# Sustainable Housing Policy

## Preliminary and Incomplete

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**Abstract**

Investment in housing is often thought of as an important tool for household wealth accumulation and for stimulating economic activity. As such, many policies aim to promote investment in housing and support housing prices. However, empirical research on the effect of such policies on wealth accumulation and investment in the economy is mixed. In this paper, we develop a comprehensive framework for studying the effect of housing investment on household wealth accumulation, the value of policies subsidizing investment in housing and the form that such policies should take. We find that when households are financially constrained, the price of housing generates externalities on investment in the economy and subsequently on household wealth accumulation. At times, it can be optimal to decrease the price of housing rather than to support high housing prices. Contrary to standard economic theory, we find that subsidizing the demand-side of the housing market and the supply-side of the market have different effects on welfare. When the return from real estate investment is high, a combination of subsidies for construction companies and taxes on purchases of houses can be optimal.

# 1    Introduction

The claim that investing in housing is an important tool for wealth accumulation has been used to justify policies subsidizing investment in housing and policies aimed at maintaining high real estate prices. The rationale behind this argument is that households obtain a durable asset when they buy a house, and they can use this asset to obtain funds to invest and build up their wealth. Following the collapse of the housing market in 2008, the United States government has taken many measures to encourage investment in real estate and maintain high housing prices including large scale purchases of mortgage-backed securities, the HARP program to increase refinancing of mortgages, and promoting the continuation of the 30-year fixed rate mortgage. In a 2013 policy brief, the White House explained the importance of these policy measures in building wealth and as a result stimulating the economy, "Housing wealth is growing again, with owners equity up $2.8 trillion since hitting a low at the beginning of 2009. This in turn has contributed to increased economic activity through consumer spending, small business investment, and more."

However empirical research on the positive effect of investment in housing on wealth accumulation and economic activity has found mixed results with many papers finding that owning real-estate does not always build wealth as effectively as renting and investing in other asset classes (Goetzmann and Spiegel (2000), Rappaport (2010)). Moreover, many policies which try to increase real estate investment via subsidization have also been associated with negative effects such as the creation of asset price bubbles, financial fragility, and investment mis-allocation. These issues calls for a comprehensive framework for studying the effect of housing investment on household wealth accumulation, the value of policies subsidizing investment in housing and the form that such policies should take.

In this paper, we analyze the optimal design of housing subsidization programs when governments wish to build household wealth and stimulate investment in the economy. Subsidization programs are often targeted at increasing the *demand* for mortgages by either incentivizing banks to lend more to real estate loans or incentivizing people to buy housing. Examples include the mortgage interest tax deduction and low insurance payments to qualify for government guarantees on mortgages in the United States and right-to-buy schemes in the United Kingdom. Contrary to this common practice, our analysis demonstrates that in many cases, it can be preferable to actually *tax* investment in real estate and subsidize the *supply-side* of the housing market instead. Such subsidies include but are not limited to subsidizing construction companies directly, a tax-credit to the sellers of houses and decreasing red-tape around land rights.

The typical channel through which investment in real estate leads to wealth accumulation for a household is the collateral channel. Households are able to borrow against the value of housing stock that they own and spend or invest the proceeds. This leads to policy goals by the government taking the form of supporting real estate investment and keeping housing prices high in an attempt to keep the net worth of agents coming from household equity high. As Chairman Bernanke mentioned during his September 13, 2012, press conference announcing the FOMC action: "To the extent that home prices begin to rise, consumers will feel wealthier, they'll feel more disposed to spend...if people feel that their financial situation is better...or for whatever reason, their house is worth more, they are more willing to go out and spend, and that's going to provide the demand that firms need in order to be willing to hire and to invest."

Following this commonly used rationale for supporting housing prices, we develop a model in which a representative household borrows from banks to invest in the real estate and commercial sectors. The household is financially constrained because of an inability to commit to repaying loans using future income and therefore must divide its income between the commercial and real estate sector even if both provide positive investment opportunities. The household can use housing purchases as collateral to help it obtain loans from a bank and thus can relax its financial constraint. Housing thus serves a dual purpose as an asset in the model - giving the household returns on investing and also relaxing the household's borrowing constraint. A government wishing to help the household build more wealth can implement demand side subsidies by promising the household a greater return on housing and supply side subsidies by subsidizing the production of houses.

The market price of housing in the model generates externalities on investment and subsequently household wealth accumulation when the household is financially constrained and cannot invest in all productive NPV opportunities, An increase in housing prices, allows household's to borrow more against household equity thus generating a positive externality on the funds available for investment. However, an increase in housing prices also causes the cost of investment in real estate to increase taking away funds from investing in the commercial sector. This generates a negative externality on investment. When the household is a price taker and financially constrained, these externalities prevent the first-best level of investment from being achieved.

The key novel insight of the model is that an increase in real estate prices does not necessarily increase household wealth even when the household actively uses it's house as an asset

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to borrow against. In our model the typical narrative of rising real estate prices helping to relax borrowing constraints by increasing the value of collateral, thus encouraging greater lending and investment is indeed true. However in addition to this, we also show that the effect of an increase in real estate prices on household wealth accumulation and aggregate investment can be negative due to a possible *crowding-out* of commercial investment as households are incentivized to invest more in mortgages. Although in our framework, an increase in real estate prices helps encourage investment by relaxing borrowing constraints, they also increase the amount of money that has to be invested for all housing units purchased. This second effect takes resources away from commercial investment in the presence of financial constraints and if big enough, can decrease the aggregate level of investment in the economy, subsequently reducing the household's wealth. The government may want to either subsidize or tax the household's investment in real estate depending on which effect dominates.

This negative investment effect of a boom in asset prices is in opposition to the collateral effect that is usually discussed in the literature. We additionally study the difference in subsidizing the supply-side of the real estate market rather than the demand-side and find that the two are not equal in our model due to their opposite effect on housing prices. A key novel insight of the model is that demand- and supply-side mortgage subsidization policies affect real estate and commercial investment in distinct ways. Interestingly this result goes against the traditional economic insight that it does not matter what side of the market is taxed or subsidized - the real effects of such interventions are the same. The difference between the two interventions arises in the model because of their opposite effects on the price of housing. While demand-side subsidies increase the price of housing by shifting the demand curve out, supply-side subsidies decrease the price of housing by shifting the housing supply curve out. In the presence of financial constraints and price externalities from housing, the real effects in the economy are not price-insensitive.

