Implications of Default Recovery Rates for Aggregate Fluctuations*

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Abstract

We document that default recovery rates in the United States are highly volatile and strongly pro-cyclical. These facts are hard to reconcile with the existing financial friction literature. Indeed, models with limited enforceability à la Kiyotaki and Moore (1997) do not have defaults and recovery rates, while agency costs models following Bernanke, Gertler, and Gilchrist (1999) underestimate the volatility of recovery rates by one order of magnitude. We extend the standard agency costs model allowing liquidation costs for creditors to depend on the tightness of the market for physical capital. Creditors do not have expertise in selling entrepreneurial assets, but when buyers are plentiful, this disadvantage is minimal. Instead when sellers are abundant, the disadvantage of being an outsider is higher. Following a negative shock, entrepreneurs sell capital and liquidation costs for creditors increase. Creditors cut lending and cause entrepreneurs to sell more capital. This liquidity channel works independently from standard balance sheet effects and amplifies the impact of financial shocks on output by up to 50 percent.

Keywords: Financial accelerator; financial frictions; recovery rates; liquidity channel.

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1 Introduction

Default recovery rates for corporate bank loans in the United States are strongly procyclical and highly volatile, ranging from 53 to 88 percent over the last 25 years. This finding does not seem surprising at first. Shleifer and Vishny (1992) pointed out that during a recession it is harder for a bank to sell the assets of a firm in financial distress, since the most productive use of these assets would be exercised by similar firms, which are likely to experience comparable financial difficulties. Furthermore, in times of recession, other financial institutions are trying to sell similar assets due to widespread bankruptcies. All of these factors make markets less liquid and cause recovery rates to deteriorate sharply during economic downturns.

General equilibrium models with financial frictions since Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999, BGG) have so far focused on explaining the dynamics of spreads and defaults, but put little emphasis on the behavior of recovery rates. By itself this is not an issue, since these models might be able to generate realistic patterns of recovery rates without explicitly trying to match them. However, we demonstrate that in the existing models recovery rates are almost flat over the cycle and rarely move by more than two percent from their average value.[1] This suggests that current models tend to underestimate the cost of bankruptcy in a recession and overestimate them in a boom. So long as bankruptcy costs impede the flow of funds from lenders to borrowers, these results imply that current frameworks might be understating the severity of financial frictions and their effects on macroeconomic aggregates.

The natural research question of this paper is how can existing models be modified in order to explain the behavior of the recovery rates and other business cycle variables. One of the simplest and most natural approaches consists in incorporating the Shleifer and Vishny (1992) insight into a dynamic general equilibrium model. Indeed, if liquidating an asset requires a match with a potential buyer, then during a recession when markets are illiquid, finding a corresponding match becomes harder, which would make liquidation costs

[1]Here we mean costly state verification approach following Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999) , Christiano, Motto, and Rostagno (2014) and others, since models following the costly state enforcement approach after Kiyotaki and Moore (1997) do not have default or recovery rates.

countercyclical and recovery rates procyclical. When markets are very liquid and buyers are plentiful, liquidation costs should decrease, while illiquidity should make it more difficult to find a match, driving liquidation costs upward. We denote this effect by liquidity channel and we embed it into a state-of-the-art model of financial frictions.

Formally, we extend a standard agency cost model by allowing liquidation costs for creditors to depend on the tightness of the market for physical capital. Building on Blanchard and Galí (2010), we assume that banks pay liquidation costs that depend on the ratio of the capital sold to capital purchased by entrepreneurs. These costs are small when the majority of entrepreneurs are trying to buy capital; in this case banks can sell liquidated assets relatively easy. On the other hand, when most of entrepreneurs sell physical capital, liquidation costs for banks increase. Naturally in the agency cost framework, most entrepreneurs are net buyers of capital in a boom and net sellers in a recession, making liquidation costs countercyclical. It turns out that this additional friction allows us to successfully explain the existing dynamics of defaults, spreads and balance sheets, as well as recovery rates.

In a related paper, Choi and Cook (2012) study the effect of a concave production function for liquidation services in a small-scale financial accelerator model, and show that this concavity can generate higher volatility of recovery rates. We differ from their work in three respects. First, we use a different approach to modeling liquidation, which relies on the tightness of the market for capital goods. Second, we build on the medium-scale DSGE model by Christiano, Motto, and Rostagno (2014), which explains the joint behavior of macroeconomic and financial variables. Third, we use Moody's dataset instead of FDIC's, which focuses on corporate debt and grants a tighter link between spreads, defaults and recoveries in the data.

We make several contributions to the literature. First, we show that standard nominal rigidities and balance sheet channels in agency costs models are not sufficient to generate the pattern of recovery rates observed in the data. Second, we introduce a liquidity channel to the agency costs model, which allows us to reconcile the model and the data. Third, we demonstrate that the liquidity channel strengthens the effect of financial shocks on output and asset prices. Indeed, when a negative shocks hits the economy, not only do markups go up and balance sheets deteriorate, but markets also become less liquid due to the fact that most entrepreneurs are trying to sell physical capital. As a result, banks become more reluctant to lend to entrepreneurs even if the latter have strong balance sheets, since, even if the probability of default for the entrepreneur is the same, the illiquidity of the markets drives down the potential recovery rate for the bank. In other words, expected bankruptcy costs go up for all borrowers, regardless of their balance sheets. We find that the liquidity channel amplifies the impact of financial shocks by a factor that is between 25 and 50 percent, depending on the nature of the shock. Finally, we find these additional negative effects to be persistent and present up to 20 quarters after the shock has hit the economy.

2 Recovery Rates and the Business Cycle

In this section we document the cyclical properties of recovery rates and investigate whether current macroeconomic models with financial frictions are able to explain them. Recovery rates measure the extent to which the creditor recovers the principal and accrued interest due on a defaulted debt. Our data come from Moody's (2016) "Annual Default Study: Corporate Default and Recovery Rates", which contains information about defaulted corporate bonds and loans recoveries, measured by the market value of defaulted debt as a percentage of par one month after default. The aggregate data are available at annual frequency from 1990 until 2014 and reflect the experience of over 20,000 corporate issuers in Moody's proprietary database. Figure 1 shows the dynamics of the recovery rate on first-lien loans along with those of real GDP growth for the United States.[2]

As the Figure shows, recovery rates tend to vary systematically over time. Recovery rates on first-lien loans exhibit substantial volatility, ranging from 53.4 percent in 1993 to 87.7 percent in 2004 within our sample. The Figure also makes clear that recovery rates closely track the business cycle. Loan recovery rates rose above 80 percent in the early 2000s while the economy was booming. As the financial crisis unravelled, recoveries started plummeting,

[2]While the recovery rates are available for different types of assets, including secured and unsecured bonds, in this study we focus on bank loans, which have been the traditional focus of the financial accelerator literature. Nevertheless, other types of assets' recovery rates exhibit strong pro-cyclicality and even higher volatility. In this sense, we take a conservative stance on volatility of recovery rates both in terms of frequency and type of assets.

reaching about 53 percent in the midst of the Great Recession. These findings are consistent with previous evidence by Frye (2000a,b) and Schuermann (2004), who show that in a recession, recovery is about a third lower than in an expansion. Over our sample period, recoveries and GDP growth exhibit a contemporaneous correlation of 0.41.[3]

Aggregate recovery rates exhibit a systematic relationship with defaults, as documented by Altman et al. (2005). As one may expect, recovery rates are lower when the aggregate default rate increases. In Figure 2 we provide some evidence of this relationship by showing the pattern of recoveries and the delinquency rates on business loan for the United States.[4] Between 2006 and 2009, delinquency rates increased from 1.27 to 3.91 percent, while recovery rates fell from 83.6 to 53.6 percent. The two time series are negatively correlated with a correlation coefficient of -0.42, while the correlation of delinquencies with real GDP growth is -0.33. Our evidence on default and recoveries is in line with previous research which highlights a similar macroeconomic dependence of recovery rates (Mora, 2012).

We now examine the behavior of aggregate recovery rates through the lens of a general equilibrium model with financial frictions. A strand of the macroeconomic literature has focused on the ability of these models to explain the behavior of spreads and defaults over the business cycle but so far their implications for recoveries remains unexplored. For our analysis we use the model of Christiano, Motto, and Rostagno (2014, CMR), which builds on the seminal contribution of BGG in the financial accelerator literature. Our choice is guided by two facts. First, this class of models features equilibrium defaults and associated bankruptcy costs. Hence, it is straightforward to construct a measure of the aggregate recovery rate in the model that can be compared with the data. Second, the estimated model of CMR is successful at explaining the time-variation in defaults observed in the data. Indeed, in one of their posterior predictive checks, the authors show that this model successfully accounts for the dynamics of delinquency rates for the United States over the last two decades. Therefore it is natural to ask whether the model is able to explain the dynamics of recovery rates, conditional on its success in accounting for fluctuations in defaults.

