

LIFETIME-LAFFER CURVES AND THE EUROZONE CRISIS*

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In a finite-horizon model of sovereign debt and default, I show that long-term debt and the lack of ability to commit to future debt issuance can give rise to a multiplicity of debt and spread trajectories despite the ability to commit to contemporaneous debt issuance in terminal periods. This multiplicity bears resemblance to recent events in the Peripheral Eurozone. In a simple calibrated exercise, I find that 380 basis points (84.6%) of the spread during the crisis may be imputable to such coordination failures; if the model is extended to include bank bailouts, it can also explain 46.63 percentage points (37.8%) of the debt-to-GDP build-up. Policy analysis reveals that both austerity measures and liquidity provision by the central bank can eliminate malignant debt trajectories, but that the latter is more likely to have resolved the crisis.

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1 Introduction

The sovereign debt crisis in the Peripheral Eurozone was unique among financial crises and has consequently generated much debate with regard to its origins. Many regard the crisis as primarily fundamental, intrinsic to the design of the monetary union or driven by the delay or abandonment of reform implementation (Lane [2012] or Fernández-Villaverde et al. [2013]); others regard the crisis as driven largely by market sentiments or illiquidity (De Grauwe [2011] or Aizenman et al. [2013]). Both stories, however, find it difficult to explain the explosive build-up in debt-to-GDP ratios that occurred *during the crisis*. Figure 1 shows this pattern among the peripheral economies starting around 2007.

The excessive debt build-up became so severe that it caused some to discuss strategies for its dissolution (Barro [2012] or Mody [2015]); yet most models of both fundamental and sentiment-driven sovereign default generate the sharp prediction that sovereign borrowers ought to delever in the face of unexpected high spreads, not lever up. Some examples of the former include Aguiar and Gopinath (2006), Arellano (2008), and Hatchondo and Martinez (2009); some examples of the latter include Cole and Kehoe (1996) and Aguiar and Amador (2013).

In this paper, I outline a new dynamic coordination failure in sovereign debt markets that naturally generates borrowing into high spreads. This coordination failure stems from a multiplicity of debt and spread trajectories, all of which satisfy the same sequence of budget constraints in a dynamic model. In this framework, all default risk is fundamental, yet expectations play a key role in selecting a given trajectory. There will be a low-debt, low-spread trajectory and a high-debt, high-spread trajectory, and these distinct trajectories remain *even when* the sovereign has the ability to commit to contemporaneous debt issuance in terminal periods and not just contemporaneous revenue issuance. In this sense, this paper generalizes the recent findings of Lorenzoni and Werning (2013).

The logic is most transparent in a three period setting: If lenders in period zero anticipate high borrowing in period one, then they will demand a risk premium on long-term bonds for the dilution caused by increased default risk in period two; as a result, the sovereign faces lower bond prices in period zero and is forced to borrow more, which implies a larger stock of debt to roll over in period one, causing higher debt issuance in period one, which justifies lenders' initial expectations.

There are two key ingredients that give rise to this multiplicity: The presence of long-term debt and a front-loaded deficit stream. Long-term debt is necessary since short-term debt does not suffer from dilution risk; accordingly, the borrowing behavior of a short-term borrower in period one is of no consequence to a period zero lender and so there is no room for dynamic coordination failures.

A front-loaded deficit stream will imply that the bulk of the sovereign's financing needs are in period zero, rather than period one. A smaller deficit in period one makes it easier to commit to contemporaneous debt issuance in period one.¹ Thus, when a deficit-stream is sufficiently front-loaded, there are no contemporaneous commitment issues in period one; they all arise in period zero, in which the sovereign lacks commitment.

Since the crisis in the peripheral Eurozone, several noteworthy pieces have emerged that have tried to explain the unusual phenomenon of borrowing into high spreads and its concomitant drastic effect on debt-to-GDP ratios. Conesa and Kehoe (2012) have termed this phenomenon 'gambling for redemption,' and have argued that being mired in a deep but temporary recession is a necessary condition for such behavior. Broner et al. (2014) and Corsetti and Dedola (2013) have also built models featuring borrowing into high spreads. The former emphasizes the crowding out effect of sovereign debt issuance when there is domestic preference for debt and the latter argues that the access to liquidity that the central bank provides is more important for preventing such crises than the printing press.

To the author's knowledge, the only others that have highlighted the role of dynamic coordination problems in the recent Eurozone crisis are Lorenzoni and Werning (2013). These authors also argue for a Laffer-curve type multiplicity in the spirit of Calvo (1988) with the explicit inclusion of long-term debt. In their environment, as in this paper, a dynamic lender coordination failure can place the economy on a malignant trajectory of high spreads and debt ratios. They term such events 'slow-moving crises.' A key difference between their paper and this is that I assume explicitly that in the terminal period, which is the period in which a contemporaneous Laffer curve arises, the sovereign can commit to both auction revenue and debt issuance, whereas in their paper the sovereign can only commit to auction revenue at any point in time.² By generating multiplicity in this way, I will demonstrate that the debt trajectories can be solved

¹By this I mean that it is easier to ensure that the sovereign is on the left-hand side of the period one 'Laffer' curve in the event of multiplicity in the vein of Calvo (1988).

²In periods prior to the terminal period, I will assume as Lorenzoni and Werning (2013) do, that the sovereign lacks the ability to commit to contemporaneous debt issuance.

for in closed form, rather than expressed as a system of differential equations. Further, as I will show, this difference highlights the non-trivial impact that austerity policies can have on the potential for dynamic coordination failures.

The methodological demonstration of this multiplicity is remarkably simple: A contemporaneous Laffer Curve emerges in the terminal period, and the sovereign is always assumed to be on the Pareto-superior left-hand side of it. However, if one collapses the sequence of flow budget constraints into a lifetime budget constraint, then another Laffer curve emerges that dictates the required lifetime issuance of debt to cover the present value of lifetime expenditures. I call this the ‘Lifetime-Laffer Curve,’ and the sovereign may be on its Pareto-dominated right-hand side while being on the left-hand side of the terminal contemporaneous Laffer curve as a result of his inability to commit not to issue large amounts of debt in the future.

Multiplicity of this kind resembles the Eurozone crisis since it naturally generates two of its puzzling features. First, malignant market sentiments seemed to play a strong role. This is not only suggested in empirical work by De Grauwe and Ji (2013) or Aizenman et al. (2013), but also by the apparent success of the OMT Program in 2012, which seemed to alleviate the crisis without direct intervention. Second, borrowing into high spreads and ever-increasing debt-to-GDP ratios, both of which are hallmark features of the crisis and difficult to generate in a structural model, are both the cause and consequence of the coordination failure.

Crises of this nature would prove difficult to detect empirically since there will be no observable difference between the two financing trajectories save the explosive build-up of debt and high spreads associated with the latter. But despite the lack of a clear identification strategy, the model provides a useful framework for counterfactual experiments. I perform one such experiment using Irish fiscal data during the crisis. Under the assumption that the country actually experienced such a malignant coordination failure and assuming the exact same sequence of fundamentals, the model imputes at least 3.47 percentage points (2.8%) of the debt-to-GDP build-up by the end of 2013 and 380 basis points (84.6%) of the average spread to that coordination failure. The reason why the mechanism generates a much stronger impact on spreads than debt levels is because in early periods, when the divergence in the trajectories begins, the debt has a relatively long maturity and is thus relatively insensitive to the large implied movements in spreads. This accords with Lorenzoni and Werning (2013), who note that feedback mechanisms in debt build-up

from high spreads are more powerful at shorter maturities since more debt needs to be rolled over at these higher spreads.

The differences in this numerical exercise would be estimated lower bounds, since the model assumes no coupon or principal payments in the interim and no feedback effects from debt levels to output, which could potentially be very significant. In fact, in an extension, it is shown that if the massive rise in spreads was the primary cause of the massive bailout of the banking sector, then the model can explain far more of the debt build-up: Up to 46.63 percentage points (37.8%).

Given the similarity of these crises to those that took place in the Peripheral Eurozone economies, the latter part of the paper considers the implications of different policies that were brought to bear during the crisis. First, it is shown that even though a crisis of this kind is due to a malignant sort of multiplicity, immediate austerity measures, which are generally designed to solve fundamental problems, *can in fact be successful*. The reason is because the front-loading of the deficit stream is an important prerequisite for the existence of multiplicity, and immediate austerity will help violate the condition. In fact, if the austerity is severe enough as to cause the need for debt buybacks, then there can be no coordination failures, since debt dilution makes it *easier* to buy back debt, not harder. It is important, however, that austerity measures are not put off until later if they are to work, since back-loading austerity measures will tend front-load the deficit stream and make the economy *more susceptible* to multiplicity. In fact, if the sovereign is issuing debt in period zero and repurchasing debt in period one, then we are guaranteed that multiple financing trajectories exist.

Second, central bank programs that are targeted toward eliminating malignant sentiments, such as the Outright Monetary Transactions (OMT) Program, can in fact work even though the defaults are fundamental in nature. The central bank, as a large potential buyer with the capacity to choose the price at which it would purchase debt, can help to rule out unpleasant equilibria even if they never purchase any debt much like deposit insurance in Diamond and Dybvig (1983).

When confronted with the two policy options, the model suggests that it is more likely that OMT eliminated the coordination failure, since almost no austerity measures throughout the peripheral economies during the crisis were so demanding as to require immediate buybacks, which is the most immediate sufficient condition for success.

The rest of the paper is divided as follows: Section 2 outlines the benchmark model and key results; Section 3 provides a calibrated example; Section 4 discusses policy implications; Section 5 generalizes the benchmark model by introducing a handful of relevant extensions; and Section 6 concludes.

2 Model

The benchmark environment is a simple, deterministic, three-period model in which a sovereign borrower issues debt to a mass of external lenders. Zero coupon debt is issued to foreign creditors in periods zero and one and pays back an amount \hat{B} in period two. Thus, in period zero the debt is long-term and the period one it is short-term.³ The sovereign begins in period zero with some initial stock of debt, b_0 .

In periods zero and one, the sovereign issues or buys back debt to fill some exogenously predetermined sequence of primary deficits, (d_0, d_1) . The sovereign takes this sequence as a primitive. Presumably he has committed to this sequence of taxation and expenditures beforehand and cannot easily alter it. Debt issuance on the other hand is not predetermined. In both of these periods, the sovereign either issues debt to or buys back debt from foreign creditors at an auction: (b_1, b_2) . At both auctions, the sovereign can commit to contemporaneous debt issuance, but in period zero, he lacks the ability to commit to period *one* debt issuance. It is this commitment problem that can give rise to a multiplicity debt and spread trajectories.

The sovereign issues or buys back debt at a sequence of prices, (q_0, q_1) . Much like debt issuance, these prices are not pre-determined and there may be more than one price trajectory that satisfies the sovereign's budget constraint. Thus, the sovereign's budget constraints in each period imply the **Budget Condition**:

$$\begin{aligned} d_0 &= q_0(b_1 - b_0) \\ d_1 &= q_1(b_2 - b_1) \end{aligned} \tag{1}$$

The sovereign suffers from some risk of complete default in period two, which is a function only of b_2 . In particular, the probability of default in this period is given by some function $g(b_2)$. It is assumed that $g(\cdot)$ is increasing and bounded between zero and one; further, there exists some debt level, $\bar{b} < \infty$ such

³This particular maturity structure of debt is not necessary for the main results; it merely simplifies the exposition greatly. The results can be generalized provided the sovereign issues some long-term debt in period zero.

that $g(\bar{b}) = 1$ i.e. there is a debt level at which the sovereign defaults with probability one. Finally, the function $g(b)$ is assumed to be weakly convex and twice differentiable whenever $b < \bar{b}$.

