

Information Percolation, Momentum, and Reversal*

Daniel Andrei[†] Julien Cujean[‡]

January 6, 2016

Abstract

We propose a rational model to explain time-series momentum. The key ingredient is word-of-mouth communication, which we introduce in a noisy rational expectations framework. Word-of-mouth communication accelerates information revelation through prices and generates short-term momentum and long-term reversal. Social interactions allow investors with heterogeneous trading strategies—contrarian and momentum traders—to coexist in the marketplace. As a result, momentum is not completely eliminated, although a significant proportion of investors trade on it. We also show that word-of-mouth communication spreads rumors and generates price run-ups and reversals. Our theoretical predictions are in line with several empirical findings.

*The authors are grateful for comments received from Bruce Carlin, Nicolae Gârleanu (WFA discussion), Cam Harvey, David Hirshleifer, Steve Heston, Eric Hughson, Aditya Kaul, Hanno Lustig, Avanidhar Subrahmanyam, Mungo Wilson, Russ Wermers, Fernando Zapatero, and from seminar and conference participants at Kellogg School of Management, University of Lausanne, Boston University, MIT Sloan School of Management, the Princeton-Lausanne Workshop on Quantitative Finance, HEC Winter Finance Workshop, UCLA Anderson, Southern California Finance Conference, Miami Behavioral Finance Conference, Banque de France, Florida International University, the 17th SGF Conference, the 2014 SFS Cavalcade, the 10th Annual Asset Pricing Retreat, the 2014 WFA meetings, the 2014 World Finance Conference, the 2014 NFA conference, and the 2015 Jackson Hole Finance Conference. Financial support from the Swiss Finance Institute, NCCR FINRISK of the Swiss National Science Foundation, UCLA, and University of Maryland is gratefully acknowledged.

[†]UCLA Anderson Graduate School of Management, 110 Westwood Plaza, Suite C420, Los Angeles, CA 90095, USA; +1 (310) 825-3544; daniel.andrei@anderson.ucla.edu; danielandrei.info

[‡]Robert H. Smith School of Business, University of Maryland, 4466 Van Munching Hall, College Park, MD 20742, USA; +1 (301) 405-7707; jcujean@rhsmith.umd.edu; juliencujean.com

1 Introduction

One of the most pervasive facts in finance is price momentum. It is documented everywhere, both across and within countries and asset classes (Asness, Moskowitz, and Pedersen, 2013; Rouwenhorst, 1998). It appears in the cross-section of returns, where it refers to securities’ *relative performance* (Jegadeesh and Titman, 1993), but also in the time-series of returns, where it refers to a security’s *own performance* (Moskowitz, Ooi, and Pedersen, 2012).¹ This latter form of momentum, referred to as “time-series momentum,” is the focus of this paper.

Time-series momentum challenges rational explanations—rational investors can easily detect momentum and trade on it, thereby eliminating it. Leading theories of momentum are therefore mostly behavioral: “momentum traders,” “conservative investors,” or “attribution bias” generate momentum, whereas “newswatchers,” “representativeness heuristic,” or “overconfidence” generate reversals.² But the weak link between momentum and various measures of investor sentiment (Moskowitz et al., 2012) indicates that behavioral models based on sentiment have yet to identify the main source driving momentum.

This paper provides an explanation for time-series momentum in which no behavioral bias is necessary for momentum to arise and persist. Our theory simultaneously produces short-term momentum and long-term reversal, consistent with evidence in Moskowitz et al. (2012). The building block is a noisy rational expectations economy (Grossman and Stiglitz, 1980) in which a large population of risk-averse agents trade a risky asset over several trading rounds. In this standard setup, investors trade based on private information but also based on public information conveyed by equilibrium prices. Our contribution is to introduce an additional channel of information acquisition: we assume that private information diffuses among the population of investors through word-of-mouth communication. Investors therefore trade in centralized markets, but also search for each other’s private information—trading is centralized, but information exchange is decentralized.

We model communication among agents through the information percolation theory (Duffie and Manso, 2007), according to which agents exchange information in random, bilateral private meetings. When embedded into a centralized trading model, information percolation has two effects. First, as agents accumulate information through random meetings, the average precision of information in the economy increases at an accelerated (exponential) rate. Second, through these random meetings, agents acquire heterogeneous amounts of information. The percolation mechanism therefore dictates both how the average precision

¹Cross-sectional and time-series momentum are related, but distinct empirical anomalies. Notably, Moskowitz et al. (2012) show that time-series momentum is not fully explained by cross-sectional momentum.

²See Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998) and Hong and Stein (1999).

evolves over time and how individual precisions are distributed across agents.

The exponential increase in average market precision produces momentum. To understand this, notice that at each trading round the market makes an error in forecasting the fundamental value of the asset. As new information becomes available after each trading round, the market revises its own past errors. As an illustration, suppose that the market overvalued the asset yesterday and keeps overvaluing the asset today. Clearly, overvaluation today implies high current returns. Because the market will receive new information tomorrow, today's overvaluation will be partially revised, generating low future returns. Today's market error therefore produces reversal. Consider now yesterday's market error. As the market receives new information today, it revises yesterday's overvaluation, reducing returns today. Because the market becomes even more precise tomorrow, it keeps revising yesterday's overvaluation, thus further reducing returns tomorrow. Past market errors therefore produce momentum. By increasing the average market precision at an exponential rate, information percolation helps generate momentum in two ways: it reduces the reversal effect associated with the current market error and amplifies the momentum effect associated with past market errors. Importantly, this result does not obtain if the precision increase is linear, a customary assumption in rational-expectations models—in these models, the revision of the current market error always dominates, producing reversals.

The heterogeneity in individual precisions causes investors to implement heterogeneous trading strategies. Specifically, investors who gather little information observe momentum in prices and trade on it. In contrast, agents who gather large amounts of information build an aggressive position on the stock mispricing today, which they expect to unwind tomorrow as the stock price corrects. Doing so, these contrarian investors effectively “front run” momentum traders. Hence, while everyone (including the econometrician) observes momentum, not everyone is a momentum trader—momentum persists in the presence of momentum traders because better informed investors trade against it. To emphasize this result, which is key to our theory of momentum, notice that in the absence of information percolation, all investors in our model are market neutral.

Our theory generates predictions that are consistent with several empirical findings. For instance, in our model the cost of capital is decreasing in the precision of information. This prediction finds support in the accounting literature (Lambert, Leuz, and Verrecchia, 2011). Furthermore, information percolation creates a hump-shaped pattern of momentum, similar to that documented by Hong, Lim, and Stein (2000). We also show that our model can simultaneously generate short-term momentum and long-term reversal (Moskowitz et al., 2012). Additionally, our model suggests that momentum measures should include several lags, as in Novy-Marx (2012), who finds that momentum is primarily driven by intermediate

past returns. Finally, our theory is consistent with the empirical finding that stock returns exhibit strong reversals at shorter horizons (Jegadeesh, 1990; Lehmann, 1990).

We extend our benchmark model along three dimensions. A first extension is based on the idea that word-of-mouth communication is a natural propagator of rumors (Shiller, 2000). When private information contains a rumor, this rumor circulates among investors, who are aware of its existence but cannot observe it, creating a disconnect between the stock price and the fundamental. Ultimately, the rumor subsides, leading to a price reversal. Second, while we derive our results in a model with a finite horizon, we show that they carry over to a fully dynamic setup. In particular, momentum obtains whether the asset pays a single liquidating dividend or an infinite stream of dividends. Finally, we introduce a large, unconstrained, risk-neutral arbitrageur who could conceivably eliminate momentum. We find that this is not the case—the arbitrageur must also consider that her trades move prices adversely.

Among the large theoretical literature on momentum, leading rational theories are based on growth-options models (Berk, Green, and Naik 1999; Johnson 2002; Sagi and Seasholes 2007). Our theory abstracts from firm decisions and directly builds on information transmission as a driver of investors’ decisions and thereby of stock returns.³ Previous rational-expectations models (Holden and Subrahmanyam 2002; Cespa and Vives 2012) suggest that an increase in information precision generates momentum. Our model offers a unified explanation for short-term momentum, long-term reversal and the persistence of momentum, despite the presence of investors who profitably trade on it. As in Biais, Bossaerts, and Spatt (2010), investors in our model follow different investment strategies and extract information from prices. Our focus, however, is on the role that word-of-mouth communication plays in generating momentum and reversal.⁴ We adopt the definition of “price drift”, as well as portfolio

³We believe that private exchange of information is linked to momentum for several reasons. First, private information is an important driver of stock price variations (French and Roll, 1986) and provides an incentive for investors to implement heterogeneous trading strategies. Public news, instead, do not predict prices (Roll, 1988), nor do they explain price changes (Chan, Fong, Kho, and Stulz, 1996) or generate trading heterogeneity (Gropp and Kadareja, 2007; Koudijs, 2010; Mitchell and Mulherin, 1994; Tetlock, 2010). Second, word-of-mouth communication is an innate channel of information processing, “*a central part of economic life*” (Stein, 2008), and plays an important role in stock market fluctuations and in investors’ decisions (Brown, Ivkovic, Smith, and Weisbenner, 2008; Cohen, Frazzini, and Malloy, 2008; Feng and Seasholes, 2004; Grinblatt and Keloharju, 2001; Hong, Kubik, and Stein, 2004; Ivkovic and Weisbenner, 2005; Shiller, 2000; Shiller and Pound, 1989; Shive, 2010). Furthermore, evidence indeed suggests that word-of-mouth communication is related to momentum: momentum profits are decreasing in analyst coverage, supporting the notion that momentum is caused by slow information diffusion (Hong et al., 2000; Hou and Moskowitz, 2005; Verardo, 2009).

⁴Biais et al. (2010) build a dynamic model in which momentum may arise depending on the relative persistence of the fundamental and noise trading risk. Instead, in our model, persistence arises endogenously through the acceleration of information through word-of-mouth communication among investors. Other papers related to momentum, but unrelated to social interactions and information diffusion, include Albuquerque and Miao (2014), Vayanos and Woolley (2010), Makarov and Rytchkov (2009), and Wang (1993).

decompositions from Banerjee, Kaniel, and Kremer (2009), who show that stock returns can exhibit momentum when investors have higher order differences of opinions. Finally, among well-established behavioral explanations for momentum, Barberis et al. (1998), Daniel et al. (1998) and Hong and Stein (1999) are related to this paper. Our study provides an alternative explanation in which agents are rational, make profits on average, learn from prices, and optimally decide whether or not to trade on momentum. The mechanism analyzed in this paper could be amplified by behavioral biases, which makes our work complementary to existing behavioral theories of momentum.

The remainder of the paper is organized as follows. Section 2 presents and solves the model, Sections 3 and 4 contain the main results on momentum and momentum trading, Section 5 presents extensions of the model and Section 6 concludes. Derivations and computational details are relegated to the Appendix.

2 Information Percolation in Centralized Markets

In this section, we build a model of *centralized* trading (a noisy rational expectations equilibrium) with *decentralized* information gathering (information percolation). We start by describing the information diffusion mechanism.

2.1 Information Percolation

Consider an economy with T trading dates, indexed by $t = 0, 1, \dots, T-1$, and a final liquidation date, T . The economy is populated by a continuum of investors indexed by $i \in [0, 1]$. There is a risky security with payoff \tilde{U} realized at the liquidation date. The payoff of this security is *unobservable* and follows a normal distribution with zero mean and precision H .⁵

Immediately prior to each trading session, each investor i obtains a private signal about the asset payoff, \tilde{z}_t^i :

$$\tilde{z}_t^i = \tilde{U} + \tilde{\epsilon}_t^i \tag{1}$$

where $\tilde{\epsilon}_t^i$ is distributed normally and independently of \tilde{U} , has zero mean, precision S , and is independent of $\tilde{\epsilon}_j^k$ if $k \neq i$ or $j \neq t$. The precision of individual private signals is constant over time and is the same across investors.

We now introduce a mechanism whereby information diffuses over time and agents become heterogeneous in terms of their precision of information. To do so, we use the information

⁵We refer to the precision of a random variable as the inverse of its variance. The zero mean assumption is without loss of generality.

percolation theory (Duffie, Malamud, and Manso, 2009). From date $t = 0$ onward, agents meet each other randomly and share their information. Meetings take place at Poisson arrival times with intensity λ —the *only* parameter we add to this standard equilibrium model.

When agents meet, they exchange their initial signal and other signals that they received during previous meetings (if any). Agents are infinitesimally small and therefore are indifferent between telling the truth or lying—if they attempt to lie, they will not be able to move prices, and therefore will not benefit from their lies. For this reason, we assume that they tell the truth. Moreover, because signals are normally distributed, an agent’s private information is completely summarized by two statistics: her total number of signals and her posterior expectation of the fundamental. These two statistics are what agents actually exchange when they meet and “talk.”

To illustrate how information percolation works, pick two agents—say D and J —out of the crowd. D and J start with one signal. Suppose the first time they meet someone, they meet each other. They exchange their signals truthfully and therefore end up with two signals after the meeting. Suppose further that D meets someone else, say M , who also has two signals (i.e., M also met someone before). Since D and M are part of an infinite crowd of agents, the person that M has met cannot be J , it must be someone else, i.e., meetings do not overlap.⁶ Hence, after the meeting, D and M both part with four signals each. Signals keep on adding up randomly in the exact same way for every agent in the economy.

Random meetings introduce heterogeneity in information precisions: while agents start off holding one signal, they end up with different numbers of signals as soon as they meet each other. This heterogeneity is captured by the cross-sectional distribution of the number of additional signals, π_t . Formally, between time $t - 1$ and t each agent i collects a number $\omega_t^i \in \mathbb{N}^*$ of signals, *including* the one she received at time t , and *excluding* the signals she received up to time $t - 1$.⁷ Furthermore, an important statistic in this economy is the cross-sectional average of the number of additional signals at time t :

$$\Omega_t \equiv \sum_{\omega_t \in \mathbb{N}^*} \pi_t(\omega_t) \omega_t. \quad (2)$$

⁶In other words, there is a zero probability that the set of agents that D has met before time t overlaps with the set of agents that J has met before time t . This eliminates the concern that we are introducing *persuasion bias* in the terms of Demarzo, Vayanos, and Zwiebel (2003): an agent might share her signals to another agents who passes those signals at subsequent meetings to other agents and maybe the same signals will come back to the first agent—without her knowledge. The infinite mass of agents prevents this double accounting of signals to happen, since the probability for an agent to meet in the future precisely those agents who got her signals is zero. Thus, for every pair (i, j) of agents, their signal sets are always disjoint.

⁷Notice that both the distribution over the total number of signals and the distribution over additional signals may be equivalently used; we choose to use distribution of additional signals because it helps us better separate and understand the effects of information percolation on the equilibrium price and trading strategies.

Since agents are initially endowed with a single signal, the initial distribution of signals has 100% probability mass at $n = 1$, and therefore $\omega_0^i = \Omega_0 = 1, \forall i \in [0, 1]$. As information diffuses (at dates $t > 0$), the distribution π_t takes positive values over \mathbb{N}^* . For example, an agent who did not meet anyone between $t - 1$ and t receives only one additional signal at t and thus is of type $n = 1$. An agent who collected ten signals between $t - 1$ and t receives one more signal at t and thus is of type $n = 11$, and so on. Following Duffie et al. (2009), the cross-sectional distribution of the number of additional signals satisfies

$$\frac{d}{dt}\pi_t(n) = \lambda\pi_t * \mu_t - \lambda\pi_t = \lambda \sum_{m=1}^{n-1} \pi_t(n-m)\mu_t(m) - \lambda\pi_t(n) \quad (3)$$

where $*$ denotes the discrete convolution product and μ represents the cross-sectional distribution of the *total* number of signals, which we define in Appendix A.1. The summation term on the right hand side in (3) represents the rate at which new agents of a given type are created, and the second term in (3) captures the rate at which agents leave a given type.

This setup has the advantage of leading to a closed-form solution for both the cross-sectional distribution π_t and the cross-sectional average Ω_t of the number of additional signals. The solution has a recursive form and is provided in Proposition 1.⁸

Proposition 1. *The probability density function π_t over the additional number of signals collected by each agent between $t - 1$ and t is given by*

$$\pi_t(n) = \sum_{i_1=0}^{n-1} \sum_{i_2=0}^{i_1} \dots \sum_{i_t=0}^{i_{t-1}} \left[\prod_{j=1}^{t-1} \binom{i_j - 1}{i_{j+1} - 1} \mathbb{1}_{\{\sum_{j=1}^t i_j = n-1\}} e^{-\lambda - \sum_{j=1}^t i_j \lambda} (e^\lambda - 1)^{i_1} \right] \quad (4)$$

The cross sectional averages of the number of additional signals for $t \geq 0$ are given by:

$$\Omega_t = e^{\lambda t}. \quad (5)$$

Proof. See Appendix A.1. □

Figure 1 illustrates the evolution of the cross-sectional distribution $\pi_t(n)$ of the number of additional signals for a given value of the meeting intensity λ , at time 1 (upper panel) and time 2 (lower panel). Both the average precision and the precision heterogeneity change over time. First, the mass of the distribution gradually shifts towards larger number of signals. As a result, the average number of signals, and therefore the average precision, increases over time. Second, while the distribution is initially concentrated at 1 (each agent starts off with

⁸Although we present here closed-form solutions for the distribution of signals, we also compute this distribution numerically through inverse Fourier transform. We provide the details on this efficient numerical procedure in Appendix A.1.

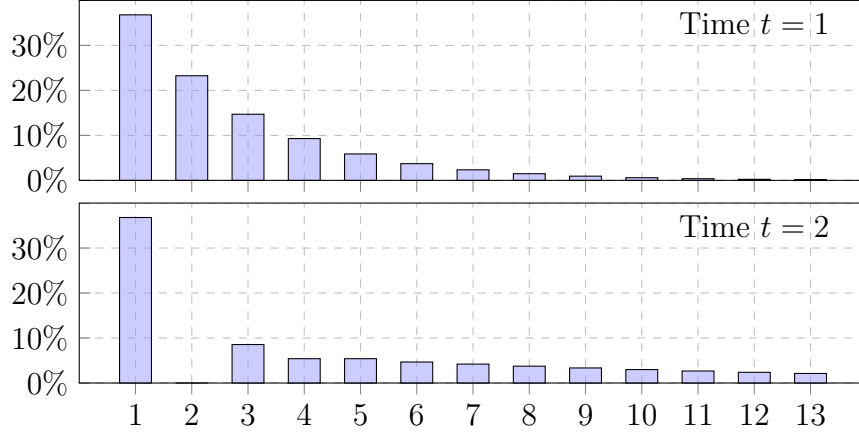


Figure 1: Evolution of Cross-Sectional Densities

The Figure depicts the evolution of the probability density function of the number of additional signals through time when agents are endowed with one signal at each period. Each graph depicts $\pi(\cdot)$ at time $t = 1$ and $t = 2$, respectively, with $\lambda = 1$.

one signal), it rapidly spreads to reflect the growing heterogeneity in precision across the population. This heterogeneity itself varies through time, as shown in Figure 1.⁹

2.2 The Economy

We embed the information percolation mechanism of the previous section within a model of centralized trading (Grossman and Stiglitz, 1980).

Investors have exponential utility with common coefficient of absolute risk aversion $1/\tau$, where τ denotes investors' risk tolerance. The asset payoff is realized and consumption takes place at time $t = T$, while trading takes place at times $t = 0, 1, \dots, T - 1$. Each investor i is endowed at time $t = 0$ with a quantity of the risky asset represented by X^i . At each trading date, investor i chooses a position in the risky asset, \tilde{D}_t^i , to maximize her expected utility of terminal wealth, denoted by \tilde{W}_T^i :

$$\begin{aligned} & \max_{\tilde{D}_t^i} \mathbb{E} \left[e^{-\frac{1}{\tau} \tilde{W}_T^i} \middle| \mathcal{F}_t^i \right] \\ & \text{subject to } \tilde{W}_T^i = X^i \tilde{P}_0 + \sum_{t=0}^{T-2} \left[\tilde{D}_t^i (\tilde{P}_{t+1} - \tilde{P}_t) \right] + \tilde{D}_{T-1}^i (\tilde{U} - \tilde{P}_{T-1}) \end{aligned} \quad (6)$$

The information set of investor i at time t , \mathcal{F}_t^i , contains (i) private signals received at each

⁹Other aspects are worth noting. First, the probability mass at $n = 1$ stays constant for all periods (it equals $e^{-\lambda}$); these are investors who do not meet anyone during the last period and consequently end up with only one additional signal at each trading date. Second, at time 2 there is zero probability mass for $n = 2$. The reason is that no investor could gather 2 additional signals between dates 1 and 2, since that would require meeting an investor with only one signal during that period, and no such investor exists after time 1.

date and collected through information percolation, and (ii) prices (endogenously determined in equilibrium and denoted by \tilde{P}_t) as public signals.¹⁰

The aggregate per capita supply of the risky asset at time $t = 0$, $\tilde{X}_0 = \int_0^1 X^i di$, is normally and independently distributed with zero mean and precision Φ . New liquidity traders enter the market in trading sessions $t = 1, \dots, T - 1$. The incremental net supply of liquidity traders, \tilde{X}_t , is normally distributed with zero mean and precision Φ .

The noisy supply prevents asset prices from fully revealing the final payoff \tilde{U} . We adopt a random walk specification for the noisy supply (or, equivalently, we assume that increments in the noisy supply are *i.i.d.*), i.e., the total supply at time t is $\sum_{j=0}^t \tilde{X}_j$.¹¹ Under this specification and in the absence of additional private information at dates $t > 1$, prices are martingales. As a result, any pattern in the correlation of returns depends *only* on the pattern of private information arrival. That is, our setup allows us to isolate the link between the diffusion of information and the serial correlation of returns.¹²

The solution method for finding a linear, partially revealing rational-expectations equilibrium is standard and is relegated to Appendix A.2. We describe the equilibrium below.