[3]The correlation between recoveries and HP-filtered GDP is 0.30.

[4]This data correspond to the series “Delinquency Rate On Business Loans, All Commercial Banks” on the FRED database.

To answer this question, we compute the aggregate implied recovery rate from CMR using their publicly available codes and compare it against the data.[5] In their paper, the authors do not try to match recovery rates, but they do a good job at explaining other financial variables such as credit, spreads, net worth and the slope from the term structure, as well as a set of traditional macroeconomic variables. As can be seen from the results presented in Figure 3, the implied recovery rate from CMR do not exceed 72 percent or fall below 68 percent even during 2008-2009 Great Recession. On the other hand, the recovery rate from the Moody's dataset features a much higher volatility.

These findings suggest that current models of financial frictions tend to underestimate the cost of bankruptcy in a recession and overestimate them in a boom. So long as bankruptcy costs impede the flow of funds from lenders to borrowers, these results imply that current frameworks might be understating the severity of financial frictions and their effects on macroeconomic aggregates. In the next sections we introduce a new channel in the financial accelerator model that is able to explain the behavior of recoveries and we study its effect on aggregate fluctuations.

3 The Liquidity Channel

In this section we introduce the contracting problem between financial intermediaries on the lending side and entrepreneurs on the borrowing side, and we demonstrate how the contract is affected by liquidity of the markets. While the contracting problem closely follows BGG, bankruptcy costs per unit of asset (or capital) are going to be affected by the liquidity of capital markets. Here the price of capital goods and the expected returns to capital are taken as given by lenders and borrowers. The subsequent section will endogenize these prices and returns in our general equilibrium environment.

[5] We derive a formula for the model-implied recovery rate in the Appendix.

3.1 Entrepreneurs

There is a continuum of entrepreneurs indexed by j. Entrepreneurs are the only agents accumulating capital in the model. At time t, entrepreneur j purchases raw capital, $\bar{K}_{t+1}(j)$, at a unit price of Q_t. The entrepreneur uses his net worth, $N_t(j)$, and a one-period loan $B_{t+1}(j)$ from a financial intermediary (or bank) to purchase his desired level of capital:

$$Q_t\bar{K}_{t+1}(j) = N_t(j) + B_{t+1}(j). \tag{1}$$

After the purchase, the entrepreneur converts the raw capital into effective capital services. At the beginning of period $t+1$, the entrepreneur is hit with an idiosyncratic shock that the lender cannot directly observe. Entrepreneur j earns income by supplying capital services and from capital gains; he then goes to another bank and gets a new loan in order to refinance his previous loan. The new bank perfectly observes the balance sheet of the entrepreneur, as the entrepreneur reveals his private information to the potential creditor. If a new loan is not extended and the entrepreneur is not able to refinance, he defaults, the old banks seizes his assets and tries to sell them in the market for physical capital. On the contrary, if the entrepreneur is able to refinance, he repays the loan to the old bank and uses his residual resources to buy the additional capital. Sometimes the new loan is not sufficient to repay the old loan, and in this case the entrepreneur covers the difference by selling some units of physical capital.

There are two differences relative to the standard agency cost frameworks in macroeconomic models. First, in our model most of the physical capital stays with entrepreneurs, who go to the market to buy or sell only the additional units. In the standard framework, physical capital moves back and forth between households or capital agencies and entrepreneurs. The assumption that entrepreneurs keep and accumulate capital is important for the notion of liquidity. In bad states of the world the majority of entrepreneurs are selling capital and are making markets very illiquid, which puts additional pressure on banks, that try to sell seized assets. In the standard framework, this effect would be much weaker, since the continuous movement of the entire capital stock between households and entrepreneurs would make capital markets always liquid. Our assumption also matches more closely real life phenomena. Indeed, the stock of capital moves very slowly with business cycles, while the flows of capital are very pro-cyclical and volatile, which perfectly serves the concept of liquidity in our framework.

Second, we assume that when the entrepreneur refinances the loan, he reveals his private information to the new lender, but not to the old lender. We consider this assumption highly plausible, since the borrower has a strong motivation to reveal his situation ex-ante in order to get the loan and to respond to all information requests from the lender, otherwise the lender could simply reject the loan application. On the other hand, once the loan is approved, entrepreneur has smaller incentives ex-post to provide the lender with his private information. Under these assumptions, we have a standard costly state verification setup and we follow the contracting problem between lenders and borrowers after BGG.

3.2 The Loan Contract

The contracting between the entrepreneur and the financial intermediary is subject to a typical agency problem. In period $t+1$, entrepreneur j is hit with an idiosyncratic shock, $\omega_{t+1}(j)$, and an aggregate shock, R^k_{t+1}, so that he is able to deliver $Q_t K_t(j) R^k_{t+1}\omega_{t+1}(j)$ units of assets. The idiosyncratic shock $\omega_{t+1}(j)$ is a log-normal random variable with distribution $\log(\omega_{t+1}(j)) \sim \mathcal{N}(-\frac{1}{2}\sigma^2_{\omega,t}, \sigma^2_{\omega,t})$ so that the mean of ω is equal to 1. We denote by $f_t = f(\omega, \sigma_{\omega,t})$ and by $F_t = F(\omega, \sigma_{\omega,t})$ the probability density function and cumulative distribution function of ω_t, respectively.[6] The realizations of ω are independent across entrepreneurs and over time. We assume that the lender cannot observe the realization of the idiosyncratic shock to the entrepreneurs unless he pays monitoring costs μ_{t+1} which are expressed in percentage of total assets. The loan obtained by the entrepreneur takes the form of a standard debt contract, where Z_{t+1} denotes the promised gross rate of return on the loan. Let, $\bar{\omega}_{t+1}$, be the value of ω below which an entrepreneur is not able to repay the principal and the interest on

[6]The timing is meant to capture the fact that the variance of ω_{t+1} is known at the time of the financial arrangement, t.

the loan. This cutoff is defined by

$$B_{t+1}(j)Z_{t+1} = Q_t\bar{K}_{t+1}(j)R^k_{t+1}\bar{\omega}_{t+1} \tag{2}$$

Entrepreneurs with $\omega < \bar{\omega}$ are not able to refinance and, hence, declare bankruptcy. In this case, the lender seizes the entrepreneurial assets and tries to find a match on the market in order to sell these assets. The ex-post $t+1$ payoff to the entrepreneur with net worth $N_t(j)$ is given by

$$\int_{\bar{\omega}_{t+1}}^{\infty} [Q_t\bar{K}_{t+1}(j)R^k_{t+1}\omega - B_{t+1}(j)Z_{t+1}]dF_t(\omega) = [1-\Gamma_t(\bar{\omega}_{t+1})]R^k_{t+1}\kappa_t N_t(j) \tag{3}$$

where

$$\kappa_t \equiv \frac{Q_t\bar{K}_{t+1}(j)}{N_t(j)}$$
$$\Gamma_t(\bar{\omega}_{t+1}) \equiv [1-F_t(\bar{\omega}_{t+1})]\bar{\omega}_{t+1} + G_t(\bar{\omega}_{t+1})$$
$$G_t(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega)$$

and κ_t denotes leverage, from which we have dropped the index j in anticipation of the result that leverage is independent of net worth (see below).

Financial intermediaries collect deposits from the household, to which they promise a competitively determined, non-state contingent, nominal interest rate R_t. The financial intermediary diversifies his lending across a large number of entrepreneurs. Thus, its participation constraint can be written as

$$[1-F_t(\bar{\omega}_{t+1})]Z_{t+1}B_{t+1}(j) + (1-\mu_{t+1})\int_0^{\bar{\omega}_{t+1}} K_{t+1}(j)Q_tR^k_{t+1}\omega dF_t(\omega) \geq R_tB_{t+1}(j) \tag{4}$$

where the left hand-side of (4) is the expected gross return on the loan and the right hand side is the opportunity cost of lending for the financial intermediary. This equation states that returns to lenders consist of the payoff from firms that did not default, and from the seized assets of entrepreneurs that could not repay their loans net of liquidation costs.

