On the Assignment of Workers to Occupations and the Human Capital of Countries

Veronica Mies
P. Universidad Catolica de Chile

Alexander Monge-Naranjo
Federal Reserve Bank of St. Louis & Washington University

Matias Tapia
Banco Central de Chile

St Louis, 2018
Disclaimer. The views expressed here are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of St. Louis or the Federal Reserve System.
The Aggregate Human Capital (AHK) of Countries?

- **Data:**
    - Uneven Expansions across countries.
  - IPUMS: Differences in Skills of Workers Across Occupations.

- **Models of Aggregate Human Capital**
  - **Standard Approach**
    - Efficiency Units: Perfect substitutes/Absolute Advantage.
  - **This paper:**
    - Assignment Imperfect substitution & Comparative Advantage.
    - Distortions of workers to occupations.
What We Do

- **Data:** IPUMS: Workers Groups & Assignment to Occupations
  - Cross-country and cross-periods differences.

- **Model:** Equilibrium Assignment: Workers to Jobs/Occupations
  - Determination of AHK from a General Equilibrium Roy model.
  - Inference:
    - Technology Shifters.
    - Comparative and Absolute advantage factors.
    - Distortions (workers' types to occupations)

- **Counterfactuals:**
  - Development and Growth Accounting.
  - Impact of Labor Market Distortions on AHK
  - *To Do:* Technology shifts and Expansion in Schooling groups.
Key Take-Aways

- **Data:** Patterns in the Assignment of Workers to Occupations

- **Model:**
  - AHK determination in a General Equilibrium Roy model.
    - Aggregate & distribution consequences: HK expansions & distortions.
  - Inference of AHK from Data
    - Simple & usable formulas on observable data.
    - ...with & without Distortions.

- **Counterfactuals:**
  - *Accounting Income Differences: Larger ‘success ratios’*
  - *Large Impact of Labor Market Distortions AHK.*
Related Literature

  - *Our focus is on AHK, no income distribution.*

- **AHK and Cross-country Differences:** Ben Jones (2014).
  - *Our AHK from equilibrium not exogenous assignment.*
    - *Impact of distortions, technological shifts and endowment changes.*

- **Assignment and AHK:** Hsieh et al. (2016).
  - *We consider groups by schooling and age race/gender groups.*
  - *We consider cross-country data, not only the US.*
Road Map

1. Model Environment and Equilibrium.

2. Using Equilibrium for Inference.
   - Key Formulas for AHK


4. Accounting for Income Differences

5. Labor Market Distortions and AHK.
The Model

- **Aggregate Output:**

\[ Y_t = Z_t (K_t)^{\alpha} (H_t)^{1-\alpha}, \]

- **Aggregate Human Capital:**

\[ H_t = \left[ \sum_{j=1}^{J} M_t (j) [H_t (j)]^\rho \right]^{\frac{1}{\rho}}. \]

  - \( H_t (j) \): Services in jobs/tasks \( j = 1 \ldots J \).
  - \( M_t (j) \geq 0; \sum_{j=1}^{J} M_t (j) = 1 \).

- **Workers:** Distributed in ‘Human capital groups’ \( e = 1, \ldots, E \).

\[ S_t = [S_t (1), \ldots S_t (e) \ldots S_t (E)]. \]

  - \( S_t (e) \geq 0 \text{ all } e; \sum_{e=1}^{E} S_t (e) = 1 \).
Assignment of Workers to Occupations

1. **Workers Choose Occupations According to:**

   1.1 the *unitary skill price* in each occupation, $w_t(j)$;
   
   1.2 *average potential productivities* $T_t(e,j) > 0$;
   
   1.3 *a random component*, $\eta(j)$, for each $j$;
   
   1.4 *wedges/taxes/barriers* $D_t(e,j)$.
Assignment of Workers to Occupations

- **Wages** $w_t(j)$: Equilibrium; common for all.

- **Skills of a Worker $e$ in Occupation $j$:** $\eta(j) \times T_t(e, j)$:
  - $\eta(j)$, $j$-th entry of random $1 \times J$ vector,
  - $\eta = [\eta(1), \ldots, \eta(J)] \in \mathbb{R}_+^J$.
  - from a multidimensional Frechet: $\theta > 1$.

  $$Q(\eta) = \prod_{j=1}^{J} \exp \left\{ -[\eta(j)]^{-\theta} \right\},$$

- **Wedges** $D_t(e, j) \geq 1$: Net earnings:
  - Gross Earnings $\times \frac{1}{D(e, j)}$ for workers $e$ in occupation $j$. 
Competitive Equilibrium Assignment

Given Skill (job) Prices: $w_t(j)$.

