Unbundling Labor *

Chris Edmond† Simon Mongey‡

February 2019

Abstract

In this paper we provide a theory for the role of technological change in the relative substitutability of workers in the economy. We show the theory to be useful for understanding new trends that we identify in the wages paid to workers. We extend Rosen (1983) to study when the *bundled* talents of workers that define their comparative advantage lead them to earn rents, and when these talents may be *unbundled* such that workers are more substitutable and rents are competed away. Allowing firms to choose their technology as in Caselli and Coleman (2006) endogenizes this substitutability. When technologies are adapted to labor supply, an unbundled economy is more likely, rents to workers shrink, and workers get paid more similarly. We provide empirical evidence consistent with the theory. In the U.S., workers in low skill occupations are paid more similarly now than in the 1980s. Over-time premia, part-time penalties and experience premia have disappeared.

*__INCOMPLETE, DO NOT CIRCULATE__

†University of Melbourne, cedmond@unimelb.edu.au.
‡University of Chicago and NBER, mongey@uchicago.edu.

1 Introduction

In this paper we provide a theory for the role of technological change in the relative substitutability of workers in the economy. We show the theory to be useful for understanding trends in the wages paid to workers. We extend Rosen (1983) to study when the *bundled* talents of a worker that define their comparative advantage lead them to earn rents, and when these talents may be *unbundled* such that workers are more substitutable across firms and sectors such that these rents are competed away. Allowing firms to choose their technology as in Caselli and Coleman (2006) we endogenize this substitutability. When technologies are chosen, an unbundled economy is more likely, rents to workers shrink, and workers get paid more similarly.

We provide empirical evidence consistent with the theory. In the U.S. workers in low skill occupations have are paid more similarly now than in the 1980s. Over-time premia, part-time penalties and experience premia have disappeared. We show that this trend is pervasive, holding for both male and female workers and workers that work both full- and part-time. Both have contributed to, but do not fully explain declining within differences in wages and earnings in low skill occupations.

2 Model

We first present a simple model based on Rosen (1983), extended to accommodate a continuum of heterogeneous firms. We characterize equilibria in this model through a simple example that describes when *bundled* and *unbundled* equilibria obtain. We then extend the model. We endogenize demand through a monopolistically competitive structure. We endogenize technology adoption by adapting the approach taken by Caselli and Coleman (2006).

Workers. There is a continuum of workers $i \in [0,1]$ and two labor tasks (or skills). Each individual is endowed with given *capacities* in each task:

$$l(i) = (l_1(i), l_2(i)) \tag{1}$$

Production. There are two sectors in the economy, x and y, each with CES production functions. In particular, a firm with productivity z in sector x produces

$$q_x = zF_x = \Big[\alpha_{1x}(a_{1x}l_{1x})^\sigma + \alpha_{2x}(a_{2x}l_{2x})^\sigma\Big]^{1/\sigma}, \qquad \alpha_{1x} + \alpha_{2x} = 1 \tag{2}$$

A firm with productivity z in sector y produces

$$q_y = zF_y = \Big[\alpha_{1y}(a_{1y}l_{1y})^\sigma + \alpha_{2y}(a_{2y}l_{2y})^\sigma\Big]^{1/\sigma}, \qquad \alpha_{1y}+\alpha_{2y}=1 \tag{3}$$

where the inputs into production are capacities of workers used in each sector, l_{1x}, l_{2x} in sector x and and l_{1y}, l_{2y} in sector y. Let $z \sim \Psi_x(z)$ in sector y and $z \sim \Psi_y(z)$ in sector y. For now the technology coefficients a_{1x}, a_{2x} and a_{1y}, a_{2y} are taken as given.

Market structure. Firms in each sector are monopolistically competitive. In particular, each firm faces an isoelastic demand curve and sets a price that is a constant markup over its marginal cost. Because the production functions F_x and F_y are both CRS, all firms have constant marginal cost. Labor markets are competitive.

Trade. Individual workers can only be employed in one sector and their capacities in each task cannot be traded.

2.1 Unbundled economy

Consider an economy in which the the aggregate capacities $\bar{L}_1$ and $\bar{L}_2$ are in fixed supply and can be purchased in continuous increments by firms in each sector.

Given that each firm sets the same markup over marginal cost, the key optimality condition governing the allocation of labor between sectors can be written

$$\frac{F_{x1}}{F_{x2}} = \frac{w_1}{w_2} = \frac{F_{y1}}{F_{y2}} \tag{4}$$

that is

$$\frac{\alpha_{1x}}{\alpha_{2x}}\left(\frac{a_{1x}}{a_{2x}}\right)^\sigma\left(\frac{l_{2x}}{l_{1x}}\right)^{1-\sigma} = \frac{w_1}{w_2} = \frac{\alpha_{1y}}{\alpha_{2y}}\left(\frac{a_{1y}}{a_{2y}}\right)^\sigma\left(\frac{l_{2y}}{l_{1y}}\right)^{1-\sigma} \tag{5}$$

It turns out to be convenient to write this as

$$\gamma_{xy}\left(\frac{l_{2x}}{l_{1x}}\right) = \left(\frac{l_{2y}}{l_{1y}}\right) \tag{6}$$

where

$$\gamma_{xy} := \left(\frac{\frac{\alpha_{1x}}{\alpha_{2x}}\left(\frac{a_{1x}}{a_{2x}}\right)^\sigma}{\frac{\alpha_{1y}}{\alpha_{2y}}\left(\frac{a_{1y}}{a_{2y}}\right)^\sigma}\right)^{\frac{1}{1-\sigma}} \tag{7}$$

Since these conditions hold for all productivity levels z we can aggregate them simply to write

$$\gamma_{xy}\left(\frac{L_{2x}}{L_{1x}}\right) = \left(\frac{L_{2y}}{L_{1y}}\right) \tag{8}$$

Aggregate feasibility requires that across sectors $L_{1x}+L_{1y} = \bar{L}_1$ for capacity 1 and $L_{2x}+L_{2y} = \bar{L}_2$ for capacity 2. This gives us the *contract curve*, i.e., the set of L_{1x}, L_{2x} describing the set of potential equilibria in this unbundled economy

$$L_{2x} = \left\{ \frac{\frac{L_{1x}}{\bar{L}_1 - L_{1x}}}{\gamma_{xy} + \frac{L_{1x}}{\bar{L}_1 - L_{1x}}} \right\} \bar{L}_2 \tag{9}$$

We assume that $\gamma_{xy} < 1$. As $\gamma_{xy} \nearrow 1$, then the substitution patterns become equal. As $\gamma_{xy} \searrow 0$, then the substitution patterns would lead to a larger allocation of $\bar{L}_2$ to sector-x. Given $\gamma_{xy} < 1$, the contract curve mapping L_{1x} to L_{2x} is strictly increasing and concave.