The assumption that default risk only emerges in the last period can be rationalized by the fact that no debt comes due until then. Thus, there would be no gain from defaulting earlier for a wide range of potential sovereign preferences. An alternative assumption could be to impose that default can occur in the case of insolvency in periods before the last. This is the approach of Lorenzoni and Werning (2013). However, making this assumption in this environment is not necessary to generate the relevant multiplicity of financing trajectories. If indeed the sovereign becomes insolvent at any point before the final period, I simply term the associated deficit stream infeasible. Throughout the analysis, I restrict attention to deficit streams that are feasible.

There is a unit mass of risk-neutral foreign creditors that purchase sovereign debt and price default risk and are required to break even in expectation. They are assumed to be deep-pocketed and have access to an asset that yields a risk-free return, $R > 1$ in each period. This implies that the pricing schedule must satisfy the **No-Arbitrage Condition**:

$$\begin{aligned} q_0 &= \frac{1}{R^2} [1 - g(b_2)] \hat{B} \\ q_1 &= \frac{1}{R} [1 - g(b_2)] \hat{B} \end{aligned} \tag{2}$$

From this pricing schedule, one can observe the dilution risk: The period zero price is affected by period one debt issuance, b_2 , even though the sovereign has yet to issue this debt. Since the sovereign cannot commit to b_2 in period zero, expectations regarding its size will impact period zero prices and thus the financing trajectory.

The timing of events is as follows: In period zero, the lenders form expectations regarding terminal debt levels and offer the sovereign a scalar price, q_0 . The sovereign must issue debt b_1 at this price to satisfy its period zero budget constraint. In this sense, the sovereign lacks commitment in period zero and mindlessly fills his fiscal deficit at the observed price.

In period one, however, the order of actions is allowed to be reversed. In particular, it is assumed that the sovereign can commit to debt issuance in this period. This is modeled in the sovereign treating q_1 as

a schedule and having control over the choice of b_2 should multiplicity arise.

The amount of revenue raised at the auction is given by

$$\frac{\hat{B}}{R} [1 - g(b_2)] [b_2 - b_1]$$

We can infer from the second order condition of this expression that when $b_1 \leq b_2 \leq \bar{b}$ i.e. the sovereign is issuing new debt and (weakly) raising revenue by doing so, this function is concave in b_2 by virtue of the convexity of $g(\cdot)$. This concavity need not hold for buybacks, but we can say from first-order conditions that the cost of buying back debt is monotone in the quantity of debt repurchased. Thus, any second period revenue schedule will have a shape such as that given by Figure 2.

From the curvature of $g(\cdot)$ therefore, it is clear that given b_1 and for any feasible level of revenue collected at this auction except for the maximum possible level of revenue, there are exactly two ways of collecting this revenue.⁴ This point was famously made by Calvo (1988). In this case with long-term debt, however, it is possible for the sovereign to repurchase debt. And unlike the case of new issuance, there is only ever one way of repurchasing debt. Thus, there is no Laffer multiplicity when $b_2 \leq b_1$.

When issuing new debt, the left-hand side of this Laffer curve will always Pareto-dominate the right side so long as default is in some capacity costly. Since the sovereign is assumed to be able to commit to contemporaneous debt issuance, it is assumed that he will be on the left-hand side in any solution. In other words, any solution must satisfy the following **Commitment Condition**:

$$g'(b_2)(b_2 - b_1) + g(b_2) \leq 1 \tag{3}$$

There will necessarily be another solution in which the Commitment Condition does not hold,⁵ but the sovereign is assumed to be capable of avoiding that unpleasant outcome.

Despite this ability to commit to current debt issuance in period one, there may yet be multiplicity of a nefarious sort that arises because the sovereign cannot commit to b_2 in period *zero*, even though he can commit to it in period one.

⁴At the maximum possible revenue, there will either be a unique b_2 or a range of b_2 in the event that the peak of the Laffer curve is ‘flat.’

⁵Except, of course, at the revenue maximizing b_2 .

2.1 Solution

A solution to the model outlined above will consist of four objects that constitute a financing and price schedule: $\langle\langle (b_1, b_2), (q_0, q_1) \rangle\rangle$. This solution will satisfy the system of equations given by the Budget, No-Arbitrage, and Commitment Conditions i.e. Equations 1, 2, and 3. Multiplicity of solutions to this system will imply the sort of Lifetime-Laffer multiplicity outlined in the introduction.

To help in the establishment of solutions, a few new definitions are in order. Let $\mathbf{D} = d_0 + \frac{1}{R}d_1$ denote present value of the government's primary deficit stream discounted by the creditors' time preference. Let $\mathbf{D}^*(b_0)$ denote the maximum possible such present value given some initial b_0 .⁶

The following proposition establishes conditions under which this sort of multiplicity exists.

Proposition 1. *Suppose that $0 < \mathbf{D} < \mathbf{D}^*(b_0)$. Then two solutions to System (1)-(3) exist if and only if the sovereign's primary deficit stream is sufficiently front-loaded.*

Proof. First, notice that we can substitute in the No-Arbitrage conditions into the prices and eliminate the need to separately compute (q_0, q_1) . The system is now defined solely in terms of (b_1, b_2) . We can further collapse the system by eliminating b_1 . Notice that one can write b_1 as a function of b_2 as follows from the period one Budget Condition:

$$b_1 = b_2 - \frac{Rd_1}{[1 - g(b_2)]\hat{B}}$$

This expression can be substituted into the period zero Budget Condition to derive:

$$d_0 + \frac{d_1}{R} = \frac{[1 - g(b_2)]\hat{B}}{R^2} [b_2 - b_0]$$

Equation 4 provides a lifetime budget constraint for the sovereign. It states that the total issuance of debt by the end of both auctions must satisfy the sovereign's lifetime revenue requirements, which are

⁶It will be soon be established that \mathbf{D}^* is a function only of b_0 .

discounted at $d_0 + \frac{d_1}{R}$. Alternatively, we could write

$$\mathbf{D} = \frac{[1 - g(b_2)]\hat{B}}{R^2} [b_2 - b_0] \quad (4)$$

Notice that the RHS of Equation 4 looks very much like the expression for second period auction revenue. In fact, it is trivial to show that this revenue function will have the exact same shape as Figure 2, with multiple solutions when $b_2 > b_0$ and a unique solution otherwise. We can use this right-hand side expression to derive the bounds of feasibility for lifetime revenue streams i.e.

$$\mathbf{D}^*(b_0) = \max_{b_2} \frac{[1 - g(b_2)]\hat{B}}{R^2} [b_2 - b_0]$$

Given that $0 < \mathbf{D} < \mathbf{D}^*(b_0)$, we know that exactly two solutions to Equation 4 will exist: A low b_2 inducing a positive revenue derivative (b_L) and a high b_2 inducing a negative revenue derivative (b_H). It now remains only to be shown that both solutions satisfy the second period Commitment Condition. Let us begin with the right-hand side solution, since this will require more work. For this solution, the first-order condition tells us that

$$g'(b_H) (b_H - b_0) + g(b_H) > 1$$

since it will be on the downsloping portion of the Lifetime-Laffer Curve. Notice that we can substitute $b_H - b_0$ from the lifetime budget constraint to arrive at

$$\frac{g'(b_H)}{[1 - g(b_H)]} \frac{R^2}{\hat{B}} \mathbf{D} + g(b_H) > 1 \quad (5)$$

We seek to find conditions under which the Commitment Condition holds. We can use the period one budget constraint to rewrite the Commitment Condition as follows:

$$\frac{g'(b_H)}{1 - g(b_H)} \frac{R}{\hat{B}} d_1 + g(b_H) \leq 1$$

To do so, we first note that b_H can be entirely determined from \mathbf{D} and is independent of the composition of d_0 and d_1 . Thus, $b_H(\mathbf{D})$ is a well-defined function for $\mathbf{D} \in (0, \mathbf{D}^*(b_0))$. Given this, we can define the

following useful function:

$$\kappa(b) = \frac{\hat{B}}{R^2} \times \frac{[1 - g(b)]^2}{g'(b)} > 0$$

Notice that $\kappa(b)$ is necessarily positive and decreasing in b . Using this new function, we can rewrite Inequality 5 as

$$\mathbf{D} > \kappa(b_H(\mathbf{D})) > 0 \tag{6}$$

In the same vein, we can re-write the Commitment Condition to observe that it will be satisfied if and only if

$$\frac{d_1}{R} \leq \kappa(b_H(\mathbf{D})) \tag{7}$$

Equation 7 is the relevant **Front-Loading Condition**, since it is clear that in order for both 6 and 7 to hold simultaneously, d_1 must be a relatively small fraction of the overall \mathbf{D} . Since \mathbf{D} is simply a linear combination of d_0 and d_1 , it is easy to see that, for any fixed and feasible \mathbf{D} , a sufficiently large d_0 accompanied by the concomitant small or negative d_1 can always be found to satisfy Inequality 7. This can be seen in Figure 3, in which the green, solid line displays a composition that satisfies the front-loading condition while the red, dotted line displays a composition for the same \mathbf{D} that violates it.

Notice that when Inequality 7 is satisfied, we will also have that the Commitment Condition is satisfied for b_L , since $b_L < b_H$ and $\kappa(\cdot)$ is a decreasing function. Thus, b_L also satisfies the System (1)-(3) provided the Front-Loading Condition is satisfied and two solutions exist. □

Proposition 1 tells us that so long a sovereign's deficit stream is sufficiently front-loaded, it can be subject to a multiplicity of financing trajectories despite its ability to commit to contemporaneous debt issuance. In particular, if lenders anticipate low debt issuance tomorrow, then there will not be much dilution risk and today's price will be higher; as a result, the sovereign can issue less debt today and has less debt to roll over tomorrow. However, if lenders anticipate high debt issuance tomorrow, then there

will be more dilution risk and today's price will be lower; as a result, the sovereign must issue more debt today and thus has more to roll over tomorrow.

This mechanism remains despite the sovereign's ability to commit to contemporaneous debt issuance because the coordination failure arises as a result of a shift in expectations in *future* behavior. The sovereign cannot commit to future default or borrowing behavior, and thus cannot escape these malignant expectational shifts.

This is illustrated graphically in Figure 4. The top graph is the Lifetime-Laffer Curve, which maps the choice of b_2 into an implied discounted lifetime deficit, \mathbf{D} ; the bottom curve is the revenue curve from the second period auction. We can see from the top graph that there are two solutions for \mathbf{D} . Further, we can see from the bottom graph that each of these solutions imply a second period auction that generates the same d_1 but using two different debt levels: The green, dashed line corresponds to the low-debt solution and the red, solid line corresponds to the high-debt solution. While these two solutions imply radically different debt levels, both solutions are on the left-hand side of their respective Laffer curves because both satisfy the Commitment Condition.

The reason why these two solutions are so radically different is because of the implied b_1 . We can infer this from where the upsloping portion of the revenue curve intersects the axis. We can see that the green line intersects at a very low b_1 , while the red line intersects at a much higher one. In both cases, the sovereign is maintaining the same primary deficit; the difference between them is that in the red line solution the sovereign has substantially more debt to roll over.

Why is a front-loaded deficit stream the condition for such crises? Because front-loading a deficit stream without changing \mathbf{D} will not change the Lifetime-Laffer Curve at all, and thus not the solution two solutions, b_L and b_H ; nevertheless, it will work to increase the amount of debt issued in period zero, b_1 , and the peak of revenue function at the second auction is increasing in b_1 . Thus, front-loading the deficit stream will shift the peak of the second period Laffer curve to the right, and at a certain point the fixed point b_H will necessarily land on the upsloping portion. This can be seen in Figure 5: The blue, Lifetime-Laffer Curve at the top remains the same as we alter the composition of the deficit stream, so the right-hand solution is always b_H ; however, the pink curves on the lower graph shift down and to the right as we front-load the deficit stream by reducing d_1 for the same \mathbf{D} . The less dashed the line is, the more

front-loaded the schedule. At some point, b_H shifts to the left-hand side of the period one Laffer curve and the Commitment Condition is satisfied.

