Nominal Debt and the Heterogeneous Effects of Forward Guidance [*]

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Abstract

We develop an incomplete-markets heterogeneous agent New-Keynesian (HANK) model in which households are allowed to borrow using nominal debt. We show that, in this framework, forward guidance, that is the promise by the central bank to lower future interest rates, can be a powerful policy tool, especially when the economy is in a liquidity trap. In our model, expected lower rates imply a future transfer of wealth from savers to borrowers, reducing precautionary motives and stimulating current demand and inflation. In addition, at the time of the policy announcement, debt deflation generates also a wealth transfer towards constrained agents, who have high marginal propensity to consume, further increasing aggregate consumption and inflation, and igniting a positive feedback loop. These results contrast with previous research on HANK models, which focused on frameworks where agents were not allowed to borrow, and which found negligible effects of forward guidance.

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[*] The views expressed in this extended abstract are those of the authors, and not necessarily those of the Board of Governors of the Federal Reserve System, its staff.

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Introduction

A growing recent literature has examined New Keynesian models with heterogeneous agent and incomplete markets, widely known as HANK models. The HANK literature has in part been motivated by the widespread agreement that representative agent new Keynesian (RANK) models suffer from a forward guidance puzzle. That is, the effects of communication about the future path of monetary policy are too strong to be credible, see Carlstrom, Fuerst and Paustian (2015) or Del Negro, Giannoni and Patterson (2012). The forward guidance puzzle also implies that central banks that find themselves in a temporary liquidity trap can rely on promises to extend the duration of the ZLB only by small number of quarters in order to achieve large improvements in macroeconomic activity and inflation while at the ZLB.

The recent paper by McKay, Nakamura and Steinsson (2016) (MNS henceforth) documents that HANK models can potentially solve the forward guidance puzzle. In their framework, households may become borrowing constrained and the resulting aggregate dynamics for consumption mimic that of a representative agent who discounts future interest rates. As a result, they claim that movements in interest rates in the future have a smaller impact on current consumption than in RANK models, which mitigates the forward guidance puzzle.

However, as pointed out by Werning (2015) and by Hagedorn et al. (2019), the results in MNS are heavily influenced by assumptions on tax progressivity and on the distribution of dividends, which imply that forward guidance redistributes resources towards wealthier agents with lower marginal propensity to consume (MPC), hence having a limited effect on aggregate demand. Hagedorn et al. (2019) introduce in the framework of MNS wage stickiness and nominal government bonds and taxes, and find that the impact of foreward guidance in a liquidity trap is still negligible. On the other hand, Werning (2015) shows that in a model with extreme market incompleteness, meaning no borrowing and no publing debt, forward guidance has the same impact in RANK and HANK models, if we assume that individual income moves proportionally to aggregate income.

Nonetheless, existing HANK models have largely abstracted from the role of nominal debt on the part of households. Both in McKay et al (2016) and in Hagedorn et al. (2019), there is a no-borrowing constraint, so that households are savers and the borrower is the government. In this paper, we revisit the power of forward guidance in HANK models under the more realistic assumption that households are allowed to hold nominal debt. This framework introduces novel amplification mechanisms for forward guidance in a HANK model, operating through debt deflation and reduced precautionary motives for borrowing households, which are absent in a RANK model. As our results show, these channels can make forward guidance a relevant policy tool also in incomplete markets models.

In particular, we consider a setup very similar to MNS, but we allow households to borrow up to a certain multiple of average labor income, which we calibrate in line with the evidence on unsecured debt in U.S. data. In equilibrium, some households hold positive assets while others take on debt positions. Furthermore, we make two additional assumptions in order to minimize the impact of the non-monetary redistributive channels operating in the model of MNS. First, we assume there is no government debt so that interest rates do not directly affect the government budget constraint, and hence there are no indirect redistributive effects of monetary policy operating through taxes and transfers. Second, in the spirit of Werning (2015) and Fahri and Werning (2018), in order to prevent fluctuations in aggregate profits to have meaningful redistributive effects, we use a dividend distribution scheme that essentially results in individual income to move proportionally with aggregate income. We believe that this approach allows us to abstract from indirect effects of forward guidance, operating through fiscal assumptions or unrealistic profit distributions, and to focus on the heterogeneous effects of forward guidance that work through borrowing/lending and through debt deflation, which represent more direct channels of monetary policy propagation.

The effects of forward guidance in our HANK model on inflation and output are then crucially affected by the presence of nominal debt. For simplicity, consider the announcement of a one time decline in the real rate in the future. Since at least some households are sufficiently unconstrained by the borrowing limit, they react to promises of lower future rates immediately. As a consequence aggregate demand rises and results in some upward pressure on inflation. The consumption of households with nominal debt positions reacts strongly to such forward guidance for several reasons. First, the (unexpected) rise in prices lowers the real value of their nominal debt and results in an immediate positive real wealth transfer from savers to borrowers. That effect can be relatively large for constrained households, because the stock of debt is a multiple of their quarterly consumption. Importantly, these households have a high MPC, potentially exerting strong upward pressure on aggregate demand. Second, the wealth of constrained agents also rises because higher demand by unconstrained households drives real wages up. Finally, the precautionary motives of unconstrained borrowers, who are more likely to hit the constraint in the future, are reduced both because of the initial transfer due to debt deflation and because of the future wealth transfer, from creditors to debtors, implied by future lower rates promised by the monetary authority. As a result, aggregate demand and inflation are stimulated further, amplifying the first two channels and igniting a positive feedback loop. In this paper we show that these mechanisms can be large enough to overturn the prediction from standard HANK models in MNS and Hagedorn et al. (2019).

In a first set of experiments, we show that a simple promise of a lower real rate in the future can generate an increase in output at time zero in our HANK model that is larger than what would occur in a RANK model. This is due to the interaction of the three channels described above. In addition, we show that, because of the increasing impact on inflation and consequently of the positive debt-deflation effect on borrowers, the initial impact of this simple forward guidance policy increases with the horizon of the announcement. Furthermore, we show that even if we remove nominal debt, our HANK model still generates forward guidance effects at least as large as in the RANK model.

Finally, we perform a forward guidance experiment in presence of the zero lower bound (ZLB), by studying the effects of the central bank keeping nominal rates to zero for a few periods after the end of the liquidity trap. Consistent with our earlier results, also in this case forward guidance is more powerful in our baseline HANK model, so that a few additional periods of zero nominal rates can prevent the recession caused by the ZLB. Summing up, even without the aggregate Euler equation, which is responsible for the power of forward guidance in RANK models, HANK frameworks can generate similar effects through additional channels linked to debt deflation, general equilibrium effects on prices and wages, and lower precautionary motives.