Prices and wages. In this unbundled economy the price of each task will be w_1 and w_2 such that the relative price w_1/w_2 satisfies equation (4) above. Given these prices, since workers are only paid according to their skill capacities, and these payments are the same in both sectors, a worker of type i will be indifferent over work in each sector (there is no specialization), and will be paid

$$w(i) = w_1 l_1(i) + w_2 l_2(i) \tag{10}$$

2.2 Bundled economy

We now consider possible allocations in an economy where the underlying labor capacities cannot be simply bought in continuous increments but instead have to be obtained in bundles (by hiring actual workers who possess capacities at both tasks).

Comparative advantage. We assume that individuals are ranked in terms of their *comparative advantage* at task 2. In particular, suppose that

$$l(i) = (l_1(i), l_2(i)) = (i, 1-i) \tag{11}$$

so that the aggregate supplies of each capacity are

$$\bar{L}_1 = \int_0^1 l_1(i)\,di = \int_0^1 i\,di = \frac{1}{2} \tag{12}$$

and

$$\bar{L}_2 = \int_0^1 l_2(i)\,di = \int_0^1 (1-i)\,di = \frac{1}{2} \tag{13}$$

Best allocation to sector x. Let $\overline{L}_{1x}(i)$ and $\overline{L}_{2x}(i)$ denote the allocations to sector x if workers with the highest comparative advantage in x are used first in sector x

$$\overline{L}_{1x}(i) = \int_0^i l_1(i')\, di' = \frac{1}{2}i^2 \tag{14}$$

and

$$\overline{L}_{2x}(i) = \int_0^i l_2(i')\, di' = i - \frac{1}{2}i^2 \tag{15}$$

Using the first of these to write $i = \sqrt{2\overline{L}_{1x}}$ and substituting this into the second we get the curve

$$\overline{L}_{2x} = \sqrt{2\overline{L}_{1x}} - \overline{L}_{1x} \tag{16}$$

Like the contract curve, this traces out an implied allocation of L_{2x} given an allocation of L_{1x}.

Worst allocation to sector x. Now consider the other extreme. Let $\underline{L}_{1x}(i)$ and $\underline{L}_{2x}(i)$ denote the allocations to sector x if workers with the lowest comparative advantage in x are used first in sector x

$$\underline{L}_{1x}(i) = \int_i^1 l_1(i')\, di' = \frac{1}{2}(1 - i^2) \tag{17}$$

and

$$\underline{L}_{2x}(i) = \int_i^1 l_2(i')\, di' = \frac{1}{2}(1 + i^2) - i \tag{18}$$

Using the first of these to write $i = \sqrt{1 - 2\underline{L}_{1x}}$ and substituting this into the second we get the curve

$$\underline{L}_{2x} = 1 - \sqrt{1 - 2\underline{L}_{1x}} - \underline{L}_{1x} \tag{19}$$

Like the contract curve, this traces out an implied allocation of L_{2x} given an allocation of L_{1x}.

Feasible allocations. The set of feasible allocations are all $(L_{1x}, Z_{2x}) \in \left[0, \bar{L}_1\right] \times \left[0, \bar{L}_2\right]$ such that the allocation of workers to sector 1 is between its 'worst' and 'best' (these can be constructed by some intermediate 'rule')

$$L_{2x} \in \left[1 - \sqrt{1 - 2L_{1x}} - L_{1x},\ \sqrt{2L_{1x}} - L_{1x}\right] \tag{20}$$

Both the lower and upper bounds are strictly increasing. The lower bound is convex and the upper bound is concave.

Note that since the contract curve is *increasing and concave*, while the upper bound is also *increasing and concave*, whereas the lower bound is *increasing and convex*. Then any intersection of the unbundled equilibrium contract curve with either of the bounds of the feasible set in the bundled economy would occur at the upper bound. This makes sense. The unbundled economy shifts task capacity in task 2 to sector-x (contract curve increasing and concave), while the distribution of task capacity in individuals in the bundled economy may place a constraint on how much of this allocation of task 2 skill to sector-x can occur.

Equilibrium. Define the following three curves

$$\mathcal{C}(\ell) := \left\{ \frac{\frac{\ell}{\bar{L}_1 - \ell}}{\gamma_{xy} + \frac{\ell}{\bar{L}_1 - \ell}} \right\} \bar{L}_2 \qquad \text{(contract curve)}$$

$$\mathcal{B}(\ell) := \sqrt{2\ell} - \ell \qquad \text{(best allocation to } x\text{)}$$

$$\mathcal{W}(\ell) := 1 - \sqrt{1 - 2\ell} - \ell \qquad \text{(worst allocation to } x\text{)}$$

The set of equilibria in the bundled economy are allocations $(L_{1x}, L_{2x}, L_{1y}, L_{2y})$ such that

$$L_{2x} = \min\left\{ \max\left[\mathcal{W}(L_{1x}), \mathcal{C}(L_{1x}) \right], \mathcal{B}(L_{1x}) \right\} \tag{21}$$

Prices. In the cases where $\mathcal{C}(L_{1x}) \notin (\mathcal{W}(L_{1x}), \mathcal{B}(L_{1x}))$, the competitive equilibrium requires that within each sector the marginal rate of substitution is still equated to the ratio of the prices of each task

$$\frac{w_{1x}}{w_{2x}} = \frac{F_{1x}}{F_{2x}}, \qquad \frac{w_{1y}}{w_{2y}} = \frac{F_{1y}}{F_{2y}} \tag{22}$$

