A Quantitative Analysis of Real Exchange Rates and Primary Commodity Prices*

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Abstract

In this paper, we show that a substantial fraction of the volatility of real exchange rates between developed economies such as Germany, Japan, and the United Kingdom against the US dollar can be accounted for by shocks that affect the prices of primary commodities such as oil, aluminum, maize, or copper. Our analysis implies that existing models used to analyze real exchange rates between large economies that mostly focus on trade between differentiated final goods could benefit, in terms of matching the behavior of real exchange rates, by also considering trade in primary commodities.

Keywords: primary commodity prices, real exchange rate disconnect puzzle.
JEL classification: F31, F41.

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1 Introduction

This is a data paper: it shows that shocks that generate fluctuations in a small number of primary commodity prices can explain a substantial fraction of the movements in real exchange rates (RER) among industrialized countries. Specifically, we study the behavior of the bilateral RER of Germany, Japan, and the United Kingdom with the United States for the 1960-2014 period. A rough summary of the results is that with just four primary commodity prices (PCP), we can account for between one-third and one-half of the volatility of the RER between the United States and those three countries.

The relevance of these results is highlighted by the so-called *exchange rate disconnect puzzle*: the fact that real exchange rates across developed economies are very volatile, very persistent, and very hard to relate to fundamentals.[1] This difficulty opened the door for theoretical explorations of models with nominal rigidities as the source of RER movements, as in, for example Chari, Kehoe, and McGrattan (2002). We will ignore nominal rigidities in our analysis and explore how far one can go with shocks that affect relative prices of the main primary commodities.

The disconnect puzzle is not present in small open economies where exports of a few primary commodities are a sizable share of total exports.[2] For countries such as Australia, Chile, or Norway, changes in the international prices of the commodities each country exports are highly correlated with their real exchange rates.

As we show, a very similar idea can go a long way in explaining movements in RER among developed economies. The idea that we exploit in the paper is very simple: fluctuations in the prices of commodities affect manufacturing costs and therefore manufacturing prices, which in turn induce changes in final good costs. These cost fluctuations translate into price fluctuations at the country level. If changes in commodity prices have differential effects on the domestic cost of any two countries, primary commodity price changes will affect the real exchange rate between those two countries.

Relating PCP changes to RER changes is a promising avenue to explore for several reasons. First, PCP are very volatile (even more volatile than real exchange rates, as we show below) and very persistent, a feature that, as we mentioned, real exchange rates also exhibit. Second, the share of trade in primary commodities in total world trade is far from trivial: total trade in a few commodities (10) accounts for between 12% to 18% of total world trade in goods, depending on the year chosen.[3] This number clearly underestimates the true

[1] See, for example, in Meese and Rogoff (1983), Engel (1999), Obstfeld and Rogoff (2001), and Betts and Kehoe (2004).

[2] See Chen and Rogoff (2003) and Hevia and Nicolini (2013).

[3] It is close to 12% in 1990 and 18% in 2012. The main difference is that the first is a year of particularly

share of commodities, since trade shares are not value-added measures. Thus, when steel is exported, it is fully counted as a manufactured good, even though an important component of its cost depends on iron. The same happens when a car is exported. Third, primary commodities are at the bottom of the production chain, so they directly affect final good prices.[4] In addition, they may directly affect the prices of other domestic inputs – such as some types of labor and services in general – that are used jointly with primary commodities in the production of intermediate goods, and thus they may indirectly affect the costs of final goods. Because just a few commodities make up a high share of total trade, we only need to focus on a handful of prices. Finally, it is well known that the law of one price on those primary commodities holds, so no ambiguity with respect to their tradability exists. The literature on RER has struggled to separate the set of final goods into two categories: the ones that are traded, for which the law of one price is assumed to hold, and the ones that are not. We only need to assume that for the few commodities we analyze, the law of one price holds, and it is precisely for these prices that independent evidence that the law of one price holds is the strongest.

The exchange rate disconnect puzzle has been widely studied in the literature. Two recent attempts at quantitatively explaining several facts related to the puzzle are Itskhoki and Mukhin (2017), and Eaton, Kortum, and Neiman (2016). They provide very good descriptions of the state of the literature. The connection between RER and PCP has largely been ignored for the countries we focus on, and not only on the empirical side. Ever since the seminal contribution of Obstfeld and Rogoff (1995), the theoretical literature developed to study RER between the countries we consider in this paper has focused exclusively on the production and trade of final goods. Our evidence suggests that theoretical models of RER among developed economies that ignore primary commodity markets may fall short of providing a comprehensive explanation of RER movements.

In Section 2 we describe the data and motivate the analysis by showing some descriptive statistics. We also discuss several issues related to the empirical methodology used in the paper. In Section 3 we present the main results. In Section 4 we perform a series of Monte Carlo exercises to address issues related to small sample properties of the moments we estimate in Section 3. In Section 5 we partially spell out the production side of a totally standard model of an open economy that makes explicit the production of commodities and the use of commodities in the production of manufactured goods. We derive an equilibrium condition relating the bilateral RER between two countries to PCP that, in the case of

low primary commodity prices, while the second is a period of particularly high prices.

[4] This direct effect is substantial enough for monetary authorities in developed countries to focus attention on measures of "core" inflation, which abstract from the "volatile" effect of primary commodity prices (food and energy).

Cobb-Douglas production functions, is linear in the logarithms of the variables. This log-linear relationship rationalizes the one used in the empirical analysis presented in Section 3. Finally, in Section 6, we present an additional exercise, motivated by the model of Section 5. A brief discussion of the implications of the results is presented in a final concluding section.

2 Model

This section presents a general equilibrium model that delivers a simple equation relating bilateral real exchange rates to primary commodity prices in equilibrium. The model consists of three countries and three sectors of production: nontradable final goods, tradable intermediate goods, and tradable primary commodities.

Country 1 produces final good Y_1, intermediate good Q_1, and primary commodities X_1 and X_3, according to the following Cobb-Douglas production functions:

$$\begin{aligned} Y_1 &= Z_1 \left(q_1^1\right)^{\alpha_1^1} \left(q_2^1\right)^{\alpha_2^1} \left(n_1^y\right)^{\alpha_3^1}, \\ Q_1 &= Z_1 \left(x_1^1\right)^{\beta_1^1} \left(x_2^1\right)^{\beta_2^1} \left(x_3^1\right)^{\beta_3^1} \left(n_1^q\right)^{\beta_4^1}, \\ X_1^1 &= Z_1 \left(e_1^1\right)^{1-\phi_1^1} \left(n_1^{x_1}\right)^{\phi_1^1}, \\ X_3^1 &= Z_1 \left(e_3^1\right)^{1-\phi_3^1} \left(n_1^{x_3}\right)^{\phi_3^1}, \end{aligned}$$

in which Z_1 denotes the total factor productivity (TFP) in country 1; q_1^1 and q_2^1 are the inputs of intermediate goods produced in countries 1 and 2; x_1^1, x_2^1, and x_3^1 are the inputs of primary commodities X_1, X_2, and X_3; n_1^y, n_1^q, $n_1^{x_1}$, and $n_1^{x_3}$ are the labor inputs; and e_1^1 and e_3^1 are the fixed endowments of natural resources, which are commodity specific. The exponents correspond to the factor shares in production.

The production structure of **country 2** is analogous. It produces final good Y_2, intermediate good Q_2, and primary commodities X_2 and X_3, according to the Cobb-Douglas production functions:

$$\begin{aligned} Y_2 &= Z_2 \left(q_1^2\right)^{\alpha_1^2} \left(q_2^2\right)^{\alpha_2^2} \left(n_2^y\right)^{\alpha_3^2}, \\ Q_2 &= Z_2 \left(x_1^2\right)^{\beta_1^2} \left(x_2^2\right)^{\beta_2^2} \left(x_3^2\right)^{\beta_3^2} \left(n_2^q\right)^{\beta_4^2}, \\ X_2^2 &= Z_2 \left(e_2^2\right)^{1-\phi_2^2} \left(n_2^{x_2}\right)^{\phi_2^2}, \\ X_3^2 &= Z_2 \left(e_3^2\right)^{1-\phi_3^2} \left(n_2^{x_3}\right)^{\phi_3^2}. \end{aligned}$$

We are interested in the bilateral real exchange rate between countries 1 and 2, so we choose **Country 3** to represent the rest of the world. We make the simplifying assumptions that it receives endowments of each of the primary commodities, X_1^3, X_2^3, and X_3^3, and produces the final good Y_3 directly from inputs of primary commodities according to the Cobb-Douglas production function:

$$Y_3 = \left(x_1^3\right)^{\pi_1} \left(x_2^3\right)^{\pi_2} \left(x_3^3\right)^{\pi_3}.$$

Households in each country consume their respective final goods and have no access to financial markets. They inelastically supply their fixed endowments of labor and natural resources in countries 1 and 2, and inelastically supply their stochastic endowments of primary commodities in country 3. We leave the complete description of the model and its equilibrium computation to Appendix X, and we turn next to the discussion of how the real exchange rate is determined in equilibrium.

The log of the bilateral **real exchange rate** between countries 1 and 2, rer, is the difference in the logs of their nominal final good prices, $p^{y_1} - p^{y_2}$. Under the assumption of perfect competition, the cost-minimization problem of firms implies that the nominal price of each good is equal to its respective marginal cost, which in turn is also a Cobb-Douglas function of its respective input prices, leading to a linear relationship in logs. After some algebra, described in Appendix X, we can express the log of the real exchange rate as:

$$\begin{aligned} rer_t \; = \; & \left(1 - \left(\alpha_2^1 - \alpha_2^2\right) + \frac{\left(\alpha_2^1 - \alpha_2^2\right)\beta_4^2 - \alpha_3^2}{\phi_2^2}\right) z_{2,t} - \left(1 + \left(\alpha_1^1 - \alpha_1^2\right) - \frac{\left(\alpha_1^1 - \alpha_1^2\right)\beta_4^1 + \alpha_3^1}{\phi_1^1}\right) z_{1,t} \\ & + \left(\left(\alpha_1^1 - \alpha_1^2\right)\beta_1^1 + \left(\alpha_2^1 - \alpha_2^2\right)\beta_1^2 + \frac{\left(\alpha_1^1 - \alpha_1^2\right)\beta_4^1 + \alpha_3^1}{\phi_1^1}\right) p_t^{x_1} \\ & + \left(\left(\alpha_1^1 - \alpha_1^2\right)\beta_2^1 + \left(\alpha_2^1 - \alpha_2^2\right)\beta_2^2 + \frac{\left(\alpha_2^1 - \alpha_2^2\right)\beta_4^2 - \alpha_3^2}{\phi_2^2}\right) p_t^{x_2} \\ & + \left(\left(\alpha_1^1 - \alpha_1^2\right)\beta_3^1 + \left(\alpha_2^1 - \alpha_2^2\right)\beta_3^2\right) p_t^{x_3} \\ & - \frac{1 - \phi_1^1}{\phi_1^1}\left(\left(\alpha_1^1 - \alpha_1^2\right)\beta_4^1 + \alpha_3^1\right) p_t^{e_1} - \frac{1 - \phi_2^2}{\phi_2^2}\left(\left(\alpha_2^1 - \alpha_2^2\right)\beta_4^2 - \alpha_3^2\right) p_t^{e_2}, \end{aligned} \tag{1}$$

in which z^1 and z^2 denote the log of the TFPs in countries 1 and 2; p^{x_1}, p^{x_2}, p^{x_3} denote the primary commodity prices in logs; and p^{e_1} and p^{e_2} denote the prices of natural resource endowments in countries 1 and 2, also in logs, that are used to produce primary commodities X_1 and X_2, respectively. The subscript t denotes the time period.

Equation (1) is the main equation of the paper and it will guide our quantitative analysis in the following sections. It relates the bilateral real exchange rate not only to the TFP in countries 1 and 2, but also to the prices of primary commodities and endowments of natural resources.[5] The main assumptions are that (i) final goods are nontradable, (ii) there is a primary commodity sector, and (iii) countries have heterogeneous input-output matrices.

Previous studies that established the real exchange rate disconnect puzzle have shown that TFPs have low explanatory power over real exchange rates for developed economies. That is, regressions of real exchange rates on TFPs have low R^2's. Equation (1) suggests that the explanatory power of such regressions can be high once we consider the fluctuations in primary commoditiy prices, for which we have good data. That is what we explore.

Of course primary commodity prices are endogenous variables in this model, meaning that in equilibrium they are functions of the underlying shocks in the economy, which includes the TFP shocks in countries 1 and 2 but potentially many others.[6] If there are other shocks that are relevant to explaining the fluctuations in primary commodity prices, then it follows from equation (1) that it is possible to have a regression of the real exchange rate on primary commodity prices with high R^2 while having the regression on TFPs with low R^2.[7]

In our case, we emphasize the role of other shocks in generating fluctuations in primary commodity prices. In particular, we allow for shocks to the endowments of primary commodities in country 3 in addition to the standard TFP shocks in countries 1 and 2. We assume the following AR1 processes:

$$\begin{aligned} z_{1,t} &= \rho^{z_1} z_{1,t-1} + \varepsilon_t^{z_1}, \\ z_{2,t} &= \rho^{z_2} z_{2,t-1} + \varepsilon_t^{z_2}, \\ x_{1,t}^3 &= \rho^{x_1} x_{1,t-1}^3 + \varepsilon_t^{x_1}, \\ x_{2,t}^3 &= \rho^{x_2} x_{2,t-1}^3 + \varepsilon_t^{x_2}, \\ x_{3,t}^3 &= \rho^{x_3} x_{3,t-1}^3 + \varepsilon_t^{x_3}, \end{aligned}$$

in which $\varepsilon_t^{z_1}$, $\varepsilon_t^{z_2}$, $\varepsilon_t^{x_1}$, $\varepsilon_t^{x_2}$, and $\varepsilon_t^{x_3}$ follow a multivariate normal distribution with zero mean, and $x = \ln X$.

If shocks to the endowments in country 3 generate large fluctuations in primary commodity prices, we may observe a high R^2 in the regression when we use only primary commodity prices as regressors. For that to be the case, it follows from equation (1) that we need the heterogeneity in input-output matrices. As a simple example, suppose that $\alpha_1^1 = \alpha_1^2$, $\alpha_2^1 = \alpha_2^2$, and $\alpha_3^1 = \alpha_3^2 = 0$. In this case, the real exchange rate is simply $rer_t = z_{2,t} - z_{1,t}$.

[5] In Ayres, Hevia and Nicolini (2015) we derive a similar equation in a multi-country setting with multiple types of labor inputs, intermediate goods, and primary commodities.

[6] The assumptions of financial autarky and no capital accumulation implies that the exogenous shocks are the only state variables in this economy.

[7] That would also be the case if the TFP shocks in countries 1 and 2 have strong nonlinear effects on the prices of primary commodities in logs.

Therefore, even if there are large fluctuations in commodity prices due to other shocks that are orthogonal to TFP shocks, they will have no impact on the real exchange rate, and the regression of the real exchange rate on primary commodity prices is likely to have low explanatory power, that is, low R^2.

In Section 5, we calibrate the model using data on the input-output matrices for the United States and Japan and show that there is enough heterogeneity such that fluctuations in primary commodity prices can account for a large share of the fluctuation in their bilateral real exchange rate in our simulations. But before that, the next section presents evidence that this relationship between primary commodity prices and real exchange rates is indeed observed in the data.