To keep the model tractable, we follow the approach of Ragot and Le Grand (2018) and approximate the incomplete markets model with a truncated history strategy. Instead of keeping track of an infinite dimensional state vector of the wealth distribution, it is assumed that only the last T periods of idiosyncratic income shocks are relevant for decision making due to an assumed transfer scheme.[1] As such, a finite dimensional state vector arises but the key aspects of heterogeneous agent models remain: namely a precautionary savings motive, a distribution of wealth and heterogeneity in the marginal propensity to consume. The main benefit of this approach is that it enables us to solve the model with (piecewise) linearization techniques, allowing us to deal easily with aggregate shocks, the ZLB, and potentially with Ramsey-optimal monetary policy. Furthermore, we show that this simplifying assumption has no bearing on the main findings as we replicate essentially the same dynamics in response to forward guidance shocks when we follow the original McKay et al. (2016).

The rest of the paper is organized as follows. Section 1 describes the model and its truncated history representation. Section 2 presents different types of forward guidance experiments. Section 3 concludes.

1 Model

Our modeling framework is very close to the one used by MNS, and features households' uninsurable income risk, borrowing constraints and nominal rigidities. The main difference between our model and the setup of MNS, where agents are subject to a no-borrowing constraint, is that we allow agents to borrow through nominal debt, subject to a positive borrowing constraint.

Furthermore, as we explain more in detail below, we depart from MNS also with respect to their assumptions regarding the aggregate availability of liquidity and the distribution of dividends. As pointed out by Werning (2015) and Hagedorn et al. (2019), the results in MNS are likely driven by the redistributive effects caused by fluctuations in taxes and profits in response to forward guidance. In particular, MNS assume: i) a fixed stock of government debt, financed by levying taxes only on high productivity agents; ii) profits, arising from monopolistic competition, equally distributed across agents in lump sums. Because of these assummptions, forward guidance produces a redistribution of wealth from high MPC agents towards low MPC households: lower interest rates imply lower returns on savings but also a lower debt servicing cost for the goverment and consequently lower taxes for low MPC agents; in addition, the positive effect of forward guidance on real wages implies a decline in dividends that hits disproportionately the agents with low productivity. As a result of these

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[1] As T converges to infinity the model converges back to the original incomplete market structure.

redistributive forces, forward guidance produces only small effects on aggregate demand in the MNS model.

In order to abstract from these confounding effects, we make assumptions aimed at minimizing indirect redistributive effects that can drive the relative power of forward guidance within an HANK model compared to its RANK counterpart. First, we assume that there is no supply of public debt, so that there is no role for fiscal policy in our model. Second, we assume that dividends are distributed lump sum but proportionally to agents' individual labor income.[2] As a result, fluctuation in aggregate income, wages plus dividends, have minimal impact on the distribution of individual income. Using the terminology of Werning (2015), we can think of household non-financial income risk as being "acyclical". Given these assumptions, we can focus on the redistributive channels of forward guidance that operate only through borrowing/lending relationships and debt deflation.

Finally, in order to solve the model, which potentially requires to keep track of an infinite-dimensional wealth distribution, we use the "truncated history" approach, suggested by Ragot (2018) and Le Grand and Ragot (2018). This solution strategy introduces a partial insurance scheme which implies that the wealth distribution can be characterized by an arbitrarily large, but finite, set of agents.

1.1 Households

The economy is populated by a continuum of households deriving utility from consumption and leisure, according to

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_{j,t}, l_{j,t}\right) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{j,t}^{1-\sigma}}{1-\sigma} - \frac{l_{j,t}^{1+\varphi}}{1+\varphi} \right]$$

where $c_{j,t}$ is consumption of household j at time t, $l_{j,t}$ is labor supply. Households receive idiosyncratic labor productivity $z_{j,t}$ which follows a Markov chain with transition probabilities $P\left(z_{j,t+1}, z_{j,t}\right)$. Households can borrow and lend using a risk-free nominal bond, subject to a borrowing constraint. The problem of household j can be written recursively as

$$V_t\left(b_{j,t}, z_{j,t}\right) = \max_{c_{j,t}, l_{j,t}, b_{j,t}} \left\{ U\left(c_{j,t}, l_{j,t}\right) + \beta \sum_{z_{j,t+1}} \Pr\left(z_{j,t+1}, z_{j,t}\right) E_t V_{t+1}\left(b_{j,t+1}, z_{j,t+1}\right) \right\}$$

subject to

$$c_{j,t} + b_{j,t} \leq z_{j,t} W_t l_{j,t} + R_t^b b_{j,t-1} + \alpha_j D_t \tag{1}$$

$$b_{j,t} \geq -\bar{b} \tag{2}$$

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[2] A similar assumption is used in Fahri and Werning (2018).

where W_t is the real wage and D_t are aggregate profits. R_t^b is the realized gross real interest rate on bonds, and, since we are considering nominal bonds, it is given by $R_t^b = \frac{i_{t-1}}{\pi_t}$, where i_t and π_t, represent the gross nominal rate and inflation rate respectively. Equation (2) represents the borrowing constraint, which depends on the parameter $\bar{b}$. Each household receives a share α_j of aggregate profits and, as mentioned above, we assume that this fraction is proportional to the household's labor income in steady state, $z_j l_j$.

The first order conditions for bond holdings and labor supply are given by

$$U_c\left(c_{j,t}\right) = \mu_{j,t} + \beta E_t \sum_{z_{j,t+1}} \Pr\left(z_{j,t+1}, z_{j,t}\right) U_c\left(c_{j,t+1}\right) R_{t+1}^b \tag{3}$$

$$z_{j,t} W_t U_c\left(c_{j,t}\right) = l_{j,t}^{\varphi} \tag{4}$$

where $U_c\left(c_{j,t}\right) = c_{j,t}^{-\sigma}$, and $U_l\left(l_{j,t}\right) = l_{j,t}^{\varphi}$ represent the marginal utility of consumption and marginal disutility of labor, whereas $\mu_{j,t}$ is the multiplier on the borrowing constraint.

1.2 Production

The final good Y_t is produced from a continuum of intermediate inputs, indexed by $h \in [0,1]$, according to the production function

$$Y_t = \left[\int_0^1 Y_t\left(h\right)^{(\varepsilon-1)/\varepsilon} dh\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{5}$$

where $Y_t\left(h\right)$ is the production of intermediate good h.