Using the definition of leverage and equation (2), the participation constraint can be re-written as

$$R^k_{t+1}[\Gamma_t(\bar{\omega}_{t+1}) - F_t(\bar{\omega}_{t+1})] = \frac{\kappa_t - 1}{\kappa_t} R_t \tag{5}$$

Following BGG, we assume that entrepreneurs go out of business with exogenous probability $(1-\gamma_t)$. In this event, after collecting their earnings from renting their capital, the entrepreneur sells his capital, pays back his loan and consumes his residual net worth. The exiting entrepreneurs are replaced by an inflow of new entrepreneurs that receive an initial start-up transfer from the household, W^e_t. Therefore, the entrepreneurial objective function is described by

$$\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\left[\left(\Pi^s_{i=0}\gamma_{t+i}\right)N_{t+s}(j)\right]\right\} \tag{6}$$

The law of motion for aggregate net worth, N_t, is given by

$$N_t = \gamma_t[1 - \Gamma_{t-1}(\bar{\omega}_t)]R^k_t Q_{t-1}\bar{K}_t + W^e_t \tag{7}$$

The debt contract specifies a pair $(B_{t+1}(j), Z_{t+1})$ that maximizes the utility of the entrepreneur given by (6) subject to the participation constraint of lenders defined by (5). As it is evident, the problem of choosing B_{t+1} is equivalent to choosing κ_t, independently of net worth. Furthermore, using (2) we can re-express Z_{t+1} in terms of $\bar{\omega}_{t+1}$, so that our contract is described by the pair $(\kappa_t, \bar{\omega}_{t+1})$. Dmitriev and Hoddenbagh (2013) show that maximization of intertemporal utility with linear preferences is identical to the maximization of the next period expected payoff in (3) to the first order approximation. As in BGG, in this model we can solve for the aggregate variables N_t, κ_t and $\bar{\omega}_{t+1}$ without keeping track of the distribution of net worth.

3.3 Financial Intermediaries

Financial intermediaries accept deposits from households and provide one-period loans to entrepreneurs. While able to diversify the idiosyncratic risk by lending to a large number of entrepreneurs, financial intermediaries are still subject to aggregate risk. Financial intermediaries play also an important role in liquidating the assets of entrepreneurs who go bankrupt. The liquidation cost that they face, μ_t, is going to be proportional to the market value of the assets.

Liquidating assets is costly for banks. Banks specialize in financial intermediation, not in selling distressed assets, hence they lack sufficient skills to properly assess the value of capital goods. When markets are very liquid, the lack of skills is not very problematic and banks can easily find buyers who would pay competitive prices. When markets are very illiquid and banks need to liquidate assets they are instead forced to take a discount on the true market value.

The notion of asset market liquidity that we consider, θ_t, is defined as the ratio of aggregate net sales over net purchases of capital by entrepreneurs in the capital goods market

$$\theta_t = \frac{\int_0^1 \max[\bar{K}_t(j) - \bar{K}_{t+1}(j), 0]dj}{\int_0^1 \max[\bar{K}_{t+1}(j) - \bar{K}_t(j), 0]dj} \tag{8}$$

An analytical expression for θ_t is derived in the Appendix. In equation (8) the term $\max[\bar{K}_t(j) - \bar{K}_{t+1}(j), 0]$ in the numerator defines the net sales of capital units by entrepreneur j. When the entrepreneur is a net purchaser of capital on the market, this term becomes zero. Correspondingly, the term $\max[\bar{K}_{t+1}(j) - \bar{K}_t(j), 0]$ in the denominator denotes net purchases of physical capital by entrepreneur j. In the steady state entrepreneurs with high idiosyncratic productivity realizations become net buyers of capital, while unproductive entrepreneurs become net sellers. However, during recessions the fall in asset prices reduces aggregate returns, and makes most of entrepreneurs start selling capital. This effect will make markets very illiquid for banks that try to sell seized assets. We assume that liquidation cost of the banks are a decreasing function of market liquidity

$$\mu_t = \mu\big(\theta_t/\theta_{ss}\big)^{\varphi} \tag{9}$$

where $\varphi > 0$ and θ_{ss} is the steady state value of θ_t. This approach parallels Blanchard and Galí (2010), who model frictions in labor markets by assuming that hiring costs are an increasing function of labor market tightness. Our formalization implies that banks can liquidate assets immediately, as long as they are willing to pay the liquidation cost, μ_t, which is a function of the liquidity of the market for capital goods. An alternative formulation of the problem would see banks paying a search cost to find a match with a potential buyer. In this case some of the capital will not be matched and stay on the balance sheets of the banks, which would add a lot of complexity by making the problem of financial intermediaries dynamic in the presence of agency costs. Moreover, the gains of developing such a model are limited by the absence of data on "vacancies" for capital, which make the matching approach less attractive in capital markets relative to labor markets. Nevertheless, both approaches share the idea that the cost of liquidating the capital is a decreasing function of the liquidity in the capital market. Our approach has the advantage of keeping the model much more tractable and compatible with the current generation of agency costs setups.

4 General Equilibrium

Our general equilibrium model extends CMR, allowing liquidation cost to vary with the business cycle, as developed in the previous section. This medium-scale DSGE model has been shown to be well suited to explain the joint co-movement of financial and traditional key macroeconomic variables. The New Keynesian backbone of the model follows Christiano, Eichenbaum, and Evans (2005), augmented with technology shocks in the production of installed capital, following the contribution of Justiniano, Primiceri, and Tambalotti (2010). The agency problem between lenders and entrepreneurs comes from BGG. Our framework is isomorphic to CMR's model when we set $\varphi = 0$.

4.1 Final Goods Producers

Perfectly competitive firms produce a homogeneous final good, Y_t, from a continuum of intermediate goods, $Y_{j,t}, j \in [0,1]$ using the following Dixit-Stiglitz aggregator

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{1}{\lambda_{f,t}}} dj \right)^{\lambda_{f,t}}, \qquad 1 < \lambda_{f,t} < \infty \tag{10}$$

where $\lambda_{f,t}$ is a price markup shock. All the shocks processes will be described below. Maximization of profits, together with the zero-profit condition, implies that the price of the final good, P_t, is the familiar CES aggregate of intermediate goods' prices.

The homogenous final good can be converted into consumption goods, C_t, one for one. A different technology converts $\Upsilon^t \mu_{\Upsilon,t}$ units of final goods into one unit of investment goods, where $\mu_{\Upsilon,t}$ is an investment-specific technology (IST) shock. These two technologies are operated by perfectly competitive firms so that the equilibrium price of investment goods is $P_t/(\Upsilon^t \mu_{\Upsilon,t})$.

4.2 Intermediate Goods Producers

Each intermediate good j is produced by a monopolist using the following production function

$$Y_{j,t} = \max[\epsilon_t K_{j,t}^{\alpha} (z_t l_{j,t})^{1-\alpha} - \Phi z_t^*, 0] \tag{11}$$

where $K_{j,t}$ and $l_{j,t}$ denote the amount of effective capital and labor employed by firm j. ϵ_t is a stationary technology shock, while the variable z_t follows a process with a stationary growth rate. Φ is a fixed cost in production chosen so that profits are zero in steady state and $z_t^* = z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha}\right)t}$ to ensure the existence of a balanced growth path. Supplier j sets his price to maximize his profits subject to Calvo-style frictions (Calvo, 1983). In particular, in every period t a random subset ξ_p of suppliers cannot optimally set its price, but adjusts it according to $P_{j,t} = \tilde{\pi}_t P_{j,t-1}$, where the indexation follows $\tilde{\pi}_t = (\pi_t^{target})^{\iota} (\pi_{t-1})^{1-\iota}$ and $\pi_{t-1} \equiv P_{t-1}/P_{t-2}$. π_t^{target} represents a target inflation rate for the monetary policy rule, described below. The complementary set of suppliers $1-\xi_p$ re-optimizes prices to maximize the profit function:

$$E_t \sum_{s=0}^{\infty} \beta^s \xi_p^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[P_{j,t} \left(\prod_{k=1}^{s} (\pi_{t+k}^{target})^{\iota} (\pi_{t+k-1})^{1-\iota} \right) Y_{j,t+s} - W_{t+s} l_{j,t+s} - P_{t+s} r_{t+s}^k K_{j,t+s} \right] \tag{12}$$

where the demand for the intermediate product $Y_{j,t}$ comes from the final goods producers, W_t indicates the nominal wage and Λ_t is the marginal utility of nominal income for the representative household.

4.3 Capital Goods Producing Sector

Perfectly competitive firms purchase investment goods and transform them into new capital. The technology used by these firms takes I_t units of investment goods and transforms them into $(1-S(\zeta_{I,t} I_t / I_{t-1})) I_t$ units of new capital. Thus, the flow profit function for a capital good producer is given by

$$Q_t(1 - S(\zeta_{I,t} I_t / I_{t-1})) I_t - P_t / (\Upsilon^t \mu_{\Upsilon,t}) I_t \tag{13}$$

The function $S(x)$ captures the presence of adjustment costs in investment, and is such that $S(x) = S'(x) = 0$ and $S''(x) = S''$, where x denotes the steady-state value of $\zeta_{I,t} I_t / I_{t-1}$ and S'' will be a model parameter. $\zeta_{I,t}$ is a shock to the marginal efficiency of investment (MEI) in producing capital goods.

