0.00245*** (0.0002)	-0.00301*** (0.0002)	-0.00296*** (0.0002)
$\bar{\pi}^{\text{input}} \times$ Size group	0.0524*** (0.0118)	0.0570*** (0.0118)	0.0564*** (0.0195)	0.0148*** (0.0028)	0.0170*** (0.0028)	0.0145*** (0.0046)
Average D.I.		0.00156*** (0.0004)	0.00155*** (0.0004)		0.000528*** (0.0001)	0.000516*** (0.0001)
Industry RS		-0.0024 (0.0144)	-0.0024 (0.0143)		-0.00766** (0.0034)	-0.00768** (0.0034)
Constant	0.0881*** (0.0125)	0.133 (0.1830)	0.133 (0.1810)	0.0211*** (0.0030)	0.123*** (0.0429)	0.123*** (0.0426)
Year/Industry FE	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes
Obs.	2940	2880	2880	2940	2880	2880
\bar{R}^2	0.179	0.181	0.181	0.27	0.289	0.289
Underidentification			798.2			796.9
Weak identification			83.43			83.25

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

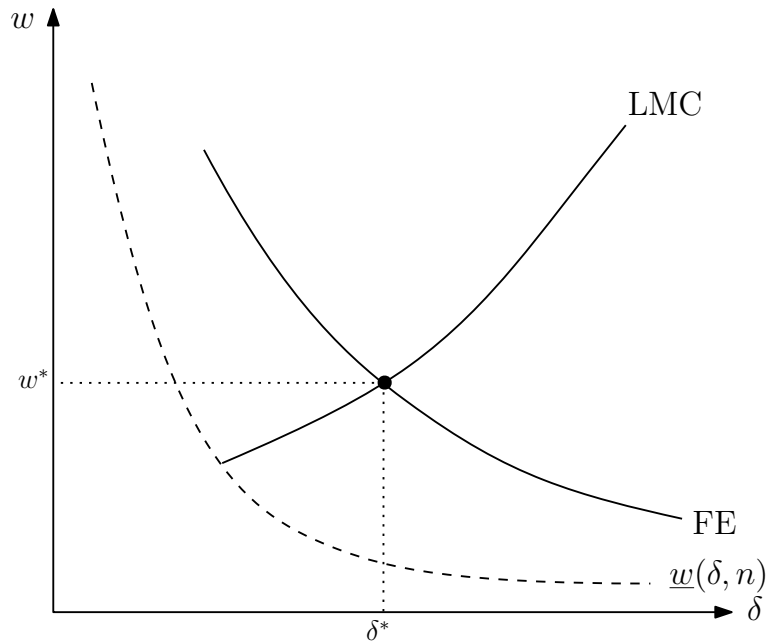


Figure 1: Stationary Equilibrium. FE and LMC stand for the free-entry condition and the labor market clearing condition, respectively.

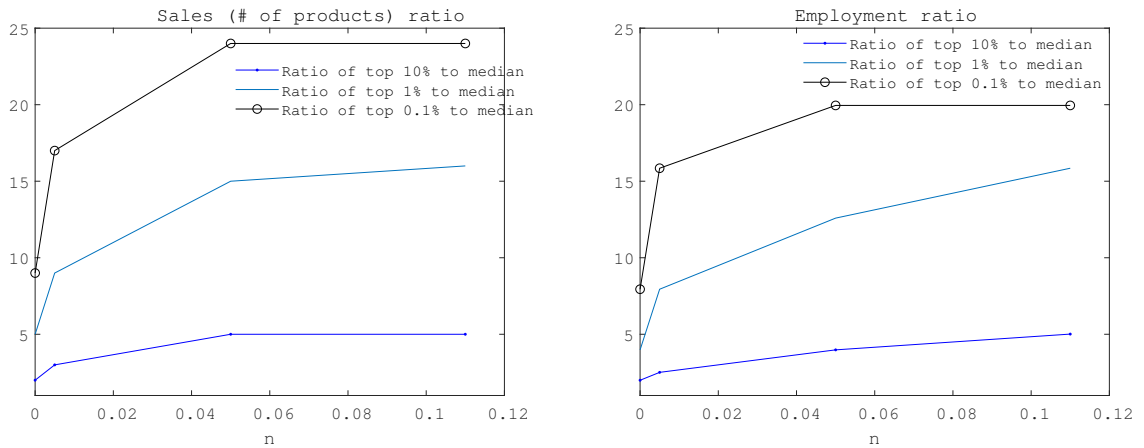


Figure 2: Sales and Employment Dispersion for Changes in the Nominal Growth Rate
 Note: The figures show changes in economic variables when the nominal growth rate, n , changes.