As a result the demand for intermediate good h will be given by

$$Y_t(h) = \left(\frac{P_t\left(h\right)}{P_t}\right)^{-\varepsilon} Y_t \tag{6}$$

where the aggregate price level P_t satisfies

$$P_t = \left[\int P_t\left(h\right)^{1-\varepsilon} dz\right]^{\frac{1}{1-\varepsilon}} \tag{7}$$

Intermediate goods are produced by monopolistically competitive producers using only labor as input, according to

$$Y_t(h) = N_t\left(h\right) \tag{8}$$

Intermediate good producers are owned by a risk-neutral manager discounting the future at rate $1/R_t$, where $R_t = \frac{i_t}{\pi_{t+1}}$ represents the gross real rate given by the Fisher relation. As is standard in New-Keynesian models, they can reset prices only occasionally, with probability

$(1-\gamma^p)$, as in Calvo (1983). As a result, their problem will consist in choosing the price P_t^*, in order to solve

$$\max_{P_t^*} E_t \sum_{s=0}^{\infty} \gamma_p^s \left[\prod_{i=0}^{s} \left(\frac{1}{R_{t+i}} \right) \right] \left[\frac{P_t^*}{P_{t+s}} - W_t \right] Y_{t+s}^*(h)$$

subject to

$$Y_{t+s}^*(h) = \left(\frac{P_t^*}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} \tag{9}$$

Aggregate profits of intermediate firms are

$$D_t = Y_t - W_t N_t \tag{10}$$

where N_t represent aggregate labor demand.

1.3 Equilibrium

Define $\Gamma_t\left(b, z\right)$ as the distribution of households over their bond holdings and labor productivity at time t. Then the equilibrium in the bond market requires

$$\int b_t\left(b, z\right) d\Gamma\left(b, z\right) = 0 \tag{11}$$

Furthermore, labor market clearing requires

$$N_t = L_t = \int z l_t\left(b, z\right) d\Gamma\left(b, z\right) \tag{12}$$

Aggregate production will be given by

$$S_t Y_t = \int N_t\left(h\right) dh \tag{13}$$

where S_t represents price dispersion.[3]

Finally, aggregating over households' budjet constraints we can obtain the aggregate resource constraint

$$C_t = Y_t \tag{14}$$

where $C_t = \int c_t\left(b, z\right) d\Gamma\left(b, z\right)$, represents aggregate consumption.

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[3] Price dispersion is given by $S_t = \left[\left(1-\gamma_p\right) \left(\pi_t^*\right)^{-\varepsilon} + \gamma_p \pi_t^{\varepsilon} S_{t-1} \right]$. However, since we will be solving our model using linearization techniques, this second order term will not affect aggregate dynamics.

1.4 Truncated History Representation

In incomplete markets model (Bewley, Huggett 1993, Aiyagari 1994) the distribution of households' wealth represents a relevant state variable. Keeping track of this time-varying, infinite dimensional object poses significant computational challanges, especially when dealing with additional non-linearities like the zero lower bound on nominal interest rates. In order to simplify our analysis, we follow the approach proposed by Le Grand and Ragot (2018) which allows to represent incomplete market economies with finite support distributions. The main idea of this approach consists in introducing some level of partial insurance implying that agents with the same history of idiosyncratic shocks for the past T periods pool resources and choose the same allocations. This strategy essentially truncates to T the length of idiosyncratic history relevant for agents' decisions. As a result, we need to keep track of an arbitrarily large, but finite, set of wealth realizations, given by $\Sigma^T = (N_z)^T$, where N_z represents the number of possible idiosyncratic shocks. As T increases, the partial insurance scheme becomes less and less relevant and the model gets closer and closer to the original incomplete markets setting. The main benefit of this approach is that it allows to solve the model with perturbation methods, and consequently it also allows to deal with the ZLB with a piecewise linear algorithm, reducing dramatically computation times. In addition, as shown by Le Grand and Ragot (2018), this representation can potentially be used to gain theoretical insights from HANK models and to solve optimal policy problems with a Ramsey approach. In figure A1 in the appendix we show how, in the original MNS model, the truncated history method generates forward guidance responses very similar to the ones obtained by MNS using a more involved nonlinear algorithm to keep track of the wealth distribution. Below we describe how to implement the truncated history insurance scheme in our model.

The key assumption is that agents with the same realizations of idiosyncratic shocks over the the past T periods can pool their assets/liabilities, and consequently make the same choices at time t. We can think of this partial insurance scheme as resulting from a framework where agents with the same truncated histories transit to the same island and share resources. If $T = 0$, we have complete insurance across all agents and we are in the world of a representative agent model. For $T = \infty$, we have an infinite number of histories/islands and we are considering the original Bewley setup.

Define e^T as a specific history of T idiosyncratic shocks from $t-T+1$ to t, and define S_{t,e^T} as the mass of agents with history $e^T = (z_{t-T+1}....z_t)$. Define $\Pi\left(e^T, \tilde{e}^T\right)$ as the probability of going from history $\tilde{e}^T$ to e^T, and z_{t,e^T}, $\tilde{z}_{t,e^T}$ as the last realization of the idiosyncratic shock in each history, then we can write

$$\Pi\left(e^T, \tilde{e}^T\right) = 1_{e^T \succeq \tilde{e}^T} P\left(z_{t,e^T}, \tilde{z}_{t,e^T}\right)$$

where $1_{e^T \succeq \tilde{e}^T} = 1$ if e^T is a possible continuation history for $\tilde{e}^T$. Hence we can derive the law of motion for S_{t,e^T} as

$$S_{t,e^T} = \sum_{\tilde{e}^T} S_{t-1,\tilde{e}^T} \Pi\left(e^T, \tilde{e}^T\right) \text{ for all for all } e^T \in \Sigma^T \tag{15}$$

As the Markov chain transition matrix P is constant over time, also the transition matrix Π, and the distribution of S_{e^T} will be time-invarying.

As anticipated, we assume that at the beginning of time t, after the realization of the idiosyncratic shock, all agents with history e^T, share the same initial per-capita bond holdings $\tilde{b}_{t,e^T}$, which evolve according to

$$\tilde{b}_{t,e^T} = \frac{1}{S_{t,e^T}} \sum_{\tilde{e}^T} S_{t-1,\tilde{e}^T} \Pi\left(e^T, \tilde{e}^T\right) b_{t-1,\tilde{e}^T} \tag{16}$$

where b_{t,e^T} represents the bond holdings chosen at the end of time t, by an agent with history e^T. Alternatively, as suggested by Le Grand and Ragot (2018), this allocation can be implemented by assuming that agents receive transfers equal to $\omega_{e^T} = \tilde{b}_{t,e^T} - b_{t-1,e^T}$.

