Monetary Transmission with Segmented Markets*

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Abstract

In this paper we build a monetary model with segmented markets in which the transmission of monetary shocks to the real economy is consistent with the empirical behavior of profits shares, interest rates, and risk premium.

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1 Introduction

A common assumption in conventional monetary models (for example Gali (2015)) is that changes in nominal interest rates directly affect conditional growth rates of inflation and aggregate output. This monetary transmission mechanism is fundamentally at odds with asset market data.

For example, Canzoneri et al. (2007) uses bond market data and VAR evidence that find a delayed fall in aggregate consumption, and increases in nominal (and real) short rates, and a near-zero effect on inflation after a monetary tightening is inconsistent with an Euler equation that determines the aggregate demand in those models.

Bernanke and Kuttner (2005) use stock market and fed funds futures data to identify monetary tightening episodes. They find that dividends and stock market valuations fall after an unanticipated increase in the policy rate. Using a Campbell and Shiller (1988) type decomposition, they conclude that most of the fall in stock valuations in accounted for by a fall in aggregate earnings/dividends and an increase in the risk premia with a negligible component coming from higher real rates. In contrast to this, the conventional monetary models has dividend shares going up with fall in firm valuations accounted for more than 100% by higher real rates.

In a similar spirit and using similar methods as in Bernanke and Kuttner (2005) , Bekaert et al. (2013) examine the effect of a monetary tightening on VIX data. They find uncertainty and premium for uncertainty both respond, and are higher after a few quarters of the monetary tightening. Mumtaz and Theodoridis (2018) show evidence that volatility of output and inflation go up too. Conventional monetary models have very little to say about second moments and hence usual implementations work with log-linearized versions of the key equations.

In this paper, we use a segmented markets model in the spirit of Alvarez et al. (2001) to construct a monetary economy where the transmission of monetary shocks is consistent with the evidence from asset markets.

1.1 Basic structure

There is measure λ of traders/capitalists and measure $1-\lambda$ of non-traders/workers. Workers are simple hand-to-mouth drones subject to Cash-In-Advance(CIA) constraints:

$$\max \mathbb{E} \sum \beta^t \left[\ln C_t^N - \frac{\beta}{1+\varphi} N_t^{1+\varphi} \right]$$

s.t.

$$P_t C_t^N \leq W_{t-1} N_{t-1}.$$

So, worker makes his labor earnings and spends them all on consumption next period. On the background, there are differentiate goods, with usual structures

$$\begin{aligned} C^N &= \left[\int C^N (i)^{1-1/\varepsilon} \, di \right]^{\varepsilon/(\varepsilon-1)} \\ C^N (i) &= \left(\frac{P (i)}{P} \right)^{-\varepsilon} C^N. \end{aligned}$$

One immediate implication of this set up is that labor supply is fixed.

$$\begin{aligned} U_l (N_t) + \beta \mathbb{E}_t \frac{W_t}{P_{t+1}} U_c \left(C_{t+1}^N \right) &= 0 \\ N_t^\varphi &= \mathbb{E}_t \frac{W_t}{P_{t+1} C_{t+1}^N} \\ N_t^\varphi &= \mathbb{E}_t \frac{W_t}{W_t N_t} \\ N_t &= 1 \end{aligned}$$

Capitalists live in a complete market world but also operate subject to CIA constraint. It is without loss of generality to assume that all the trading is actually done by firms, so capitalists simply have

$$P_t C_t^T \leq D_t$$

where D_t are dividends of firms and again we have on background

$$\begin{aligned} C^T &= \left[\int C^T (i)^{1-1/\varepsilon} \, di \right]^{\varepsilon/(\varepsilon-1)} \\ C^T (i) &= \left(\frac{P (i)}{P} \right)^{-\varepsilon} C^T. \end{aligned}$$

Dividends per capitalist are given by

$$D_t = \left[P_{t-1} Y_{t-1} - W_{t-1} (1 - \lambda) N_{t-1} \right] / \lambda + \left[Q_t B_t - B_{t-1} \right]$$

The first piece is profit per capitalist, and second piece are changes in bond positions that firms hold on capitalists' behalf. [so the timing is that firms received it money at the end of the period and payed out wages, after that it traded on asset markets and gave money to capitalists as dividends]

The government budget constraint is

$$Q_t B_t - B_{t-1} = M_t - M_{t-1},$$

so that the government just prints money. It is easy to introduce additional transfers/taxes in here, and in some sense that could be a reasonable thing to do as it will allows us to match various things better.