2.3 Equilibrium

We first introduce notation and terminology for further use. At each date t , agent i receives ω_t^i new signals. From Gaussian theory, these signals are equivalent to a single signal with precision $S\omega_t^i$. We denote this signal by \tilde{Z}_t^i :

$$\tilde{Z}_t^i = \tilde{U} + \tilde{\varepsilon}_t^i, \quad \text{where } \tilde{\varepsilon}_t^i \equiv \left(\frac{1}{\omega_t^i} \sum_{j=1}^{\omega_t^i} \tilde{\varepsilon}_t^j \right) \sim N \left(0, \frac{1}{S\omega_t^i} \right) \quad (7)$$

The conditional precision of agent i about the final payoff \tilde{U} , given all available information, is denoted by K_t^i ,

$$K_t^i \equiv \text{Var}^{-1} \left[\tilde{U} \mid \mathcal{F}_t^i \right], \quad (8)$$

¹⁰Our model bears similarities with Brennan and Cao (1997), with the main difference that it embeds an information diffusion mechanism. To keep the setup comparable to leading momentum theories, such as Daniel et al. (1998) and Hong and Stein (1999), we focus on a single asset economy, featuring several trading dates and a final liquidation date.

¹¹*i.i.d.* incremental changes in noisy supply are likely to happen when time between consecutive trading dates is small.

¹²Other specifications, such as an AR(1) noise trading process, give qualitatively similar results, but complicates unnecessarily the analysis.

whereas the cross-sectional average of conditional precisions over the entire population of agents is denoted by K_t ,

$$K_t \equiv \sum_{\omega \in \mathbb{N}^*} K_t^i(\omega) \pi_t(\omega). \quad (9)$$

Throughout the paper, we will often refer to the “average agent” as the agent whose precision of information equals the average precision at time t , K_t . Note that, without information percolation, the average agent represents any agent in the economy, because $K_t^i = K_t$, $\forall i, t$. Finally, we refer to the normalized price signals as

$$\tilde{Q}_t \equiv \tilde{U} - \frac{1}{\tau S \Omega_t} \tilde{X}_t. \quad (10)$$

Observing the signals $\{\tilde{Q}_j\}_{j=0}^t$ or past prices $\{\tilde{P}_j\}_{j=0}^t$ generates equivalent information sets. These information sets represent the information available to an econometrician at time t .

Theorem 1 describes the risky asset prices at each date in a noisy rational expectations equilibrium with information percolation.

Theorem 1. *There exists a partially revealing rational expectations equilibrium in the T trading session economy in which the risky asset price, \tilde{P}_t , for $t = 0, \dots, T - 1$, is given by:*

$$\tilde{P}_t = \frac{K_t - H}{K_t} \tilde{U} - \sum_{j=0}^t \frac{1 + \tau^2 S \Omega_j \Phi}{\tau K_t} \tilde{X}_j. \quad (11)$$

The individual and average precisions, K_t^i and K_t , are given by

$$K_t^i = H + \sum_{j=0}^t S \omega_j^i + \sum_{j=0}^t \tau^2 S^2 \Omega_j^2 \Phi, \quad (12)$$

$$K_t = H + \sum_{j=0}^t S \Omega_j + \sum_{j=0}^t \tau^2 S^2 \Omega_j^2 \Phi. \quad (13)$$

The individual asset demands, \tilde{D}_t^i , are given by

$$\tilde{D}_t^i = \tau K_t^i \left(\mathbb{E}[\tilde{U} | \mathcal{F}_t^i] - \tilde{P}_t \right) \quad (14)$$

$$= \tau \left(\underbrace{S \sum_{j=0}^t \omega_j^i \tilde{Z}_j^i}_{\text{private information}} + \underbrace{\tau^2 S^2 \Phi \sum_{j=0}^t \Omega_j^2 \tilde{Q}_j - K_t^i \tilde{P}_t}_{\text{public information}} \right). \quad (15)$$

Proof. See Appendix A.2. □

The asset price in Equation (11) is a linear function of the final payoff and supply shocks, as is customary in the noisy rational-expectations literature. Without information percolation ($\lambda = 0$), we recover a standard rational-expectations equilibrium: average precision is constant ($\Omega_t = 1, \forall t$) and there is no precision heterogeneity across agents ($K_t^i = K_t, \forall i, t$). With information percolation ($\lambda > 0$), the average precision increases exponentially over time, as can be seen by replacing $\Omega_j = e^{\lambda j}$ in (13). Finally, Equation (15) shows how agents build their demands based on both private and public information.

3 Momentum and Reversal

In this section we analyze the implications of information percolation for the serial correlation of returns. Without new information arriving at each trading date, prices are martingales. Hence, return predictability arises only if information accumulates over time. If information accumulates linearly, returns *always* exhibit reversals, as is customary in rational-expectations models. Information percolation, instead, causes information to accumulate exponentially and returns *always* exhibit momentum beyond a certain threshold of the meeting intensity. The goal of this section is to fully characterize this threshold and relate the pattern of the serial correlation of returns it implies to empirical evidence.

3.1 Predictability of Returns

The following proposition establishes the condition under which future returns are predictable.

Proposition 2. *Agent i 's expectation regarding future returns conditional on \mathcal{F}_t^i satisfies:*

$$\mathbb{E}[\tilde{P}_{t+1} - \tilde{P}_t | \mathcal{F}_t^i] = \underbrace{\frac{K_{t+1} - K_t}{K_{t+1}}}_{\geq 0} (\mathbb{E}[\tilde{U} | \mathcal{F}_t^i] - \tilde{P}_t). \quad (16)$$

Returns are predictable only if average market precision, K , is strictly increasing over time.

Proof. See Appendix A.3.1. □

The first term in Equation (16) represents the relative evolution of average market precision K over time. Clearly, if average market precision is constant, prices are martingales and no agent, even perfectly informed, can predict future returns. If, instead, average market precision increases over time, Equation (16) shows that an agent can predict future returns by comparing the current price she observes to her current expectations of the fundamental.

When she perceives that the stock is overvalued, she predicts negative future returns and when she perceives that the stock is undervalued, she predicts positive future returns.

Future returns can be further decomposed into three main sources of predictability. To perform this decomposition, we denote by

$$\tilde{E}_t \equiv \tilde{Q}_t - \tilde{U} \quad (17)$$

the error in the price signal \tilde{Q} , as defined in (10). We interpret this variable as the error that the market makes at time t in estimating the fundamental: a positive error means that the market overvalues the fundamental and vice-versa. Errors are independent across time and are normally distributed with mean zero and precision $\tau^2 S^2 \Omega_t^2 \Phi$.¹³ We therefore refer to \tilde{E}_t as the “market error” and accordingly obtain the following decomposition of future returns.

Proposition 3. *Stock returns from time t to time $t + 1$ admit the following decomposition:*

$$\tilde{P}_{t+1} - \tilde{P}_t = \underbrace{\left(\bar{\mathbb{E}}_{t+1}[\tilde{U}] - \bar{\mathbb{E}}_t[\tilde{U}] \right)}_{\text{Evolution of Market Consensus}} - \frac{K_{t+1} - K_t}{K_t K_{t+1}} \underbrace{\sum_{j=0}^t S \Omega_j \tilde{E}_j}_{\text{Past and Current Market Errors}} + \frac{S \Omega_{t+1}}{K_{t+1}} \underbrace{\tilde{E}_{t+1}}_{\text{Future Market Error}} \quad (18)$$

where $\bar{\mathbb{E}}_t[\cdot] \equiv \int_i \mathbb{E}[\cdot | \mathcal{F}_t^i] di$ denotes average market expectation at time t .

Proof. See Appendix A.3.2. □

According to Proposition 3, three elements drive future returns: (i) the evolution of the market consensus, (ii) current and past market errors, and (iii) the market error occurring in the future. Notice that no agent, even perfectly informed, can predict future market errors. Hence, predictability must arise through the first two components of returns. In general, agents’ expectations regarding these two components differ, because they have heterogenous information sets. Agents, however, share a common source of information: the history of normalized price signals. This common information set, which we define as

$$\mathcal{F}_t^c = \left\{ \tilde{Q}_j : 0 \leq j \leq t \right\}, \quad (19)$$

is equivalent to the information set of the econometrician, who observes all past prices. From the point of view of the econometrician, the evolution of market consensus is not predictable (see Appendix A.3 for a proof):

$$\mathbb{E} \left[\bar{\mathbb{E}}_{t+1}[\tilde{U}] - \bar{\mathbb{E}}_t[\tilde{U}] \mid \mathcal{F}_t^c \right] = 0. \quad (20)$$

¹³Each market error is exclusively driven by the contemporaneous supply shocks. Hence, market errors are independent across time.

Therefore, observing past prices only, predictability arises exclusively through the inference of the current and past market errors.

Proposition 4. *From the point of view of the econometrician, return predictability arises solely from the inference of current and past market errors:*

$$\mathbb{E} \left[\tilde{P}_{t+1} - \tilde{P}_t \mid \mathcal{F}_t^c \right] = -\frac{K_{t+1} - K_t}{K_t K_{t+1}} \sum_{j=0}^t S\Omega_j \mathbb{E} \left[\tilde{E}_j \mid \mathcal{F}_t^c \right] \quad (21)$$

$$= \frac{K_{t+1} - K_t}{K_{t+1} K_t^c} \sum_{j=0}^t S\Omega_j \left(\tilde{P}_t - \tilde{Q}_j \right), \quad (22)$$

where we denote by $K_t^c \equiv H + \sum_{j=0}^t \tau^2 \Phi S^2 \Omega_j^2$ the precision of the econometrician.

Proof. See Appendix A.3.3. □

To understand how returns become serially correlated in our model, use Proposition 3 to write current returns as

$$\tilde{P}_t - \tilde{P}_{t-1} = \mathbb{E}_t[\tilde{U}] - \mathbb{E}_{t-1}[\tilde{U}] - \frac{K_t - K_{t-1}}{K_{t-1} K_t} \sum_{j=0}^{t-1} S\Omega_j \tilde{E}_j + \frac{S\Omega_t}{K_t} \tilde{E}_t. \quad (23)$$

Inspecting Equations (18) and (23), we observe that the current market error, \tilde{E}_t , affects current and future returns in opposite directions, inducing reversal in stock returns. The reason is that the market revises its own past errors as new information becomes available after each trading round t . Past market errors, $(\tilde{E}_j)_{j=0}^{t-1}$, instead, produce momentum in stock returns as they continue to be revised in the future. As the number of additional signals increases, the impact of current market errors becomes increasingly smaller, whereas the impact of the revision of past market errors becomes increasingly larger. That is, an increase in the number Ω of additional signals helps generate momentum in two ways: it reduces the reversal effect associated with current market errors and magnifies the momentum effect associated with past market errors.¹⁴

The relations in Equations (21)-(21) are not based on a standard notion of past returns and therefore do not clearly identify momentum or reversal. Hence, we follow the convention introduced by Banerjee et al. (2009) and condition future returns on past returns, as opposed to the common information set \mathcal{F}^c .¹⁵ Letting $\tilde{P}_{-1} = \Omega_{-1} \equiv 0$, we define the information set

¹⁴A similar intuition can be derived in terms of supply shocks by replacing (10) in (17): the current supply shock produces a reversal, whereas past supply shocks continue to be absorbed by the market, producing momentum. An increase in precision weakens the reversal effect and strengthens the momentum effect.

¹⁵Note that these two information sets are observationally equivalent, but only by conditioning on the information set \mathcal{F}_t^r are we able to conclude whether returns exhibit momentum or reversal.

containing past returns as

$$\mathcal{F}_t^r = \left\{ \tilde{P}_{t-l+1} - \tilde{P}_{t-l} : 1 \leq l \leq t+1 \right\}. \quad (24)$$

Since this information set is equivalent to \mathcal{F}^c , we obtain from Equation (22) an expression for the serial correlation of returns at different lags, which we describe in Proposition 5.

Proposition 5. *Conditional on past returns, expected future returns satisfy*

$$\mathbb{E} \left[\tilde{P}_{t+1} - \tilde{P}_t \mid \mathcal{F}_t^r \right] = \sum_{l=1}^{t+1} \underbrace{\frac{K_{t+1} - K_t}{K_{t+1} K_t^c} m_{t-l}}_{\substack{\text{Serial Correlation of Returns} \\ \text{at the } l\text{-th Lag}}} \left(\tilde{P}_{t-l+1} - \tilde{P}_{t-l} \right), \quad (25)$$

where the coefficients m_{t-l} are defined as:

$$m_{t-l} \equiv \frac{\tau^2 S \Omega_{t-l+1} \Phi}{1 + \tau^2 S \Omega_{t-l+1} \Phi} K_{t-l+1} - K_{t-l+1}^c. \quad (26)$$

Proof. See Appendix A.3.4. □

Proposition 5 shows that the sign of the serial correlation of stock returns at the l -th lag (equivalently, the sign of the coefficient m_{t-l}) depends on the average number of additional signals received from time $t-l$ to $t-l+1$. As a result, while an increase in average precision is sufficient to make returns predictable (Proposition 2), this increase needs to be *sufficiently* large to induce momentum in stock returns. In Corollary 1, we derive the increase in precision that is necessary for momentum to arise.

Corollary 1. *Returns exhibit momentum at the l -th lag if and only if the average number of additional signals received at time $t-l+1$, Ω_{t-l+1} , satisfies*

$$\Omega_{t-l+1} > \frac{K_{t-l+1}^c}{\tau^2 \Phi S (K_{t-l+1} - K_{t-l+1}^c)}. \quad (27)$$

At lag $l = t+1$, the serial correlation of returns is always negative, or zero if $t \rightarrow \infty$:

$$m_{-1} = -\frac{K_{t+1} - K_t}{K_{t+1} K_t^c} \frac{H}{1 + \tau^2 \Phi S \Omega_0} \leq 0, \quad \lim_{t \rightarrow \infty} m_{-1} = 0 \quad (28)$$

Proof. See Appendix A.3.5. □

We now study the role of information percolation in enforcing the condition (27) and thus in generating momentum.

3.2 Information Percolation and Momentum

We formalize the effect of information percolation on the serial correlation of returns in Theorem 2.

Theorem 2. *For each finite horizon $t - l \geq 0$,*

1. *There exists a unique threshold, $\lambda^*(H, S, \Phi, \tau, t - l) \in \left(0, \log\left(\frac{H + \Phi\tau^2 S^2}{\Phi\tau^2 S^2}\right)\right]$, of the meeting intensity above which stock returns always exhibit momentum.*
2. *This threshold is increasing in H and decreasing in S, Φ, τ and $t - l$.*
3. *As a function of the meeting intensity, the serial correlation of returns is hump-shaped in the momentum region, $\lambda \in [\lambda^*, \infty)$.*

Proof. See Appendix A.4. □

For lower values of the meeting intensity, $\lambda < \lambda^*(H, S, \Phi, \tau, t - l)$, returns always exhibit reversal at horizon $t - l$. This result, which commonly obtains in noisy rational expectations models, originates from inventory considerations (Grossman and Miller, 1988): because risk-averse informed investors act as market makers and accommodate the noninformational demand of noise traders, they require a risk premium for holding the asset. A positive supply shock today (i.e., noise traders sell the stock) then simultaneously decreases the stock price today and increases the risk premium for holding a larger supply of the asset. As a result, returns exhibit reversal. Alternatively, the same phenomenon can be described using market errors: a revision of the current market error tomorrow generates a reversal effect, which, in this case, dominates the persistent effect of the revision of past market errors.

For values of the meeting intensity above the threshold $\lambda^*(H, S, \Phi, \tau, t - l)$, information percolation creates an increase in market precision that is sufficient to enforce the momentum condition (27) and thus to overturn the reversal effect commonly obtained in rational-expectations models. The reason is that information percolation creates an exponential increase in the average number of additional signals, $\Omega_t = e^{\lambda t}$, as opposed to the linear increase, which prevails in the absence of information percolation. Proposition 4 provides the economic intuition for this result: by increasing the average market precision, information percolation significantly improves learning, which in turn reduces the reversal effect of the current market error (by reducing the risk of holding the asset) and strengthens the momentum effect of all past market errors (by strengthening their revision).

The negative relationship between the precision of information and the risk premium is consistent with empirical findings. In particular, there is consensus in the accounting

literature that increasing the precision of information reflected in prices decreases the cost of capital (Lambert et al., 2011).¹⁶

The second part of Theorem 2 shows how the threshold λ^* reacts to changes in the parameters of the model. While an increase in fundamental precision requires information percolation to play a larger role in generating momentum, the other parameters have the opposite effect. For instance, decreasing noise trading or decreasing the risk aversion help information percolation generate momentum. To understand this, notice that less noise trading reduces the magnitude of momentum by allowing more information to be revealed through prices. However, less noise trading also decreases the risk premium and the reversal effect associated with it, thus helping information percolation to generate momentum. The same reasoning applies to risk aversion and the precision of individual signals.

The momentum threshold decreases with the horizon $t - l$ and therefore increases with the lag l . Hence, as we increase the lag, we need a higher meeting intensity to generate momentum, i.e., $\lambda^*(\cdot, t - l - 1) > \lambda^*(\cdot, t - l)$. An immediate consequence of this result, which is empirically appealing, is that there always exists a meeting intensity $\lambda \in (\lambda^*(\cdot, t - l), \lambda^*(\cdot, t - l - 1))$, such that we simultaneously obtain short-term momentum and long-term reversal. A second consequence is that the serial correlation of returns in Equation (25) decays with the lag in the momentum region, thus generating a downward-sloping term structure of momentum.

The last part of Theorem 2 shows that momentum is hump-shaped in the meeting intensity. To see this, pick the momentum threshold as the meeting intensity, $\lambda = \lambda^*(\cdot, t - l)$. At this point, the reversal effect associated with current market errors exactly offsets the momentum effect associated with the revision in past market errors. An increase in the meeting intensity beyond the momentum threshold, λ^* , weakens the reversal effect and strengthens the momentum effect, thereby creating an increasing relation between momentum and the meeting intensity. As the meeting intensity becomes infinite, not only does the reversal effect die out, but the momentum effect also disappears, because past market errors completely vanish. As a result, the relation between momentum and the meeting intensity becomes decreasing as the meeting intensity increases, resulting in a hump-shaped pattern.

To illustrate the different points of Theorem 2, we plot in Figure 2 the serial correlation of returns as a function of the meeting intensity for different lags. In the absence of information percolation, stock returns exhibit reversals, as indicated in Figure 2.¹⁷ As information percolation intensifies and the meeting intensity rises above the thresholds of Theorem 2, stock returns become positively autocorrelated (the momentum thresholds are indicated in

¹⁶See also Botosan, Plumlee, and Xie (2004), Francis, LaFond, Olsson, and Schipper (2005), and Amir and Levi (2014).

¹⁷Without percolation, information accumulates at a linear rate and the serial correlation coefficients are negative and have the same value at all lags. See Appendix A.3.4, Equation (A.105).

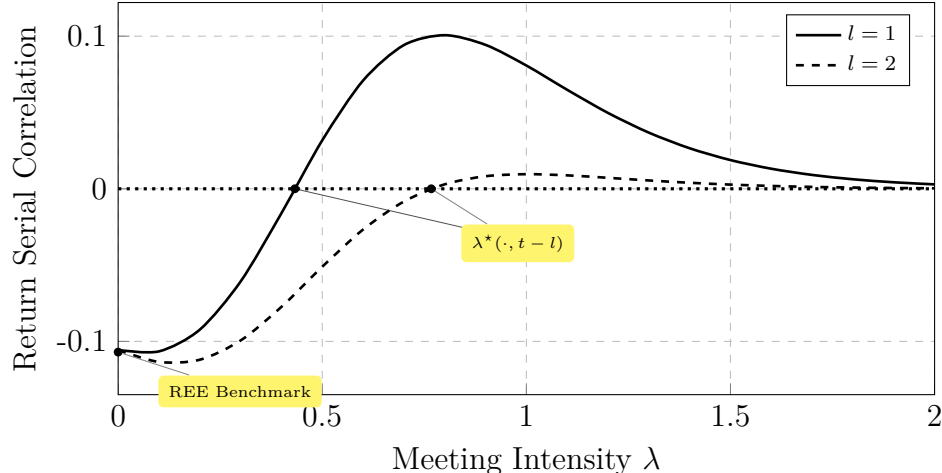


Figure 2: Information Percolation and Serial Correlation in Returns

The figure depicts the serial correlation of returns as function of the meeting intensity λ . Serial correlation is represented at time $t = 3$ for two different lags. The solid line corresponds to lag $l = 1$, and the dashed line to lag $l = 2$. The calibration used for the illustration is $H = S = \Phi = 1$ and $\tau = \frac{1}{3}$.

Figure 2). Furthermore, the momentum thresholds increase with the lag (the second point of Theorem 2), while the magnitude of momentum decays with the lag. Finally, for larger values of the meeting intensity, the effect of past market errors gradually disappears and the magnitude of momentum therefore decreases (the last part of Theorem 2).

3.3 Model Predictions and Empirical Evidence

Our model predicts a hump-shaped relation between momentum and the meeting intensity λ (Theorem 2). This prediction is consistent with Hong et al. (2000), who use firm size as a proxy for the speed of information diffusion and document a hump-shaped relation between firm size and the magnitude of momentum.¹⁸ Specifically, Hong et al. (2000) find reversals for the smallest firms. Our model is in line with their conjecture that these reversals are driven by liquidity shocks.¹⁹ Furthermore, the relation between the meeting intensity and momentum in our model mimics their empirical relation between momentum and firm size.

Another empirical finding is that the magnitude of momentum is large for short lookback periods (one to six months) and decays as the lookback period increases, with weaker evidence of reversal for periods longer than 12 months (Moskowitz et al., 2012).²⁰ Information

¹⁸Note that this empirical finding is related to cross-sectional momentum, whereas our theoretical study is about time-series momentum. These two forms of momentum are strongly related but not identical. See Moskowitz et al. (2012) for a discussion.

