Means of Payment

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Abstract

When consumers or firms purchase goods or pay bills, they must choose a means of payment: cash, credit card, check, electronic transfer, etc. What governs those choices? In particular, how do their choices vary with the inflation rate, and how do total transaction costs change?

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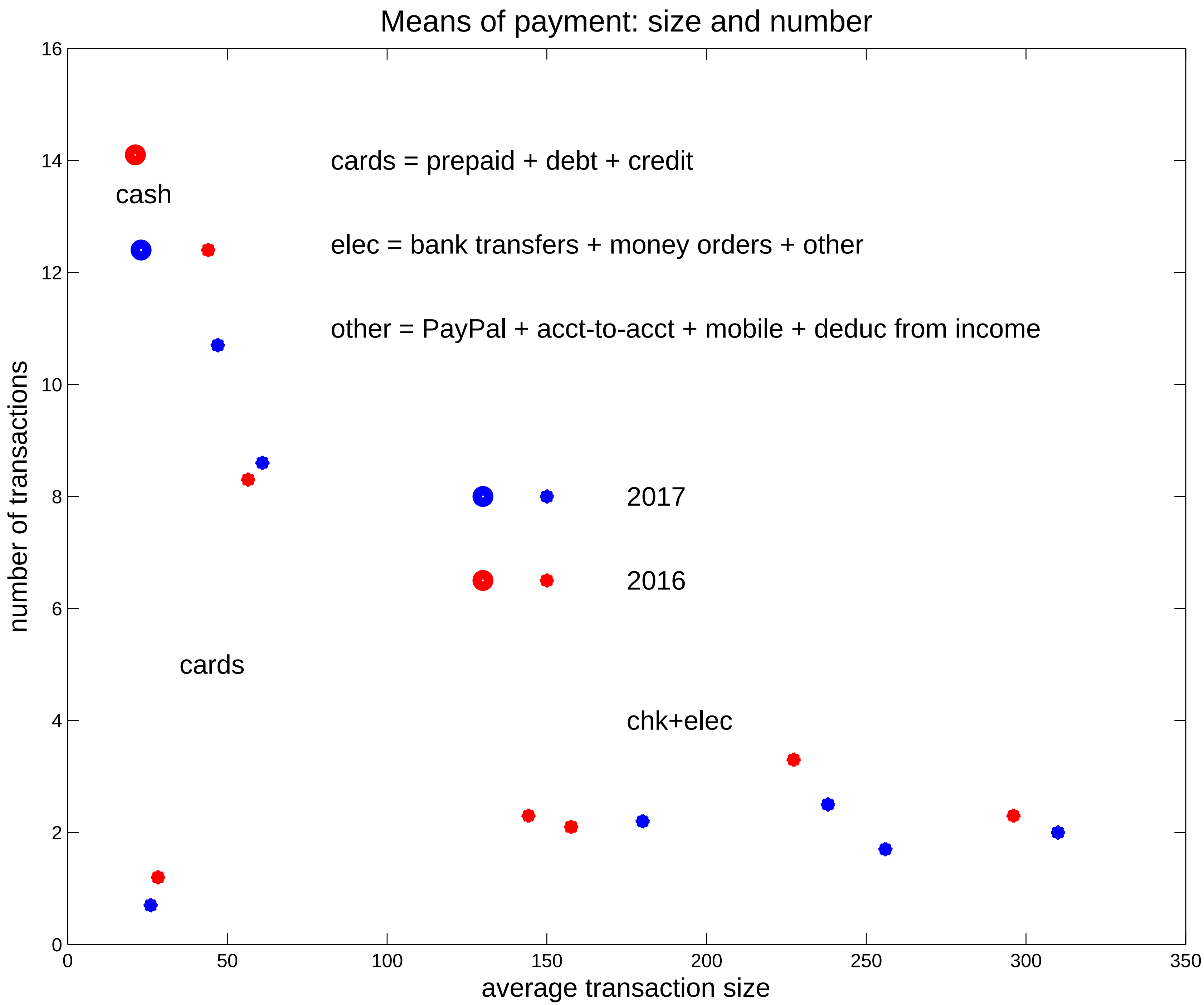
1. INTRODUCTION

When consumers purchase goods or pay bills, they must choose a means of payment: cash, credit card, check, electronic transfer, etc. What governs those choices, and how have they been affect by recent innovations in technology? For example, as credit and debit cards have become more widely available to households, more retail establishments have chosen to accept those means of payment, and for smaller purchases. At the same time, electronic debits have become commonly used for recurring payments, like utility bills, mortgage payments, and condo assessments. Figure 1 displays evidence about these choices from the Diaries of Consumer Payment Choice for 2016 and 2017.

Decisions about the means of payment have two potential channels for affecting macroeconomic aggregates. One is in determining the nominal price level. When the quantity theory of money was proposed as a model of the nominal price level, most payments were made by cash or check, and the hypothesis that the money supply—as measured by M1—was a major fact in determining the nominal price level worked well for many decades. McCandless and Weber (1995) show how well it worked, for both cross-country evidence and U.S. time series. But in the U.S., the relationship between M1 and other aggregates broke down after 1990, prompting a search for a suitable replacement.

A second channel is in the propagation of macroeconomic shocks. Non-financial firms also purchase goods and make payments, and the volume of these payments is even larger than those of households, and financial businesses make an extraordinary number and volume of payments. The financial system, the economy's "plumbing," must accommodate all of these transactions. Frictions in the payments system may play a role in exacerbating economic fluctuations. To understand how, a better framework is needed for understanding payments flows.



Means of payment: size and number
cards = prepaid + debt + credit
cash
elec = bank transfers + money orders + other
other = PayPal + acct-to-acct + mobile + deduc from income
2017
2016
cards
chk+elec
number of transactions
average transaction size
0
2
4
6
8
10
12
14
16
0
50
100
150
200
250
300
350

2. RELATED LITERATURE

The model here is a continuous-time variant of the one in Lucas and Nicolini (2015), which in turn is based on Freeman and Kydland (2000), which in turn is related to Prescott (1987).

It is also related to the classic Baumol (1952) and Tobin (1956) models of cash holding, as well as the more recent literatures on money demand, including Svensson (1985), Teles and Zhou (2005), Alvarez and Lippi (2009), and on means of payment, including Karni (1973), McCallum and Goodfriend (1987), Piazzesi and Schneider (2016).

[To be completed.]

A model without capital is described in section 3. In section 4 the steady state is characterized, and calibration to data on consumer payment choices is discussed. In section 5 the model is extended to include a production sector. Proofs and derivations are gathered in Appendix A, and the data on consumer payments choice is described in Appendix B.

3. THE MODEL WITHOUT CAPITAL

A. Overview

The economy has three types of agents: households, a (competitive) banking sector, and a (passive) monetary authority. There is a single consumption good, which is nonstorable and is produced using labor as the only input. There are two assets, cash and nominal government debt. Purchases of goods can be carried out by three means of payment: cash (c), cards (d) and electronic transfers (e). Cash purchases have no time cost, but are subject to a standard CIA constraint. Purchases with cards and electronically entail a time cost as well as a payment. In addition, the payment has cash component, which represents a period of time when funds must be held in an account that pays less interest than government debt. To economize on notation, this fact is represented by requiring the household to finance a fraction of card and electronic payments with cash. The share of purchases carried out with each means of payment is chosen by the household.