3 The data and the methodology

We collected monthly data on consumer price levels and nominal exchange rates for the United States, Germany, the United Kingdom, and Japan. The bilateral RER are defined as the nominal exchange rates between Germany (DEU), Japan (JPN), and the United Kingdom (UK) against the US dollar multiplied by the ratio of CPIs. In the case of Germany, we use the mark until 2000 and the euro thereafter. We also collected price data for the 10 primary commodities with the largest shares in world trade in 1990 and for which monthly prices are available from January 1960 to December 2014. Appendix A describes the data. Throughout the paper we show results for the data in four-year differences, but show in Appendix XXX that all results hold using the data in levels.[8]

Table 1 shows the volatility (standard deviation) of the monthly data in four-year differences on the US bilateral (log) real exchange rates against the United Kingdom, Germany, and Japan between 1960 and 2014, as well as for four subperiods.[9] We also report the average volatility (simple and trade-weighted) of the (log) prices of the commodities listed in Table A.1. As can be seen, the volatility of PCP is substantially higher than that of RER.[10]

In addition, it is apparent that in the subperiods in which the volatility of PCP is high, so is the volatility of the RER. We find this issue particularly interesting, since the substantial

[8]We performed unit root tests for the data in (log) levels and in three-, four-, and five year differences. There is evidence of unit roots for the raw data that vanishes for the data in four-years differences. Still, the real exchange rates and commodity prices remain very persistent even for the data in four-year differences. See Appendix XXX.

[9]When specifying the subperiods, we opted for isolating 1960–1972, the period during which the Bretton Woods system was active. Then we chose the next subperiods so that they would have similar lengths.

[10]This is also the case for small open economies, where the ratio of the volatility of the relevant PCP is between 2.5 and 3.5 the volatility of the RER. These values are similar to the ones that can be obtained from Table 1.

increase in the volatility of real exchange rates after the breakdown of the Bretton-Woods system of fixed exchange rates is accompanied by an equally substantial increase in the volatility of commodity prices. The conventional interpretation has been that the increase in volatility after 1972 was the result of the regime change from fixed to flexible exchange rates (Musa, 1986). An alternative interpretation is that the fundamentals that cause real exchange rates and commodity prices to move together were more volatile after 1973 than before. To the extent that PCP are independent of the exchange rate regimes, our evidence points toward an alternative explanation of the increase in volatility post Bretton-Woods.

As a complementary piece of evidence, Figure 1 shows rolling volatilities computed using windows of 10 years of data for the real exchange rates and for the average of the 10 primary commodity prices. The positive association between the volatilities of the real exchange rates and commodity prices reinforces, in our view, the interest in associating RER with PCP.[11]

Table 1: Volatilities of real exchange rates and primary commodity prices

	1960–2014	1960–1972	1973–1985	1986–1998	1999–2014
Real Exchange Rates					
US-UK	0.18	0.11	0.26	0.17	0.14
US-DEU	0.22	0.07	0.31	0.16	0.21
US-JPN	0.23	0.07	0.27	0.26	0.17
Average across commodities					
Simple	0.38	0.17	0.44	0.30	0.36
Trade weighted	0.46	0.17	0.55	0.35	0.36

Notes: Variables are in logs and commodity prices are normalized by US CPI. Weights are based on the share of total trade in 1990. The set of primary commodities is oil, fish, meat, aluminum, copper, gold, wheat, maize, timber, and cotton.

3.1 Methodology

Our goal is to assess how much of the variability of the US bilateral real exchange rates with the United Kingdom, Germany, and Japan can be accounted for by a set of primary commodity prices. In following the small open economy literature, and pairing the United States and United Kingdom as an example, we analyze the following regression equation:

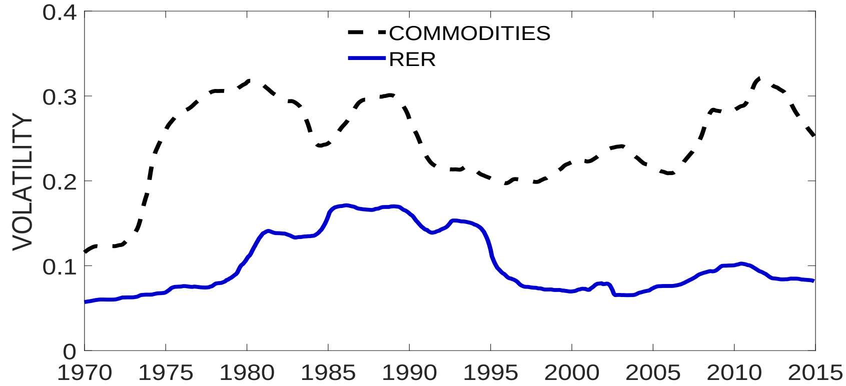
$$r_t^{USA,UK} = \beta' \mathbf{p}_t^{X,USA} + v_t. \tag{2}$$

The left-hand side of equation (2) is the log of the bilateral real exchange rate, $r_t^{USA,UK} = \ln P_t^{USA} - \ln P_t^{UK} + \ln S_t$, where P_t^{USA} denotes the price level in the United States, P_t^{UK}

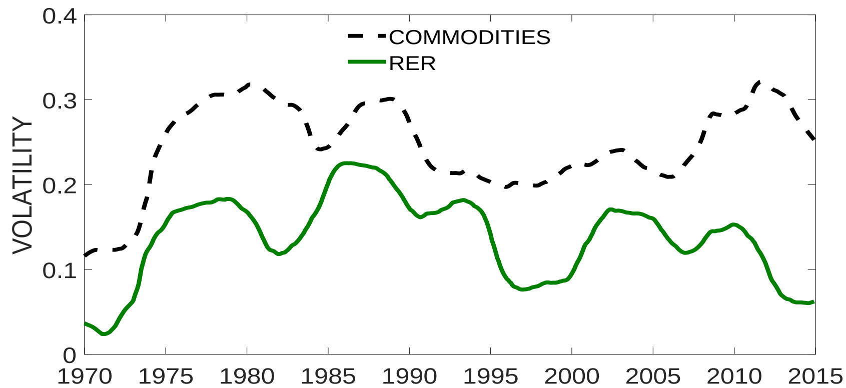
[11]The correlations are 0.40, 0.54, and 0.39 for the United Kingdom, Germany, and Japan, respectively.

Figure 1: Rolling volatilities of real exchange rates and commodity prices (10-year windows)

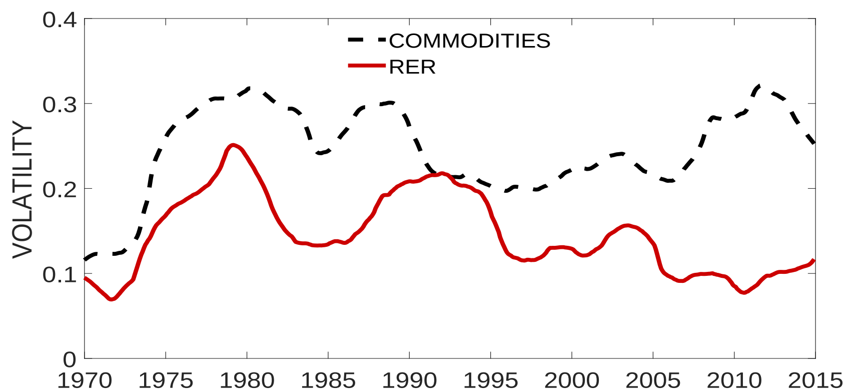
(a) United Kingdom



(b) Germany



(c) Japan



denotes the price level in the United Kingdom, and S_t denotes the nominal exchange rate between US dollars and British pounds. On the right-hand of the equation, $\mathbf{p}_t^{X,USA}$ is a vector if log primary commodity prices normalized by the US price level, β is a vector of coefficients, and v_t is the error term.[12] The subscript t denotes the time period.

[12]Since we use PCP in constant dollars, one might be worried that the US price level enters both sides of the equation. The results, however, do not depend on that normalization (see Appendix XXX).

Following the small open economy literature at this stage entails a major difficulty. The assumption that PCP are exogenous for economies such as Norway or New Zealand is reasonable, but it is clearly unacceptable for the pairs of economies we analyze. Indeed, we have no hope of obtaining consistent estimators of the vector β. But the R^2 on that regression still contains valuable information, precisely the information we need to answer the question of the paper. We now discuss why this is so.

The RER and the PCP in equation (2) are determined simultaneously. Suppose that $\mathbf{p}_t^{X,USA} \in R^m$ and let $\xi_t \in R^n$ be a vector representing the state of the economy. The state vector may include endogenous state variables, such as stocks of capital, and exogenous state variables, such as productivity, preference, and policy shocks. In any model, the equilibrium values for the RER and the PCP are (possibly nonlinear) functions of the state variables ξ_t.[13] A linear approximation to those functions implies

$$\begin{aligned} r_t^{USA,UK} &= \theta' \xi_t, \\ \mathbf{p}_t^{X,USA} &= \Omega \xi_t, \end{aligned} \tag{3}$$

where $\theta \in R^n$, Ω is an $m \times n$ matrix, and variables are measured as deviations from their long-run means. We treat ξ_t as unobserved, so we can interpret the state variables as orthogonal with an identity covariance matrix without loss of generality.[14]

Consider projecting the real exchange rate onto the commodity prices,

$$Proj(r_t^{USA,UK} | \mathbf{p}_t^{X,USA}) = \beta' \mathbf{p}_t^{X,USA}.$$

Equations (3) and the orthogonality principle imply $\beta' = (\theta'\Omega')(\Omega\Omega')^{-1}$. It then follows that projecting $r_t^{USA,UK}$ onto $\mathbf{p}_t^{X,USA}$ is equivalent to decomposing the real exchange rate into two orthogonal components:

$$r_t^{USA,UK} = \beta'\Omega\xi_t + (\theta' - \beta'\Omega)\xi_t. \tag{4}$$

The first term of the projection measures how much of the variability of the real exchange rate can be accounted for by fundamental shocks that affect primary commodity prices. The second term of the projection is orthogonal to the first and measures how much of the variability of the real exchange rate is accounted for by fundamental shocks that do not manifest themselves as fluctuations in commodity prices correlated with the real exchange

[13] In section XXX, we describe a model that implies an equilibrium relationship between the RER and the PCP that rationalizes equation (2).

[14] If the shocks ξ_t have a non-diagonal covariance matrix $E(\xi_t \xi_t') = \Sigma$, there is an observationally equivalent system with orthogonal state variables by letting $\tilde{\xi}_t = \Sigma^{-1/2}\xi_t$, $\tilde{\theta}' = \theta'\Sigma^{1/2}$, and $\tilde{\Omega} = \Omega\Sigma^{1/2}$.

rate. In terms of this decomposition, the R^2 of the regression (2) can be written as

$$R^2 = \frac{E[\beta'\Omega\xi_t\xi_t{}'\Omega'\beta]}{E[(\theta'\xi_t\xi_t{}'\theta)]} = \frac{\beta'\Omega\Omega'\beta}{\theta'\theta}. \tag{5}$$

The underlying (implicit) assumption in much of the literature on bilateral real exchange rates between developed countries is that the component associated with commodity prices can be safely ignored. We can express this no-relevance-of-commodities assumption as the requirement that the R^2 of the regression of real exchange rates on commodity prices is zero, which is true whenever $\beta'\Omega = 0$.

Let us split the state variables as $\xi_t = [\xi'_{1t}\ \xi'_{2t}]'$, so that $r_t^{USA,UK} = \theta'_1\xi_{1t} + \theta'_2\xi_{2t}$ and $\mathbf{p}_t^{X,USA} = \Omega_1\xi_{1t} + \Omega_2\xi_{2t}$. It then follows that $\beta'\Omega = \theta'_1\Omega_1 + \theta'_2\Omega_2$. A necessary and sufficient condition for the R^2 of the regression to be zero is thus $\theta'_1\Omega_1 = -\theta'_2\Omega_2$. This equality holds, for example, when $\theta_1 = 0$ and $\Omega_2 = 0$. This implies a block-recursive structure in which the set of state variables that determine the real exchange rate are different from (and orthogonal to) those that determine primary commodity prices. If these conditions do not hold, then commodity prices will be (generically) correlated with the real exchange rate.

Measuring how much of the variability of the RER can be explained by this common component – the R^2 of equation (2) – is the objective of the following sections.

3.1.1 Higher-order terms and time-varying coefficients

Linear approximations work well when the shocks are small. The large and persistent movements in RER and PCP suggests that the approximation error may be large. One way out of this could be to add nonlinear terms in the regression. We opted for simplicity and consider only the linear terms. Since adding variables can only increase the R^2, we can interpret our results as a lower bound on the fraction of the volatility of the RER that can be accounted for by shocks that also move PCP.

Another concern is that the coefficients in the linearized system (3) are evaluated at the equilibrium around which the linearization is made. The economies we consider have experienced major transformations in their production structures and trade patterns during the period that we study. As such, it is reasonable to imagine that the equilibrium around which the linearization is made in the 1960s is different from the one in the 1980s.[15] Thus, there is no reason to believe that the coefficients β would remain constant over time. To capture this possibility, we present our results for the whole period and for several subperiods. We also use this idea to motivate the out-of-sample fit exercises we do in the next section.

[15] The model in Section XXX makes these statements precise.

4 Results

We start our analysis by reporting the R^2 of the OLS regression of equation (2) using the primary commodity prices prices listed in Table A.1. We run the regression for the whole period and also for four subperiods. The results are reported in Table 2.

Table 2: Coefficients of determination R^2

	1960–2014	1960–1972	1973–1985	1986–1998	1999–2014
(a) 10 commodities, 4-year differences					
United Kingdom	0.48	0.90	0.90	0.81	0.60
Germany	0.63	0.95	0.87	0.83	0.75
Japan	0.57	0.92	0.84	0.92	0.82
(b) 4 commodities (best fit), 4-year differences					
United Kingdom	0.33	0.72	0.82	0.63	0.58
Germany	0.56	0.84	0.87	0.81	0.74
Japan	0.48	0.88	0.76	0.86	0.80

Table 2.a shows regression results for the data in four year differences using 10 commodities as regressors. The R^2 are 0.48, 0.63, and 0.57 for the United Kingdom, Germany, and Japan, respectively. The R^2s are larger when we consider the subperiods, although as we argue below, they are largely the effect of smaller samples.

As we show in Table B.4, the prices of the commodities that we are using are highly correlated. One could then guess that it is possible to account for a large fraction of the real exchange rate volatility even if we considerably reduce the number of PCP. To explore this possibility, we pick the four commodities (out of the ten) with highest t-statistics and rerun the regressions.[16] The results are reported in Table 2.b.[17]

By selecting only four commodity price series, we can still account for between 33% and 56% of the volatility of real exchange rates in four-year differences. It is important to emphasize that PCP can account for a large share of real exchange rate fluctuations in all subperiods we consider, and, in particular, there are no systematic differences in the relationship between PCP and real exchange rates before and after the Bretton Woods system. This goes in line with the alternative hypothesis about the increase in real exchange rate volatility following 1972: that it coincided with an increase in the volatility of fundamentals.

[16]Throughout the paper, we compute t-statistics using the Newey-West heteroskedasticity-and-autocorrelation-consistent standard errors.

[17]Tables B.8–B.10 in Appendix E report the coefficients of the regressions in levels and in 4-year differences. We also the results for the case in which we choose only three commodities.