¹⁹See also French and Roll (1986) for further evidence on this fact.

²⁰Moskowitz et al. (2012) find that the magnitude of momentum is large for short lookback periods (one

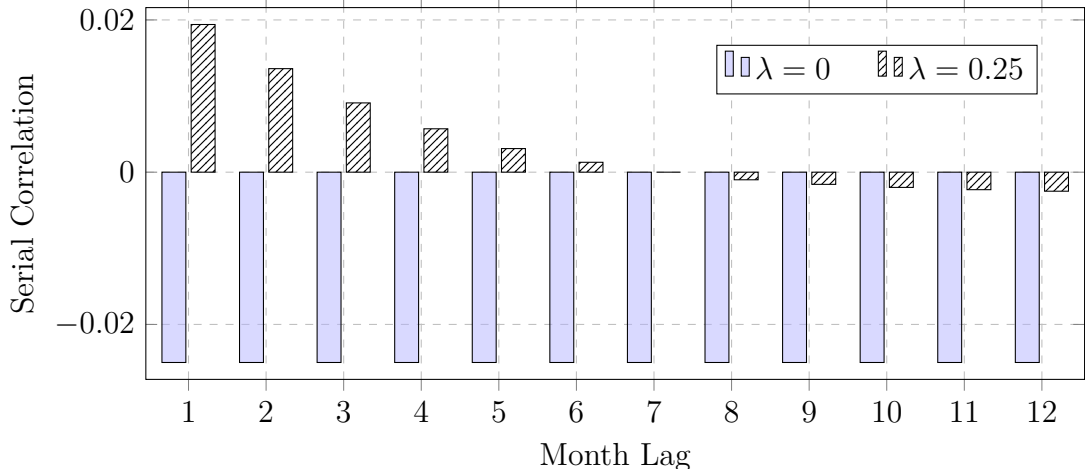


Figure 3: Term Structure of Momentum

The figure depicts the serial correlation of returns when the lookback period varies from one to twelve months, as in (29)-(30). There are two sets of bars, one corresponding to $\lambda = 0$ and the second to $\lambda = 0.25$. The calibration is $H = S = \Phi = 1$, and $\tau = \frac{1}{3}$.

percolation bears similar time-series implications. To see this, note that the specification of Proposition 5 can also be written for different lookback periods:

$$\mathbb{E} \left[\tilde{P}_{t+1} - \tilde{P}_t \mid \mathcal{F}_t^r \right] = \sum_{l=1}^t \frac{K_{t+1} - K_t}{K_{t+1} K_t^c} m'_{t-l} (\tilde{P}_t - \tilde{P}_{t-l}) + m_{-1} \tilde{P}_0, \quad (29)$$

where m_{-1} is defined in (28) and

$$m'_{t-l} \equiv K_{t-l} \left(\frac{1}{1 + \tau^2 S \Omega_{t-l} \Phi} - \frac{1}{1 + \tau^2 S \Omega_{t-l+1} \Phi} \right) - \frac{H}{1 + \tau^2 \Phi S \Omega_0}. \quad (30)$$

Equation (29) indicates that momentum also arises for larger lookback windows. In particular, combining Equations (28) and (30) suggests a decaying “term-structure” of momentum, whereby returns exhibit momentum for short lookback periods and reversal over longer lookback periods. To illustrate this decaying pattern of momentum, we plot the serial correlation in Equations (29)-(30) as a function of the lookback window in Figure 3. Without information percolation, returns exhibit reversals at all horizons (the solid blue bars). Information percolation instead generates short-run momentum and long-run reversals (the dashed bars), consistent with the finding of Moskowitz et al. (2012).

Another way of measuring momentum is proposed by Novy-Marx (2012): keeping the

to six months) and usually decays as the lookback period increases, with weak or no evidence of momentum for periods longer than 12 months. Not all asset classes display the same decaying pattern. The pattern is decaying for commodities, equities, and currencies and U-shaped for other asset classes.

lookback window constant, one instead increases the lag. This way of measuring momentum actually corresponds to our momentum measure in Proposition 5. In our model, we use all past returns to obtain a complete description of momentum. Ignoring lags leads to an omitted-variable bias and could potentially result in overestimating the magnitude of time-series momentum.²¹ Our model therefore suggests that momentum measures should include several lags, as in Novy-Marx (2012), who finds that momentum is primarily driven by intermediate past returns (i.e. the second lag return).

Finally, stock returns exhibit strong reversals at frequencies less than one month, as shown by Lehmann (1990) and Jegadeesh (1990). In our model, the amount of information that agents accumulate depends on the time elapsed between trading rounds—the longer the time within trading rounds is, the more information agents accumulate through random meetings. Consequently, fixing the meeting intensity, our model predicts that the sign of serial correlation varies for different trading frequencies. While the information percolation mechanism is strong enough to generate momentum at lower trading frequencies, agents have little time to talk between trading rounds at high trading frequencies and short-term reversal therefore prevails at high trading frequencies.

4 Trading Strategies

In this section we analyze investors’ trading strategies. We first decompose investors’ demand into two components, a short-term and long-term component. We show that information percolation induces investors to trade on short-term price moves, as opposed to long-term fundamentals. We then show that information percolation generates heterogeneity in precision, which induces better informed investors to front run those lesser informed. Specifically, better informed investors act as “profit takers”, while lesser informed investors follow the public opinion. As a result, better informed investors systematically trade against the serial correlation of returns: when returns exhibit momentum, better informed investors are contrarians, while lesser informed investors are momentum traders. In contrast, with homogeneous precisions, all agents are market neutral in the eyes of the econometrician.

²¹Intuitively, consider the following relation:

$$\Delta\tilde{P}_{t+1} = b\Delta\tilde{P}_t + c\Delta\tilde{P}_{t-1} + \eta \tag{31}$$

and assume that b and c are positive (returns exhibit momentum). Assume further that $\Delta\tilde{P}_{t-1} = f\Delta\tilde{P}_t + \eta'$. Because returns exhibit momentum, f is also positive. Omitting $\Delta\tilde{P}_{t-1}$ in (31) yields

$$\Delta\tilde{P}_{t+1} = (b + cf)\Delta\tilde{P}_t + (\eta + c\eta') \tag{32}$$

thus the momentum coefficient is over-estimated. The same argument holds if the omitted variable is $\Delta\tilde{P}_t$.

By trading at the expense of momentum traders, contrarians optimally allow momentum to persist, despite the existence of momentum traders. This result is key to our theory of momentum: while an exogenous increase in average market precision is sufficient to generate momentum (Holden and Subrahmanyam, 2002), investors do not trade on it if they have homogeneous precisions. But what makes momentum a puzzle is that it persists, despite the presence of momentum traders. Our model offers a potential answer to this puzzle based on heterogeneity in individual precisions—momentum survives in the presence of momentum traders because better informed investors trade against it.

4.1 Predictability of Trading Strategies

We start by decomposing investors' trading strategies in (15) into two components, which we highlight in Proposition 6.

Proposition 6. *At date t , agent i 's optimal demand is given by*

$$\tilde{D}_t^i = \frac{\tau K_t^i}{K_t} \left(\underbrace{\frac{K_t^2}{K_{t+1}} (\mathbb{E}[\tilde{U} | \mathcal{F}_t^i] - \tilde{P}_t)}_{\text{Long-Term Position}} + K_t \underbrace{(\mathbb{E}[\tilde{P}_{t+1} | \mathcal{F}_t^i] - \tilde{P}_t)}_{\text{Short-Term Position}} \right). \quad (33)$$

Proof. See Appendix A.5.1. □

Agent i 's optimal demand is the product of two terms. The first term, $\tau K_t^i / K_t$, shows that each agent i compares the precision of her own information with the average market precision and trades more aggressively when her precision is higher. The second term (in brackets) has the same structure for all agents and consists of two components (see also Banerjee et al. (2009) for a similar decomposition):

- (1) A *long-term position*, reflecting the agent's view about the long-term payoff.
- (2) A *short-term position*, reflecting the agent's view about short-term price moves.

When average market precision remains constant over time (i.e., $K_{t+1} = K_t$), prices are martingales (see Proposition 2) and agents' short-term position drops out of (33): because the price tomorrow does not contain more information than the price today, agents focus on their long-term view of the fundamental. With information percolation, in contrast, the average market precision increases exponentially over time. The price tomorrow therefore incorporates increasingly precise information about the fundamental, causing investors' short-term position to dominate their demand (in the extreme case whereby agents collect an infinite amount

of information between t and $t + 1$, their optimal demand at time t becomes myopic and consists of their short-term position exclusively). Hence, the decomposition in (33) shows that information percolation, by generating an increase in average market precision, induces investors to optimally adopt trading strategies based on short-term views. This short-term trading activity arises as investors anticipate an increase in average market precision next period, K_{t+1} , which they can perfectly predict.²²

We now analyze how investors expect to trade in the future. To do so, we first establish in Proposition 7 a key relationship necessary to understand investors' trading behavior.

Proposition 7. *The portfolio of any agent i in the economy, rescaled by the inverse of her relative precision, K_t/K_t^i , is a martingale:*

$$\mathbb{E} \left[\frac{K_{t+1}}{K_{t+1}^i} \widetilde{D}_{t+1}^i \middle| \mathcal{F}_t^i \right] = \frac{K_t}{K_t^i} \widetilde{D}_t^i. \quad (34)$$

Proof. See Appendix A.5.2. □

Proposition 7 has two important implications, which we present as corollaries. First, an agent whose current precision coincides with average market precision, K_t , cannot predict how she will trade next period.

Corollary 2. *If $K_t^i = K_t$ then*

$$\mathbb{E} \left[\widetilde{D}_{t+1}^i - \widetilde{D}_t^i \middle| \mathcal{F}_t^i \right] = 0. \quad (35)$$

As a consequence, the average agent is neutral to the market: she is neither a momentum trader nor a contrarian.²³ This agent therefore serves as a useful benchmark when analyzing the heterogeneity of trading strategies generated by information percolation.

Second, the expected trading strategy of any agent i conditioned on the common information set, \mathcal{F}^c , can be written as follows.

Corollary 3. *The trading strategy of agent i , as measured by the econometrician, satisfies*

$$\mathbb{E} \left[\widetilde{D}_{t+1}^i - \widetilde{D}_t^i \middle| \mathcal{F}_t^c \right] = \tau \frac{(K_t - K_t^i)(K_{t+1} - K_t)}{K_{t+1}} \left(\mathbb{E}[\widetilde{U} | \mathcal{F}_t^c] - \widetilde{P}_t \right). \quad (36)$$

Proof. See Appendix A.5.2. □

²²The cross-sectional average of investors' precision for the next period, K_{t+1} , is known at time t , because it is just a function of time. In other words, investors know today how precise they will be *on average* next period, although they do not know what their *individual* precision will be next period.

²³One can see this by applying the law of iterated expectations on (35) and conditioning on the last period price move, $\widetilde{P}_t - \widetilde{P}_{t-1}$. This directly implies that $\text{cov}(\widetilde{D}_{t+1}^i - \widetilde{D}_t^i, \widetilde{P}_t - \widetilde{P}_{t-1}) = 0$.

From now on we adopt the point of view of the econometrician and describe agents' trading strategies with respect to the common information set \mathcal{F}_t^c , under which strategies are comparable directly. The econometrician can predict how agent i trades only if average market precision improves over time *and* if agent i 's precision differs from the average market precision. In particular, better informed agents ($K_t^i > K_t$) trade against the “public opinion”, as measured by the term $(\mathbb{E}[\tilde{U}|\mathcal{F}_t^c] - \tilde{P}_t)$, whereas less informed agents follow the public opinion. We draw two conclusions from this observation. First, a model in which all agents have the same precision does not produce predictable trading—even though an exogenous increase in precision can generate momentum (Corollary 1), no one would trade on it. Second, heterogeneous precisions create an additional layer of trading activity, whereby informed agents not only trade against noise traders, but also trade against less informed traders.

That better informed agents trade against lesser informed agents can be interpreted as a competitive form of “front running”. To illustrate this, suppose that the public opinion today is that the stock is undervalued, $\mathbb{E}[\tilde{U}|\mathcal{F}_t^c] > \tilde{P}_t$. All agents then buy the stock today—the better informed they are, the more aggressively they buy.²⁴ Equation (36) in turn indicates that lesser informed investors expect to further increase their position tomorrow, thus following the public opinion (their trades tomorrow are positively correlated with the public opinion today). In contrast, while better informed investors build a large position today, they expect to partly unwind it tomorrow at the expense of the lesser informed agents, thus acting as “profit takers”. We emphasize, however, that this “front-running” behavior is not strategic, since investors, who belong to a continuum, do not have price impact in our model.

4.2 Information Percolation and Momentum Trading

To identify trend-following and contrarian strategies, we follow the convention introduced by Brennan and Cao (1997). This approach is consistent with the convention we adopted to measure momentum: we condition future trading strategies on the information set containing past returns, \mathcal{F}_t^r . We provide this trading measure in Proposition 8.

Proposition 8. *Conditional on past returns, the expected trading strategy of investor i from time t to $t + 1$ satisfies*

$$\mathbb{E} \left[\tilde{D}_{t+1}^i - \tilde{D}_t^i | \mathcal{F}_t^r \right] = \sum_{l=1}^t \underbrace{\tau(K_t - K_t^i) \frac{K_{t+1} - K_t}{K_{t+1} K_t^c} m_{t-l}}_{\text{Momentum Trading Coefficient}} (\tilde{P}_{t-l+1} - \tilde{P}_{t-l}), \quad (37)$$

²⁴To see this, notice that $\tilde{D}_t^i = \tau K_t^i (\mathbb{E}[\tilde{U}|\mathcal{F}_t^i] - \tilde{P}_t)$. Thus, $\mathbb{E}[\tilde{D}_t^i | \mathcal{F}_t^c] = \tau K_t^i (\mathbb{E}[\tilde{U}|\mathcal{F}_t^c] - \tilde{P}_t)$, and therefore if $\mathbb{E}[\tilde{U}|\mathcal{F}_t^c] - \tilde{P}_t > 0$ investors buy the stock today.

where the coefficients m_{t-l} are defined in Proposition 5.

Proof. See Appendix A.5.3. □

An investor’s trading behavior is tightly connected to the serial correlation of returns: Equation (5) shows that the trading coefficient of an investor i is the serial correlation of returns multiplied by a factor $\tau(K_t - K_t^i)$, measuring how investor i ’s precision compares to average market precision. As a result, better informed investors trade systematically against the serial correlation of returns: when returns exhibit reversals they are momentum traders and when returns exhibit momentum they are contrarians. The opposite mechanism applies for lesser informed investors. This trading behavior is consistent with the front-running pattern we previously discussed: better informed agents speculate against the public opinion and front run the trades of the lesser informed agents, who, they expect, trade on momentum.

To illustrate these points, we plot in Figure 4 the “trading coefficient” at lag $l = 1$, as a function of the meeting intensity and for two investor types: (i) the 5% percentile *least informed investor* (solid line) and (ii) the 95% percentile *best informed investor* (dashed line). The area between the lines therefore captures 90% of the investor population. In the absence of information percolation, all investors are neutral. Because they have information with identical precision, they are neither momentum traders nor contrarians. For positive values of the meeting intensity, optimal trading strategies differ through the diversity of informational advantages. Better informed investors are momentum traders in the reversal region and contrarians in the momentum region, whereas the opposite holds for the lesser informed investors. Finally, the spectrum of trading strategies expands as the magnitude of momentum increases and contracts as the magnitude of momentum decreases.

These trading patterns are consistent with empirical evidence. In particular, specialists and commercial investors, or hedgers—who presumably have a broad experience on how the market operates—are contrarians and liquidity providers (Hendershott and Seasholes, 2007; Moskowitz et al., 2012). In contrast, mutual funds—who presumably have little knowledge of the market and delegate their portfolio—chase past performance and further exacerbate market anomalies (Akbas, Armstrong, Sorescu, and Subrahmanyam, 2014; Lou, 2009). Finally, Kelley and Tetlock (2013) observe that informed retail trades predict returns, all the more so in markets with higher investor heterogeneity, consistent with our idea that heterogeneity is a key element in understanding return predictability.

Overall, our model provides an explanation to the puzzling observation that time-series momentum persists, even though investors trade on it (Moskowitz et al., 2012). In our setup, better informed individuals trade systematically against the serial correlation of returns, front running the lesser informed agents. Conversely, the main force that could eliminate

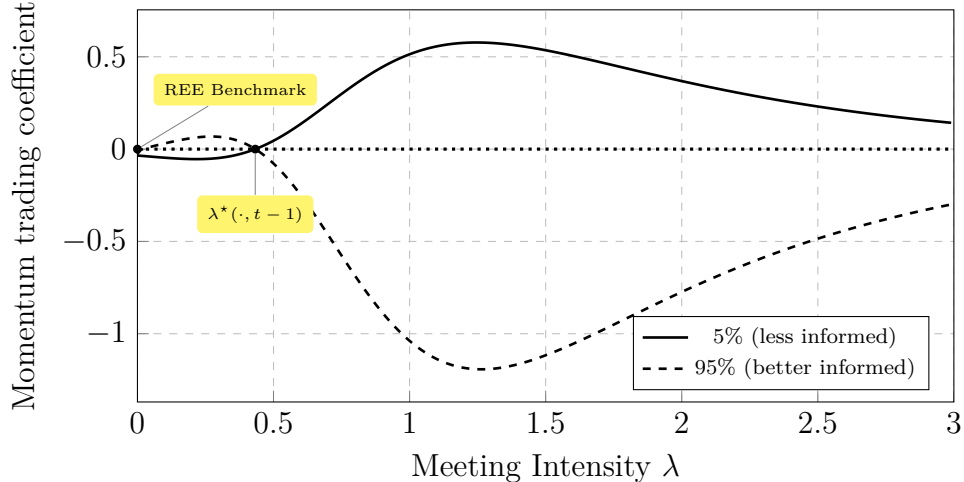


Figure 4: Information Percolation and Momentum Trading

The figure depicts the momentum trading coefficient from Equation (37) as a function of the meeting intensity λ . A positive coefficient means momentum trading, whereas a negative coefficient means contrarian trading. The solid line corresponds to the 5% percentile less informed investor, and the dashed line to the 95% percentile better informed investor. Thus, the area between the line represents 90% of the investor population. The coefficient is represented at time $t = 3$ with one lag, $l = 1$. The calibration used for the illustration is $H = S = \Phi = 1$ and $\tau = \frac{1}{3}$.

momentum—the lesser informed investors—is also the weakest one. A potential caveat is that an unconstrained, risk-neutral arbitrageur could enter the market and conceivably eliminate momentum. We consider this possibility in Appendix A.6 and show that this arbitrageur must necessarily impact prices to eliminate momentum. Since her trades move prices adversely, she faces a tradeoff between trading aggressively on momentum and moderating her price impact. Hence, she optimally decides not to eliminate momentum completely.

5 Extensions

In this section, we extend our model along two dimensions. First, we extend our single liquidation date equilibrium to a stationary equilibrium. In particular, we build a setup in which the asset pays an infinite stream of dividends, as opposed to a single liquidating dividend. In this setup, we demonstrate that information percolation generates and amplifies momentum, thus generalizing our previous results. Second, we incorporate a “rumor” in our benchmark model and show that it can generate a phase of price over-shooting followed by a phase of price correction. Under certain conditions, this convergence pattern can jointly produce short-term momentum and long-term reversal.

5.1 Dynamic Setup

Our theory for momentum relies on an economy with a single liquidation date. In this section, we show that our results carry over to a fully dynamic setup. In particular, we build a stationary version of the model and show that, even in this case, information percolation generates or amplifies momentum. We present a simplified version of the model and relegate all technical details to Appendix A.7. We consider an economy that goes on forever and in which the stock pays a stochastic dividend D_t per share. As in the finite version of the model, new liquidity traders enter the market in every trading session. To keep things simple, let us assume that the dividend process D_t and the supply process X_t follow random walks:

$$D_t = D_{t-1} + \varepsilon_t^d \quad (38)$$

$$X_t = X_{t-1} + \varepsilon_t^x \quad (39)$$

All investors observe the past and current realizations of dividends and of the stock prices. Each investor observes a signal about the dividend innovation 3-steps ahead.²⁵

$$\tilde{z}_t^i = \varepsilon_{t+3}^d + \tilde{\varepsilon}_t^i \quad (40)$$

As in the baseline model, investors meet and share private information over time. A fundamental difference, however, is that investors do not talk about a single liquidation value, but about several dividends revealed at different times in the future. That is, not only do they share information about the dividend 3-steps ahead, but they also share information about the dividend 2-steps ahead, and so on.

Unlike the baseline model, we consider an overlapping generation of agents, as in Bacchetta and Wincoop (2006), Watanabe (2008), Banerjee (2010), and Andrei (2013). This assumption considerably simplifies the analysis by ruling out dynamic hedging demands.²⁶ The solution method, which follows Andrei (2013), proceeds by specifying an equilibrium price that is a linear function of model innovations:

$$P_t = \alpha D_t + \beta X_{t-3} + (a_3 \ a_2 \ a_1)\varepsilon_t^d + (b_3 \ b_2 \ b_1)\varepsilon_t^x \quad (41)$$

²⁵Note that the model can be extended to a general case in which investors receive information about the dividend T -steps ahead at the expense of analytical complexity and without altering the main intuition presented here.

²⁶In the infinite-horizon case the portfolio maximization problem is substantially more complicated. The fixed point problem cannot be reduced to a finite dimensional one, but Bacchetta and Wincoop (2006) and Andrei (2013) show how to approximate the problem to a desired accuracy level by truncating the state space. The (numerical) results for the infinite horizon model are very close to those obtained in the overlapping generations model. See also Albuquerque and Miao (2014).