The government collects a (real) lump-sum tax on households, pays interest on its outstanding debt, and uses open market operations to increase or decrease the money supply.

Households supply labor and manage their assets to carry out transactions. As in the classic Baumol-Tobin model, the household makes periodic trips to the bank to make cash withdrawals, which it uses to finance the cash portion of its purchases. As a household makes purchases, it uses both its cash and its bank balance as payments. At the same time, both the cash and the non-cash payments it receives as a seller flow into the bank, where they accrue as wage payments. Thus, the household's wage, net of any tax levied, flows into its bank account, and the non-cash portion of its expenditure on consumption flows out.

The bank holds government bonds, acts as a clearing house for cash, and records non-cash receipts and payments. It receives and records cash and non-cash inflows from sellers of goods, and records wage payments net of any tax levied. When a household replenishes its cash supply, it makes a withdrawal from its account. The bank is also the government's (only) counterparty for open market operations.

If the money supply is increasing over time, the inflation rate is positive. In this event households are making ever-larger cash withdrawals when they make trips to the bank, thereby soaking up the new cash injections.

B. Households: payments method

First consider the household's decision about how to carry out transactions. As in Freeman and Kydland (2000) and Lucas and Nicolini (2015), consumption requires a continuum of purchases of various sizes, where for simplicity the relative size distribution of purchases $F(z)$ is fixed and exogenous. Assume F is continuously differentiable, with density $f(z)$, and define

$$\Omega(z) \equiv \int_0^z \zeta f(\zeta) d\zeta, \qquad \text{all } z \geq 0.$$

Then $\Omega(z)$ is the total **volume** of transactions of relative size no greater than z. Define $\nu \equiv \lim_{z\to\infty} \Omega(z)$.

Higher or lower levels of consumption require proportionately larger or smaller transactions of each relative size. That is, all adjustments in the consumption level appear on the *intensive* margin in terms of transactions. Thus, real consumption $c > 0$ scales the *absolute size* of each transaction by c, so the total transaction *volume* is $c\nu$. The total number of transactions of each type does not vary with c. With this convention, changes in c can be interpreted as changes in the quality of the purchased bundle: a bigger apartment, better car, better quality clothing, food, etc. In this respect the model here differs from FK and LN, who assume that c scales the number of transactions of each size.

As noted above, there are three means of payment: cash, cards, and electronic. Cash transactions have no time cost, but it can be lost or stolen. Let $\eta \geq 0$ denote the rate at which these losses occur. Cards and electronic payments entail no such losses, but they nevertheless have two costs components. In particular, they have time costs k^d and k^e, respectively, paid per *transaction,* and in addition each requires a fraction of the transaction *value* to be held as cash. These amounts represent reserve requirements on these transaction modes. Let θ^d and θ^e be those fractions, and define $\theta^c \equiv 1/(1-\eta)$. We will assume throughout that $0 < k^d < k^e$ and

$1 \geq \theta^c > \theta^d > \theta^e \geq 0$.

Clearly, efficiency requires that small transactions be made in cash, middle-size transactions with cards, and large transactions electronically. Hence the consumer chooses two thresholds, γ and δ, that define the maximum size transactions carried out with cash and cards. Define

$$\begin{aligned} L(\gamma, \delta) &\equiv \Omega(\gamma)\theta^c + [\Omega(\delta) - \Omega(\gamma)]\, \theta^d + [\nu - \Omega(\delta)]\, \theta^e, \qquad (1) \\ T(\gamma, \delta) &\equiv F(\gamma) \cdot 0 + [F(\delta) - F(\gamma)]\, k^d + [1 - F(\delta)]\, k^e, \end{aligned}$$

as the liquidity and time required for transactions, for consumption $c = 1$. For higher or lower consumption levels c, the volume of transactions is scaled proportionately, so the required liquidity is cL and the non-cash volume of transactions is $c\,[\nu - L(\gamma, \delta)]$. The time cost T, which involves the *number* of transaction of each type does not change with c. The derivatives of L and T are

$$\begin{aligned} L_\gamma(\gamma, \delta) &= \gamma f(\gamma) \left(\theta^c - \theta^d\right) > 0, \qquad T_\gamma(\gamma, \delta) = -f(\gamma) k^d < 0, \\ L_\delta(\gamma, \delta) &= \delta f(\delta) \left(\theta^d - \theta^e\right) > 0, \qquad T_\delta(\gamma, \delta) = -f(\delta) \left(k^e - k^d\right) < 0. \end{aligned}$$

An increase in γ or δ raises the required liquidity and reduces the transaction time.

C. Households: consumption, bank withdrawals

Time is continuous. Let π denote the (constant) rate of money growth, and suppose that the price level also grows at that rate,

$$P(t) = e^{\pi t} P_0, \qquad t \geq 0,$$

where P_0 is the nominal price level at $t = 0$. We will focus on steady states, equilibria where all real variables are constant and all nominal variables grow at the constant rate π.

Suppose a household begins at date $t = 0$ with nominal wealth $A_0 > 0$ but no cash. The household a has a endowment of one unit of time, a flow, which it allocates to work and transactions. Let w denote its real wage rate per unit of time spent working, and $\phi > 0$ the lump-sum time cost of making a cash withdrawal. At $t = 0$ the household makes a cash withdrawal $M_0 > 0$, pays the value $P_0 w\phi$ of the fixed time cost for its withdrawal, and plans the date $\tau > 0$ for its next withdrawal. The household also chooses a (flow) consumption rate c, and a purchasing strategy (γ, δ), both of which are assumed to be constant until date τ.

At $t = 0$ its post-withdrawal nominal bank balance is

$$B_0 = A_0 - M_0 - P_0 w\phi. \tag{2}$$

Over the period until its next trip to the bank, this balance grows due to receipts from the household's flow of labor income, less a (constant, real) tax t_x and withdrawals for the non-cash portion of its purchases. Let

$$z_0 \equiv w(1-T) - t_x - c(\nu - L), \tag{3}$$

denote this inflow, in real terms. Let i denote the nominal interest rate, and $r = i - \pi$ the real interest rate. Over time the price level grows at the rate π and the bank balance accrues interest at the rate i. Hence the bank balance grows like

$$\dot{B}(s) = iB(s) + e^{\pi s} P_0 z_0, \qquad s \geq 0,$$

and s units of time after a trip to the bank it is

$$\begin{aligned} B(s) &= e^{is} B_0 + \int_0^s e^{i(s-v)} e^{\pi v} P_0 z_0 dv \\ &= e^{is}\left(B_0 + P_0 z_0 \frac{1 - e^{-rs}}{r}\right), \qquad s \in [0, \tau]. \end{aligned} \tag{4}$$

Similarly, the household's cash at date s is

$$M(s; 0) = M_0 - P_0 cL \frac{e^{\pi s} - 1}{\pi}, \qquad s \in [0, \tau]. \tag{5}$$