Specialization. Workers will now no longer be indifferent across markets in which they work. For any worker i

$$w_x(i) = w_{1x} l_1(i) + w_{2x} l_2(i) = w_{2x} - i(w_{2x} - w_{1x})$$
$$w_y(i) = w_{1y} l_1(i) + w_{2y} l_2(i) = w_{2y} - i(w_{2y} - w_{1y})$$

Since we assumed $\gamma_{xy} < 1$ we know that $\mathcal{C}(\ell)$ is strictly increasing and strictly concave, so if $\mathcal{C}(L_{1x}) \notin (\mathcal{W}(L_{1x}), \mathcal{B}(L_{1x}))$ then it will be the case that $\mathcal{C}(L_{1x}) > \mathcal{W}(L_{1x})$ so that

$$\frac{w_{1y}}{w_{2y}} = \frac{F_{1y}}{F_{2y}} > \frac{F_{1x}}{F_{2x}} = \frac{w_{1x}}{w_{2x}} \tag{23}$$

and hence

$$\frac{w_{2x}}{w_{2y}} < \frac{w_{1x}}{w_{2x}} \tag{24}$$

such that task 2 is paid more in sector x, where it is scarce relative to the unbundled economy, and task 1 is paid more in sector y, where it is scarce relative to the unbundled economy. Therefore worker i works in sector x if i is sufficiently low:

$$w_x(i) > w_y(i) \tag{25}$$

or equivalently

$$i < \frac{(w_{2x} - w_{2y})}{(w_{2x} - w_{2y}) + (w_{1y} - w_{1x})} =: i^*(\boldsymbol{w}) \tag{26}$$

which is clearly between $(0, 1)$ since $w_{2x} > w_{2y}$ and $w_{1y} > w_{1x}$. In short, if $i < i^*$, the worker works in sector x, else if $i \geq i^*$, the worker works in sector y.

Note that if the marginal rate of substitution in the y sector, F_{y1}/F_{y2} is much larger than the marginal rate of substitution in the x sector, F_{x1}/F_{x2}, then $i^*(\boldsymbol{w})$ declines

$$\downarrow i^*(\boldsymbol{w}) = \frac{\uparrow (w_{2x} - w_{2y})}{\uparrow (w_{2x} - w_{2y}) + \uparrow (w_{1y} - w_{1x})}$$

Wages are then

$$w(i) = \begin{cases} w_{2x} - i(w_{2x} - w_{1x}) & , \quad \text{if} \quad i < \frac{(w_{2x} - w_{2y})}{(w_{2x} - w_{2y}) + (w_{1y} - w_{1x})} \\ w_{2y} - i(w_{2y} - w_{1y}) & , \quad \text{if} \quad i \geq \frac{(w_{2x} - w_{2y})}{(w_{2x} - w_{2y}) + (w_{1y} - w_{1x})} \end{cases}$$

2.3 Examples

To illustrate we assume Cobb-Douglas technologies $\sigma = 0$ and normalize the coefficients $a_{1x} = a_{2x} = a_{1y} = a_{2y} = 1$ and set $\alpha_{1x} = 0.22$ with various cases for α_{1y}.

In Figure 1, $\alpha_{1y} = 1 - \alpha_{1x}$, so the marginal rates of substitution between L_1 and L_2 are perfectly asymmetric across sectors. In this case when production is very low in either sector, allocations along the contract curve in the unbundled economy are feasible. With an intermediate range of production, we get specialization of labor across industries.

In Figure 2, $\alpha_{1y} = 0.90 > 1 - \alpha_{1x}$, so that production in sector y is relatively *more specialized* in task 1, than production in sector x is *specialized* in task 2. In this case the contract curve of the unbundled economy sits outside the feasible region for nearly all L_1^x. The unbundled economy wants to employ the capacity in task 1 in sector y, and the capacity in task 2 in sector x, but the embodiment of those skills in individuals does not allow any of these equilibria to be feasible in the bundled economy.

In Figure 3, $\alpha_{1y} = 0.5 < 1 - \alpha_{1x}$, and the contract curve lies nearly always inside the feasible set. The embodiment of tasks in individuals in the bundled economy is not a restriction on the competitive equilibrium lying on the contact curve in the bundled economy