Feasibility is

$$\lambda C_t^T + (1-\lambda) C_t^N = Y_t$$

Let

$$C = \lambda C^T + (1-\lambda) C^N$$

Now, firms that freely adjust prices solve

$$\max_p p \left(\frac{p}{P}\right)^{-\varepsilon} C - \frac{W}{A} \left(\frac{p}{P}\right)^{-\varepsilon} C$$

that gives

$$\frac{1}{p}\frac{W}{A} = \frac{\varepsilon - 1}{\varepsilon}$$

Assume that $A = 1$ and $\Phi_t \equiv \frac{\varepsilon-1}{\varepsilon}$ is an i.i.d. random variable. Then

$$P_t = \Phi_t^{-1} W_t.$$

Finally, the government follows a money supply rule

$$M_t = X_t M_{t-1}$$

where X_t is money supply shock. Our economy has two exogenous shocks, X_t and Φ_t.

1.2 Key equations

Substituting a few feasibility constraints in, we have 6 equations and 6 unknowns $(\{Y_t, C_t^T, C_t^N, W_t, P_t, N_t\})$

$$P_t = \Phi_t^{-1} W_t.$$

$$N_t = 1$$

$$Y_t = (1-\lambda) N_t.$$

$$P_{t+1} C_{t+1}^N = W_t N_t$$

$$P_{t+1} C_{t+1}^T = (P_t Y_t - (1-\lambda) W_t N_t)/\lambda + (M_{t+1} - M_t)/\lambda$$

$$\lambda C_t^T + (1-\lambda) C_t^N = Y_t$$

Two equations trivially drop out: $N_t = 1$, $Y_t = (1-\lambda)$. If we take a weighted sum of the two CIA constraints and substitution into feasibility, we get

$$P_{t+1} Y_{t+1} - P_t Y_t = M_{t+1} - M_t,$$

which implies

$$\begin{aligned} P_t Y_t &= M_t \\ (1-\lambda) P_t &= M_t. \end{aligned}$$

The CIA of trader is:

$$\begin{aligned}
\lambda P_{t+1} C_{t+1}^T &= (P_t Y_t - (1-\lambda) W_t N_t) + (X_{t+1} - 1) M_t \\
\lambda P_{t+1} C_{t+1}^T &= (1-\lambda)(P_t - W_t) + (X_{t+1} - 1) M_t \\
\lambda P_{t+1} C_{t+1}^T &= (1-\lambda) P_t \left(1 - \frac{W_t}{P_t}\right) + (X_{t+1} - 1) M_t \\
\lambda P_{t+1} C_{t+1}^T &= (1-\lambda) P_t (1 - \Phi_t) + (X_{t+1} - 1) M_t \\
P_{t+1} C_{t+1}^T &= \frac{1-\lambda}{\lambda} P_t (1 - \Phi_t) + \frac{1}{\lambda} (X_{t+1} - 1) M_t \\
C_{t+1}^T &= \frac{\frac{1-\lambda}{\lambda}(1-\Phi_t) + \frac{1}{\lambda}(X_{t+1} - 1)\frac{M_t}{P_t}}{P_{t+1}/P_t}
\end{aligned}$$

Substitute $P_t = \frac{M_t}{1-\lambda}$ to get

$$C_{t+1}^T = \frac{1-\lambda}{\lambda} \frac{(1-\Phi_t) + (X_{t+1} - 1)}{X_{t+1}}$$

or

$$C_{t+1}^T = \frac{1-\lambda}{\lambda}\left(1 - \frac{\Phi_t}{X_{t+1}}\right).$$

Throughout we focus on the simple thought experiment. Set $X_t = 1$ for all t, and consider an MIT shock: in period 0 there is an expected persistent increase in X_0, so that the new X_t follows a deterministic path, in log form

$$x_t = \rho^t x_0.$$

Mark up shocks are simply

$$\ln \Phi_t = \bar{\phi} + \sigma \epsilon_t,$$

and assume that

$$\epsilon_0 = 0.$$

Also observe that

$$\frac{\Phi}{1-\Phi} = \frac{\frac{\varepsilon-1}{\varepsilon}}{1-\frac{\varepsilon-1}{\varepsilon}} = \frac{\frac{\varepsilon-1}{\varepsilon}}{\frac{1}{\varepsilon}} = \varepsilon - 1 > 0.$$