At date $s = \tau$, the beginning of the following transaction period, any unspent cash is added to the household's bank balance, and the household's total nominal assets are

$$\begin{aligned} A' &= \left[M_0 - P_0 c L(\gamma, \delta) \frac{1}{\pi} \left(e^{\pi\tau} - 1 \right) \right] + e^{i\tau} \left(A_0 - M_0 - P_0 w \phi \right) \\ &\quad + \frac{e^{i\tau} - e^{\pi\tau}}{i - \pi} \left\{ P_0 \left[w \left(1 - T \right) - t_x - c \left(\nu - L \right) \right] \right\}. \end{aligned} \tag{6}$$

Let $u(\cdot)$ denote the household's utility function, and $\rho > 0$ its rate of time preference. Then the Bellman equation for its decision problem is

$$V(A_0; P_0) = \max_{M_0, \tau, c, \gamma, \delta} \left[u(c) \frac{1}{\rho} \left(1 - e^{-\rho\tau} \right) + e^{-\rho\tau} V(A'; e^{\pi\tau} P_0) \right] \tag{7}$$

$$\text{s.t.} \qquad P_0 c L(\gamma, \delta) \frac{1}{\pi} \left(e^{\pi\tau} - 1 \right) - M_0 \leq 0, \tag{8}$$

where A' is as in (6) and (8) is the cash constraint. Let μ be the multiplier on the cash constraint.

Note that a household receives its income flow as a direct deposit to its bank account, and is never constrained by the fact that a "paycheck" will not arrive until later. Thus, the specification here precludes "hand-to-mouth" households, who live paycheck to paycheck.

We begin with a preliminary result.

Lemma 1: If $P(t) = e^{\pi t} P_0$, all t, then the value function $V(A; P)$ is h.o.d. zero.

The first order condition for M_0 is

$$\mu = e^{-\rho\tau} V_A(A'; e^{\pi\tau} P_0) \left(e^{i\tau} - 1 \right), \tag{9}$$

and μ is the marginal cost of holding cash.

Using (9), the first order conditions for (γ, δ) are

$$\begin{aligned} \gamma &= \frac{w}{c} \frac{k^d - 0}{\theta^c - \theta^d} \left[\frac{i - \pi}{\pi} \frac{e^{\pi\tau} - 1}{1 - e^{-(i-\pi)\tau}} - 1 \right]^{-1}, \\ \delta &= \frac{w}{c} \frac{k^e - k^d}{\theta^d - \theta^e} \left[\frac{i - \pi}{\pi} \frac{e^{\pi\tau} - 1}{1 - e^{-(i-\pi)\tau}} - 1 \right]^{-1}. \end{aligned} \tag{10}$$

The two thresholds depend on the ratio w/c, the interest and inflation rates i, π, and the length of the transaction period τ. Both thresholds are increasing in w/c: a higher cost of time relative to consumption encourages the use of transaction methods that are more cash-intensive and economize on time use. With $i - \pi = r$ constant, a higher inflation rate has the opposite effect, encouraging transaction methods that are less cash-intensive. For $i > \pi$, they are also decreasing in τ: a longer transaction period raises the cost of using cash.

The following assumption is maintained throughout.

ASSUMPTION 1: (a) $\rho > 0$ and $\rho + \pi > 0$;

(b) the transaction parameters satisfy

$$\begin{aligned} 0 &< k^d < k^e, \\ 1 &\geq \theta^c > \theta^d > \theta^e \geq 0; \end{aligned}$$

and

$$\frac{k^d - 0}{k^e - k^d} < \frac{\theta^c - \theta^d}{\theta^d - \theta^e}. \tag{11}$$

LEMMA 2: If $i > \pi$ and (11) holds, the solutions for γ and δ in (10) are strictly positive, with $0 < \gamma < \delta$.

Define

$$K_\gamma \equiv \frac{k^d - 0}{\theta^c - \theta^d}, \qquad K_\delta \equiv \frac{k^e - k^d}{\theta^d - \theta^e},$$

and note that Ass. 1 implies $K_\gamma < K_\delta$. From (10), clearly $\delta = \gamma K_\delta / K_\gamma$. As usual, we will find below that in steady state $r = \rho$, so the first part of (a) in Assumption 1 insures $i > \pi$.

Using (9) again, the first order condition for c is

$$u'(c)\frac{1}{\rho}\left(1 - e^{-\rho\tau}\right) = e^{-\rho\tau} V_A(A'; e^{\pi\tau} P_0) P_0 \tag{12}$$

$$\times \left[L \frac{e^{\pi\tau} - 1}{\pi} e^{i\tau} + (\nu - L) \frac{e^{i\tau} - e^{\pi\tau}}{i - \pi} \right],$$

which equates the marginal utility of consumption with its marginal cost: the value of the cash needed to carry out the transactions, plus the non-cash payment. Since a change in c does not change the time required for transacting, the time cost does not appear.

The first order condition for τ can be written as

$$\begin{aligned} u(c) - \rho V &= V_A \cdot \left[\left(e^{i\tau} - 1 \right) e^{\pi\tau} P_0 c L + \pi M_0 + P_0 c L \right. \\ &\quad \left. + (\pi - i) e^{i\tau} B_0 - e^{i\tau} P_0 z_0 \right]. \end{aligned} \tag{13}$$

The term on the left is the direct effect of increasing τ: getting utility $u(c)$ longer and receiving the continuation value ρV later. The term on the right is the marginal value of assets multiplied by the change in end-of-period (EoP) assets—additional erosion of cash holdings from inflation, plus the cost of the extra cash required to finance transactions over the longer period, adjusted for additional interest on beginning-of-period (BoP) assets and a longer flow of earnings and transfers less consumption.

The envelope condition is

$$V_A(A_0; P_0) = e^{(i-\rho)\tau} V_A(A'; e^{\pi\tau} P_0). \tag{14}$$

D. Other equilibrium conditions

The government gets revenue from the tax and from new issues of money and debt, and its only expense is interest on the outstanding debt. Hence the government budget constraint is

$$\dot{M}^S(t) + \dot{B}^S(t) + P(t) t_x = i B^S(t), \tag{15}$$

where $B^S(t)$ is the supply of government bonds.

There are also market clearing conditions for goods, cash and nominal debt,

$$\nu \overline{c} \;=\; w \left(1 - \overline{T} - \frac{\phi}{\tau} \right), \tag{16}$$

$$\overline{M}(t) = M^s(t),$$
$$\overline{B}(t) = B^s(t), \qquad \text{all } t,$$

where $\overline{c}(t)$ and $\overline{T}$ are the average levels of consumptions and time spent transacting across households, and $\overline{M}(t)$ and $\overline{B}(t)$ are average holdings of cash and bonds.