Figure 1: $\alpha_{1x} = 0.22, \alpha_{1y} = 0.78$

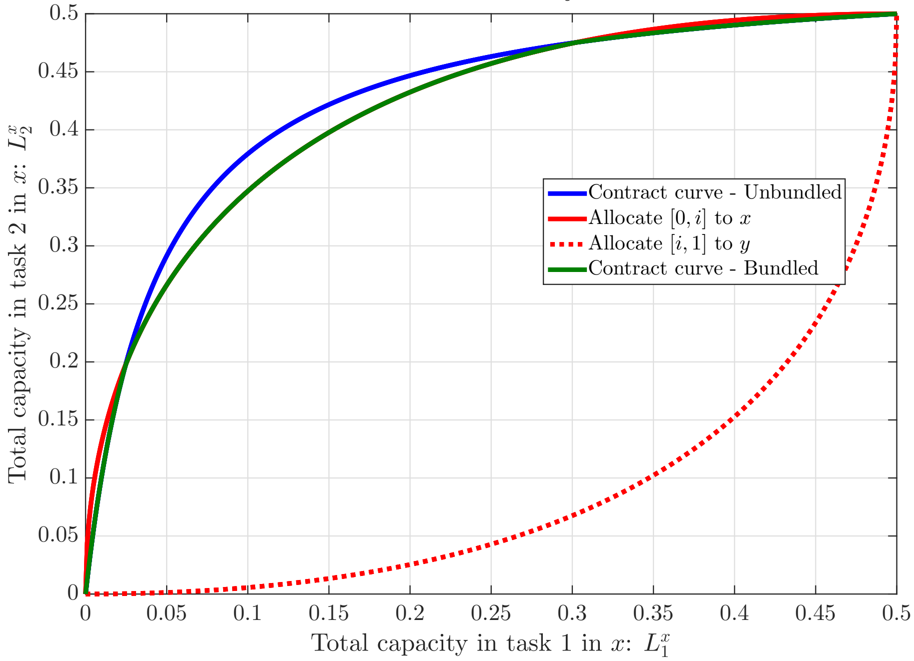


Figure 2: $\alpha_{1x} = 0.22, \alpha_{1y} = 0.90$

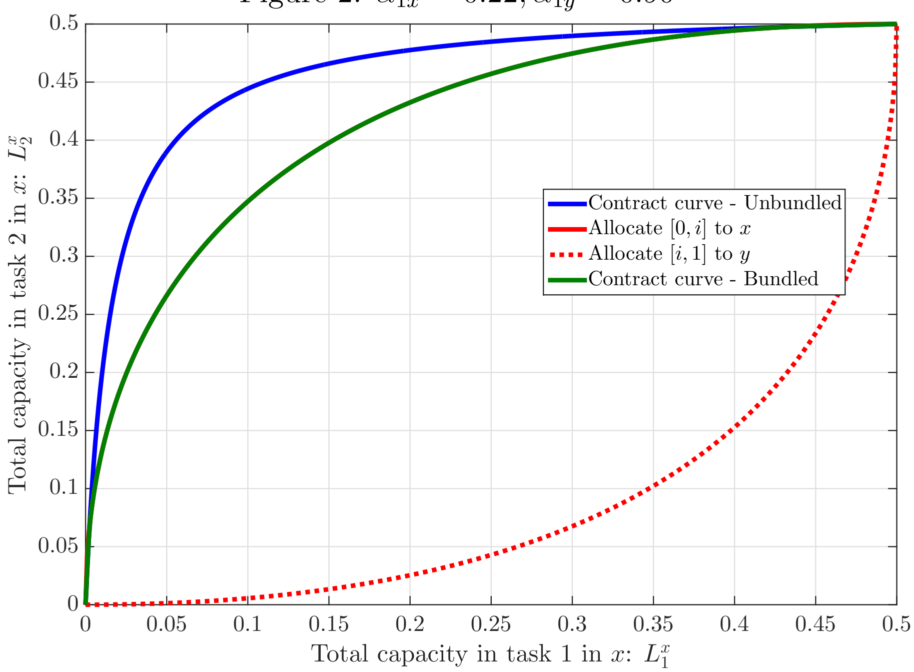
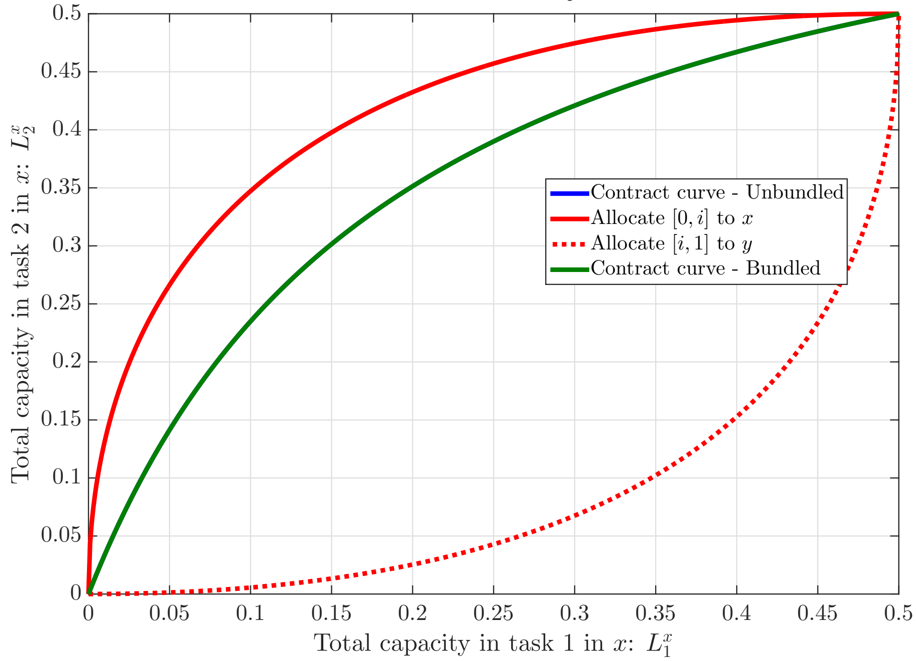


Figure 3: $\alpha_{1x} = 0.22, \alpha_{1y} = 0.50$



2.4 Demand and technology adoption

We now generalize this model in two directions. First, we briefly outline how to embed this model in an otherwise standard model of firm heterogeneity with monopolistic competition. This allows us to pin down the demand facing each firm. Second, we allow each firm to decide whether to keep their existing technology or, at a cost, to optimize their technology to the factor conditions they face. This latter feature of the model is an "appropriate technology" setup in the spirit of Caselli and Coleman (2006).

To simplify notation, here we suppress the sector subscripts x or y.

Monopolistic competition. Output in each key sector is a CES aggregate of differentiated varieties

$$Q = \left(\int_\Omega q(\omega)^{\frac{\theta-1}{\theta}} \, d\omega \right)^{\frac{\theta}{\theta-1}}, \qquad \theta > 1 \tag{27}$$

The producer of each differentiated variety then faces a standard demand curve and price index

$$q(\omega) = \left(\frac{p(\omega)}{P} \right)^{-\theta} Q, \qquad PQ = \int_\Omega p(\omega) q(\omega) \, d\omega \tag{28}$$

Local technology. The technology operated by a firm is endogenous. In particular, each producer has access to a "local" production function

$$y = zF(a_1 l_1, a_2 l_2) = z\Big[(a_1 l_1)^\sigma + (a_2 l_2)^\sigma\Big]^{1/\sigma} \tag{29}$$

By paying a technology adoption cost $\kappa \geq 0$ each firm can optimize the coefficients a_1, a_2 subject to the CES *technology frontier*

$$\bar{A} \geq G(a_1, a_2) = \Big(\gamma a_1^\omega + (1-\gamma) a_1^\omega\Big)^{1/\omega} \tag{30}$$

where $\bar{A} > 0$ is a given parameter describing the state of technology.