4.4 Labor Market

The structure of the labor market follows Erceg, Henderson, and Levin (2000). The specialized labor types, $h_{i,t}$, are combined by perfectly competitive employment agencies into a homogenous labor input using the following technology:

$$l_t = \left(\int_0^1 (h_{i,t})^{\frac{1}{\lambda_w}} di \right)^{\lambda_w}, \qquad 1 < \lambda_w < \infty \tag{14}$$

The homogenous labor input is then sold to the intermediate firms. The wage paid by these firms for the homogenous labor input

$$W_t = \left(\int_0^1 (W_t^i)^{\frac{1}{1-\lambda_w}} di \right)^{1-\lambda_w} \tag{15}$$

can be obtained by solving the profit maximization problem of the employment agencies.

4.5 Households

The representative household maximizes its lifetime utility by choosing the optimal path of consumption and labor input

$$E_t \sum_{s=0}^{\infty} \beta^s \zeta_{c,t+s} \left\{ \log(C_{t+s} - bC_{t+s-1}) - \Psi_L \int_0^1 \frac{h_{i,t+s}^{1+\sigma_L}}{1+\sigma_L} di \right\} \qquad b, \sigma_L > 0 \tag{16}$$

where C_t denotes household consumption, b parameterizes the degree of consumption habits and $\zeta_{c,t}$ indicates a preference shock. The household provides a continuum of differentiated labor inputs, $h_{i,t} \in [0, 1]$.

We can write the flow budget constraint for the household as

$$(1 + \tau^c) P_t C_t + B_{t+1} \leq (1 - \tau^l) \int_0^1 W_t^i h_{i,t} di + R_t B_t + \Pi_t \tag{17}$$

The left-hand side of the budget constraint encompasses the sources of expenditure. The household purchases consumption goods, C_t, that are taxed at a rate τ^c,at price P_t, and bonds, B_{t+1}. The household sources of revenues are the earnings from labor and from bonds. Π_t denotes lump-sum payments to the household, including profits from intermediate goods, transfers from entrepreneurs, and lump-sum transfers from the government net of lump-sum taxes. Following Erceg, Henderson, and Levin (2000), we assume that there is a monopoly union for each type of labor input that sets the wage rate, W_t^i, according to a Calvo-style friction. Specifically, in every period a random subset of unions $1 - \xi_w$ sets their wage optimally by maximizing

$$E_t \sum_{s=0}^{\infty} \beta^s \xi_w^s \left\{ \Lambda_{t+s} W_t^i h_{i,t+s} - \zeta_{c,t+s} \Psi_L \frac{h_{i,t+s}^{1+\sigma_L}}{1+\sigma_L} \right\} \tag{18}$$

subject to the labor demand function coming from the intermediate goods producers. The complementary set of unions adjusts their wage according to $W_t^i = (\mu_{z^*,t})^{\iota_\mu} (\mu_{z^*})^{1-\iota_\mu} \tilde{\pi}_{w,t} W_{t-1}^i$, where μ_{z^*} is the growth rate of z_t^* in the non-stochastic steady state and

$$\tilde{\pi}_{w,t} = (\pi_t^{target})^{\iota_w} (\pi_{t-1})^{1-\iota_w}, \quad 0 < \iota_w < 1. \tag{19}$$

4.6 Aggregate Returns and Law of Motion for Capital

While the entrepreneur's problem and timing were described in the previous section, here we explicitly determine the aggregate returns, R_{t+1}^k, and the law of motion for capital. At the beginning of period $t+1$, after observing the aggregate rate of returns and prices in period $t+1$, each entrepreneur determines the utilization rate, u_{t+1}, of its capital and supplies effective capital services $u_{t+1} \omega \bar{K}_{t+1}(j)$ for a competitive market rental rate, r_{t+1}^k.[7] At the end of period $t+1$ the entrepreneur is left with $(1-\delta)\omega K_{t+1}(j)$, which is sold in a competitive market for the price Q_{t+1}. Hence, the aggregate component of the entrepreneurs' return, R_{t+1}^k, is given by

$$R_{t+1}^k \equiv \frac{(1-\tau^k)[u_{t+1} r_{t+1}^k - a(u_{t+1})] \Upsilon^{-(t+1)} P_{t+1} + (1-\delta) Q_{t+1} + \tau^k \delta Q_t}{Q_t} \tag{20}$$

where a is an increasing and convex function capturing the cost of capital utilization and τ^k indicates the tax rate on capital income. The utilization rate is set to its optimal level, which satisfies

$$\Upsilon^{-(t+1)} a'(u_{t+1}) = r_{t+1}^k \tag{21}$$

In steady state, $u = 1$, $a(1) = 0$ and $\sigma_a \equiv a''(1)/a'(1)$.

In equilibrium, aggregate demand for capital goods must be equal to aggregate supply. Aggregate demand is given by the demand for capital goods by all entrepreneurs, $\bar{K}_{t+1}$. Ag-

[7] The utilization rate is not indexed by j as it is independent of the entrepreneur's net worth. This can be seen below in equation (21).

gregate supply is given by the undepreciated capital of all entrepreneurs, $(1-\delta)\bar{K}_t$, plus the new capital goods produced in period t, $(1 - S(\zeta_{I,t}I_t/I_{t-1}))I_t$. Hence the law of motion for aggregate capital is

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + (1 - S(\zeta_{i,t}I_t/I_{t-1}))I_t \tag{22}$$

The utilization rate transforms raw capital into effective capital services according to

$$K_t = u_t\bar{K}_t \tag{23}$$

4.7 The Government and The Resource Constraint

A monetary authority sets the nominal interest rate, in linearized form, following the feedback rule:

$$R_t - R = \rho_p(R_{t-1} - R) + (1-\rho_p)\left[\alpha_\pi(\pi_{t+1} - \pi_t^{target}) + \alpha_{\Delta y}\frac{1}{4}(g_{y,t} - \mu_z^*)\right] + \frac{1}{400}\epsilon_t^p, \tag{24}$$

where ϵ_t^p is a shock to monetary policy in annual percentage points and ρ_p is a smoothing parameter in the policy rule. The monetary authority responds to deviation of expected inflation from target, $\pi_{t+1} - \pi_t^{target}$, and to deviations of quarterly growth in gross domestic product from its steady state, $g_{y,t} - \mu_z^*$.

Fiscal policy is fully Ricardian. Government consumption expenditure, G_t, is given by

$$G_t = z_t^* g_t \tag{25}$$

where g_t follows a stationary stochastic process.

Finally, the resource constraint can be written as

$$Y_t = D_t + G_t + C_t + \frac{I_t}{\Upsilon^t \mu_{\Upsilon,t}} + a(u_t)\frac{\bar{K}_t}{\Upsilon^t} \tag{26}$$

The last term on the constraint indicates the output cost of adjusting capital utilization. D_t

represents the resource cost associated with liquidation by financial intermediaries

$$D_t = \mu_t G(\bar{\omega}_t) R_t^k \frac{Q_{t-1}\bar{K}_t}{P_t} \tag{27}$$

where, relative to CMR, μ_t is determined endogenously.

4.8 Shocks and Information

The model described above includes 11 aggregate shocks: $\epsilon_t, \mu_{z,t}, \lambda_{f,t}, \pi_t^*, \zeta_{c,t}, \zeta_{I,t}, \mu_{\Upsilon,t}, \gamma_t, \sigma_t, \epsilon_t^p$ and g_t. Relative to CMR we abstract from modeling long-term interest rates and its associated shock . Each shock is modeled as a first-order autoregressive process. The autocorrelation of monetary policy and equity shock is set to zero. Following CMR baseline specification, we allow agents to receive information about the realization of risk shocks before innovation is realized. In particular we consider the following representation of the risk shock:

$$\sigma_t = \rho_\sigma \sigma_{t-1} + \underbrace{\xi_{0,t} + \xi_{1,t-1} + \ldots + \xi_{p,t-p}}_{=u_t} \tag{28}$$

The innovation u_t is a sum of i.i.d. random variables with zero mean that are orthogonal to $x_{t-j}, j \geq 1$. In period t, agents observe $\xi_{j,t}, j = 0, 1, \ldots, p$ and we refer to $\xi_{j,t}, j > 0$ as news shocks. The news shocks exhibit the following correlation:

$$\rho_{\sigma,n}^{|i-j|} = \frac{E\xi_{i,t}\xi_{j,t}}{\sqrt{(E\xi_{i,t}^2)(E\xi_{j,t}^2)}}, \qquad i, j = 0, \ldots, p. \tag{29}$$

where $-1 \leq \rho_{\sigma,n}^{|i-j|} \leq 1$. Under this specification, the parameters associated with the risk shock are: $\rho_\sigma, \rho_{\sigma,n}, \sigma_\sigma$ and $\sigma_{\sigma,n}$. The other shocks have only two parameters: an autocorrelation and a standard deviation parameter.