3 Calibrated Example: Ireland

How malignant can the debt build-up become? I calibrate a version of the model to data from Ireland to perform the following counterfactual: Supposing that Ireland experienced a coordination failure of this nature, what would have happened had they not?

Though the model can reasonably be applied to any Peripheral Eurozone country, Ireland is a natural choice since its public debt to GDP ratio increased by a factor of nearly 5 during the course of the crisis. In percentage points, the trough-to-peak change in the public debt to GDP ratio was the largest among the peripheral economies. This was not simply a product of falling GDP; Ireland consistently ran fiscal deficits in the face of growing spreads. In this sense it epitomized the phenomenon of borrowing into high spreads that the model is intended to explain.

In the 20 years prior to the crisis Ireland had been consistently reducing its public debt ratio, both in anticipation of joining the Eurozone and in the first decade of the currency union (see Figure 1). In the early to mid-2000's, it experienced a boom, particularly in its housing and construction sectors (Lane [2012]). During this period, its housing investment as a share of gross capital formation was the highest in the EU (Fernández-Villaverde et al. [2013]). Relatively little investment in their more tradable sectors made it particularly difficult for Ireland build up their export capacity (? [?]).

As a consequence, tax revenues remained relatively low throughout the crisis despite internal devaluations and the government had to rely on borrowing and rescue packages from the IMF, the European Commission, and the ECB. Nevertheless, the high degree of borrowing and the magnitude of the spreads seemed unreasonably high despite the weak fundamental state of the economy (De Grauwe and Ji [2013]). Indeed, the model counterfactual will demonstrate that for the same sequence of weak fundamentals, a trajectory with much lower spreads and debt levels exists and satisfies the government's budget constraint.

To reasonably take the model to the data, we will require one natural extension. Instead of a 3-period model, we consider a general T -period model with a primary deficit stream $\{d_0, d_1, \dots, d_{T-1}\}$. The only risk

involved remains the default risk in period T , which is again given by the same function $g(b_T)$. b_0 is again taken as given. We can define $\mathbf{D}^*(b_0)$ as in the 3-period case (details in proof) and claim the following

Proposition 2. *Suppose that $0 < \mathbf{D} < \mathbf{D}^*(b_0)$. Then two solutions to the T -period model exist if and only if the sovereign's primary deficit stream is sufficiently front-loaded.*

Proof. See Appendix A □

Here, the front-loading condition simply places an upper bound on the deficit in period $T - 1$, so it is actually easier to satisfy than the three-period model, since it applies to a smaller fraction of the overall deficit stream.

A T -period model will allow us to fit more data points. We consider a simple, piecewise single-term polynomial default rule in which

$$g(b) = \begin{cases} \min\{\alpha(b - \underline{b})^\kappa, 1\}, & b \geq \underline{b} \\ 0, & b < \underline{b} \end{cases} \quad (8)$$

for some strictly positive α and κ . \underline{b} denotes a lower bound on the set of debt levels for which positive default risk exists. Note that $g(\cdot)$ is weakly convex for $b \leq \bar{b} = 1/\alpha^{1/\kappa} + \underline{b}$ and is continuously differentiable on this region as well.

We will take $T = 6$ and interpret the periods as being 2008-2013. World risk-free rates were fairly low at this time, so we will assume $R = 1.01$. Rather than scaling debt and deficit levels by GDP, which fluctuates over the sample, we will scale them by a common scale factor of $\zeta = 150000$, which is moderately less than GDP over this horizon in millions of Euros. Given this scale factor, Ireland's scaled fiscal deficit⁷ stream is $\{0.08721, 0.15579, 0.35734, 0.14482, 0.09329, 0.06759\}$; their initial government debt was $b_{2007} = 0.314798$ and their final debt was $b_{2013} = 1.435619$.⁸

We will perform the counterfactual by calibrating $g(\cdot)$ to simultaneously target $b_{2013} = 1.435619$; an

⁷Although the model developed assumes a stream of *primary* deficits, in application the relevant target is the *fiscal* deficit, since the fiscal deficit determines whether the sovereign needs to raise additional funds at a sovereign debt auction or whether it will be buying back debt. It is this buyback distinction in the model that determines whether or not the coordination failure is possible.

⁸Debt and deficit data comes from Eurostat.

average spread of 450 basis points;⁹ a default-risk threshold, \underline{b} , that implies 100% debt-to-GDP; and a maximum debt-capacity, \bar{b} , that implies 130% debt-to-GDP. This exercise will prove easy, since there are closed-form expressions for model parameters in terms of these objects. First, \underline{b} and \bar{b} can easily be determined by the scale factor and GDP in 2013. Further, if p is the probability of default and s is the spread, then we can alter the spread formula, $s = \frac{\hat{B}}{q_{N-1}} - R$, to derive

$$p = \frac{s/R}{1 + s/R}$$

$$\kappa = \frac{\ln(p)}{\ln\left(\frac{b_{2013} - \underline{b}}{\bar{b} - \underline{b}}\right)}$$

$$\alpha = (\bar{b} - \underline{b})^{-\kappa}$$

Once we have calibrated $g(\cdot)$ in this way, we can read \hat{B} off of the debt-issuance requirements from the Lifetime-Laffer curve i.e.

$$\hat{B} = \frac{R^N \times \mathbf{D}}{[1 - g(b_{2013})][b_{2013} - b_{2007}]}$$

and verify that b_{2013} is in fact on the RHS of this curve. Implementing this procedure for Ireland, we arrive at the parameter values in Table 1.

Once we have these parameters, we can verify whether or not the front-loading condition holds: For the case of Ireland, it does so easily. Thus, there are two solutions for b_2 in accordance with Proposition 1. There is a high debt solution, which was targeted and in which $b_H = b_{2013}$; and there is a low debt solution, which is the model's counterfactual prediction for what would have happened. This solution is given by $b_L = 1.3952$. In other words, assuming the same GDP sequence, the model predicts that for the same fundamentals and in the absence of such a coordination failure, the debt-to-GDP build-up would have been only 119.73% by the end of 2013, or about 3.47 percentage points (2.82%) lower.

While the contraction in the debt level seems relatively small, the model's counterfactual prediction for the spread is quite substantial. The counterfactual spread is $s_L = R \left(\frac{g(b_L)}{1 - g(b_L)} \right) = 0.0069$ i.e. very close to

⁹According to Datastream, the 10-year bond spread in Ireland ranged widely over the course of the crisis from nearly zero early on to more than 900 basis points at its peak. The model implies only a single spread, so here for simplicity I choose the midpoint. The results regarding the fraction of the spread explained are not sensitive to the choice of target spread, as I show in a sensitivity analysis in Appendix E.

zero. This suggests that the coordination failure can account for over 380 basis points of the spread, or over 84% of it. This very large figure is a result of the estimated convexity of $g(\cdot)$, which can be seen in Figure 6: Even a small decrease in the estimated debt burden can result in a substantial decrease in the default probability and consequently the spreads. Such convex spread functions are a property shared by many canonical models in the literature that involve endogenous default, such as Aguiar and Gopinath (2006), Arellano (2008), and Chatterjee and Eyigungor (2012).

Further, these estimates are model-based *upper bounds* on the counterfactual debt-to-GDP ratio and spread for a couple of reasons: First, if the primary deficit stream was the same, then the second period fiscal deficit would actually be lower when investors are coordinating on b_L due to lower interest and principal payments on outstanding debt; second, if there is any causality traveling from high debt levels to economic growth e.g. the crowding out of private investment, then we would expect GDP to be higher in the b_L regime and thus the debt-to-GDP ratio to be even lower. It is even possible that government expenditures would not have been as large in the absence of such a malignant spread trajectory since, for instance, the banking sector may not have required a large bailout had the yields on their dominant ‘safe’ asset not skyrocketed. In an extension, I will show that multiple solutions will continue to exist if deficits increase in the face of large debt burdens and that they can account for a substantially larger fraction of Irish debt build-up in this case.

It is worth noting that Ireland was shut out of bond markets from late-2010 until mid-2012 and relied on rescue loans from the Eurozone Financial Stability Mechanism for external finance during this time. These loans were marked below the market rate. As a robustness exercise, I have computed the model with a ‘rescue discount’ on the spread applied to the relevant periods. The results are in Appendix E. Accounting for a rescue discount does little to change the counterfactual predictions.

Even though this version of the model is stylized, it demonstrates in very simple terms the potential for these coordination failures to cause unnecessary build-ups of debt and substantial spreads. While model misspecifications may diminish this figure some, even a fraction of an unnecessary spread jump of this magnitude would present a concern for policymakers.

4 Policy Prescriptions

Given the clear similarities between Lifetime-Laffer Curve multiplicity and the Eurozone crisis, in this section, I examine some of the policy implications and find that two of the principal policies that were brought to bear on the crisis, austerity measures and central bank intervention, can be effective if implemented correctly.

4.1 Austerity Measures

Although the general Front-Loading Condition is a bit involved mathematically, there is one easy and policy-relevant special case that always satisfies it.

Corollary 1. *If $0 < \mathbf{D} < \mathbf{D}^*(b_0)$ and $d_1 \leq 0$, then two solutions to System (1)-(3) exist.*

Proof. If $d_1 \leq 0$, then we will have

$$\frac{d_1}{R} \leq 0 \leq \kappa(b_H(\mathbf{D}))$$

since $\kappa(\cdot)$ is a positive-valued function. Thus, the Commitment Condition holds for b_H , which implies that it holds for b_L . □

If we were to apply this model to the Eurozone, Corollary 1 tells us an important policy prescription: Expected austerity is not sufficient to free the economy from malignant market sentiments. Even if the government promises to run enormous surpluses in the future (negative d_1), so long as deficits are being run today ($\mathbf{D} > 0$), the economy can still be placed on this malignant, high-debt trajectory. Austerity measures, if they are to work at all, must be implemented immediately. This is made clear in the following result.

Corollary 2. *If $d_0 \leq 0$ then at most one solution to System (1)-(3) exists in which $b_2 < \bar{b}$. If a solution does exist, it will be on the Pareto-superior left-hand side of the Lifetime-Laffer Curve.*

Proof. We will break this into two cases: First, consider the case of $0 < \mathbf{D} < \mathbf{D}^*(b_0)$. In this case, the Lifetime-Laffer Curve will exhibit two solutions, as before, with b_H satisfying

$$\mathbf{D} > \kappa(\mathbf{D})$$

Because $\mathbf{D} = d_0 + \frac{d_1}{R}$ and $d_0 \leq 0$, this directly implies that

$$\frac{d_1}{R} > \kappa(\mathbf{D})$$

which violates the Commitment Condition. Thus, of the two solutions, b_H and b_L , only b_L may satisfy the Commitment Condition. However, we are not guaranteed that b_L satisfies the Commitment Condition. If d_1 is very large, it may be that no solution exists.

In the second case, we consider $\mathbf{D} \leq 0$, the Lifetime-Laffer curve yields a unique solution in which $b_2 < \bar{b}$, since it is strictly monotone over this range.¹⁰ However, as with the previous case, this unique solution need not satisfy the Commitment Condition if d_1 is very large. \square

Corollary 2 tells us that immediate implementation of austerity measures can in fact eliminate the malignant financing trajectory. The reason is because the multiplicity that arises on the Lifetime-Laffer Curve hinges on the model feature that low prices in period zero, which are caused by dilution, increase the amount of debt that needs to be issued in period zero. However, if the sovereign is buying back debt in period zero, then low prices are advantageous and have the reverse effect: They can increase the amount of debt the sovereign can repurchase and thus they *reduce* his indebtedness in period one rather than increase it. Thus, expectations of dilution cannot be self-fulfilling.