To operate, producers hire labor l_1, l_2 at factor prices w_1, w_2 and need to pay a fixed cost f. Let $C_0(\boldsymbol{a}, \boldsymbol{w})$ denote the cost function of a firm that uses the local technology

$$C_0(\boldsymbol{a}, \boldsymbol{w}) := \min_{\boldsymbol{l}} \Big[w_1 l_1 + w_2 l_2 \quad \Big| \quad F(a_1 l_1, a_2 l_2) = 1 \Big] \tag{31}$$

so that the total variable costs of a firm with productivity z are $C_0(\boldsymbol{a}, \boldsymbol{w}) y/z$ with marginal cost $C_0(\boldsymbol{a}, \boldsymbol{w})/z$. Since $F(a_1 l_1, a_2 l_2)$ has the CES form given above, this cost function is

$$C_0(\boldsymbol{a}, \boldsymbol{w}) = \Big[\Big(\frac{w_1}{a_1}\Big)^{\frac{\sigma}{\sigma-1}} + \Big(\frac{w_2}{a_2}\Big)^{\frac{\sigma}{\sigma-1}} \Big]^{\frac{\sigma-1}{\sigma}} \tag{32}$$

Now let $C_1(\boldsymbol{w}) y/z$ denote the cost function of a firm that uses the global technology

$$C_1(\boldsymbol{w}) := \min_{\boldsymbol{a}} \Big[C_0(\boldsymbol{a}, \boldsymbol{w}) \quad \Big| \quad G(a_1, a_2) = \bar{A} \Big] \tag{33}$$

Pricing. Given the CES demand, each firm charges a constant markup over its marginal cost. For firms that operate the local technology this is

$$p_0(z) = \frac{\theta}{\theta - 1} \frac{C_0(\boldsymbol{a}, \boldsymbol{w})}{z} \tag{34}$$

For firms that operate the global technology this is

$$p_1(z) = \frac{\theta}{\theta - 1} \frac{C_1(\boldsymbol{w})}{z} \tag{35}$$

Profits. Let $\pi_0(z)$ and $x_0(z) = p_0(z) y$ denote the profits and revenue of a firm that uses the local technology. Using the CES demand

$$\begin{aligned} \pi_0(z) &= p_0(z) y - C_0(\boldsymbol{a}, \boldsymbol{w}) y/z - f \\ &= \frac{1}{\theta} x_0(z) - f \\ &= \frac{1}{\theta} \left(\frac{\theta - 1}{\theta} \frac{Pz}{C_0(\boldsymbol{a}, \boldsymbol{w})} \right)^{\theta - 1} - f \end{aligned}$$

Let $\pi_1(z)$ and $x_1(z) = p_1(z)y$ denote the profits and revenue of a firm that uses the global technology. Again using the CES demand

$$\pi_1(z) = p_1(z)y - C_1(\boldsymbol{w})y/z - f$$

$$= \frac{1}{\theta}x_1(z) - f$$

$$= \frac{1}{\theta}\left(\frac{\theta-1}{\theta}\frac{Pz}{C_1(\boldsymbol{w})}\right)^{\theta-1} - f$$

Adopting the appropriate technology. Given the technology adoption cost κ, a firm will choose the global technology if

$$\pi_1(z) - \kappa \geq \pi_0(z) \tag{36}$$

Equivalently, if

$$\frac{1}{\theta}\left(\frac{\theta-1}{\theta}\frac{Pz}{C_1(\boldsymbol{w})}\right)^{\theta-1} - \frac{1}{\theta}\left(\frac{\theta-1}{\theta}\frac{Pz}{C_0(\boldsymbol{a},\boldsymbol{w})}\right)^{\theta-1} \geq \kappa \tag{37}$$

Collecting terms

$$Pz\left(\left(\frac{1}{C_1(\boldsymbol{w})}\right)^{\theta-1} - \left(\frac{1}{C_0(\boldsymbol{a},\boldsymbol{w})}\right)^{\theta-1}\right)^{\frac{1}{\theta-1}} \geq \frac{\theta}{\theta-1}(\theta\kappa)^{\frac{1}{\theta-1}} \tag{38}$$

Since $C_1(\boldsymbol{w}) \leq C_0(\boldsymbol{a},\boldsymbol{w})$ and $\theta > 1$ the LHS is an increasing function of z while the RHS is constant. Hence this implicitly defines a cutoff z^*, say, such that only those firms with $z \geq z^*$ would be willing to pay the cost of adopting the global technology. The cutoff firm z^* depends on the aggregates $\boldsymbol{w}, P$, which need to be determined in equilibrium.

Factor demands: non-adopters. For firms $z < z^*$ that use the local technology $C_0(\boldsymbol{a},\boldsymbol{w})$, factor demands are given by the optimality conditions

$$w_1 = \lambda F_1 \tag{39}$$

$$w_2 = \lambda F_2 \tag{40}$$

and

$$y = zF(a_1 l_1, a_2 l_2) \tag{41}$$

where by the envelope theorem $\lambda = C_0(\boldsymbol{a},\boldsymbol{w})$. Hence for these firms, as usual,

$$\frac{w_1}{w_2} = \frac{F_1}{F_2} \tag{42}$$

where the marginal rate of substitution F_1/F_2 is evaluated at the initially given local technology $\boldsymbol{a}$. Using the CES form for F this is just

$$\frac{w_1}{w_2} = \left(\frac{a_1}{a_2}\right)^{\sigma} \left(\frac{l_1}{l_2}\right)^{\sigma-1} \tag{43}$$

so that relative factor demand is

$$\frac{l_1}{l_2} = \left(\frac{a_1}{a_2}\right)^{\frac{\sigma}{1-\sigma}} \left(\frac{w_1}{w_2}\right)^{-\frac{1}{1-\sigma}} \tag{44}$$

conditional on a given a_1/a_2, the elasticity of substitution between labor inputs is $1/(1-\sigma)$. Using the CES form for F we can write the conditional factor demands for firms $z < z^*$ as

$$l_1 = \left(C_0(\boldsymbol{a}, \boldsymbol{w})\frac{a_1}{w_1}\right)^{\frac{1}{1-\sigma}} \frac{y}{a_1 z} \tag{45}$$

$$l_2 = \left(C_0(\boldsymbol{a}, \boldsymbol{w})\frac{a_2}{w_2}\right)^{\frac{1}{1-\sigma}} \frac{y}{a_2 z} \tag{46}$$