5 Calibration

Standard Parameters. Our calibration for the standard part of the model follows CMR. The values for the parameters that are related to the long-run properties of our model are summarized in Table 1. We set the capital's share, α, to 0.4, the Frisch elasticity of labor supply, σ_L to 1, and the depreciation rate for capital to 0.025. The mean growth rate of the unit root technology, μ_z, is fixed at 0.41 percent and the rate of investment specific technological change at 0.42 percent. These values are chosen to be consistent with the average growth rate of per capita GDP over the sample period and to account for the average rate of decline in the price of investment goods. We set the steady state value of g_t so that government expenditure is 20 percent of GDP in steady state, consistent with the data. Steady-state inflation is fixed at 2.4 percent on an annual basis. The household's discount factor is set to 0.9987. As in Christiano, Eichenbaum, and Evans (2005) we set the steady state markups in the product market $\lambda_{f,t}$ and in the labor market λ_w to 1.2 and 1.05. The tax rates on consumption, capital income and labor follow CMR. We fix the habit formation parameter, b, to 0.74, CMR posterior mode.

For the part of the model that relates to financial frictions, we set the steady-state probability of entrepreneurs exiting business, $1-\gamma$, is set to $1-0.985$. Liquidation costs in steady state, μ, are set to 0.21 and the steady-state volatility of idiosyncratic productivity to 0.26. These values imply a steady-state leverage $\bar{K}/N$ of 2.015, an annualized default probability of 2.24 percent and a value of R^k/R of 1.0073 corresponding to annualized excess return to capital of 4 percent. Furthermore, our calibration implies a share of consumption and investment in GDP of 0.52 and 0.27, in line with the data. Note that our modification to the CMR framework does not affect the steady state of the model but only its dynamics.

The remainder of the parameters affect the model's dynamics. We calibrate these parameters using CMR's posterior mode. CMR estimate were pinned down by Bayesian techniques to match the dynamics of eight macroeconomic series and four financial series. The parameter values are summarized in Table 2. The persistence parameters tend to be relatively high for all shocks, except for the persistent component of technology growth, implying that $\log z_t$ follows roughly a random walk. CMR estimates point to sizable nominal rigidities both in prices and wages. Overall these parameter values are reasonable and in line with previous research.

Calibrating φ. The elasticity of liquidation costs, φ, is calibrated using our data on recovery rates. While our model is parameterized at a quarterly frequency, recovery rates are only available at annual frequency. We address the issue by constructing in the model the following moving average that we map directly into the recovery data

$$R_{\tau}^{c} = \frac{R_{t}^{c} + R_{t-1}^{c} + R_{t-2}^{c} + R_{t-3}^{c}}{4R^{c}} \tag{30}$$

where τ is a yearly time subscript. We choose a value of φ equal to 16, such that the volatility of recovery rates in the model matches the volatility of recovery rates in the data. This value of φ implies that that if the ratio of sellers to buyers of capital increases by 1 percent, liquidation costs will go up by 2 percentage points.

6 Impulse Response Analysis

Figure 4 outlines the dynamic effect of a positive entrepreneurial survival shock on the economy. Following the shock, fewer entrepreneurs exit the economy, which drives aggregate entrepreneurial net worth up and allows entrepreneurs to invest more. Consequently investment increases, driving asset prices, output and hours upward. Higher asset prices boost aggregate returns, which leads to a rise of worth and a decrease in defaults. This is an example of standard financial accelerator mechanism, and it holds for the basic model and the extended version with liquidity channel. Despite these similarities two models generate very different dynamics of recovery rate. While in the basic model recovery rate stays practically flat, in the model with liquidity channel recovery rate skyrockets by 25 percent. This spike in recovery follows from the fact that with a higher survival rate fewer entrepreneurs exit the industry and sell their businesses. In turn, this implies that capital markets are more liquid, making it much easier for banks to sell seized assets. The surge in recovery rates decreases the cost of lending, which in turn drives investment, net worth, output and asset prices further up. As a result, the impact of a survival shock on output is almost twice as high for the model with liquidity channel.

The dynamic effect of risk shocks on the economy is demonstrated on Figure 5. Higher risk causes the increase in defaults and bankruptcy costs, and as a result investment contracts and drive asset prices down, causing net worth and output to fall. Decrease in asset prices leads to the decline of recovery rate by 2 percentage point in the basic model and by 8 percentage point in the model with the liquidity channel. This sharper decrease in the recovery rate is caused by markets becoming less liquid due to the surge in defaults and reduction in investment. The fall in recovery rate for the extended model makes cost of lending higher, and this leads to the next contraction of investment, net worth, price of capital and output. Overall, the presence of the liquidity channel amplifies the impact response of output to the risk shock by 25 percent.

The mechanism is similar for a contractionary monetary shock, illustrated in Figure 6. In the case of the basic model the negative demand shock causes a contraction in investment and asset prices. This initial effect translates into net worth losses and leads to the next round of financial tightening, decreasing in investment, prices, and net worth and leading to a surge in defaults. All of these processes make markets less liquid and, therefore, the model with the liquidity channel generates a stronger decline in recovery rates relative to the basic model, where it essentially stays at the steady state level. The additional fall in recovery rate makes external financing more costly and causes investment and asset prices to go down, which again leads to the deterioration of net worth and strengthens the recession.

The liquidity channel amplifies shocks for all the cases above. This is not a coincidence, since investment are procyclical, making capital markets less liquid during a recession and more liquid during a boom. The illiquidity causes the recovery rates to fall during the recession, increasing the financial wedge stronger, which, through a deterioration of balance sheets, has an additional negative impact on the economy. Balance sheet effects work in a similar way through a procyclicality of investment that causes asset prices and, hence, net worth to be procyclical as well. Although these two channels are distinct, they tend to reinforce each other. Lower net worth causes a decline in investment and makes market less liquid, but the liquidity channel causes recovery rates to go down, reducing lending and investment, which leads to a drop in asset prices and, consequently, net worth.

7 Conclusion

In this paper we document that recovery rates from default in the United States are very volatile and strongly pro-cyclical. We demonstrate that current models of financial frictions significantly understate this volatility by one order of magnitude relative to the data. This finding suggests that current models may underestimate the severity of frictions in financial markets. We therefore extend the financial friction model of Christiano, Motto, and Rostagno (2014), allowing liquidation costs for defaulted assets to depend on the liquidity of the market for these assets. This modification allows us to explain the behavior of standard business cycle variables as well as recovery rates. Our impulse response analysis suggests that the effect of financial shocks on output and asset prices is strongly amplified in the presence of liquidity channel.

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Appendix

Expression for Recovery Rates

In keeping with the data, we measure recovery rates in the model as the market value of defaulted debt as a percentage of its face value (or par). In the financial accelerator model of BGG, there is a continuum of borrowers (or entrepreneurs), indexed by (j) who purchase raw capital, $\bar{K}$, at a unit price of Q. The entrepreneur j uses his net worth, $N(j)$, and a one-period loan $B(j)$ from a financial intermediary to purchase his desired level of capital. The entrepreneur is subject to an aggregate return, R^k, and an idiosyncratic return, ω, where $\log(\omega) \sim \mathcal{N}(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2)$ so that the mean of ω is equal to 1. We denote by $f(\omega)$ and by $F(\omega)$ the probability density function and cumulative distribution function of ω, respectively. Thus, the value of the entrepreneur's assets ex-post is $Q\bar{K}(j)R^k\omega$. The loan obtained by the entrepreneur takes the form of a standard debt contract, where Z denotes the promised gross rate of return on the loan. Let, $\bar{\omega}$, be the value of ω below which an entrepreneur is not able to repay the principal and the interest on the loan. This cutoff is defined by

$$B(j)Z = Q\bar{K}(j)R^k\bar{\omega} \tag{31}$$

Entrepreneurs with $\omega < \bar{\omega}$ are not able to refinance and, hence, declare bankruptcy. Due to bankruptcy costs, the financial intermediary is only able to recover a fraction $(1-\mu)$ of the entrepreneur's asset. Thus the average recovery rate, conditional on default is given by:

$$R_c = \int_0^1 \int_0^\infty \frac{(1-\mu)\omega Q\bar{K}(j)R^k}{F(\bar{\omega})B(j)Z} dF(\bar{\omega})dj \tag{32}$$

Now multiply both the numerator and denominator by $\bar{\omega}$, and substitute out for $B(j)Z$ using (31) to obtain

$$R_c = \int_0^1 \int_0^\infty \frac{(1-\mu)\omega}{F(\bar{\omega})\bar{\omega}} dF(\bar{\omega})dj = \frac{(1-\mu)G(\bar{\omega})}{F(\bar{\omega})\bar{\omega}} \tag{33}$$

where $G(\bar{\omega}) \equiv \int_0^\infty \omega dF(\omega)$.