Figure 2 plots the data versus the respective fitted values for the regressions in four-year differences for the cases of both 10 and 4 PCPs, and also reports the respective correlation between the data and fitted values (equivalent to the square root of the R^2).[18] As can be seen, the match is very good in all cases.

The online Appendix shows that these results are robust to several variations in the specification of the regressions: using data in log-levels considering the ten commodities and selecting the best four commodities as above; regressions without subtracting the log of US CPI from either side of equation (2); regressions using non-US pairs of bilateral real exchange rates, such as the bilateral real exchange rate between the United Kingdom and Germany; and regressions selecting commodities based on US trade data rather than by a statistical criterion. This last procedure can be rationalized by the theory shown in section XXX, which shows that, in order for the commodity price to explain a large fraction of real exchange fluctuations, a necessary condition is that the commodity must be an important input in the production structure of one of the economies in the country pair.

4.1 Out-of-sample fit

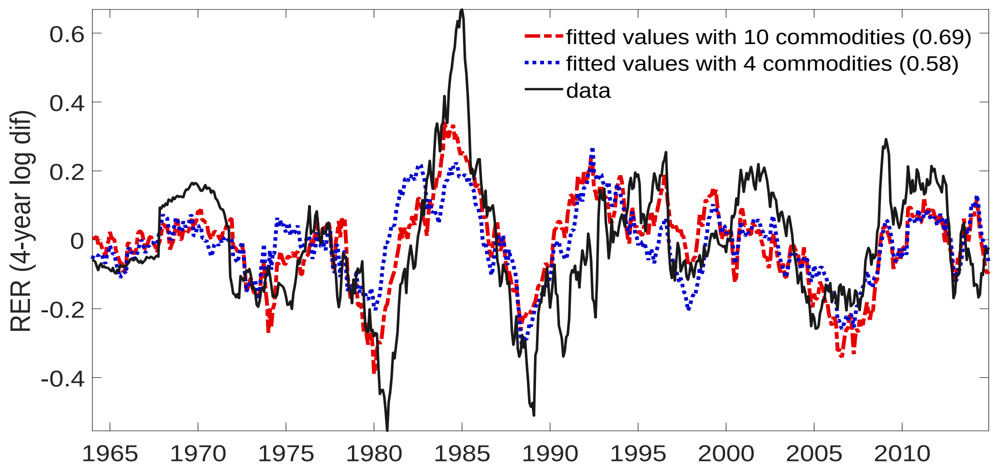
In the previous regressions, we chose the four primary commodities that obtain a good fit with the real exchange rate, so one could argue that the regressors have been chosen precisely to match the data. To check the robustness of our results to the in-sample selection, we adopt the following procedure. We start by running a regression using data in four-year differences over the period 1960-1972. We drop the six commodities with the lowest t-statistics and rerun the regression. Based on the four commodities selected by this procedure and their estimated coefficients, we use observed commodity prices over the following h periods to fit the real exchange rate and store the h fitted values. We next add one observation to the sample and repeat previous regressions to fit the real exchange rates over the following h periods. Repeating this procedure until the end of the sample, we construct time series of out-of-sample fitted real exchange rates over the following $h = 1, 2, ..., 60$ periods.

The logic behind this exercise is related to the discussion in Subsection 3.1. We interpret the linear regression as a linear approximation of the solution of a model in which the RER and the PCP are jointly determined, as described in equations (3). The parameters on those equations are evaluated at the point equilibrium which the linearization is made. The maintained assumption in this exercise is that those values will not change much in a relatively short period of time, so that the reduced-form estimates could work reasonably well for an interval of time that is not too long, particularly if no major changes occurred.

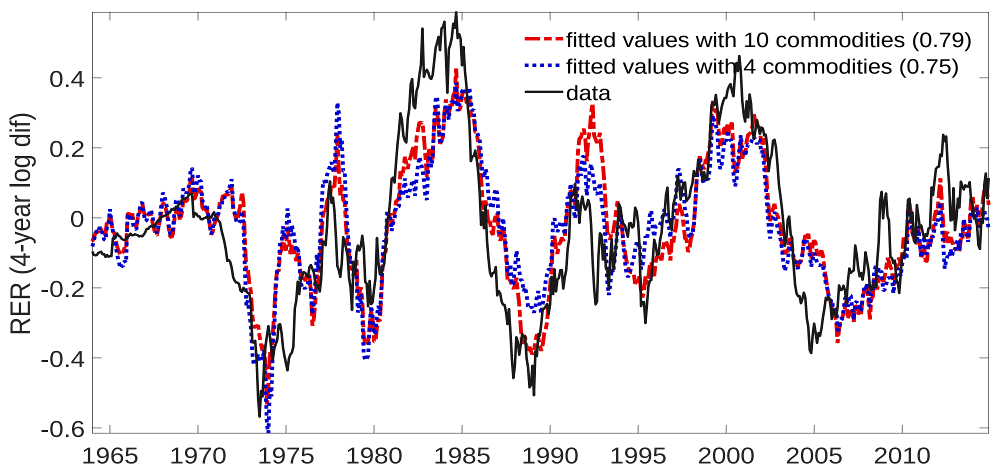
[18]For the case of the data in levels, see Figure 8 in Appendix C. The results are very similar, except for Japan, which is the country for which the unit root evidence is very high.

Figure 2: Real exchange rates and fitted values, four-year differences.

(a) United Kingdom



(b) Germany



(c) Japan

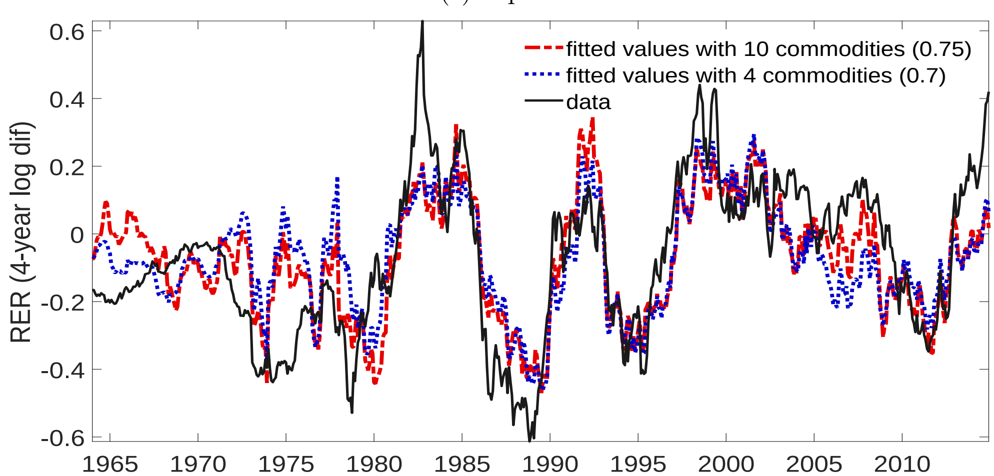


Figure 3 shows the actual and fitted real exchange rates for the case $h = 6$ months ahead. The out-of-sample fit is remarkable, with a correlation between the fitted and actual values of 0.45 for the United Kingdom, 0.73 for Germany, and 0.64 for Japan.

We summarize the results in Figure 4, in which we show the correlation between fitted and actual real exchange rates as we vary the forward window from $h = 1$ to $h = 60$ months ahead. Although the correlations decrease as the fitting horizon increases, they decrease slowly. There is a good out-of-sample fit even using data that are several years old to select the commodities and coefficients to fit real exchange rates today. Overall, we interpret these results as supporting our initial findings that shocks that affect just four commodity prices account for a substantial fraction of real exchange rate movements.

4.2 Are the results spurious?

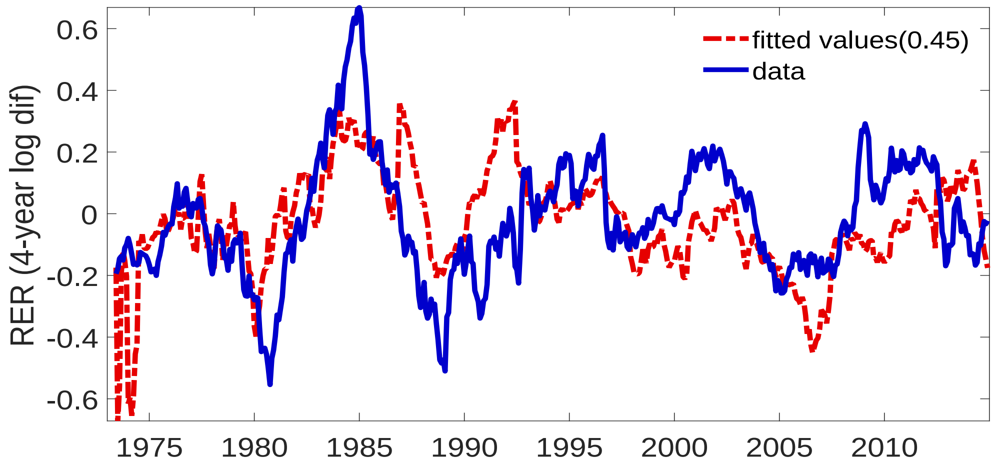
A concern with the previous regressions is to what extent the results could be due to a problem of small sample size. It is well known that, even with stationary series, regressing two orthogonal but highly persistent series could lead to a spurious correlation for moderate sample sizes. To explore this issue, we perform small sample inference by using a parametric bootstrap procedure that generates artificial data under the null hypothesis that commodity prices are orthogonal to real exchange rates. By construction, commodity prices and real exchange rate are orthogonal, which implies that the R^2 converges to zero as the artificial sample size grows towards infinity. But for finite samples, the R^2 is positive.

Take, for example, the Germany-US real exchange rate regression with an R^2 of 0.56 in Table 2.b. The bootstrap procedure is as follows. We first estimate an autoregressive process for the Germany-US real exchange rate and an independent vector autoregression with the four commodity prices used in the regression (we use the Schwarz information criterion to select the lag lengths). To compute the small sample distribution of the R^2, we draw 10,000 samples of length 660 (the number of months between January 1960 and December 2014) by resampling from the residuals of the estimated processes and compute artificial real exchange rate and commodity price data. Next, for each artificial sample, we run a regression of the real exchange rate on the commodity prices and store the associated R^2. Finally, we compare the estimated R^2 using actual data with its small sample distribution to assess how common is to observe an R^2 of 0.56 under the null hypothesis of orthogonality.

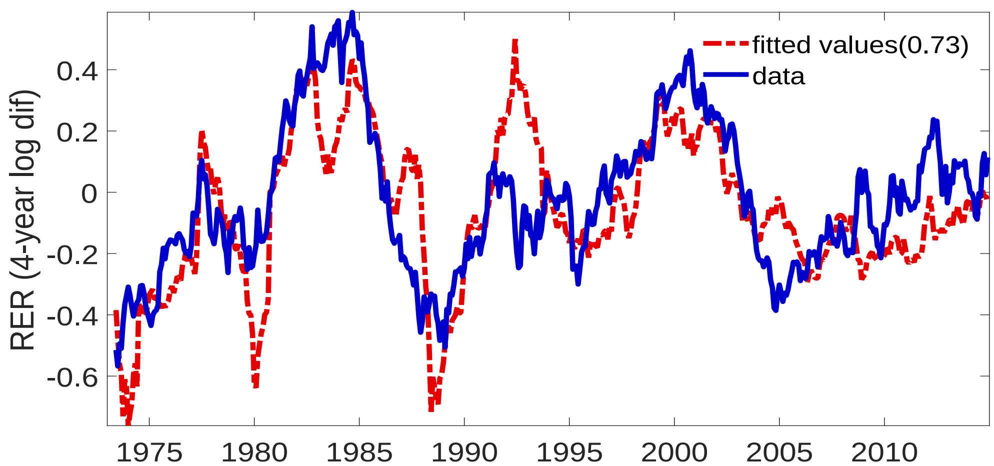
Figure 5 shows the small sample distributions of the R^2 over the entire sample period. The vertical lines are the estimated R^2 using the actual data. In all cases, the probability of obtaining an R^2 as large as that estimated in Table 2.b is smaller than 5% and as low as 0% for the case of Germany. The three distributions under the null hypothesis are positively

Figure 3: Out-of-sample fit six months ahead with four commodities (best fit)

(a) United Kingdom



(b) Germany



(c) Japan

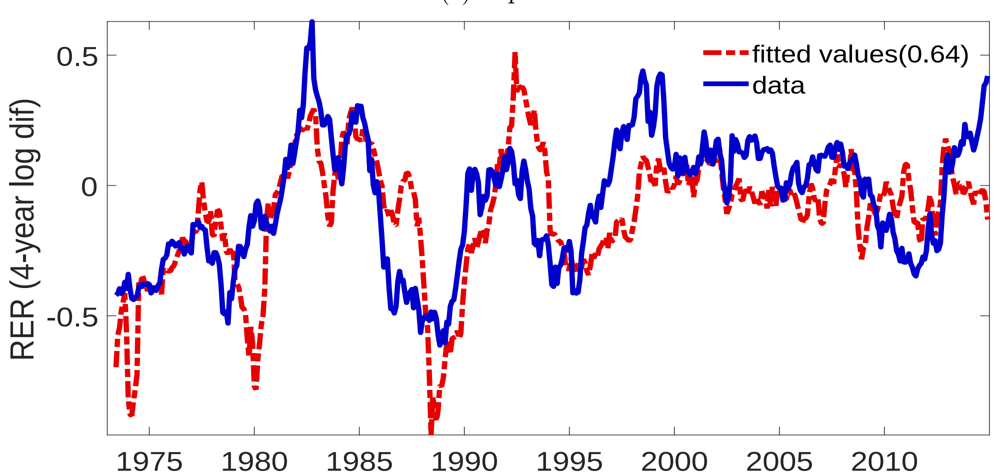
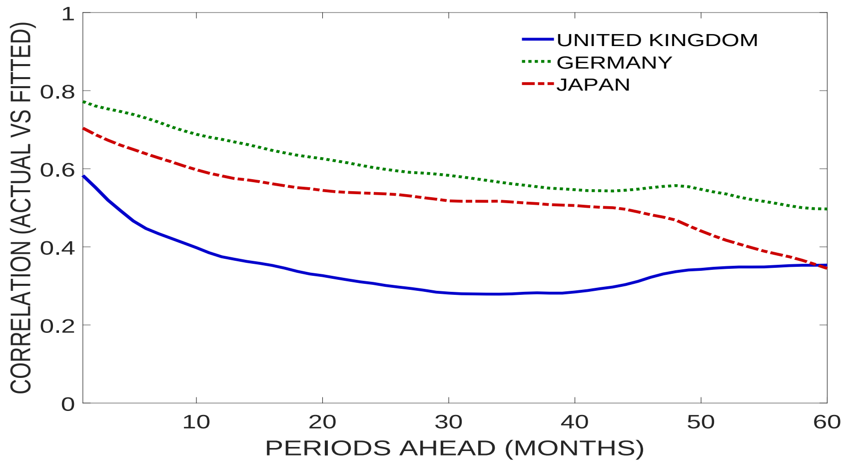


Figure 4: Out-of-sample fit, four commodities, correlations as a function of r (months ahead)



skewed with a mode of about 0.1, which is much smaller than the estimated R^2s in the table.