(summing $w_1 l_1 + w_2 l_2$ and noting $y/z = F$ gives the CES cost function $C_0(\boldsymbol{a}, \boldsymbol{w})$ above). Hence given scale y and factor prices w_1, w_2 we know how much of each type of labor these firms will use.

Factor demands: adopters. For firms $z > z^*$ that pay κ to use the global technology $C_1(\boldsymbol{w})$ the ratio a_1/a_2 is endogenous to w_1/w_2. Recall that

$$C_1(\boldsymbol{w}) = \min_{\boldsymbol{a}} \left[C_0(\boldsymbol{a}, \boldsymbol{w}) \quad \middle| \quad G(a_1, a_2) = \bar{A} \right] \tag{47}$$

The optimality conditions for this problem can be combined to get

$$\frac{C_{0,1}}{C_{0,2}} = \frac{G_1}{G_2} \tag{48}$$

which implicitly defines a_1/a_2 in terms of w_1/w_2. Calculating the various derivatives

$$\frac{C_{0,1}}{C_{0,2}} = \left(\frac{a_1}{a_2}\right)^{\frac{\sigma}{1-\sigma}-1} \left(\frac{w_1}{w_2}\right)^{-\frac{\sigma}{1-\sigma}} \tag{49}$$

and

$$\frac{G_1}{G_2} = \frac{\gamma}{1-\gamma}\left(\frac{a_1}{a_2}\right)^{\omega-1} \tag{50}$$

so this optimality condition becomes

$$\left(\frac{a_1}{a_2}\right)^{\frac{\sigma}{1-\sigma}-1} \left(\frac{w_1}{w_2}\right)^{-\frac{\sigma}{1-\sigma}} = \frac{\gamma}{1-\gamma}\left(\frac{a_1}{a_2}\right)^{\omega-1} \tag{51}$$

which simplifies to

$$\frac{a_1}{a_2} = \left(\frac{\gamma}{1-\gamma}\right)^{\frac{1-\sigma}{\sigma-\omega(1-\sigma)}} \left(\frac{w_1}{w_2}\right)^{\frac{\sigma}{\sigma-\omega(1-\sigma)}} \tag{52}$$

as in Caselli and Coleman (2006). Plugging this solution into the relative factor demand for the local technology gives the Caselli and Coleman (2006) result that for the global technology the elasticity of substitution between l_1 and l_2 is *greater* than for the local technology. In particular,

$$\frac{l_1}{l_2} = \left(\frac{a_1}{a_2}\right)^{\frac{\sigma}{1-\sigma}} \left(\frac{w_1}{w_2}\right)^{-\frac{1}{1-\sigma}}$$

$$= \left(\left(\frac{\gamma}{1-\gamma}\right)^{\frac{1-\sigma}{\sigma-\omega(1-\sigma)}} \left(\frac{w_1}{w_2}\right)^{\frac{\sigma}{\sigma-\omega(1-\sigma)}}\right)^{\frac{\sigma}{1-\sigma}} \left(\frac{w_1}{w_2}\right)^{-\frac{1}{1-\sigma}}$$

$$= \left(\frac{\gamma}{1-\gamma}\right)^{\frac{\sigma}{\sigma-\omega(1-\sigma)}} \left(\frac{w_1}{w_2}\right)^{\frac{\omega-\sigma}{\sigma-\omega(1-\sigma)}}$$

Having optimally chosen $a_1(\boldsymbol{w}), a_2(\boldsymbol{w})$ in terms of w_1, w_2 these firms with $z > z^*$ have factor demands given by

$$l_1 = \left(C_1(\boldsymbol{w})\frac{a_1(\boldsymbol{w})}{w_1}\right)^{\frac{1}{1-\sigma}} \frac{y}{a_1(\boldsymbol{w})z} \tag{53}$$

$$l_2 = \left(C_1(\boldsymbol{w})\frac{a_2(\boldsymbol{w})}{w_2}\right)^{\frac{1}{1-\sigma}} \frac{y}{a_2(\boldsymbol{w})z} \tag{54}$$

(i.e., the same as non-adopters but with marginal cost $C_1(\boldsymbol{w})$ instead of $C_0(\boldsymbol{a}, \boldsymbol{w})$ and with the optimized $a_1(\boldsymbol{w}), a_2(\boldsymbol{w})$).

Unbundling labor. Now consider an economy like this with $\kappa = +\infty$ so that all firms have the local technology with elasticity of substitution $1/(1-\sigma)$. Suppose that the parameters are such that, as in Figure 1, the contract curve lies nearly always above the feasible set. There is specialization and tasks that are in relatively short supply earn premia within each sector. Now suppose that the costs of technology adoption fall to $\kappa = 0$ such that all firms optimize their coefficients. If this shifts the contract curve in (recall that the best and worst allocations depend on the supplies of each task but not the technology), then the resulting equilibrium will *unbundle labor* so that the marginal rates of substitution are equalized across sectors and the different premia paid to each task disappear. In this equilibrium, it is as if each labor task is completely commodified, priced as if it can be bought in continuous increments, even though the labor capacities are embodied in actual workers.

3 Evidence

Our model describes an economy in which changes in technology can lead to workers in certain sectors of the economy getting paid more similarly. In equilibrium, technology changes in order to make workers with a particular comparative advantage in one subsector more substitutable with workers in another subsector. The former workers previously earned premia, but now, unbundled, the labor market thickens, workers face greater competition and earn similar wages regardless of subsector.