For bonds, B_0 is as in (2), and the household's bond holding s units of time after a trip to the bank is as in (4). It follows immediately that in a steady state with inflation rate π, the average across households at t is

$$\overline{B}(t) = e^{\pi t}\frac{1}{\tau}\int_0^{\tau} e^{-\pi s}B(s,0)ds.$$

Similarly, average across households must equal the money supply. But here the market clearing condition determines M_0. The cash held by a household s units of time after a trip to the bank is as in (5). Hence market clearing for cash at date t requires

$$\begin{aligned} M_0^s e^{\pi t} &= \overline{M}(t) = e^{\pi t}\frac{1}{\tau}\int_0^{\tau} e^{-\pi s}M(s,0)ds \\ &= e^{\pi t}\frac{1}{\tau}\int_0^{\tau} e^{-\pi s}\left[M_0 - P_0 cL\frac{e^{\pi s}-1}{\pi}\right]ds. \end{aligned}$$

4. THE STEADY STATE

We will look at steady states of economies that have a common, constant rate of growth π for money and nominal bonds, and where households have uniformly distributed dates for trips to the bank. In steady state the real variables $c^{ss}, \tau^{ss}, \gamma^{ss}, \delta^{ss}, \mu^{ss}$, are constant, as well as the normalized variables $p^{ss} \equiv P/M^S$, $m_0^{ss} \equiv M_0/M^S$, $a^{ss} \equiv A/M^S$, etc. The flow of households into the bank for withdrawals is constant at $1/\tau^{ss}$, and all have the consumption flow c^{ss}, all incur the (flow) time cost T^{ss} for purchases, and all have cash and non-cash expenditure flows $c^{ss}L^{ss}$ and $c^{ss}\left(\nu - L^{ss}\right)$. Note that the normalized money supply per household is unity, $m^S = 1$, and that m_0^{ss}

is the initial cash balance of a household, relative to the money supply per person, just after making a withdrawal.

A. Conditions for a steady state

A steady state consists of levels for the real variables ($c^{ss}, \tau^{ss}, \gamma^{ss}, \delta^{ss}, p^{ss}, m_0^{ss}$, $a_0^{ss}, i^{ss}, r^{ss}, \overline{b}^{ss}$). For notational convenience we will drop the superscripts.

In steady state the nominal and real interest rates are

$$\begin{aligned} i &= \rho + \pi, \\ r &= \rho, \end{aligned} \tag{17}$$

and (10) determines γ, δ as functions τ, c.

To determine $\left(c, \tau, p, m_0, a_0, \overline{b}\right)$, we have (4), (6), (8), (13), the government budget constraint, and the market clearing conditions for goods and bonds. By Walras' Law one condition is redundant.

Each cash withdrawal incurs the (lump-sum) time cost ϕ. Thus, with uniformly distributed trips to the bank, market clearing for goods requires

$$\nu c = w\left(1 - T - \frac{\phi}{\tau}\right). \tag{18}$$

Under Assumption 1 $i > 0$, so the cash constraint (8) binds and

$$pcL = \frac{\pi m_0}{e^{\pi\tau} - 1}. \tag{19}$$

In steady state $A' = e^{\pi\tau} A_0$, so (6) implies

$$\begin{aligned} a_0 &= \frac{m_0 + p_0 w\phi}{1 - e^{-r\tau}} - \frac{1}{r}\left\{p\left[w\left(1 - T\right) - t_x - c\left(\nu - L\right)\right]\right\} \\ &= m_0\left(\frac{1}{1 - e^{-r\tau}} - \frac{\pi}{r}\frac{1}{e^{\pi\tau} - 1}\right) + pw\phi\left(\frac{1}{1 - e^{-r\tau}} - \frac{1}{r\tau}\right) + \frac{pt_x}{r}, \end{aligned} \tag{20}$$

where the second line uses (18) and (19). The first order condition for τ, (13), can be written as

$$a_0 = \left(1 + \frac{\pi}{r}\right) m_0 + \left(1 - \frac{1}{r\tau}\right) pw\phi + \frac{1}{r} pt_x. \tag{21}$$

Market clearing for cash, $\overline{m} = 1$, requires

$$m_0 = \begin{cases} \left[1/\pi\tau - 1/\left(e^{\pi\tau} - 1\right)\right]^{-1}, & \text{if } \pi \neq 0, \\ 2, & \text{if } \pi = 0, \end{cases} \tag{22}$$

so m_0 depends only on π and τ. The average bond holding $\overline{b}$ across households is

$$\overline{b} = \frac{\left(e^{r\tau} - 1\right)/r\tau - e^{-\pi\tau}}{\left(1 - e^{-\pi\tau}\right)/\pi}\frac{1}{r}m_0 - pw\phi\frac{1}{r\tau} + \frac{1}{r}pt_x. \tag{23}$$

The government budget constraint is

$$rb^S = \pi + pt_x.$$

Here we will assume that the tax is zero, $t_x = 0$, and that the government has just enough debt so that seigniorage—revenue from money creation—finances the required interest, $b^S = \pi/r$. Then market clearing for bonds, $b^S = \overline{b}$, requires

$$\frac{\pi}{r} = \frac{\left(e^{r\tau} - 1\right)/r\tau - e^{-\pi\tau}}{\left(1 - e^{-\pi\tau}\right)/\pi}\frac{1}{r}m_0 - pw\phi\frac{1}{r\tau}. \tag{24}$$

Then (18)-(23) provide six equations for $\left(c, \tau, p, m_0, a_0, \overline{b}\right)$, and (24) can be used as a check (Walras' Law).

B. Solving the model

To determine (γ, δ, τ), (18)-(21) can be combined to get one condition,

$$e^{\pi\tau}\left(\frac{e^{r\tau} - 1}{r} - \frac{1 - e^{-\pi\tau}}{\pi}\right)\frac{1}{\nu} = \frac{\phi}{L}\frac{1}{1 - T - \phi/\tau}. \tag{25}$$

Given $L(\gamma, \delta)$ and $T(\gamma, \delta)$, (25) determines τ. Two more conditions are in (10),

$$\left[\frac{\left(e^{\pi\tau} - 1\right)/\pi}{\left(1 - e^{-r\tau}\right)/r} - 1\right]\frac{\gamma}{\nu} = \frac{K_\gamma}{1 - T - \phi/\tau}, \tag{26}$$

$$\delta = \gamma K_\delta / K_\gamma. \tag{27}$$

Define

$$\hat{L}(\gamma) \equiv L(\gamma, \gamma K_\delta / K_\gamma),$$
$$\hat{T}(\gamma) \equiv T(\gamma, \gamma K_\delta / K_\gamma),$$

with $\hat{L}' > 0$, $\hat{T}' < 0$, and write (25) and (26) as

$$0 = \frac{e^{\pi\tau}}{\nu}\left(\frac{e^{r\tau}-1}{r} - \frac{1-e^{-\pi\tau}}{\pi}\right) - \frac{\phi}{\hat{L}(\gamma)}\frac{1}{1-\hat{T}(\gamma)-\phi/\tau}, \tag{28}$$

$$0 = \frac{\gamma}{\nu}\left[\frac{r\left(e^{\pi\tau}-1\right)}{\pi\left(1-e^{-r\tau}\right)} - 1\right] - \frac{K_\gamma}{1-\hat{T}(\gamma)-\phi/\tau}. \tag{29}$$

Then (28)-(29) determine (τ, γ) and (27) determines δ.

Lemma 3: There exists a unique pair (τ, γ) satisfying (28) and (29).

Note that since w and c do not appear in (28) and (29), the choice of (τ, γ, δ) would be the same across heterogeneous households with different wage rates w.