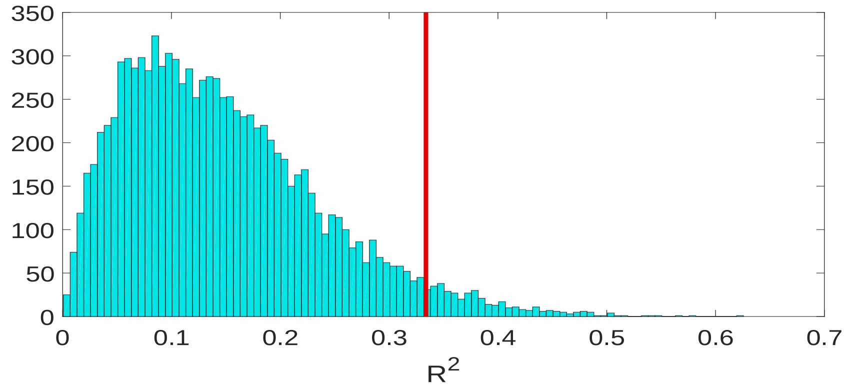
Table 3: Bootstraped distributions of R^2 under the null hypothesis of orthogonality.

		Percentiles distribution of R^2				
	$\hat{R}^2$	Median	75	90	95	$\Pr(R^2 \geq \hat{R}^2)$
United Kingdom						
1960-2014	0.33	0.13	0.20	0.27	0.31	0.037
1960-1972	0.72	0.52	0.66	0.75	0.80	0.143
1973-1985	0.82	0.37	0.52	0.64	0.70	0.004
1986-1998	0.63	0.37	0.50	0.61	0.67	0.077
1999-2014	0.58	0.29	0.41	0.53	0.59	0.059
Germany						
1960-2014	0.56	0.13	0.19	0.26	0.31	0.000
1960-1972	0.84	0.56	0.69	0.79	0.83	0.032
1973-1985	0.87	0.49	0.63	0.73	0.78	0.005
1986-1998	0.81	0.40	0.54	0.65	0.71	0.007
1999-2014	0.74	0.30	0.43	0.55	0.61	0.007
Japan						
1960-2014	0.48	0.14	0.21	0.29	0.34	0.003
1960-1972	0.88	0.59	0.72	0.81	0.85	0.022
1973-1985	0.76	0.46	0.60	0.70	0.75	0.045
1986-1998	0.86	0.41	0.55	0.66	0.71	0.001
1999-2014	0.80	0.33	0.46	0.57	0.63	0.002

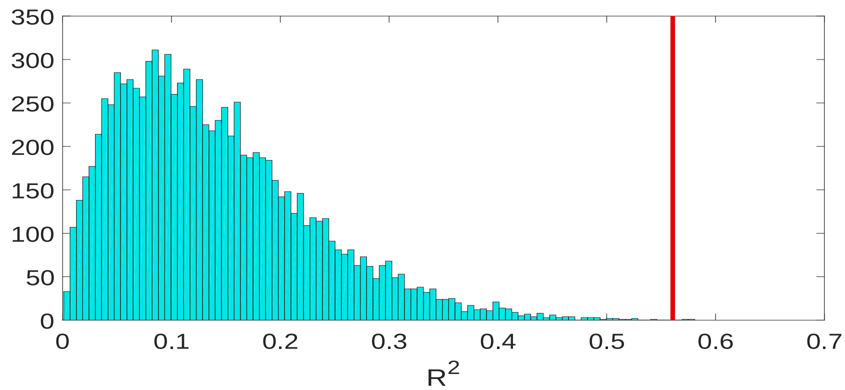
Table 3 shows statistics of the small sample distributions under the null of orthogonality for the three bilateral real exchange rates and for the five subperiods, together with the

Figure 5: Small sample distribution of the R^2 over the period 1960–2014

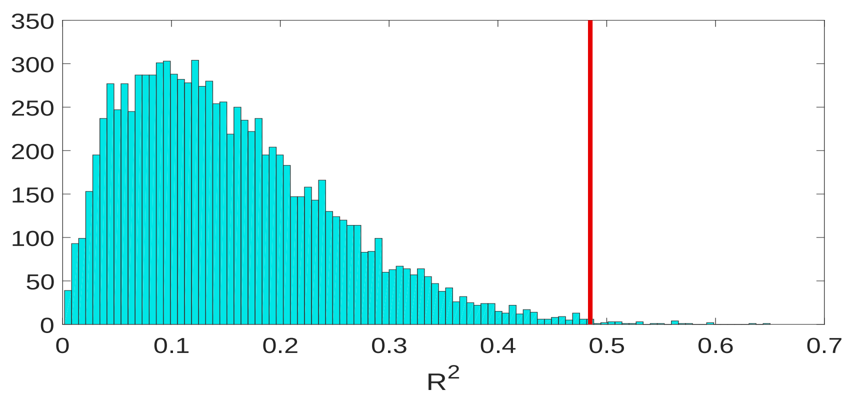
(a) United Kingdom



(b) Germany



(c) Japan



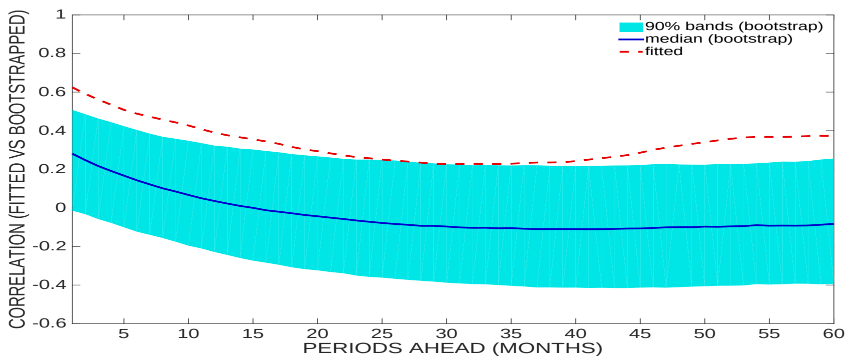
probability of observing an R^2 as large as that estimated with the actual data under the null of orthogonality. For comparison, the table also includes the estimated R^2s from Table 2.b. Overall, these results suggest that the estimated correlations are robust for every subperiod and bilateral real exchange rate. Of course, for some subperiods and countries, the small sample distributions are more dispersed and it is not uncommon to observe a relatively large

R^2 under the null of orthogonality, especially for smaller sample sizes. For example, although the estimated R^2 for Germany over the period 1964-1972 is 0.84, the median R^2 under the null of orthogonality is 0.56.

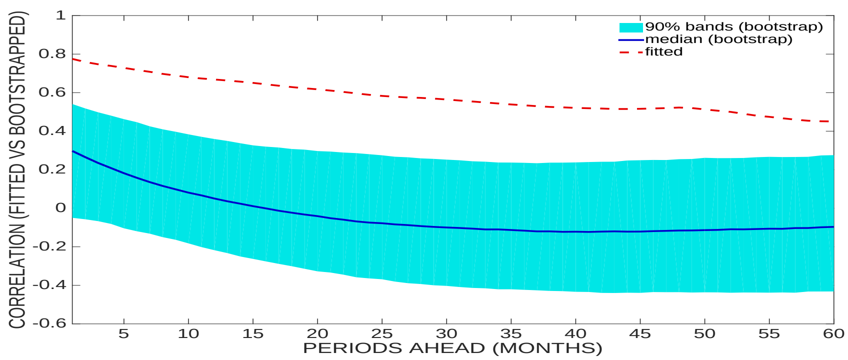
We also computed the small sample distributions of the out-of-sample fit exercise of subsection 4.1. For each country, we created 2,000 artificial correlations as a function of $h = 1, 2, ..., 60$ replicating the procedure in Figure 4 but imposing that real exchange rates are orthogonal to commodity prices, as we did before. Figure 6 displays the median correlation for each country under the null hypothesis (solid line), and the shaded areas represent the 5th and 95th percentiles of the small sample distribution of the correlation as a function of the horizon $h = 1, 2, ..., 60$. The dashed lines represents the estimated correlations from Figure 4. In all the cases, we comfortably reject the null hypothesis of orthogonality.

Figure 6: Fitted correlations and bootstrap bands under the null hypothesis of orthogonality (with four commodities, best fit)

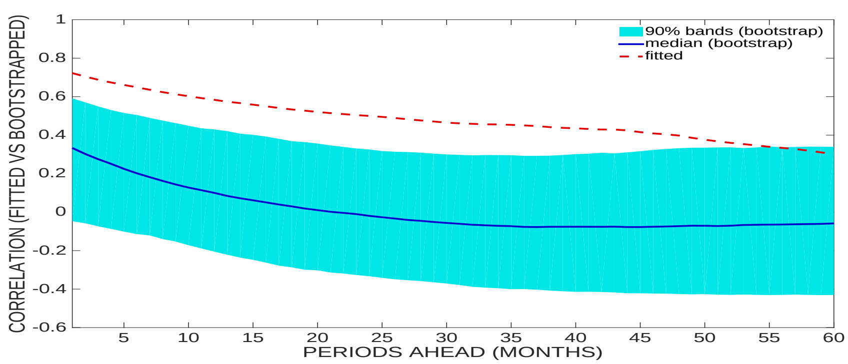
(a) United Kingdom



(b) Germany



(c) Japan



5 Calibration and simulations

In this section, we calibrate the model presented in Section 2 for the United States as country 1 and Japan as country 2, and run simulations to study the relationship between their bilateral real exchange rate and primary commodity prices in the model economy. The results show that they are closely related, resembling the empirical findings of Section X.

We use the 2000 Japan-U.S. Input-Output Table published by the Ministry of Economy, Trade and Industry (METI) of Japan to calibrate the factor shares in the production functions of both countries. We map each of the 174 sectors in the input-output table into the three sectors considered in our model: final goods, intermediate goods, and primary commodities. The exact mapping is presented in Appendix A. The group of all final goods in the United States is assumed to be Y_1 and the group of all its intermediate goods is assumed to be Q_1. We do the same for Y_2 and Q_2 in the case of Japan. Regarding the primary commodities, we assume that X_1 corresponds to the sector "petroleum and natural gas," and that X_2 corresponds to the sectors "fishing" and "seafood" together. The rest of the primary commodities are grouped into X_3.

The input-output table contains data on the payments to each of the factors of production such as intermediate inputs, compensation of employees, and operating surplus. We compute the shares of each factor of production considered in the model to pin down the parameters of the Cobb-Douglas production functions described in Section 2. For the intermediate and final good sectors, we assume that the payments to labor input are equal to the value added in the data. In the primary commodity sector, on the other hand, the labor share is computed as the share of compensation of employees in value added.

We do not have a similar dataset for the input-output table of country 3, the rest of the world (ROW), so we use data from Comtrade to compute the factor shares of each primary commodity based on their respective shares in total world trade in primary commodities in 2000. The resulting factor shares are presented in Table 4.

Next, we calibrate the relative size of each economy. We normalize the TFP level in countries 1 and 2 to be equal to one, so the relative sizes of the economies are determined by their relative endowments of labor, natural resources, and primary commodities. We normalize country 1, the United States, to have size equal to one and calibrate the other parameters in order to exactly match the average relative size of each economy in terms of nominal GDP between 1960 and 2014. We use data on nominal GDP in US dollars from the World Development Indicators (WDI) from the World Bank to decompose world GDP between the United States, Japan, and ROW in the 1960–2014 period. The calibration is presented in Table 5. The relative size of the endowments of natural resources with respect

Table 4: Calibration - factor shares (%)

	Country 1 (USA)	**Country 2 (JPN)**
Final good		
intermediate good Q_1	$\alpha_1^1 = 20.2$	$\alpha_1^2 = 0.3$
intermediate good Q_2	$\alpha_2^1 = 0.1$	$\alpha_2^2 = 23.5$
labor n^q	$\alpha_3^1 = 79.7$	$\alpha_3^2 = 76.2$
Intermediate good		
primary commodity X_1	$\beta_1^1 = 6.5$	$\beta_1^2 = 5.9$
primary commodity X_2	$\beta_2^1 = 0.0$	$\beta_2^2 = 0.1$
primary commodity X_3	$\beta_3^1 = 4.7$	$\beta_3^2 = 11.5$
labor n^q	$\beta_4^1 = 88.8$	$\beta_4^2 = 82.5$
Primary commodity X_i		
labor $n_i^{x_i}$	$\phi_1^1 = 27.8$	$\phi_2^2 = 33.0$
natural resource e_i^i	$1 - \phi_1^1 = 72.2$	$1 - \phi_2^2 = 67.0$
Primary commodity X_3		
labor $n_i^{x_3}$	$\phi_3^1 = 50.0$	$\phi_3^2 = 28.5$
natural resource e_3^i	$1 - \phi_3^1 = 50.0$	$1 - \phi_3^2 = 71.5$
	Country 3 (ROW)	
Final good		
primary commodity X_1	$\pi_1 = 58.8$	
primary commodity X_2	$\pi_2 = 5.0$	
primary commodity X_3	$\pi_3 = 36.2$	

to the endowment of labor within each country is set to be one, and the same is done for the relative endowments of primary commodities in the ROW. All the results presented here are robust to changes in the relative size of endowments within countries while keeping the relative sizes of their economies constant.

Table 5: Calibration - relative sizes

		Share of world GDP (%)	
	Parameters	**Data**	**Model**
Country 1 (USA)	$n_1 = e_1^1 = e_3^1 = 1$	30	30
Country 2 (Japan)	$n_2 = e_2^2 = e_3^2 = 0.95$	10	10
Country 3 (ROW)	$X_1^3 = X_2^3 = X_3^3 = 3.87$	60	60

Finally, we calibrate fourteen parameters related to the stochastic processes of the TFP shocks in countries 1 (USA) and 2 (JPN) and of the endowment shocks in country 3 (ROW) to minimize the (Euclidean) distance between fourteen moments in the model and in the data with respect to fluctuations in real GDP and primary commodity prices. The time period of our model is chosen to be one year. The parameters, moments, and their respective values are described in Table 6.

We assume that shocks to TFP in countries 1 and 2 are orthogonal to shocks to the endowments of primary commodities in country 3. We use the cyclical component of the annual real GDP per capita series for the US and Japan using the HP filter with parameter 6.25. For the primary commodity prices, we use the price of oil as p^{x_1}, the price of fish as p^{x_2}, and the price of aluminum as a proxy for the price of the rest of the commodities p^{x_3}, all described in Section X. We choose the price of aluminum as p^{x_3} because the group of metals is the largest group of primary commodities in terms of trade volume excluding oil. Table 6 shows that the moments implied by our model closely match their counterparts in the data. The results are based on a simulation of 6,000 periods, in which we drop the first 1,000 periods.

5.1 Results

With the calibrated model, we can assess how well the model performs with respect to non-targeted moments. The results are presented in Table 7. We start by analyzing the sectoral composition of GDP in each country. The main difference between the model and the data is with respect to the shares of the final and intermediate good sectors. For example, in the case of the United States, the share of the intermediate good sector in US GDP is 47.9 percent in the data, while it is just 18 percent in the model. This is not surprising, since we have a simplified model of each economy. What is most important in the sectoral composition in Table 7 is that we are not overstating the relative size of the primary commodity sector in these countries, which is the main feature of our analysis. In the case of the United States, for example, the primary commodity sector accounts for 3.6 percent of GDP in the data and 2.3 percent in the model.

Next, we analyze the standard deviation of the RER that is implied by the model. The result is striking. In the data, the standard deviation of the bilateral RER has the same order of magnitude as the standard deviation of primary commodity prices. In the model, that is exactly what we observe. In particular, by targeting only the volatilities in primary commodity prices, the model is able to deliver a volatility of the RER (40%) that is very close to the data (37%). This result suggests that the mechanism that we propose in this paper, that is, the fluctuations in primary commodity prices and their sources, has a great potential in explaining the fluctuations in the bilateral real exchange rates of these economies.

