Firms, Failures, and Fluctuations*

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The production of goods and services in any modern economy is organized around complex, interlocking supply chains, as firms rely on a variety of different inputs for production. This observation has led to a growing literature that studies the role of input-output linkages as a mechanism for the propagation and amplification of shocks and whether microeconomic shocks can result in sizable fluctuations at the aggregate level.

Even though the input-output linkages are formed and are operational at the level of individual firms, by focusing on competitive or monopolistically competitive frameworks, most models serve as better approximations to the nature of interactions at the level of industries, thus ignoring important firm-level issues such as market power, firm-specific relationships, and endogenous bankruptcies.

This paper develops a theory of firm-level production networks, with firm-specific relationships, endogenous bankruptcies, and market power. Firms in each industry have access to a production technology that uses relationship-specific intermediate inputs produced by their "customized" suppliers, with prices determined via pairwise bargaining between suppliers and customers. Operating the customized technology, however, requires paying fixed costs of entry. Hence, negative shocks can result in a cascade of firm failures in the economy.

We establish the existence of an equilibrium and provide comparative static results on how prices, firm failures, and macroeconomic aggregates respond to changes in parameters. We then study how the interplay between firm-level linkages and firm failures shape the propagation of shocks over the economy's production network. Our theoretical results indicate that understanding network-originated aggregate fluctuations may require moving beyond models of sectoral linkages and focusing on how firm-level interactions can lead to chains of failures.

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Input-Output Linkages and Propagation of Shocks

- Modern economies organized as complex production networks:
  - Expenditure on intermediate goods & services in the U.S. $\approx$ 1 GDP.

- A growing literature argues that input-output linkages...
  (i) function as mechanism for propagation & amplification of shocks (micro);
  (ii) can translate micro shocks into aggregate fluctuations (macro).

- Even though linkages are between firms, most models...
  (i) focus on interactions at the industry level;
  (ii) ignore the possibility of firm failures (all the action is at the intensive margin)

Firm-Level Linkages

- In reality, failures of firms' suppliers and customers can be first order"
  - the U.S. auto industry in 2008–09
  - bankruptcies due to spillovers over credit linkages (Jacobson and Von Schedvin, 2015)
  - the aftermath of the Great East Japan Earthquake (Carvalho et al., 2016)

- Firm-level linkages are highly persistent:
  - Chile: median firm retains 41% and 46% of its suppliers and customers from year to year (Huneeus, 2018).
  - Japan: similar persistence patterns

- But if there is a lot of action at the firm-level, the sectoral focus can miss the most important elements.

This "Paper"

- A theoretical model of firm-level interactions with (i) firm-specific relationships, (ii) endogenous bankruptcies, and (iii) market power.
  - Failures are the main channel via which negative shocks propagate.

- Study how firm-level linkages and firm failures shape the nature of how shocks propagate in the economy and impact aggregate fluctuations.

- The aggregated economy at the sectoral level is isomorphic to an industry-level model with distortions, but these distortions are *endogenous* and depend on the extent of firm failures.

- Main take-away: to understand network-originated fluctuations, we may have to go beyond sectoral linkages and study how firm-level interactions cause chains of failures.

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- Main take-away: to understand network-originated fluctuations, we may have to go beyond sectoral linkages and study how firm-level interactions cause chains of failures.

Related Literature

- Production networks and the origins of aggregate fluctuations
  - Long and Plosser (1983); Horvath (1998, 2000); Acemoglu et al. (2012, 2017); Atalay (2017); Baqaee (2018); Baqaee and Farhi (2017), and many more...
  - Jones (2013), Bigio and La'O (2018), Baqaee and Farhi (2018), Liu (2018)

- Endogenous production networks
  - Carvalho and Voigtländer (2014); Oberfield (2018); Acemoglu and Azar (2018)

- Empirical evidence
  - Acemoglu et al. (2016); Barrot and Sauvagnat (2016); Carvalho et al. (2016)

- Models of firm-level interactions
  - Taschereau-Dumouchel (2018); Tintelnot et al. (2018); Kikkawaa et al. (2018)

Model

- An economy with $N+1$ industries.
    - industries $1,\dots,N$ produce intermediate goods
    - industry 0 produces the final good.