As a result, we can rewrite the households' problem in terms of the individual state variables e^T and $\tilde{b}_{t,e^T}$ as

$$V_t\left(\tilde{b}_{t,e^T}, e^T\right) = \max_{c_{t,e^T}, l_{t,e^T}, b_{t,e^T}} \left\{ U\left(c_{t,e^T}, l_{t,e^T}\right) + \beta \sum_{\hat{e}^T \in \Sigma^T} \Pi\left(\hat{e}^T, e^T\right) E_t V_{t+1}\left(\tilde{b}_{t+1,\hat{e}^T}, \hat{e}^T\right) \right\}$$

$$c_{t,e^T} + b_{t,e^T} \leq z_{t,e^T} W_t l_{t,e^T} + R_t^b \tilde{b}_{t,e^T} + \alpha_{e^T} D_t \text{ for all } e^T \in \Sigma^T \quad \text{(per capita BC)} \tag{17}$$

$$b_{t,e^T} \leq -\bar{b} \text{ for } e^T \in \Psi_t \tag{18}$$

subject to (15) and (16). All agents with the same truncated history will choose the same consumption, labor and bond holdings.

The first order conditions for bonds and labor supply are given by

$$U_c\left(c_{t,e^T}\right) = \mu_{t,e^T} + \beta E_t \sum_{\hat{e}^T \in \Sigma^T} \Pi\left(\hat{e}^T, e^T\right) U_c\left(c_{t+1,\hat{e}^T}\right) R_{t+1}^b \tag{19}$$

$$U_c\left(c_{t,e^T}\right) z_{t,e^T} W_t = U_l\left(l_{t,e^T}\right) \tag{20}$$

where μ_{t,e^T} represents the lagrange multiplier on the borrowing constraint for agents with history e^T.

Importantly, this specification still allows to capture precautionary motives, since, as it can be seen from equation (19), households internalize the possibility of facing the borrowing constraint in the future, that is of transitioning to an island with constrained agents. The main advantage of the truncated history approach is that we only need to keep track of the wealth distribution of $(N_z)^T$ agents, and we can easily derive the time invarying transition probabilities across these groups of households by using the original Markov matrix P.

Finally, the market clearing conditions for labor and bonds are given by

$$N_t = L_t = \sum_{e^T \in \Sigma^T} z_{t,e^T} S_{t,e^T} l_{t,e^T} \tag{21}$$

$$\sum_{e^T \in \Sigma^T} S_{t,e^T} b_{t,e^T} = 0 \tag{22}$$

and the implied aggregate resource constraint is

$$Y_t = \sum_{e^T \in \Sigma^T} S_{t,e^T} c_{t,e^T} \tag{23}$$

2 Forward Guidance Experiments

2.1 Calibration

Apart from our assumptions on the supply of government bonds, on the distribution of dividends, and on the borrowing constraint, we calibrate the model exactly as in MNS. We use a value for the risk aversion parameter σ, equal to 2. The Frish elasticity $1/\varphi$ is set equal to 0.5. We choose a parameter for the elasticity of substitution of intermediate goods, ε, equal to 6, which implies a steady state markup of 20 percent. The Calvo parameter γ^p is set to 0.85, resulting in a 15 percent probability of resetting prices every quarter. We set the discount factor $\beta = 0.947$ in order to obtain a steady state value of the real interest rate of 2 percent annually.

As described above, we depart from MNS with respect to three assumptions. First, we assume that bonds are in zero net supply in order to avoid redistributive effects linked to assumptions on tax progressivity. Second, we assume, as in Fahri and Werning (2018), that dividend distribution is proportional to labor income in steady state, that is, in the truncated history notation, α_{e^T} is proportional to $z_{e^T} l_{e^T}$.[4] This assumption helps to avoid that fluctuations in aggregate income might have unrealistic redistributive effects, possibly overcoming the direct effects of forward guidance. In addition, we introduce a fixed cost Φ, calibrated in order to have zero profits in steady state, so that assumptions on profits distribution do not affect the steady state wealth distribution. Third, we set a value for the borrowing constraint, $\bar{b} = 1.5$, that is equal to about 5 times average labor income in the steady state equilibrium. This multiple was suggested by MNS as a reasonable approximation of the evidence provided by Kaplan, Violante and Weidner (2014) on the distribution of households' unsecured debt in the U.S. This calibration results in about 20 percent of agents being constrained in the model's steady state.

As in MNS, we calibrate the idiosyncratic wage risk to match the wage process estimated in Flode and Linde' (2001), by using an AR(1) process with an autoregressive coefficient of 0.966 and an innovation variance of 0.17. This process is approximated with a three states

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[4] The shares α_{e^T} are normalized to sum up to one, that is $\alpha_{e^T} = z_{e^T} l_{e^T} / L$.

Markov chain using the Rouwenhorst (1995) method.[5] Given our truncated history approach, a novel parameter to be calibrated is the length of idiosyncratic histories that we keep track of, T. As suggested by Le Grand and Ragot (2018), a good measure for the amount of risk sharing that is artificially introduced with the truncated history approach, can be given by the standard deviation of the implied wealth transfers ω_{e^T} across agents, normalized by total income $Inc_{e^T} = z_{e^T} l_{e^T} W + R_t^b b_{,e^T} + \alpha_{e^T} D$. For our experiments, we choose $T = 2$, which implies a standard deviation of transfers of about 2 percent in steady state, a pretty low number. As a result, we have to keep track of $(N_z)^T = 9$ groups of agents.[6] Finally, unless otherwise specified, we use a standard Taylor rule to determine the nominal interest rate log$i_t = \log R + \phi \log \pi_t$, with $\phi = 1.5$.

2.2 Simple Forward Guidance Experiment

We begin by replicating a simple forward guidance experiment proposed by MNS. For simplicity, let's assume that monetary policy is characterized by an exogenous rule for the real interest rate, that is $R_t = i_t / E_t \pi_{t+1} + \varepsilon^r_{t,t-j}$, where $\varepsilon^r_{t,t-j}$ represents a shock to the real interest rate announced j periods in advance. We consider an experiment in which the monetary authority promises a 50 basis points decline in the real interest rate 5 years in the future.[7] Figure 1 reports the response of our HANK model and of the corresponding RANK model to this type of anticipated shock. In our HANK model with nominal debt, output and consumption increase about 50 percent more than in the RANK model on impact. After the initial jump, the path of output in HANK is slightly higher than in the RANK model, until the realization of the monetary shock, after that the two economies behave similarly.[8] There are several forces that explain why forward guidance is so powerful in our HANK framework. First, in addition to the standard intertemporal motive, lower future interest rates stimulate the demand of uncostrained agents by reducing precautionary savings. In fact, lower real rates imply a future transfer from savers towards borrowers, who are the agents with a higher likelihood of becoming constrained. The top right panel of figure 2 reports the consumption response of a set of uncostrained agents in our truncated history representation. These agents have a similar impact response as in the RANK model, and then their consumption increases as the rate cut approaches. Higher unconstrained demand, has positive effects on impact also on the wealth of constrained agents through two general equilibrium effects. Higher wages imply

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[5] As pointed out by Hagedorn et al. (2018), MNS use values for their Markov chain that imply an aggregate productivity greater than one. We adjust slightly the values so that aggregate productivity is equal to 1 as in the complete market model.