The close relationship between the RER and primary commodity prices is also present when we run the OLS regressions of the RER on primary commodity prices in the simulated

Table 6: Calibration - stochastic processes

Moments	**Data**	**Model**
Standard deviation of US Real GDP per capita (%)	1.3	1.5
Standard deviation of Japan RGDP per capita (%)	1.6	3.6
Autocorrelation of US Real GDP	0.31	0.31
Autocorrelation of Japan Real GDP	0.18	0.18
Correlation between US and Japan Real GDP	0.41	0.41
Standard deviation of the price of oil (%)	66.9	68.2
Standard deviation of the price of fish (%)	36.5	37.7
Standard deviation of the price of aluminum (%)	32.2	43.5
Autocorrelation of the price of oil	0.99	0.99
Autocorrelation of the price of fish	0.99	0.99
Autocorrelation of the price of aluminum	0.99	0.99
Correlation between the prices of oil and fish	0.28	0.28
Correlation between the prices of oil and aluminum	-0.20	-0.62
Correlation between the prices of fish and aluminum	0.36	0.36

Parameters	**Values**
$100 \times$ standard deviation of $\varepsilon_t^{z_1}$	1.0
$100 \times$ standard deviation of $\varepsilon_t^{z_2}$	2.5
ρ^{z_1}: autocorrelation of $z_{1,t}$	0.00
ρ^{z_2}: autocorrelation of $z_{2,t}$	0.00
correlation between $\varepsilon_t^{z_1}$ and $\varepsilon_t^{z_2}$	0.31
$100 \times$ standard deviation of $\varepsilon_t^{x_1}$	9.2
$100 \times$ standard deviation of $\varepsilon_t^{x_2}$	3.2
$100 \times$ standard deviation of $\varepsilon_t^{x_3}$	9.6
ρ^{x_1}: autocorrelation of $x_{1,t}^3$	0.99
ρ^{x_2}: autocorrelation of $x_{2,t}^3$	0.99
ρ^{x_3}: autocorrelation of $x_{3,t}^3$	0.99
correlation between $\varepsilon_t^{x_1}$ and $\varepsilon_t^{x_2}$	0.00
correlation between $\varepsilon_t^{x_1}$ and $\varepsilon_t^{x_3}$	-0.31
correlation between $\varepsilon_t^{x_2}$ and $\varepsilon_t^{x_3}$	0.81

Notes: Real GDP per capita corresponds to the cyclical component of the HP-filtered data with smoothing parameter 6.25. Primary commodity prices are normalized by US CPI both in the model and in the data. The correlation between TFP shocks and shocks to the endowments of primary commodities are set to zero.

data. The R^2 of the OLS regression is 0.99, meaning that the fluctuations in primary commodity prices account for virtually all of the RER fluctuation. In this case, it is important to notice that we are assuming that we know exactly what the relevant prices of primary

Table 7: Non-targeted moments

	Data	Model
(1) Share of country GDP (%)		
United States		
Final good Y_1	48.5	79.7
Intermediate good Q_1	47.9	18.0
Primary commodity X_1	1.2	0.6
Primary commodity X_3	2.4	1.7
Japan		
Final good Y_2	46.3	78.9
Intermediate good Q_2	49.9	20.4
Primary commodity X_2	0.4	0.0
Primary commodity X_3	3.4	0.6
(2) Standard deviation of RER (%)	37.0	40.1
(3) OLS regressions	$\mathbf{R^2}$	
Baseline regression	0.99	
Regression without $p_t^{x_3}$	0.59	

Notes: We report the standard deviation of the log of the RER. OLS regressions are based on the four-year differences of the log of RER and primary commodity prices. The latter are normalized by US CPI.

commodities are. In the data, however, there are many primary commodities. For example, Table 7 also shows the R^2 of the OLS regression when we use only the prices of X_1 and X_2. In this case, the R^2 drops to 0.59.

Finally, we can use the OLS regressions in the simulated data to interpret the estimated coefficients on the primary commodity prices. The coefficients are reported in Table 8. The table shows that there is no relevant information that we can extract from the estimated coefficients. The first reason is that we do not know what the relevant set of primary commodity prices in the data is. As we can see in Table 8, the coefficients change significantly when we estimate the regression without including the price of X_3 as a regressor, for example. The coefficient on p^{x_1} changes its sign from -0.005 to 0.376. The second reason is that even when including the correct set of primary commodity prices, which is the case in our baseline regression, the estimated coefficients are not the same as the ones implied by Equation 1. Table 8 shows that, based on our benchmark calibration, Equation 1 would imply a coefficient equal to 1.347 on the price of primary commodity X_1, whereas the regression in the simulated data delivers a coefficient equal to -0.005.

Table 8: Coefficients of the OLS regressions

	p^{x_1}	p^{x_2}	p^{x_3}
Baseline regression	-0.005	-0.006	-0.920
Regression without $p_t^{x_3}$	0.376	-0.809	
Coefficients implied by Equation 1	1.347	-1.427	-0.017

6 Conclusion

In this paper, we provide empirical evidence that points toward a common factor that moves a handful of primary commodity prices on the one hand and real exchange rates between the United States and the United Kingdom, Germany, and Japan on the other. Both sets of variables are volatile and very persistent. Moreover, during decades in which commodity prices are particularly volatile, so are commodity prices. More specifically, with only four of these prices, one can account for between one-third to one-half of the volatility of the real exchange rates for a period that lasts more than half a century.

This relationship has been known to hold for small open economies, as has been long recognized in the literature. The results of this paper imply that similar relationships have the potential to go a long way in rationalizing the behavior of the three real exchange rates between these large developed economies.

Theoretical models of real exchange rates for large developed economies have almost exclusively focused on trade in final differentiated goods. A challenge for the literature has been to deliver large fluctuations in prices (the real exchange rates) without large movements in quantities, since quantities do not seem to move nearly as much in the data. A natural way out of the puzzle is to impose small enough elasticities in the models, since these elasticities deliver large changes in prices without large changes in quantities. However, independent estimates of the substitutability between final goods deliver numbers that are much higher than what would be required to match the data. On the contrary, the elasticities of demand and supply for the primary commodities that we consider in the paper are known to be very small, so that could be a way out of the puzzle. Incorporating the commodities into a model with multiple large economies places a long research path ahead of us. The results of this paper suggest that the path is worth pursuing.

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A Data

We used (end of period) monthly series for the nominal exchange rates and the Consumer Price Index (CPI) of each country, both obtained from Global Financial Data. The commodity price series are from the World Bank *Commodity Price Data* (Pink Sheet) and the United Nations (UNCTAD*stat*). We excluded natural gas, coal, and iron because of data availability.[19] The data sources for the price series of each commodity are as follows:

(1) Petroleum - Brent crude oil. Source: Global Financial Data, Ticker: BRT_ D.

(2) Fish - price of fish meal. Source: UNCTAD*stat*.

(3) Meat - price of beef. Source: World Bank *Commodity Price Data*.

(4) Aluminum - Source: World Bank *Commodity Price Data*.

(5) Copper: - Source: World Bank *Commodity Price Data*.

(6) Gold - Source: World Bank *Commodity Price Data*.

(7) Wheat - US, $n°1$, hard red winter. Source: World Bank *Commodity Price Data*.

(8) Maize - Source: World Bank *Commodity Price Data*.

(9) Timber - Logs, Malaysia. Source: World Bank *Commodity Price Data*.

(10) Cotton - Cotton Outlook A index. Source: World Bank *Commodity Price Data*.

Table A.1 shows the selected primary commodities and their respective definitions and shares in world trade.[20]

Table A.1: List of Primary Commodities

Commodity	(%) share of world trade in 1990	SITC (rev.3)	Commodity	(%) share of world trade in 1990	SITC (rev.3)
(1) Petroleum	7.22	33	(6) Gold	0.42	971.01
(2) Fish	1.05	03	(7) Wheat	0.35	041
(3) Meat	0.89	011/012	(8) Maize	0.28	044
(4) Aluminium	0.49	285.1/684.1	(9) Timber	0.26	24
(5) Copper	0.45	283.1/682.1	(10) Cotton	0.22	263

Note: SITC (rev3) stands for Standard International Trade Classification (revision 3).
Source: Comtrade.

[19] We also performed all the experiments in the paper using sugar instead of gold (which also serves as a store of value) and the results are virtually the same.

[20] We repeated the analysis using trade data in 2000 and the results remain the same. In this case, maize and cotton are replaced by platinum and coffee.

A.1 Trade data

Trade data were obtained from the United Nations *Comtrade* Database.[21] World trade (exports+imports) for each commodity and its total were computed as the sum of trade over all the countries in the dataset.

Table A.2: Share of imports and exports in each country (% average in 1990–1999)

	United States		**United Kingdom**		**Germany**		**Japan**	
	Imports	Exports	Imports	Exports	Imports	Exports	Imports	Exports
Petroleum	8.5	1.2	3.4	6.0	5.2	0.7	13.1	0.4
Fish	1.0	0.6	0.7	0.5	0.5	0.2	5.0	0.2
Meat	0.3	0.9	0.7	0.6	1.1	0.4	2.4	0.0
Aluminium	0.4	0.2	0.3	0.2	0.5	0.1	1.5	0.0
Copper	0.2	0.2	0.3	0.0	0.4	0.1	1.2	0.1
Gold	0.3	0.9	0.0	0.1	0.3	0.2	0.8	0.1
Wheat	0.0	0.8	0.1	0.3	0.1	0.2	0.4	0.0
Maize	0.0	1.1	0.1	0.0	0.1	0.0	0.8	0.0
Timber	0.9	1.0	0.8	0.0	0.5	0.2	3.5	0.0
Cotton	0.0	0.5	0.0	0.0	0.1	0.0	0.3	0.0
SUM	**11.8**	**7.2**	**6.5**	**7.7**	**8.7**	**2.1**	**29.0**	**0.8**

[21] Available online at https://comtrade.un.org/data.

ONLINE APPENDIX

(not for publication)

Table B.1 reports the results of unit root tests for the data in levels and in three-, four-, and five-year differences. While there is evidence of unit roots for the raw data, it vanishes for the data in four-year differences.

Table B.1: Unit root tests (p-values)

	Level	three-year differences	four-year differences	five-year differences
Real Exchange Rates				
US-UK	0.018	0.001	0.003	0.003
US-DEU	0.117	0.005	0.028	0.027
US-JPN	0.809	0.001	0.018	0.027
Commodities				
Oil	0.485	0.079	0.128	0.356
Fish	0.352	0.001	0.027	0.009
Meat	0.523	0.019	0.047	0.304
Aluminium	0.145	0.001	0.001	0.003
Copper	0.319	0.009	0.025	0.103
Gold	0.508	0.001	0.016	0.025
Wheat	0.226	0.001	0.005	0.009
Maize	0.269	0.001	0.010	0.013
Timber	0.047	0.003	0.018	0.047
Cotton	0.592	0.005	0.016	0.015

Notes: variables are in logs and commodity prices are normalized by US CPI. We use the Dickey-Fuller test, in which the p-values are under the null hypothesis that the series follows a unit root process. The lag length is selected according to the Ng-Perron test. We assume a trend in the case of Japan. Cointegration tests such as Johansen (1991) or Stock and Watson (1993) do not provide evidence of cointegration between real exchange rates and primary commodity prices.

In Table B.2, we report the first-order autocorrelation for all the series in four-year differences, which is our benchmark case. As can be clearly seen, the high persistence of real exchange rates is also present in the commodity prices.

Table B.3 shows the volatility (standard deviation) of the bilateral real exchange rates and the commodity prices in log-levels. This table provides the same messages as Table 1 in the body of the paper.

Table B.4 shows the simple correlations of each of the bilateral RER and all the commodity prices we use. As can be seen, all simple correlations between the prices and the RER are sizable. In addition, the correlations across the PCP are also sizable in many cases.

Table B.2: First order autocorrelation of four-year differences

US-UK	US-DEU	US-JPN	Oil	Fish	Meat	Aluminium
0.98	0.98	0.98	0.97	0.98	0.97	0.98
Copper	Gold	Wheat	Maize	Timber	Cotton	
0.98	0.99	0.98	0.97	0.97	0.98	

Note: Variables are in logs and commodity prices are normalized by US CPI.

Table B.3: Volatilities of real exchange rates and primary commodity prices

	1960–2014	1960–1972	1973–1985	1986–1998	1999–2014
(a) Levels					
Real Exchange Rates:					
US-UK	0.12	0.06	0.15	0.08	0.08
US-DEU	0.18	0.07	0.21	0.09	0.13
US-JPN	0.37	0.13	0.13	0.12	0.11
Average across commodities:					
Simple	0.44	0.13	0.31	0.22	0.36
Trade weighted	0.57	0.13	0.37	0.23	0.46

Notes: Variables are in logs and commodity prices are normalized by US CPI. Weights are based on the share of total trade in 1990. The set of primary commodities is oil, fish, meat, aluminum, copper, gold, wheat, maize, timber, and cotton.

Table B.4: Contemporaneous correlations (1960–2014)

	Oil	Fish	Meat	Alum.	Copper	Gold	Wheat	Maize	Timber	Cotton
RER										
US-UK	-0.47	0.00	0.30	0.11	0.09	-0.53	0.26	0.36	-0.40	0.30
US-DEU	-0.51	-0.24	0.16	0.08	-0.08	-0.62	0.06	0.14	-0.58	0.11
US-JPN	-0.49	0.25	0.59	0.52	0.41	-0.63	0.59	0.63	-0.44	0.55
Commodities										
Oil	1.00									
Fish	0.28	1.00								
Meat	-0.17	0.45	1.00							
Alum.	-0.20	0.36	0.73	1.00						
Copper	0.07	0.72	0.57	0.52	1.00					
Gold	0.88	0.25	-0.16	-0.22	0.03	1.00				
Wheat	-0.05	0.57	0.78	0.70	0.60	-0.07	1.00			
Maize	-0.11	0.58	0.81	0.70	0.62	-0.13	0.94	1.00		
Timber	0.39	0.10	0.18	0.17	0.10	0.56	0.21	0.15	1.00	
Cotton	-0.23	0.39	0.83	0.78	0.43	-0.21	0.84	0.86	0.24	1.00

Note: Variables are in logs and primary commodity prices are normalized by US CPI.