The key observation is that this shock decreases conditional variance of relevant variables. We have

$$\begin{aligned} c_{t+1}^T &= \ln\left(\frac{1-\lambda}{\lambda}\right) + \ln\left(1 - \exp\left(\bar{\phi} + \sigma\epsilon_t - \rho^{t+1}x_0\right)\right) \\ &= \frac{1-\lambda}{\lambda}\left(1 - \frac{\bar{\Phi}}{X_{t+1}}\right) + \sigma\frac{\exp\left(\bar{\phi} - \rho^{t+1}x_0\right)}{1 - \exp\left(\bar{\phi} - \rho^{t+1}x_0\right)}\epsilon_t \\ &\quad + \frac{\sigma^2}{2}\frac{\exp\left(\bar{\phi} - \rho^{t+1}x_0\right)}{\left[1 - \exp\left(\bar{\phi} - \rho^{t+1}x_0\right)\right]^2}\epsilon_t^2 \\ &\quad + O\left(\sigma^3\right) \end{aligned}$$

therefore

$$var_{t-1}\left(c_{t+1}^T\right) = \sigma^2\frac{\exp\left(\bar{\phi} - \rho^{t+1}x_0\right)}{1 - \exp\left(\bar{\phi} - \rho^{t+1}x_0\right)}var\left(\epsilon\right) + O\left(\sigma^3\right)$$

and this object is decreasing in x_0. From here we go to all other results

Stochastic discount factor is

$$S_0^1 = \beta\left(\frac{C_1^T}{C_0^T}\right)^{-\gamma}\frac{P_0}{P_1}$$

will be used to price assets. Given what we showed before, can be written as

$$S_0^1 = \beta\left(\frac{C_1^T}{C_0^T}\right)^{-\gamma}\frac{1}{X_1}$$

In the model the risk behaves with a one period lag, so we also will be interested in

$$S_0^t = \beta \left(\frac{C_t^T}{C_0^T} \right)^{-\gamma} \frac{P_0}{P_t} = \beta \left(\frac{C_t^T}{C_0^T} \right)^{-\gamma} \frac{1}{X_t}$$

Fact 1: interest rates decrease Consider a (log) price of a t period ahead bond in period 0:

$$q_0^t = \ln \mathbb{E}_0 S_0^{t+1}$$

The effect on interest rates actually comes from the zeroth order expansion already. We have

$$\begin{aligned}
\bar{q}_0^t &= \ln \beta - \gamma \left(\bar{c}_{t+1}^T - \bar{c}_0^T \right) - \bar{x}_{t+1} \\
&= \ln \beta - \gamma \left[\ln \left(1 - \exp \left(\bar{\phi} - \rho^{t+1} x_0 \right) \right) - \ln \left(1 - \exp \left(\bar{\phi} - x_0 \right) \right) \right] - \rho^{t+1} x_0 \\
&= \ln \beta + \gamma \frac{\exp \left(\bar{\phi} \right)}{1 - \exp \left(\bar{\phi} \right)} \left(1 - \rho^{t+1} \right) x_0 - \rho^{t+1} x_0 + O \left(x_0^2 \right).
\end{aligned}$$

If ρ is sufficiently low or γ is sufficiently high, this expression is increasing in x_0, so expansionary money supply shock increases nominal price of bond/decreases nominal interest rate. This is the standard mechanism in liqudity-effect literature, such asAlvarez et al. (2001).

Fact 2: dividends/profits raise Since we have $D_t = C_t^T$ and C_t^T increases in X_0, already from the zeroth order expansion we can see that dividends go up.

Fact 3: stock market raise For now we price period 2 dividends. Their value is

$$\begin{aligned}
V_0 &= \mathbb{E}_0 S_0^2 D_2 \\
&= \mathbb{E}_0 \left(\frac{C_2^T}{C_0^T} \right)^{-\gamma} \frac{1}{X_2} C_2^T.
\end{aligned}$$

From zero order expansion we have that both $\left(\frac{C_2^T}{C_0^T} \right)^{-\gamma} \frac{1}{X_2}$ and C_2^T increase in x_0, so stock market raises.

