Lifetime-Laffer Curves and the Eurozone Crisis

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Motivation

- Eurozone crisis: Key features
  1. Sentiments seemed to play role (OMT)
     - Not liquidity Bocola and Dovis (2015)
  2. Borrowing into high spreads → Debt-to-GDP exploded

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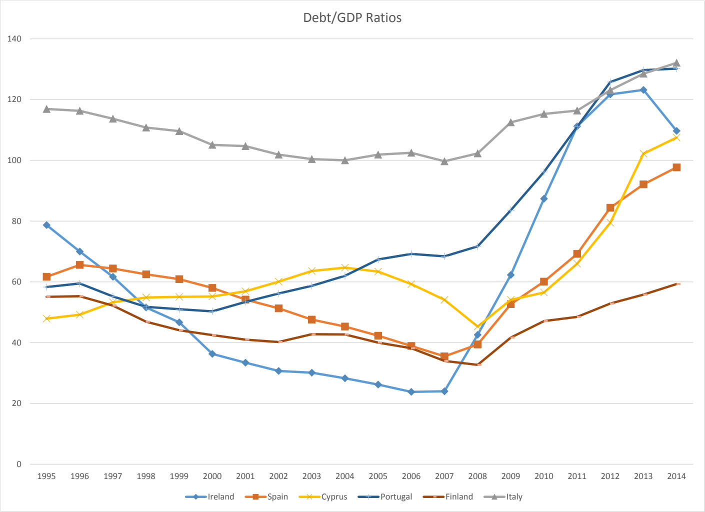
Debt/GDP Ratios
140
120
100
80
60
40
20
0
1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014
Ireland
Spain
Cyprus
Portugal
Finland
Italy

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    4. Extensions

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  5. Sovereign borrows more tomorrow (expectations fulfilled)

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- **Budget Condition**

$$d_0 = q_0(b_1 - b_0)$$

$$d_1 = q_1(b_2 - b_1)$$

Model Outline: Lenders

- Default risk in period 2: $g : \mathcal{R} \to [0, 1]$
  - g is increasing, continuous, differentiable, and convex up to $\bar{b} < \infty$ s.t.

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- Lenders are risk-neutral, deep-pocketed, price against risk-free R
- Implies **No-Arbitrage Condition**

$$q_0 = \frac{\hat{B}}{R^2}[1 - g(b_2)]$$

$$q_1 = \frac{\hat{B}}{R}[1 - g(b_2)]$$

Model Outline: Commitment

- Period one auction revenue given by

$$\frac{\hat{B}}{R}[1 - g(b_2)] \times [b_2 - b_1]$$

Under our assumptions, concave in b_2

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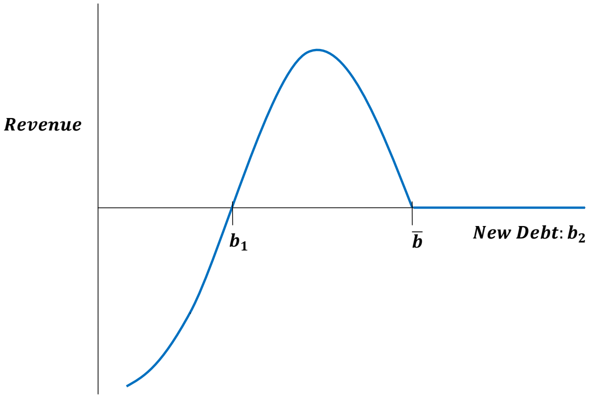

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Ensures *contemporaneous commitment to debt issuance*

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- Where $\mathbf{D} = d_0 + \frac{d_1}{R}$

Multiplicity via the Lifetime-Laffer Curve

- Let

$$\mathbf{D}^{\star}(b_0) = \max_{b_2} \frac{[1 - g(b_2)]\hat{B}}{R^2}[b_2 - b_0]$$

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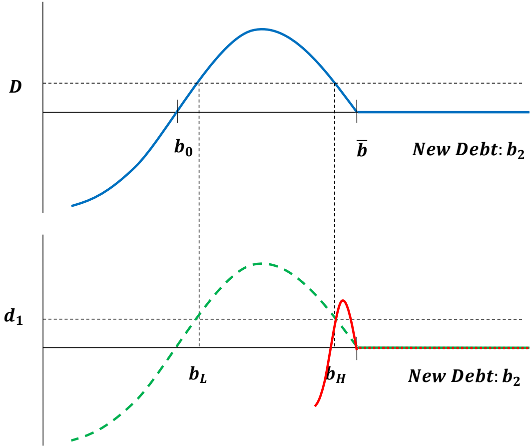
Proposition