Supply- and demand- side subsidies are not equivalent in the model because they affect the agents' budget constraints differently when agents are financially constrained through their effect on housing prices. A change in price affects the agents' budget constraint in two ways. The first is a *direct* effect. A decrease in prices makes housing cheaper for the agent to invest it thus aiding the financially constrained agent. This channel makes supply-side subsidies preferable. The second effect is an *indirect* effect coming through the borrowing constraint. The representative agent is born with an existing set of real estate assets that he can use as collateral allowing loans to be made against future earnings. An increase in real

estate prices helps encourage investment by relaxing borrowing constraints since the value of the agents' collateral increases. This makes demand-side subsidies preferable. Interestingly, we find that in our setting, a social planner will want to combine expansionary supply-side policies like subsidies with contractionary demand-side policies such as a tax on household real-estate investment and vice-versa. In particular, supply-side subsidies are optimally combined with demand-side taxes when the return from real-estate investment is high. This leads to a very interesting and counter-intuitive implication of our framework - when the government believes that there are high returns from investing in housing then it is optimal to tax investment in housing and subsidize the production of housing due to the externalities generated by housing prices.

We additionally find that the timing of when the benefits of different subsidization policies accrue is of first-order importance in the model and find generally that when the household is financially constrained, policies that front-load their benefits are better. This gives support to policies such as the government directly subsidizing the down-payment of houses over policies in whose benefits accrue later such as the mortgage interest rate tax deduction or government insurance provisions. These policy recommendations are in line with those recently proposed by Mian and Sufi (2015). However, while they propose this based on the inherent risk associated with taking on debt we abstract from risk entirely and demonstrate another channel that would favour this policy. Interestingly, even without any risk, our model demonstrates that investing in housing is not always an efficient way to build wealth.

Finally, our model also has implications for the effect of high real estate prices on aggregate investment. In a recent paper, Chakraborty, Goldstein, and MacKinlay (2014) empirically document the effect our model describes. They find that banks in areas that experienced a significant real estate price boom, increased mortgage lending while simultaneously decreasing commercial lending. They also find that their effects are more pronounced for banks and firms that are relatively financially constrained. This is in line with the predictions of our model as we find that crowding-out of commercial investment only occurs in the presence of financial constraints which may prevent positive NPV projects from being undertaken. Our model can thus shed light on their empirical findings.

The rest of the paper is arranged as follows. Section 2 discusses other literature that is related to our paper. Section 3 outlines the main model. Section 4 discusses a benchmark case of our model in which the household is not financially constrained. Section 5 discusses the features of the model equilibrium. Section 6 compares demand and supply based policy

interventions. Section 7 discusses some key features and mechanisms of the model. Section 8 discusses optimal housing policy. The last section concludes.

## 2  Related Literature

Surprisingly, the literature considering the supply side of the housing market when considering housing policy is quite sparse. Glaeser, Gyourko, and Saiz (2008) argue that we must consider the supply side of the housing market to understand fluctuations in housing cycles. They present a model in which areas with more elastic housing supply have fewer and shorter housing bubbles. They also do an empirical analysis in which they find that areas with a more inelastic housing supply were the ones that experiences a large run-up in housing prices in the 1980s. Our model predictions are in line with theirs, also predicting that housing prices in areas with a more elastic supply will be less affected by shifts in housing demand. However, we focus on the optimal housing policy given different elasticities of the supply curve and actually consider supply-side policies which may not cause a price boom at all in order to increase homeownership.

The closest paper to ours is by Romer (2000), in which he argues that government policy programs aimed at increasing innovation should focus on subsidizing the supply of scientists and engineers rather than the demand for them. Romer makes the simple observation that if the supply of scientists and engineers is inelastic, subsidizing their labor demand may simply push wages up without increasing the equilibrium amount of innovation by much. He therefore recommends policies that would make the supply of such labour more elastic. While this is obviously true in our model, we find a difference in demand- and supply- side subsidies even if we keep the elasticity of the supply curve constant. This is due to the fact that financial constraints respond differently to increases versus decreases in price.

Our paper also contributes to the literature on the real effects of real estate price fluctuations. In particular, the collateral effect whereby an increase in real estate prices helps alleviate borrowing constraints in the economy has been shown theoretically by Holmstrom and Tirole (1997). Owners of real estate will be able to post more collateral and raise more capital for investment when prices of real estate are high. Chaney, Sraer, and Thesmar (2012) provide some empirical support for this.

While the collateral effect does operate in our model, we also have an investment crowd-out that can occur in the commercial sector as the price of real estate increases due to resources

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being diverted away from firms. Chakraborty et al. (2014) and Jorda et al (2014) find empirical support for this by documenting negative real effects on commercial investment of housing price appreciation. Theoretically, our model highlights a substitution effect amongst effects in a world with financial constraints which resembles the papers on internal capital markets by Stein (1997) and Scharfstein and Stein (2000). In these papers, a financially constrained headquarters makes an investment decision on how to allocate resources across divisions. A positive shock to the investment opportunities in one division diverts resources away from other divisions. In our model, agents face similar constrains and allocate resources to mortgages at the expense of commercial investment following price appreciations.

This negative real crowding-out effect of price booms we find in our paper also relates to Tirole (1985) in which bubbles in asset prices crowd out productive real investment by raising interest rates and reducing firm incentives to invest. In a similar vein, in a recent paper Farhi and Tirole (2012) find that the rise in interest rates might further restrict credit availability for financially constrained firms. In our paper, we do not require a bubble to produce the negative real effects accompanying a price-boom. We simply require that the household is financially constrained.

Finally, our paper also contributes to the macro literature which tries to understand the role of asset prices for the real economy and how price changes amplify shocks to investment. Seminal papers in this field such as Bernanke and Gertler (1989) and Moore and Kiyotaki (1997) discuss the amplification of choice via the effect of price on collateral and the ability of firms to borrow. Our proposed mechanism suggests that asset price booms may also serve to cause negative shocks to investment.

## 3    The Model

There are two time periods $(1, 2)$, a representative risk-neutral household, a representative bank, and two investment goods in the economy. The household consumes at $t = 2$ and tries to maximize the terminal value of its wealth. At $t = 1$, the household receives an endowment $\omega$ which it can invest in two sectors - housing and commercial firms. The representative household is also born with an existing housing stock, $B$, and can use this stock along with any new housing purchases, $x_m$, as collateral to borrow an amount $l$ from banks. The household additionally has access to a storage technology that has a return of 1.

At $t = 1$, a representative bank can make a loan, $l$, to the household to invest in housing and commercial firms. Loans need to be collateralized because of moral hazard in the repayment of loans. The household can use its housing stock as collateral and can commit to repaying a portion of the value of this stock $\phi(B + x_m)P$ at $t = 2$, where $P$ represents the price of housing in the economy and $\phi \leq 1$. $\phi$ represents the degree of pledgability of collateral. We assume that the banking sector in the economy is competitive and therefore the equilibrium interest rate on bank loans is 1.