- Each industry $I \in \{1,\dots,N\}$ consists of two types of firms
    - a competitive fringe of firms producing a generic variant of the good
    - a unit mass of specialized firms producing specialized/customized inputs

- A unit mass of households
    - log utilities over the final good
    - one unit of labor supplied inelastically

Generic Inputs

- A competitive fringe of firms in each industry produces a generic variant of the good using labor and other generic inputs.

- Constant returns to scale technology:

$$\widetilde{y}_I = F_I(\widetilde{\ell}_I, B_{I1}\widetilde{x}_{I1}, \ldots, B_{IN}\widetilde{x}_{IN})$$

  - generic good producers can be indexed by the industry they belong to
  - $\widetilde{x}_{IJ}$: quantities
  - B_{IJ}: productivity shock

- Production of generic goods can be represented by an *industry-level* network

Customized Inputs

- Specialized firms can produce inputs that are customized to specific customers.
- Firms are matched to potential suppliers via a matching function $\phi_{IJ} : I \to J \cup \{\emptyset\}$

$$\phi_{IJ}(i) := \begin{cases} j & j \in J \text{ is a matched supplier of } i \in I \\ \emptyset & i \in I \text{ is not matched to a supplier in } J \end{cases}$$

- Each firm can be matched to...
  - suppliers in its input-producing industries;
  - at most one supplier in any given industry;
  - at most one customer in the entire economy.
- Not all customized firms may be active.

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Customized Inputs

- Let S denote the set of active firms.
- Production technology of firm $i \in I$:

$$y_i = F_I\left(\ell_i, \{A_{ij}x_{ij}\}_{j\in S}, \{B_{ij}\tilde{x}_{ij}\}_{j\notin S}\right).$$

Assumption

Customized inputs are more productive than generic inputs:

$$A_{ij} \geq B_{ij}.$$

Customized Inputs

- Production of customized goods entails fixed costs, borne by the supplier j.
- z_j units of labor, where $z_j \sim G_J$ and G_J has full support over $[0, \infty)$.
- Costs are sunk once the firm customizes its technology to its matched customer.

- Suppliers that cannot meet this fixed cost "fail".
  - The set of active firms S may be different from the set of all firms.
  - S is determined endogenously

Customized Inputs: Terms of Trade

- A pair of matched firms (i, j).

- Price p_{ij} is determined as the SPNE of the Rubinstein bargaining game:
  - the supplier j makes an offer with probability δ_{ij}
  - the customer i makes an offer with probability $1 - \delta_{ij}$
  - common discount factor $\eta \to 1$

- Commitment by j to deliver as many units demanded by i at price p_{ij}.

- Remarks:
  - Customer i has the outside option of using the generic variant of input J
  - When negotiating, firms take all other (generic and customized) prices as given.

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Consumption Good Sector

- A firm combining outputs from various firms in an industry into industry-level bundles, which are then combined into a single consumption good:

$$x_{0I} = H_I\left((x_{0i})_{i\in\phi_{0I}(0)}\right)$$
$$y_0 = F_0(x_{01},\ldots,x_{0N})$$

- Generic variants from industry I are perfect substitutes for all goods produced using the customized technologies
- The supplier has all the bargaining power $\delta_{0i} = 1$.
- Various inputs in H_I are gross complements.

Summary and Timing

- Timing:
  (1) Firms are matched with potential suppliers/customers in other industries.
  (2) Productivities A_{IJ} and B_{IJ} and customization costs z_i are realized.
  (3) Firms decide whether to operate the customized technology.
  (4) Customized firm-level p_{ij} prices are determined.
  (5) Firms choose their inputs, production occurs.
  (6) Customized firms that cannot cover their fixed costs fail.