This result would be generalizable to an augmentation of the model that allowed for coupon payments

¹⁰When $\mathbf{D} = 0$, there will always be a continuum of solutions for $b_2 > \bar{b}$, but these solutions are neither interesting nor relevant since they are tantamount to the sovereign running a balanced budget over the course of its life while issuing an enormous amount of debt only to default on it completely.

on debt as well as debt maturing at the time of new debt issuance provided that the sovereign undertook immediate *fiscal* surpluses and not just *primary* surpluses, since it is the act of repurchasing debt that obviates the multiplicity.

Thus, while multiplicity via the Lifetime-Laffer Curve is fundamentally an issue of coordination and beliefs, well-placed austerity measures can in fact work to eliminate the malignant, high-debt trajectory. However, in order for such measures to work, it is crucial that they be implemented immediately, since ‘kicking the can down the road’ and implementing such measures in the future will have precisely the opposite effect and *guarantee* the presence of the high-debt trajectory.

4.2 Liquidity Provision

Another policy that seemed effective in alleviating the crisis was the ECB’s institution of the Outright Monetary Transaction (OMT) bond-buying program in the summer of 2012. In this policy, the central bank pledged to purchase risky sovereign debt in secondary markets, which would essentially provide those countries with liquidity, should a troubled Eurozone country request it and meet certain conditions. The announcement of this program seemed to quickly and dramatically reduce spreads and debt build-up. At the time of this writing, OMT has yet to be drawn on by a troubled member state.

Since Lifetime-Laffer Curve multiplicity is fundamentally an issue of coordination and beliefs, such action from the central bank can in fact eliminate the high-debt trajectory. Here, we will think of the central bank as a deep-pocketed actor with the capacity to purchase sovereign debt at a price of its choice in any period.

The addition of a central bank program of this kind will place an additional restriction on the system. Namely, we will assume that if the sovereign has multiple debt trajectories available to it in period zero, it will choose the one that induces the smallest probability of default i.e. default is in some capacity, costly. With this addition to the model, we can say the following:

Proposition 3. *The central bank can costlessly immediately eliminate the high-debt trajectory if this solution exists.*

Proof. First, we know from Proposition 1 that if the high-debt solution exists, then so does the low-debt solution. Let $\ll (q_{0,L}, q_{1,L}), (b_{1,L}, b_L) \gg$ be the low-debt solution. The central bank can pledge in period zero to purchase any quantity of debt from the sovereign at the price schedule $(q_{0,L}, q_{1,L})$. If the sovereign were to borrow from the central bank, it would issue $(b_{1,L}, b_L)$ to fill its primary deficit. Given this, the high-debt trajectory is no longer sustainable: It induces a strictly higher probability of default and so the sovereign will always prefer to issue debt to the central bank.

However, because the price schedule offered by the central bank constitutes an equilibrium pricing schedule, it can choose to purchase zero debt and lenders will provide the full amount of $(b_{1,L}, b_L)$. Thus, the central bank does not need to purchase any debt or incur any costs. \square

Thus, the provision of liquidity as in OMT can prove effective in the case of a crisis of this nature, *even though all defaults are fundamental and not driven by liquidity concerns*. Notice that OMT here is not interpreted as a bailout or a rescue package. Even in the low-debt trajectory, there is some probability that the sovereign will default in period two, and the central bank will not come to the rescue in this model. This is in many senses true to the way that OMT is actually set up, since stringent conditions need to be met for a member state to apply for OMT, and these conditions are least likely to be met when the sovereign is experiencing a crisis: For instance, in recent years turbulent Greece has not met the conditions to apply for OMT help since they have not managed to regain complete access to private lending markets, which is a necessary condition of the OMT program.

Thus, the model predicts that providing liquidity in this way may not be an effective tool for preventing all future defaults (at least not costlessly), but it *is* an effective way to prevent a malignant, sentiment-driven build-up of debt.

Of the two employed policy options, it is more likely that OMT and programs like it helped to eliminate the multiplicity rather than austerity measures. This is for two reasons: First, the timing of OMT in the summer of 2012 seemed to align perfectly with a drastic fall in spreads, which is suggestive of a shift to a Pareto-superior equilibrium; second, almost no austerity measures throughout the peripheral economies during the crisis were so demanding as to require immediate buybacks, which is the most immediate sufficient condition for success in accordance with Corollary 2. Rather, much of the austerity

was expected, which can actually worsen crises by front-loading the deficit stream in accordance with Corollary 1. In fact, by front-loading the deficit stream, a host of *scheduled* austerity measures introduced early in the crisis may have contributed to causing the coordination failure.

5 Extensions

While the model presented in the previous section was intentionally simple and intended for illustration, the general principle of multiplicity via Lifetime-Laffer Curves is far more robust. In this section, the model is augmented along two relevant dimensions to demonstrate this.

First, I consider how the baseline model with T periods behaves when the sovereign responds to debt build-up with fiscal policy. This framework will allow me to expand the calibrated example of Ireland and construct a counterfactual in which a bailout of the banking sector is not required. In this case, under the Pareto-superior counterfactual, debt-to-GDP build-up over the course of the crisis falls by 46.63 percentage points (37.85%).

Second, I allow for ex ante uncertainty regarding states in future periods i.e. rollover risk. Here, the multiplicity of trajectories generalizes to a multiplicity of state-contingent borrowing policies. These results generalize even further to the case with fiscal response to indebtedness.

5.1 Responding to Debt Build-Up

We begin the analysis by allowing the stream of primary deficits to respond to indebtedness in the T -period case. In particular, we assume that the primary deficit stream in period t is given by some continuous and differentiable function, $d_t(b_T)$. We further assume that each $d_t(b_T)$ is bounded below by small but strictly positive constants i.e. no buybacks are allowed. In the calibration, this assumption is not at all restrictive.

For any t , this function could be either increasing or decreasing in b_T . While this specification rules out explicit an explicit Markovian structure, which would condition on current debt in the policy rule, it allows the sovereign to respond to *expected levels of future indebtedness*, which has similar implications and comes with similar intuition. We could also interpret the deficit streams as responding to default risk, by assuming that the deficit stream is some function $\hat{d}_t(p)$, where p is the default probability in period T .

In this case, we can define $d_t(b_T) = \hat{d}_t(g(b_T))$. If instead, the deficit responded to the spread on sovereign debt with some function $\hat{d}_t(s)$, then we simply define $d_t(b_T) = \hat{d}_t\left(\frac{Rg(b_T)}{1-g(b_T)}\right)$. This last case is the one we will consider in the calibrated example, since an increase in deficits during times of high spreads can be interpreted as the banking sector requiring a bailout as the value of a vast swath of their assets, government bonds, falls.

With this set-up, we can establish the following result:

Proposition 4. *Under a feasibility condition, a positive economy $\ll b_0, \{d_t(\cdot)\}_{t=0}^{T-1} \gg$ will have at least two distinct financing trajectories.¹¹ Further, if each $d_t(\cdot)$ is increasing and convex and a front-loading condition holds, then there are exactly two solutions to the system.*

Proof. See Appendix B □

Proposition 4 simply generalizes the first existence result to the case where there is some feedback from debt levels or spreads onto the primary deficit stream. This is a useful result since we can augment our calibrated example to allow for the possibility of a very plausible counterfactual: The absence of a banking sector bailout.

5.1.1 Bank Bailouts in the Calibrated Example: Ireland

Our calibration in the baseline model suggested that such coordination failures can account for an enormous fraction of the spread build-up, over 84% in the case of Ireland, but relatively small debt build-ups given the same fundamentals. However, high spreads on sovereign debt can be devastating to an economy's banking sector by tightening bank balance sheets, since domestic banks tend to hold large fractions of sovereign debt (see Bocola [2014], Sosa-Padilla [2012]). It is entirely plausible, therefore, that without the spread spike observed during the crisis, domestic banks would not have required as large of a bailout, or even a bailout at all.

¹¹These trajectories are not guaranteed to satisfy the Commitment Condition, and thus are not guaranteed to be full solutions.

The banking sector bailout was fairly substantial, costing the Irish government some 62.8 billion Euros. By late 2010, about 46.3 billion Euros had been spent and most of the rest was spent the following year. We can incorporate this bailout in our model in a fairly simple way. In particular, we can assume in the model that the deficit in 2010 and 2011 was some linear function of the spread on sovereign bonds as follows:

$$d_{2010} = d_{NB,2010} + \delta_{2010}s \tag{9}$$

$$d_{2011} = d_{NB,2011} + \delta_{2011}s$$

Other deficits in the sequence are simply assumed to be constant. The assumption here is that no bailout is required if there is no risk-spread on sovereign debt and, in the presence of a positive spread, that expected bailout costs are linear in that spread. We can scale everything as in the previous example and calibrate both δ_{2010} and δ_{2011} directly from the data assuming the high debt solution. Doing so yields $\delta_{2010} = 6.86$ and $\delta_{2011} = 2.44$. The model continues to satisfy the Commitment Condition with this addition.

The counterfactual, low-debt solution in this case is much lower: $b_L = 0.8922$, which translates to a 2013 debt-to-GDP ratio of 76.57%, which is more than 46.63 percentage points (37.85%) lower. The counterfactual spread is $s_L = 0$, and so no bank bailout is required, *despite the fact that it was allowed for in the calibration*. The divergent paths for the debt levels can be seen in Figure 7.

The intuition for why adding this bailout rule in the economy amplifies substantially the debt-to-GDP build-up is the following: The degree of tightness in banks' balance sheets, and thus the potential need for bailouts, is an increasing function of the spread, which is highly non-linear and very sensitive to the coordination failure. When the high-spread coordination failure disappears, so too does the need for an expensive and costly bailout.

5.2 Introducing Uncertainty

The next natural extension to the model is to introduce uncertainty prior to the default shock and show that the multiplicity of financing trajectories outlined before translates to a multiplicity of state-contingent borrowing policies. For this section, I revert back to the 3-period case for simplicity of exposition, but

there is nothing that prevents the results from holding in the T -period case.

The state in period zero is assumed to be known and thus d_0 remains a constant. States in period one, however, are drawn randomly from a finite set S with dimension, N and a distribution $\{\pi_s\}_{s=1}^N$. N can be arbitrarily large. The primary deficit in period one is dependent on s i.e. $d_1(s)$.

Again, the sovereign enters period zero with some debt stock, b_0 , that is due in period two. Thus, any economy is specified by model parameters and $\langle\langle b_0, d_0, \{d_1(s)\}_{s=1}^N \rangle\rangle$. A solution will be an $2N + 2$ tuple of the following form: $\langle\langle q_0, b_1, \{q_1(s)\}_{s=1}^N, \{b_2(s)\}_{s=1}^N \rangle\rangle$. In this section, we will show that the same basic principles from the first section apply and that multiple solutions of this kind will exist under a front-loading condition.

Proposition 5. *Under a feasibility condition and a front-loading condition, a positive economy $\langle\langle b_0, d_0, \{d_1(s)\}_{s=1}^N \rangle\rangle$ will have at least two distinct solutions. Further, if $\langle\langle b_{L,1}, \{b_L(s)\}_{s=1}^N \rangle\rangle$ and $\langle\langle b_{H,1}, \{b_H(s)\}_{s=1}^N \rangle\rangle$ are components of those two distinct solutions and wlog it must be the case that $b_L(s) \leq b_H(s)$ for any $s \in S$ and that only the lesser solution will be numerically stable.¹²*

Proof. See Appendix C □

Proposition 5 generalizes Proposition 1 to a higher dimension. Although the result requires much more work to show, the economic intuition from the one-dimensional case is unchanged: Investors anticipate high borrowing tomorrow, $b_H(s)$, and as a consequence demand a lower price today to compensate them for the dilution risk.