Investing in firms gives a return of $r_f(x_f)$ at $t = 2$ for every $x_f$ units invested at $t = 1$. The function $r_f$ has the following standard properties, $r'_f(x_f) > 0$ and $r''_f(x_f) < 0$ for all $x_f$, $r'_f(0) = \infty$ and $r'_f(\infty) = 0$.

Investing in housing gives a return $r_m(x_m)$ at $t = 2$ for every $x_m$ units invested at $t = 1$. The function $r_m$ has the following standard properties, $r'_m(x_m) > 0$ and $r''_m(x_m) < 0$ for all $x_m$, $r'_m(0) = \infty$ and $r'_m(\infty) = 0$. The price per-unit of housing, $P$, is determined by demand and supply in the housing market. The representative construction firm is competitive and has a strictly increasing and convex cost of housing production given by $K(x_m)$.

Housing is a unique asset as it serves a dual role - as a consumption and as an investment good. As a consumption good, the return on housing can be a convenience yield derived from the benefits of owning rather than renting a house such as the ability to customize one's residence, entering into long-term service contracts and the option of reselling in the future amongst others. As an investment good, the return on housing reflects rental income, beneficial tax treatment and resale value.

The main results of the model are not dependent on decreasing returns to scale in the production functions. However, empirically this seems to be the more relevant case since it would imply investment taking place in both the housing and the commercial sector in equilibrium. Linear technologies would favor all the investment going into the more productive sector.

The government may have subsidization schemes in place to affect the amount of investment in the real-estate sector. This can be done by a demand side-subsidy which increases the $t = 2$ per-unit return on housing by an amount $r_g$. In practice most government interventions to increase real-estate investment tend to fall into demand-side interventions. We also consider supply-side subsidies in the form of a per-unit tax-rebate, $b$, that the government

can give to real estate firms as an alternative intervention. The government must have a balanced budget and households are taxed an amount $\tau$ at $t = 2$ to cover the cost of the subsidy.

To close the model, we finally make two more assumptions. First, households, firms and banks are price-takers in the economy. They therefore do not internalize their effect on $P$ and $\tau$ when making decisions. Second, households have limited funds. We define $x_m^*$ and $x_f^*$ as the quantities of $x_m$ and $x_f$ when the marginal return from investing is 1, i.e. $\frac{r_m^{'}(x_m^*)}{K^{'}(x_m^*)} = 1$ and $r_f^{'}(x_f^*) = 1$. Then we assume that,

$$K^{'}(x_m^*)x_m^* + x_f^* > \phi(B + x_m^*)K^{'}(x_m^*) + \omega \qquad \text{(Limited Funds)}$$

The above equation implies that even the household is financially constrained and cannot invest in all positive investment opportunities even if it borrows to its full capacity. [1] Therefore the household's borrowing constraint will bind in equilibrium. This is a key assumption that drives the main results of the model. We will discuss its importance throughout the paper.

## 3.1 The Household's Problem

The representative household borrows an amount $l$ from banks and maximizes the terminal value of its wealth. Since due to subsidies the price the household faces for buying houses may differ from the price the firm gets for selling houses, we refer to the former as $P^h$ and the latter as $P^f$. It therefore solves the following portfolio allocation problem, where $x_m$, $x_f$ and $x_s$ are the units of housing, commercial investment and storage purchased,

$$max_{(x_f,x_m,x_s,l) \geq 0} \quad r_f(x_f) + r_m(x_m) + r_g x_m + x_s - P^h x_m - x_f - \tau - l(1 + r_l)$$

$$s.t \quad x_f + P^h x_m + x_s \leq \omega + l$$

$$s.t \quad l \leq \phi(B + x_m)P^h$$

The first four terms in the household's $t = 2$ wealth represent the value of the household's investment portfolio, the next two terms represent the cost of the investment portfolio and

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[1] Note that if the equilibrium quantity of housing demanded by the household is $x_m^*$, then in equilibrium (unless a construction firm is subsidized), $K^{'}(x_m^*) = P^*$, where $P^*$ is the equilibrium price of housing. Therefore in an equilibrium in which the equilibrium quantity of housing demanded by the household is $x_m^*$, the above assumption can also be written as $P^* x_m^* + x_f^* > \phi(B + x_m^*)P^* + \omega$.

the last two terms represent the payments that the household must make at $t = 2$ to the government and the bank. The first constraint is the household's budget constraint while the second constraint is its borrowing constraint.

Since there is no uncertainty in the economy, the household never defaults in the model. At $t = 2$, a household's funds are deterministic conditional on its chosen investment portfolio at $t = 1$. In equilibrium the bank will never lend more than the household's ability to repay and therefore there default will never occur. Competition between banks therefore drives the equilibrium rate of interest on loans, $r_l$, to 0.

If additionally, the Limited Funds assumption is satisfied, the household's borrowing constraint binds in equilibrium and $l = \phi(B + x_m)P^h$. In this case, the household will never invest in the storage technology since it has unexploited NPV projects at $t = 1$ that yield a return strictly greater than 1. Since this is the pertinent case for government intervention, we focus on this going forward and assume the Limited Funds assumption always holds. This allows us to simplify the portfolio allocation problem to the following,

$$max_{(x_f, x_m) \geq 0} \quad r_f(x_f) + r_m(x_m) + r_g x_m - P^h x_m - x_f - \tau - \phi B P^h$$

$$s.t \quad x_f + P^h(1 - \phi)x_m \leq \omega + \phi B P^h \tag{1}$$

This yields the following first order condition that determines the amount of investment given the price of housing,

$$r'_m(x_m) + r_g = P^h + P^h(r'_f(x_f) - 1)(1 - \phi) \tag{2}$$

Market clearing requires that $P^f = P^h + b$ and that $x_m^s = x_m^d$ where $x_m^s$ is the amount of housing produced by the firm and $x_m^d$ is the amount of housing demanded by the household.