Production Equilibrium

Definition

Given the set of active firms S and customized prices $\boldsymbol{p}$, a *production equilibrium* is a collection of quantities $x(S, \boldsymbol{p})$ such that

(a) all firms maximize their profits while meeting their output obligations;

(b) households maximize profits taking prices as given;

(c) all markets clear.

- The equilibrium notion treats prices as exogenous.
- The only requirement on the firms is to minimize production costs to meet demand from their customized customers.

Bargaining Equilibrium

Definition

For a set of active firms S, a *bargaining equilibrium* is a collection of prices $\boldsymbol{p}(S) = (p_{ij})_{i,j \in S}$ such that there does not exist a pair of matched supplier-customer firms in E such that one party would rather deviate by entering into a bargaining process with the other, taking all other prices as given.

- For $\boldsymbol{p}$ to be a bargaining equilibrium, no firm would want to unilaterally
  (i) renegotiate a price
  (ii) terminate an agreement
  (iii) enter into a new agreement

Full Equilibrium

Definition

A *full (subgame perfect) equilibrium* consist of a collection of active firms S^*, firm-level prices $\boldsymbol{p}^*(S)$, and quantities $x^*(S, \boldsymbol{p})$ such that

(a) given any S and $\boldsymbol{p}$, the quantities $x^*(S, \boldsymbol{p})$ form a production equilibrium;

(b) given any S, the price vector $\boldsymbol{p}^*(S)$ is a bargaining equilibrium;

(c) no firm in S^* fails and no firm outside of S^* would rather start operating:

$$\begin{aligned} \pi_i(S^*) &\geq 0 \qquad && \forall i \in S^* \\ \pi_i(S^* \cup \{i\}) &< 0 \qquad && \forall i \notin S^*. \end{aligned}$$

- Solve the model using backward induction.

Generic Inputs

- Generic inputs are produced by a competitive fringe of firms in each industry.
- Competitive sub-economy with constant returns to scale & a single factor of production

Non-substitution theorem $\longrightarrow$ generic prices determined irrespective of the matching, relationship-specific productivities, bargaining, etc.

- System of N equations and N unknowns:

$$\widetilde{p}_I = c_I\left(w, \frac{\widetilde{P}_1}{B_{I1}}, \frac{\widetilde{P}_2}{B_{I2}}, \dots, \frac{\widetilde{P}_N}{B_{IN}}\right).$$

- Generic technologies also pin down the real wage.

Production Equilibrium

- Given the set of active firms S and generic and customized prices, the production equilibrium is determined via cost minimization, market clearing, and household's utility maximization.

- cost minimization:

$$(\ell_i, x_{i1}, \ldots, x_{in}) = \arg\max \quad w\ell_i + \sum_{j\in S} p_{ij}x_{ij} + \sum_{j\notin S} \tilde{p}_j x_{ij}$$
$$\text{subject to} \quad y_i = F_i(\ell_i, (A_{ij}x_{ij})_{j\in S}, (B_{ij}\tilde{x}_{ij})_{j\notin S})$$

- market clearing:

$$y_j = x_{ij}$$
$$\tilde{y}_J = \int_{i\in S, j\notin S} \tilde{x}_{ij}\, di + \sum_I \tilde{x}_{IJ}$$

- Household budget constraint:

$$y_0 = wL + \sum_{J=1}^{N} \int_0^\infty \pi_j\, dj$$

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Assumption

(Generic or customized) inputs from different industries are gross complements with another and labor in F_i.

$$y_i = F_i \left(\ell_i, \{A_{ij} x_{ij}\}_{j \in S}, \{B_{ij} \tilde{x}_{ij}\}_{j \notin S} \right).$$