Given (τ, γ, δ), (18) and (22) determine c and m_0, (19) determines p, (20) or (21) determines a_0, and (23) determines $\bar{b}$. The following is then immediate.

Proposition 1: A steady state exists and it is unique.

C. Calibration

For preferences, standard conventions from the macro literature can be used: $u(c) = c^{1-\sigma}/\left(1-\sigma\right)$, with $\sigma = 2$, and $\rho = 0.04$.

The rate of inflation π is the single policy variable.

The transaction parameters θ^d, θ^e, k^d, k^e, are more difficult, as there is little direct evidence on any of them. The values θ^d, θ^e, represent the fraction of time that money (non-interest bearing) must be held to finance a unit of purchases with DD's and MMDA's, while k^d, k^e, represent the time costs.

However, there is some evidence from transactions data about the thresholds γ, δ, as well as τ, the length of time between trips to the bank.

Given values for γ, δ, τ, and for the average inflation rate π during the relevant period, the model can be used in either of two ways. Plausible guesses can be made for θ^d, θ^e, and the model—eq. (10) and (21)— can be used to back out values for k^d, k^e; or the roles of θ^d, θ^e and k^d, k^e can be reversed.

Either way, the resulting calibration can then be used to calculate the response of payments choices to changes in the inflation rate.

The Diaries of Consumer Payment Choice (DCPC) provides data on the shares of transactions—both number and volume—by various means of payment. These data come from surveys in 2015, 2016, and 2017, of about 2000 consumers during the month of October.[1] Respondents were asked to record each purchase by size and means of payment:

> cash; prepaid, debit and credit cards; checks; bank account number pay (BANP); online banking bill pay (OBBP); money order; and other (PayPal, account-to-account transfers, mobile payments, and deductions from income).

Figure 1 shows the relationship between average size (value) and number of transactions of each type for 2016 and 2017. The data clearly display an inverse relationship between size and number. They also show that the average transaction size differs across different means of payment. More precisely, the transactions fall into two groups, with cash and cards used for small transactions, and checks and electronic payments used for large ones. Thus, the data suggest breaking non-cash payments into those made by card, which are of modest size, and those made by check and by various electronic means, which are substantially larger. Table 1 show the figures, aggregated this way.

[1] There was also a survey in 2012, which involved a smaller number of consumers and covered a slightly different time window. The data are broadly similar, but reported a higher share for cash and lower for cards and electronic payments.

Table 1: Share of transaction (%) by various means of payment

		cash	cards	chk, elec.
number	2017	30.3	48.9	20.8
	2016	30.6	47.5	21.9
	2015	32.0	48.0	20.0
value	2017	8.4	30.7	60.9
	2016	8.6	30.2	61.2
	2015	9.0	34.0	57.0

Figure 1 also suggests an exponential distribution can be used. Since the units of z are arbitrary, it is harmless (and convenient) to choose $\alpha = 1$ as the single parameter. Then $\nu = 1$, and

$$\begin{aligned} 1 - F(z) &= e^{-z}, \\ 1 - \Omega(z) &= e^{-z}(1+z), \qquad \text{all } z. \end{aligned}$$

so

$$z = \frac{F(z) - \Omega(z)}{1 - F(z)}, \qquad \text{all } z.$$

Using the figures in Table 1, we find that the values for γ and δ are

$$\begin{aligned} \gamma &= (0.315,\ 0.316,\ 0.36), \\ \delta &= (1.93,\ 1.79,\ 2.28). \end{aligned}$$

so the cutoffs are similar in all three years.

5. THE MODEL WITH PRODUCTION

The model can be extended to include productive capital and (non-financial) firms. The economy then has four types of agents: households, firms, banks and a government. It has one type of output and three assets: physical capital, interest-bearing government debt, and cash.

Households hold capital and government debt as stores of value, and they hold cash to carry out transactions. They supply capital and labor inelastically and make consumption-savings decision. The household's decision problem is unchanged, except that A and B are now interpreted as total wealth. There are no shocks to the production technology, so physical capital and government debt are both safe assets, and r is their common (real) rate of return.

The government issues money and interest-bearing debt, and uses lump sum taxes or transfers to balance its budget each period.

As before banks, which are also perfectly competitive, record transactions, act as a clearing-house for cash, and are the single counterparty of the government in open-market transactions.

We turn next to firms.

A. Firms

Firms, which are perfectly competitive, hire capital and labor to produce output, and hold cash to carry out transactions. Their decision problem about means of payment is similar to the household's. For simplicity assume that they face the same costs $\left(k^d, k^e, \theta^c, \theta^d, \theta^e\right)$.

However, their distribution and volume of transactions is different, for many reasons. Goods go through several stage of production before they are offered for sale as final goods, leading to a larger overall volume of transactions. Because firms are large, they may have a larger share of large transactions. Finally, retail firms must hold stocks of cash to carry out transactions with households.

To capture these three effects, we will make two adjustments. First, we will assume that firms face a different distribution of transaction sizes, call it F_f with density f_f.

Define

$$\Omega_f(z) \equiv \int_0^z \zeta f_f(\zeta) d\zeta, \qquad \text{all } z \geq 0,$$

so $\Omega_f(z)$ is the total volume of transactions of size no greater than z, and define $\nu_f \equiv \lim_{z\to\infty} \Omega_f(z)$. Second, we will assume that the production and sale of a unit of final goods requires carrying out μ transactions with total volume $\mu\nu_f$, so $\mu > 0$ scales both the total number *and* total volume of transactions. The transaction period τ_f for firms will also be different from the one for households, but the time cost for their transactions is the wage rate w, as for consumers. Let χ_f denote the total cost of carrying out transactions per unit of final goods.

The production function

$$y = F(k, \ell) = \left[\hat{F}(k, \ell) - \delta k\right],$$

has constant returns and is defined net of depreciation on capital. Since transaction costs use up the share χ_f of final output, the real return on capital and the real wage are

$$\begin{aligned} r(\kappa) &= (1 - \chi_f)\, F_k(\kappa, 1), \qquad (30) \\ w(\kappa) &= (1 - \chi_f)\, F_\ell(\kappa, 1), \end{aligned}$$

where $\kappa = k/\ell$ is the aggregate capital/labor ratio. Factor payments exhaust net output, so there are no profits.

B. Market clearing

Market clearing for goods requires

$$\begin{aligned} c + \dot{k} &= (1 - \chi_{fk})\, F(k, \ell) \\ &= w\left(1 - T - \frac{\phi}{\tau}\right) + rk, \end{aligned}$$

where the left side is total demand, including net additions to the capital stock, and the right side is the supply of goods, net of real transaction costs incurred by firms. In steady state $\dot{k} = 0$.

In steady state, market clearing for cash and interest-bearing assets requires

$$\begin{aligned} M^S &= \overline{M}_h + \overline{M}_f, \\ B_g^S + Pk &= \overline{B}_h + \overline{B}_f, \end{aligned}$$

where $\overline{M}_h$, $\overline{M}_f$ denote average cash balances of households and firms, $\overline{B}_h$, $\overline{B}_f$ denote their average interest-bearing assets, and B_g^S is government debt.