## 3.2 The Firm's Problem

The representative construction firm solves the following maximization problem,

$$max_{(x_m) \geq 0} \quad P^f x_m - K(x_m)$$

The first order condition for the firm gives us the equilibrium quantity of housing supplied,

$$K'(x_m^s) = P^f$$

## 3.3 The Government Problem

The government sets a subsidy level $\{r_g, b\}$ to maximize the utility of the representative household. Representing the utility of the household by $U$ and the equilibrium utility of the household by $U^e$ and the equilibrium quantity of housing investment by $x_m^e$, the government solves,

$$max_{r_g, b, \tau} \qquad U^e(r_g, b, \tau)$$

$$s.t \qquad \tau = (r_g + b)x_m^e(r_g, b)$$

where

$$U^e = r_f(x_f^e) + r_m(x_m^e) + r_g x_m^e - P^h x_m^e - x_f^e - \tau - \phi B P^h$$

and

$$\{x_m^e, x_m^f\} = \text{argmax}_{(x_f, x_m) \geq 0} \; r_f(x_f) + r_m(x_m) + r_g x_m - P^h x_m - x_f - \tau - \phi B P^h$$

$$s.t \qquad x_f + (1 - \phi)P^h x_m \leq \omega + \phi(B + x_m^e)P^h$$

## 3.4 Equilibrium

An equilibrium of this economy is given by, (i) A subsidization level $\{r_g^e, b^e\}$, (ii) A tax schedule $\tau^e$ given a subsidization level $\{r_g^e, b^e\}$ such that $\tau^e = (r_g^e + b^e)x_m^e$, (iii) The household's portfolio allocation $x_m^d$, $x_f^e$ and $x_s^e$ given the price of housing the household faces $P^{h^e}$ and subsidization level $r_g^e$, (iv) The firm's choice of housing production $x_m^s$ given price $P^{f^e}$ ,(v) Prices $P^{h^e}$ and $P^{f^e}$ such that the housing market clears, $P^{f^e} = P^{h^e} + b^e$ and $x_m^s = x_m^d = x_m^e$.

# 4 Benchmark Case - No Financial Constraint

We start by outlining a benchmark case, in which the household is not financially constrained. In the model, this is equivalent to the household being able to borrow from the bank without a problem of repayment. Therefore conditional on the expected return from an investment being above 1, the household can borrow the funds available to invest in it. The presence of the storage technology ensures the household never invests with an expected return of below 1 and therefore the household has no limit to the amount it can borrow. In the benchmark case, the household solves the following problem,

$$max_{(x_f,x_m,x_s,l)\geq 0} \quad r_f(x_f) + r_m(x_m) + x_s - Px_m - x_f - l$$

$$s.t \quad x_f + Px_m + x_s \leq \omega + l$$

The first order conditions yield the following equilibrium quantities of housing and commercial investment in the economy where the superscript $b$ refers to the benchmark case,

$$x_f^b = r_f^{'-1}(1)$$

$$x_m^b = \frac{r_m^{'-1}(1)}{P^b} \quad (3)$$

Assuming households do not borrow simply to store (i.e. they have a weak preference for not taking a loan), storage is used in equilibrium if $\omega \geq P^b x_m^b + x_f^b$. If $\omega < P^b x_m^b + x_f^b$, then the household will borrow an amount $l = P^b x_m^b + x_f^b - \omega$ from banks to fund its additional investment. The limited funds assumption corresponds to this second case with the household borrowing in equilibrium.

## 4.1 Social Planner - Benchmark Case

In the benchmark case, we can consider the allocation preferred by a social planner who takes price effects into account. The planner solves,

$$max_{(x_f,x_m,x_s,l)\geq 0} \quad r_f(x_f) + r_m(x_m) + x_s - x_f - K(x_m)$$

$$s.t \quad x_f + P(x_m)x_m + x_s \leq \omega + l$$

The social planner has the following first order conditions, where $\lambda$ is the lagrange multiplier on the budget constraint,

$$r_f'(x_f) = 1 + \lambda$$

$$r_m'(x_m) = K'(r_m) + \lambda(P + P'x_m - P'\phi B) \quad (4)$$

$$x_f + Px_m - \omega - \phi BP = 0$$

Since the household is not constrained in equilibrium $\lambda = 0$, giving the following equilibrium allocations chosen by the social planner,

$$x_f^b = r_f^{'-1}(1)$$

$$x_m^b = \frac{r_m^{'-1}(1)}{K'(x_m)}$$

Since $K'(x_m) = P^b$, this corresponds to the decentralized equilibrium allocation from (3). The usual welfare theorems apply in this benchmark economy. Now that we understand the benchmark equilibrium when the household is not financially constrained, we can see how in the presence of financial constraints the household not internalizing its effects on the price of housing causes inefficiencies in equilibrium.

# 5    Equilibrium Analysis

We now move to the equilibrium analysis of the main model in which the household is financially constrained. The decentralized equilibrium allocation here simply corresponds to setting $r_g = 0$ and $b = 0$ in (2). To establish the inefficiencies in the decentralized equilibrium, we first solve the constrained social planner's problem in this economy.

The social planner in this economy takes into account how housing prices affect the financial constraint of the household. The social planner therefore solves,

$$max_{(x_f,x_m,x_s)\geq 0} \quad r_f(x_f) + r_m(x_m) + x_s - x_f - K(x_m)$$

$$s.t \quad x_f + P(x_m)x_m + x_s \leq \omega + l$$

$$s.t \quad l \leq \phi(B + x_m)P(x_m)$$

When the Limited Funds assumption is satisfied, the household is financially constrained in equilibrium, it's borrowing constraint binds and $x_s = 0$. This problem simplifies to,

$$max_{(x_f,x_m)\geq 0} \quad r_f(x_f) + r_m(x_m) - x_f - K(x_m)$$

$$s.t \quad x_f + P(x_m)(1 - \phi)x_m \leq \omega + \phi BP(x_m)$$

The FOCs are similar to (4) however now $\lambda > 0$,

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$$r'_f(x_f) = 1 + \lambda$$

$$r'_m(x_m) = K'(x_m) + \lambda(P(1 - \phi) + P'(1 - \phi)x_m - P'\phi B)$$

$$x_f + P(1 - \phi)x_m - \omega - \phi BP = 0$$

The firm's FOC requires that $P = K'(x_m)$. This simplifies the above equation to the following equilibrium condition,

$$r'_m(x_m) = P + P(r'_f(x_f) - 1)(1 - \phi) + P'(x_m)((1 - \phi)x_m - \phi B)(r'_f(x_f) - 1) \qquad (5)$$

Looking at the above equation, note that the case in which the household is not financially constrained corresponds to $r'_f(x_f) = 1$. In this case, the social planner optimum corresponds to the the decentralized choice of the household in (2). When the household borrowing constraint binds, $r'_f(x_f) > 1$, the third term in (7) is not zero. In this case internalizing the price effects of housing, will make the social planner's optimum differ from that of the household. Therefore, the first welfare theorem does not hold in this case.

Before we move on, we make the following assumption $\forall (1 - \phi)x_m \neq \phi B$,

$$\text{abs}(K''(x_m)(\phi B - (1 - \phi)x_m)) > \text{abs}(K'(x_m)) \qquad \text{(Aggregate Price Effects)}$$

This assumption means that in the economy the aggregate impact of marginal changes in price outweight the local impact. The price changes that accompany an additional unit of housing being bought in the economy affect the value of the aggregate stock of collateral and the total aggregate cost of housing purchases. This assumption implies that the net effect of these aggregate changes are greater than the extra cost of purchasing the additional house.