Bargaining Equilibrium: Existence and Uniqueness

Theorem

For any given set of active firms S,

(a) *a bargaining equilibrium* $\boldsymbol{p}(S)$ *always exists;*

(b) *the vector of prices* $\boldsymbol{p}(S)$ *is determined independently of the quantities.*

Theorem

Suppose either $\delta_{ij} = 1$ *or production functions are Leontief. Then, the bargaining equilibrium is unique.*

Bargaining Equilibrium: Comparative Statics

Proposition

Suppose production functions are Leontief. If $S \subseteq S'$, then

$$p_i(S') - c_i(S') \geq p_i(S) - c_i(S)$$

Proposition

Suppose production functions are Leontief. An increase in productivity A_{ij}

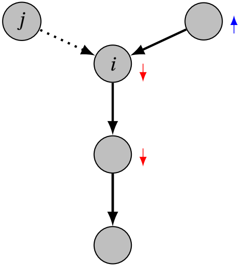
(a) *weakly increases all prices upstream to j*

(b) *weakly decreases all prices downstream to i*

(c) *weakly increases all prices that are horizontal to the pair (j, i).*

Bargaining Equilibrium: Comparative Statics

- Benefits of higher productivity or more entry spill over to all other firms in the form of bigger difference between marginal cost and price.



Proof: Pairwise Bargaining

Lemma

Supplier-customer pair (j, i) reach an agreement if and only if $\boldsymbol{p}_{-ij}$ satisfies

$$c_i(\boldsymbol{p}_{-ij}, c_j(\boldsymbol{p}_{-ij})) \leq p_i.$$

- It guarantees that there are gains from trade between supplier j and customer i.
- Firm i's marginal cost is smaller than its output price if j sells at marginal cost.
- If violated, there are no gains from trade: the two firms would take the outside option of not trading with one another.

Pairwise Bargaining

Lemma

Suppose $c_i(\boldsymbol{p}_{-ij}, c_j(\boldsymbol{p}_{-ij})) \leq p_i$. The SPNE of the bargaining game entails the price

$$p_{ij} = \begin{cases} p_{ij}^{\dagger} & \text{if } \psi_{ij}(\min\{\bar{p}_{ij}, p_{ij}^{o}\}) \geq 0 \\ \min\{\bar{p}_{ij}, p_{ij}^{o}\} & \text{if } \psi_{ij}(\min\{\bar{p}_{ij}, p_{ij}^{o}\}) < 0 \end{cases},$$

where $p_{ij}^{\dagger}$ is the solution to the equation

$$\psi_{ij}(p_{ij}^{\dagger}) = \delta_{ij} \frac{\bar{\pi}_i(p_{ij}^{\dagger})}{\bar{\pi}_i'(p_{ij}^{\dagger})} + (1 - \delta_{ij}) \frac{\bar{\pi}_j(p_{ij}^{\dagger})}{\bar{\pi}_j'(p_{ij}^{\dagger})} = 0,$$

$\bar{p}_{ij} = A_{ij}\widetilde{p}_j / B_{ij}$ is the outside option of firm i, and p_{ij}^{o} is the price at which firm i makes zero profits.

- The price depends on the production functions, but not the quantities.
- $\psi_{ij}(\min\{\bar{p}_{ij}, p_{ij}^{o}\}) \geq 0$ is the condition that implies outside options do not bind.
- Otherwise, firm i either uses the generic variant or may not produce at all.

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Pairwise Bargaining and Bargaining Equilibrium

$$\psi_{ij}(p_{ij}) = \delta_{ij} \frac{\bar{\pi}_i(p_{ij})}{\bar{\pi}_i'(p_{ij})} + (1 - \delta_{ij}) \frac{\bar{\pi}_j(p_{ij})}{\bar{\pi}_j'(p_{ij})}$$

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- Can now use Kakutani's fixed point theorem to establish the existence of a bargaining equilibrium.
- p_{ij} is increasing in supplier bargaining power δ_{ij}, supplier's marginal cost c_j, customer's output price p_i, and the relationship-specific productivity A_{ij}.
- p_{ij} is decreasing in the productivity of generic technology B_{ij}.