[6] We have tried our experiments also with $T = 3$ and $T = 4$, and have obtained very similar results.

[7] Alternatively, it is possible to replicate this experiment by using a standard Taylor rule, and by feeding a sequence of anticipated monetary policy shocks that delivers a constant path for the real interest rate until period 19, and then produces a 50 basis points decline in period 20. Given the linear solution of our model this approach can be easily implemented in our framework, and we checked that it delivers basically identical results.

[8] We assume that the same fixed costs, used in the HANK model to obtain zero profits in steady state, are present in the RANK model. As a result, output moves less than aggregate consumption in both models.

higher income for constrained agents, for the first 20 periods of the experiment. In addition, due to the nominal nature of debt contracts in our model, initial unexpected higher inflation implies a transfer from savers to borrowers at time zero, as suggested by the initial decline in the realized bond rate reported in the bottome left panel of figure 1. This debt deflation effect is responsible for the spike in consumption and output in our HANK model, in the first period of the experiment. In fact, constrained households behave as hand-to-mouth agents and have a higher marginal propensity to consume, compared to unconstrained households. As a result, their consumption reacts much more in period 1, as shown in the middle panel of the top row of figure 2.[9] In addition, from this figure we can see the effect of the positive transfer from savers to borrowers that occurs once real rates are lowered in period 21.

The bottom row of figure 2 reports the behavior of consumption in our model when we consider real debt instead of nominal debt. Apart from the debt-deflation effect, the other two positive effects, working through the demand of unconstrained agents and higher wages, are still present. As a result, the economy does not experience the same initial jump in output and consumption as in our baseline model, but still the response to forward guidance is slightly larger than in the RANK model.

Next, figure 3 reports the initial impact of forward guidance at different horizons for the RANK model, for which the impact is constant, and for our HANK model, with and without nominal debt.[10] A new result in our baseline model is that, unlike in MNS, the impact of forward guidance increases with the horizon. This result is due to the increasing impact of forward guidance on inflation, a behavior shared with MNS, which in our model generates larger and larger debt-deflation effects, transferring resources towards constrained agents. On the other hand, in this simple experiment the HANK model with real debt generates slightly larger effects than the RANK models, which disappear as the forward guidance horizon increases.

Finally, figure A1 replicates the experiments of figure 1 using the original MNS calibration while solving the model with truncated history, and compare the results with the original responses obtained by MNS with their nonlinear perfect foresight algorithm.[11] In both cases the results are quite close, suggesting that our solution method does not generate any large bias that might drive our results.[12]

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[9] In our truncated history representation, constrained agents are the ones with a history made only of low productivity shocks. These agents remain the only constrained group in all of our dynamic experiments.

[10] Figure 3 reports the initial impact, on output and inflation, of an announced decline of 50 basis points in the real interest rate at different dates in the future.

[11] In particular in this case we use a value of $T = 5$, which generates 243 groups of agents and a standard deviation of transfers below 5 percent.

[12] Compared to McKay et al. (2016), who use a perfect foresight nonlinear algorithm, our results for the RANK model and HANK model are slightly different also because our solution approach allows to use loglinearization, hence abstracting from the effects of price dispersion.

2.3 Forward Guidance in a Liquidity Trap

In this section, we consider the effect of forward guidance in a liquidity trap in our model. Figure 4 considers an experiment in which the discount factor increases for a known number of periods, before reverting to its steady state value, as done in MNS or Hagedorn et al. (2019). In this experiment, the nominal rate is governed by a standard "naive" Taylor rule, responding only to inflation, and the path of the discount factor is calibrated to obtain a ZLB episode of eight quarters and a 4% decline in output both in the RANK and in the HANK model (solid red and blue line).[13] The dashed line represents the behavior of the two models under an "extended" monetary policy rule which keeps the nominal rate to zero for four extra periods, compared to the "naive" rule.

In the RANK model, forward guidance reduces by half the decline in output and inflation. In our HANK model, the effect of the four extra periods of low rates is twice as powerful, with output declining only by about 1 percent, and inflation only by 0.5 percent. Hence, contrary to the findings in MNS and Hagedorn et al. (2019), we find that forward guidance can be a powerful policy tool, when conventional monetary policy is constrained by the zero lower bound, also in a HANK model. This result is due to the three positive channels outlined above: a reduction of precautionary motives for unconstrained agents, and positive general equilibrium effects on the wealth of constrained agents due to higher wages and lower real value of outstanding debt.

It is important to note that, for the positive effects of forward guidance to be visible in the HANK setting, it is crucial to introduce modeling assumptions that prevent fluctuations in dividends to generate unrealistic negative effects on high MPC agents, as it was the case in MNS. Assuming equal lump sum distribution of dividends, which decline on impact as real wages increase, would result in a much weaker effect of forward guidance also in our setting, despite the positive effect of debt-deflation.[14]

Figure 5 reports the impact of forward guidance policies with different durations, given that the economy is in the liquidity trap described in figure 4. The horizontal axis reports the announced periods of additional ZLB, after the 8 periods of endogenous zero interest rates. Similarly to the results in figure 3, the gap between the effect of forward guidance in the

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[13] We exploit the linear solution of our model and we implement the ZLB by usinc the OccBin toolbox developed by Guerrieri and Iacoviello (2015). This approach allows to easily filter shocks that deliver the required decline in output with the economy subject to the ZLB constraint.

[14] Hagedorn et al. (2019) introduce in their HANK model wage rigidities in order to attenuate dividend fluctuations, while still assuming that dividends are equally distributed in lump sums. In their framework they find that forward guidance has negligible effects in a liquidity trap. However, there are several assumptions that could be driving their result. First of all, it is still possible that, for realistic levels of wage stickiness, fluctuations in wages still result in large changes in dividends for high MPC agents. Furthermore, in order to introduce wage stickiness in their model, Hagedor et al. (2019) eliminate labor supply heterogeneity, fundamentally altering the original HANK framework. Finally, in their main expeiment of forward guidance in a liquidity trap, they assume a Taylor rule with a coefficien on inflation of only 0.5. This assumption could generate a lower path of the federal funds rate at the end of the ZLB episode, implying that additional periods of ZLB correspond to small anticipated monetary policy shocks, hence pontentially having a small impact also in the corresponding RANK model.