We can now establish the following proposition,

**Proposition 1** *Let $x_m^{dc}$ be the decentralized equilibrium demand for housing and $x_m^{sp}$ be the optimal amount of housing in the constrained social planner equilibrium. If $(1 - \phi)x_m^{dc} > \phi B$ then $x_m^{sp} < x_m^{dc}$ and if $(1 - \phi)x_m^{dc} < \phi B$ then $x_m^{sp} > x_m^{dc}$.*

**Proof.** Appendix. ∎

The inefficiency in the decentralized equilibrium in this model arises because the representative household is acting like a price-taker and does not internalize the effect his housing demand has on prices. The price of housing affects both the value of the household's collateral and the amount of funds the household has to fund investment opportunities. When the existing amount of housing is high, an increase in the price of housing affects the household's ability to borrow. The social planner internalizes this effect and therefore wants to increase the demand for real-estate investment to push up the aggregate price of housing. When the existing amount of housing in the economy is low, an increase in the amount of housing causes resources to be diverted away from investment in the commercial sector when the household has borrowing constraints. The social planner internalizes this effect and therefore wants to decrease the demand for real-estate investment to decrease the aggregate price of housing.

As a corollary to the above proposition, we can then establish the following about the level of commercial investment in the economy,

**Corollary 1** *If $(1 - \phi)x_m^{dc} > \phi B$ then increasing housing investment crowds investment out of the commercial sector. Conversely if $(1 - \phi)x_m^{dc} < \phi B$ then increasing housing investment crowds investment in to the commercial sector.*

To study the effect that that increasing housing investment has on commercial investment we take the derivative of firm investment w.r.t $x_m$,

$$\frac{\partial x_f}{\partial x_m} = \phi B \frac{\partial P}{\partial x_m} - (1 - \phi)\frac{\partial}{\partial x_m}(Px_m) \tag{6}$$

An increase in the price of housing affects commercial investment in two ways - the *collateral effect* and the *income effect*. The *collateral effect* is captured by the first term on the right hand side of (6). An increase in the price of housing loosens the households budget constraint as the existing stock of homeownership is now worth more. The household can therefore borrow more against their future income and invest more in commercial firms at $t = 1$. The *income effect* is captured by the last two terms of (6). The boom in the price of houses causes the household to spend relatively more on housing and therefore they must compensate by reducing the amount spent on firm investment. Which effect dominates depends on relative increase in the value of collateral $\left(\frac{\partial}{\partial r_g}\phi BP\right)$ versus the relative increase in the amount that the household needs to pay for the extra investment in housing $\left((1 - \phi)\frac{\partial}{\partial x_m}(Px_m)\right)$. If the existing housing in an economy is high, a small increase in price can relax borrowing constraints enough to lead to a crowding-in of commercial investment.

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Finally, we can establish a demand subsidization scheme that can restore the social planner optimum. The following proposition states this,

**Proposition 2** *A demand-based subsidization scheme in which $r_g^* = P'(x_m^{sp})((1-\phi)x_m^{sp}\phi B)(r_f'(x_f^{sp}) - 1)$ and $b^* = 0$ restores the socially optimum level of housing and commercial investment chosen by the constrained social planner.*

    **Proof.** Appendix. ■

A particularly interesting aspect of this proposition is that if this investment-driven collateral effect is not large enough in the economy (a low $B$), the optimum policy for the government would be to have a negative tax on real estate investment. The size of the government subsidy or tax is larger when quality of investment opportunities in the commercial investment sector ($r_f'$) is higher. This is because better commercial investment opportunities increase the social cost of the externality on investment coming from housing prices. The size of the government subsidy or tax is also larger when the supply curve is more inelastic. This arises because of the non-neutrality of prices in the model. The price effects of housing drive the investment externalities and a more inelastic supply curve leads to greater price movements as we change the amount of real estate investment.

One of the key takeaways of our model so far is that when the household is financially constrained, price effects are not welfare-neutral. Therefore when we consider government interventions, how they affect the price of housing is critical. Subsequently, we show that we can restore the first-best level of welfare corresponding to the unconstrained social planner's outcome with a mix of demand and supply interventions. In the next section, we help explain the reasons demand and supply interventions differ in the model which will help understand the mechanism behind the optimal policy we propose.

# 6   Comparing Demand and Supply Subsidies

Traditional economic theory has long established that under general conditions, it does not matter whether we subsidize (tax) supply or demand from a welfare perspective. The gains to consumers and suppliers are the same and depend only on the relative elasticities of the supply and demand curves. However in our model, we find that taxing the supply and demand sides of the market can be quite different due to the effects that are caused by the increase or decrease in the price of housing when households are financially constrained and when there is some inter-temporal opportunity cost of capital. In our model, this cost is the

presence of a second sector in the economy since this seems to be an empirically relevant case. However, this cost can be much more general, and could include labor income costs, costs associated with moral hazard of lending such as monitoring costs, etc.

In this economy, the introduction of subsidies ($r_g > 0$) that increase the demand for housing always lead to an appreciation in the price of housing. The price for housing increases because each additional home is more expensive to produce giving rise to an upward sloping supply curve in the housing market. For the housing market to clear and respond to the increase in demand, the price of housing must consequently appreciate. Conversely, the introduction of subsidies ($b > 0$) that increase the supply for housing have an opposite effect on price to that of demand subsidies. They lead to a decrease in the price of housing. Supply side interventions hence do not cause the boom in housing prices that demand subsidies do.

In most literature on externalities, the socially optimum level of the good in question is affected by the existence of externalities. This level is typically independent of price movements. However, in our paper the externalities themselves are generated due to prices and therefore the different effect on prices that the demand- and supply- subsidies causes them to have different welfare implications. This will be discussed more and formalized in the rest of this section. Since typically externalities lead to the social planner choosing a particular optimal level of a good, in this case housing $x_m^*$, and then supply and demand subsidies produce the same result, we will compare these two subsidies holding fixed the level of housing investment in the economy. This will help clarify the main forces in the model that are driving the difference between these two policy interventions.

## 6.1   Demand and Supply Equivalence

As discussed earlier, in classic economic theory the welfare implications of taxing or subsidizing the supply and demand side of the market are the same. We will therefore start this section with a discussion of when subsidizing the supply and the demand side of the market in our model are welfare equivalent. We will then discuss which frictions cause the welfare implications of these two policies to differ.