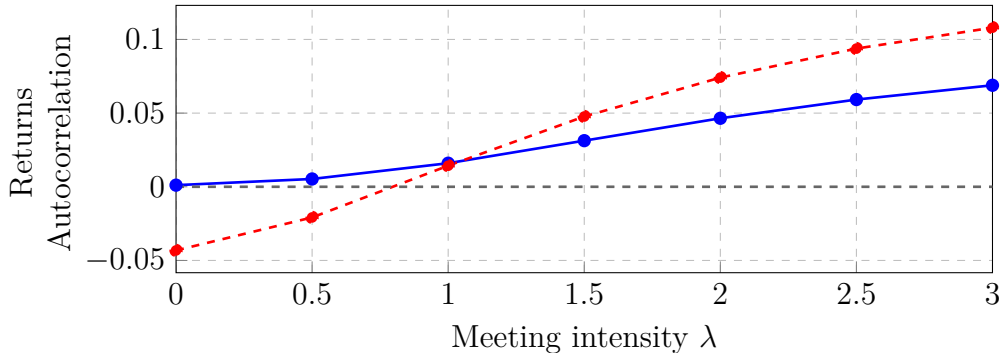


Figure 5: Serial Correlation of Returns in the Stationary Model

The figure depicts the serial correlation of returns, $\text{corr}(P_{t+1} - P_t, P_{t+2} - P_{t+1})$, for different levels of the meeting intensity λ . There are two cases: (i) the dividend and supply processes are random walks (solid blue line) and (ii) the dividend and supply processes are mean-reverting with AR(1) parameter 0.9. The calibration for the rest of the parameters ensures the existence of an equilibrium in the stationary model: $R = 1.1$, $H = 1$, $S = 10$, $\Phi = 1/100$, and $\tau = 3$, although most of the calibrations we have tried give the same qualitative results.

where $\epsilon_t^d \equiv (\epsilon_{t+1}^d \ \epsilon_{t+2}^d \ \epsilon_{t+3}^d)^\top$ are the 3 future unobservable dividend innovations and $\epsilon_t^x \equiv (\epsilon_{t-2}^x \ \epsilon_{t-1}^x \ \epsilon_t^x)^\top$ are the last 3 supply innovations. The main difference with respect to our baseline model is that equilibrium prices are now stationary. That is, the coefficients α , β , a , and b do not change over time, in contrast with the price coefficients in Theorem 1 which change as the economy approaches the finite end-point.

We now show that information percolation generates momentum, even though prices are stationary. The random walk specification (38) - (39) helps us isolate the effect of information percolation on prices. In particular, when investors do not have private information, this specification directly implies that stock returns are serially uncorrelated. The solid blue line in Figure 5 then shows that, once investors receive private information, stock returns exhibit momentum, which information percolation further amplifies.²⁷

The same intuition applies when the dividend and supply processes are not random walks. Because these processes are now mean-reverting, stock returns exhibit reversals in most cases (without information percolation). Our calculations show that information percolation can also generate momentum, much as in the baseline case. For instance, the dashed red line in Figure 5 shows how information percolation can turn reversals into momentum when the dividend and supply processes are mean-reverting with AR(1) parameters 0.9.

²⁷To be consistent with our main model, we compute the serial correlation of returns using ex-dividend prices. Alternatively, one could assume several trading rounds in-between dividend payment dates (which would bring this extension even closer to our baseline case), with similar results. See Makarov and Rytchkov (2009) for a detailed analysis when returns are computed using cum-dividend prices.

5.2 Rumors

Our baseline model can jointly generate short-term momentum, consistent with the empirical finding of Jegadeesh and Titman (1993), and long-term reversal, consistent with the over-reaction phenomenon of De Bondt and Thaler (1985). However, an important question is whether these effects can be amplified by rumors. It is natural to think of social interactions as natural propagators of rumors.²⁸ We introduce a rumor in our model by assuming that agents receive signals of the form:

$$\tilde{z}_t^i = \tilde{U} + \tilde{V} + \tilde{\epsilon}_t^i \quad (42)$$

where \tilde{V} is normally distributed with zero mean and precision ν . We build a simplified version of the model in which we assume that the asset is liquidated at time $T = 4$.

The *common* noise, \tilde{V} , satisfies two important properties of a rumor: (i) it circulates from person to person and (ii) it is unverifiable. The first property arises as private signals now contain the variable \tilde{V} , which now circulates from one agent to another through word-of-mouth communication. The second property results from the signal specification in (42): on average, private signals only reveal the sum of the fundamental value and the rumor ($\tilde{U} + \tilde{V}$). As a result, the rumor is unverifiable as agents cannot distinguish fundamental information from the rumor, either using prices or their private signals.

Rumors do not last forever, but eventually subside. To incorporate this aspect, we assume that each agent receives a signal at time $t = 3$ that is centered on the fundamental:

$$\tilde{Z}_3^i = \tilde{U} + \tilde{\epsilon}_3^i.$$

Using this signal, agents can back out the content of the rumor (on average) at time $t = 3$ and the rumor subsides. Overall, agents are aware of the rumor, but cannot learn about its content until time $t = 3$.

In the presence of a rumor, asset prices and investors' asset demands do not have a closed-form solution. Theorem 3 describes a system of recursive equations for the equilibrium price coefficients. We provide the proof of Theorem 3 and we solve this system of equations through an efficient numerical scheme that we describe in Appendix A.8.

Theorem 3. *In the presence of a rumor, the adjusted price signals can be written*

$$\tilde{Q}_t = \tilde{U} + \frac{\Lambda_t}{\tau S \Omega'_t} \tilde{V} - \frac{1}{\tau S \Omega'_t} \tilde{X}_t$$

²⁸Peterson and Gist (1951) define a rumor as “an unverified account or explanation of events circulating from person to person and pertaining to an object, event, or issue in public concern.”

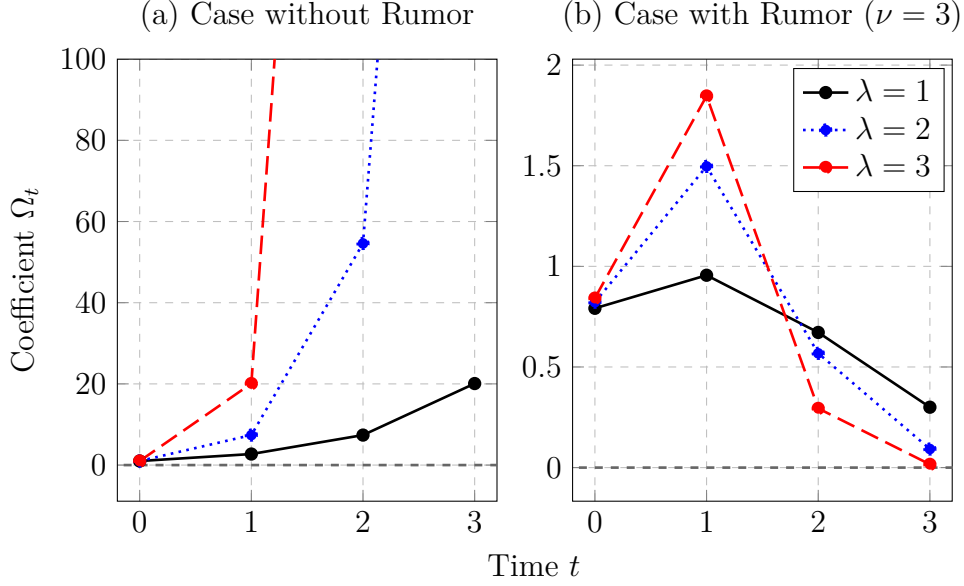


Figure 6: Price Coefficient Ω with and without Rumor

The Figure depicts the same coefficient Ω over time. Each line corresponds to a meeting intensity of $\lambda = 1$, $\lambda = 2$, and $\lambda = 3$. Panel (a) shows the coefficient Ω in the case of no rumor, while panel (b) shows the same coefficient in the case involving a rumor.

where the equilibrium coefficients Ω' and Λ solve a fixed-point problem given by a system of recursive equations:

$$\begin{aligned}\Omega'_t &= \frac{1}{\tau S} \sum_{j=0}^t \bar{\theta}_j - \sum_{j=0}^{t-1} \Omega'_j \\ \Lambda_t &= \sum_{j=0}^t \bar{\theta}_j - \sum_{j=0}^{t-1} \Lambda_j\end{aligned}\tag{43}$$

in which $\bar{\theta}$ denotes the average coefficients of agents' private signals in their optimal demand.

Proof. See Appendix A.8. □

The rumor has two important effects on prices, the first of which is immediately apparent when looking at the coefficient Ω' . We plot this coefficient in Figure 6, both when signals contain a rumor (panel (b) with $\nu = 3$) and when they do not (panel (a)).

When signals do not contain a rumor, the coefficient Ω' represents the average number of incremental signals and thus equals Ω from our baseline model. Panel (a) shows that this average increases exponentially in λ . When signals contain a rumor (panel (b)), the coefficient Ω increases initially but then reverts back to zero. Intuitively, agents know they possess information of lower quality due to the presence of the rumor and therefore apply

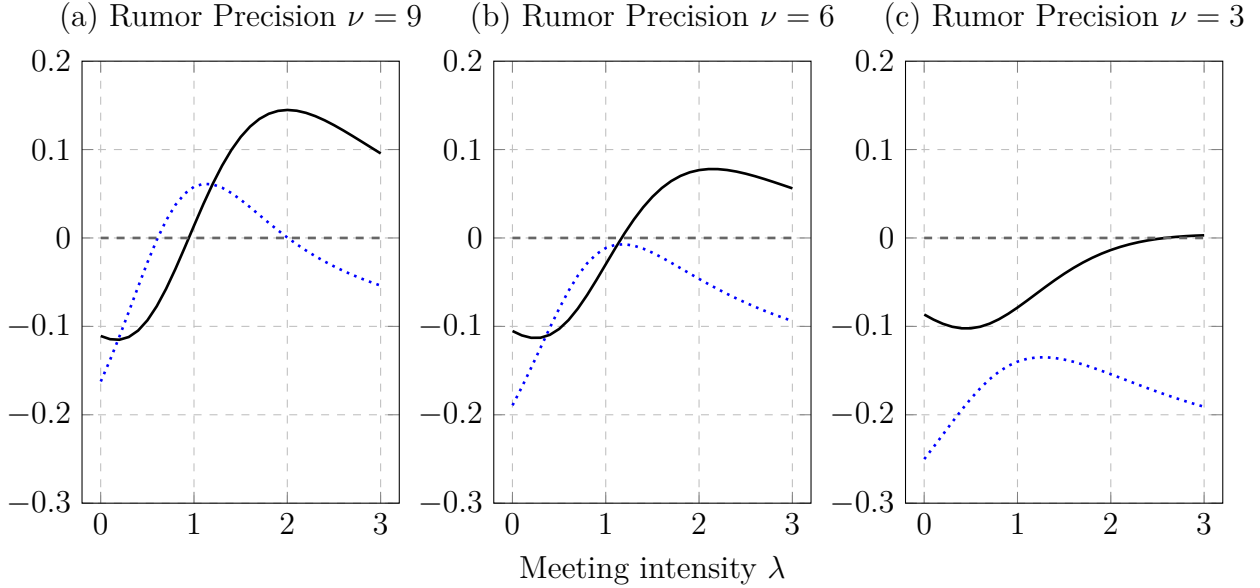


Figure 7: Serial Correlation of Return and Rumors

The Figure depicts the serial correlation of stock returns over the first (the solid black line) and the second period (the blue dotted line) as a function of the meeting intensity. The first period serial correlation is defined as $\text{corr}(\tilde{P}_1 - \tilde{P}_0, \tilde{P}_2 - \tilde{P}_1)$, whereas the second period serial correlation is defined as $\text{corr}(\tilde{P}_2 - \tilde{P}_1, \tilde{P}_3 - \tilde{P}_2)$. Each panel corresponds to a different rumor precision ν .

a discount on the actual number of signals they have—the coefficient Ω now represents a *discounted* average of incremental signals.

At time $t = 2$, the discounted average Ω declines: agents anticipate that they will get better information at time $t = 3$ and apply a stronger discount on their number of signals. At time $t = 3$, the discounted average number of signals reaches zero for $\lambda = 3$: when agents have collected a vast number of signals, they can accurately forecast $\tilde{U} + \tilde{V}$. Hence, when they get the signal that is centered on the fundamental, they ignore their other signals. Overall, the rumor induces agents to interpret their information with some caution.

We now investigate how this convergence pattern relates to the serial correlation of stock returns. Intuitively, the first phase of price “over-shooting” generates short-term momentum and the second phase of price correction generates long-term reversal. To show this, we plot the serial correlation of returns in Figure 7.

When the rumor is fairly precise (panel (a)), returns mostly exhibit momentum: despite the presence of the rumor, agents’ precision rises over time, generating momentum. As the precision of the rumor decreases (panel (b)), agents discount their actual number of signals more strongly. As a result, agents progressively cut back their positions—they adjust their trades to reflect that their information is of lower quality. While these portfolio adjustments

do not prevent returns to exhibit momentum in the first-period, they induce reversal in the second period as the price gradually corrects. Finally, when the rumor's precision is low (panel (c)), agents become extremely cautious about their information and the improvement in their precision is not sufficient to generate momentum.

6 Conclusion

This paper suggests several interesting avenues for future research. For instance, in this paper we abstract from individual behavioral biases, but we believe that individual biases, such as in [Daniel et al. \(1998\)](#) or [Barberis et al. \(1998\)](#), would amplify the effects we analyze. Other questions are worthwhile investigating, such as extending the setup to multiple assets, where information percolation could generate rich dynamics of the conditional correlation among assets. It is also interesting to study precisely the mechanism of information transmission and find conditions under which investors find it beneficial to tell the truth ([Stein, 2008](#)).

A legitimate question is what empirical exercise would validate our model. We believe that natural experiments capturing an exogenous increase or decrease in the intensity of word-of-mouth communication could make a worthwhile empirical point. For example, [Shiller \(2000\)](#) relates the increase in the word-of-mouth communication intensity once the telephone became effective during the 1920s with the steady increase of volatility during the same period. Another option is to study the consequences of the Regulation Fair Disclosure, promulgated by the U.S. Securities and Exchange Commission in August 2000. This regulation forbids firms and their insiders to provide information to some investors (often large institutional investors). Hence, after August 2000 there should be less information propagated through the word-of-mouth communication channel.

References

- [1] Admati, A. R. (1985). A noisy rational expectations equilibrium for multi-asset securities markets. *Econometrica* 53(3), pp. 629–658. [39, 41]
- [2] Akbas, F., S. Armstrong, S. Sorescu, and A. Subrahmanyam (2014). Smart money, dumb money, and capital market anomalies. *Working Paper*. [23]
- [3] Albuquerque, R. and J. Miao (2014). Advance information and asset prices. *Journal of Economic Theory* 149(0), 236 – 275. [4, 25]
- [4] Amir, E. and S. Levi (2014). Precision of information in daily stock prices and cost of equity. *Working Paper*. [16]
- [5] Andrei, D. (2013). Information percolation driving volatility. [25, 62, 63]
- [6] Asness, C. S., T. J. Moskowitz, and L. H. Pedersen (2013, 06). Value and Momentum Everywhere. *Journal of Finance* 68(3), 929–985. [2]
- [7] Bacchetta, P. and E. V. Wincoop (2006, June). Can information heterogeneity explain the exchange rate determination puzzle? *American Economic Review* 96(3), 552–576. [25]
- [8] Banerjee, S. (2010). Learning from prices and the dispersion in beliefs. *Review of Financial Studies*. [25]
- [9] Banerjee, S., R. Kaniel, and I. Kremer (2009). Price drift as an outcome of differences in higher-order beliefs. *Review of Financial Studies* 22(9), 3707–3734. [5, 13, 20]
- [10] Barberis, N., A. Shleifer, and R. Vishny (1998, September). A model of investor sentiment. *Journal of Financial Economics* 49(3), 307–343. [2, 5, 30]
- [11] Berk, J. B., R. C. Green, and V. Naik (1999, October). Optimal investment, growth options, and security returns. *Journal of Finance* 54(5), 1553–1607. [4]
- [12] Biais, B., P. Bossaerts, and C. Spatt (2010). Equilibrium asset pricing and portfolio choice under asymmetric information. *Review of Financial Studies* 23(4), 1503–1543. [4]
- [13] Botosan, C., M. Plumlee, and Y. Xie (2004). The role of information precision in determining the cost of equity capital. *Review of Accounting Studies* 9(2-3), 233–259. [16]
- [14] Brennan, M. J. and H. H. Cao (1997, December). International portfolio investment flows. *Journal of Finance* 52(5), 1851–80. [9, 22, 39, 64]
- [15] Brown, J. R., Z. Ivkovic, P. A. Smith, and S. Weisbenner (2008, 06). Neighbors matter: Causal community effects and stock market participation. *Journal of Finance* 63(3), 1509–1531. [4]
- [16] Cespa, G. and X. Vives (2012). Dynamic trading and asset prices: Keynes vs. hayek. *Review of Economic Studies* 79(2), 539–580. [4]
- [17] Chan, K. C., W.-M. Fong, B.-C. Kho, and R. Stulz (1996, August). Information, trading and stock returns: Lessons from dually-listed securities. *Journal of Banking & Finance* 20(7), 1161–1187. [4]

- [18] Cohen, L., A. Frazzini, and C. Malloy (2008, October). The small world of investing: Board connections and mutual fund returns. *Journal of Political Economy* 116(5), 951–979. [4]
- [19] Daniel, K., D. Hirshleifer, and A. Subrahmanyam (1998, December). Investor psychology and security market under- and overreactions. *Journal of Finance* 53(6), 1839–1885. [2, 5, 9, 30]
- [20] De Bondt, W. F. M. and R. Thaler (1985, July). Does the stock market overreact? *Journal of Finance* 40(3), 793–805. [27]
- [21] Demarzo, P. M., D. Vayanos, and J. Zwiebel (2003, August). Persuasion bias, social influence, and unidimensional opinions. *The Quarterly Journal of Economics* 118(3), 909–968. [6]
- [22] Duffie, D., S. Malamud, and G. Manso (2009). Information percolation with equilibrium search dynamics. *Econometrica* 77, 1513–1574. [6, 7]
- [23] Duffie, D. and G. Manso (2007, May). Information percolation in large markets. *American Economic Review* 97(2), 203–209. [2, 38]
- [24] Feng, L. and M. S. Seasholes (2004, October). Correlated trading and location. *Journal of Finance* 59(5), 2117–2144. [4]
- [25] Francis, J., R. LaFond, P. Olsson, and K. Schipper (2005, June). The market pricing of accruals quality. *Journal of Accounting and Economics* 39(2), 295–327. [16]
- [26] French, K. R. and R. Roll (1986, September). Stock return variances : The arrival of information and the reaction of traders. *Journal of Financial Economics* 17(1), 5–26. [4, 17]
- [27] Grinblatt, M. and M. Keloharju (2001, 06). How distance, language, and culture influence stock-holdings and trades. *Journal of Finance* 56(3), 1053–1073. [4]
- [28] Gropp, R. E. and A. Kadareja (2007). Stale information, shocks and volatility. ZEW Discussion Papers 07-012, ZEW - Zentrum für Europäische Wirtschaftsforschung / Center for European Economic Research. [4]
- [29] Grossman, S. J. and M. H. Miller (1988, July). Liquidity and market structure. *Journal of Finance* 43(3), 617–37. [15]
- [30] Grossman, S. J. and J. E. Stiglitz (1980). On the impossibility of informationally efficient markets. *The American Economic Review* 70(3), pp. 393–408. [2, 8]
- [31] He, H. and J. Wang (1995). Differential information and dynamic behavior of stock trading volume. *Review of Financial Studies* 8(4), 919–72. [39]
- [32] Hendershott, T. and M. S. Seasholes (2007, May). Market Maker Inventories and Stock Prices. *American Economic Review* 97(2), 210–214. [23]
- [33] Holden, C. W. and A. Subrahmanyam (2002, January). News events, information acquisition, and serial correlation. *Journal of Business* 75(1), 1–32. [4, 20]
- [34] Hong, H., J. D. Kubik, and J. C. Stein (2004, 02). Social interaction and stock-market participation. *Journal of Finance* 59(1), 137–163. [4]

- [35] Hong, H., T. Lim, and J. C. Stein (2000). Bad news travels slowly: Size, analyst coverage, and the profitability of momentum strategies. *The Journal of Finance* 55(1), pp. 265–295. [3, 4, 17]
- [36] Hong, H. and J. C. Stein (1999, December). A unified theory of underreaction, momentum trading, and overreaction in asset markets. *Journal of Finance* 54(6), 2143–2184. [2, 5, 9]
- [37] Hou, K. and T. J. Moskowitz (2005). Market Frictions, Price Delay, and the Cross-Section of Expected Returns. *Review of Financial Studies* 18(3), 981–1020. [4]
- [38] Ivkovic, Z. and S. Weisbenner (2005, 02). Local does as local is: Information content of the geography of individual investors’ common stock investments. *Journal of Finance* 60(1), 267–306. [4]
- [39] Jegadeesh, N. (1990, July). Evidence of Predictable Behavior of Security Returns. *Journal of Finance, American Finance Association* 45(3), 881–98. [4, 19]
- [40] Jegadeesh, N. and S. Titman (1993, March). Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance* 48(1), 65–91. [2, 27]
- [41] Johnson, T. C. (2002, 04). Rational momentum effects. *Journal of Finance* 57(2), 585–608. [4]
- [42] Kelley, E. K. and P. C. Tetlock (2013). Retail Short Selling and Stock Prices. *Working Paper*. [23]
- [43] Koudijs, P. (2010). The boats that did not sail. news, trading and asset price volatility in a natural experiment. Working paper. [4]
- [44] Lambert, R. A., C. Leuz, and R. E. Verrecchia (2011). Information Asymmetry, Information Precision, and the Cost of Capital. *Review of Finance* 16(1), 1–29. [3, 16]
- [45] Lehmann, B. N. (1990, February). Fads, Martingales, and Market Efficiency. *The Quarterly Journal of Economics, MIT Press* 105(1), 1–28. [4, 19]
- [46] Lou, D. (2009). A flow-based explanation for return predictability. LSE Research Online Documents on Economics 29310, London School of Economics and Political Science, LSE Library. [23]
- [47] Makarov, I. and O. Rytchkov (2009). Forecasting the forecasts of others: Implications for asset pricing. *Working Paper*. [4, 26]
- [48] Mitchell, M. L. and J. H. Mulherin (1994, July). The impact of public information on the stock market. *Journal of Finance* 49(3), 923–50. [4]
- [49] Moskowitz, T. J., Y. H. Ooi, and L. H. Pedersen (2012). Time series momentum. *Journal of Financial Economics* 104(2), 228–250. [2, 3, 17, 18, 23]
- [50] Novy-Marx, R. (2012). Is momentum really momentum? *Journal of Financial Economics* 103(3), 429–453. [3, 18, 19]
- [51] Peterson, W. A. and N. P. Gist (1951). Rumor and public opinion. *The American Journal of Sociology* 57(2), pp. 159–167. [27]
- [52] Roll, R. (1988, July). R-S1-2. *Journal of Finance* 43(3), 541–66. [4]

- [53] Rouwenhorst, K. G. (1998, 02). International momentum strategies. *Journal of Finance* 53(1), 267–284. [2]
- [54] Sagi, J. S. and M. S. Seasholes (2007, May). Firm-specific attributes and the cross-section of momentum. *Journal of Financial Economics* 84(2), 389–434. [4]
- [55] Shiller, R. J. (2000). *Irrational Exuberance*. Princeton University Press. [4, 30]
- [56] Shiller, R. J. and J. Pound (1989). Survey evidence on diffusion of investment among institutional investors. *Journal of Economic Behavior and Organization* 12, 47–66. [4]
- [57] Shive, S. (2010, February). An epidemic model of investor behavior. *Journal of Financial and Quantitative Analysis* 45(01), 169–198. [4]
- [58] Stein, J. C. (2008). Conversations among competitors. *American Economic Review* 98(5), 2150–62. [4, 30]
- [59] Tetlock, P. C. (2010). Does Public Financial News Resolve Asymmetric Information? *Review of Financial Studies* 23(9), 3520–3557. [4]
- [60] Vayanos, D. and P. Woolley (2010). An institutional theory of momentum and reversal. *Working Paper*. [4]
- [61] Verardo, M. (2009, August). Heterogeneous Beliefs and Momentum Profits. *Journal of Financial and Quantitative Analysis* 44(04), 795–822. [4]
- [62] Wang, J. (1993, April). A model of intertemporal asset prices under asymmetric information. *Review of Economic Studies* 60(2), 249–82. [4]
- [63] Watanabe, M. (2008, 02). Price volatility and investor behavior in an overlapping generations model with information asymmetry. *Journal of Finance* 63(1), 229–272. [25]

A Appendix

A.1 Information Percolation

A.1.1 Distribution of Incremental Signals: Closed-Form Solution

To obtain the closed-form solution for the distribution π of incremental signals, we first derive the equation for its dynamics.