HANK and RANK model increases with the lenght of the policy experiment. In addition, from figure 5 we see that, in a liquidity trap, forward guidance is more powerful also in our HANK model without nominal debt (dashed blue line).[15] Compared to figure 3, where the path of the real rate was fixed across models, in this experiment the real rate (not shown) declines more in the HANK model with real debt than in the RANK model, because of the larger positive effect on inflation in the HANK model. Lower real rates further reduce precautionary motives and stimulate aggregate demand in our HANK setting, as explained above.[16]

3 Conclusions

This paper is aimed at illustrating a potentially relevant channel for the transmission of unconventional monetary policy in HANK models, operating through private nominal debt. Once we eliminate confounding factors linked to fiscal policy or to the redistribution of profits, our HANK model introduces new channels for the transmission of forward guidance which are absent in a standard representative agent model. These channels operate through future or current wealth redistribution from savers towards borrowers, and through a consequent reduction in precautionary motives. These effects could potentially be even larger if we introduced collateralized debt, because of the positive effect of lower rates on asset prices and on agents' borrowing capacity.

Our results hence support the view of Werning (2015), who suggests that, under realistic assumptions on aggregate liquidity and income risk, incomplete market models generates effects of forward guidance that are at least as large as in the representative agent framework. The forward guidance puzzle is still alive in HANK models as well, especially in presence of nominal debt. Farhi and Werning (2018), propose a solution for the forward guidance puzzle which relies on the interaction between incomplete markets and bounded rationality (in the form of "level-k thinking"). They focus on a framework without borrowing or nominal debt, where forward guidance has the same power in HANK and RANK. Studying whether deviations from rational expectations can be strong enough to mitigate the effects of future monetary policy in HANK models with nominal debt is an interesting topic for future research.

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[15] Also for the HANK model with real debt we filter shocks that deliver the same decline in output and ZLB duration as in the HANK model with nominal debt.

[16] There are several reasons why the behavior of our HANK model with and without nominal debt does not produce a divergent path as stark as in the experiment in figure 3. First, unlike figure 3, in this experiment the paths of real rates is not fixed across models. In addition, the initial set of discount factor shocks, responsible for bringing the economy to the ZLB, has different implications for the distribution of wealth and consumption in the two versions of our HANK model, as the decline in inflation negatively affects constrained agents much more in the model with nominal debt.

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Figure 1: **Simple Forward Guidance Experiment in HANK and RANK: Aggregate Variables**

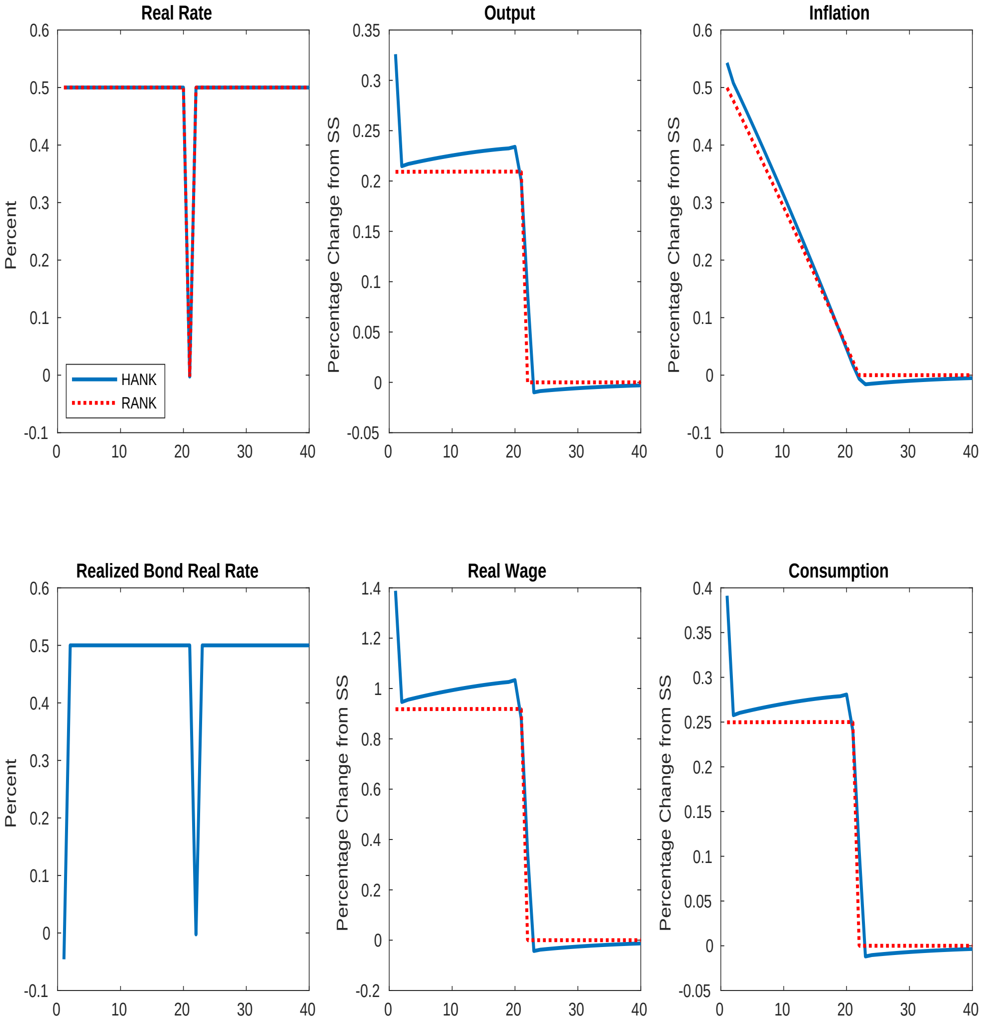

Figure 2: **Simple Forward Guidance Experiment in HANK with and without nominal debt: Consumption**