If the household is not financially constrained, then demand and supply subsidization are equivalent policies. For any level of housing that can be achieved in the economy, subsidizing either the supply or demand generate the same utility for the representative household. This is because the household will simply invest until all the productive investment opportunities in the economy are realized. We can then establish the following proposition,

**Proposition 3 (Demand and Supply Equivalence)** *Suppose the household is unconstrained. Then for any $r_g'$ generates $x_m'$, $x_f'$ and $U'$, there exists a $b'$ that also generates $x_m'$, $x_f'$ and $U'$. The converse is also true.*

**Proof.** Appendix. ∎

Proposition 3 states the conditions under which supply and demand subsidies are welfare equivalent. The household in this model is financially constrained which prevents all productive investment opportunities from being realized in equilibrium. Once this constraint is taken away, the costs of being financially constrained i.e. the investment externalities that investing more in housing causes on the commercial sector, dissapear as well. The welfare gains from supply and demand subsidies therefore come from how they each affect the household's ability to borrow and invest.

From Proposition 3 , we see that under no financial constraints, subsidizing the demand- and supply-side are welfare equivalent. The household's constraints prevent it from investing in all productive investment opportunities. To understand the difference in the two policy interventions, we therefore need to look at their effect on financial constraints and subsequent investment in the economy. Having established when demand and supply subsidies produce the same effects, we now establish two more propositions that explain why they differ in our model.

Supply subsidies may allow the household to invest more by lowering the price of housing at $t = 1$. This effectively allows the household to "borrow" against some of its future income since taxes to pay for the subsidies are paid in the future. Alternatively demand side subsidies directly increase the value of collateral that households have thus helping them to borrow more. Subsequently, without a collateral effect in the model demand subsidies lose their advantage over supply-side subsidies. The following proposition formalizes this result,

**Proposition 4** *If $\phi = 0$, subsidizing the supply side of the housing market pareto dominates subsidizing the demand side of the housing market. That is, for any $r_g$ that is associated with utitily $U$, there exists a supply-subsidy $b$ that generates higher $U' > U$.*

**Proof.** Appendix. ∎

This proposition makes a strong statement - without a collateral effect, supply subsidies are always preferable to demand subsidies. The key assumptions driving this result are financial

constrains and an inter-temporal advantage of having more capital early. In the case of our model, the advantage is being able to invest in positive NPV projects of commercial firms. Consider the case when $\phi = 0$. In this case the income available for the household to invest does not change with the price of housing. Therefore the introduction of demand subsidies will push up price and always cause a reallocation of household investment in favour of mortgages and away from firm investment due to a negative income effect. Supply subsidies, on other hand, will free up household funds to invest in more projects by pushing the cost of housing down and have a positive income effect.

We can establish an analogous proposition for demand-based subsidization schemes. Namely,

**Proposition 5** *If $r_g x_m$ is given to the household at $t = 1$, subsidizing the demand side of the housing market pareto dominates subsidizing the supply side of the housing market. That is, for any b that is associated with utility $U$, there exists a demand-subsidy $r_g$ that generates $U' > U$.*

One of the key differences between supply- and demand- interventions in the model are that the benefits of the supply-subsidies are realized at $t = 1$ while those of the demand-based subsidization schemes are realized later. If the benefits of demand-subsidization subsidies are moved earlier then they pareto-dominate supply side-subsidies since this movement neutralizes the income effect while keeping the stronger collateral effect that demand subsidies naturally have since they push up the price of housing. This type of intervention is similar to the government providing aid to households to help in the downpayment of a mortgage rather than providing incentives that come due later like the mortgage interest tax deduction. We will come back to specific policy interventions more later.

# 7   Discussion

**Investment:**   In the model understanding how supply and demand subsidies affect investment through their effect on price is the key to understanding the differences between these policy interventions. This may provide a theoretical basis for the empirical results found in Chakraborty et al. (2014). We find that a housing price increase is only good for investment if the existing stock of home-ownership is large and households are financially constrained in terms of their ability to invest in investment opportunities. When the existing stock of housing is low, increasing the price of housing can lead to negative externalities on investment and crowd-out investment from the commercial sector. In such a case, policies aimed at reducing the price of housing are preferable for investment.

**Price Externalities:** An important novel feature of our model is looking at price externalities in a general equilibrium framework. When externalities exist because of prices the standard result of the irrelavance of using supply or demand subsidies no longer applies, because these two interventions move prices in the opposite direction.

**Inter-temporal Effects:** A key mechanism at play in our model is that the costs and benefits of supply and demand subsidies are distributed differently across time. Supply subsidies have their "direct" benefits delivered today in terms of lower prices while demand subsidies have their "direct" benefits accrue in the future in terms of a higher return on housing. Without financial constraints, these different inter-temporal properties of the two schemes do not matter. However, in the presence of financial constraints, the inter-temporal properties matter because there is a positive effect of front-loading benefits of subsidies since the household can use the extra funds to invest.

# 8 Optimal Housing Policy

From the analysis so far, we have highlighted an important feature of this analysis - in our setting housing price movements determine the optimal policy rather than a specific level of housing investment due to externalities on investment caused by housing prices. Based on this, we establish the key proposition of the paper,

**Proposition 6** *The first-best level of welfare can be achieved by a subsidy pair $\{r_g, b\} = \{\frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*} - r_m'(x_m^*), r'(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*}\}$.*

**Proof.** Appendix. ∎

This proposition is the key result of our paper and yields a very surprising result. In the presence of price externalities from housing, the optimal way to build wealth in the economy is to combine expansionary housing supply subsidies with contractionary housing demand taxes and vice-versa. From the proposition we can see that the in optimal subsidy scheme $r_g = -b$. The intuition behind this is due to price externalities being the key source of inefficiency in the model. The optimal policy therefore needs to target price movements which require opposite susbsides to demand and supply. An interesting corollary to the above result follows,

**Corollary 2** *The size of optimal subsidies and taxes increase as the demand and supply curves for housing becomes more inelastic.*

This corollary is quite intuitive given that our mechanism is driven by price externalities. Greater inelasticities means that the magnitude of price changes when $x_m$ changes are larger causing the externalities generated to also be larger. Therefore, the appropriate policy response is also larger.

We can modify the housing production functions by adding a technological progress parameter, $A_m r_m$, and establish an important proposition which follows from the above result,

**Proposition 7** *As $A_m$ increases or equivalently as the optimal $x_m^*$ increases, the optimal housing policy in the economy favours expansion of supply-side housing subsidies and demand-side housing taxes.*

**Proof.** Appendix. ∎

This result says that when the returns from real estate investment are large in the economy, the optimal policy to increase real estate investment involves taxing housing demand and subsidizing housing supply instead. We should be pushing housing prices down rather than trying to support housing prices.