- **Firms:**

  \[
  \max_{\{H_t(j), K_t\}} \pi = \left\{ \begin{aligned}
  & Z_t(K_t)^{\alpha} \left( \left[ \sum_{j=1}^{J} M_t(j) [H_t(j)]^{\rho} \right]^{\frac{1}{\rho}} \right)^{1-\alpha} \\
  & - \sum_{j=1}^{J} w_t(j) H_t(j) - R_t K_t.
\end{aligned} \right.
\]

- **Workers:**

  \[
  \max_{i \in \{1, \ldots, J\}} \left\{ \eta(i) T_t(e, i) \frac{w_t(i)}{D_t(e, i)} \right\}.
\]
Competitive Equilibrium Assignment

- **Skill (job) Prices**: \( w_t (j) \).

\[
    w_t (j) = \tilde{w}_t \times M_t (j) [H_t (j)]^{\rho-1},
\]

\[
    \tilde{w}_t \equiv (1 - \alpha) Z_t (K_t / H_t)^{\alpha} \times (H_t)^{1-\rho}.
\]

- **Workers**: Probability a worker \( e \) goes to occupation \( j \):

\[
    p_t (e, j) = \frac{\left[ w_t (j) \frac{T_t (e,j)}{D_t (e,j)} \right]^\theta}{\sum_{i=1}^{J} \left[ w_t (i) \frac{T_t (e,i)}{D_t (e,i)} \right]^\theta};
\]

\[
    \sum_{j=1}^{J} p_t (e, j) = 1 \text{ for all } e.
\]
Undistorted Equilibrium: $D(e,j)=1$

- Services from workers $e$ to occupation $j$:

\[
H_t(j, e) = [S_t(e) p_t(e, j)] \left[ \Gamma (1 - \theta^{-1}) T_t(e, j) p_t(e, j)^{-1/\theta} \right],
\]

- Summing over all $e$:

\[
H_t(j) = \Gamma (1 - \theta^{-1}) \sum_{e=1}^{E} S_t(e) [p_t(e, j)]^{(\theta-1)/\theta} T_t(e, j).
\]

- Participation shares: Inserting $w_t(j)$, taking out $\bar{w}_t$:

\[
p_t(e, j) = \frac{\left[ M_t(j) [H_t(j)]^{\rho-1} T_t(e, j) \right]^\theta}{\sum_{i=1}^{J} \left[ M_t(i) [H_t(i)]^{\rho-1} T_t(e, i) \right]^\theta}.
\]
Undistorted Equilibrium: $D(e,j)=1$

- Fixed point to determine $\{H_t(j)\}_{j=1}^J$

\[
H_t(j) = \Theta \left\{ \sum_{e=1}^E \frac{S_t(e) M_t(j)^{(\theta-1)} T_t(e,j)^\theta}{\left[ \sum_{i=1}^J \left( M_t(i) [H_t(i)]^{\rho-1} T_t(e,i) \right)^\theta \right]^\frac{\theta-1}{\theta}} \right\}^{\frac{1}{1-(\rho-1)(\theta-1)}}
\]

\[
\Theta \equiv \left[ \Gamma \left( 1 - \theta^{-1} \right) \right]^{\frac{1}{1-(\rho-1)(\theta-1)}},
\]
Distorted Equilibrium: Pure Wedges

‘Pure Wedges’

- Workers receive $1 / D_t(e, j)$ earnings.
- Firms receive $1 / D_t(e, j)$ of services

Assignment:

$$p_t(e, j) = \frac{\left[ M_t(j) [H_t(j)]^{\rho-1} \frac{T_t(e,j)}{D_t(e,j)} \right]^{\theta}}{\sum_{i=1}^{J} \left[ M_t(i) [H_t(i)]^{\rho-1} \frac{T_t(e,j)}{D_t(e,j)} \right]^{\theta}}.$$

$H_t(j)$: Fixed point of

$$H_t(j) = \Theta \left\{ \frac{\sum_{e=1}^{E} S_t(e) M_t(j)^{(\theta-1)} \left[ \frac{T_t(e,j)}{D_t(e,j)} \right]^{\theta}}{\left( \sum_{i=1}^{J} \left[ M_t(i) [H_t(i)]^{\rho-1} \frac{T_t(e,j)}{D_t(e,j)} \right]^{\theta} \right)^{\frac{\theta-1}{\theta}}} \right\} \frac{1}{1-(\rho-1)(\theta-1)}.$$
Distorted Equilibrium: Pure Taxes

**‘Pure Taxes’**

- Workers receive $\frac{1}{D_t(e,j)}$ earnings.
- Firms receive $\frac{1}{D_t(e,j)}$ of services.