Derivation of θ_t

Here we derive an analytical expression for θ. Consider an entrepreneur with net worth $N_t(j)$ His returns are given by

$$N_t(j) = [\omega_t R_t^k N_{t-1}(j)\kappa_{t-1} - (\kappa_{t-1} - 1)Z_t] \tag{34}$$

where κ is the common leverage across entrepreneurs. The new amount of capital chosen by the entrepreneur is $\bar{K}_{t+1}(j) = N_t(j)\kappa_t/Q_t$, so we have

$$\bar{K}_{t+1} = [\omega_t R_t^k N_{t-1}(j)\kappa_{t-1} - N_{t-1}(j)(\kappa_{t-1} - 1)Z_t]\frac{\kappa_t}{Q_t} \tag{35}$$

and $\bar{K}_t(j) = N_{t-1}(j)k_{t-1}/Q_{t-1}$. Thus, net purchases or sales of capital for entrepreneur j are equal to

$$\bar{K}_{t+1}(j) - \bar{K}_t(j) = [\omega_t R_t^k N_{t-1}(j)k_{t-1} - N_{t-1}(j)(k_{t-1} - 1)Z_t]\frac{\kappa_t}{Q_t} - N_{t-1}(j)\frac{\kappa_{t-1}}{Q_{t-1}} \tag{36}$$

Define $\tilde{\omega}$ as the value of ω for which an entrepreneur neither buys nor sells capital. $\tilde{\omega}$ is pinned down by

$$0 = [\tilde{\omega}_t R_t^k N_{t-1}(j)\kappa_{t-1} - N_{t-1}(j)(\kappa_{t-1} - 1)Z_t]\frac{\kappa_t}{Q_t} - N_{t-1}(j)\frac{\kappa_{t-1}}{q_{t-1}} \tag{37}$$

Then

$$\tilde{\omega}_t R_t^k N_{t-1}(j)k_{t-1} = (N_{t-1}(j)(k_{t-1} - 1)Z_t)\frac{\kappa_t}{Q_t} + N_{t-1}(j)\frac{\kappa_{t-1}}{Q_{t-1}} \tag{38}$$

Using this last expression, we can rewrite (36) as

$$\bar{K}_{t+1}(j) - \bar{K}_t(j) = R_t^k N_{t-1}(j)\kappa_{t-1}(\omega_t - \tilde{\omega}_t) \tag{39}$$

Summing across all entrepreneurs and taking into account the the new entrants as well as those who leave business we can write

$$\theta_t = \frac{\int_0^1 \max[\bar{K}_t(j) - \bar{K}_{t+1}(j), 0]dj}{\int_0^1 \max[\bar{K}_{t+1}(j) - \bar{K}_t(j), 0]dj} \tag{40}$$

$$= \frac{-\gamma_t R_t^k N_{t-1}\kappa_{t-1}\frac{\kappa_t}{Q_t}\int_0^{\tilde{\omega}_t}(\omega - \tilde{\omega}_t)dF(\omega) + (1-\gamma_t)N_{t-1}\kappa_{t-1}R_t^k\frac{\kappa_t}{Q_t}}{\gamma_t R_t^k N_{t-1}\kappa_{t-1}\frac{\kappa_t}{Q_t}\int_{\tilde{\omega}_t}^{\infty}(\omega - \tilde{\omega}_t)dF(\omega) + W_t^e\frac{\kappa_t}{Q_t}} \tag{41}$$

$$= \frac{-\gamma_t R_t^k N_{t-1}\kappa_{t-1}\int_0^{\tilde{\omega}_t}(\omega - \tilde{\omega}_t)dF(\omega) + (1-\gamma_t)N_{t-1}\kappa_{t-1}R_t^k}{\gamma_t R_t^k N_{t-1}\kappa_{t-1}\int_{\tilde{\omega}_t}^{\infty}(\omega - \tilde{\omega}_t)dF(\omega) + W_t^e} \tag{42}$$

$$= \frac{-\gamma_t \int_0^{\tilde{\omega}_t}(\omega - \tilde{\omega}_t)dF(\omega) + (1-\gamma_t)}{\gamma_t \int_{\tilde{\omega}_t}^{\infty}(\omega - \tilde{\omega}_t)dF(\omega) + \frac{W_t^e}{R_t^k N_{t-1}\kappa_{t-1}}} \tag{43}$$

$$= \frac{-\gamma_t(\Gamma_t(\tilde{\omega}_t) - \tilde{\omega}) + 1 - \gamma_t}{\gamma_t(1 - \Gamma(\tilde{\omega}_t)) + \frac{W_t^e}{R_t^k N_{t-1}\kappa_{t-1}}} \tag{44}$$

where

$$\tilde{\omega}_t = \frac{\frac{\kappa_{t-1}}{\kappa_t}\frac{Q_t}{Q_{t-1}} + (\kappa_{t-1} - 1)Z_t}{R_t^k \kappa_{t-1}} = \bar{\omega}_t + \frac{1}{\kappa_t}\frac{Q_t}{Q_{t-1}R_t^k} \tag{45}$$

and

$$\Gamma(\tilde{\omega}) = \int_0^{\tilde{\omega}} \omega f(\tilde{\omega})d\omega + \tilde{\omega}(1 - F(\tilde{\omega})) \tag{46}$$

$$= \Phi\left(\frac{\log(\tilde{\omega})}{\sigma} - \frac{\sigma}{2}\right) + \tilde{\omega}\left[1 - \Phi\left(\frac{\log(\tilde{\omega})}{\sigma} + \frac{\sigma}{2}\right)\right] \tag{47}$$

where $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution.

Tables

Table 1: Calibration - Parameters Related to the Steady State

Name	Description	Value
β	Discount rate	0.9987
σ_L	Inverse Frisch elasticity of labor supply	1
Ψ_L	Disutility weight on labor	0.7705
b	Habit formation	0.74
λ_w	Steady-state mark-up for suppliers of labor	1.05
λ_f	Steady-state mark-up for intermediate goods firms	1.2
μ_z	Mean growth rate of unit root technology process	0.41
Υ	Steady-state rate of investment-specific technological change	0.42
δ	Capital depreciation rate	0.025
α	Share of capital in production function	0.4
γ	Fraction of entrepreneurial net worth retained	0.985
μ	Steady-state monitoring costs	0.21
σ	Steady-state standard deviation of idiosyncratic productivity	0.26
W^e	Transfers received by entrepreneurs	0.005
η_g	Share of government spending in GDP in steady state	0.2
π^{target}	Steady-state inflation rate (APR)	2.43
τ^c	Tax rate on consumption	0.05
τ^k	Tax rate on capital income	0.32
τ^l	Tax rate on labor income	0.24

Table 2: Calibration - Other Parameters

Name	Description	Value
ξ_w	Calvo wage stickiness	0.81
σ_a	Curvature, utilization cost	2.54
S''	Curvature, investment adjustment cost	10.78
ξ_p	Calvo price stickiness	0.74
α_π	Policy weight on inflation	2.4
ρ_p	Policy smoothing parameter	0.85
ι	Price indexing weight on inflation target	0.90
ι_w	Wage indexing weight on inflation target	0.49
Υ	Wage indexing weight on technology shock	0.94
$\alpha_{\Delta y}$	Policy weight on output growth	0.36
φ	Elasticity of liquidation costs	16
$\rho_{\sigma,n}$	Correlation among signals	0.39
ρ_{λ_f}	Autocorrelation, price markup shock	0.91
ρ_{μ_Υ}	Autocorrelation, IST shock	0.99
ρ_g	Autocorrelation, government spending shock	0.94
ρ_{μ_z}	Autocorrelation, persistent technology	0.15
ρ_ϵ	Autocorrelation, transitory technology	0.81
ρ_σ	Autocorrelation, risk shock	0.97
ρ_{ζ_c}	Autocorrelation, preference shock	0.90
ρ_{ζ_I}	Autocorrelation, MEI shock	0.91
Standard deviations		
$\sigma_{\sigma,n}$	Anticipated risk shock	0.028
$\sigma_{\sigma,0}$	Unanticipated risk shock	0.07
σ_{λ_f}	Price markup shock	0.011
σ_{μ_Υ}	IST shock	0.004
σ_g	Government spending shock	0.023
σ_{μ_z}	Persistent technology shock	0.0071
σ_γ	Survival probability shock	0.0081
σ_γ	Temporary technology shock	0.0046
σ_{ϵ^p}	Monetary policy shock	0.49
σ_{ζ_c}	Preference shock	0.023
σ_{ζ_I}	MEI shock	0.055
σ_{ζ_I}	Measurement error, net worth	0.018

Figures

Figure 1: Moody's Recovery Rates (left axis) and U.S. real GDP growth (right axis).