Figure 8 provides a simple, non-calibrated example of two such borrowing rules for exposition. I consider the case in which $N = 25$ and d_1 ranges from 0.01 to 0.1. All gaps $d_1(s) - d_1(s - 1)$ are equidistant and each state is allotted equal weight. I consider, as in the calibrated example, a linear default rule: In this simple case, $g(b) = \min\{0.5b, 1\}$. Finally, $b_0 = 0.05$, $d_0 = 0.4$, $R = 1.01$, and $\hat{B} = 1.0$.

Both borrowing policies in Figure 8 (which have been linearly interpolated) constitute solutions to the system. Importantly, both satisfy the Commitment Condition in each of the N potential states. We can

¹²Numerical stability is formally defined in the proof.

see a fairly drastic difference between these two rules, with the high-debt solution inducing substantially higher debt levels in every state.

We can generalize this result further by allowing for deficits to depend on debt levels, as we did before. First, suppose that the sovereign has some ability to respond to elevated debt levels by reducing d_1 . Let us suppose that in each of the N states in period one, the primary deficit is given by some function $d_1(s, b_2)$, which will be decreasing, continuous, and differentiable in b_2 . Thus, if the sovereign finds itself having to issue a large portion of debt, it will partially offset this issuance by reducing b_2 . Under the Commitment Condition, this will be isomorphic to a situation in which d_1 responds to initial debt rather than debt issued, since there will be a one-to-one map between the two. However, for tractability, it is much easier to focus on the former case.

To help us expost the results, let us first recall the *Hazard Rate* of a function:

$$H[S(b)] = -\frac{S'(b)}{S(b)}$$

for any differentiable function S . We can now state the result, which is a generalization of Proposition 4 when $T = 3$ to the stochastic case:

Proposition 6. *If, for every $s = 1, \dots, N$, the hazard rate, $H[d_1(s, b)]$ is sufficiently bounded for $b \in [0, \bar{b})$, then under the conditions of Proposition 5, a positive economy $\ll b_0, d_0, \{d_1(s, \cdot)\}_{s=1}^N \gg$ will exhibit two distinct solutions.*

Proof. See Appendix D □

Proposition 6 is intuitive: It states that the multiplicity will remain provided that the sovereign does not respond too dramatically to higher debt levels with fiscal consolidation. In fact, if deficits *increase* in response to debt levels, as was the case in our previous example, then we are guaranteed multiple solutions. This result generalizes Proposition 4 when $T = 3$, since here we can guarantee the existence of solutions even in cases where deficits respond negatively to debt levels.

Barring technical details, variations on the policy results derived in the deterministic case can be applied to this more general case as well.

6 Conclusion

In this paper, I outlined conditions under which a multiplicity of debt and spread trajectories can exist in a simple finite-horizon model while maintaining the assumption of commitment to contemporaneous debt issuance in the terminal period. Such crises generate quite naturally the sentiment-driven build-up of debt levels that seems to have characterized the crisis in the periphery Eurozone.

I argued further that only immediate austerity measures can be effective; backloading the austerity measures can in fact make such crises more likely. On the contrary, liquidity provision by the central bank can always be implemented effectively, since the underlying problem is one of coordination failure. I then demonstrated the generality of this multiplicity by adding fiscal rules and rollover risk to the model.

Nefarious multiplicity of this kind was clearly not the entire force behind the crisis in the Peripheral Eurozone. Many other factors were likely at play, from the delay or abandonment of reforms, to problems of bailouts and moral hazard, to instances of illiquidity. However, many of these other stories fail to generate the remarkable and seemingly sentiment-driven build-up in debt-to-GDP ratios that proved a hallmark of the crisis.

There is still much room for further work. In particular, a method of identifying such sentiment-driven build-ups as in this work or Lorenzoni and Werning (2013) in the data would prove quite useful in ascertaining the applicability of my model or theirs to the data. An investigation of the potential for these crises in other types of financial markets suffering from similar frictions, such as the market for commercial paper or municipal debt, could also be in order; whether that entails investigating historical incidences of such failures or the potential for failures to come.

Appendices

A Proof of Proposition 2

Notice that for any period, $t - 1$, we can write the Budget and No-Arbitrage Condition jointly to derive b_{t-1} as a function of b_t .

$$b_{t-1} = b_t - \frac{R^{T-t}d_{t-1}}{[1 - g(b_T)]\hat{B}}$$

We can recursively substitute this expression until we arrive at the period zero budget constraint

$$d_0 = \frac{[1 - g(b_T)]\hat{B}}{R^T} \left(b_T - \frac{Rd_{T-1}}{[1 - g(b_T)]\hat{B}} - \frac{R^2d_{T-2}}{[1 - g(b_T)]\hat{B}} \cdots - \frac{R^{T-1}d_1}{[1 - g(b_T)]\hat{B}} - b_0 \right) \quad (10)$$

which can be rearranged as

$$\mathbf{D} = \frac{[1 - g(b_T)]\hat{B}}{R^T} [b_T - b_0]$$

The RHS is the Lifetime-Laffer Curve and \mathbf{D} is, as before, the present value of the dividend stream discounted at R i.e. which can be rearranged as

$$\mathbf{D} = \sum_{t=0}^{T-1} \frac{d_t}{R^t}$$

As before, the Lifetime-Laffer Curve will have two solutions provided $0 < \mathbf{D} < \mathbf{D}^*(b_0)$: b_L and b_H . The Commitment Condition need only be satisfied in period $T - 1$, and thus the Front-Loading Condition that guarantees it can be written as

$$\frac{d_{N-1}}{R} \leq \kappa(D) \quad (11)$$

where $\kappa(\mathbf{D})$ comes exactly from the Front-Loading Condition for Proposition 1.

B Proof of Proposition 4

If we substitute our d_t functions into the Lifetime Budget Constraint of the T -period case, Equation 10 from Appendix A, we arrive at:

$$d_0(b_T) = \frac{[1 - g(b_T)]\hat{B}}{R^T} \left(b_T - \frac{Rd_{T-1}(b_T)}{[1 - g(b_T)]\hat{B}} - \frac{R^2d_{T-2}(b_T)}{[1 - g(b_T)]\hat{B}} \dots - \frac{R^{T-1}d_1(b_T)}{[1 - g(b_T)]\hat{B}} - b_0 \right)$$

We can manipulate this expression to arrive at

$$b_T = \frac{R^T}{[1 - g(b_T)]\hat{B}} \sum_{t=0}^{T-1} \frac{d_t(b_T)}{R^t} + b_0 \quad (12)$$

Let us define the right-hand side of this expression to be $F(b_T)$. Notice first that since each deficit level is bounded below by a positive constant and it is assumed that $b_0 \geq 0$, we can safely claim both of the following

$$\begin{aligned} F(0) &> 0 \\ \lim_{b \rightarrow \bar{b}} F(b) &= \infty \end{aligned}$$

We further know that F is continuous and differentiable by the assumptions on its underlying components.

Notice that a solution to the economy is simply a fixed point of b . Thus, if we make the following assumption, which is simply that *Strict Feasibility Condition* in this context, we can claim the existence of two solutions using only the Intermediate Value Theorem:

$$\exists b' \in (0, \bar{b}) \text{ such that } F(b') < b' \quad (13)$$

If the weaker version of this condition does not hold i.e. a weak inequality instead of a strict one, then there is no solution to the model, which is why it can be suitably called a feasibility condition. In the model with an endogenous deficit stream, is the equivalent of the condition in Proposition 1 that requires $0 < \mathbf{D} < \mathbf{D}^*(b_0)$.

With this assumption, the multiplicity is easy to establish. We simply invoke the Intermediate Value

Theorem twice on the function $X(b) = F(b) - b$. First, we know that

$$X(0) > 0$$

$$X(b') < 0$$

Since X is a continuous function, it follows that $\exists b_L \in (0, b')$ such that $F(b_L) = b_L$. This is the low-debt solution.

To see the high-debt solution, we invoke the fact that $F(\cdot)$ travels to infinity. In particular, we know that

$$X(b') < 0$$

$$\lim_{b \rightarrow \bar{b}} X(b) = \infty$$

Since $X(\cdot)$ is continuous, it must be the case that some $b_H \in (b', \bar{b})$ exists such that $X(b_H) = 0$. To see this, suppose not. Then this implies that $X(b) < 0$ for all $b \in (b', \bar{b})$, which in turn implies, by continuity, that $\lim_{b \rightarrow \bar{b}} X(b) \leq 0$. However, this contradicts $\lim_{b \rightarrow \bar{b}} X(b) = \infty$.

Thus, at least two solutions, b_L and b_H exist and satisfy $b_L < b_H$. In general, there may be more solutions depending on the nature of the deficit response. However, it is easily shown that if every d_t is both increasing and convex, and given the assumptions already in place on g , then there are exactly two solutions, since the function F will also be convex. Any more solutions would require points of inflection, which would violate convexity. This can be seen in Figure B.1, where $F(\cdot)$ is overlaid on a dashed forty-five degree line.

With multiplicity established we now turn to the Commitment Condition. As before, it is given by

$$\frac{d_{t-1}(b_T)}{R} \leq \kappa(b_T)$$

The *Front-Loading Condition* will again be

$$\frac{d_{t-1}(b_H)}{R} \leq \kappa(b_H) \tag{14}$$

When this condition holds, the Commitment Condition will hold for the unique b_H ¹³ solution by definition, but it will also hold for the b_L solution, since $d_{t-1}(b_L) \leq d_{t-1}(b_H)$ and the right-hand side is decreasing in b . Thus, when Equations 13 and 14 hold, there are at least two solutions, b_L and b_H , both of which satisfy the Commitment Condition.

C Proof of Proposition 5

We begin again by collapsing the price expressions into functions of debt levels alone and using the fact that debt issued in period zero can be described as a function of debt issued in period one through the period one budget constraint in each state i.e.

$$b_1 = b_2(s) - \frac{Rd_1(s)}{[1 - g(b_2(s))] \hat{B}} \quad \forall s \in S$$

We can further substitute b_1 into the period zero budget constraint to derive the following

$$d_0 = \frac{\hat{B}}{R^2} E_{\tilde{s}} [1 - g(b_2(\tilde{s}))] \left[b_2(s) - \frac{Rd_1(s)}{[1 - g(b_2(s))] \hat{B}} - b_0 \right] \quad \forall s \in S$$

Which can be rearranged as

$$b_2(s) = \frac{R^2 d_0}{E_{\tilde{s}} [1 - g(b_2(\tilde{s}))] \hat{B}} + \frac{Rd_1(s)}{[1 - g(b_2(s))] \hat{B}} + b_0 \quad \forall s \in S$$

Let us define the function $f : \mathcal{R}^N \rightarrow \mathcal{R}^N$ as the right-hand side of this expression, taking as its argument the vector b_2 i.e.

$$f_s(b_2) = \frac{R^2 d_0}{E_{\tilde{s}} [1 - g(b_2(\tilde{s}))] \hat{B}} + \frac{Rd_1(s)}{[1 - g(b_2(s))] \hat{B}} + b_0 \quad (15)$$

for $s = 1, \dots, N$. We restrict attention to positive debt levels, which for positive-value economies is without loss of generality. Equation 15 and variations on it will be at the core of the analysis, since fixed points of f will present solutions to the economy with uncertainty.

I proceed with a short series of lemmas to highlight the key properties of f . To do so, I first define E

¹³ b_H and b_L are unique when the sequence d_t are convex functions, and thus these are well-defined.

to be the real Banach Space defined by \mathcal{R}^N equipped with the Euclidean norm. I then provide this space with an order cone $P = \mathcal{R}_+^N \subset E$. P will define an order \geq on E : We will say that $b_1 \leq b_2$ if $b_2 - b_1 \in P$. In other words, we will have $b_1 \leq b_2$ iff $b_1(s) \leq b_2(s)$ for $s = 1, \dots, N$.