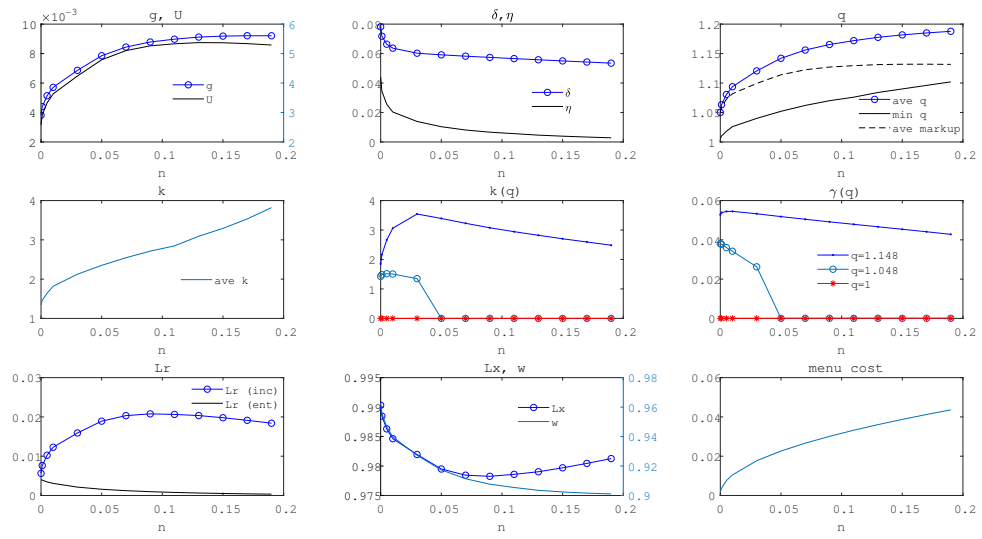


Figure 3: Effects of Nominal Growth

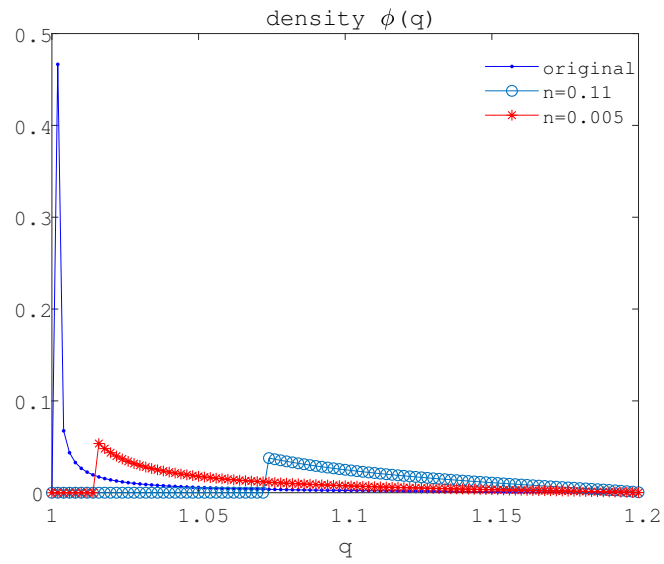


Figure 4: Changes in the Ex Post Quality Distribution for Changes in the Nominal Growth Rate

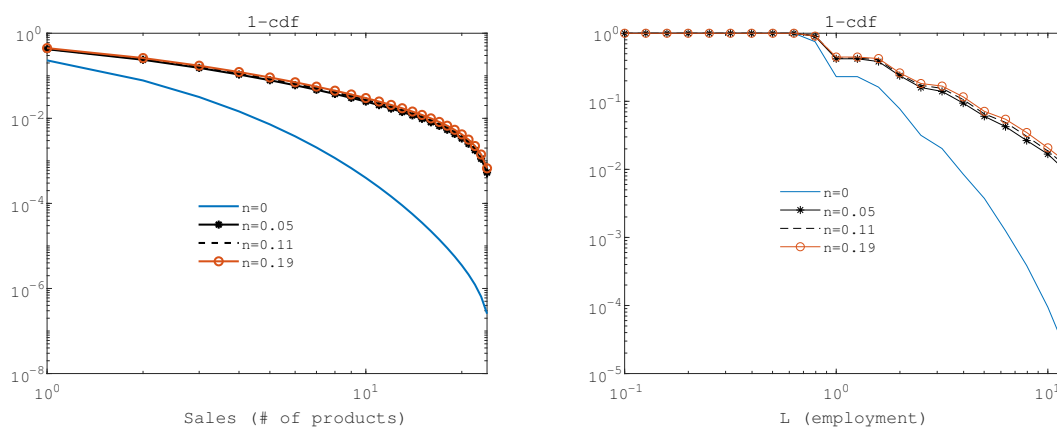


Figure 5: Changes in the Tail Distributions of Sales and Employment for Changes in the Nominal Growth Rate