We present two pieces of evidence consistent with this idea. We think of labor markets being occupation specific and these changes flattening wages within an occupation, across sectors. We focus on low skill occupations where we think that the role of capital in making workers more substitutable is perhaps most prevalent. Using the CPS we document that over the last 30 years, within low skill occupations (i) overall earnings inequality has declined, driven by a decline in earnings inequality within occupations, (ii) the cross-sectional wage premia previously earned by workers that worked longer hours or had more experience have declined. We find this to be robust across male and female workers. While these trends are pervasive for low skill occupations, no such trends are identified for high skill occupations.

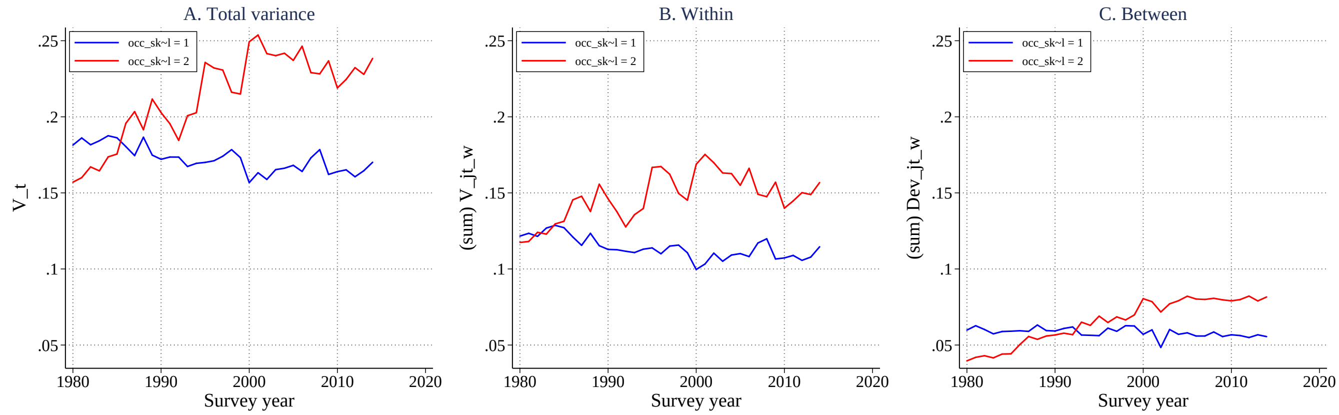
Data. We use the CPS and follow Heathcote et al. (2010) in terms of sample restrictions and construction of earnings, hours and wages. We classify occupations into different skill groups as follows. We rank occupations by the fraction of workers with a high school education or less. We then take employment weighted quantiles of this measure to define low and high skill occupations. Throughout we use David Dorn's harmonized *occ1990* occupation codes.

Regarding our classification of occupations, our results are robust to the following. We classify high and low skill occupations according to the above measure in 1980. Reclassifying occupations each year does not make a difference. We can also classify occupations by fraction of workers with a college degree, by non-routine skill content or by manual skill content and again obtain similar results.

(i) Compression in low skill occupations. First we consider a simple variance decomposition of overall earnings inequality as measured through the CPS. Let j denote an occupation. Then the economy-wide year t variance of log earnings y_{ijt} across individuals i can be decomposed into within and between occupation components:

$$var_t(\log y_{ijt}) = \underbrace{\sum_j \omega_{jt} var_{jt}(\log y_{ijt})}_{\text{Within}} + \underbrace{\sum_j \omega_{jt} \left(E_{jt}[\log y_{ijt}] - E_t[\log y_{ijt}]\right)^2}_{\text{Between}}, \quad (55)$$

Figure 4: Decomposition of overall income inequality



Notes Plots the components of the decomposition (55). The occupational categories *occskill1* (blue line) and *occskill2* (red line) are respectively *low* and *high* skill occupations, determined by whether an occupation has a fraction of individuals employed with a high school degree or less, that is above or below the median.

where ω_{jt} is the employment share of occupation j.

We split this decomposition into four parts, by summing over occupations j within two groups: low skill occupations and high skill occupations. Figure 4 plots the components of total, within and between occupation inequality for these groups.

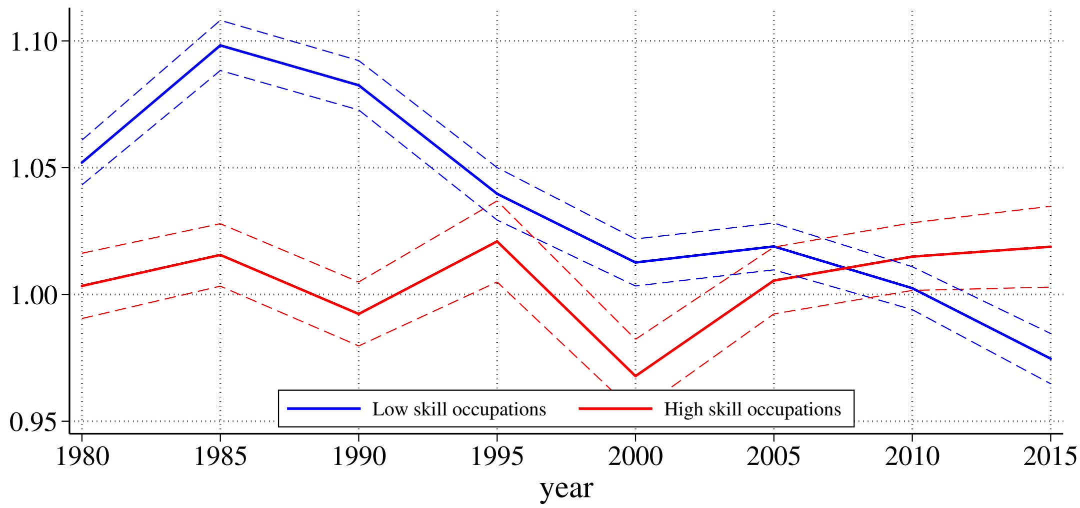
It is well known that overall earnings inequality—the sum of the lines in panel A—has increased over this period. What is less well known is that this masks a compression of earnings inequality among low skill occupations (panel B). Moreover, this is driven by a decrease in the dispersion in earnings *within* occupations (panel C). Not pictured here, these patterns are consistent across alternative specifications. Whether we consider dispersion in earnings, wages, or residuals of earnings or wages after controlling for a standard set of observables, we find that dispersion has decreased in low skill occupations, driven by a compression of within occupation dispersion.