We first assert some useful and trivial properties of this space. Definitions

Lemma 1. *The order cone P is nonempty, normal, regular, and minihedral.*

The Banach Space provided is a very well-behaved space, so in the interest of brevity I will omit the proof and simply provide the definitions according to Guo and Lakshmikantham (1988): An order cone is *normal* if there exists a constant, $N > 0$, such that $x \leq y$ and $x, y \in P$ implies $\|x\| \leq N\|y\|$; an order cone is *regular* if and only if every increasing and bounded in order sequence in E has a limit; and an order cone is *minihedral* if $\sup\{x, y\}$ exists for any pair $\{x, y\}$ which is bounded above in order.

Lemma 2. *If $\langle\langle d_0, \{d_1(s)\}_{s=1}^N \rangle\rangle$ is weakly positive, then the function f is continuous and monotone increasing with respect to the order cone, P .*

Proof. Continuity follows from continuity along each dimension and continuity in g . As for monotonicity, suppose that $b_1 \leq b_2$. Then it is clear from Equation 15 that, provided the deficit stream is weakly positive, for every s we will have that $f_s(b_1) \leq f_s(b_2)$, since g is an increasing function. Thus, $f(b_1) \leq f(b_2)$. \square

Now, we are prepared to establish the multiplicity. First, we can formally define here the notion of feasibility in a way that will guarantee us not only a solution, but also a multiplicity of solutions. As in the case of Proposition 4, an economy will be *strictly feasible* provided

$$\exists \hat{b} < \bar{b} \text{ such that } \hat{b} > f(\hat{b}) \tag{16}$$

Looking carefully at the definition of f , Equation 15, we can see that this condition essentially puts an effective cap on the size of the initial debt, b_0 , and subsequent deficit stream: If they are too large, then they will be impossible to finance over the lifetime of the sovereign and no solution exists. The strict inequality will guarantee a multiplicity of solutions. If a \hat{b} existed such that the condition only held with

equality, then we could only ensure one solution, namely itself, \hat{b} . This would correspond to the peak of the Lifetime-Laffer Curve in the deterministic case, for which only one solution exists. This notion simply generalizes to higher dimensions.

To establish the existence of these solutions, I will rely on two fixed point theorems that generalize the 1-Dimensional Intermediate Value Theorem to this environment. The first is drawn from Guo and Lakshmikantham (1988). Let A be an operator defined by the function f but restricted to the domain $[u_0, \nu_0] = [0, \hat{b}]$. Notice that $Au_0 \geq u_0$ under the assumption that $b_0 \geq 0$ and that $A\nu_0 \leq \nu_0$. With this notation in place, we can apply Theorem 2.1.1 from Guo and Lakshmikantham (1988) to derive.

Lemma 3. *There exists at least one fixed point of f , b_L , such that $0 \leq b_L \leq \hat{b}$. Further, this fixed point is stable.*

Proof. See Theorem 2.1.1 in Guo and Lakshmikantham (1988). Condition (H_2) is satisfied. This theorem actually guarantees us a minimal and a maximal fixed point of A , which may coincide. A fixed point of A is of course just a fixed point of f on the restricted domain $[0, \hat{b}]$. By stability, I simply mean that the theorem also provides us with the feature that repeated iteration of A from below will converge to the minimal fixed point and repeated iteration of A from above will converge to the maximal fixed point. \square

With one fixed point established, I now turn to the other one, referencing a similar result from Kostykin and Oleynik (2012), which holds when we flip which endpoint is the supersolution and which is the subsolution. Before we can use their result, there is one more claim to establish: The existence of a supersolution. We begin as follows:

Lemma 4. *$\exists b' \in E$ such that $\hat{b} \leq b' < \bar{b}^{14}$ and $f(b') \geq b'$.*

Proof. To see this, note that if we took a monotone increasing sequence of vectors, $b_n \in E$, such that $b_n \rightarrow \bar{b}$, then we will have that $f(b_n) \rightarrow \infty$. By the continuity and monotonicity of f , it must be that there exists an $N > 0$ such that for all $n \geq N$, we will have $f(b_n) \geq b_n$. We take $b' = b_N$. \square

¹⁴ \bar{b} here is taken to mean an N -dimensional vector in which every element is \bar{b} .

Now, we again define the function f on a different domain as an operator $T : [u_0, \nu_0] \rightarrow E$, where $[u_0, \nu_0] = [\hat{b}, b']$. We need only one more claim before our result.

Lemma 5. *The operator T is compact.*

Proof. T is compact if and only if for every set $S \subset [u_0, \nu_0]$, the image $T(S)$ is relatively compact. A set is relatively compact if its closure is compact. For any S , the closure of $T(S)$ will certainly be bounded below by Tu_0 and bounded above by $T\nu_0$, since T is a monotone operator. Since the closure of $T(S)$ is necessarily closed, it is both closed and bounded. Thus, by the Heine-Borel theorem, the closure of $T(S)$ is compact and so T is a compact operator. \square

With this result in hand (and barring a few notational discrepancies), we can apply the result of Kostykin and Oleynik (2012) to deduce:

Lemma 6. *There exists a fixed point of f , b_H , such that $\hat{b} \leq b_H < \bar{b}$. This fixed point is distinct from b_L .*

Proof. Existence follows from Theorem 1 of Kostykin and Oleynik (2012). To get distinctness, suppose otherwise i.e. $b_L = b_H$. In this case, the weak inequality $b_H \leq b_L$ must hold. By transitivity, therefore, we will have that $b_H \leq \hat{b}$. Since we have from the theorem that $\hat{b} \leq b_H$, by antisymmetry we must have that $b_L = b_H = \hat{b}$. But that implies that $f(\hat{b}) = \hat{b}$, which contradicts the assumption of strong feasibility: $f(\hat{b}) < \hat{b}$. Thus, b_H is a distinct fixed point and $b_H \geq b_L$. \square

While this latter fixed point certainly exists, it will generally not exhibit the stability properties of b_L . This can make finding this solution numerically a bit more challenging since many algorithms will favor stable fixed points in the presence of multiplicity. Further a *distinct* fixed point may not exist if strict feasibility does not hold i.e. if we were only certain that a \hat{b} exists such that $f(\hat{b}) \leq \hat{b}$, since in this case b_L and b_H could coincide at \hat{b} .

As a brief aside, the arguments outlined thus provide ground for a multiplicity of solutions i.e. at least two. There may in fact be more than this. In the event that this is so, I will consider the maximal fixed point from Lemma 3 to be b_L and the minimal fixed point from Lemma 6 to be b_H . Tarski's Fixed Point

Theorem can be applied to both cases to demonstrate that these extrema actually exist. Thus, we have two well-defined functions that map the economy into two potential solutions. What remains to be shown is whether the Commitment Condition is satisfied for these solutions. To conclude the proof, I provide conditions under which this is so.

Note first that, once a state in period one is realized, the problem becomes identical to the deterministic case in period one, and thus the Commitment Condition, as in that case, can be written in terms of a front-loading condition.

$$\frac{d_1(s)}{R} \leq \frac{\hat{B}[1 - g(b_2(s))]^2}{R^2 g'(b_2(s))}$$

With this, we can arrive at the Lemma that completes the proof.

Lemma 7. *The Commitment Condition for both solutions will be satisfied provided that*

$$\frac{d_1(s)}{R} \leq \frac{\hat{B}[1 - g(b_H(s))]^2}{R^2 g'(b_H(s))} \quad \text{for } s = 1, \dots, N \quad (17)$$

Proof. The Commitment Condition is clearly satisfied for b_H under this assumption. We need only note that since $b_L \leq b_H$ and the RHS of this expression is decreasing in b , then we must also have that

$$\frac{d_1(s)}{R} \leq \frac{\hat{B}[1 - g(b_L(s))]^2}{R^2 g'(b_L(s))} \quad \text{for } s = 1, \dots, N$$

which is the Commitment Condition for the low-debt solution. □

Thus, under Conditions 16 and 17, two solutions to the System with uncertainty will exist.

D Proof of Proposition 6

Note that with the inclusion of this prudence mechanism, the only thing from the previous problem that changes is the definition of f , which becomes

$$f_s(b_2) = \frac{R^2 d_0}{E_{\tilde{s}} [1 - g(b_2(\tilde{s}))] \hat{B}} + \frac{R d_1(s, b_2(s))}{[1 - g(b_2(s))] \hat{B}} + b_0 \quad (18)$$

So long as this operator satisfies all of the conditions of the former one, the previously invoked fixed point theorems will apply, since it operates on the same space. Continuity and compactness follows immediately from differentiable nature of d_1 . We need only show that f is monotone, which occurs so long as the altered second term remains increasing in b despite the decreasing nature of d_1 . i.e. we need that for every s ,

$$\begin{aligned} 0 \leq \frac{d}{db} \left[\frac{R d_1(s, b)}{[1 - g(b)] \hat{B}} \right] &= \frac{R d_1'(s, b) [1 - g(b)] + g'(b) d_1(s, b)}{\hat{B} [1 - g(b)]^2} \\ &\iff H[d_1(s, b)] \leq H[1 - g(b)] \end{aligned}$$

This provides one necessary bound on the hazard rate of d_1 . Given this, we can guarantee two solutions again provided that $\exists \hat{b} \leq \bar{b}$ such that $f(\hat{b}) < \hat{b}$. The Commitment Condition will again be satisfied for both solutions provided that

$$d_1(s, b^*(s)) \leq \frac{\hat{B} [1 - g(b^*(s))]^2}{R^2 g'(b^*(s))}$$

for $b^* \in \{b_L, b_H\}$. Before, we could say that if this condition held for b_H , it would also hold for b_L as well. However, if d_1 responds to lower debt levels be increasing, this need not be true. A sufficient condition for it to remain true is if the absolute value of the slope of d_1 is less than that of the right-hand-side of the above expression. Simple derivatives tell us that this will be true provided that $\frac{\partial d_1}{\partial b}$ is bounded below by

$$-\frac{\hat{B}}{R^2} \left[2[1 - g(b)] + \frac{g''(b)}{H[1 - g(b)]^2} \right] < 0$$

Thus, if for every s and for all $b \in [0, \bar{b}]$, the following inequality holds:

$$H[d_1(s, b)] \leq \min\left\{ H[1 - g(b)], \frac{\hat{B}}{R^2 d_1(s, b)} \left[2[1 - g(b)] + \frac{g''(b)}{H[1 - g(b)]^2} \right] \right\} \quad (19)$$

then two solutions to the system above will exist. Notice that this condition would always hold when the hazard rate of d_1 was a constant zero, which it was in Proposition 5, and in regions of b with default risk it would hold with strict inequality. By continuity, a small increase in this rate in the regions with default risk would also satisfy this inequality, and thus policy rules exist that satisfy this criterion.

E Sensitivity Analysis: Calibrated Example

In this section, I perform two robustness exercises regarding the calibrated examples.

E.1 Alternative Spread Specifications

I consider a variety of different spreads for both calibrated examples i.e. with (B) and without (NB) the feedback of spreads to debt levels through banking bailouts. The results are given in Table E.1 which provides for each column both the counterfactual figure and the percentage of its empirical counterpart explained by the coordination failure. It is apparent that the results are largely unchanged, especially with regard to the percentage impact on spreads and debt-to-GDP in the bailout case.

It is worth noting that the magnitude of the explained debt and spread build-up is increasing in all cases in the assumed initial spread. This is quite intuitive, since the greater is the assumed spread, the larger is the empirical debt build-up that the model attributes to the spread, and thus the larger the gap when that spread is reduced.