Lemma 1. *The probability density function π over the additional number of signals collected by each agent satisfies*

$$\frac{d}{dt}\pi_t(n) = -\lambda\pi_t(n) + \lambda(\pi_t * \mu_t)(n), \quad \pi_0 = \delta_{n=1}. \quad (\text{A.1})$$

Proof. We compute the distribution of new signals that have been gathered between time 0 and time T . Denote by X_{t_i} the number of new signals gathered if a meeting occurs at time t_i and observe that it is distributed as

$$X_{t_i} \sim \mu(t_i, \cdot)$$

where the distribution $\mu(t, x)$ satisfies the differential equation

$$\frac{d}{dt}\mu_t(n) = -\lambda\mu_t(n) + \lambda(\mu_t * \mu_t)(n), \quad \mu_0 = \delta_{n=1}. \quad (\text{A.2})$$

Furthermore, the number $N(T)$ of meetings that took place between time 0 and T is a Poisson counter with intensity λ ; accordingly, the total number Y_T of new signals gathered between time 0 and T is given by $\sum_{i=1}^{N(T)} X_{t_i}$. We now characterize its distribution. First, observe that Y_T , conditional on the set of times $\{0 \leq t_1 \leq t_2 \leq \dots \leq t_{N(T)} \leq T\}$ at which a meeting occurs (up to time T) and the total number of meetings $N(T)$ (that is, conditioning on the whole trajectory A_T^N of the Poisson process), is distributed as

$$\begin{aligned} Y_T | A_T^N &\sim \int_{\mathbb{R}^{N-1}} \mu(Y_{t_N} - Y_{t_{N-1}}, t_N) d\mu(Y_{t_{N-1}} - Y_{t_{N-2}}, t_{N-1}) \dots d\mu(Y_{t_1} - 0, t_1) \\ &\equiv \int_{\mathbb{R}^{N-1}} \mu(X_{t_N}, t_N) d\mu(X_{t_{N-1}}, t_{N-1}) \dots d\mu(X_{t_1}, t_1). \end{aligned} \quad (\text{A.3})$$

Second, observe that the distribution $\mu(X_{t_i}, t_i)$ of increment can be expressed as a translation \mathcal{T} of the type measure μ and the increment x . Hence, the distribution in (A.3) may be written as

$$Y_T | A_T^N \sim \Gamma_{i=1}^{N(T)} \mu_{t_i}$$

where, for any probability measures $\alpha_1, \dots, \alpha_k$, we write $\Gamma_{i=1}^k = \alpha_1 * \alpha_2 * \dots * \alpha_k$.

Now, observe that each t_i in the sequence of meetings $\{0 \leq t_1 \leq t_2 \leq \dots \leq t_{N(T)} \leq T\}$ conditional on $N(T)$ is uniformly distributed over T ; accordingly, we have that

$$Y_T | N(T) \sim \Gamma_{i=1}^{N(T)} \frac{1}{T} \int_0^T \mu_{t_i} dt_i = \left(\frac{1}{T^{N(T)}} \left(\int_0^T \mu_s ds \right)^{*N(T)} \right)$$

where $*n$ denotes the n -fold convolution.

Finally, since $N(T)$ is a Poisson(λ) counter, we have

$$Y_T \sim \sum_{k=0}^{\infty} e^{-\lambda T} \frac{(\lambda T)^k}{k!} \frac{1}{T^k} \left(\int_0^T \mu_s ds \right)^{*k} = \sum_{k=0}^{\infty} e^{-\lambda T} \frac{\lambda^k}{k!} \left(\int_0^T \mu_s ds \right)^{*k}.$$

Using the fact that, by Taylor expansion, e^x is equivalently written as $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$, we can write

$$\pi_T = e^{\lambda \left(\int_0^T \mu_s ds - T \right)}.$$

Differentiating this expression, we obtain (A.1), which corresponds to Equation (3) in the paper. \square

A.1.2 Proof of Proposition 1

To obtain the closed form, it is convenient to think of individual signals received at times $t = 0, 1, 2$ separately. That is, even though all are signals about the same fundamental, we will treat them separately. Assume that signals received at time $t = 0, 1, 2$ are of type s_0, s_1 , and s_2 respectively.

Let us fix some notation:

- 0. $n_0 \equiv$ the total number of type s_0 signals received between time 0 and t .
- 1. $n_1 \equiv$ the total number of type s_1 signals received between time 1 and t .
- 2. $n_2 \equiv$ the total number of type s_2 signals received between time 2 and t .

and:

- 0. $m_0 \equiv$ the incremental number of type s_0 signals received between *the last trading date* and t .
- 1. $m_1 \equiv$ the incremental number of type s_1 signals received between *the last trading date* and t .
- 2. $m_2 \equiv$ the incremental number of type s_2 signals received between *the last trading date* and t .

Let us focus first on the total number of signals, n_0, n_1 , and n_2 . From time 0 to 1, the distribution of n_0 has the support $[1, \infty)$. Recursive computations show that this distribution is

$$\mu_{\text{total}, n_0} = e^{-n_0 \lambda \tau} \left(e^{\lambda \tau} - 1 \right)^{n_0 - 1} \quad (\text{A.4})$$

where $0 \leq \tau \leq 1$.

From time 1 to 2, the distribution of $\{n_0, n_1\}$ has the support $[1, \infty) \times [1, \infty)$. Further recursive computations show that this distribution is

$$\mu_{\text{total}, n_0, n_1} = \begin{cases} \binom{n_0-1}{n_1-1} e^{-n_0 \lambda - n_1 \lambda \tau} \left(e^{\lambda} - 1 \right)^{n_0 - n_1} \left(e^{\lambda \tau} - 1 \right)^{n_1 - 1}, & \text{if } n_0 \geq n_1 \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.5})$$

From time 2 to 3, the distribution of $\{n_0, n_1, n_2\}$ has the support $[1, \infty) \times [1, \infty) \times [1, \infty)$. Further recursive computations show that this distribution is

$$\mu_{\text{total}, n_0, n_1, n_2} = \begin{cases} \binom{n_0-1}{n_1-1} \binom{n_1-1}{n_2-1} e^{-n_0 \lambda - n_1 \lambda - n_2 \lambda \tau} \left(e^{\lambda} - 1 \right)^{n_0 - n_2} \left(e^{\lambda \tau} - 1 \right)^{n_2 - 1}, & \text{if } n_0 \geq n_1 \geq n_2 \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.6})$$

Focus now on the distribution of increments, m_0 , m_1 , and m_2 . From time 0 to 1, the distribution of m_0 has the support $[0, \infty)$. Recursive computations show that this distribution is

$$\mu_{\text{incr},n_0} = e^{(-m_0+1)\lambda\tau} \left(e^{\lambda\tau} - 1 \right)^{m_0} \quad (\text{A.7})$$

From time 1 to 2, the distribution of $\{m_0, m_1\}$ has the support $[0, \infty) \times [0, \infty)$. Further recursive computations show that this distribution is

$$\mu_{\text{incr},m_0,m_1} = \begin{cases} \binom{m_0-1}{m_1-1} e^{-m_0\lambda-(m_1+1)\lambda\tau} \left(e^\lambda - 1 \right)^{m_0-m_1} \left(e^{\lambda\tau} - 1 \right)^{m_1}, & \text{if } m_0 \geq m_1 \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.8})$$

From time 2 to 3, the distribution of $\{m_0, m_1, m_2\}$ has the support $[0, \infty) \times [0, \infty) \times [0, \infty)$. Further recursive computations show that this distribution is

$$\mu_{\text{incr},m_0,m_1,m_2} = \begin{cases} \binom{m_0-1}{m_1-1} \binom{m_1-1}{m_2-1} e^{-m_0\lambda-m_1\lambda-(m_2+1)\lambda\tau} \left(e^\lambda - 1 \right)^{m_0-m_2} \left(e^{\lambda\tau} - 1 \right)^{m_2}, & \text{if } m_0 \geq m_1 \geq m_2 \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.9})$$

We can now group the signals of the same type, since they are all informative about the same fundamental. From time 0 to 1, signals are only of type s_0 and thus the probability density function over the additional number of signals follows from (A.7) with $\tau = 1$, with $m_0 = n - 1$:

$$\pi_1(n) = e^{-n\lambda} \left(e^\lambda - 1 \right)^{n-1} \quad (\text{A.10})$$

From time 1 to 2, given a number n of additional signals one has to find all the combinations of m_0 and m_1 for which $m_0 + m_1 = n - 1$, and then make the sum of all the corresponding terms in (A.8) with $\tau = 1$:

$$\pi_2(n) = \sum_{i=0}^{n-1} \sum_{j=0}^i \left[\binom{i-1}{j-1} \mathbb{1}_{\{i+j=n-1\}} e^{-i\lambda-(j+1)\lambda} \left(e^\lambda - 1 \right)^i \right] \quad (\text{A.11})$$

From time 2 to 3, given a number n of additional signals one has to find all the combinations of m_0 , m_1 , and m_2 for which $m_0 + m_1 + m_2 = n - 1$, and then make the sum of all the corresponding terms in (A.9) with $\tau = 1$:

$$\pi_3(n) = \sum_{i=0}^{n-1} \sum_{j=0}^i \sum_{k=0}^j \left[\binom{i-1}{j-1} \binom{j-1}{k-1} \mathbb{1}_{\{i+j+k=n-1\}} e^{-(i+j)\lambda-(k+1)\lambda} \left(e^\lambda - 1 \right)^i \right] \quad (\text{A.12})$$

and thus we obtain the recursive form of Proposition 1. The cross-sectional average of additional signals collected between $t = 0$ and $t = 1$ is

$$\Omega_1 = \sum_{n=0}^{\infty} n\pi_1(n) = e^\lambda \quad (\text{A.13})$$

The cross-sectional average of additional signals collected between $t = 2$ and $t = 2$, Ω_2 is computed as follows

$$\Omega_2 = \mathbb{E} [n_0(t = 2) + n_1(t = 2)] - \mathbb{E} [n_0(t = 1)] \quad (\text{A.14})$$

$$= \mathbb{E}[n_0(t=2)] + \mathbb{E}[n_1(t=2)] - \mathbb{E}[n_0(t=1)] \quad (\text{A.15})$$

$$= e^{2\lambda} + e^\lambda - e^\lambda \quad (\text{A.16})$$

$$= e^{2\lambda} \quad (\text{A.17})$$

where $\mathbb{E}[n_x(t=\tau)]$ represent the average at time τ of the total number of signals of type x . The same calculus applies for Ω_t , with any $t \geq 3$:

$$\Omega_t = e^{t\lambda}. \quad (\text{A.18})$$

A.1.3 Numerical Approach: Fourier Transform

To obtain the distribution of incremental signals numerically, we proceed through discrete Fourier transforms. Denote the Fourier transform of μ and π by $\hat{\mu}_t(z) := \int_{\mathbb{R}} e^{izn} d\mu_t(n)$ and $\hat{\pi}_t(z) := \int_{\mathbb{R}} e^{izn} d\pi_t(n)$, respectively, where $i = \sqrt{-1}$ and $z \in \mathbb{R}$. As in [Duffie and Manso \(2007\)](#), $\hat{\mu}$ is given in closed form:

$$\hat{\mu}_t(z) = \frac{\hat{\mu}_0(z)}{e^{\lambda t}(1 - \hat{\mu}_0(z)) + \hat{\mu}_0(z)}$$

where $\hat{\mu}_0(z) = e^{iz}$ since $\mu_0(n)$ is a Dirac mass at 1.

To obtain $\hat{\pi}$, we integrate [\(A.1\)](#) and get the following equation

$$\frac{d}{dt} \hat{\pi}_t(z) = -\lambda \hat{\pi}_t(z) + \lambda \hat{\mu}_t(z) \hat{\pi}_t(z),$$

the solution of which is also available in closed form:

$$\hat{\pi}_t(z) = \frac{\hat{\pi}_0(z) e^{i\pi}}{e^{\lambda t}(\hat{\mu}_0(z) - 1) - \hat{\mu}_0(z)}$$

where $\hat{\pi}_0(z) = e^{iz}$ since $\pi_0(n)$ is a Dirac mass at 1.

We can now recover π numerically by using the inverse Fourier formula. To do so, notice that both distributions $\mu(n)$ and $\pi(n)$ are so-called *lattice* distributions for which every possible realization of n can be represented as $a + bk$ where k only takes integral values—in our case, $n \in \mathbb{N}$, $a = \frac{N}{2}$, and $b = \frac{1}{2}$. For this class of discrete distributions, the inverse Fourier formula writes

$$P[X = x_k] = \frac{b}{2\pi} \int_{-\pi/b}^{\pi/b} e^{-izx_k} \hat{\mu}_t(z) dz. \quad (\text{A.19})$$

To compute the integral in [\(A.19\)](#), we use fast Fourier Transform: we rewrite [\(A.19\)](#) as

$$P[X = x_k] = \frac{b}{\pi} \int_0^{\pi/b} e^{-izx_k} \hat{\mu}_t(z) dz.$$

We then use the trapezoid rule to discretize the integral:

$$P[X = x_k] \approx \frac{b}{\pi} \frac{e^{-i\frac{\pi}{b}x_k} \hat{\mu}_t(\frac{\pi}{b}) \Delta z}{2} + \frac{b}{\pi} \sum_{j=0}^{M-1} \delta_j e^{-ij\Delta z x_k} \hat{\mu}_t(j\Delta z) \Delta z$$

with $\delta_j = \frac{1}{2}$ if $j = 1$ and $\delta_j = 1$ otherwise. We need to choose Δz such that the upper integration bound $\frac{\pi}{b} = 2\pi$ is reached by z ; accordingly, we set $\Delta z \equiv \frac{2\pi}{M}$. Furthermore, the grid points for x_k are $x_k = -d + \lambda k$, $j \in \mathbb{N}$ and the fast Fourier transform method imposes that $\lambda \Delta z = \frac{2\pi}{M}$, i.e., $\lambda = 1$.

Since $x_k \in \mathbb{N}$, we must have that $d = 0$. As a result, $P[X = x_k]$ takes the form of a discrete Fourier transform:

$$P[X = x_k] \approx \frac{1}{M} \sum_{j=0}^{M-1} g_j e^{-jk \frac{2\pi}{M} i}$$

with

$$g_j = \delta_j \frac{M}{2\pi} \hat{\mu}_t \left(j \frac{2\pi}{M} \right) \frac{2\pi}{M}.$$

Finally, since each agent gets an additional signal every period, we must use the discrete Fourier transform sequentially: to derive the distribution π at time $t + 1$, we use the numerical expression for $\mu_t(n)$ that we computed at time t and compute $\hat{\mu}_0(z) = \sum_{k=1}^N e^{izk} f_0(k)$ where

$$f_0(n) = \begin{cases} \mu_t(n-1) & \text{if } n \geq t+1 \\ 0 & \text{otherwise} \end{cases}.$$

We then substitute $\hat{\mu}_0(n)$ into $\hat{\pi}_t(n)|_{t=1}$ and apply the inverse Fourier formula, which yields π_{t+1} .

A.2 Proof of Theorem 1

We provide the proof for a two trading session economy. Once the equilibrium quantities are written in a recursive form, as in [Brennan and Cao \(1997\)](#), or in [He and Wang \(1995\)](#), it is straightforward to derive the full recursive equilibrium solution. The model is solved backwards, starting from date 1 and then going back to date 0. First, we conjecture that prices in period 0 and period 1 are

$$\tilde{P}_0 = \beta_0 \tilde{U} - \alpha_{0,0} \tilde{X}_0 \tag{A.20}$$

$$\tilde{P}_1 = \beta_1 \tilde{U} - \alpha_{1,0} \tilde{X}_0 - \alpha_{1,1} \tilde{X}_1 \tag{A.21}$$

Consider the normalized price signal in period zero (which is informationally equivalent to \tilde{P}_0):

$$\tilde{Q}_0 = \frac{1}{\beta_0} \tilde{P}_0 = \tilde{U} - \frac{\alpha_{0,0}}{\beta_0} \tilde{X}_0 \tag{A.22}$$

Replace \tilde{X}_0 from [\(A.22\)](#) into [\(A.21\)](#) to obtain

$$\tilde{P}_1 = \varphi_1 \tilde{U} + \xi_1 \tilde{Q}_0 - \alpha_{1,1} \tilde{X}_1 \tag{A.23}$$

where $\varphi_1 = \beta_1 - \alpha_{1,0} \frac{\beta_0}{\alpha_{0,0}}$ and $\xi_1 = \alpha_{1,0} \frac{\beta_0}{\alpha_{0,0}}$. These coefficients are to be determined in equilibrium.