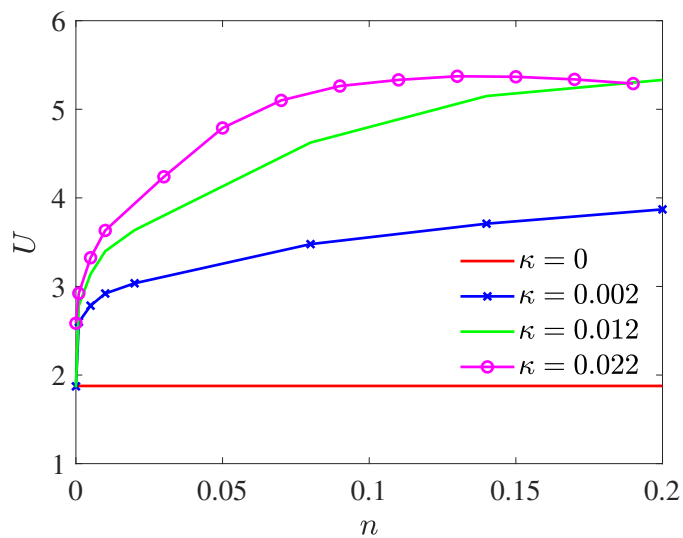


Figure 6: Effects of Nominal Growth on Welfare for Different Menu Cost Values

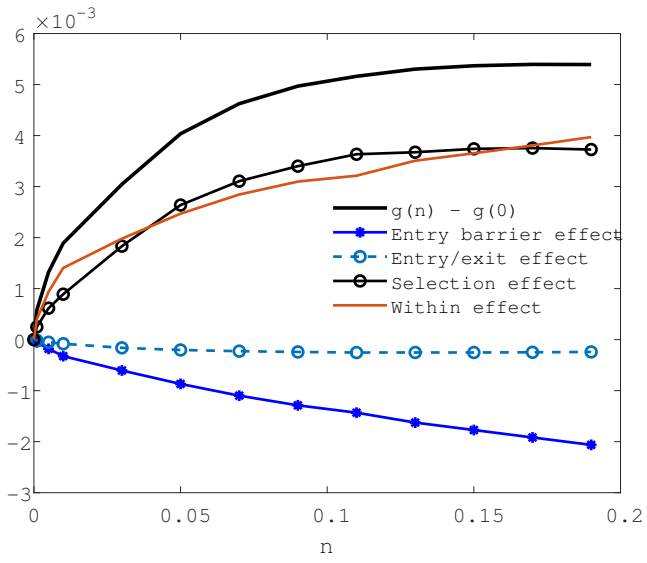


Figure 7: Growth Decomposition of $g(n) - g(0)$

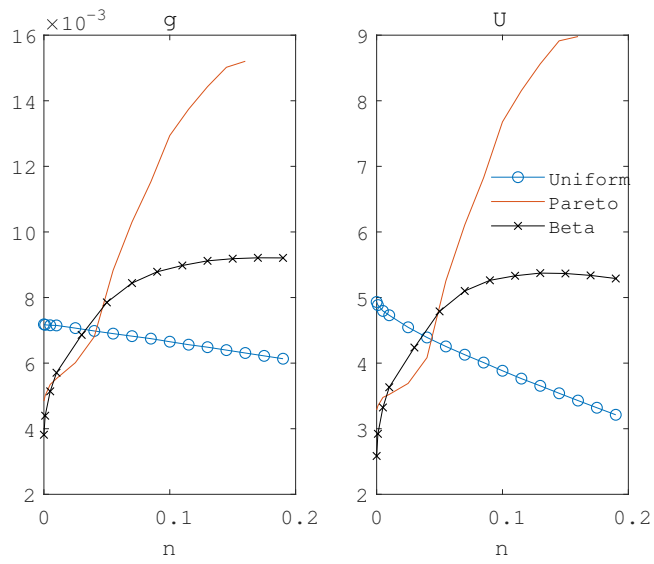


Figure 8: Effects of Nominal Growth under Different Distributions $\bar{\phi}(q)$