E.2 Applying a Rescue Package Discount

For most of 2011-2012, Ireland did not borrow from international capital markets, but rather from the ‘Troika’ of the IMF, the ECB and the European Commission. These loan packages were priced below what the implied market rate would have been. One can account for this by applying a ‘rescue discount.’ For

instance, if the market price is q_t , then the rescue package price is $\frac{q_t}{\psi_t}$ for some $\psi_t \leq 1$. Using the sovereign's Budget Condition (Equation 1), we can see that this rise in the price is isomorphic to a fall in required funds. In other words, we may assume that the sovereign borrows at the market rate but must only fill a deficit of $\psi_t d_t$. Thus, we can account for the rescue package by redefining \mathbf{D} as follows:

$$\mathbf{D} = \sum_{t=0}^T \frac{d_t}{R^t} \psi_t$$

In the benchmark case, we set $\psi_t = 1.0$ for all t . As a robustness exercise, we allow $\psi_t < 1$ (and constant) over the second half of the sample. Table E.2 presents these results for a host of different potential rescue discounts. Notice that since the rescue package only affects \mathbf{D} , in the benchmark model, the only thing that will change is \hat{B} , and thus given the calibration strategy, the implied counterfactual spreads and debt-to-GDP ratios will not change. In contrast, the results change in the extension with banking sector bailouts, but only modestly relative to the assumed rescue discount.

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Tables

Table 1:
Model Parameters in Calibrated Example

Parameter	Value
\underline{b}	0.9905
\bar{b}	1.5149
p	0.0427
κ	19.2558
α	250,347.9767
\hat{B}	0.8768

Table E.1:
Alternative Spread Specifications and Implied Counterfactuals

Assumed Spread: s_H	NB: s_L ($\Delta\%$)	NB: $[\frac{B}{Y}]_{2013,L}$ ($\Delta\%$)	B: s_L ($\Delta\%$)	B: $[\frac{B}{Y}]_{2013,L}$ ($\Delta\%$)
.02	.0136 (32.0%)	1.226 (0.4%)	0.0 (100.0%)	.7778 (36.9%)
.06	.0049 (91.0%)	1.182 (5.0%)	0.0 (100.0%)	.7584 (38.4%)
.09	.0028 (97.0%)	1.156 (6.2%)	0.0 (100.0%)	.7454 (39.5%)

Table E.2:

Implied Counterfactuals: Extension with Troika Rescue Discount.

Rescue Discount: ψ	s_L ($\Delta\%$)	$\left[\frac{B}{Y}\right]_{2013,L}$ ($\Delta\%$)
.975	0.0 (100.0%)	.7698 (37.5%)
.95	0.0 (100.0%)	.7740 (37.2%)
.90	0.0 (100.0%)	.7827 (36.5%)