We normalize the price signal in period $t = 1$ and obtain \tilde{Q}_1 :

$$\tilde{Q}_1 = \frac{1}{\varphi_1} \left(\tilde{P}_1 - \xi_1 \tilde{Q}_0 \right) = \tilde{U} - \frac{\alpha_{1,1}}{\varphi_1} \tilde{X}_1 \tag{A.24}$$

Thus, observing $\{\tilde{Q}_0, \tilde{Q}_1\}$ is equivalent with observing $\{\tilde{P}_0, \tilde{P}_1\}$. We conjecture the following relationships (see [Admati 1985](#)):

$$\frac{\alpha_{0,0}}{\beta_0} = \frac{1}{\tau S \Omega_0} \tag{A.25}$$

$$\frac{\alpha_{1,1}}{\varphi_1} = \frac{1}{\tau S \Omega_1} \tag{A.26}$$

In our setup, $\Omega_0 = 1 \forall \lambda$, $\Omega_1 = 1$ if $\lambda = 0$, and $\Omega_1 > 1$ if $\lambda > 0$. Relationships (A.25) and (A.26) make the calculations that follow straightforward and are to be verified once the solution is obtained. Thus, the normalized price signals, informationally equivalent with prices, are:

$$\tilde{Q}_0 = \tilde{U} - \frac{1}{\tau S \Omega_0} \tilde{X}_0 \quad (\text{A.27})$$

$$\tilde{Q}_1 = \tilde{U} - \frac{1}{\tau S \Omega_1} \tilde{X}_1 \quad (\text{A.28})$$

Period 1 Consider an investor i who, at date $t = 1$, collects $\omega_1^i \geq 1$ additional signals. At date $t = 1$, investor i chooses \tilde{D}_1^i to maximize expected utility of final wealth:

$$\max_{\tilde{D}_1^i} \mathbb{E} \left[-e^{-\frac{1}{\tau} \tilde{W}_2^i} \middle| \mathcal{F}_1^i \right] \quad (\text{A.29})$$

where the final wealth at date $t = 2$ (at liquidation) is

$$\tilde{W}_2^i = X^i \tilde{P}_0 + \tilde{D}_0^i (\tilde{P}_1 - \tilde{P}_0) + \tilde{D}_1^i (\tilde{U} - \tilde{P}_1) \quad (\text{A.30})$$

and \mathcal{F}_1^i represents the total information available at date $t = 1$. This information is given by \tilde{Z}_1^i , \tilde{Z}_0^i (private signals) and \tilde{Q}_1 , \tilde{Q}_0 (public signals, informationally equivalent with prices). Note that \tilde{Z}_0^i represent only one signal of precision S , but \tilde{Z}_1^i represent the average of the ω_1^i additional signals collected by the investor at date $t = 1$ (ω_1^i signals of equal precision S are informationally equivalent with a signal equal to their average with precision $\omega_1^i S$). With this information at hand at date $t = 1$, investor i forecasts \tilde{U} . The state variables corresponding to investor i are therefore

Precision \rightarrow	$\frac{1}{H}$	$\frac{1}{\omega_1^i S}$	$\frac{1}{S}$	$\frac{1}{\Phi}$	$\frac{1}{\Phi}$	
Variable \downarrow	\tilde{U}	$\tilde{\varepsilon}_1^i$	$\tilde{\varepsilon}_0^i$	\tilde{X}_1	\tilde{X}_0	$\mathbb{E}[\cdot]$
\tilde{U}	1	0	0	0	0	0
\tilde{Z}_1^i	1	1	0	0	0	0
\tilde{Z}_0^i	1	0	1	0	0	0
\tilde{Q}_1	1	0	0	$-\frac{1}{\tau S \Omega_1}$	0	0
\tilde{Q}_0	1	0	0	0	$-\frac{1}{\tau S \Omega_0}$	0

We compute

$$K_1^i = \text{Var}^{-1} \left[\tilde{U} \middle| \tilde{Z}_1^i, \tilde{Z}_0^i, \tilde{Q}_1, \tilde{Q}_0 \right] \quad (\text{A.31})$$

$$\tilde{\mu}_1^i = \mathbb{E} \left[\tilde{U} \middle| \tilde{Z}_1^i, \tilde{Z}_0^i, \tilde{Q}_1, \tilde{Q}_0 \right] \quad (\text{A.32})$$

by using the projection theorem:

Theorem 4 (Projection Theorem). Consider a n -dimensional normal random variable

$$(\theta, s) \sim N \left(\begin{bmatrix} \mu_\theta \\ \mu_s \end{bmatrix}, \begin{bmatrix} \Sigma_{\theta,\theta} & \Sigma_{\theta,s} \\ \Sigma_{s,\theta} & \Sigma_{s,s} \end{bmatrix} \right)$$

The conditional density of θ given s is normal with conditional mean

$$\mu_\theta + \Sigma_{\theta,s} \Sigma_{s,s}^{-1} (s - \mu_s)$$

and variance-covariance matrix

$$\Sigma_{\theta,\theta} - \Sigma_{\theta,s} \Sigma_{s,s}^{-1} \Sigma_{s,\theta}$$

provided $\Sigma_{s,s}$ is non-singular.

We obtain

$$K_1^i = H + S (1 + \omega_1^i) + \tau^2 S^2 \Phi (\Omega_0^2 + \Omega_1^2) \quad (\text{A.33})$$

$$\tilde{\mu}_1^i = \frac{1}{K_1^i} \left[S \tilde{Z}_0^i + S \omega_1^i \tilde{Z}_1^i + \tau^2 S^2 \Phi (\Omega_0^2 \tilde{Q}_0 + \Omega_1^2 \tilde{Q}_1) \right] \quad (\text{A.34})$$

The optimal demand of trader i in period 1 has a standard form (from the normality of distribution assumption in conjunction with the exponential utility function):

$$\tilde{D}_1^i = \tau K_1^i (\tilde{\mu}_1^i - \tilde{P}_1) \quad (\text{A.35})$$

Replace (A.34) in (A.35) to obtain

$$\tilde{D}_1^i = \tau \left[S \tilde{Z}_0^i + S \omega_1^i \tilde{Z}_1^i + \tau^2 S^2 \Phi (\Omega_0^2 \tilde{Q}_0 + \Omega_1^2 \tilde{Q}_1) - K_1^i \tilde{P}_1 \right] \quad (\text{A.36})$$

We can now integrate the optimal demands to get the total demand. We follow the convention used by Admati (1985) that implies $\int_0^1 \tilde{Z}_j^i = \tilde{U}$, a.s.. More important, we have now to keep track of the heterogeneity in information endowments when aggregating all individual demands. In particular, at time $t = 1$ there is an infinity of types of investors with respect to their number of signals, and in each such type there is a continuum of investors. Consequently, the total demand at time $t = 1$ is

$$\tilde{D}_1 = \int_0^1 \tilde{D}_1^i = \sum_{\omega_1^i=1}^{\infty} \left[\pi_1(\omega_1^i) \int_{\omega_1^i} \tilde{D}_1^i \right] \quad (\text{A.37})$$

which yields

$$\tilde{D}_1 = \tau \left[S (\Omega_0 + \Omega_1) \tilde{U} + \tau^2 S^2 \Phi (\Omega_0^2 \tilde{Q}_0 + \Omega_1^2 \tilde{Q}_1) - K_1 \tilde{P}_1 \right] \quad (\text{A.38})$$

where K_1 is the average precision across the entire population of agents:

$$K_1 \equiv \sum_{\omega_1^i=1}^{\infty} K_1^i(\omega_1^i) \pi_1(\omega_1^i) = H + S (\Omega_0 + \Omega_1) + \tau^2 S^2 \Phi (\Omega_0^2 + \Omega_1^2) \quad (\text{A.39})$$

Replace (A.28) in (A.38) to obtain

$$\tilde{D}_1 = \tau \left[\left(S \Omega_0 + S \Omega_1 + \tau^2 S^2 \Phi \Omega_1^2 \right) \tilde{U} + \tau^2 S^2 \Phi \Omega_0^2 \tilde{Q}_0 - \tau \Phi S \Omega_1 \tilde{X}_1 - K_1 \tilde{P}_1 \right] \quad (\text{A.40})$$

The market clearing condition is $\tilde{D}_1 = \tilde{X}_0 + \tilde{X}_1$. Once we impose market clearing, we can use the conjectured \tilde{P}_1 equation (A.23) to get the undetermined coefficients φ_1 , ξ_1 , and $\alpha_{1,1}$:

$$\varphi_1 = \frac{S \Omega_1 (1 + \tau^2 S \Phi \Omega_1)}{K_1}, \quad (\text{A.41})$$

$$\xi_1 = \frac{S \Omega_0 (1 + \tau^2 S \Phi \Omega_0)}{K_1} \quad (\text{A.42})$$

$$\alpha_{1,1} = \frac{1 + \tau^2 S \Phi \Omega_1}{\tau K_1} \quad (\text{A.43})$$

We can now verify that, indeed, $\frac{\alpha_{1,1}}{\varphi_1} = \frac{1}{\tau S \Omega_1}$. Hence, (A.26) is now verified. The undetermined coefficients of \tilde{P}_1 from the conjectured form (A.21) are

$$\beta_1 = \frac{K_1 - H}{K_1} \quad (\text{A.44})$$

$$\alpha_{1,0} = \frac{1 + \tau^2 S \Phi \Omega_0}{\tau K_1} \quad (\text{A.45})$$

$$\alpha_{1,1} = \frac{1 + \tau^2 S \Phi \Omega_1}{\tau K_1} \quad (\text{A.46})$$

and thus

$$\tilde{P}_1 = \frac{K_1 - H}{K_1} \tilde{U} - \frac{1 + \tau^2 S \Phi \Omega_0}{\tau K_1} \tilde{X}_0 - \frac{1 + \tau^2 S \Phi \Omega_1}{\tau K_1} \tilde{X}_1 \quad (\text{A.47})$$

which can also be written as

$$\tilde{P}_1 = \frac{S \Omega_0 + \tau^2 S^2 \Phi \Omega_0^2}{\tau K_1} \tilde{Q}_0 + \frac{S \Omega_1 + \tau^2 S^2 \Phi \Omega_1^2}{\tau K_1} \tilde{Q}_1 \quad (\text{A.48})$$

Period 0 The problem of investor i at time $t = 0$ is

$$\max_{\tilde{D}_0^i} \mathbb{E} \left[-e^{-\frac{1}{\tau} \tilde{W}_2^i} \middle| \tilde{Z}_0^i, \tilde{Q}_0 \right] \quad (\text{A.49})$$

where the final wealth is given in (A.30). Observe that, at time $t = 0$, investor i needs to estimate \tilde{U} , \tilde{P}_1 and \tilde{D}_1^i , after observing \tilde{Z}_0^i and \tilde{Q}_0 . \tilde{P}_1 and \tilde{D}_1^i are given by (A.47) and (A.35). The maximization problem then becomes:

$$\max_{\tilde{D}_0^i} \mathbb{E} \left[-e^{-\frac{1}{\tau} [X^i \tilde{P}_0 + \tilde{D}_0^i (\tilde{P}_1 - \tilde{P}_0) + \tilde{D}_1^i (\tilde{U} - \tilde{P}_1)]} \middle| \tilde{Z}_0^i, \tilde{Q}_0 \right] \quad (\text{A.50})$$

Lemma 2. *When an agent builds her portfolio, her future number of signals is irrelevant.*

Proof. We prove the claim in a slightly more general context: for the proof only, suppose agent i starts with an arbitrary number n_0 of signals at time $t = 0$. The value function V^i of agent i is then given by

$$\begin{aligned} V^i(n_0, W_0) &= e^{-\frac{1}{\tau} W_0} \max_{\tilde{D}_0^i(n_0)} \mathbb{E} \left[-e^{-\frac{1}{\tau} (\tilde{D}_0^i(n_0) (\tilde{P}_1 - \tilde{P}_0) + \tilde{D}_1^i(n_1) (\tilde{U} - \tilde{P}_1))} \middle| \tilde{Z}_0^i, \tilde{Q}_0 \right] \\ &= e^{-\frac{1}{\tau} W_0} \max_{\tilde{D}_0^i(n_0)} \sum_{k=1}^{\infty} \mu_1(k) \underbrace{\mathbb{E} \left[-e^{-\frac{1}{\tau} (\tilde{D}_0^i(n_0) (\tilde{P}_1 - \tilde{P}_0) + \tilde{D}_1^i(k) (\tilde{U} - \tilde{P}_1))} \middle| \tilde{Z}_0^i, \tilde{Q}_0; n_1 = k \right]}_{g(n_0, k, \tilde{D}_0^i)}. \end{aligned}$$

The function g represents an expectation of an exponential affine quadratic normal variable. To derive its explicit form, we use the following theorem.

Theorem 5. Consider a random vector $z \sim N(0, \Sigma)$. Then,

$$E \left[e^{z'Fz + G'z + H} \right] = |I - 2\Sigma F|^{-\frac{1}{2}} e^{\frac{1}{2}G'(I - 2\Sigma F)^{-1}\Sigma G + H}.$$

Tedious computations then show that

$$g(n_0, k, \tilde{D}_0^i) = -|I - 2\Sigma(n_0, k)F(k)|^{-\frac{1}{2}} e^{\frac{1}{2}G(n_0, k, \tilde{D}_0^i)'(I - 2\Sigma(n_0, k)F(k))^{-1}\Sigma(n_0, k)G(n_0, k, \tilde{D}_0^i) + H(n_0, k, \tilde{D}_0^i)} \quad (\text{A.51})$$

where

$$\begin{aligned} \Sigma(n_0, k) &= \begin{pmatrix} \frac{1}{K_0^i(n_0)} & \frac{K_1 - H}{K_1 K_0^i(n_0)} & \frac{1}{K_0^i(n_0)} - \frac{1}{K_1^i(k)} \\ \frac{K_1 - H}{K_1 K_0^i(n_0)} & \frac{(K_1^i(k) - S(k - n_0))(1 + \tau^2 S\Phi\Omega_{t+1})^2}{K_1^2 K_0^i(n_0)\tau^2\Phi} & \frac{K_1 - H}{K_1 K_0^i(n_0)} \\ \frac{1}{K_0^i(n_0)} - \frac{1}{K_1^i(k)} & \frac{K_1 - H}{K_1 K_0^i(n_0)} & \frac{1}{K_0^i(n_0)} - \frac{1}{K_1^i(k)} \end{pmatrix}, \\ F(k) &= \begin{pmatrix} 0 & \frac{1}{2}K^i(k) & -\frac{1}{2}K^i(k) \\ \frac{1}{2}K^i(k) & -K^i(k) & \frac{1}{2}K^i(k) \\ -\frac{1}{2}K^i(k) & \frac{1}{2}K^i(k) & 0 \end{pmatrix}, \\ G(n_0, k, \tilde{D}_0^i) &= \begin{pmatrix} \frac{K_1^i(k)S(\Omega_0(H + n_0S(1 + \tau^2 S\Omega_0\Phi))\tilde{Q}_0 - K_0n_0\tilde{Z}_0^i)}{K_1 K_0^i(n_0)} \\ -\frac{\tilde{D}_0^i}{\tau} - \frac{2K_1^i(k)S(\Omega_0(H + n_0S(1 + \tau^2 S\Phi\Omega_0))\tilde{Q}_0 - K_0n_0\tilde{Z}_0^i)}{K_1 K_0^i(n_0)} \\ \frac{K_1^i(k)S(\Omega_0(H + n_0S(1 + \tau^2 S\Omega_0\Phi))\tilde{Q}_0 - K_0n_0\tilde{Z}_0^i)}{K_1 K_0^i(n_0)} \end{pmatrix}, \\ H(n_0, k, \tilde{D}_0^i) &= -\frac{\tilde{P}_0}{\tau}\tilde{D}_0^i - \frac{S \begin{pmatrix} K_1^i(k)\tau S((K_0n_0 + H(\Omega_0 - n_0))\tilde{Q}_0 - K_0n_0\tilde{Z}_0^i)^2 \\ +K_1 K_0^i(n_0)\tilde{D}_0^i \begin{pmatrix} (K_0n_0 + K_1\tau^2 S\Phi\Omega_0^2 + H(\Omega_0 - n_0))\tilde{Q}_0 \\ +n_0S\Omega_1(1 + \tau^2 S\Phi\Omega_1)\tilde{Z}_0^i \end{pmatrix} \end{pmatrix}}{\tau K_1^2 K_0^i(n_0)^2}. \end{aligned}$$

Further computations show that

$$h(n_0, k) \equiv |I - 2\Sigma(n_0, k)F(k)| = \frac{K_1^i(k)}{K_1^2 K_0^i(n_0)\tau^2\Phi} \begin{pmatrix} H^2\tau^2\Phi + H(1 + \tau^2\Phi(K_0 + K_1 - 2H + S\Omega_1)) \\ +S \begin{pmatrix} n_0(1 + \tau^2 S\Phi\Omega_1)^2 \\ +\tau^2 S\Phi\Omega_0^2(2 + \tau^2\Phi(K_1 - H + S(\Omega_0 + \Omega_1))) \end{pmatrix} \end{pmatrix}$$

and

$$\begin{aligned} q(n_0, \tilde{D}_0^i) &\equiv \frac{1}{2}G(n_0, k, \tilde{D}_0^i)'(I - 2\Sigma(n_0, k)F(k))^{-1}\Sigma(n_0, k)G(n_0, k, \tilde{D}_0^i) + H(n_0, k, \tilde{D}_0^i) \\ &= \frac{1}{2K_0^i(n_0)\tau^2(H^2\tau^2\Phi + HX + S(n_0Z_1^2 + \tau^2 S\Phi Y\Omega_0^2))} \\ &\quad \times \begin{pmatrix} 2K_0^i(n_0)\tau\tilde{D}_0^i \begin{pmatrix} \tilde{P}_0(H^2\tau^2\Phi + HX + S(n_0Z_1^2 + \tau^2 S\Phi Y\Omega_0^2)) \\ -S(\tau^2\Phi\Omega_0\tilde{Q}_0(H + S\Omega_0(\tau^2\Phi(K_1 + S(\Omega_0 + \Omega_1)) + 2)) + n_0Z_1^2\tilde{Z}_0^i) \end{pmatrix} \\ +K_0^i(n_0)Z_1^2(\tilde{D}_0^i)^2 - \tau^4\Phi(S\Omega_0\tilde{Q}_0(H + n_0SZ_0) - K_0n_0S\tilde{Z}_0^i)^2 \end{pmatrix} \end{pmatrix} \end{aligned}$$

with

$$\begin{aligned} X &= 1 + \tau^2\Phi(K_0 + K_1 - 2H + S\Omega_1), \\ Y &= 2 + \tau^2\Phi(K_1 - H + S(\Omega_0 + \Omega_1)), \end{aligned}$$

$$Z_t = 1 + \tau^2 S \Phi \Omega_t.$$

Plugging these expressions into (A.51), agent i solves

$$V^i(n_0, W_0) = e^{-\frac{1}{\tau} W_0} \underbrace{\left(\sum_{k=1}^{\infty} \mu_1(k) h(n_0, k) \right)}_{\text{anticipation of future signals}} \max_{\tilde{D}_0^i(n_0)} -e^{q(n_0, \tilde{D}_0^i)} \quad (\text{A.52})$$

and it follows that her portfolio decision is independent of her expectation regarding her future number of signals. \square

To obtain agent i 's optimal demand, we solve the problem in (A.52) and impose optimality

$$\frac{\partial}{\partial \tilde{D}_0^i} q(n_0, \tilde{D}_0^i) = 0.$$

We integrate the resulting optimal demand and impose market clearing in order to solve for the undetermined coefficients of \tilde{P}_0 , i.e., β_0 and $\alpha_{0,0}$. The solutions for these coefficients are:

$$\begin{aligned} \beta_0 &= \frac{K_0 - H}{K_0} \\ \alpha_{0,0} &= \frac{1 + \tau^2 S \Phi \Omega_0}{\tau K_0} \end{aligned} \quad (\text{A.53})$$

where

$$K_0 = K_0^i = H + S + \tau^2 S^2 \Phi \Omega_0^2 \quad (\text{A.54})$$

$$\tilde{\mu}_0^i = \frac{1}{K_0} \left(S \tilde{Z}_0^i + \tau^2 S^2 \Phi \Omega_0^2 \tilde{Q}_0 \right) \quad (\text{A.55})$$

We have $K_0 = K_0^i$ in (A.54) because investors start with homogeneous information endowments at time 0, i.e., all have one signal. In other words, the cumulative informational (dis)advantage is zero for all investors: $A_0^i = 0, \forall i$. The optimal demand of investor i at time $t = 0$ is

$$\begin{aligned} \tilde{D}_0^i &= \tau \left(S \omega_0^i \tilde{Z}_0^i - S \Omega_0 \tilde{Q}_0 - (K_0^i - K_0) \tilde{P}_0 \right) \\ &= \tau K_0^i \left(\mathbb{E}_0^i[\tilde{U} | \mathcal{F}_0^i] - \tilde{P}_0 \right) \end{aligned} \quad (\text{A.56})$$

At this point, we can use (A.53) and (A.54) to verify that, indeed, $\frac{\alpha_{0,0}}{\beta_0} = \frac{1}{\tau S \Omega_0}$. Hence, (A.25) is now verified. Then

$$\tilde{P}_0 = \beta_0 \tilde{Q}_0 = \frac{K_0 - H}{K_0} \tilde{U} - \frac{K_0 - H}{K_0} \frac{1}{\tau S \Omega_0} \tilde{X}_0 \quad (\text{A.57})$$

$$= \frac{K_0 - H}{K_0} \tilde{U} - \frac{1 + \tau^2 S \Phi \Omega_0}{\tau K_1} \tilde{X}_0 \quad (\text{A.58})$$

The solution can then be written in a recursive form and extended to more than 2 trading periods, as done in Theorem 1. The recursive form for prices follows from (A.58) and (A.47); the recursive form for individual precisions follows from (A.54) and (A.33); the recursive form for individual demands follows from (A.56) and (A.35)-(A.36).

A.3 Predictability of Returns

In this Appendix, we provide the proof of Propositions 2, 3, 4 and 5 and Corollary 1.

A.3.1 Proof of Proposition 2

We start with the following lemma.