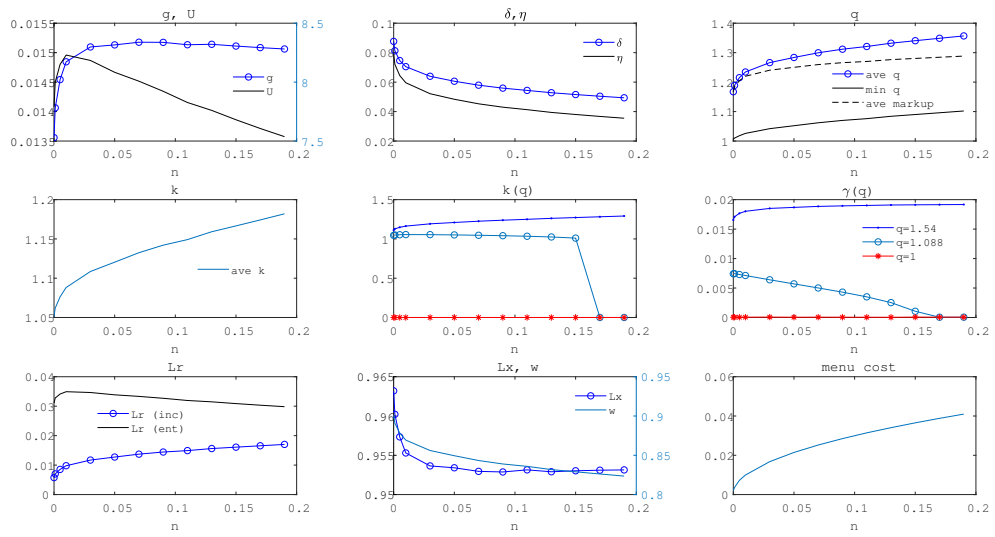


Figure 9: Effects of Nominal Growth for Japan

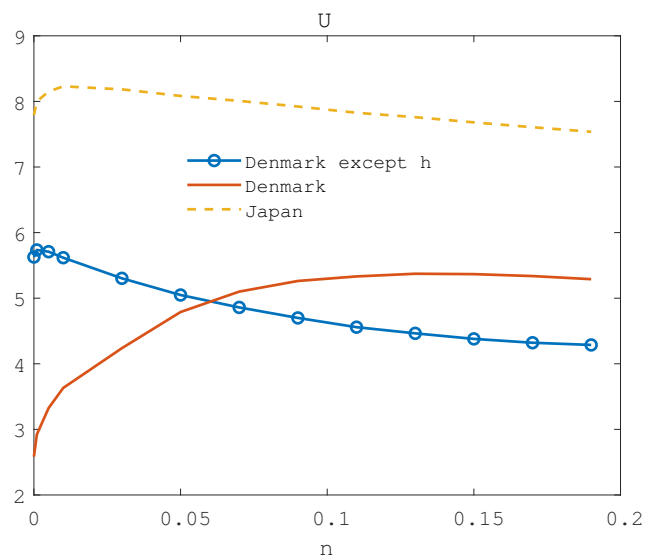


Figure 10: Danish Economy with Potential Entrants, h , in Japan

Table 3: Summary Table for Input Inflation and Firm Size Dispersion by Industries

	$\bar{\pi}^{\text{input}}$		T/M ratio				T/B ratio			
			Sales		Employment		Sales		Employment	
	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.
Foods	0.29%	2.17%	6.48	0.32	4.43	0.15	24.45	1.72	10.57	0.65
Textile	0.68%	2.51%	4.58	0.36	2.90	0.25	17.05	1.45	5.42	0.62
Pulp, paper and wooden products	0.46%	2.34%	5.22	0.56	3.66	0.18	14.15	1.85	7.14	0.41
Chemical products	2.19%	4.50%	9.00	0.65	6.40	0.55	35.47	4.55	16.59	1.25
Petroleum and coal products	6.59%	14.19%	68.59	62.87	7.81	2.90	305.93	196.72	19.90	7.93
Ceramics, stone and clay products	0.82%	2.10%	5.04	0.28	3.51	0.21	14.98	1.19	6.98	0.43
Steel	2.73%	7.23%	5.61	0.51	4.34	0.42	23.30	3.81	9.10	0.95
Non-ferrous metal	3.40%	11.19%	7.42	1.22	4.36	0.34	28.49	6.25	9.42	0.89
Metal products	0.83%	3.43%	4.94	0.38	3.56	0.18	13.16	0.77	6.56	0.49
General machinery	-0.02%	1.42%	6.59	0.48	4.26	0.21	19.27	1.90	8.94	0.46
Electrical machinery	-1.90%	1.77%	9.30	0.63	5.14	0.27	35.17	2.14	12.81	0.59
Transportation equipments	-0.49%	1.23%	8.96	0.51	5.69	0.25	34.94	3.04	14.98	0.87
Precision instruments	-1.16%	1.15%	7.56	1.07	4.79	0.58	22.96	2.94	11.03	1.35
Miscellaneous	0.61%	2.25%	5.35	0.25	3.72	0.10	16.32	1.08	7.61	0.23