C. Steady state

The existence and uniqueness of a steady state can be established as before. Data on payments and/or transactions can be used to calibrate the distribution F_f. The assets held by households and firms can be used to back out balances that could be interpreted as demand deposits. Data on cash and demand deposits can also be incorporated as a cross-check, as well as data on B-to-B transaction.

The most obvious application of the model is to ask how the choices of housholds and firms vary with the inflation rate π, and how total transaction costs change.

APPENDIX A: PROOFS AND DERIVATIONS

Proof of Lemma 1: Clearly the constraints in (6) and (8) are h.o.d. zero in (P_0, A_0, M_0, A'). Thus, if $[M_0, \tau, c, \gamma, \delta]$ is feasible for initial assets and price level (A_0, P_0) and leads to assets A' at the end of the transaction period, then for initial assets and price level $(\Lambda A_0, \Lambda P_0)$, the choice $[\Lambda M_0, \tau, c, \gamma, \delta]$ is feasible and leads to assets $\Lambda A'$ at the end of the period. Hence the two problems have the same solution for $[\tau, c, \gamma, \delta]$, and initial cash M_0 and final assets A' are scaled like P_0. Since consumption is unchanged, $V(\Lambda A_0, \Lambda P_0) = V(A_0, P_0)$, all Λ, all (A_0, P_0), as claimed. ■

Derivation of (10): Using (9), the first order condition for γ is

$$\begin{aligned} 0 &= -P_0 c L_\gamma \frac{e^{\pi\tau}-1}{\pi} + e^{i\tau} P_0 \left[-wT_\gamma + cL_\gamma\right] \frac{1-e^{-(i-\pi)\tau}}{i-\pi} - \left(e^{i\tau}-1\right) P_0 c L_\gamma \frac{e^{\pi\tau}-1}{\pi} \\ &= e^{i\tau} P_0 \left[-wT_\gamma + cL_\gamma\right] \frac{1-e^{-(i-\pi)\tau}}{i-\pi} - e^{i\tau} P_0 c L_\gamma \frac{e^{\pi\tau}-1}{\pi}, \end{aligned}$$

or

$$cL_\gamma \frac{e^{\pi\tau}-1}{\pi} = \left[-wT_\gamma + cL_\gamma\right] \frac{1-e^{-(i-\pi)\tau}}{i-\pi},$$

or

$$c\gamma\left(\theta^c - \theta^d\right)\left[\frac{i-\pi}{\pi}\frac{e^{\pi\tau}-1}{1-e^{-(i-\pi)\tau}} - 1\right] = w\left(k^d - 0\right).$$

The first order condition for δ has the same form, so the pair are as in (10).

Proof of Lemma 2: If (11) holds, $\gamma, \delta > 0$ if the term on the right in brackets in (10) is strictly positive. If $i > \pi$, the condition is

$$\left(\frac{i}{\pi} - 1\right)\left(1 - e^{-\pi\tau}\right) > e^{-\pi\tau} - e^{-i\tau},$$

or

$$\frac{i}{\pi}\left(1 - e^{-\pi\tau}\right) - e^{-\pi\tau} > 1 - e^{-i\tau},$$

which holds. Clearly (11) implies $\gamma < \delta$. ■

Derivation of (13): The first order condition for τ is

$$\begin{aligned} u(c) - \rho V &= -\left[V_A \frac{dA'}{d\tau} - V_A\left(e^{i\tau}-1\right)e^{\pi\tau} P_0 c L + \pi e^{\pi\tau} M_0^S V_P\right] \\ &= V_A\left[\left(e^{i\tau}-1\right)e^{\pi\tau} P_0 c L + \pi A' - \frac{dA'}{d\tau}\right], \end{aligned}$$

where the second line uses the fact that since V is h.o.d. zero, by Euler's theorem $AV_A(A,P) + PV_P(A,P) = 0$. Use (6) to find that

$$\begin{aligned} \pi A' - \frac{dA'}{d\tau} &= \pi M_0 - P_0 c L\left(e^{\pi\tau} - 1 - e^{\pi\tau}\right) + (\pi - i)\, e^{i\tau} B_0 \\ &\quad + \frac{1}{i-\pi}\left[\pi e^{i\tau} - \pi e^{\pi\tau} - \left(i e^{i\tau} - \pi e^{\pi\tau}\right)\right] P_0 z_0 \\ &= \pi M_0 + P_0 c L + (\pi - i)\, e^{i\tau} B_0 - e^{i\tau} P_0 z_0. \end{aligned}$$

Use this expression to write the first order condition for τ as in (13).

Conditions for a steady state.—

Since V is h.o.d. zero, it is convenient to define the function

$$v(a) \equiv V(a;1), \qquad \text{all } a,$$

with

$$V_A(A;P) = \frac{1}{P} v'(A/P). \tag{31}$$

Then (14) implies that in SS the nominal and real interest rates are as in (17), and (7) implies that

$$v(a) = u(c)\frac{1}{\rho}. \tag{32}$$

Use (17) and (31) in (9) to get

$$\begin{aligned} \mu &= e^{-(\rho+\pi)\tau} v'(a) \left(e^{i\tau} - 1\right), \\ &= \left(1 - e^{-i\tau}\right) v'(a). \end{aligned} \tag{33}$$

Derivation of (21): Use (17), (32) and (33) to write (13) as

$$\begin{aligned} rb + z_0 &= \frac{1}{e^{i\tau}} \left[\left(e^{i\tau} - 1\right) e^{\pi\tau} pcL + \pi m_0 + pcL\right] \\ &= \frac{1}{e^{i\tau}} \left[\frac{\left(e^{i\tau} - 1\right) e^{\pi\tau} + 1}{e^{\pi\tau} - 1} + 1\right] \pi m_0 \\ &= \frac{1}{1 - e^{-\pi\tau}} \pi m_0, \end{aligned}$$

where the second line uses (19). Recall from (2) and (3) that

$$\begin{aligned} b_0 &= a_0 - m_0 - pw\phi, \\ z_0 &= pw\phi\frac{1}{\tau} + \frac{\pi}{e^{\pi\tau} - 1} m_0, \end{aligned}$$

where the second line uses (18) and (19). Use these expressions to substitute above and find that (13) requires

$$a_0 = \left(1 - \frac{1}{r\tau}\right) pw\phi + \left(1 - \frac{\pi}{r}\frac{1}{e^{\pi\tau} - 1} + \frac{\pi}{r}\frac{e^{\pi\tau}}{e^{\pi\tau} - 1}\right) m_0 + \frac{1}{r} pt_x$$

$$= \left(1+\frac{\pi}{r}\right) m_0 + \left(1-\frac{1}{r\tau}\right) pw\phi + \frac{1}{r} pt_x,$$

as in (21).