Results 1-3 are quite mechanical. We give a transitory boost of income to capitalists via money injections to the firms, which they pay out as dividends. So profits raise, interest rates fall. The other facts will use risk and volatility.

Fact 4: excess returns go down Again, We compute excess returns on period 2 dividend stream:

$$\begin{aligned}
\text{Excess return} &\equiv \ln \frac{\mathbb{E}_0 D_2}{V_0} - \ln \frac{1}{Q_0^2} \\
&= \ln \mathbb{E}_0 C_2^T + \ln \mathbb{E}_0 S_0^2 - \ln \mathbb{E}_0 S_0^2 C_2^T .
\end{aligned}$$

Now let's do second order expansion

$$\begin{aligned}
\ln \mathbb{E}_0 \exp c_2^T &= \bar{c}_2^T + \sigma \frac{\mathbb{E}_0 \exp \bar{c}_2^T \frac{dc_2^T}{d\sigma}}{\mathbb{E}_0 \exp \bar{c}_2^T} + \frac{\sigma^2}{2} \frac{\left(\mathbb{E}_0 \exp \bar{c}_2^T \frac{d^2 c_2^T}{d\sigma^2} + \mathbb{E}_0 \exp \bar{c}_2^T \left(\frac{dc_2^T}{d\sigma} \right)^2 \right) \mathbb{E}_0 \exp \bar{c}_2^T - \left(\mathbb{E}_0 \exp \bar{c}_2^T \frac{dc_2^T}{d\sigma} \right)^2}{\left[\mathbb{E}_0 \exp \bar{c}_2^T \right]^2} \\
&= \bar{c}_2^T + \sigma \frac{\exp\left(\bar{\phi} - \rho^2 x_0\right)}{1 - \exp\left(\bar{\phi} - \rho^2 x_0\right)} \mathbb{E}_0 \epsilon_1 + \frac{\sigma^2}{2} \left(\mathbb{E}_0 \frac{d^2 c_2^T}{d\sigma^2} + \mathbb{E}_0 \left(\frac{dc_2^T}{d\sigma} \right)^2 - \left(\mathbb{E}_0 \frac{dc_2^T}{d\sigma} \right)^2 \right) \\
&= \bar{c}_2^T + \frac{\sigma^2}{2} \left(\frac{\exp\left(\bar{\phi} - \rho^2 x_0\right)}{\left[1 - \exp\left(\bar{\phi} - \rho^2 x_0\right)\right]^2} + \frac{\exp\left(\bar{\phi} - \rho^2 x_0\right)}{1 - \exp\left(\bar{\phi} - \rho^2 x_0\right)} \right) var\left(\epsilon\right)
\end{aligned}$$

Similarly

$$\begin{aligned}
\ln \mathbb{E}_0 S_0^2 &= \bar{s}_0^2 + \frac{\sigma^2}{2} \left(-\gamma \mathbb{E}_0 \frac{d^2 c_2^T}{d\sigma^2} + \gamma^2 \mathbb{E}_0 \left(\frac{dc_2^T}{d\sigma} \right)^2 \right) \\
&= \bar{s}_0^2 + \frac{\sigma^2}{2} \left(\frac{-\gamma \exp\left(\bar{\phi} - \rho^2 x_0\right)}{\left[1 - \exp\left(\bar{\phi} - \rho^2 x_0\right)\right]^2} + \frac{\gamma^2 \exp\left(\bar{\phi} - \rho^2 x_0\right)}{1 - \exp\left(\bar{\phi} - \rho^2 x_0\right)} \right) var\left(\epsilon\right)
\end{aligned}$$

and

$$\ln \mathbb{E}_0 S_0^2 C_2^T = \bar{s}_0^2 + \bar{c}_2^T + \frac{\sigma^2}{2} \left(\left(1-\gamma\right) \mathbb{E}_0 \frac{d^2 c_2^T}{d\sigma^2} + \left(1-\gamma\right)^2 \mathbb{E}_0 \left(\frac{dc_2^T}{d\sigma} \right)^2 \right) var\left(\epsilon\right)$$

Therefore

$$\begin{aligned}
\text{Excess return} &\equiv \frac{\sigma^2}{2} \mathbb{E}_0 \left(\frac{dc_2^T}{d\sigma} \right)^2 \left[\left(1+\gamma^2\right) - \left(1-\gamma\right)^2 \right] \\
&= \sigma^2 \gamma \frac{\exp\left(\bar{\phi} - \rho^2 x_0\right)}{1 - \exp\left(\bar{\phi} - \rho^2 x_0\right)} var\left(\epsilon\right)
\end{aligned}$$

Once again, high x_0 lowers $\frac{\exp\left(\bar{\phi} - \rho^2 x_0\right)}{1 - \exp\left(\bar{\phi} - \rho^2 x_0\right)}$ and therefore lowers excess returns.