Suppose that $0 < \mathbf{D} < \mathbf{D}^{\star}(b_0)$. *Then two solutions exist if and only if the sovereign's primary deficit stream is sufficiently front-loaded.*

Call b_2 for each of these solutions b_L and b_H

Graphical Example



The Logic of Front-Loading

Formal **Front-Loading Condition**

$$\frac{d_1}{R} \leq \frac{\hat{B}[1 - g(b_H(\mathbf{D}))]^2}{R^2 g'(b_H(\mathbf{D}))}$$

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  - Holds for $b_H \rightarrow$ Holds for b_L

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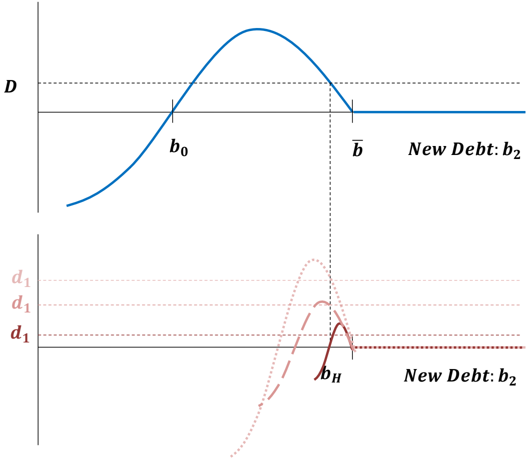

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- Can still construct Lifetime-Laffer Curve with all same properties

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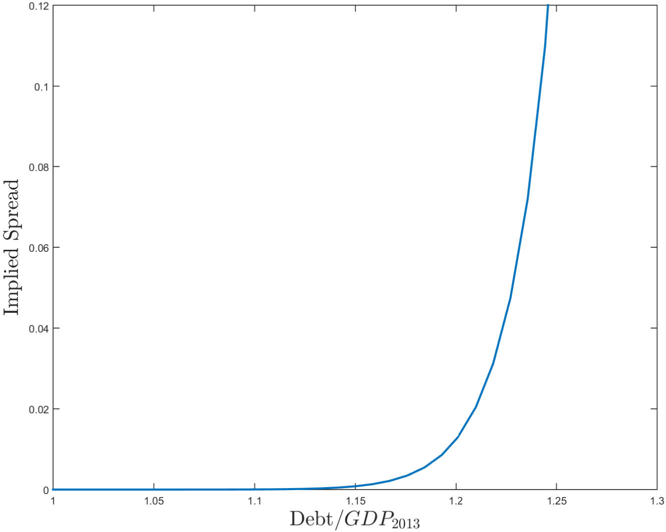
- $(\underline{b}, \alpha, \kappa, \hat{B})$ chosen to set default risk region and match spread ($450bp$), B/Y (123.2%)
- Easy to verify that $\mathbf{D} < \mathbf{D}^{\star}(b_0)$ and FL-Condition satisfied

Estimated Spread Function



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- Result is likely a *lower bound* on difference. No
  1. Coupon payments and debt maturing at auction
  2. Feedback effects from debt burdens to GDP growth

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- Delayed austerity *guarantees* existence of two solutions
- Front-loads deficit stream

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Proposition

The central bank can costlessly eliminate the high-debt solution by pledging to provide liquidity at $<q_{0,L}, q_{1,L}>$.

Deficit Response

- In T-period model, suppose that $d_t(b_T)$
  - Sovereign responds to *expected* debt build-up (or spreads, default prob, etc.)
  - Assume $d_t(\cdot)$ continuous, twice differentiable

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Under a feasibility condition, a positive economy $<< b_0, \{d_t(\cdot)\}_{t=0}^{T-1} >>$ will have at least two distinct financing trajectories. Further, if each $d_t(\cdot)$ is increasing and convex and a front-loading condition holds, then exactly two solutions exist.

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- Can augment model to include possibility of banking sector bailout

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- Re-estimate Irish model
  - Counterfactual B/Y falls: 76.57%
  - No need for bailout

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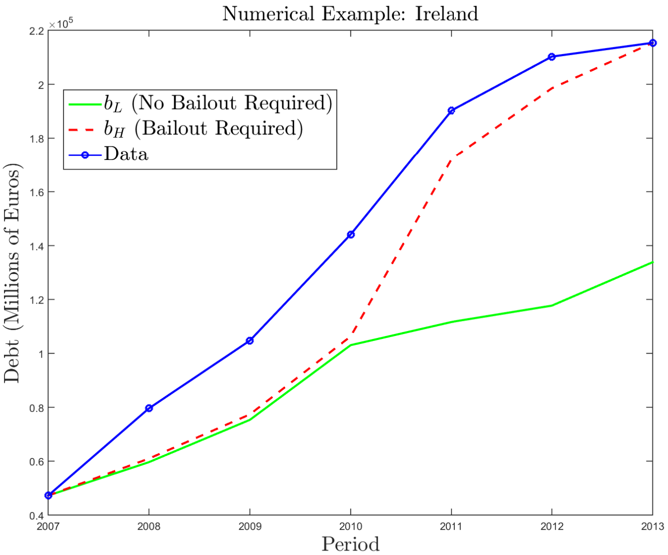