This result is very interesting because it highlights that when the return to housing is high relative to the return on other investments, it is preferable to try and reduce the price of housing, rather than to make borrowing for households easier (leverage). Other arguments in the literature also suport household's taking on less leverage (Mian and Sufi (2015)). We get the same in our model but through a completely different channel. We abstract away from the risk created by taking on leverage and show that even from a wealth accumulation perspective taking on leverage may not be optimal for households. This is driven by the fact that leverage today can actually reduce the funds available for a household to invest in other productive opportunities because it can push up the price of housing. Reducing the costs of downpayments of houses can instead free up funds and increase a household's ability to invest. Demand subsidies in the model induce taking on leverage while supply subsidies cause a reduction in the leverage required. The income effect works as a counter to taking on more leverage.

# 9   Conclusion

In this paper we develop a comprehensive framework for studying the effect of housing policy on household wealth accumulation. We find that an increase in household home

equity does not necessarily lead to efficient wealth accumulation, housing prices can generate externalities on investment, and supply and demand subsidies are not equivalent in the presence of price externalities. When the return to investing in real estate in the economy is high, the optimal policy involves an expansion of supply subsidies and a tax on housing demand.

In this paper, we abstract away from the risk of investing in real estate and focus on a different channel highlighting the effect of housing subsidies on the financial constraints of a household. Our results support policies aimed at reducing the housing debt taken on by households and encourage policies that front-load their benefits even in the absence of risk.

In practice, it appears that many households used household equity to fund consumption rather than investment. Our framework can be extended to include a measure of impatient households who would rather borrow against their home to fund immediate consumption over investment and may be an interesting area for future research.

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# 10 Appendix

**Proof of Proposition** 1. To begin the proof first note that, $P'(x_m) = K''(x_m) > 0$. The equilibrium condition for the decentralized equilibrium requires,

$$r'_m(x_m) = P + P(r'_f(x_f) - 1)(1 - \phi) + P'(x_m)((1 - \phi)x_m - \phi B)(r'_f(x_f) - 1) \quad (7)$$

$$r'_m(x_m^{dc}) = P^{dc} + P^{dc}(r'_f(x_f^{dc}) - 1)(1 - \phi)$$

If $(1 - \phi)x_m^{dc} > \phi B$, then,

$$\frac{\partial x_f}{\partial x_m} = \frac{\partial P}{\partial x_m}\left(\phi B - (1 - \phi)x_m^{dc}\right) - P(1 - \phi) < 0$$

Since $r''_f < 0$, this implies that $\frac{\partial r'_f(x_f)}{\partial x_m} > 0$. Therefore if $(1 - \phi)x_m^{dc} > \phi B$,

$$\frac{\partial P'(x_m)((1 - \phi)x_m - \phi B)(r'_f(x_f) - 1)}{\partial x_m} > 0 \quad (8)$$

If $(1 - \phi)x_m^{dc} > \phi B$ then,

$$r'_m(x_m^{dc}) < P^{dc} + P^{dc}(r'_f(x_f^{dc}) - 1)(1 - \phi) + P^{dc'}(x_m^{dc})((1 - \phi)x_m^{dc} - \phi B)(r'_f(x_f^{dc}) - 1) \quad (9)$$

Since the RHS of (9) is decreasing in $x_m$ while the LHS is increasing in $x_m$ from (8), $x_m^{sp} < x_m^{dc}$.

The proof for the second part of the proposition follows via an analogous similar argument using the Aggregate Price Effects assumption to establish that when $(1 - \phi)x_m^{dc} < \phi B$, $\frac{\partial x_f}{\partial x_m} > 0$. ∎

**Proof of Proposition** 2. When $r_g^* = P'(x_m^{sp})((1 - \phi)x_m^{sp} - \phi B)(r'_f(x_f^{sp}) - 1)$ and $b^* = 0$, the household's first order condition is,

$$r'_m(x_m) + P'((1 - \phi)x_m^{sp})(x_m^{sp} - \phi B)(r'_f(x_f^{sp}) - 1) = P + P(r'_f(x_f) - 1)(1 - \phi)$$

When $b = 0$, $P = K'(x_m)$ in equilibrium. Substituting that in,

$$r'_m(x_m) + K''((1 - \phi)x_m^{sp})(x_m^{sp} - \phi B)(r'_f(x_f^{sp}) - 1) = K'(x_m) + K'(x_m)(r'_f(x_f) - 1)(1 - \phi)$$

For this equation to hold, $x_m = x_m^{sp}$ and $x_f = x_f^{sp}$ since it is identical to (??). The household's utility under this subsidy scheme is given by,

$$U = r_m(x_m^{sp}) + r_f(x_f^{sp}) - Px_m^{sp} - x_f^{sp} + r_g x_m^{sp} - \tau$$

For the government to have a balanced budget, $\tau = r_g x_m^{sp}$. Substituting this into household utility and using the fact that $K(x_m^{sp}) = Px_m^{sp}$, we see that the utility is the same as that of the constrained social planner. ∎

**Proof of Proposition** 3. When the household can invest as it likes and is not financially constrained, the household chooses $x_f^*$ s.t. $r'(x_f^*) = 1 \; \forall \; b, r_g$. This can be proven by contradiction. Suppose the household chooses an $x_f < x_f^*$. Then taking a loan of $x_f - x_f^*$ and deviating to $x_f^*$ will provide higher terminal wealth and therefore an $x_f < x_f^*$ cannot be optimal. Suppose the household chooses an $x_f > x_f^*$. Then reducing its loan by $x_f^* - x_f$ and deviating to $x_f^*$ will provide higher terminal wealth and therefore an $x_f > x_f^*$ cannot be optimal for the household. In the analysis that follows, superscript $d$ refers to a demand-side quantities, while superscript $s$ refers to supply-side quantities.

A demand side intervention $r_g$ that generates $x_m'$ will be associated with household utility $U^d$ given by,

$$U^d = r_m(x_m') + r_f(x_f^*) - P^d x_m' - x_f^* + r_g x_m' - \tau^d - l^d \tag{10}$$

Substituting in for $\tau^d$ and $l^d = P^d x_m' + x_f^* - \omega$, this simplifies to

$$U^d = r_m(x_m') + r_f(x_f^*) - \omega$$

Similarly, a supply side subsidy $b$ that generates $x_m'$ is associated with household utility $U^s$ that is given by,

$$U^s = r_m(x_m') + r_f(x_f^*) - P^s x_m' - x_f^* - \tau^s - l^s$$

Substituting in for $\tau^s$ and $l^s = P^s x_m' + x_f^* - \omega$, this simplifies to

$$U^s = r_m(x_m') + r_f(x_f^*) - \omega$$

Say we want to achieve a level of $x_m'$. Then a demand side intervention will require $r_g$ such that,

$$r'_m(x'_m) + r_g = S^{-1}(x'_m)r'_f(r^d_f) \tag{11}$$

A supply side intervention will require $b$ such that,

$$r'_m(x'_m) = \left(S^{-1}(x'_m) - b)\right) r'_f(r^s_f) \tag{12}$$

We can rewrite $b$ as,

$$b = P^d - \frac{r'_m(x^*_m)}{r'_f(r^s_f)} = P^d - P^s \tag{13}$$

Using the fact that $b = P^d - P^s$, this simplifies to,

$$U^s = r_m(x'_m) + r_f(x^*_f) + f(x'_m) - (x^*_f + P^d x'_m - \omega) = U^d$$

Therefore, the representative household has the same utility under both supply and demand subsidies. ∎