Bargaining Equilibrium: $\delta_{ij} = 1$

- $\delta_{ij} = 1$ for all supplier-customer pairs (j, i):

$$p_{ij} = \frac{A_{ij}}{B_{ij}} \widetilde{p}_j$$

$$\mu_{ij} = p_{ij} / c_j = A_{ij} / B_{ij}.$$

- In this case, an increase in productivity A_{ij} only manifests itself as higher markups by the supplier firm:

$$\frac{d \log \mu_{ij}}{d \log A_{ij}} = 1$$

$$\frac{d \log p_{kr}}{d \log A_{ij}} = 0.$$

- Unlike a model with fixed markups, productivity shocks do not change any of the other prices $\rightarrow$ no pass-through.

Bargaining Equilibrium: Leontief Production Function

- Leontief production functions

$$y_i = \min \left\{ l_i, \{A_{ij} x_{ij}\}_{j \in S}, \{B_{ij} \tilde{x_{ij}}\}_{j \notin S} \right\},$$

- Equilibrium prices:

$$p_{ij} = \min \left\{ (1 - \delta_{ij}) c_j + \delta_{ij} A_{ij} \left(p_i - \sum_{k \neq j} \frac{p_{ik}}{A_{ik}} \right), \frac{A_{ij}}{B_{ij}} \widetilde{p}_j \right\},$$

Full Equilibrium

Definition

A *full (subgame perfect) equilibrium* consist of a collection of active firms S^*, firm-level prices $\boldsymbol{p}^*(S)$, and quantities $x^*(S, \boldsymbol{p})$ such that

(a) given any S and $\boldsymbol{p}$, the quantities $x^*(S, \boldsymbol{p})$ form a production equilibrium;

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$$\pi_i(S^*) \geq 0 \qquad \forall i \in S^*$$
$$\pi_i(S^* \cup \{i\}) < 0 \qquad \forall i \notin S^*.$$

Theorem

Suppose either $\delta_{ij} = 1$ for all (i,j) or all production functions are Leontief. Then,

(a) *a full equilibrium exists;*

(b) *the set of full equilibria is non-empty and has a maximal element;*

(c) *the set of active firms in the maximal equilibrium shrinks as fixed costs increase.*

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Proof Sketch

- Best response function:

$$\psi(S) = \{i \in E : \pi_i(S \cup \{i\}) \geq 0\}.$$

  Firm $i \in \psi(S)$ if i finds it optimal to operate the customized technology when the set of active firms is S.

- Full equilibrium: $S^* = \psi(S^*)$.

- Firm entry decisions are not strategic complements: new firm entry requires reallocation of resources from production to paying fixed entry costs, which may reduce other firms' profits. In other words,

$$S_1 \subseteq S_2 \not\Rightarrow \psi(S_1) \subseteq \psi(S_2).$$

- Cannot use Tarski's fixed point theorem!

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A Different Fixed Point Theorem

Theorem

Suppose the mapping $\psi : 2^E \to 2^E$ *satisfies the following:*

(i) *if* $S_1 \subseteq \psi(S_2)$, *then* $S_1 \subseteq \psi(S_1 \cup S_2)$.

(ii) *if* $S_r \subseteq \psi(S_r)$ *for* $r \in R$, *then* $\cup_{r\in R} S_r \subseteq \psi(\cup_{r\in R} S_r)$.

Then, $S^* = \cup_{S\in\mathcal{S}} S$ *is a maximal fixed point of* ψ, *where* $\mathcal{S} = \{S \subseteq E : S \subseteq \psi(S)\}$.
Furthermore, if $\psi_1(S) \subseteq \psi_2(S)$ *for all* S, *then* $S_1^* \subseteq S_2^*$.