Table B.5: Coefficients of determination R^2

	1960–2014	1960–1972	1973–1985	1986–1998	1999–2014
(a) 10 commodities, level					
United Kingdom	0.50	0.76	0.73	0.67	0.54
Germany	0.59	0.87	0.86	0.57	0.67
Japan	0.81	0.87	0.60	0.75	0.75
(b) 10 commodities, 4-year differences					
United Kingdom	0.48	0.90	0.90	0.81	0.60
Germany	0.63	0.95	0.87	0.83	0.75
Japan	0.57	0.92	0.84	0.92	0.82
(c) 4 commodities (best fit), level					
United Kingdom	0.39	0.66	0.70	0.51	0.51
Germany	0.56	0.77	0.83	0.38	0.66
Japan	0.79	0.85	0.57	0.67	0.66
(d) 4 commodities (best fit), 4-year differences					
United Kingdom	0.33	0.72	0.82	0.63	0.58
Germany	0.56	0.84	0.87	0.81	0.74
Japan	0.48	0.88	0.76	0.86	0.80
(e) Nominal values					
United Kingdom	0.41	0.60	0.82	0.67	0.63
Germany	0.59	0.86	0.87	0.81	0.75
Japan	0.57	0.89	0.76	0.77	0.79
(f) Non-US pairs					
UK-DEU	0.37	0.73	0.53	0.68	0.72
UK-JPN	0.42	0.64	0.56	0.80	0.68
DEU-JPN	0.35	0.79	0.46	0.69	0.68

One concern about regression (2) is that the variables are expressed in constant US dollars, so the US CPI appears on both sides of the equation. If its volatility is sufficiently large relative to the volatility of the nominal exchange rate and foreign CPI, that would imply large R^2s. In Table **??**.a we show that this is not the case. The table shows the same results as in Table 2.d, but for the case in which we use variables expressed in current US dollars; that is, we do not subtract the log of US CPI from either side of equation (2).[22] Table **??**.a shows that the results are invariant to whether we use variables in current or in

[22] This procedure is correct to the extent that the sum of the coefficients that multiply the price of the US CPI on the right-hand side is 1 in all cases. The model in Section XXX rationalizes that restriction.

constant US dollars.

Another concern regarding regression (2) is that commodity prices are expressed in US dollars, so they might contain the nominal exchange rate, which in turn would imply that the nominal exchange rate appears on both sides of equation (2). Again, Table **??**.b shows that this is not the case. Table **??**.b shows the results for the case in which we run the regressions for the bilateral real exchange rates without including the United States. That is, we run the regression in (2) for the bilateral real exchange rates of the United Kingdom versus Germany and Japan, and for Germany versus Japan. The results show that four primary commodities still account for a large fraction of these bilateral real exchange rate fluctuations. And this result holds true for the whole period as well as the subperiods.

B Selecting commodities based on US trade data

In Section 4 we showed the results with the four PCP that make the best fit with the real exchange rates. This set (possibly) varies by country pair and subperiod, and whether we use data in levels or in four-year differences. In this section, we explore an alternative approach based on the theory presented above: we choose the set of commodities based on US trade data and keep it fixed for all subperiods and country pairs.

Equation (**??**) shows that in order for the primary commodity price to explain a large fraction of real exchange rate fluctuations, a necessary condition is that the commodity price must be an important input (has a large share) in the production structure of one of the economies in the country pair. As we mentioned before, the difference in shares is what is crucial, but it can only be observed if the primary commodity is an important input in at least one of the economies. Based on that information, in this section we show the same set of results as in Section 4, but we choose as regressors the four commodities with the largest trade share for the United States. This is, admittedly, a very crude approximation to the data using the model of the previous section, but it has the advantage that the four commodities have not been chosen to fit the data. We see this exercise as a first approximation to using the model to discipline our choices in the empirical analysis.

Table A.2 in Appendix A.1 shows the trade data for each country and commodity that we analyze in this paper. We choose the four commodities that are the most traded in the United States according to Table A.2: petroleum, fish, timber, and gold. We report the results in Tables B.6 and B.7, and in Figure 7.

As can be seen, the results when we choose the set of four PCP based on US trade data are still very good. However, it is important to discuss some of the differences. First, the R^2s in Table B.7 are lower than in the case for the best-fit commodities (Table 3). This should be expected, since the four commodities were chosen to maximize R^2. But, with the exception of the United Kingdom, the differences are not very large. Second, the bootstrap exercise based on the out-of-sample fit exercise shows very similar results for Germany and Japan and somewhat worse for the United Kingdom. Indeed, the main difference with the previous analysis is the out-of-sample fit for the UK, in which case the curve is within the 90% confidence interval, very marginally so for the first months but getting worse as the period length grows longer.

Table B.6: R^2 with four commodities (largest US-trade share)

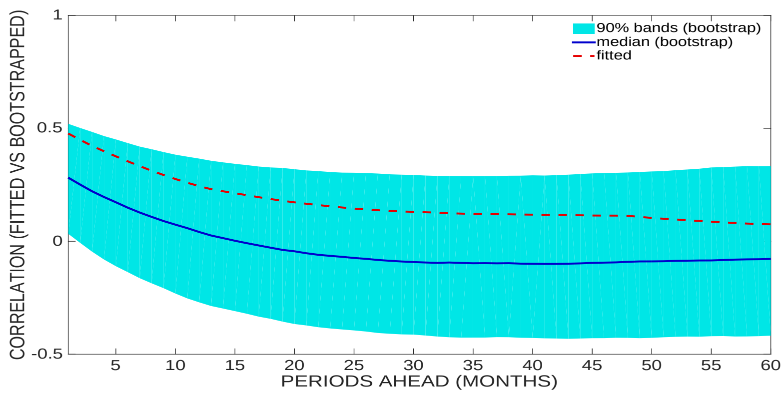
	1960–2014	1960–1972	1973–1985	1986–1998	1999–2014
		(a) Level			
United Kingdom	0.32	0.52	0.63	0.38	0.34
Germany	0.48	0.50	0.55	0.19	0.56
Japan	0.59	0.21	0.18	0.55	0.58
		(b) Four-year differences			
United Kingdom	0.25	0.63	0.73	0.25	0.46
Germany	0.53	0.89	0.71	0.50	0.69
Japan	0.44	0.71	0.52	0.82	0.68

Table B.7: Bootstrap distributions of R^2 under the null hypothesis of orthogonality, with four commodities (largest US-trade share) in four-year differences

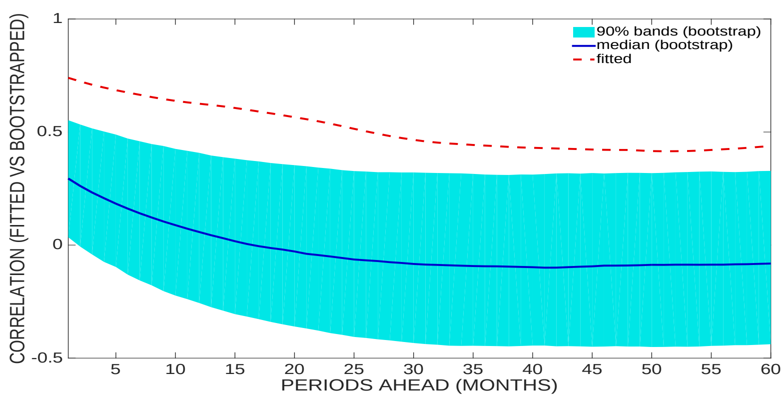
	$\hat{R}^2$	Percentiles distribution of R^2				$\Pr(R^2 \geq \hat{R}^2)$
		Median	75	90	95	
United Kingdom						
1960-2014	0.25	0.13	0.20	0.27	0.31	0.134
1960-1972	0.63	0.51	0.64	0.74	0.79	0.274
1973-1985	0.73	0.44	0.59	0.70	0.76	0.070
1986-1998	0.25	0.37	0.50	0.62	0.68	0.734
1999-2014	0.46	0.30	0.43	0.54	0.59	0.188
Germany						
1960-2014	0.53	0.13	0.20	0.27	0.33	0.000
1960-1972	0.89	0.53	0.68	0.77	0.81	0.005
1973-1985	0.71	0.45	0.60	0.72	0.77	0.106
1986-1998	0.50	0.40	0.54	0.66	0.71	0.313
1999-2014	0.69	0.33	0.47	0.58	0.64	0.024
Japan						
1960-2014	0.44	0.13	0.20	0.27	0.32	0.007
1960-1972	0.71	0.54	0.67	0.77	0.82	0.190
1973-1985	0.52	0.47	0.62	0.74	0.79	0.421
1986-1998	0.82	0.42	0.56	0.67	0.73	0.007
1999-2014	0.68	0.35	0.48	0.59	0.65	0.033

Figure 7: Fitted correlations and bootstrap bands under the null hypothesis of orthogonality (with four commodities, largest US-trade share)

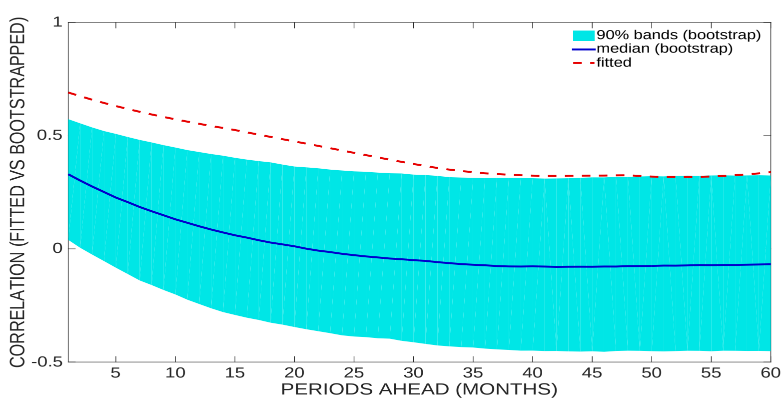
(a) United Kingdom



(b) Germany



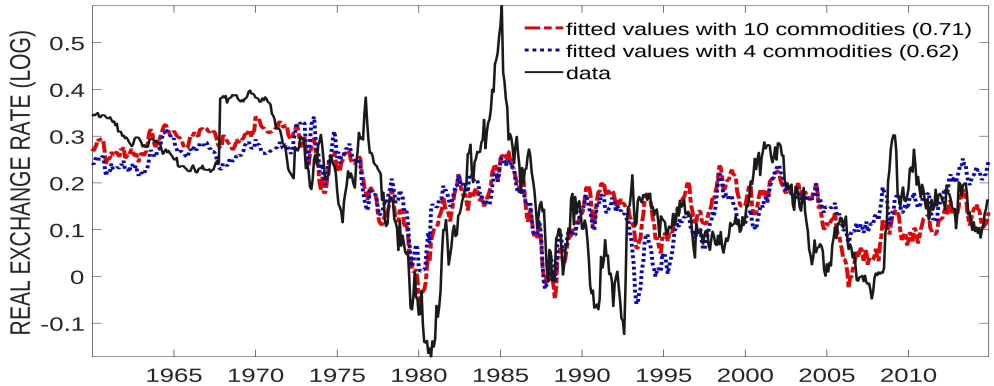
(c) Japan



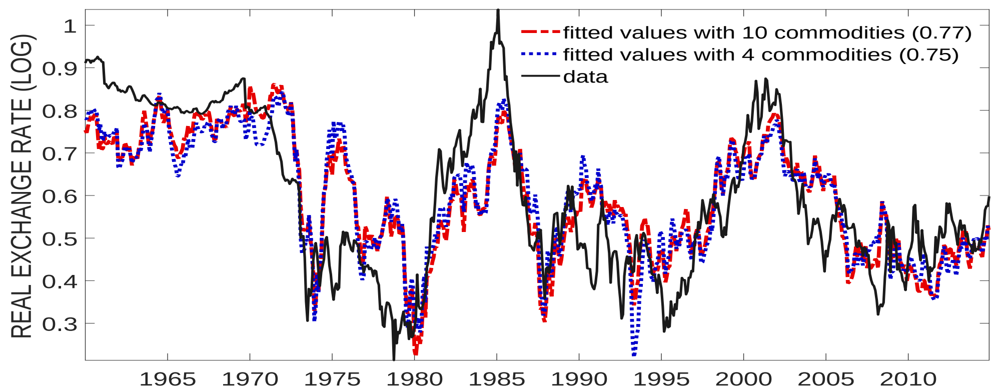
C Real exchange rates and fitted values (level)

Figure 8: Real exchange rates and fitted values, level.

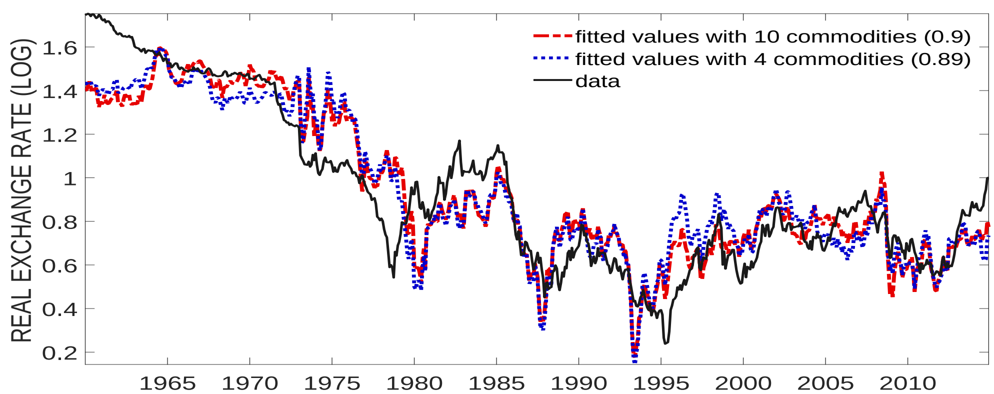
(a) United Kingdom



(b) Germany



(c) Japan



D R2s as a function of the number of months for which we take the differences

Figure 9: R2s as a function of the number of months for which we take the differences, four commodities (best fit), 1960–2014

E Regression coefficients

Table B.8: Regression coefficients: United Kingdom

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
			(a) Level		
Oil		-0.317***		-0.033	
Fish				-0.189***	
Meat	0.078		-0.204***		-0.035
Aluminium	-0.113**	-0.045	-0.198**		
Copper				-0.178***	-0.186***
Gold			-0.250***		0.228
Wheat				0.254***	-0.122***
Maize	0.120***	-0.280***			
Timber	-0.230***	-0.010			
Cotton			0.055		
			(b) Four-year differences		
Oil				-0.012	
Fish	-0.213***	-0.252***		-0.218***	-0.162**
Meat		-0.176**	-0.322***	-0.457***	-0.139***
Aluminium	-0.197***	-0.456***			
Copper			0.402***	-0.127***	-0.196***
Gold			-0.329***		0.290***
Wheat					
Maize	0.121**				
Timber			-0.211***		
Cotton	0.027	-0.439***			

Note: Superscripts *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.

Table B.9: Regression coefficients: Germany

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
			(a) Level		
Oil		-0.338***			
Fish	-0.234***				
Meat	0.095			0.218***	0.193**
Aluminium		0.575***	-0.612***		
Copper		-0.085***		-0.150**	-0.151***
Gold			0.120***	-0.063	-0.033
Wheat			-0.449***		-0.106**
Maize	0.139		0.302*		
Timber	-0.450***	0.088*		-0.138***	
Cotton					
			(b) Four-year differences		
Oil					
Fish	-0.319***	-0.095***	-0.194**		-0.375***
Meat			-0.005		
Aluminum			-0.541***		
Copper				-0.231***	
Gold				-0.138**	-0.186***
Wheat	-0.137*		-0.233***		-0.279***
Maize	0.223*	-0.032		0.252***	0.316**
Timber	-0.314***	0.282***		-0.149***	
Cotton		-0.537***			

Note: Superscripts *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.

Table B.10: Regression coefficients: Japan

	1960-2014	1960-1972	1973-1985	1986-1998	1999-2014
			(a) Level		
Oil		-0.331***	0.018		
Fish					0.393***
Meat		-0.313**	-0.090*	0.299***	
Aluminium		0.932***	-0.375***		
Copper		-0.124**		-0.137**	
Gold	-0.238***			-0.122**	-0.327***
Wheat	0.289**				0.200***
Maize	0.282***		0.239***		
Timber	-0.529***			-0.371***	
Cotton					-0.171***
			(b) Four-year differences		
Oil	0.173**			0.270***	
Fish			-0.318***	0.241***	0.346***
Meat			-0.027		
Aluminium	-0.170**		-0.434***	-0.260***	
Copper		-0.029***	0.109		
Gold		-0.290***			-0.532***
Wheat					0.205***
Maize		-0.244***			
Timber	-0.396***			-0.352***	
Cotton	-0.124*	-0.318***			-0.130***

Note: Superscripts *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.