**Proof of Proposition** 4. In the analysis that follows, superscript $d$ refers to a demand-side quantities, while superscript $s$ refers to supply-side quantities. Say we want to achieve a level of $x'_m$. Then a demand side intervention will require $r_g$ such that,

$$r'_m(x'_m) + r_g = K'(x'_m)r'_f(r^d_f) \tag{14}$$

A supply side intervention will require $b$ such that,

$$r'_m(x'_m) = \left(K'(x'_m) - b)\right) r'_f(r^s_f) \tag{15}$$

The utility of the household under demand-side intervention is then given by,

$$U^d = r_m(x'_m) + r_f(x^d_f) - P^d x'_m - x^d_f + r_g x'_m - \tau \tag{16}$$

Using the fact that $\tau = r_g x'_m$ and that $P^d x'_m = K(x'_m)$, this simplifies to,

$$U^d = r_m(x'_m) + r_f(x^d_f) - K(x'_m) - x^d_f \tag{17}$$

Similarly, the utility of the household under supply-side intervention is given by,

$$U^s = r_m(x'_m) + r_f(x^s_f) - P^s x'_m - x^s_f - \tau \tag{18}$$

Using the fact that $\tau = bx'_m$ and that $P^s x'_m = K(x'_m) - bx'_m$, this simplifies to,

$$U^s = r_m(x'_m) + r_f(x^s_f) - K(x'_m) - x^s_f \tag{19}$$

Comparing (17) and (19), we can see that since $r'_f(x_f) \geq 1$, if $x^s_f > x^d_f$, supply side subsidies pareto-dominate demand side subsidies. Using the budget constraint, we see that,

$$x^d_f = \omega - K'(x'_m)x'_m$$

$$x^s_f = \omega - (K'(x'_m) - b)x'_m \tag{20}$$

Since $b > 0$, $x^s_f > x^d_f$ and therefore supply subsidies pareto dominate demand subsidies. ∎

**Proof of Proposition** 5. In the analysis that follows, superscript $d$ refers to a demand-side quantities, while superscript $s$ refers to supply-side quantities. Say we want to achieve a level of $x'_m$. Then a demand side intervention will require $r_g$ such that,

$$r'_m(x'_m) + r_g = K'(x'_m) + K'(x'_m)(r'_f(x^d_f) - 1)(1 - \phi) \tag{21}$$

A supply side intervention will require $b$ such that,

$$r'_m(x'_m) = \left(K'(x'_m) - b\right)\left(1 + (r'_f(x^d_f) - 1)(1 - \phi)\right) \tag{22}$$

The utility of the household under demand-side intervention is then given by,

$$U^d = r_m(x'_m) + r_f(x^d_f) - P^d x'_m - x^d_f + r_g x'_m - \tau \tag{23}$$

Using the fact that $\tau = r_g x'_m$ and that $P^d x'_m = K(x'_m)$, this simplifies to,

$$U^d = r_m(x'_m) + r_f(x^d_f) - K(x'_m) - x^d_f \tag{24}$$

Similarly, the utility of the household under supply-side intervention is given by,

$$U^s = r_m(x'_m) + r_f(x^s_f) - P^s x'_m - x^s_f - \tau \tag{25}$$

Using the fact that $\tau = bx'_m$ and that $P^s x'_m = K(x'_m) - bx'_m$, this simplifies to,

$$U^s = r_m(x'_m) + r_f(x^s_f) - K(x'_m) - x^s_f \tag{26}$$

Comparing (24) and (26), we can see that since $r'_f(x_f) \geq 0$, if $x^d_f > x^s_f$, demand side subsidies

pareto-dominate demand side subsidies. Using the budget constraint, we see that,

$$x_f^d = \omega - (1 - \phi)K'(x_m')x_m' + r_g x_m' + K'(x_m')\phi B$$

$$x_f^s = \omega - (1 - \phi)K'(x_m')x_m' + (1 - \phi)b x_m' + (K'(x_m') - b)\phi B$$

(27)

From (19) we can rewrite $r_g$ as $r_g = b(1 + r_f'(x_f^s) - 1)(1 - \phi)) > b(1 - \phi)$. Substituting this into (27), we see that demand subsidies pareto dominate supply subsidies. ∎

**Proof of Proposition** 6. At optimal,

$$x_f^* = \omega + \phi B P - P(1 - \phi)x_m^*$$

(28)

This gives us

$$P = \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*}$$

(29)

At that level household doesn't invest anymore in $x_f$ since $r'(x_f^*) = 1$. Looking at the household's FOC, we also require that,

$$\frac{r_m'(x_m^*) + r_g}{P} = 1$$

(30)

This gives us an $r_g$ of

$$r_g = \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} - r_m'(x_m^*)$$

(31)

and it gives us a $b$ of,

$$K'(x_m^*) = \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} + b$$

(32)

therefore $b$ is,

$$b = K'(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*}$$

(33)

We know that at $x_m^*$, $K'(x_m^*) = r_m'(x_m^*)$ and therefore we can rewrite $b$ as,

$$b = r'(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} = -r_g$$

(34)

Interesting: expansionary supply-side policy (positive $b$) has to be accompanied by contractionary demand-side intervention (negative $r_g$) to achieve the optimum. $b$ is positive when,

$$r'(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} > 0$$

(35)

Rewriting,

$$r^{'}(x_m^*) > \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} \tag{36}$$

Under this subsidy scheme the household's utility is given by,

$$U = r_m(x_m^*) + r_f(x_f^*) - Px_m^* - x_f^* + r_g x_m^* - \tau$$

For the government to have a balanced budget, $\tau = (b + r_g)x_m^*$. Substituting this into household utility and using the fact that $Px_m^{sp} = K(x_m^{sp}) - b^* x_m^{sp}$, we see that the utility is the same as that of the unconstrained social planner. ∎

**Proof of Proposition** 7.    We can repeat the analysis in the paper with the new technology enhanced production functions and find that the optimal subsidy scheme is given by,

$$\{r_g, b\} = \left\{ \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} - A_m r_m^{'}(x_m^*), A_m r^{'}(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} \right\} \tag{37}$$

We can see that when $A_m$ increases $b$ increases and $r_g$ decreases. This proves the proposition. ∎