- When $\delta_{ij} = 1$ or production functions are Leontief the best response mapping $\psi(S) = \left\{i \in E : \pi_i(S \cup \{i\}) \geq 0\right\}$ satisfies these conditions.
- Therefore, we get existence, existence of a maximal fixed point, and monotonicity.

Comparative Statics

Corollary

If G_I is replaced by a distribution that first-order stochastically dominates G_I, then

(a) *the likelihood of failure in all industries weakly increases;*

(b) *aggregate output declines;*

(c) *more firms stop operating.*

- PE effect: an increase in the likelihood of failures in other industries
- GE effect: less demand for all goods in the economy, thus lower (gross) profits

Firm-Level Analysis: Failure Propagations

- The matching ϕ induces a distribution over firm-level production trees:

$$Q(I_0, I_1, \ldots, I_k) = \left\{ i_0 \in I_0 : \exists (i_1, \ldots, i_k) \text{ s.t. } i_r = \phi_{I_{r+1} I_r}(i_{r+1}) \text{ and } \nexists j \text{ s.t. } i_k = \phi_{J I_k}(j) \right\}$$

  - ▷ $Q(I_0)$: set of firms in I_0 with no matched customers.
  - ▷ The firm-level trees can be infinitely long ($k = \infty$).
  - ▷ Not sufficient statistics for firm-level variables, but nonetheless useful.

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Failure Propagation

- A specialized firm $i \in I$ survives only if its profits exceed the fixed costs wz_i, where $z_i \sim G_I$.
- Any revenue a firm earns is obtained from sales to its matched customer in a downstream industry.
- Therefore, as long as G_I has no mass point at 0, firm $i \in I$ fails almost surely if its designated customer fails.

$\rightarrow$ Implication: failures propagate upstream from a firm to its suppliers
(may also propagate downstream depending on parameters)

Failure Propagation

Lemma

An intermediate good producing firm $i_0 \in I_0$ fails if either of the following two conditions are satisfied:

(i) $i_0 \in Q(I_0, I_1, \ldots, I_k)$ *for some finite k such that* $I_k \neq 0$.

(ii) $i_0 \in Q(I_0, I_1, \ldots, I_k)$ *for* $k = \infty$.

- A firm can only survive if there is a finite production tree connecting it to the consumption good sector.
  - ▷ We can limit our attention to such structures.

Failure Propagation

- Suppose $\delta_{ij} = 1$. Then, failures only propagate upstream.
- Suppose $\delta_{ij} < 1$. Then, failures propagate both upstream and downstream.

Decomposition

- So far: existence, (partial) characterization, and (micro) propagations
- Next step: aggregate implications

Partial Equilibrium Decomposition

- $S = \cup_{r \in R} T_r$: partition of the set of firms to various production trees
- $s_r = m(T_r)$: the mass of active firms in tree T_r

- Partial equilibrium decomposition:

$$d \log \text{GDP} = \underbrace{\sum_{(i,j)} \frac{\partial \log \text{GDP}}{\partial \log A_{ij}} d \log A_{ij}}_{\text{productivity}} + \underbrace{\sum_{r \in R} \frac{\partial \log \text{GDP}}{\partial s_r} ds_r}_{\text{extensive margin movements}}$$

- productivity: direct technology effect + reallocation + changes in markups
- extensive margin: changes in the set of active firms changes the production possibility frontier, the total expenditure on fixed costs, household demand.

Partial Equilibrium Decomposition: Productivity

- Keeping the set of active firms constant, the effect of productivity shocks manifests itself as two separate terms:

$$\frac{d\log \mathrm{GDP}}{d\log A_{ij}} = \underbrace{\frac{\partial \log \mathrm{GDP}}{\partial \log A_{ij}}}_{\substack{\text{shifts in technology frontier} \\ \text{\& reallocation}}} + \sum_k \underbrace{\frac{\partial \log \mathrm{GDP}}{\partial \log \mu_k} \frac{d\log \mu_k}{d\log A_{ij}}}_{\text{endogenous changes in markup of firm } k}$$