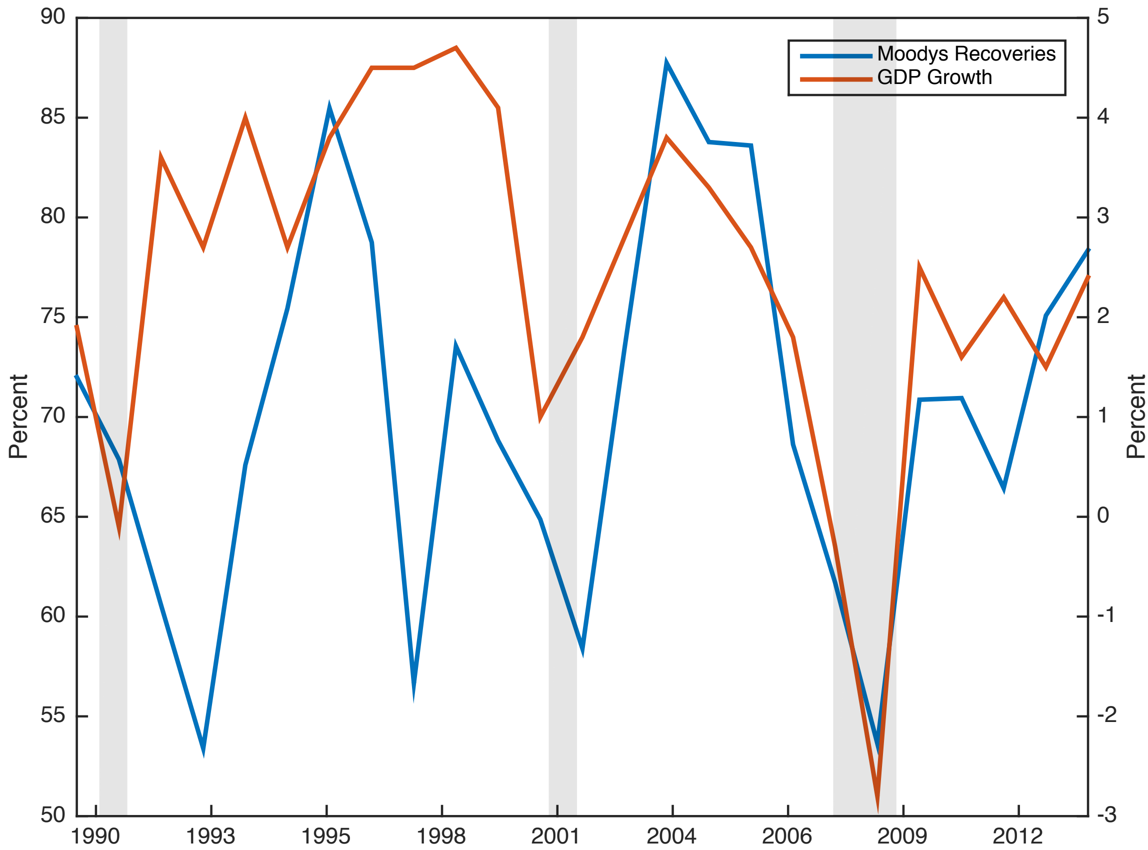

Figure 2: Moody's Recovery Rates (left axis) and Business Loan Delinquency Rates from FRED (right axis).

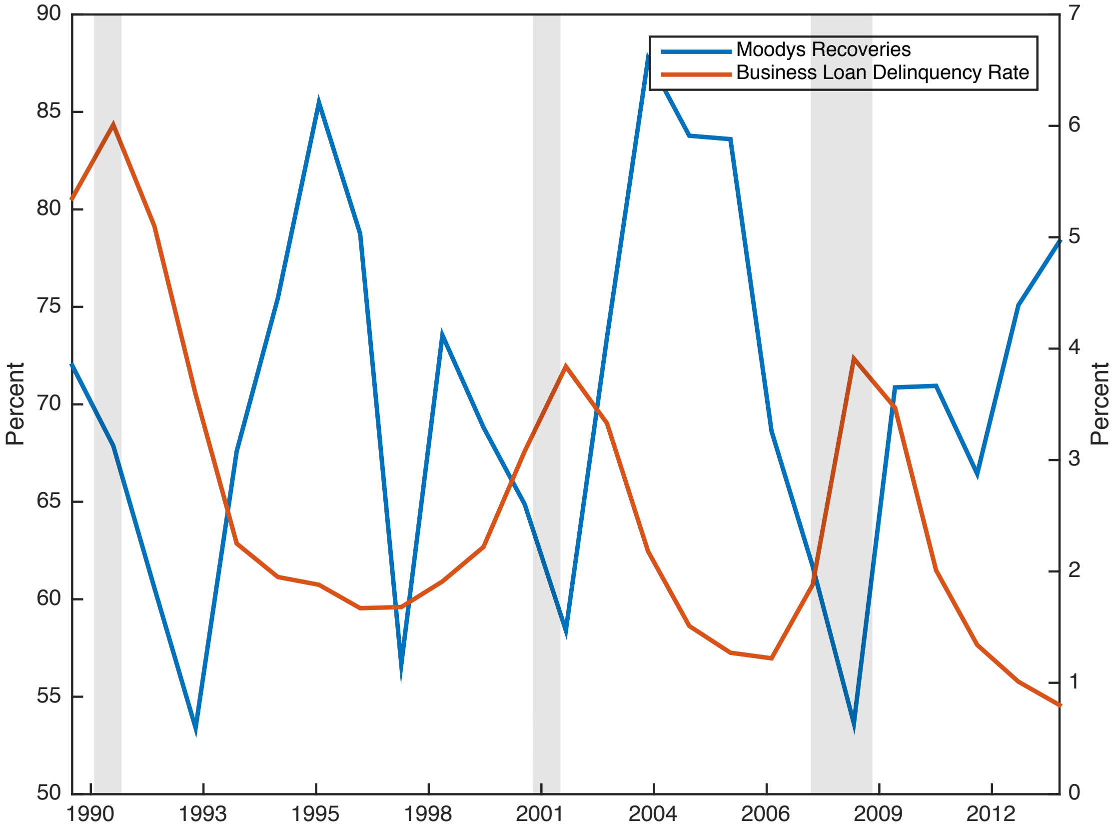

Figure 3: Recovery rates from Christiano, Motto and Rostagno(2014) and from Moody's.

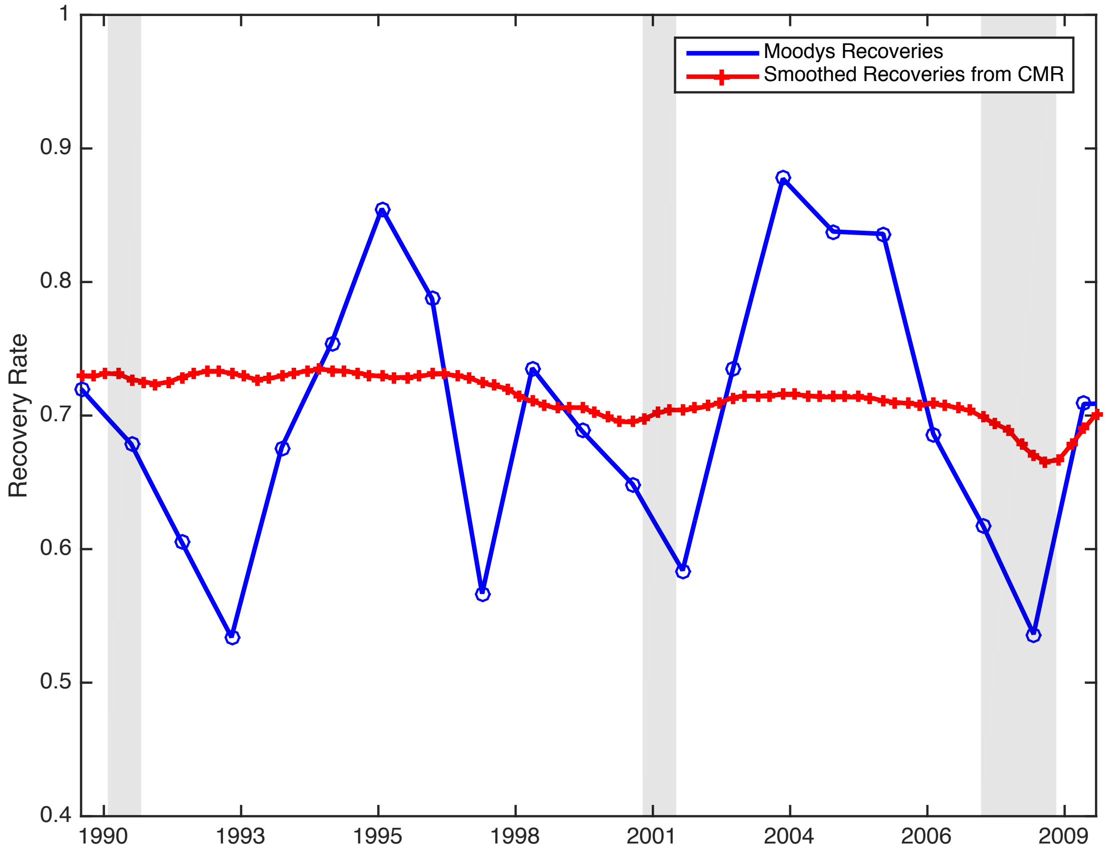

Figure 4: Effect of Entreprenerial Exit Shocks on the Economy in the Baseline CMR and in the Model with Time-Varying Liquidation Costs (CD).