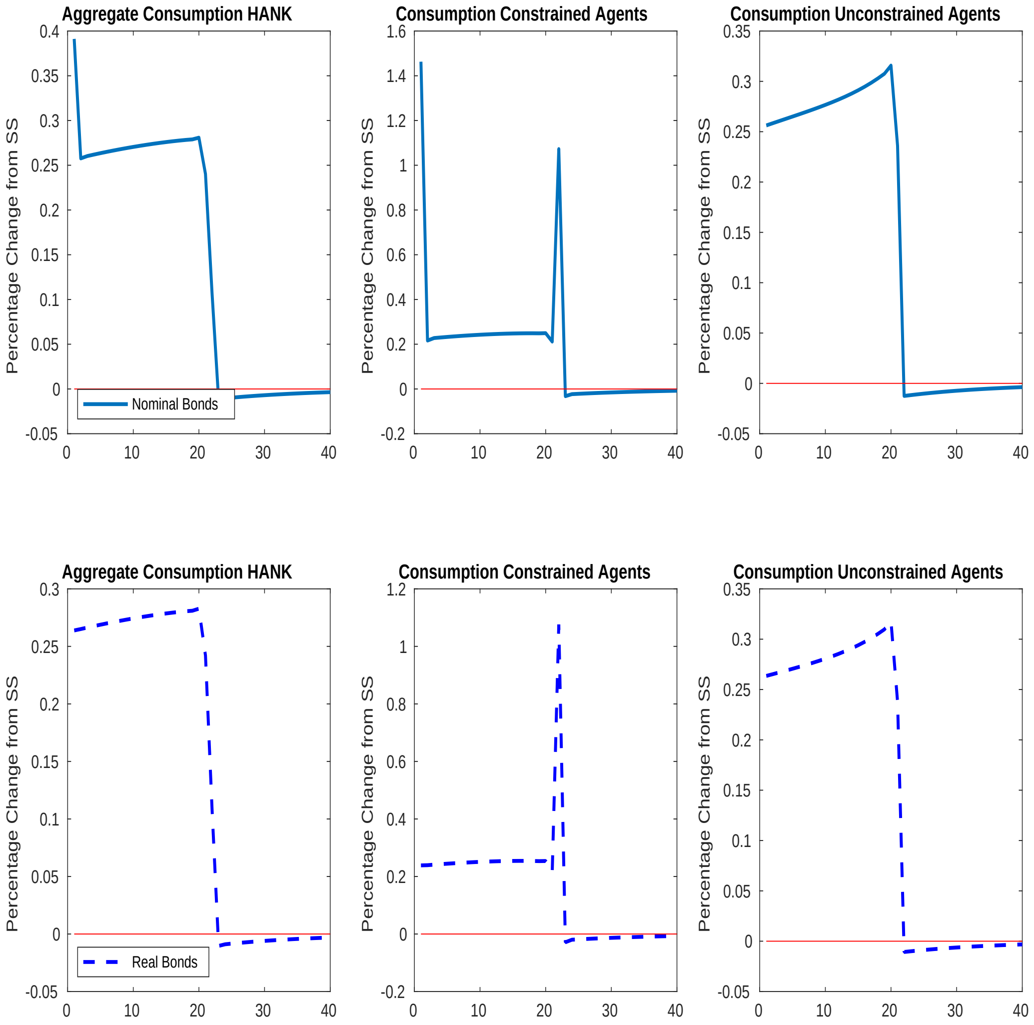

Figure 3: **Simple Forward Guidance Experiment: Initial Responses for Different Horizons of FG**