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  - Commitment condition, budget condition, and no-arbitrage condition hold in each state
- Economy is described by $<< b_0, d_0, \{d_1(s)\}_{s=1}^N >>$ and the distribution across s
- Solution given by $<< b_1, \{b_2(s)\}_{s=1}^N >>$

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Under a feasibility condition and a front-loading condition, a positive economy $<< b_0, d_0, \{d_1(s)\}_{s=1}^N >>$ *will have at least two distinct solutions. Further, if* $<< b_{L,1}, \{b_L(s)\}_{s=1}^N >>$ *and* $<< b_{H,1}, \{b_H(s)\}_{s=1}^N >>$ *are components of those two distinct solutions and wlog it must be the case that* $b_L(s) \leq b_H(s)$ *for any* $s \in S$ *and that only the lesser solution will be numerically stable.*

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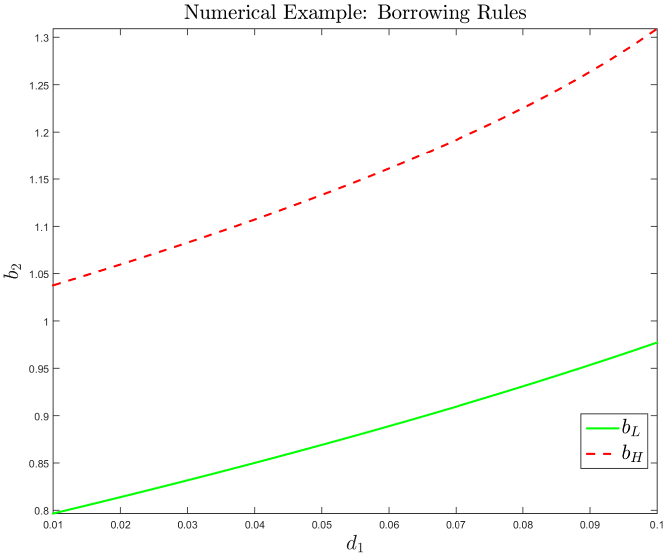
Generalizes deterministic existence result

Example with Uncertainty: $N = 25$



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Proposition

If, for every $s = 1, \ldots, N$, *the hazard rate,* $H[d_1(s, b)]$ *is sufficiently bounded for* $b \in \left[0, \bar{b}\right)$, *then under the conditions of stochastic case result, a positive economy* $<< b_0, d_0, \{d_1(s, \cdot)\}_{s=1}^{N} >>$ *will exhibit two distinct solutions.*

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- Natural to assume that $\frac{\partial d_1}{\partial b_2} < 0$

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If, for every $s = 1, \ldots, N$, *the hazard rate,* $H[d_1(s, b)]$ *is sufficiently bounded for* $b \in \left[0, \bar{b}\right)$, *then under the conditions of stochastic case result, a positive economy* $<< b_0, d_0, \{d_1(s, \cdot)\}_{s=1}^{N} >>$ *will exhibit two distinct solutions.*

- So long as d_1 does not respond too strongly to movements in requisite debt issuance, result still holds

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- Assume exogenous, state-dependent tax s.t.

$$d_1(s, b_1) = g_1 - \tau(s)$$

- Limited commitment
  1. Solve maximization in period 1
  2. Take policy as given in period 0

Moving Toward Equilibrium II

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$$\max_{b_2} u_1(g_2; s) - \beta\kappa g(b_2)$$

$$\text{s.t.} \quad g_2 - \tau(s) = \frac{1}{R}[1 - g(b_2)][b_2 - b_1]$$

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  - Simple, extreme example

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    - d_1 does not respond to b_1 $\rightarrow$ Previous Proposition holds
    - Intuition: Fiscal consolidation very costly/difficult

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- **Again, very concave utility disincentivizes delevering**
  - Multiple financing paths $\rightarrow$ Multiple equilibria

Conclusion

- Tractable, three period model in which multiple financing trajectories arise as a result of coordination failures with long-term debt
- Calibrated simple model to Ireland $\rightarrow$ Substantial impact on spreads
  - Translates to debt-to-GDP when accounting for feedback to bank bailouts

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