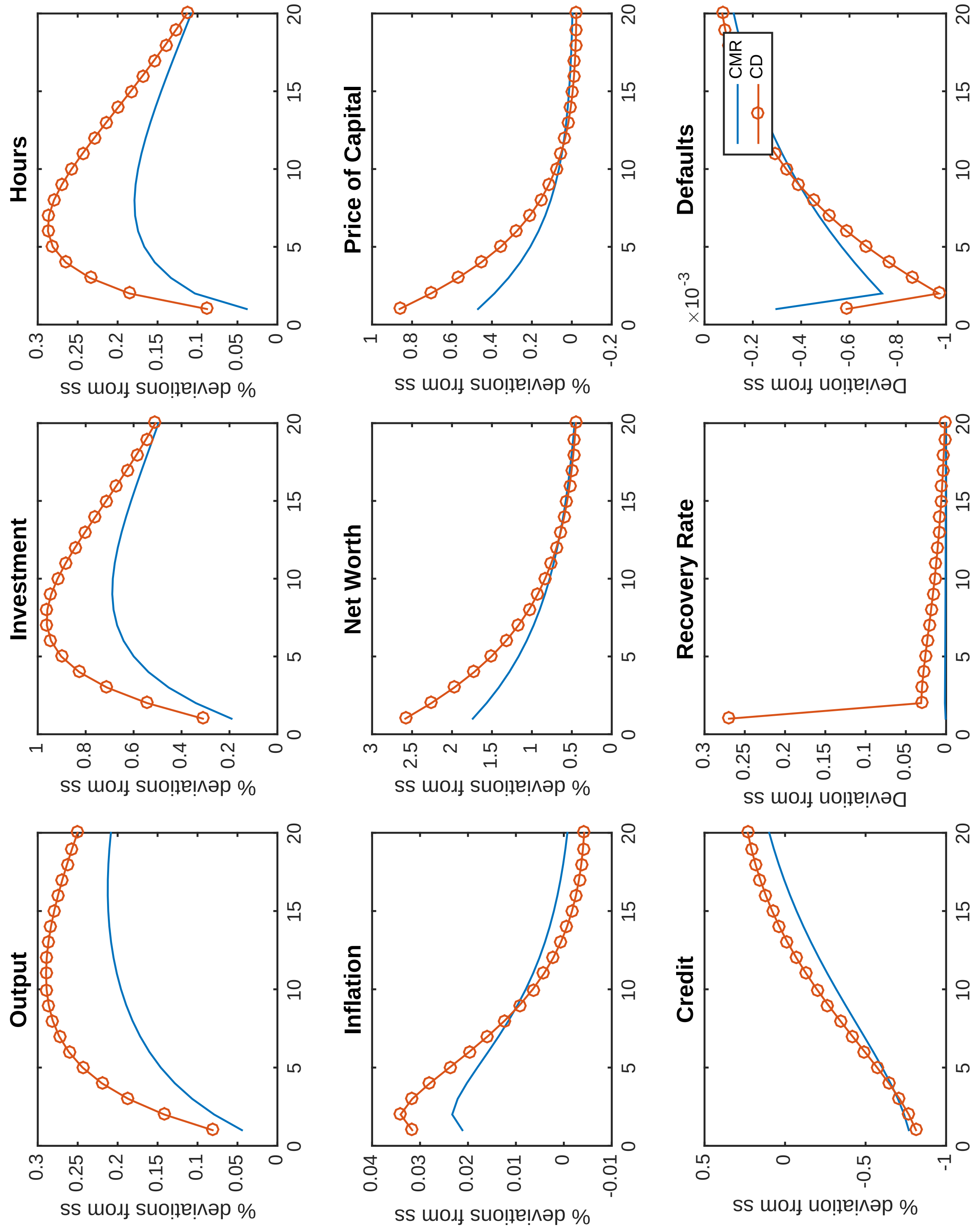

Figure 5: Effect of Risk Shocks on the Economy in the Baseline CMR and in the Model with Time-Varying Liquidation Costs (CD).

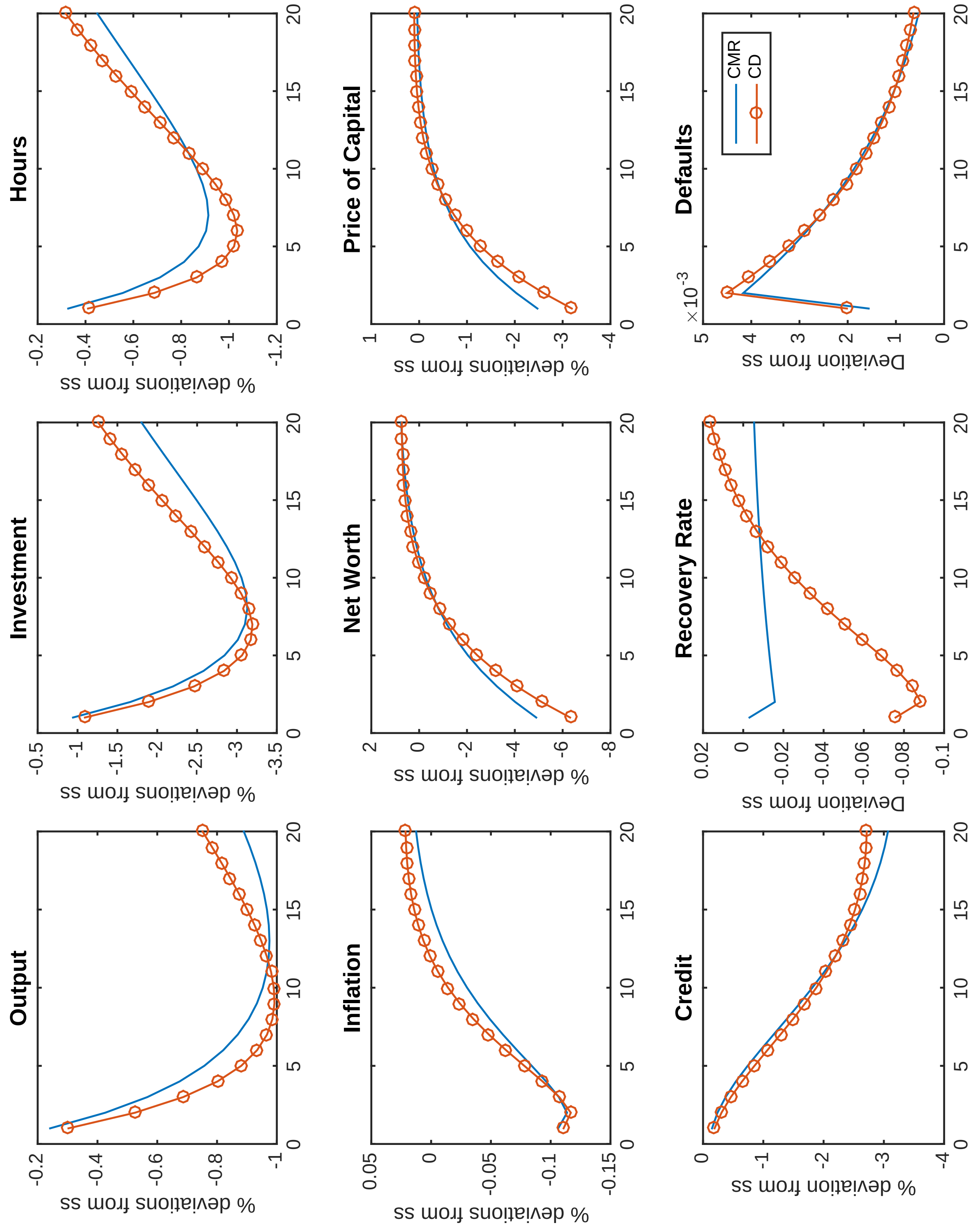

Figure 6: Effect of Monetary Shocks on the Economy in the Baseline CMR and in the Model with Time-Varying Liquidation Costs (CD).