Derivation of m_0.—

The market clearing condition for cash is, by construction, $\overline{m} = 1$. This fact can be used as follows to determine m_0. The cash balance at $s \in [0,\tau]$ of a household who made a withdrawal at $t = 0$ is

$$M(s,0) = \begin{cases} M_0 - P_0 cL\left(e^{\pi s} - 1\right)/\pi, & \text{if } \pi \neq 0, \\ M_0 - P_0 cLs, & \text{if } \pi = 0. \end{cases}$$

The balance at t of a household who made a withdrawal at s' is

$$M(t,s') = e^{\pi s'} M(t-s',0), \qquad t \in \left[s'+0, s'+\tau\right].$$

Hence the average across households at t is

$$\begin{aligned} \overline{M}(t) &= \frac{1}{\tau}\int_{t-\tau}^{t} e^{\pi s'} M(t-s',0)ds' \\ &= \frac{1}{\tau} e^{\pi t}\int_0^{\tau} e^{-\pi s} M(s,0)ds. \end{aligned}$$

Market clearing for cash at $t = 0$, requires $\overline{M}(0) = M_0^S$. For the normalized variables, if $\pi \neq 0$ the condition is

$$\begin{aligned} 1 &= \frac{1}{\tau}\int_0^{\tau} e^{-\pi s}\left[m_0 - pcL\frac{1}{\pi}\left(e^{\pi s}-1\right)\right] ds \\ &= m_0 \frac{1}{\tau}\int_0^{\tau} e^{-\pi s}\left(1 - \frac{e^{\pi s}-1}{e^{\pi\tau}-1}\right) ds \\ &= m_0 \frac{1}{\tau}\frac{1}{e^{\pi\tau}-1}\int_0^{\tau}\left(e^{\pi(\tau-s)}-1\right) ds \\ &= m_0 \frac{1}{\tau}\frac{1}{e^{\pi\tau}-1}\left(\frac{e^{\pi\tau}-1}{\pi} - \tau\right) \\ &= m_0\left(\frac{1}{\pi\tau} - \frac{1}{e^{\pi\tau}-1}\right), \end{aligned}$$

where the second line uses (19). Hence m_0 is as in the first line of (22) if $\pi \neq 0$.

If $\pi = 0$, the condition is

$$\begin{aligned} 1 &= \frac{1}{\tau}\int_0^\tau (m_0 - pcLs)\, ds \\ &= \frac{1}{\tau} m_0 \int_0^\tau \left(1 - \frac{s}{\tau}\right) ds \\ &= \frac{1}{\tau} m_0 \left(\tau - \frac{1}{\tau}\frac{\tau^2}{2}\right) \\ &= \frac{1}{2} m_0, \qquad \text{if } \pi = 0, \end{aligned}$$

where the second line uses (19). Hence m_0 is as in the second line of (22) if $\pi = 0$.

Alternatively, equating the outflow and inflow of cash in steady state implies

$$\begin{aligned} \frac{1}{\tau} m_0 &= pcL + \pi \\ &= \frac{\pi}{e^{\pi\tau} - 1} m_0 + \pi, \end{aligned}$$

where the second line uses (19). This expression agrees with (22) (use L'Hopital's rule if $\pi = 0$).

Derivation of $\bar{b}$.—

To construct $\bar{b}$, use $B(s)$ in (4). The bonds $B(s,0)$ held at s by a household who made a withdrawal at 0 are as in (4). Hence the bonds held at t by a household who made a withdrawal at s' are

$$B(t, s') = e^{\pi s'} B(t - s', 0), \qquad t \in [s' + 0, s' + \tau].$$

Consequently the average across households at t is

$$\begin{aligned} \overline{B}(t) &= \frac{1}{\tau}\int_{t-\tau}^{t} B(t, s') ds' \\ &= \frac{1}{\tau}\int_{t-\tau}^{t} e^{\pi s'} B(t - s', 0) ds' \\ &= e^{\pi t}\frac{1}{\tau}\int_0^\tau e^{-\pi s} B(s, 0) ds, \end{aligned}$$

and the normalized average balance in SS is

$$\begin{aligned}\bar{b} &= \frac{\overline{B}(t)}{M^S(t)} \\ &= \frac{1}{\tau}\int_0^\tau e^{-\pi s}\frac{B(s,0)}{M^S(0)}ds \\ &= \frac{1}{\tau}\int_0^\tau e^{rs}\left[b_0 + z_0\frac{1}{r}\left(1-e^{-rs}\right)\right]ds \\ &= \frac{e^{r\tau}-1}{r\tau}\left(b_0 + z_0\frac{1}{r}\right) - z_0\frac{1}{r}.\end{aligned}$$

Use (2) to get

$$\begin{aligned}b_0 &= a_0 - m_0 - pw\phi \\ &= \frac{\pi}{r}m_0 - \frac{1}{r\tau}pw\phi + \frac{1}{r}pt_x,\end{aligned}$$

where the second line uses (21), and use (3) to get

$$\begin{aligned}z_0 &= p\left[w\left(1-T\right) - c\nu + cL\right] - pt_x \\ &= pw\phi\frac{1}{\tau} + \frac{\pi}{e^{\pi\tau}-1}m_0 - pt_x,\end{aligned}$$

where the second line uses (18) and (19). Hence

$$\begin{aligned}b_0 + z_0\frac{1}{r} &= \left(1 + \frac{1}{e^{\pi\tau}-1}\right)\frac{\pi}{r}m_0 \\ &= \frac{e^{\pi\tau}}{e^{\pi\tau}-1}\frac{\pi}{r}m_0,\end{aligned}$$

and

$$\begin{aligned}\bar{b} &= e^{\pi\tau}\frac{1}{r}\frac{e^{r\tau}-1}{r\tau}\frac{\pi}{e^{\pi\tau}-1}m_0 \\ &\quad -\frac{1}{r}\left[pw\phi\frac{1}{\tau} + \frac{\pi}{e^{\pi\tau}-1}m_0 - pt_x\right] \\ &= \frac{e^{\pi\tau}\left(e^{r\tau}-1\right)/r\tau - 1}{e^{\pi\tau}-1}\frac{\pi}{r}m_0 - \frac{1}{r\tau}pw\phi + \frac{1}{r}pt_x. \qquad (34)\end{aligned}$$

If $\pi \neq 0$, use (**??**) to get

$$\bar{b} = \frac{e^{\pi\tau}\left(e^{r\tau}-1\right)/r\tau - 1}{\left(e^{\pi\tau}-1\right)/\pi\tau - 1}\frac{\pi}{r} - \frac{1}{r\tau}pw\phi + \frac{1}{r}pt_x. \qquad (35)$$

If $\pi = 0$, use L'Hopital's rule and (**??**) to get

$$\bar{b} = \frac{1}{r\tau}\left[2\left(\frac{e^{r\tau}-1}{r\tau}-1\right)-pw\phi\right]+\frac{1}{r}pt_x. \tag{36}$$

Solving the model.—

Use (18) in (19) to get

$$p = \frac{\nu}{w}\frac{1}{L}\frac{1}{(1-T)\tau-\phi}\frac{\pi\tau}{e^{\pi\tau}-1}m_0,$$

and combine (20) and (21) to get

$$\begin{aligned} 0 &= m_0\left[\left(\frac{1}{1-e^{-r\tau}}-1\right)-\frac{\pi}{r}\left(1+\frac{1}{e^{\pi\tau}-1}\right)\right]+pw\phi\left(\frac{1}{1-e^{-r\tau}}-1\right) \\ &= \left(\frac{1}{e^{r\tau}-1}-\frac{\pi/r}{1-e^{-\pi\tau}}\right)m_0+pw\phi\frac{1}{e^{r\tau}-1}, \end{aligned}$$

or

$$p = \frac{1}{w\phi}\left[\frac{(e^{r\tau}-1)/r}{(1-e^{-\pi\tau})/\pi}-1\right]m_0. \tag{37}$$

Combine these two equations to get

$$\frac{(e^{r\tau}-1)/r}{(1-e^{-\pi\tau})/\pi}-1 = \frac{\phi\nu}{L}\frac{1}{1-T-\phi/\tau}\frac{\pi}{e^{\pi\tau}-1},$$

or

$$e^{\pi\tau}\left(\frac{e^{r\tau}-1}{r}-\frac{1-e^{-\pi\tau}}{\pi}\right)\frac{1}{\nu} = \frac{\phi}{L}\frac{1}{1-T-\phi/\tau}.$$

as in (25).