Figures

Figure 1:
Debt-to-GDP Ratios of Peripheral Eurozone Economies

Data from Eurostat

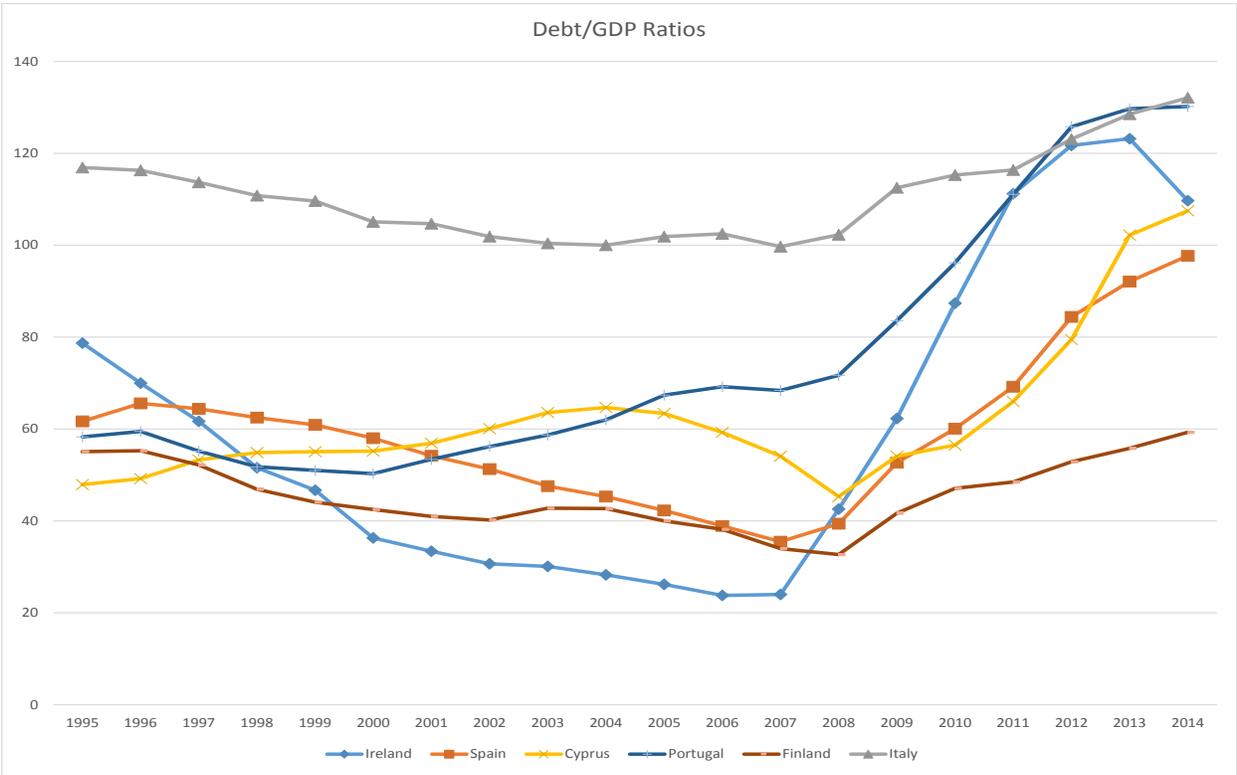


Figure 2:
Shape of Sample Revenue schedule

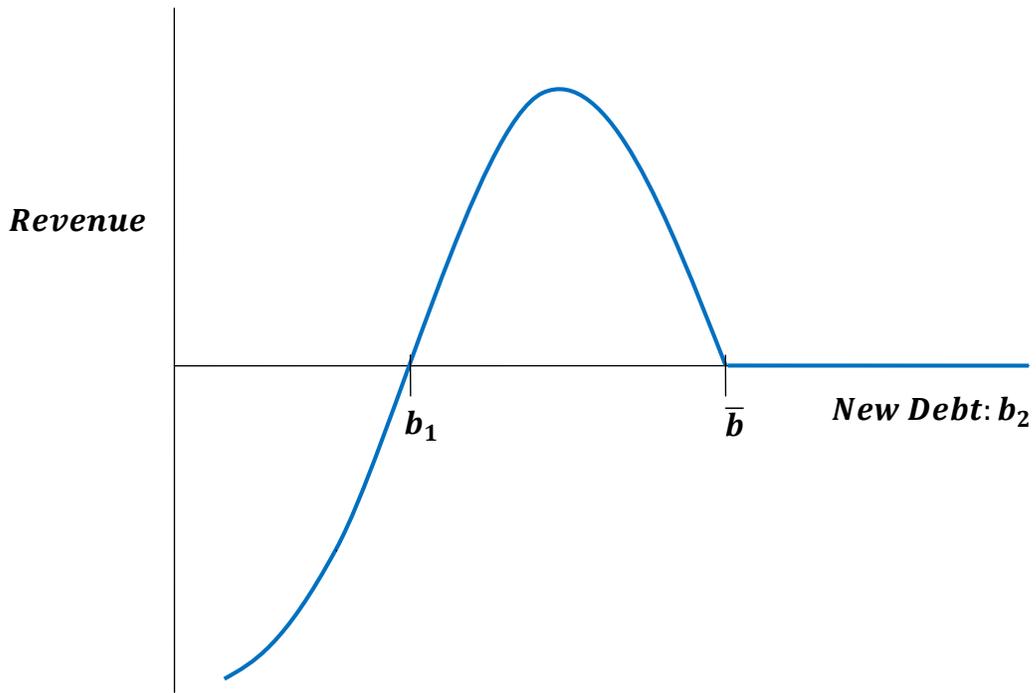


Figure 3:
The Front-Loading Condition

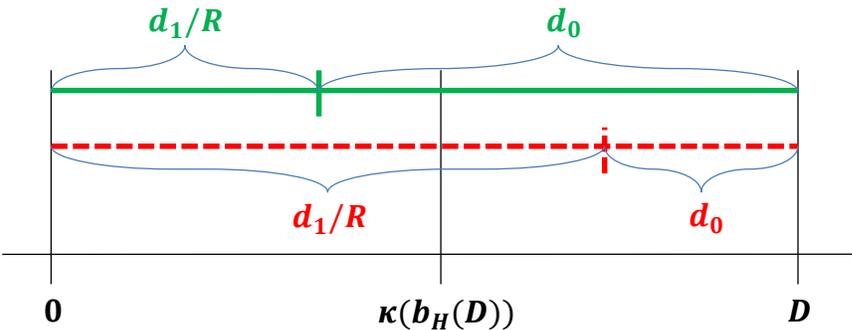


Figure 4:
Multiplicity via the Lifetime-Laffer Curve

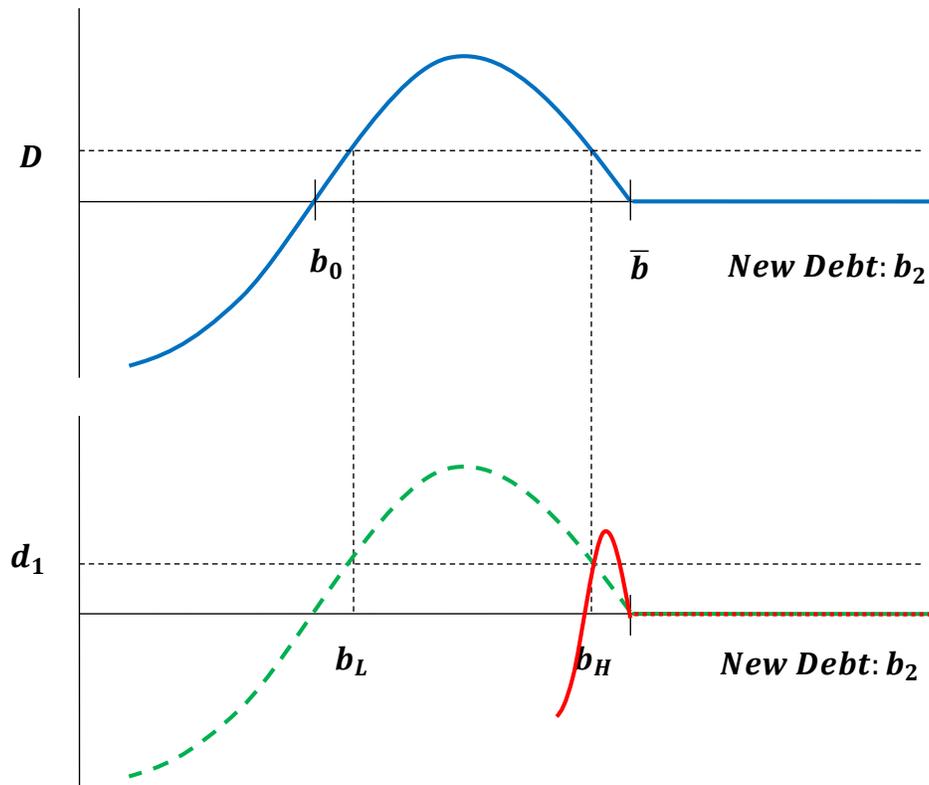


Figure 5:
The Logic of Front-Loading

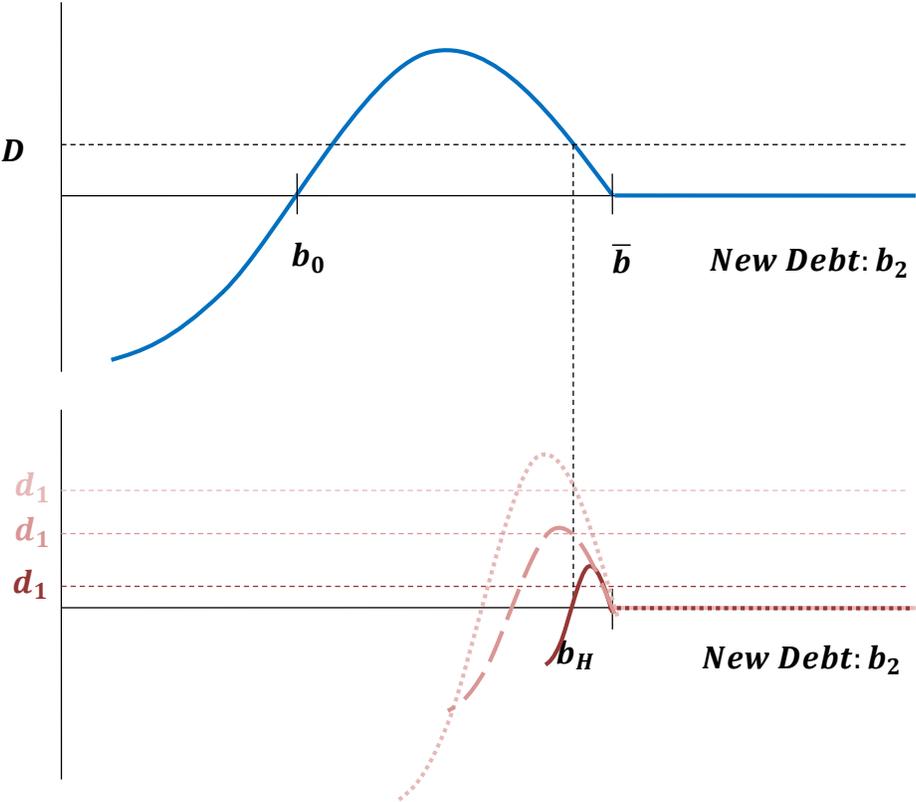


Figure 6:
Estimated Spread Function

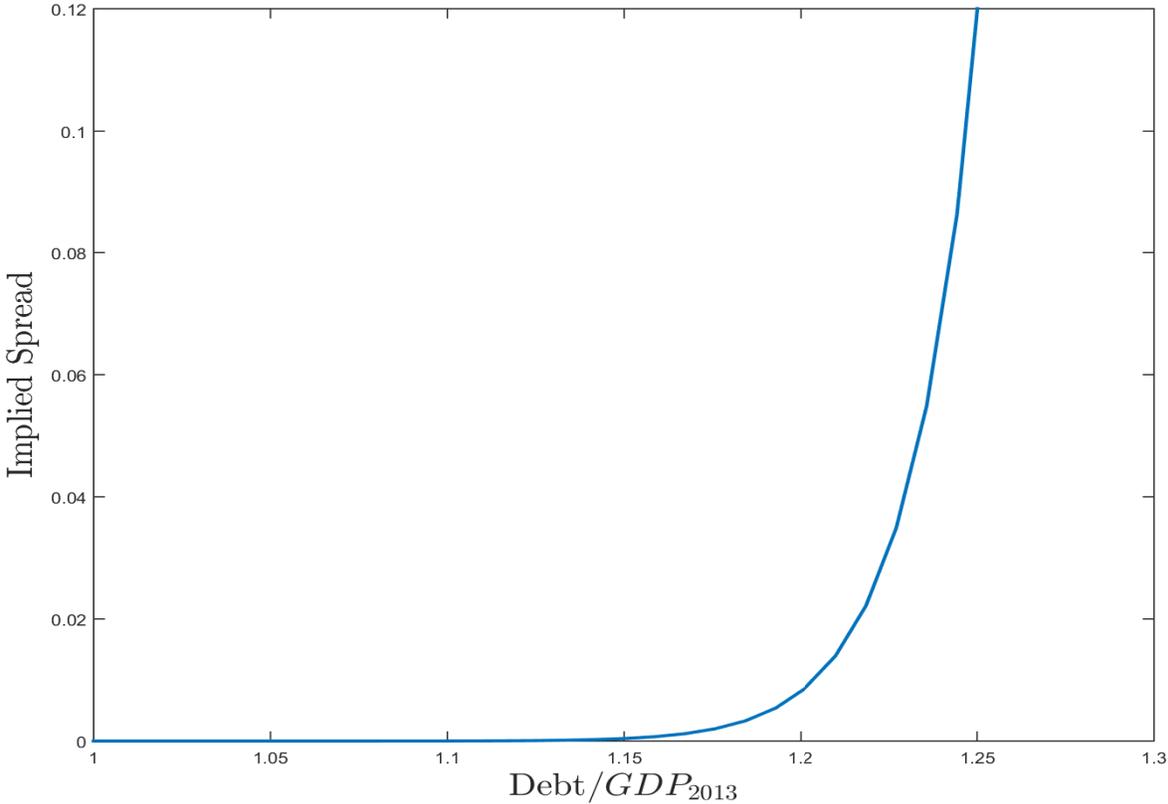


Figure 7:
Diverging Debt Trajectories

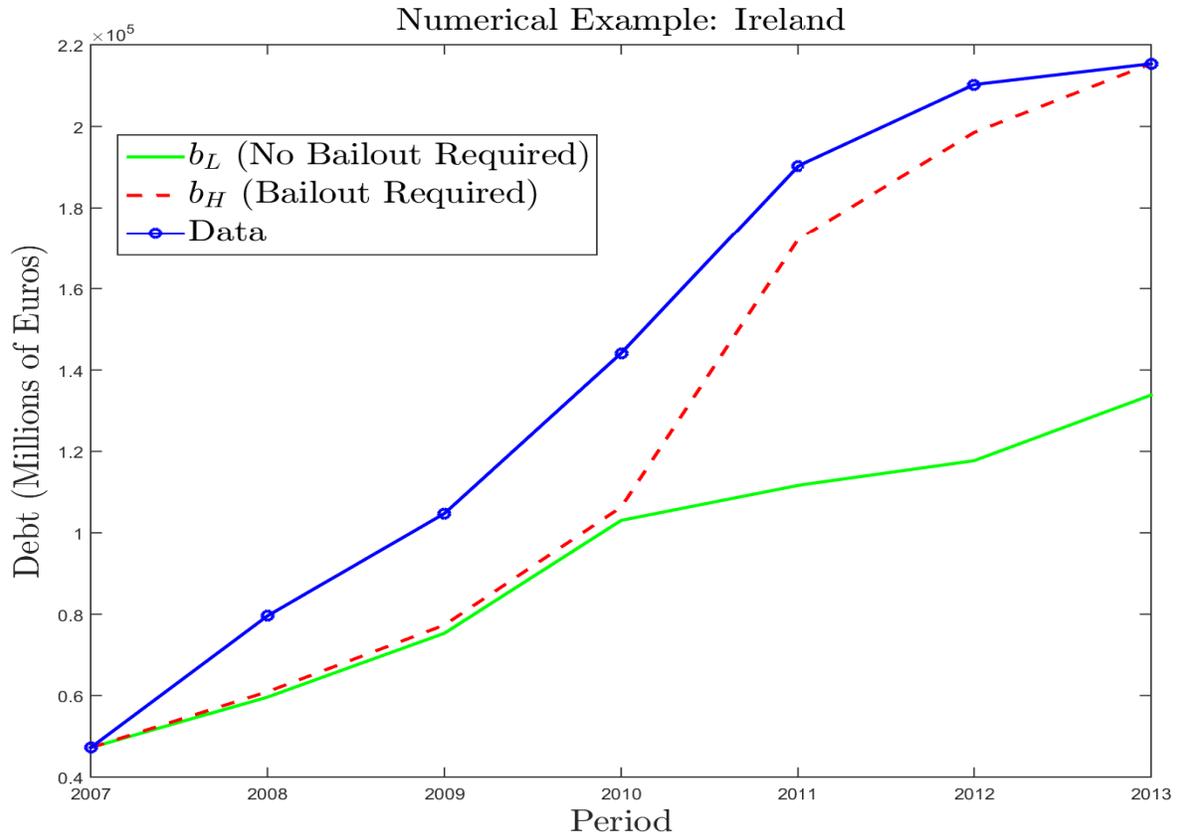


Figure 8:
Borrowing Policies in Stochastic Case Example

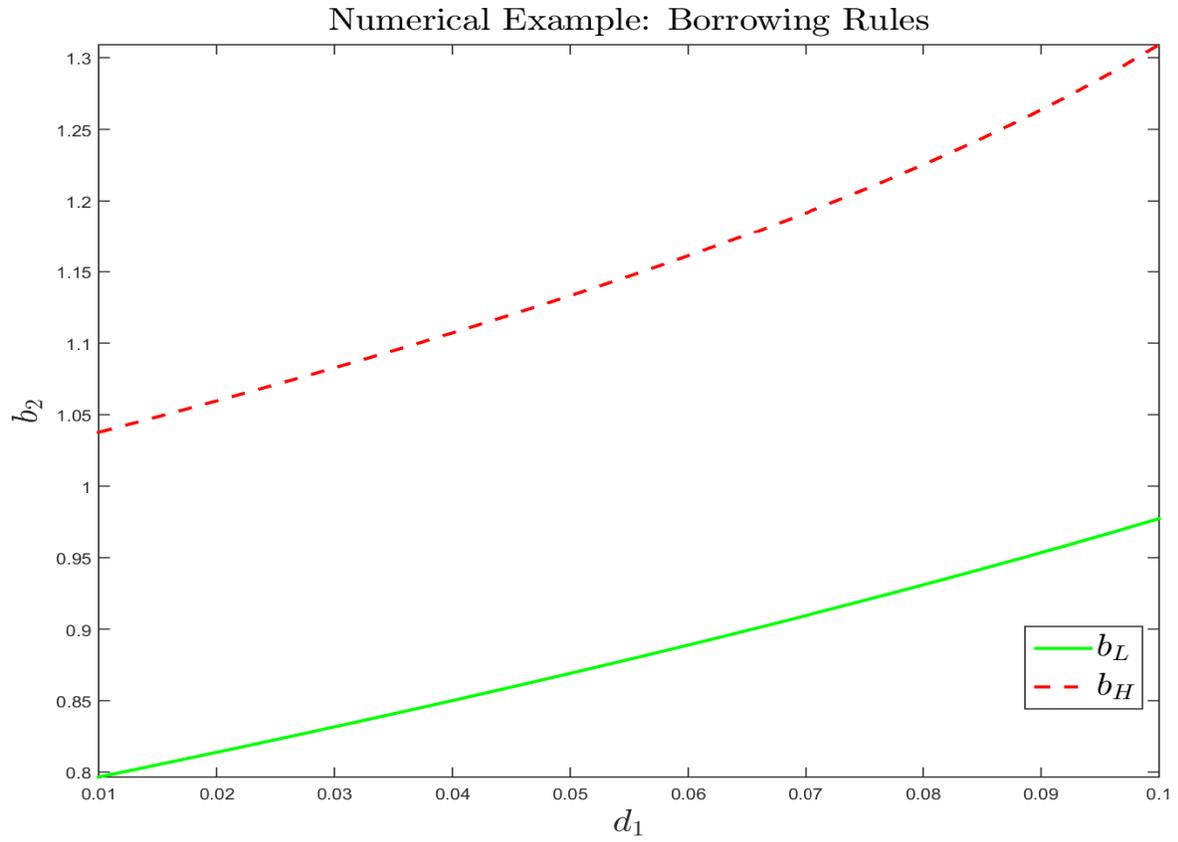


Figure B.1:
Graph of F With Convex, Increasing d_t