F Selecting three commodities (best fit)

Table B.11: R^2 with three commodities (best fit)

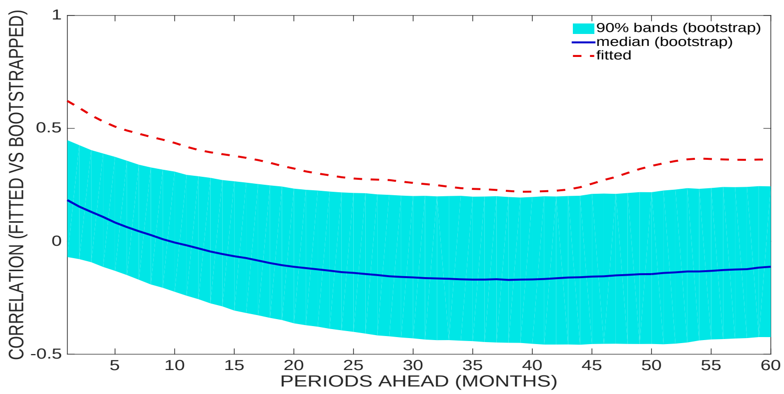
	1960–2014	1960–1972	1973–1985	1986–1998	1999–2014
			(a) Level		
United Kingdom	0.37	0.66	0.70	0.51	0.51
Germany	0.54	0.66	0.81	0.37	0.65
Japan	0.78	0.78	0.42	0.66	0.38
			(b) Four-year differences		
United Kingdom	0.24	0.68	0.78	0.63	0.39
Germany	0.53	0.84	0.80	0.72	0.72
Japan	0.46	0.81	0.58	0.73	0.69

Table B.12: Bootstrap distributions of R^2 under the null hypothesis of orthogonality, with three commodities (best fit) in four-year differences

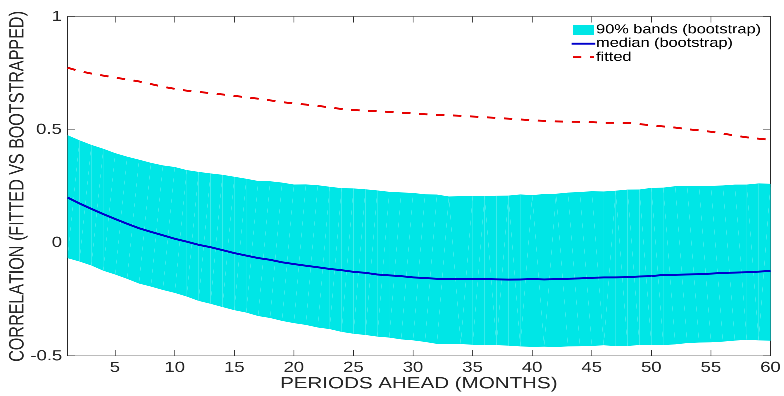
		Percentiles distribution of R^2				
	$\hat{R}^2$	Median	75	90	95	$\Pr(R^2 \geq \hat{R}^2)$
United Kingdom						
1960-2014	0.24	0.09	0.16	0.23	0.28	0.088
1960-1972	0.68	0.48	0.64	0.75	0.80	0.187
1973-1985	0.78	0.32	0.47	0.60	0.67	0.006
1986-1998	0.63	0.30	0.45	0.57	0.63	0.050
1999-2014	0.39	0.24	0.36	0.47	0.54	0.203
Germany						
1960-2014	0.53	0.10	0.17	0.24	0.29	0.000
1960-1972	0.84	0.37	0.52	0.64	0.70	0.002
1973-1985	0.80	0.31	0.47	0.60	0.66	0.005
1986-1998	0.72	0.33	0.48	0.61	0.67	0.024
1999-2014	0.72	0.23	0.36	0.49	0.57	0.005
Japan						
1960-2014	0.46	0.10	0.17	0.24	0.29	0.003
1960-1972	0.81	0.43	0.59	0.71	0.77	0.024
1973-1985	0.58	0.32	0.48	0.61	0.67	0.130
1986-1998	0.73	0.30	0.44	0.56	0.63	0.011
1999-2014	0.69	0.26	0.40	0.52	0.59	0.012

Figure 10: Fitted correlations and bootstrap bands under the null hypothesis of orthogonality (with three commodities, best fit)

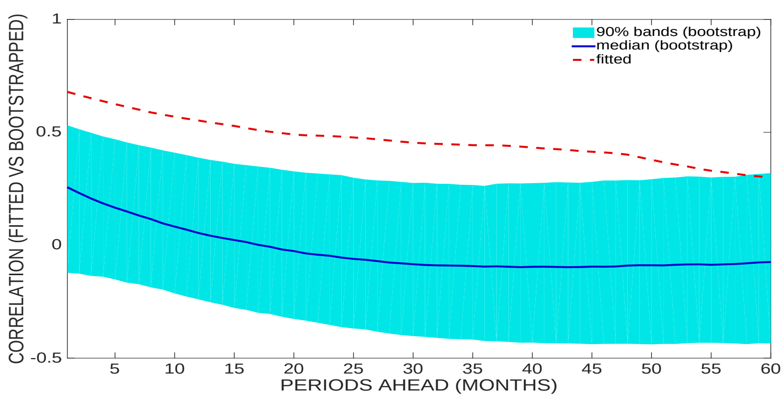
(a) United Kingdom



(b) Germany



(c) Japan



G Multiple representations of the real exchange rate

In this section we show that the model admits many representations of the bilateral real exchange rate. All of these representations hold simultaneously in equilibrium and are derived by substituting different equilibrium conditions into the definition of the real exchange rate. To make the discussion as simple as possible, we develop our arguments using an economy with one non-traded final good, Y_t, one traded intermediate good, q_t, two commodities, $x_t(1)$ and $x_t(2)$, and one type of labor. In terms of the previous notation, $J = N = 1$ and $H = 2$. We use the symbol "$\sim$" to represent variables from an arbitrary foreign country.

The nontraded final good is produced with a Cobb-Douglas technology

$$Y_t = \bar{\alpha} Z_t^y \left(n_t^y\right)^{\alpha} \left(q_t\right)^{1-\alpha},$$

where $\bar{\alpha} = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$. The technology to produce intermediate goods is

$$Q_t = \bar{\beta} Z_t^q \left(n_t^q\right)^{\beta_n} x_t(1)^{\beta_1} x_t(2)^{\beta_2},$$

where $\bar{\beta} = \beta_n^{-\beta_n}\beta_1^{-\beta_1}\beta_2^{-\beta_2}$ and $\beta_n + \beta_1 + \beta_2 = 1$. Finally, commodity $h = 1, 2$ is produced with the production function

$$X_t(h) = Z_t^x(h)\left(n_t^x(h)\right)^{\eta_h} E(h)^{1-\eta_h}.$$

As mentioned above, we set $E(h) = 0$ if the economy is unable to produce commodity h.

Competitive markets imply that nominal prices equal marginal costs. In the final and intermediate goods sectors these conditions are, respectively,

$$P_t = (W_t)^{\alpha}(P_t^q)^{1-\alpha}/Z_t^y \tag{6}$$

$$P_t^q = (W_t)^{\beta_n}\left(P_t^x(1)\right)^{\beta_1}\left(P_t^x(2)\right)^{\beta_2}/Z_t^q, \tag{7}$$

where $P_t^x(h)$ denotes the nominal price of commodity $h = 1, 2$.

The two commodities and the intermediate good can be internationally traded, possibly with some frictions represented by trade taxes; these could be actual taxes or any other implicit impediment to trade that drives a wedge between home and foreign prices. If we let S_t denote the nominal exchange rate, defined as foreign currency per unit of domestic currency, the laws of one price in the intermediate goods and commodities sectors are, respectively,

$$S_t P_t^x(1) T_t^x(1) = \tilde{P}_t^x(1) \quad (8)$$
$$S_t P_t^x(2) T_t^x(2) = \tilde{P}_t^x(2) \quad (9)$$
$$S_t P_t^q T_t^q = \tilde{P}_t^q, \quad (10)$$

where T_t^q and $T_t^x(h)$ for $h = 1, 2$ are (gross) taxes on foreign trade.

We now consider two representations of the bilateral real exchange rate in terms of commodity prices and nominal wages measured in a common currency. The two representations differ in whether we replace the law of one price in intermediate goods (10) in the price indexes (6). In particular, introducing the intermediate good price index (7) into the final good price index (6) and taking logs gives[23]

$$p_t = (\alpha + (1-\alpha)\beta_n) w_t + (1-\alpha)\beta_1 p_t^x(1) + (1-\alpha)\beta_2 p_t^x(2) - (1-\alpha) z_t^q - z_t^y.$$

Using the equivalent expression for the foreign country and the laws of one price (8) and (9), we derive the first representation of the bilateral real exchange rate in terms of commodity prices and nominal wages,

$$\begin{aligned} p_t + s_t - \tilde{p}_t &= [(1-\alpha)\beta_1 - (1-\tilde{\alpha})\tilde{\beta}_1] p_t^x(1) + [(1-\alpha)\beta_2 - (1-\tilde{\alpha})\tilde{\beta}_2] p_t^x(2) \\ & [\alpha + (1-\alpha)\beta_n] w_t - [\tilde{\alpha} + (1-\tilde{\alpha})\tilde{\beta}_n](\tilde{w}_t - s_t) + \varepsilon_{1t}, \quad (11) \end{aligned}$$

where ε_{1t} is a bundle of productivity shocks and trade taxes defined as

$$\varepsilon_{1t} = \tilde{z}_t^y - z_t^y + (1-\tilde{\alpha})\tilde{z}_t^q - (1-\alpha) z_t^q - (1-\tilde{\alpha})[\tilde{\beta}_1 t_t^x(1) + \tilde{\beta}_2 t_t^x(2)].$$

Equation (11) is one representation of the real exchange rate in terms of commodity prices and wages. We can also use the law of one price (7) to obtain a similar representation of the real exchange rate. In particular, using equations (7), (8), and (9), the law of one price for intermediate goods (10) implies the following relation between home and foreign wages:

$$\beta_n w_t + (\beta_1 - \tilde{\beta}_1) p_t^x(1) + (\beta_2 - \tilde{\beta}_2) p_t^x(2) + t_t^q - z_t^q = \tilde{\beta}_n(\tilde{w}_t - s_t) + \tilde{\beta}_1 t_t^x(1) + \tilde{\beta}_2 t_t^x(2) - \tilde{z}_t^q.$$

[23]We use lowercase letters to denote the natural logarithm of the corresponding uppercase letters. In particular, $p_t = \log P_t$, $t_t^x(h) = \log T_t^x(h)$, and so forth.

Therefore, equation (11) can also be written as

$$\begin{aligned} p_t + s_t - \tilde{p}_t &= (\tilde{\alpha}\tilde{\beta}_1 - \alpha\beta_1) p_t^x(1) + (\tilde{\alpha}\tilde{\beta}_2 - \alpha\beta_2) p_t^x(2) \\ &\quad + \alpha(1-\beta_n) w_t - \tilde{\alpha}(1-\tilde{\beta}_n)(\tilde{w}_t - s_t) + \varepsilon_{2t}, \end{aligned} \tag{12}$$

where ε_{2t} is given by

$$\varepsilon_{2t} = \tilde{z}_t^y - z_t^y + \alpha z_t^q - \tilde{\alpha}\tilde{z}_t^q + \tilde{\alpha}\tilde{\beta}_1 t_t^x(1) + \tilde{\alpha}\tilde{\beta}_2 t_t^x(2) - t_t^q.$$

The two representations (11) and (12) emphasize the relation between the bilateral real exchange rate, nominal wages measured in a common currency, productivity shocks, and trade taxes. Yet, the interpretation of the parameters multiplying wages and commodities changes depending on what equation we use to represent the real exchange rate.

It is also possible to express the real exchange rate in terms of commodity prices, allocations, shocks, and trade taxes. Suppose first that the home and foreign countries produce the same commodity, let us say $x_t(1)$. The first-order conditions for the optimal choice of labor in the commodity sector $h = 1$ in the home and foreign countries are given, respectively, by

$$\begin{aligned} w_t &= p_t^x(1) + z_t^x(1) - (1-\eta_1)\log n_t^x(1) + \log(\eta_1 E(1)^{1-\eta_1}) \\ \tilde{w}_t &= \tilde{p}_t^x(1) + \tilde{z}_t^x(1) - (1-\tilde{\eta}_1)\log \tilde{n}_t^x(1) + \log(\tilde{\eta}_1 \tilde{E}(1)^{1-\tilde{\eta}_1}). \end{aligned}$$

Using the law of one price (8) in the foreign first-order condition and replacing the two wage equations in (12) delivers the following expression for the real exchange rate in terms of commodity prices, labor allocations, productivity shocks, and trade taxes:

$$p_t + s_t - \tilde{p}_t = (\alpha\beta_2 - \tilde{\alpha}\tilde{\beta}_2)(p_t^x(1) - p_t^x(2)) + \varepsilon_{3t}, \tag{13}$$

where ε_{3t} depends on productivity shocks, trade taxes, and labor allocations as follows:

$$\begin{aligned} \varepsilon_{3t} &= \varepsilon_{2t} + \alpha(1-\beta_n)[z_t^x(1) - (1-\eta_1)\log n_t^x(1) + \log(\eta_1 E(1)^{1-\eta_1})] \\ &\quad - \tilde{\alpha}(1-\tilde{\beta}_n)[t_t^x(1) + \tilde{z}_t^x(1) - (1-\tilde{\eta}_1)\log \tilde{n}_t^x(1) + \log(\tilde{\eta}_1 \tilde{E}(1)^{1-\tilde{\eta}_1})]. \end{aligned}$$

There is, of course, a symmetric expression if both countries produce the commodity $x_t(2)$.

Suppose instead that the home country produces commodity $x_t(1)$ and the foreign country produces commodity $x_t(2)$. A similar algebra delivers the following expression for the real exchange rate:

$$p_t + s_t - \tilde{p}_t = (\tilde{\alpha}\tilde{\beta}_1 + \alpha (1 - \beta_n - \beta_1)) (p_t^x (1) - p_t^x (2)) + \varepsilon_{4t}, \tag{14}$$

where ε_{4t} is now given by

$$\begin{aligned}\varepsilon_{4t} =& \varepsilon_{2t} + \alpha (1 - \beta_n) [z_t^x (1) - (1 - \eta_1) \log n_t^x (1) + \log(\eta_1 E (1)^{1-\eta_1})] \\ & - \tilde{\alpha}(1 - \tilde{\beta}_n)[t_t^x (2) + \tilde{z}_t^x (2) - (1 - \tilde{\eta}_2) \log \tilde{n}_t^x (2) + \log(\tilde{\eta}_2 \tilde{E} (2)^{1-\tilde{\eta}_2})].\end{aligned}$$

Again, there is a symmetric representation if the home country produces $x_t (2)$ and the foreign country produces $x_t (1)$.