Proof of Lemma 3: Existence: Write the conditions in (28) and (29) as

$$0 = \Psi(\gamma,\tau), \qquad 0 = \Lambda(\gamma,\tau),$$

where Ψ and Λ are the functions on the right sides of those equations. The functions Ψ and Λ are increasing in τ and γ, so both are downward sloping in (γ,τ)-space.

As $\gamma, \delta \to 0$, all transactions are made electronically. No liquidity is required and time use approaches its maximum value, $\hat{L} \to 0$ and $\hat{T} \to k^e$. Hence (**??**) requires

$$0 = \lim_{\gamma \to 0} \frac{e^{\pi\tau_1}}{\nu} \left(\frac{e^{r\tau_1} - 1}{r} - \frac{1 - e^{-\pi\tau_1}}{\pi} \right) - \frac{\tau_1}{\hat{L}(\gamma)}.$$

The second term in Ψ diverges, and τ must grow without bound so the first term offsets it.

As γ gets small, the value of τ satisfying (**??**) grows without bound,

$$e^{\pi\tau} \approx 1 + \frac{\pi}{r} + \frac{1}{\gamma}\frac{\pi}{r}\frac{\pi\nu K_\gamma}{1 - k^e}$$

As $\gamma, \delta \to \infty$, all transactions are made with cash. A finite amount of cash is required (per unit of consumption), and no transaction time is needed, $\hat{L} \to \nu$ and $\hat{T} \to 0$. Hence (**??**)-(**??**) imply that the values for τ asymptotically approach the values τ_1 and τ_2 satisfying

$$\begin{aligned} e^{\pi\tau_1}\frac{e^{r\tau_1} - 1}{r} - \frac{e^{\pi\tau_1} - 1}{\pi} &= \frac{\phi}{1 - \phi/\tau_1}, \\ \frac{\gamma}{\nu}\left[\frac{r\left(e^{\pi\tau_2} - 1\right)}{\pi\left(1 - e^{-r\tau_2}\right)} - 1\right] &= \frac{K_\gamma}{1 - \phi/\tau_2}. \end{aligned}$$

Show that $\tau_2 > \tau_1$, so as $\gamma \to \infty$, Λ lies below Ψ.

Hence the curves cross at least one.

Uniqueness: [To be completed. Show that at any intersection, Λ is more steeply downward sloping.] ■

Use (37) in (23) to check that

$$\frac{\pi}{r} = \left[\frac{\left(e^{r\tau} - 1\right)/r\tau - e^{-\pi\tau}}{\left(1 - e^{-\pi\tau}\right)/\pi} - \frac{\left(e^{r\tau} - 1\right)/r\tau}{\left(1 - e^{-\pi\tau}\right)/\pi} + \frac{1}{\tau}\right]\frac{1}{r}m_0,$$

or

$$\pi = \left[-\frac{\pi}{e^{\pi\tau} - 1} + \frac{1}{\tau}\right]m_0,$$

which agrees with (22) for $\pi \neq 0$. For $\pi = 0$, use L'Hôpital's rule to find that the term in brackets is zero.

Determination of $v'(a)$: Use (**??**) and (**??**) in (12) to get

$$\begin{aligned} u'(c)\frac{1}{r}\left(1-e^{-r\tau}\right) &= v'(a)\Big\{\left[\left(e^{\pi\tau}-e^{-r\tau}\right)+e^{-r\tau}\right]pL\frac{1}{\pi}\left(1-e^{-\pi\tau}\right) \\ &\quad + [p\nu - pL]\frac{1}{r}\left(1-e^{-r\tau}\right)\Big\} \\ &= v'(a)\Big\{\left[\frac{1}{\pi}\left(e^{\pi\tau}-1\right)-\frac{1}{r}\left(1-e^{-r\tau}\right)\right]pL \\ &\quad + p\nu\frac{1}{r}\left(1-e^{-r\tau}\right)\Big\}, \end{aligned}$$

or

$$u'(c) = v'(a)\left[p\nu + \left(\frac{r}{\pi}\frac{e^{\pi\tau}-1}{1-e^{-r\tau}}-1\right)pL\right]. \tag{38}$$

Eq. (38) determines $v'(a)$, which doesn't appear elsewhere. The terms on the right represent the two costs of higher consumption: the goods plus the cash needed to carry out the larger transactions. If $\pi = -r$ the second term vanishes, and the only cost is the goods cost, $p\nu$.

APPENDIX B: DATA

Data for 2017 are from Greene and Stavins (2018, p. 7, Figure 1), where the authors report the number and average size of transactions for each of the nine categories. In the Figures 2 and 3 (pp. 8 and 9), they report the changes in numbers and average values from 2016 to 2017 for eight of the categories. (Money orders are not reported.) These figures are used to back out the figures used here for 2016.

The authors also report on cash holdings (pp. 15-16). They report that at 65.1% of consumers held some cash, and the average cash held on the respondent's person (pocket, purse, or wallet) averaged \$58.9 in 2017 and \$57.2 in 2016. Finally, they report that a smaller fraction of consumers (22%) held some cash elsewhere. Among those who held cash, the average amount was \$738, most of it in the form of \$100 bills. The overall average was \$198.7, so it accounts for a larger share of the total than cash in wallets.

Kumar, Maktabi and O'Brien (2018) report comparisons on transactions for 2015, 2016, and 2017 (Figures 1 and 2, pp. 5-6). They also report average cash holdings by age and income group (Figures 11 and 12, p 11). Older households (55-64 and over 65) hold substantially more cash than the youngest households (under 25 and age 25-34). The highest income households (incomes above $125K) hold substantially more cash than those with the lowest incomes (less than $25K), butin the middle range the cash holdings are quite flat.

Metheny, O'Brien, and Wang (2016) report the 2015 and 2012 figures (Figure 4, p. 4) for five categories: cash, credit and debit cards, check, electronic, and other. They also report payment instrument by transaction size (Figure 7, p. 6). Cash and debit cards domiante for small transactions (75-84% for transactions under $25), while credit and electronic dominate for large ones (71% for transactions over $100).

They also report a size distribution for cash holdings (Figures 10 and 11, p. 8), and payment instrument use by age and income group (Figures 14-17, pp. 10-11).

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