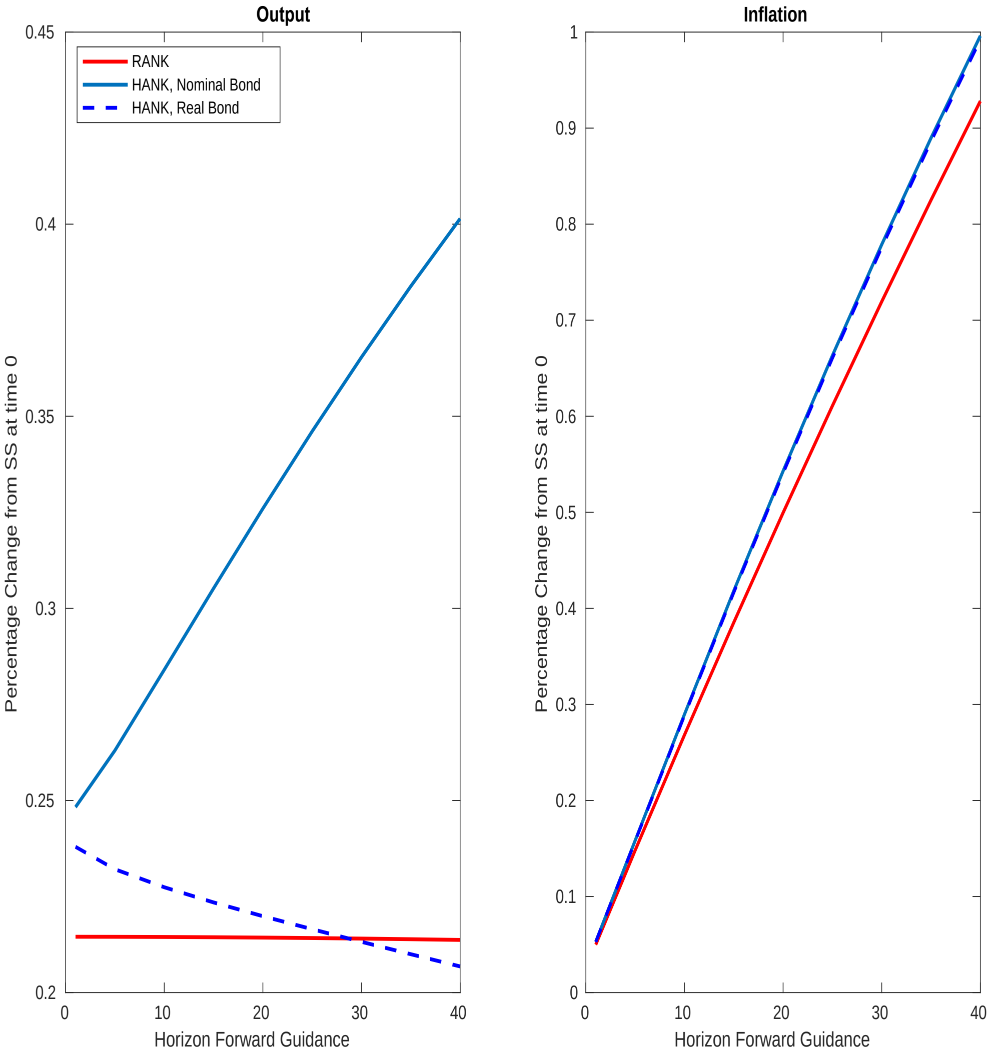

Figure 4: **Forward Guidance in a Liquidity Trap**

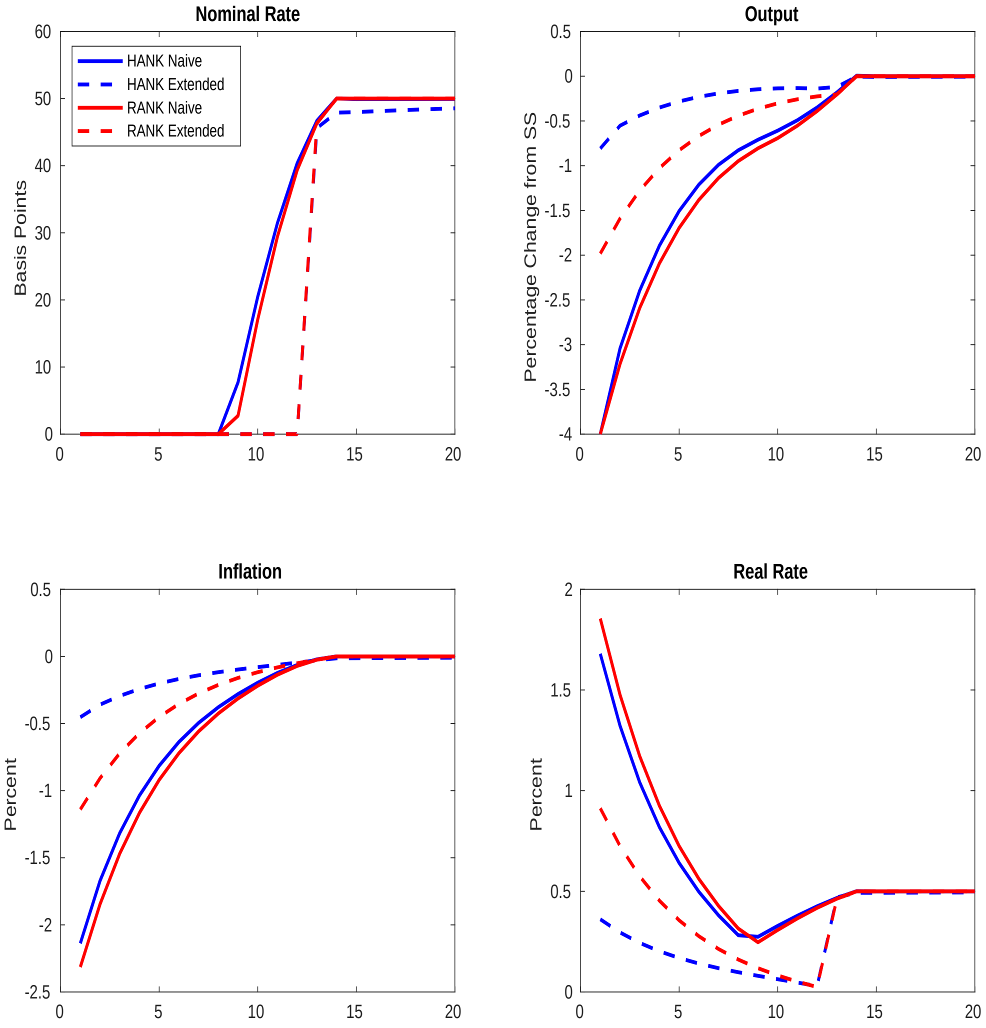

Figure 5: **Forward Guidance in a Liquidity Trap: Initial Responses for Different Horizons of FG**

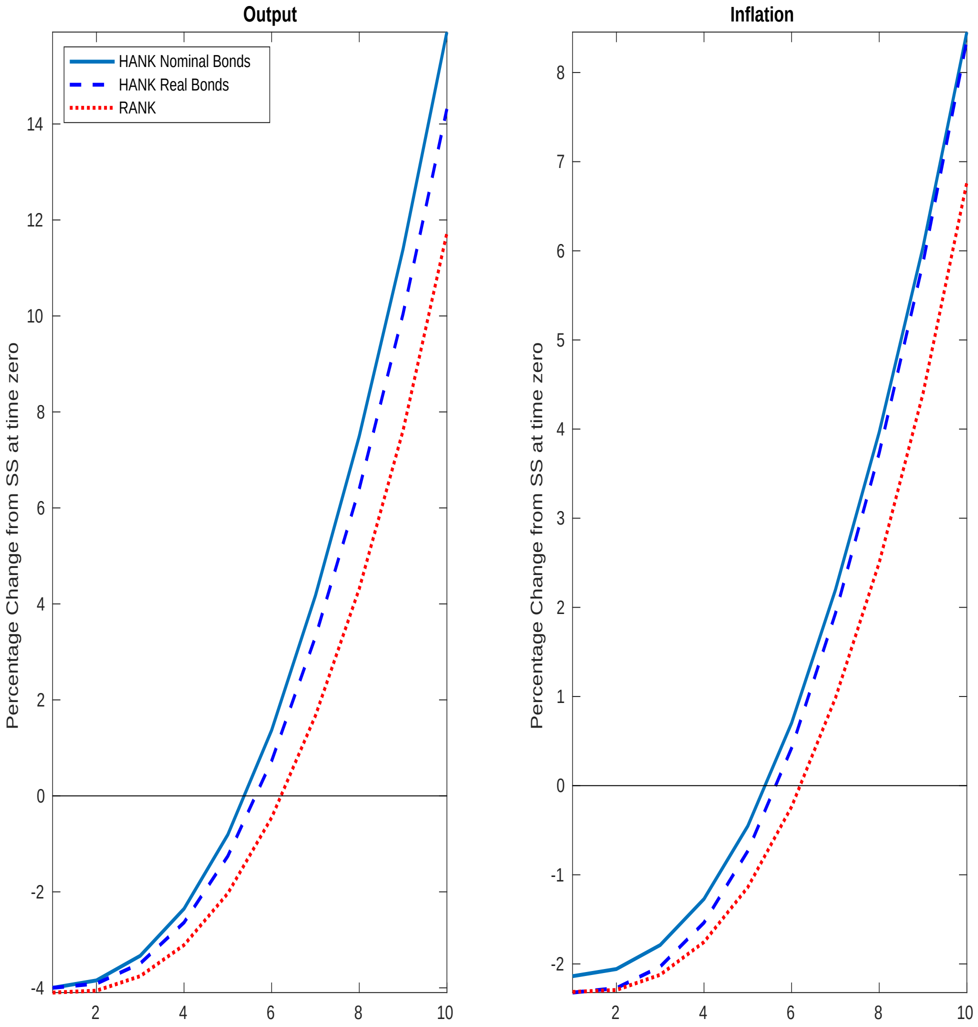

Figure A1: **Simple Forward Guidance Experiment in MNS model: Truncated History Solution**