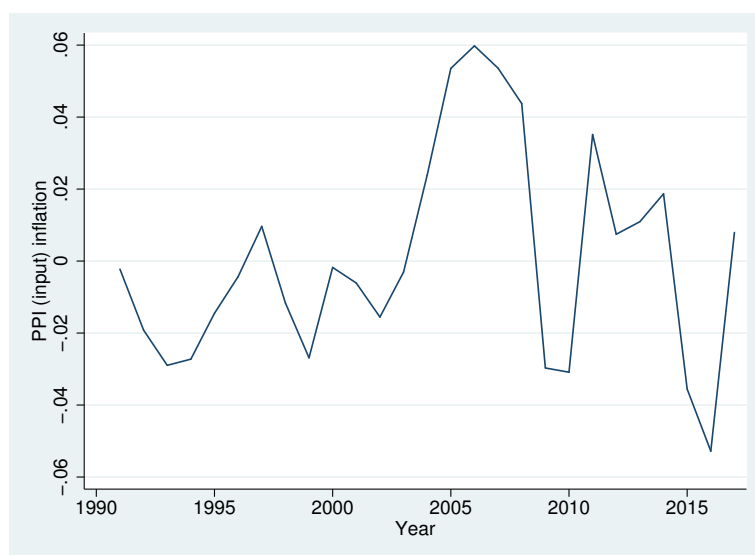


Figure 11: PPI (Input) Inflation in the Japanese Manufacturing Sector.

Table 4: Inflation and Reallocation: Domestic Sales
(a) Domestic sales dispersion:

	Top/Middle ratio			Top/Bottom ratio		
	(1) OLS	(2) OLS	(3) 2SLS	(4) OLS	(5) OLS	(6) 2SLS
$\bar{\pi}^{\text{input}}$	100.3*** (19.79)	102.2*** (20.21)	188.7.6*** (27.16)	251.8*** (59.64)	257.1*** (60.67)	556.8*** (82.07)
D.I. gap (T/M or T/B)		-0.0436 (0.18)	0.132 (0.18)		-0.120 (0.46)	0.345 (0.49)
D.I.		-0.331 (0.22)	-0.510** (0.23)		-1.060 (0.66)	-1.653** (0.68)
Industry RS		2.577 (5.59)	3.961 (5.95)		-6.574 (16.78)	-5.720 (18.06)
Constant	4.791 (5.30)	-26.09 (70.88)	-40.57 (75.63)	18.73 (15.97)	107.2 (212.8)	105.3 (229.2)
Year/Industry FE	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes
Obs.	322	316	302	322	316	302
\bar{R}^2	0.515	0.513	0.492	0.695	0.694	0.671
Underidentification			164.1			165.4
Weak identification			22.97			23.37

(b) Firm growth rate and the share of growing firms:

	Average growth			Share of growing firms		
	(1) OLS	(2) OLS	(3) 2SLS	(4) OLS	(5) OLS	(6) 2SLS
$\bar{\pi}^{\text{input}}$	-0.168 (4.307)	-0.117 (4.399)	2.823 (8.299)	-1.463*** (0.0882)	-1.546*** (0.0876)	-1.689*** (0.165)
Size group	-0.105*** (0.0394)	-0.101*** (0.0452)	-0.0946*** (0.0477)	-0.000598 (0.000807)	-0.00405*** (0.000901)	-0.00438*** (0.000949)
$\bar{\pi}^{\text{input}} \times \text{Size group}$	0.287 (0.641)	0.277 (0.653)	-0.081 (1.076)	0.122*** (0.0131)	0.134*** (0.0130)	0.152*** (0.0214)
Average D.I.		-0.00556 (0.0198)	-0.00723 (0.0201)		0.00332*** (0.000394)	0.00340*** (0.000400)
Industry RS		0.538 (0.796)	0.536 (0.790)		-0.0214 (0.0158)	-0.0213 (0.0157)
Constant	0.476 (0.681)	-6.355 (10.08)	-6.357 (10.01)	0.516*** (0.0139)	0.816*** (0.201)	0.816*** (0.199)
Year/Industry FE	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes
Obs.	2940	2880	2880	2940	2880	2880
\bar{R}^2	0.002	0.001	0.001	0.474	0.488	0.488
Underidentification			798.5			798.5
Weak identification			83.48			83.48

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$