(ii) Falling premia. We show that in low skill occupations in the 1980s, it used to be the case that longer hours were associated with a wage premium, shorter hours with a wage penalty, and more experienced workers earned more. Since then these premia and penalties have shrunk substantially.

To show this we estimate the following regression separately for low, medium and high skill occupations, and for each five year period $\tau \in \{1980-1984, 1985-1989, \dots, 2010-2014\}$:

$$\begin{aligned} \log Earnings_{it} &= \gamma_t + D_\tau^{ind(i)} + D_\tau^{ed(i)} \\ &\quad + \beta_\tau^{Male} Male_{it} + \beta_\tau^{White} White_{it} + \beta_\tau^{Large} Large_{it} \\ &\quad + \beta_\tau^{Hours} \log Hours_{it} + \beta_\tau^{Exp} Exp_{it} + \beta_\tau^{Exp^2} Exp_{it}^2 + \varepsilon_{it} \end{aligned}$$

Figure 5: Cross-sectional hours premium



Notes Plots $\hat{\beta}_{\tau}^{Hours}$ from estimated on subsamples of (56).

Our dependent variable is log annual earnings deflated to 2000 dollars. We control for seven standard industry group fixed effects, three education level fixed effects. We also control for firm size with dummies for smaller / larger than 500 workers. Our coefficients of interest are $\{\beta_{\tau}^{Hours}, \beta_{\tau}^{Exp}, \beta_{\tau}^{Exp^2}\}$. Since we are controlling for hours, these describe the cross-sectional *wage* premia associated with working longer hours and being more experienced.[1] If wages were constant in hours, then $\beta_{\tau}^{Hours} = 1$ and our regression is equivalent to a regression with wages as the dependent variable.

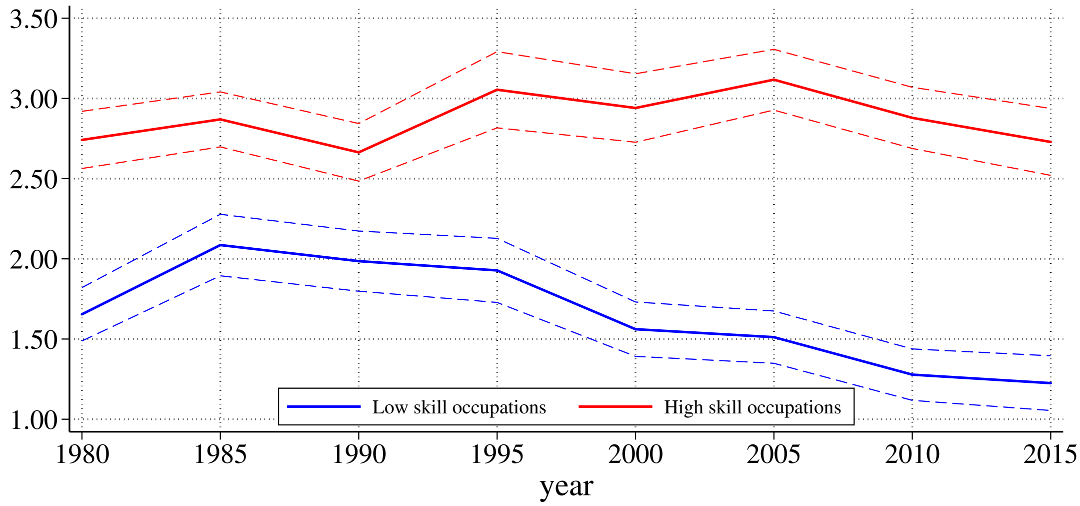
Figures 5 and 6 describe our results, plotting these coefficients over time for low and high skill occupations. Strikingly, premia decline sharply in low skill occupations, while being approximately constant in high skill occupations.

Quantitatively these differences are large. In 1980 if one worker worked two times as many hours as a coworker their wages would be 7 percent higher. In 2015 this is approximately zero. Not shown here, we find that when our sample is restricted to workers with less than 40 hours a week, or more than 40 hours a week the effects are the same. Both part-time penalties and over-time premia have disappeared. If that same worker in 1980 had an extra 10 years experience they could expect wages that were around 20 percent higher. In 2015 this has shrunk to around 10 percent.

Summary. The empirical evidence presented here is only cursory, but describes an economy in which workers in low skill occupations are becoming increasingly commodified. Previous

[1] Experience is defined in the usual way as $Exp_{it} = Age_{it} - \max\{YearsSchool_{it}, 12\} - 16$.

Figure 6: Cross-sectional experience premium



Notes Plots $\hat{\beta}_{\tau}^{Exp} + 10\hat{\beta}_{\tau}^{Exp^2}$ which gives the estimated partial effect (non-causal) of experience on wages at 10 years of experience. The y-axis can be read as follows. Estimated on the sample 1985-1990, one year extra experience at 10 years experience would be associated with a 2 percent higher wage.

channels through which low skill occupation could expect to 'get ahead' in the labor market through either accumulating experience or working longer hours have narrowed. The model presented in this paper aims to describe under what conditions such commodification can occur.

References

Caselli, Franceso and Wilbur John Coleman, "The World Technology Frontier," *American Economic Review*, June 2006, *96* (3), 499–522.

Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante, "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States: 1967-2006," *Review of Economic Dynamics*, January 2010, *13* (1), 15–51.

Rosen, Sherwin, "A Note on Aggregation of Skills and Labor Quality," *Journal of Human Resources*, 1983, *18* (3), 425–431.