- Productivity shocks shift the production possibility frontier, keeping the allocation of resources unchanged (Hulten's)
- When economy is inefficient, reallocation of resources has first-order effect (Baqaee and Farhi, 2018)
- New term: endogenous shifts in markups in the bargaining equilibrium.
  - Depends on the passthrough of the shocks

General Equilibrium Decomposition

- Productivity shocks impact the set of active firms (by changing firm profits)
- Household's budget constraint:

$$y_0 = w(L - z(s)) + (1 - c(s, A, B))y_0,$$

- $c(s, A, B)$: equilibrium marginal cost of producing one unit of consumption good, which depends on the bargaining equilibrium and set of active firms.
- $z(s)$: total fixed cost expenditure, which only depends on the set of active firms

- Hence,

$$\text{GDP} = w\frac{L - z(s)}{c(s, A, B)}.$$

- Chain rule:

$$\frac{d}{dA_{ij}} \log \text{GDP} = -\frac{1}{c}\frac{dc}{dA_{ij}} - \frac{1}{c}\sum_r \frac{ds_r}{dA_{ij}}\left(\frac{w}{\text{GDP}}\frac{dz}{ds_r} + \frac{dc}{ds_r}\right).$$

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General Equilibrium Decomposition

- The set of active firms, however, itself depends on the output.

$$\frac{ds_r}{dA_{ij}} = \frac{\partial s_r}{\partial A_{ij}} + \frac{\partial s_r}{\partial y_0}\frac{d\,\text{GDP}}{dA_{ij}}.$$

  - $\partial s_r/\partial A_{ij}$: higher productivity increases profits
  - $\partial s_r/\partial y_0$: aggregate demand channel. Higher demand increases all firms' profits

- Therefore:

$$\frac{d\log \text{GDP}}{d\log A_{ij}} = -\frac{\dfrac{\partial \log c}{\partial \log A_{ij}} + \sum_r \left(\dfrac{\partial s_r}{\partial \log A_{ij}}\right)\left(\dfrac{w}{c\text{GDP}}\dfrac{dz}{ds_r} + \dfrac{d\log c}{ds_r}\right)}{1 + \sum_r \left(\dfrac{\partial s_r}{\partial \log y}\right)\left(\dfrac{w}{c\text{GDP}}\dfrac{dz}{ds_r} + \dfrac{d\log c}{ds_r}\right)}.$$

General Equilibrium Decomposition

$$\frac{d\log \mathrm{GDP}}{d\log A_{ij}} = -\frac{\overbrace{\frac{\partial \log c}{\partial \log A_{ij}}}^{\text{partial equilibrium }(-)} + \sum_r \left(\overbrace{\frac{\partial s_r}{\partial \log A_{ij}}}^{\text{cascade effect }(+)} \right) \left(\frac{w}{c\mathrm{GDP}} \overbrace{\frac{dz}{ds_r}}^{\text{fixed cost effect }(+)} + \frac{d\log c}{ds_r} \right)}{1 + \sum_r \left(\underbrace{\frac{\partial s_r}{\partial \log y}}_{\text{aggregate demand effect }(+)} \right) \left(\frac{w}{c\mathrm{GDP}} \frac{dz}{ds_r} + \underbrace{\frac{d\log c}{ds_r}}_{\text{entry effect }(-)} \right)}.$$

- Partial equilibrium effect: direct technology effect (Hulten's), reallocation effect, and movements in markups, holding the set of active firms constant
- Cascade effect: shocks change the set of active firms
- Entry effect: changes in the set of active firms changes aggregate productivity
- Aggregate demand effect: more active firms increases households' demand, which then translates into higher profits