**Assignment: Same as before**

$$p_t(e,j) = \frac{\left[ M_t(j) [H_t(j)]^{\rho-1} \frac{T_t(e,j)}{D_t(e,j)} \right]^\theta}{\sum_{i=1}^{J} \left[ M_t(i) [H_t(i)]^{\rho-1} \frac{T_t(e,j)}{D_t(e,j)} \right]^\theta}.$$

**\( H_t(j) \): Fixed point of**

$$H_t(j) = \left\{ \frac{1}{1-(\rho-1)(\theta-1)} \right\}.$$

$$H_t(j) = \left\{ \frac{1}{1-(\rho-1)(\theta-1)} \right\}$$
A “Ratio-of-Ratios” \( \forall j, j' \) and \( e, e' \):

\[
\frac{p_t(e, j)}{p_t(e, j')} \cdot \frac{p_t(e', j)}{p_t(e', j')} = \left( \frac{T_t(e, j)}{T_t(j', e)} \right)^\theta \cdot \left( \frac{T_t(e, j)}{T_t(j', e')} \right)^\theta.
\]

A Convenient Decomposition:

\[
T_t(e, j) = A_t(e) C_t(e, j),
\]

- \( A_t(e) \) is absolute productivity of group \( e \) across tasks \( j \)
- \( C_t(e, j) \) is the comparative advantage term.
Proposition 1. If the underlying equilibrium of the economy is undistorted: (a) the comparative advantage term is given by

\[ C_t(e, j) = \bar{C}_t p_t(e, j)^{1/\theta}, \]

for some \( \bar{C}_t > 0 \); (b) the occupation shares \( M_t(j) \) are

\[ M_t(j) = \left( \frac{\sum_{e=1}^{E} S_t(e) A_t(e) p_t(e, j)}{\sum_{i=1}^{J} \left[ \sum_{e=1}^{E} S_t(e) A_t(e) p_t(e, i) \right]^{1-\rho}} \right)^{1-\rho}. \]
Proposition 2. ...with an estimated $A_t(e)$ the AHK is

$$H_t = \Gamma \left(1 - \theta^{-1}\right) \frac{\left[\sum_{e=1}^{E} S_t(e) A_t(e)\right]^{\frac{1}{\rho}}}{\left\{\sum_{i=1}^{J} \left[\sum_{e=1}^{E} S_t(e) A_t(e) p_t(e, i)\right]^{1-\rho}\right\}^{\frac{1}{\rho}}}.$$ 

Remarks:

- $H_t$ increasing, separately HD1 in:
  - $S_t(\cdot)$ (i.e. scale);
  - $A_t(\cdot)$ (i.e. absolute productivities.)

- $H_t$ independent of Frechet parameter $\theta$.

- If $\rho = 1$ (perfect substitutes), $H_t$: traditional measurement.
Inference From Data: Distorted Equilibrium

Proposition 1*

- **Inference of** $C_t(e, j)$, $M_t(j)$ **and** $H_t$ **given** $D_t(e, j)$:

  - **Pure Wedges:**
    - **Comparative Advantage Components:**
      
      \[ C_t(e, j) = \bar{C}_t D_t(e, j) \left[ p_t(e, j) \right]^{\frac{1}{\theta}} \]
    
    - **Human Capital and shares** $M_t(j)$: Same as undistorted.

  - **Pure Taxes:**
    - **Comparative Advantage Components:**
      \[ C_t(e, j) = \bar{C}_t D_t(e, j) \left[ p_t(e, j) \right]^{\frac{1}{\theta}} \]
    
    - **Human Capital and shares** $M_t(j)$:
      \[
      H_t(j) = \Gamma \left(1 - \theta^{-1}\right) \sum_{e=1}^{E} S_t(e) A_t(e) p_t(e, j) D_t(e, j). 
      \]
      \[
      M_t(j) = \frac{\left[ \sum_{e=1}^{E} S_t(e) A_t(e) p_t(e, j) D_t(e, j) \right]^{1-\rho}}{\sum_{i=1}^{J} \left[ \sum_{e=1}^{E} S_t(e) A_t(e) p_t(e, i) D_t(e, i) \right]^{1-\rho}}. 
      \]
Inference From Data: Distorted Equilibrium

**Proposition 2** Human Capital $H_t$

- **Pure Wedges:** Same as undistorted.