Lemma 3. $X_t^i = K_t(\tilde{P}_t - \tilde{\mu}_t^i)$ is a martingale under agent i 's information set

$$\mathbb{E} \left[X_{t+1}^i | \mathcal{F}_t^i \right] = X_t^i. \quad (\text{A.59})$$

Proof. Compute first the expected stock price tomorrow as

$$\mathbb{E} \left[P_{t+1} | \mathcal{F}_t^i \right] = \frac{K_{t+1} - H}{K_{t+1}} \tilde{\mu}_t^i - \sum_{j=0}^t \frac{1 + \tau^2 S \Omega_j \Phi}{\tau K_{t+1}} \mathbb{E} \left[\tilde{X}_j | \mathcal{F}_t^i \right] \quad (\text{A.60})$$

$$= \frac{K_{t+1} - H}{K_{t+1}} \tilde{\mu}_t^i - \sum_{j=0}^t \frac{S \Omega_j + \tau^2 S^2 \Omega_j^2 \Phi}{K_{t+1}} (\tilde{\mu}_j^i - \tilde{Q}_j) \quad (\text{A.61})$$

$$= \frac{K_{t+1} - K_t}{K_{t+1}} \tilde{\mu}_t^i + \sum_{j=0}^t \frac{S \Omega_j + \tau^2 S^2 \Omega_j^2 \Phi}{K_{t+1}} \tilde{Q}_j \quad (\text{A.62})$$

Moreover, observe that since $\tilde{P}_t \in \mathcal{F}_t^i$, we also have that

$$\tilde{P}_t = \mathbb{E} \left[\tilde{P}_t | \mathcal{F}_t^i \right] = \sum_{j=0}^t \frac{S \Omega_j + \tau^2 S^2 \Omega_j^2 \Phi}{K_t} \tilde{Q}_j \quad (\text{A.63})$$

Multiply (A.60) by K_{t+1} and withdraw $K_{t+1} \tilde{\mu}_{t+1}^i$ to obtain

$$\mathbb{E} \left[K_{t+1} (P_{t+1} - \tilde{\mu}_{t+1}^i) | \mathcal{F}_t^i \right] = -K_t \tilde{\mu}_t^i + \sum_{j=0}^t (S \Omega_j + \tau^2 S^2 \Omega_j^2 \Phi) \tilde{Q}_j, \quad (\text{A.64})$$

which follows from the fact that, by the law of iterated expectations

$$\mathbb{E}[\tilde{\mu}_{t+1}^i | \mathcal{F}_t^i] = \tilde{\mu}_t^i. \quad (\text{A.65})$$

Similarly, multiply (A.63) by K_t and withdraw $K_t \tilde{\mu}_t^i$ to obtain

$$K_t (\tilde{P}_t - \tilde{\mu}_t^i) = -K_t \tilde{\mu}_t^i + \sum_{j=0}^t (S \Omega_j + \tau^2 S^2 \Omega_j^2 \Phi) \tilde{Q}_j \quad (\text{A.66})$$

Clearly, comparing (A.64) and (A.66), $K_t(\tilde{P}_t - \tilde{\mu}_t^i)$ is a martingale under \mathcal{F}_t^i :

$$\mathbb{E} \left[K_{t+1} (P_{t+1} - \tilde{\mu}_{t+1}^i) | \mathcal{F}_t^i \right] = K_t (\tilde{P}_t - \tilde{\mu}_t^i). \quad (\text{A.67})$$

□

Rearranging the martingale relation of Lemma 3, we obtain

$$\mathbb{E} \left[P_{t+1} | \mathcal{F}_t^i \right] = \frac{K_t}{K_{t+1}} \tilde{P}_t + \frac{K_{t+1} - K_t}{K_{t+1}} \tilde{\mu}_t^i \quad (\text{A.68})$$

and Proposition 2 follows.

A.3.2 Proof of Proposition 3

Observe that every price is of the form

$$\tilde{P}_t = \bar{\mathbb{E}}_t[\tilde{U}] - \frac{1}{\tau K_t} \sum_{j=0}^t \tilde{X}_j. \quad (\text{A.69})$$

and thus

$$\tilde{P}_{t+1} - \tilde{P}_t = \left(\bar{\mathbb{E}}_{t+1}[\tilde{U}] - \bar{\mathbb{E}}_t[\tilde{U}] \right) + \frac{K_{t+1} - K_t}{\tau K_t K_{t+1}} \sum_{j=0}^t \tilde{X}_j - \frac{1}{\tau K_{t+1}} \tilde{X}_{t+1} \quad (\text{A.70})$$

which, after replacing $\tilde{X}_j = \tau S \Omega_j (\tilde{U} - \tilde{Q}_j)$, gives Equation (18) in the paper.

A.3.3 Proof of Proposition 4

First, notice that average market expectations at time t can be written as

$$\bar{\mathbb{E}}_t[\tilde{U}] = \frac{\sum_{j=0}^t S \Omega_j}{K_t} \tilde{U} + \frac{K_t - \sum_{j=0}^t S \Omega_j}{K_t} \mathbb{E} \left[\tilde{U} | \mathcal{F}_t^c \right] \quad (\text{A.71})$$

$$\equiv \alpha_t \tilde{U} + (1 - \alpha_t) \mathbb{E} \left[\tilde{U} | \mathcal{F}_t^c \right]. \quad (\text{A.72})$$

Substituting this expression in the first term of (18) and applying the law of iterated expectations with respect to the common information set, \mathcal{F}^c , we obtain

$$\mathbb{E} \left[\bar{\mathbb{E}}_{t+1}[\tilde{U}] | \mathcal{F}_t^c \right] = \mathbb{E} \left[\alpha_{t+1} \tilde{U} + (1 - \alpha_{t+1}) \mathbb{E} \left[\tilde{U} | F_{t+1}^c \right] | \mathcal{F}_t^c \right] \quad (\text{A.73})$$

$$= \alpha_{t+1} \mathbb{E} \left[\tilde{U} | F_t^c \right] + (1 - \alpha_{t+1}) \mathbb{E} \left[\tilde{U} | F_t^c \right] \quad (\text{A.74})$$

$$= \mathbb{E} \left[\tilde{U} | F_t^c \right]. \quad (\text{A.75})$$

Furthermore, notice that

$$\mathbb{E} \left[\bar{\mathbb{E}}_t[\tilde{U}] | \mathcal{F}_t^c \right] = \mathbb{E} \left[\alpha_t \tilde{U} | \mathcal{F}_t^c \right] + (1 - \alpha_t) \mathbb{E} \left[\tilde{U} | F_t^c \right] \quad (\text{A.76})$$

$$= \alpha_t \mathbb{E} \left[\tilde{U} | F_t^c \right] + (1 - \alpha_t) \mathbb{E} \left[\tilde{U} | F_t^c \right] \quad (\text{A.77})$$

$$= \mathbb{E} \left[\tilde{U} | F_t^c \right] \quad (\text{A.78})$$

and thus

$$\mathbb{E} \left[\bar{\mathbb{E}}_{t+1}[\tilde{U}] - \bar{\mathbb{E}}_t[\tilde{U}] | \mathcal{F}_t^c \right] = 0. \quad (\text{A.79})$$

Using (A.79) to compute the common expectation of (18), we finally get

$$= \frac{K_{t+1} - K_t}{K_t K_{t+1}} \sum_{j=0}^t S\Omega_j \mathbb{E} [\tilde{U} - \tilde{Q}_j | \mathcal{F}_t^c], \quad (\text{A.80})$$

which yields the first part of Proposition 4. To obtain the second part of the proposition, apply the law of iterated expectations to (A.68) with respect to \mathcal{F}^c to obtain

$$\mathbb{E} [P_{t+1} | \mathcal{F}_t^c] = \frac{K_t}{K_{t+1}} \tilde{P}_t + \frac{K_{t+1} - K_t}{K_{t+1}} \mathbb{E} [\tilde{U} | \mathcal{F}_t^c] \quad (\text{A.81})$$

and thus

$$\mathbb{E} [P_{t+1} - \tilde{P}_t | \mathcal{F}_t^c] = \frac{K_{t+1} - K_t}{K_{t+1}} \left(\mathbb{E} [\tilde{U} | \mathcal{F}_t^c] - \tilde{P}_t \right). \quad (\text{A.82})$$

Furthermore, observing that

$$\mathbb{E} [\tilde{U} | \mathcal{F}_t^c] = \sum_{j=0}^t \frac{\tau^2 S^2 \Omega_j^2 \Phi}{K_t^c} \tilde{Q}_j \quad (\text{A.83})$$

Moreover, observe that since $\tilde{P}_t \in \mathcal{F}_t^i$, we also have

$$\tilde{P}_t = \mathbb{E} [\tilde{P}_t | \mathcal{F}_t^i] = \sum_{j=0}^t \frac{S\Omega_j + \tau^2 S^2 \Omega_j^2 \Phi}{K_t} \tilde{Q}_j. \quad (\text{A.84})$$

Using (A.84), we can write

$$\mathbb{E} [\tilde{U} | \mathcal{F}_t^c] = \frac{1}{K_t^c} \left(K_t \tilde{P}_t - \sum_{j=0}^t S\Omega_j \tilde{Q}_j \right). \quad (\text{A.85})$$

Substituting this expression in (A.82) and factorizing yields

$$\mathbb{E} [P_{t+1} - \tilde{P}_t | \mathcal{F}_t^c] = \frac{K_{t+1} - K_t}{K_{t+1} K_t^c} \left((K_t - K_t^c) \tilde{P}_t - \sum_{j=0}^t S\Omega_j \tilde{Q}_j \right). \quad (\text{A.86})$$

Finally, observing that $K_t - K_t^c = \sum_{j=0}^t S\Omega_j$, we obtain

$$\mathbb{E} [P_{t+1} - \tilde{P}_t | \mathcal{F}_t^c] = \frac{K_{t+1} - K_t}{K_{t+1} K_t^c} \sum_{j=0}^t S\Omega_j (\tilde{P}_t - \tilde{Q}_j), \quad (\text{A.87})$$

which yields the second part of Proposition 4.

A.3.4 Proof of Proposition 5

Conditioning (A.87) on \mathcal{F}^r requires computing the following expectation

$$\mathbb{E} [\tilde{P}_t - \tilde{Q}_j | \mathcal{F}_t^r], \quad \forall j = 0, \dots, t, \quad (\text{A.88})$$

which amount to derive the recursive relation between the information sets \mathcal{F}^c and \mathcal{F}^r . To do so, we first use (A.84) and obtain

$$\tilde{P}_t = \frac{K_{t-1}}{K_t} \tilde{P}_{t-1} + \frac{K_t - K_{t-1}}{K_t} \tilde{Q}_t, \quad (\text{A.89})$$

from which we obtain

$$\tilde{P}_t - \tilde{Q}_t = \frac{K_{t-1}}{K_t} (\tilde{P}_{t-1} - \tilde{Q}_t) \quad (\text{A.90})$$

$$\tilde{P}_t - \tilde{P}_{t-1} = -\frac{K_t - K_{t-1}}{K_t} (\tilde{P}_{t-1} - \tilde{Q}_t). \quad (\text{A.91})$$

Replacing (A.91) in (A.90), we further get

$$\tilde{P}_t - \tilde{Q}_t = -\frac{K_{t-1}}{K_t} \frac{K_t}{K_t - K_{t-1}} (\tilde{P}_t - \tilde{P}_{t-1}) \quad (\text{A.92})$$

$$= -\frac{K_{t-1}}{K_t - K_{t-1}} (\tilde{P}_t - \tilde{P}_{t-1}). \quad (\text{A.93})$$

Accordingly, the expectation for $j = t$ in (A.88) writes

$$\mathbb{E} [\tilde{P}_t - \tilde{Q}_t | \tilde{P}_t - \tilde{P}_{t-1}] = -\frac{K_{t-1}}{K_t - K_{t-1}} (\tilde{P}_t - \tilde{P}_{t-1}). \quad (\text{A.94})$$

Proceeding similarly for $j = t - 1$, we obtain the following recursive relation

$$\mathbb{E} [\tilde{P}_t - \tilde{Q}_{t-1} | \tilde{P}_t - \tilde{P}_{t-1}] = \mathbb{E} [\tilde{P}_t - \tilde{P}_{t-1} + \tilde{P}_{t-1} - \tilde{Q}_{t-1} | \tilde{P}_t - \tilde{P}_{t-1}] \quad (\text{A.95})$$

$$= 1 + \mathbb{E} [\tilde{P}_{t-1} - \tilde{Q}_{t-1} | \tilde{P}_t - \tilde{P}_{t-1}]. \quad (\text{A.96})$$

Iterating over this recursive relation, the sum in (A.87) can be written as

$$\begin{aligned} \sum_{j=0}^t S\Omega_j (\tilde{P}_t - \tilde{Q}_j) &= \left(\sum_{k=0}^{t-1} S\Omega_k - S\Omega_t \frac{K_{t-1}}{K_t - K_{t-1}} \right) (\tilde{P}_t - \tilde{P}_{t-1}) \\ &+ \left(\sum_{k=0}^{t-2} S\Omega_k - S\Omega_{t-1} \frac{K_{t-2}}{K_{t-1} - K_{t-2}} \right) (\tilde{P}_{t-1} - \tilde{P}_{t-2}) \\ &+ \dots \\ &+ \left(\sum_{k=0}^{j-1} S\Omega_k - S\Omega_j \frac{K_{j-1}}{K_j - K_{j-1}} \right) (\tilde{P}_j - \tilde{P}_{j-1}) \\ &+ \dots \\ &+ \left(S\Omega_0 - S\Omega_1 \frac{K_0}{K_1 - K_0} \right) (\tilde{P}_1 - \tilde{P}_0) \\ &+ \left(-S\Omega_0 \frac{H}{K_0 - H} \right) (\tilde{P}_0 - 0), \end{aligned} \quad (\text{A.97})$$

which pins down the recursive equivalence between \mathcal{F}^c and \mathcal{F}^r . Inspecting (A.97) shows that the coefficient of $(\tilde{P}_{t-l+1} - \tilde{P}_{t-l})$ is:

$$\sum_{k=0}^{t-l} S\Omega_k - S\Omega_{t-l+1} \frac{K_{t-l}}{K_{t-l+1} - K_{t-l}} = \sum_{k=0}^{t-l+1} S\Omega_k - S\Omega_{t-l+1} - S\Omega_{t-l+1} \frac{K_{t-l}}{K_{t-l+1} - K_{t-l}} \quad (\text{A.98})$$

$$= (K_{t-l+1} - K_{t-l+1}^c) - S\Omega_{t-l+1} \frac{K_{t-l+1}}{K_{t-l+1} - K_{t-l}} \quad (\text{A.99})$$

$$= K_{t-l+1} \left(1 - S\Omega_{t-l+1} \frac{1}{K_{t-l+1} - K_{t-l}} \right) - K_{t-l+1}^c \quad (\text{A.100})$$

$$= \frac{\tau^2 S^2 \Omega_{t-l+1}^2 \Phi}{S\Omega_{t-l+1} + \tau^2 S^2 \Omega_{t-l+1}^2 \Phi} K_{t-l+1} - K_{t-l+1}^c \quad (\text{A.101})$$

$$= \frac{\tau^2 S\Omega_{t-l+1} \Phi}{1 + \tau^2 S\Omega_{t-l+1} \Phi} K_{t-l+1} - K_{t-l+1}^c \quad (\text{A.102})$$

and thus the sum in (A.87) can be written recursively as

$$\sum_{j=0}^t S\Omega_j (\tilde{P}_t - \tilde{Q}_j) = \sum_{l=1}^{t+1} \left(\frac{\tau^2 S\Omega_{t-l+1} \Phi}{1 + \tau^2 S\Omega_{t-l+1} \Phi} K_{t-l+1} - K_{t-l+1}^c \right) (\tilde{P}_{t-l+1} - \tilde{P}_{t-l}) \quad (\text{A.103})$$

$$\equiv \sum_{l=1}^{t+1} m_{t-l} (\tilde{P}_{t-l+1} - \tilde{P}_{t-l}) \quad (\text{A.104})$$

and the relation in (25) follows. If $\lambda = 0$, then we have $\Omega_t = 1, \forall t$ and the serial correlation coefficients become:

$$\frac{K_{t+1} - K_t}{K_{t+1} K_t^c} m_{t-l} = - \frac{HS}{(H + (t+1)\tau^2 S^2 \Phi)(H + (t+2)(S + \tau^2 S^2 \Phi))} \quad (\text{A.105})$$

which are always negative and do not depend on the lag l .

A.3.5 Proof of Corollary 1

Using the relation in (25), as sufficient condition for momentum to obtain at lag l is that $m_{t-l} > 0$. Rearranging the expression in (26), we can express this condition as

$$\tau^2 S\Omega_{t-l+1} \Phi (K_{t-l+1} - K_{t-l+1}^c) - K_{t-l+1}^c > 0, \quad (\text{A.106})$$

which implies

$$\Omega_{t-l+1} > \frac{K_{t-l+1}^c}{\tau^2 S\Phi (K_{t-l+1} - K_{t-l+1}^c)}, \quad (\text{A.107})$$

which gives the first relation in Corollary 1. The last part of the claim follows directly from inspecting the last lag $l = t + 1$, which satisfies

$$m_{-1} (\tilde{P}_0 - \tilde{P}_{-1}) \equiv - \frac{H}{1 + \tau^2 S\Phi\Omega_0} \tilde{P}_0. \quad (\text{A.108})$$

For the limit when $t \rightarrow \infty$, we need to compute:

$$\lim_{t \rightarrow \infty} \frac{K_{t+1} - K_t}{K_{t+1}K_t^c} = \lim_{t \rightarrow \infty} \frac{1}{K_t^c} - \lim_{t \rightarrow \infty} \frac{K_t}{K_{t+1}K_t^c} \quad (\text{A.109})$$

The first limit is zero. For the second limit, notice that:

$$\frac{1}{K_{t+1}} < \frac{K_t}{K_{t+1}K_t^c} < \frac{1}{K_t^c} \quad (\text{A.110})$$

and both bounds go to zero as $t \rightarrow \infty$. Thus, the second limit is also zero, and we obtain

$$\lim_{t \rightarrow \infty} \frac{K_{t+1} - K_t}{K_{t+1}K_t^c} = 0 \quad (\text{A.111})$$

A.4 Momentum and Reversal Regions

The proof is organized in three parts. We first prove that there exists a unique threshold such that the momentum condition in (27) is satisfied. Second, we show how this threshold is related to the parameters of the model. Finally, we show that the serial correlation of returns is hump-shaped in the momentum region.

We start by rewriting the momentum condition in (27) as

$$\Omega_{t-l+1} \tau^2 S^2 \Phi \sum_{k=0}^{t-l+1} \Omega_k - H - \tau^2 S^2 \Phi \sum_{k=0}^{t-l+1} \Omega_k^2 > 0 \quad (\text{A.112})$$

using that $K_{t-l+1} - K_{t-l+1}^c > 0, \forall t-l \geq 0$. Furthermore, using Proposition 1, we can write $\Omega_k = e^{\lambda k}$ and thus

$$\sum_{k=0}^{t-l+1} \Omega_k = \frac{e^{\lambda(t-l+2)} - 1}{e^\lambda - 1}, \quad (\text{A.113})$$

$$\sum_{k=0}^{t-l+1} \Omega_k^2 = \frac{e^{2\lambda(t-l+2)} - 1}{e^{2\lambda} - 1}. \quad (\text{A.114})$$

Plugging these expressions into (A.112), we can express the momentum condition as

$$\frac{\tau^2 S^2 \Phi}{e^{2\lambda} - 1} \left(\frac{H + \tau^2 S^2 \Phi}{\tau^2 S^2 \Phi} - \frac{H}{\tau^2 S^2 \Phi} e^{2\lambda} + e^{(t-l+1)\lambda} \left(e^\lambda \left(e^{(t-l+1)\lambda} - 1 \right) - 1 \right) \right) > 0. \quad (\text{A.115})$$

Furthermore, let $a = t - l + 1 \geq 1$. Introducing a monotonic change of variable, $x = e^\lambda$, define the function $f : [1, \infty) \rightarrow \mathbb{R}_+$ as

$$f(x) = \frac{H + \tau^2 S^2 \Phi}{\tau^2 S^2 \Phi} - \frac{H}{\tau^2 S^2 \Phi} x^2 - x^a - x^{a+1} + x^{2a+1}, \quad \forall x \in [1, \infty). \quad (\text{A.116})$$

Accordingly, the momentum condition becomes

$$\frac{\tau^2 S^2 \Phi}{x^2 - 1} f(x) > 0. \quad (\text{A.117})$$

Without information percolation (i.e., $x = 1$), this condition is always violated:

$$\lim_{x \rightarrow 1} \frac{\tau^2 S^2 \Phi}{x^2 - 1} f(x) = -H < 0. \quad (\text{A.118})$$

Hence, the momentum condition is only relevant over the domain $(1, \infty)$. Over this domain, the denominator in (A.117) is always positive and the momentum condition therefore boils down to $f(x) > 0$ for $x > 1$. Furthermore, notice that the function f in (A.116) is a polynomial of order $2a + 1$. Denote a root of this polynomial by x^* :

$$f(x^*) = 0, \quad x^* > 1. \quad (\text{A.119})$$

Our goal now is to show there exists a unique root x^* over the domain $(1, \infty)$. We first derive the following result.

Proposition 9. *If there exists a root in (A.119), then it is unique.*

Proof. To prove this result, we use the following lemma.

Lemma 4. *The number of positive roots of a polynomial with real coefficients is not greater than the number of variations of sign of the polynomial coefficients.*

Proof. The proof follows from Descartes' rule of signs. \square

Notice that the ascending-ordered coefficients of the polynomial in (A.116) have 2 consecutive changes of signs. Using Lemma 4, the function f therefore has at most 2 positive roots, one of which is $f(1) = 0$. Hence, if there exists a root over the domain $(1, \infty)$, it is unique. \square

It is therefore sufficient to establish existence and uniqueness follows. To do so, the following result is convenient.

Proposition 10. *If there exists a root in (A.119), it lies in the domain $(1, \bar{x}]$ with $\bar{x} = \frac{H + 2\Phi\tau^2 S^2}{\Phi\tau^2 S^2}$.*

Proof. To prove this result, we use the following lemma.

Lemma 5. *Let $p(x) = x^n + \sum_{k=0}^{n-1} a_k x^k$ be any polynomial with real coefficients and a leading coefficient of 1. Let $M_1 = 1 + \max\{|a_0|, \dots, |a_{n-1}|\}$ and let $M_2 = \max\left\{1, \sum_{k=0}^{n-1} |a_k|\right\}$. Finally, let $M = \min\{M_1, M_2\}$. Then, every root of $p(x)$ lies between $-M$ and M .*

Proof. The proof follows from the Single Bound Theorem. \square

Applying the above lemma to (A.116), we have

$$M_1 = 1 + \max\left\{\frac{H + \tau^2 S^2 \Phi}{\tau^2 S^2 \Phi}, \frac{H}{\tau^2 S^2 \Phi}, 1\right\} = \frac{H + 2\tau^2 S^2 \Phi}{\tau^2 S^2 \Phi}, \quad (\text{A.120})$$

$$M_2 = \max\left\{1, 2 + \frac{H}{\tau^2 S^2 \Phi} + \frac{H + \tau^2 S^2 \Phi}{\tau^2 S^2 \Phi}\right\} = \frac{2H + 3\tau^2 S^2 \Phi}{\tau^2 S^2 \Phi}. \quad (\text{A.121})$$

Hence,

$$M = \min\left\{\frac{H + 2\tau^2 S^2 \Phi}{\tau^2 S^2 \Phi}, \frac{2H + 3\tau^2 S^2 \Phi}{\tau^2 S^2 \Phi}\right\} = \frac{H + 2\tau^2 S^2 \Phi}{\tau^2 S^2 \Phi} \quad (\text{A.122})$$

and the result follows. \square

We finally provide a condition under which a positive root $x^* > 1$ exists.