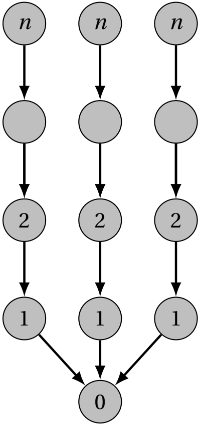
General Equilibrium Decomposition

- Suppose the distribution of fixed costs G_k is parameterized by a parameter ζ_k, with an increase in ζ_k corresponding to a first-order stochastic dominance shift in the distribution G_k.

$$
\frac{d \log \mathrm{GDP}}{d \log \zeta_k} = - \frac{\frac{w}{c\,\mathrm{GDP}} \overbrace{\frac{\partial z}{\partial \zeta_k}}^{\text{partial equilibrium } (+)} + \sum_r \left(\overbrace{\frac{\partial s_r}{\partial \zeta_k}}^{\text{cascade effect } (-)} \right) \left(\frac{w}{c\,\mathrm{GDP}} \frac{dz}{ds_r} + \frac{d \log c}{ds_r} \right)}{1 + \sum_r \left(\underbrace{\frac{\partial s_r}{\partial \log y}}_{\text{aggregate demand effect } (+)} \right) \left(\frac{w}{c\,\mathrm{GDP}} \frac{dz}{ds_r} + \underbrace{\frac{d \log c}{ds_r}}_{\text{entry effect } (-)} \right)}.
$$

Example: Production Chains

- Suppose there are n industries, with industry I_k supplying industry I_{k-1}.
- Supplier has all the bargaining power $\delta_{k-1,k} = 1$.



Failures and Aggregate Output

- m_k: measure of active firms in industry I_k.
- Aggregate output:

$$\text{GDP} = \frac{L - \bar{z}}{1 - \sum_{k=1}^{n} m_k (A_{12} \ldots A_{k-1,k} B_{k,k+1} \ldots B_{n-1,n})^{-1} \left(\frac{A_{k-1,k}}{B_{k-1,k}} - 1 \right)}$$

- failure cascades:

$$m_{k+1} = m_k G_{k+1} \left(\frac{(1 - B_{k,k+1}/A_{k,k+1})\,\text{GDP}}{A_{12} \ldots A_{k-1,k} B_{k,k+1} \ldots B_{n-1,n}} \right)$$

- expenditure on fixed costs:

$$\bar{z} = \sum_{k=1}^{N} \int_0^\infty z g_k(z) \mathbf{1} \left\{ z \leq \frac{(1 - B_{k,k+1}/A_{k,k+1})\,\text{GDP}}{A_{12} \ldots A_{k-1,k} B_{k,k+1} \ldots B_{n-1,n}} \right\} dz.$$

Failures and Aggregate Output

$$\text{GDP} = \frac{L - \bar{z}}{\sum_{k=1}^{n} (m_k - m_{k+1}) (A_{12} \ldots A_{k-1,k} B_{k,k+1} \ldots B_{n-1,n})^{-1}}$$

$$m_{k+1} = m_k G_{k+1} \left(\frac{(1 - B_{k,k+1}/A_{k,k+1})\,\text{GDP}}{A_{12} \ldots A_{k-1,k} B_{k,k+1} \ldots B_{n-1,n}} \right)$$

- Compare the output to the economy with endogenous set of active firms (GDP^*) to that of an economy with exogenous set of active firms (GDP):

$$\lim_{A_k \to \infty} \lim_{A_1 \to B_1} \frac{\text{GDP}^*}{\text{GDP}} = \infty.$$

Summary and Next Steps

- A firm-level model of input-output linkages that takes firm-specific relationships and failures into account. Failures are the main channel via which negative shocks propagate

- Expressions for the failure rates and aggregate output as a function of firm-level production chains.

- Aggregated industrial-level variables (Domar weights, sectoral markups) not sufficient statistics for understanding

  (i) the propagation of the shocks
  (ii) how various shocks shape aggregate output

- Next steps:
  - more detailed comparative statics
  - numerical estimates for the various forces in a more realistic economy.
  - measuring the various terms in the data?