- **Pure Taxes:** Incorporates $D_t(e, j)$:

  $$H_t = \Gamma \left(1 - \theta^{-1}\right) \left\{ \sum_{i=1}^{J} \left[ \sum_{e=1}^{E} S_t(e) A_t(e) p_t(e, i) D_t(e, i) \right] \frac{1}{\rho} \right\}^{1-\rho} \frac{1}{\rho}.$$
IPUMS Data

- **Occupations:** \( J = 9 \) categories:
  - (1) elementary, (2) operators, (3) agriculture, (4) traders, (5) services, (6) clerks, (7) managers, (8) technicians, (9) professionals.

- **Workers:** \( E = 42 \) groups
  - **Education:** (1) no schooling, (2) incomplete primary, (3) complete primary, (4) incomplete secondary, (5) complete secondary, (6) incomplete tertiary, (7) complete tertiary.
  - **Age:** Young, Middle, Old.
  - **Gender:** Women, Men.
Following Caselli (2005): ‘Success’ of factors-only explanation:

\[
\text{success} = \frac{\frac{Y_{\text{rich}}}{Y_{\text{poor}}}_{\text{actual}}}{\frac{Y_{\text{rich}}}{Y_{\text{poor}}}_{\text{predicted}}}
\]

where:

\( Y \) is income per worker or per person.

‘predicted’ is by adjusting AHK alone.
Accounting

Success Ratios: Standard Model (absolute adv. only) vs. Benchmark MMT:
Accounting

Success Ratios: Model with Wedges vs. Model with Pure Taxes
Aggregate Cost of Distortions

Output Gains of Removing Distortions: From $D_{\text{actual}}$ to $D=1$
## Aggregate Cost of Distortions

**Output Gains of Removing Distortions: From $D_{\text{actual}}$ to $D=1$**

### Gross Gains in AHK: From actual to $D=1$

<table>
<thead>
<tr>
<th>Country</th>
<th>Year</th>
<th>Pure Wedges</th>
<th>Pure Taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>2000</td>
<td>2.57</td>
<td>1.04</td>
</tr>
<tr>
<td>Canada</td>
<td>1971</td>
<td>1.53</td>
<td>1.02</td>
</tr>
<tr>
<td>Canada</td>
<td>2001</td>
<td>1.39</td>
<td>1.03</td>
</tr>
<tr>
<td>India</td>
<td>1999</td>
<td>1.83</td>
<td>1.06</td>
</tr>
<tr>
<td>Indonesia</td>
<td>1976</td>
<td>1.45</td>
<td>1.06</td>
</tr>
<tr>
<td>Indonesia</td>
<td>1995</td>
<td>1.51</td>
<td>1.02</td>
</tr>
<tr>
<td>Mexico</td>
<td>1960</td>
<td>2.80</td>
<td>1.05</td>
</tr>
<tr>
<td>Mexico</td>
<td>1990</td>
<td>2.16</td>
<td>1.03</td>
</tr>
<tr>
<td>Panama</td>
<td>2000</td>
<td>1.95</td>
<td>1.06</td>
</tr>
<tr>
<td>USA</td>
<td>1960</td>
<td>1.44</td>
<td>1.04</td>
</tr>
<tr>
<td>USA</td>
<td>2000</td>
<td>1.46</td>
<td>1.03</td>
</tr>
</tbody>
</table>
Conclusions

▸ **Data:** Patterns in the Assignment of Workers to Occupations

▸ **Model:**
  ▸ AHK determination in a General Equilibrium Roy model.
    ▸ Aggregate & distribution consequences: HK expansions & distortions.
  ▸ Inference of AHK from Data
    ▸ Simple & usable formulas on observable data.
    ▸ ...with & without Distortions.

▸ **Counterfactuals:**
  ▸ *Accounting Income Differences: Larger ‘success ratios’*
  ▸ *Large Impact of Labor Market Distortions AHK.*