Fact 5: Volatility of returns goes down In our model returns are

$$\ln \text{Return}_2 = \ln D_2 - \ln V_0 = c_2^T - \ln V_0$$

and since volatility of c_2^T goes down with monetary shock, so is conditional volatility $var_0 (\ln \text{Return}_2)$

Fact 6: Volatility of SDF goes down In our setting

$$s_0^2 = -\gamma c_2^T + \text{deterministic stuff}$$

so since $var_0 \left(c_2^T\right)$ goes down, so does $var_0 \left(s_0^2\right)$

1.3 Extensions

Effect on output One deficiency in the analysis above is that monetary policy has no affect on output. This can be fixed in several different ways. The easiest is to drop $\ln C$ assumption on the workers. If they also have curvature $1-\gamma$ on their preferences, we have

$$\begin{aligned} U_l (N_t) + \beta \mathbb{E}_t \frac{W_t}{P_{t+1}} U_c \left(C_{t+1}^N\right) &= 0 \\ N_t^{\varphi} + \mathbb{E}_t \frac{W_t}{P_{t+1} C_{t+1}^N} \left(C_{t+1}^N\right)^{1-\gamma} &= 0 \\ N_t^{\varphi} + \frac{1}{N_t} \mathbb{E}_t \left(C_{t+1}^N\right)^{1-\gamma} &= 0. \end{aligned}$$

A positive monetary shock lowers C_{t+1}^N. So N_t goes up in report to such shock if $\gamma > 1$.

Other ways to induce workers to work more is to add nominal state-unconingent transfers or investments.

Sticky prices If prices are sticky the only thing that changes in the analysis is that equation $P_t = \frac{\varepsilon}{\varepsilon - 1} W_t$ is replaced with

$$(1-\varepsilon) + \varepsilon \frac{W_t}{P_t} - \theta \left(\frac{P_t}{P_{t-1}} - 1\right) \frac{P_t}{P_{t-1}} + \theta \beta \mathbb{E}_t \frac{U'\left(C_{t+1}^T\right)}{U'\left(C_t^T\right)} \left(\frac{P_{t+1}}{P_t} - 1\right) \frac{P_{t+1}}{P_t} \frac{Y_{t+1}}{Y_t} = 0,$$

and equation for C_t^T is more involved since W_t / P_t is no longer constant. Our analysis will work as follows. In response to a positive monetary shock (higher x_0), real wage W_t / P_t

will increase, because this is what it does in NK models. It will have some offsetting effect on capitalists, but we can make this effect arbitrarily small by lowering θ. So then qualitatively we will be also hitting higher real wages without changing anything more (in fact, since real wages change so little empirically, that θ would not be arbitrary but "tightly calibrated").

Forward guidance It is easy to do forward guidance in this model, but considering what happens if Fed changes deterministic path of $\{X_t\}$ so that interest rates stay low for longer. One this is obvious is that it will not have a hugely stimulative effect in our baseline model (since nothing stimulates output there).

Investments If we put investments in, capitalists will increase investments to have higher output and hence consumption in the future. So aggregate consumption will be higher in later periods, producing the hump-shape often observed in the data.

This can potentially reconcile the model with Canzoneri et al. (2007) findings, that in response to the shock expected consumption growth and expected path of nominal interest rates go in the opposite directions.

Transfers, taxes It might be valuable to add nominal transfers to the model, and allow the government to satisfy its budget constraint through means other than seniorage. The easiest way to have lump-sum transfers for workers (or labor taxes). Adding such transfers would have several effects. First, nominal prices will react less to money injection since some of that being absorbed through taxes. This might be a desirable feature, since VAR evidence suggest that prices react very little to monetary shocks. Second, it may also be a simple way to hit evidence on term premia without having to introduce additional modeling implications.

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