In obtaining the real exchange rate representations (13) and (14), we introduced the labor first-order conditions in the commodities sector into the representation in terms of wages (12). We could have used instead the cost minimization conditions in the final good's sector,

$$\frac{W_t}{P_t^q} = \frac{\alpha}{1-\alpha}\frac{q_t}{n_t^y} \text{ and } \frac{\tilde{W}_t}{\tilde{P}_t^q} = \frac{\tilde{\alpha}}{1-\tilde{\alpha}}\frac{\tilde{q}_t}{\tilde{n}_t^y}, \tag{15}$$

to replace nominal wages in equation (12). Inserting these expressions into (12) gives a representation of the real exchange rate in terms of commodity prices, intermediate good prices measured in a common currency, productivity shocks, trade taxes, and allocations:

$$\begin{aligned}p_t + s_t - \tilde{p}_t =& (\tilde{\alpha}\tilde{\beta}_1 - \alpha\beta_1)p_t^x (1) + (\tilde{\alpha}\tilde{\beta}_2 - \alpha\beta_2)p_t^x (2) \\ & + \alpha (1 - \beta_n) p_t^q - \tilde{\alpha}(1 - \tilde{\beta}_n) (\tilde{p}_t^q - s_t) + \varepsilon_{5t},\end{aligned} \tag{16}$$

where

$$\begin{aligned}\varepsilon_{5t} =& \varepsilon_{2t} + \alpha (1 - \beta_n) \log (q_t/n_t^y) - \tilde{\alpha}(1 - \tilde{\beta}_n) \log (\tilde{q}_t/\tilde{n}_t^y) \\ & + \log\left(\frac{\alpha}{1-\alpha}\right) (\alpha (1 - \beta_n) - \tilde{\alpha}(1 - \tilde{\beta}_n)).\end{aligned}$$

Finally, we show a representation of the real exchange rate that is independent of commodity prices. The cost minimization conditions (15), the price index (7), and the law of one price in commodities allows us to write

$$\begin{aligned}w_t(1 - \beta_n) &= \beta_1 p_t^x (1) + \beta_2 p_t^x (2) - z_t^q + \log\left(\frac{\alpha}{1-\alpha}\frac{q_t}{n_t^y}\right) \\ (\tilde{w}_t - s_t) (1 - \tilde{\beta}_n) &= \tilde{\beta}_1 [p_t^x (1) + t_t^x (1)] + \tilde{\beta}_2 [p_t^x (2) + t_t^x (2)] - \tilde{z}_t^q + \log\left(\frac{\tilde{\alpha}}{1-\tilde{\alpha}}\frac{\tilde{q}_t}{\tilde{n}_t^y}\right).\end{aligned}$$

Replacing these expressions in (12) delivers the representation real exchange rate that is independent of commodity prices,

$$p_t + s_t - \tilde{p}_t = \tilde{z}_t^y - z_t^y - t_t^q + \alpha \log\left(q_t/n_t\right) - \tilde{\alpha} \log\left(\tilde{q}_t/\tilde{n}_t\right) + \kappa, \tag{17}$$

where $\kappa = \alpha \log\left(\alpha/\left(1-\alpha\right)\right) - \tilde{\alpha} \log\left(\tilde{\alpha}/\left(1-\tilde{\alpha}\right)\right)$ is a constant.

H General Technologies

In this section we show that the simple case with Cobb-Douglas technologies generalizes, in an appropriate fashion, to general constant returns to scale production functions. The equilibrium prices in the final good sector and the intermediate good sector can be represented by the marginal cost function

$$P_t = C^y\left(W_t, P_t^q\right)/Z_t^y \tag{18}$$

$$P_t^q = C^q\left(W_t, P_t^x\left(1\right), P_t^x\left(2\right)\right)/Z_t^q \tag{19}$$

Replacing (18) with (19) and using that the cost function is homogeneous of degree one in factor prices, we can write the price of the final good as

$$P_t = P_t^x\left(1\right) c\left(\frac{W_t}{P_t^x\left(1\right)}, \frac{P_t^x\left(2\right)}{P_t^x\left(1\right)}\right)/Z_t^y$$

with a similar expression for the foreign country,

$$\tilde{P}_t = \tilde{P}_t^x\left(1\right) \tilde{c}\left(\frac{\tilde{W}_t}{\tilde{P}_t^x\left(1\right)}, \frac{\tilde{P}_t^x\left(2\right)}{\tilde{P}_t^x\left(1\right)}\right)/\tilde{Z}_t^y.$$

The real exchange rate is then defined as

$$rer_t = \frac{c\left(\frac{W_t}{P_t^x(1)}, \frac{P_t^x(2)}{P_t^x(1)}\right)}{\tilde{c}\left(\frac{\tilde{W}_t}{\tilde{P}_t^x(1)}, \frac{\tilde{P}_t^x(2)}{\tilde{P}_t^x(1)}\right)} \frac{\tilde{Z}_t^y}{Z_t^y} \frac{P_t^x\left(1\right) S_t}{\tilde{P}_t^x\left(1\right)}.$$

Using the law of one price for the commodities,

$$rer_t = \frac{c\left(\frac{W_t}{P_t^x(1)}, \frac{P_t^x(2)}{P_t^x(1)}\right)}{\tilde{c}\left(\frac{\tilde{W}_t}{\tilde{P}_t^x(1)}, \frac{P_t^x(2)}{P_t^x(1)}\frac{T_t^x(2)}{T_t^x(1)}\right)} \frac{\tilde{Z}_t^y/Z_t^y}{T_t^x\left(1\right)}. \tag{20}$$

Let the technology to produce commodity $i = 1, 2$ be given by $X_t\left(i\right) = Z_t^x\left(i\right) F_i\left(n_t^x\left(i\right)\right)$. The first-order condition with respect to labor is then given by

$$W_t/P_t^x\left(i\right) = Z_t^x\left(i\right) F'\left(n_t^x\left(i\right)\right).$$

Suppose that both countries produce commodity $X_t\left(1\right)$. Then replacing the previous condition evaluated at $i = 1$ in (20) gives the first representation of the real exchange rate in terms of commodity prices and labor allocations,

$$rer_t = \frac{c\left(Z_t^x(1)\,F_1'(n_t^x(1))\,,\frac{P_t^x(2)}{P_t^x(1)}\right)}{\tilde{c}\left(\tilde{Z}_t^x(1)\,F_1'(\tilde{n}_t^x(1))\,,\frac{P_t^x(2)}{P_t^x(1)}\frac{T_t^x(2)}{T_t^x(1)}\right)}\frac{\tilde{Z}_t^y/Z_t^y}{T_t^x(1)}. \tag{21}$$

Suppose instead that the home country produces commodity $X(1)$ but the foreign country produces commodity $X(2)$. The real exchange rate can then be written as

$$rer_t = \frac{c\left(Z_t^x(1)\,F_1'(n_t^x(1))\,,\frac{P_t^x(2)}{P_t^x(1)}\right)}{\tilde{c}\left(\tilde{Z}_t^x(2)\,F_2'(\tilde{n}_t^x(2))\,\frac{P_t^x(2)}{P_t^x(1)},\frac{P_t^x(2)}{P_t^x(1)}\right)}\frac{\tilde{Z}_t^y/Z_t^y}{T_t^x(2)}, \tag{22}$$

which is our second representation of the real exchange rate.

To obtain our third representation of the real exchange rate, we use the cost minimization condition in the final goods' sector, which can be written as

$$\frac{W_t}{P_t^q} = h\left(\frac{n_t^y}{q_t}\right) \tag{23}$$

where h is a decreasing function. Using this expression in (20) delivers

$$rer_t = \frac{c\left(h\left(\frac{n_t^y}{q_t}\right)\frac{P_t^q}{P_t^x(1)},\frac{P_t^x(2)}{P_t^x(1)}\right)}{\tilde{c}\left(\tilde{h}\left(\frac{\tilde{n}_t^y}{\tilde{q}_t}\right)\frac{\tilde{P}_t^q}{\tilde{P}_t^x(1)},\frac{T_t^x(2)}{T_t^x(1)}\right)}\frac{\tilde{Z}_t^y/Z_t^y}{T_t^x(1)}. \tag{24}$$

This expression relates the real exchange rate to relative commodity prices, to the relative price of the intermediate good in terms of commodity $X(1)$, and to the allocation.

Finally, we obtain an expression independent of commodity prices. To that end, we use that the equilibrium price in the intermediate goods' sector (19) can be written as

$$\frac{P_t^q}{P_t^x(1)} = \phi\left(\frac{W_t}{P_t^x(1)},\frac{P_t^x(2)}{P_t^x(1)}\right)/Z_t^q.$$

But (23) implies

$$\frac{W_t}{P_t^x(1)} = \frac{P_t^q}{P_t^x(1)}h\left(\frac{n_t^y}{q_t}\right),$$

which, using the previous expression, becomes

$$\frac{W_t}{P_t^x(1)} = \phi\left(\frac{W_t}{P_t^x(1)},\frac{P_t^x(2)}{P_t^x(1)}\right)h\left(\frac{n_t^y}{q_t}\right)/Z_t^q.$$

Using the properties of the cost function,

$$1 = \phi\left(1, \frac{P_t^x(2)/P_t^x(1)}{W_t/P_t^x(1)}\right) h\left(\frac{n_t^y}{q_t}\right)\frac{1}{Z_t^q}.$$

This is an implicit function which delivers

$$\frac{W_t}{P_t^x(1)} = \frac{P_t^x(2)}{P_t^x(1)}\kappa\left(\frac{n_t^y}{q_t}, Z_t^q\right).$$

The equivalent expression for the foreign country (using the law of one price for the commodities) is

$$\frac{\tilde{W}_t}{\tilde{P}_t^x(1)} = \frac{P_t^x(2)}{P_t^x(1)}\frac{T_t^x(2)}{T_t^x(1)}\tilde{\kappa}\left(\frac{\tilde{n}_t^y}{\tilde{q}_t}, \tilde{Z}_t^q\right).$$

Using the latter two expressions in (20) gives the last representation of the real exchange rate,

$$rer_t = \frac{c\left(\frac{P_t^x(2)}{P_t^x(1)}\kappa\left(\frac{n_t^y}{q_t}, Z_t^q\right), \frac{P_t^x(2)}{P_t^x(1)}\right)}{\tilde{c}\left(\frac{P_t^x(2)}{P_t^x(1)}\frac{T_t^x(2)}{T_t^x(1)}\tilde{\kappa}\left(\frac{\tilde{n}_t^y}{\tilde{q}_t}, \tilde{Z}_t^q\right), \frac{P_t^x(2)}{P_t^x(1)}\frac{T_t^x(2)}{T_t^x(1)}\right)}\frac{\tilde{Z}_t^y/Z_t^y}{T_t^x(1)}.$$

The properties of the cost function imply that the relative prices $P_t^x(2)/P_t^x(1)$ disappear,

$$rer_t = \frac{c\left(\kappa\left(\frac{n_t^y}{q_t}, Z_t^q\right), 1\right)}{\tilde{c}\left(\frac{T_t^x(2)}{T_t^x(1)}\tilde{\kappa}\left(\frac{\tilde{n}_t^y}{\tilde{q}_t}, \tilde{Z}_t^q\right), \frac{T_t^x(2)}{T_t^x(1)}\right)}\frac{\tilde{Z}_t^y/Z_t^y}{T_t^x(1)}. \tag{25}$$

I Multiple Consumption Goods

In this section we assume that there are many different types of (nontraded) consumption goods. In particular, we assume that there are $k = 1, ..., K$ types of consumption goods. Households in each country value the basket of consumption goods

$$\prod_{k=1}^{K} C_t(k)^{\rho(k)},$$

in which the coefficients, $\rho(k)$, add up to one. Again, we allow for different coefficients, $\rho(k)$, across countries. Each consumption good is produced according to the technology

$$C_t(k) = Z_t^C(k) \left(\prod_{j=1}^{J} [n_t^{C(k)}(j)]^{\psi^C(k,j)} \right)^{\alpha} \left(\prod_{i=1}^{N} q_t^{C(k)}(i)^{\varphi(k,i)} \right)^{1-\alpha}.$$

The rest of the economy (the production of intermediate goods and commodities) has the same structure as in Section 2.

I.1 Prices

Let P_t denote the price of the basket of goods and $P_t^C(k)$ the price of the type-k consumption good. Households' cost-minimization problem over different types of consumption goods implies that, in equilibrium, the price of the basket satisfies

$$\ln P_t = \sum_{k=1}^{K} \rho(k) \ln P_t^C(k).$$

Next, we use the fact that the price of each type of consumption good will be Cobb-Douglas functions of factor prices (see Section **??**). So we reach the equivalent of equation (**??**) for the case of multiple consumption goods:

$$\ln P_t = \sum_{k=1}^{K} \rho(k) \left[\begin{array}{l} \ln\left(\frac{\kappa^{C(k)}}{Z_t^C(k)}\right) + \alpha \sum_{j=1}^{J} \psi^C(k,j) \ln W_t(j) + \\ (1-\alpha) \sum_{i=1}^{N} \varphi(k,i) \ln P_t^Q(i) \end{array} \right],$$

which is now a sum over the different types of consumption goods. Therefore, we can follow the same steps as described in Section **??** in order to derive a relationship between bilateral real exchange rates and PCP that resembles equation (**??**). The only difference in the case of multiple consumption goods is that the relationship between PCP and real exchange rates will also depend on the difference in preferences over the types of consumption goods across countries, that is, differences in the coefficients $\rho(k)$.

J Sum of coefficients

In this appendix, we show that the sum of the coefficients in prices and wages in expression (**??**), repeated here for convenience,

$$
\begin{aligned}
\ln P_t &= \ln \frac{\kappa^C}{z_t^y} + (1-\alpha)\sum_{i=1}^{N} \varphi(i)\, \frac{\kappa^{Q(i)}}{z_t(i)} \\
&+ \sum_{j=1}^{J}\left[\alpha\psi^C(j) + \sum_{i=1}^{N}(1-\alpha)\,\varphi(i)\,\beta(i)\psi^Q(i,j)\right]\ln W_t(j) \\
&+ (1-\alpha)\sum_{i=1}^{N}\varphi(i)\,(1-\beta(i))\sum_{h=1}^{N}\gamma(i,h)\ln P_t(h),
\end{aligned}
$$

add up to 1. The sum of the coefficients is

$$
\sum_{j=1}^{J}[\alpha\psi^C(j) + (1-\alpha)\sum_{i=1}^{N}\beta(i)\varphi(i)\psi^Q(i,j)] + (1-\alpha)\sum_{h=1}^{N}\sum_{i=1}^{N}(1-\beta(i))\varphi(i)\gamma(i,h),
$$

which can be written

$$
\alpha\sum_{j=1}^{J}\psi^C(j) + (1-\alpha)\sum_{i=1}^{N}\beta(i)\varphi(i)\sum_{j=1}^{J}\psi^Q(i,j) + (1-\alpha)\sum_{i=1}^{N}(1-\beta(i))\varphi(i)\sum_{h=1}^{N}\gamma(i,h)
$$

or

$$
\begin{aligned}
&\alpha + (1-\alpha)\sum_{i=1}^{N}\beta(i)\varphi(i) + (1-\alpha)\sum_{i=1}^{N}(1-\beta(i))\varphi(i) \\
= \;& \alpha + (1-\alpha)\sum_{i=1}^{N}\beta(i)\varphi(i) + (1-\alpha) - (1-\alpha)\sum_{i=1}^{N}\beta(i)\varphi(i) = 1.
\end{aligned}
$$