Proposition 11. *For every finite coefficient, a , a positive root $x^* > 1$ exists.*

Proof. We first evaluate the function f in (A.116) at \bar{x} , as defined in Proposition 10. To do so, we let $y \equiv \frac{H}{\tau^2 S^2 \Phi}$ and use the Binomial Theorem to write

$$f(\bar{x}) = 1 - y^3 - 4y^2 - 3y - \sum_{k=0}^{a+1} \binom{a+1}{k} 2^{a+1-k} y^k - \sum_{k=0}^a \binom{a}{k} 2^{a-k} y^k + \sum_{k=0}^{2a+1} \binom{2a+1}{k} 2^{2a+1-k} y^k. \quad (\text{A.123})$$

First notice that

$$\sum_{k=0}^a \left(\binom{2a+1}{k} 2^{a+1} - \binom{a+1}{k} 2 - \binom{a}{k} \right) 2^{a-k} y^k - 3y > 0, \quad \forall a \geq 1 \quad (\text{A.124})$$

since

$$\binom{2a+1}{k} \geq \binom{a+1}{k} \geq \binom{a}{k}, \quad \forall 0 \leq k \leq a, \quad \forall a \geq 1. \quad (\text{A.125})$$

and since, for $k = 1$,

$$\left(\binom{2a+1}{1} 2^{a+1} - \binom{a+1}{1} 2 - \binom{a}{1} \right) 2^{a-1} > 3, \quad \forall a \geq 1. \quad (\text{A.126})$$

Furthermore, we have

$$y^{a+1} \left(\binom{2a+1}{a+1} 2^a - 1 \right) - 4y^2 > 0, \quad \forall a \geq 1 \quad (\text{A.127})$$

since

$$\binom{2a+1}{a+1} 2^a > 4, \quad \forall a \geq 1. \quad (\text{A.128})$$

Finally, we have

$$\sum_{k=a+2}^{2a+1} \binom{2a+1}{k} 2^{2a+1-k} y^k \geq y^3, \quad \forall a \geq 1. \quad (\text{A.129})$$

Putting (A.124), (A.127) and (A.129) together, we conclude that

$$f(\bar{x}) > 0. \quad (\text{A.130})$$

Finally, using (A.118), we know that

$$\lim_{x \rightarrow 1^+} \frac{f(x)}{x^2 - 1} < 0. \quad (\text{A.131})$$

Since the function f in (A.116) is a polynomial, it is continuous. Hence, there exists a point \underline{x} arbitrarily close to 1 such that $f(\underline{x}) < 0$. Using the Intermediate Value Theorem, it follows that there

exists a point $x^* \in [x, \bar{x}] \subset (1, \bar{x}]$ such that the condition in (A.119) is satisfied, which concludes the first part of the proof. \square

To prove the second part of the result, we use the following lemma.

Lemma 6. *The function f in (A.116) is strictly increasing at x^* :*

$$\left. \frac{d}{dx} f(x) \right|_{x=x^*} > 0. \quad (\text{A.132})$$

Proof. From Lemma 4 and Proposition 11, we know that the function f crosses zero at exactly two points, $f(1) = 0$ and $f(x^*) = 0$, on \mathbb{R}_+ . Starting at $f(1) = 0$, the function f is decreasing in a neighborhood of $x = 1$:

$$\left. \frac{d}{dx} f(x) \right|_{x=1} = -\frac{2H}{S^2\tau^2\Phi} < 0. \quad (\text{A.133})$$

Moreover, since the function f only crosses 0 once for $x > 1$, namely at $x = x^*$, it must cross 0 from below and the result follows. \square

From (A.116), the root in (A.119) depends on $\mathbf{P} = (H \ S \ \Phi \ \tau \ a)$. We now make this dependence explicit by writing $x^*(\mathbf{P})$ and rewriting (A.119) as

$$f(x^*(\mathbf{P}); \mathbf{P}) = 0. \quad (\text{A.134})$$

Applying the Implicit Function Theorem to (A.134), we obtain

$$\nabla_{\mathbf{P}} x^*(\mathbf{P}) = - \left(\left. \frac{d}{dx} f(x) \right|_{x=x^*} \right)^{-1} \nabla_{\mathbf{P}} f(x^*(\mathbf{P}); \mathbf{P}). \quad (\text{A.135})$$

It remains to pin down the sign of each component of $\nabla_{\mathbf{P}} f(x^*(\mathbf{P}); \mathbf{P})$, which satisfies

$$\nabla_{\mathbf{P}} f(x^*(\mathbf{P}); \mathbf{P}) = \begin{bmatrix} \frac{1-(x^*)^2}{S^2\tau^2\Phi} < 0 \\ \frac{2H((x^*)^2-1)}{S^3\tau^2\Phi} > 0 \\ \frac{H((x^*)^2-1)}{S^2\tau^2\Phi^2} > 0 \\ \frac{2H((x^*)^2-1)}{S^3\tau^3\Phi} > 0 \\ x^a (x(2x^a - 1) - 1) \log(x) > 0 \end{bmatrix}^{\top}. \quad (\text{A.136})$$

Using Lemma 6, we conclude that

$$\frac{\partial}{\partial H} x^*(H, S, \Phi, \tau, a) > 0, \quad (\text{A.137})$$

$$\frac{\partial}{\partial S} x^*(H, S, \Phi, \tau, a) < 0, \quad (\text{A.138})$$

$$\frac{\partial}{\partial \Phi} x^*(H, S, \Phi, \tau, a) < 0, \quad (\text{A.139})$$

$$\frac{\partial}{\partial \tau} x^*(H, S, \Phi, \tau, a) < 0, \quad (\text{A.140})$$

$$\frac{\partial}{\partial a} x^*(H, S, \Phi, \tau, a) < 0. \quad (\text{A.141})$$

Finally, using this result, we obtain a refinement of the upper bound in Proposition 10, which we provide in Corollary 4.

Corollary 4. *The root in (A.119) lies in the domain $\left(1, \frac{H+S^2\tau^2\Phi}{S^2\tau^2\Phi}\right]$.*

Proof. Since $\frac{\partial}{\partial a}x^*(H, S, \Phi, \tau, a) < 0$, it follows that the largest value of the momentum threshold is

$$\max_{a \in \mathbb{N}} x^*(H, S, \Phi, \tau, a) = x^*(H, S, \Phi, \tau, 1). \quad (\text{A.142})$$

In this special case ($a \equiv 1$), the momentum threshold is explicitly given by

$$x^*(H, S, \Phi, \tau, 1) = \frac{H + S^2\tau^2\Phi}{S^2\tau^2\Phi}. \quad (\text{A.143})$$

□

For the last part of the proof, we need to take into account the first term in (25). Using (A.113), we can write this term as

$$\frac{K_{t+1} - K_t}{K_{t+1} \left(K_t - \sum_{j=0}^t \Omega_j S \right)} = \frac{Sx^{t+1} + S^2\tau^2\Phi^2x^{2(t+1)}}{\left(H + S\frac{x^{t+2}-1}{x-1} + S^2\tau^2\Phi\frac{x^{2(t+2)}-1}{x^2-1} \right) \left(H + S^2\tau^2\Phi\frac{x^{2(t+1)}-1}{x^2-1} \right)} \quad (\text{A.144})$$

Based on this expression, define a function $g : [1, \infty) \rightarrow \mathbb{R}_+$ as

$$g(x) = \frac{\tau^2 S^2 \Phi x^{t+1}}{\left(H + S\frac{x^{t+2}-1}{x-1} + S^2\tau^2\Phi\frac{x^{2(t+2)}-1}{x^2-1} \right) \left(H(x^2-1) + S^2\tau^2\Phi(x^{2(t+1)}-1) \right)} \frac{S(1 + S\tau^2\Phi x^{t+1})}{1 + \tau^2 S \Phi x^a}, \quad (\text{A.145})$$

for which we have the following lemma.

Lemma 7. *The function g in (A.145) is strictly decreasing.*

Proof. Consider the first ratio in (A.145) and notice that numerator is of order $t+1$, while the denominator is of order larger than $2(t+1)$ and that both are positive. As a result, the first ratio is strictly decreasing in $x > 1$. Similarly, consider the second ratio in (A.145) and notice that $t+1 \geq a$ and thus the second ratio is also strictly decreasing in $x > 1$. Furthermore, notice that g has an asymptote at $x = 1$. Its slope is infinite and negative at this point. □

Since the serial correlation coefficient at lag l in (25) can be expressed as $g(x)f(x)$, we also need to pin down the behavior of the function f , as we do in the following lemma.

Lemma 8. *The function f in (A.116) is U-shaped over $[1, x^*)$ and strictly increasing over $[x^*, \infty)$.*

Proof. The first part of the claim directly follows from Lemma 6 and its proof. To prove the second part, notice that the derivative of f satisfies

$$\frac{d}{dx}f(x) = (1 + 2a)x^{2a} - ax^{a-1} - (1 + a)x^a - \frac{2H}{\tau^2 S^2 \Phi}x. \quad (\text{A.146})$$

This expression shows that the ascending-ordered coefficients of the derivative of f have only one change of sign. Therefore, using Lemma 4, the derivative of f crosses zero at most once. Since f is U-shaped in $[1, x^*)$, it crosses zero at $x \in [1, x^*)$. From Lemma 6, we finally conclude that it remains positive for all $x > x^*$. □

We know that at $x = x^*$, $f(x^*) = 0$. Since $f(x^* + \epsilon) > 0$ in a small neighborhood $\epsilon > 0$ (Lemma 8) and that g is strictly positive, $f(x + \epsilon)g(x + \epsilon)$ must be positive. As a result, the serial correlation is increasing and strictly positive in the neighborhood of x^* . Finally, observe that

$$\lim_{x \rightarrow \infty} f(x)g(x) = 0. \quad (\text{A.147})$$

The product $f(x)g(x)$ must therefore become decreasing as x grows large. Hence, this product must exhibit at least one hump over $[x^*, \infty)$. Using Lemma 7 and 8, we conclude that this hump is unique since f is strictly increasing, while g is strictly decreasing over $[x^*, \infty)$.

A.5 Trading Strategies

A.5.1 Proof of Proposition 6

Lemma 9. $Y_t^i = K_t(\tilde{P}_t - \tilde{\mu}_t^i)$ is a martingale under agent i 's information set

$$\mathbb{E} \left[Y_{t+1}^i | \mathcal{F}_t^i \right] = Y_t^i. \quad (\text{A.148})$$

Proof. Compute first the expected stock price tomorrow as

$$\mathbb{E} \left[\tilde{P}_{t+1} | \mathcal{F}_t^i \right] = \frac{K_{t+1} - H}{K_{t+1}} \tilde{\mu}_t^i - \sum_{j=0}^t \frac{1 + \tau^2 S \Omega_j \Phi}{\tau K_{t+1}} \mathbb{E} \left[\tilde{X}_j | \mathcal{F}_t^i \right] \quad (\text{A.149})$$

$$= \frac{K_{t+1} - H}{K_{t+1}} \tilde{\mu}_t^i - \sum_{j=0}^t \frac{S \Omega_j + \tau^2 S^2 \Omega_j^2 \Phi}{K_{t+1}} (\tilde{\mu}_j^i - \tilde{Q}_j) \quad (\text{A.150})$$

$$= \frac{K_{t+1} - K_t}{K_{t+1}} \tilde{\mu}_t^i + \sum_{j=0}^t \frac{S \Omega_j + \tau^2 S^2 \Omega_j^2 \Phi}{K_{t+1}} \tilde{Q}_j. \quad (\text{A.151})$$

where the second equality follows from replacing $\tilde{X}_j = \tau S \Omega_j (\tilde{U} - \tilde{Q}_j)$. Moreover, observe that since $\tilde{P}_t \in \mathcal{F}_t^i$ (this is a restatement of Equation A.84),

$$\tilde{P}_t = \mathbb{E} \left[\tilde{P}_t | \mathcal{F}_t^i \right] = \sum_{j=0}^t \frac{S \Omega_j + \tau^2 S^2 \Omega_j^2 \Phi}{K_t} \tilde{Q}_j. \quad (\text{A.152})$$

Multiply (A.151) by K_{t+1} and withdraw $K_{t+1} \tilde{\mu}_{t+1}^i$ to obtain

$$\mathbb{E} \left[K_{t+1} (\tilde{P}_{t+1} - \tilde{\mu}_{t+1}^i) | \mathcal{F}_t^i \right] = -K_t \tilde{\mu}_t^i + \sum_{j=0}^t (S \Omega_j + \tau^2 S^2 \Omega_j^2 \Phi) \tilde{Q}_j, \quad (\text{A.153})$$

which follows from the fact that, by the law of iterated expectations, we have $\mathbb{E}[\tilde{\mu}_{t+1}^i | \mathcal{F}_t^i] = \tilde{\mu}_t^i$. Similarly, multiply (A.152) by K_t and withdraw $K_t \tilde{\mu}_t^i$ to obtain

$$K_t (\tilde{P}_t - \tilde{\mu}_t^i) = -K_t \tilde{\mu}_t^i + \sum_{j=0}^t (S \Omega_j + \tau^2 S^2 \Omega_j^2 \Phi) \tilde{Q}_j. \quad (\text{A.154})$$

Comparing (A.153) and (A.154), $K_t(\tilde{P}_t - \tilde{\mu}_t^i)$ yields the martingale result:

$$\mathbb{E} \left[K_{t+1}(\tilde{P}_{t+1} - \tilde{\mu}_{t+1}^i) | \mathcal{F}_t^i \right] = K_t(\tilde{P}_t - \tilde{\mu}_t^i). \quad (\text{A.155})$$

□

An application of Lemma 9 yields agent i 's expectation regarding the future price:

$$\mathbb{E} \left[\tilde{P}_{t+1} | \mathcal{F}_t^i \right] = \frac{K_t}{K_{t+1}} \tilde{P}_t + \frac{K_{t+1} - K_t}{K_{t+1}} \tilde{\mu}_t^i. \quad (\text{A.156})$$

which is obtained by rearranging (A.155). Reorganize the relation in (A.156) as

$$\tilde{\mu}_t^i = \mathbb{E} \left[\tilde{P}_{t+1} | \mathcal{F}_t^i \right] - \frac{K_t}{K_{t+1}} \tilde{P}_t + \frac{K_t}{K_{t+1}} \tilde{\mu}_t^i, \quad (\text{A.157})$$

and substitute it in individual portfolio demands $\tilde{D}_t^i = \tau K_t^i (\tilde{\mu}_t^i - \tilde{P}_t)$ to obtain

$$\tilde{D}_t^i = \tau K_t^i \left(\mathbb{E} \left[\tilde{P}_{t+1} | \mathcal{F}_t^i \right] - \frac{K_t}{K_{t+1}} \tilde{P}_t + \frac{K_t}{K_{t+1}} \tilde{\mu}_t^i - \tilde{P}_t \right). \quad (\text{A.158})$$

Reorganizing yields the following decomposition

$$\tilde{D}_t^i = \tau K_t^i \frac{K_t}{K_{t+1}} \left(\mathbb{E}[\tilde{U} | \mathcal{F}_t^i] - \tilde{P}_t \right) + \tau K_t^i \left(\mathbb{E}[\tilde{P}_{t+1} | \mathcal{F}_t^i] - \tilde{P}_t \right). \quad (\text{A.159})$$

which can also be written as in Proposition 6:

$$\tilde{D}_t^i = \frac{\tau K_t^i}{K_t} \left(\frac{K_t^2}{K_{t+1}} \left(\mathbb{E}[\tilde{U} | \mathcal{F}_t^i] - \tilde{P}_t \right) + K_t \left(\mathbb{E}[\tilde{P}_{t+1} | \mathcal{F}_t^i] - \tilde{P}_t \right) \right). \quad (\text{A.160})$$

A.5.2 Proof of Proposition 7

Recall that individual demands are given by

$$\tilde{D}_t^i = K_t^i (\tilde{\mu}_t^i - \tilde{P}_t), \quad (\text{A.161})$$

and the martingale condition of Lemma 9 is

$$\mathbb{E} \left[K_{t+1}(\tilde{P}_{t+1} - \tilde{\mu}_{t+1}^i) | \mathcal{F}_t^i \right] = K_t(\tilde{P}_t - \tilde{\mu}_t^i), \quad (\text{A.162})$$

which looks like a condition on rescaled portfolios. In particular, we can write:

$$\mathbb{E} \left[\frac{K_{t+1}}{K_{t+1}^i} \tilde{D}_{t+1}^i | \mathcal{F}_t^i \right] = \frac{K_t}{K_t^i} \tilde{D}_t^i, \quad (\text{A.163})$$

which is the result of Proposition 7.

Proof of Corollary 3 We can write

$$\mathbb{E} \left[\tilde{D}_{t+1}^i | \mathcal{F}_t^i \right] = \tau \mathbb{E} \left[K_{t+1}^i \tilde{\mu}_{t+1}^i - K_{t+1}^i \tilde{P}_{t+1} | \mathcal{F}_t^i \right] \quad (\text{A.164})$$

$$= \tau \mathbb{E} \left[K_{t+1}^i \tilde{\mu}_{t+1}^i | \mathcal{F}_t^i \right] - \tau \mathbb{E} \left[K_{t+1}^i | \mathcal{F}_t^i \right] \mathbb{E} \left[\tilde{P}_{t+1} | \mathcal{F}_t^i \right] \quad (\text{A.165})$$

where the second line follows from the fact that K_{t+1}^i and \tilde{P}_{t+1} are independent conditional on the information set \mathcal{F}_t^i . Using

$$\tilde{\mu}_t^i = \frac{1}{K_t^i} \left(\sum_{j=0}^t S \omega_j^i \tilde{Z}_j^i + \sum_{j=0}^t \Phi \tau^2 S^2 \Omega_j^2 \tilde{Q}_j \right), \quad (\text{A.166})$$

we can express conditional expectations recursively as

$$K_{t+1}^i \tilde{\mu}_{t+1}^i = K_t^i \tilde{\mu}_t^i + S \omega_{t+1}^i \tilde{Z}_{t+1}^i + \Phi \tau^2 S^2 \Omega_{t+1} \tilde{Q}_{t+1}. \quad (\text{A.167})$$

Observe that, since meetings are independent, an agent i expects to collect the average incremental number of signals next period

$$\mathbb{E} \left[\omega_{t+1}^i | \mathcal{F}_t^i \right] = \Omega_{t+1}. \quad (\text{A.168})$$

As a result, we have

$$\mathbb{E} \left[K_{t+1}^i \tilde{\mu}_{t+1}^i | \mathcal{F}_t^i \right] = K_t^i \tilde{\mu}_t^i + S \Omega_{t+1} \mathbb{E} \left[\tilde{Z}_{t+1}^i | \mathcal{F}_t^i \right] + \Phi \tau^2 S^2 \Omega_{t+1} \mathbb{E} \left[\tilde{Q}_{t+1} | \mathcal{F}_t^i \right] \quad (\text{A.169})$$

$$= K_t^i \tilde{\mu}_t^i + S \Omega_{t+1} \tilde{\mu}_t^i + \Phi \tau^2 S^2 \Omega_{t+1} \tilde{\mu}_t^i \quad (\text{A.170})$$

$$= (K_t^i + K_{t+1} - K_t) \tilde{\mu}_t^i \quad (\text{A.171})$$

and

$$\mathbb{E} \left[K_{t+1}^i | \mathcal{F}_t^i \right] = K_t^i + K_{t+1} - K_t. \quad (\text{A.172})$$

Finally, using the relation (A.156), we can write

$$\mathbb{E} \left[\tilde{D}_{t+1}^i | \mathcal{F}_t^i \right] = \tau \left(K_t^i + K_{t+1} - K_t \right) \left(\tilde{\mu}_t^i - \mathbb{E} \left[\tilde{P}_{t+1} | \mathcal{F}_t^i \right] \right) \quad (\text{A.173})$$

$$= \tau \left(K_t^i + K_{t+1} - K_t \right) \left(\tilde{\mu}_t^i - \frac{K_t}{K_{t+1}} \tilde{P}_t - \frac{K_{t+1} - K_t}{K_{t+1}} \tilde{\mu}_t^i \right) \quad (\text{A.174})$$

$$= \tau \left(K_t^i + K_{t+1} - K_t \right) \frac{K_t}{K_{t+1}} \left(\tilde{\mu}_t^i - \tilde{P}_t \right) \quad (\text{A.175})$$

$$= \left(K_t^i + K_{t+1} - K_t \right) \frac{K_t}{K_{t+1}} \frac{1}{K_t^i} \tilde{D}_t^i. \quad (\text{A.176})$$

We can therefore compute

$$\mathbb{E} \left[\tilde{D}_{t+1}^i - \tilde{D}_t^i | \mathcal{F}_t^c \right] = \left(\left(K_t^i + K_{t+1} - K_t \right) \frac{K_t}{K_{t+1}} \frac{1}{K_t^i} - 1 \right) \mathbb{E} \left[\tilde{D}_t^i | \mathcal{F}_t^c \right] \quad (\text{A.177})$$

$$= \left(\left(K_t^i + K_{t+1} - K_t \right) \frac{K_t}{K_{t+1}} - K_t^i \right) \tau \left(\mathbb{E}[\tilde{U} | \mathcal{F}_t^c] - \tilde{P}_t \right) \quad (\text{A.178})$$

$$= \frac{(K_t - K_t^i)(K_{t+1} - K_t)}{K_{t+1}} \tau \left(\mathbb{E}[\tilde{U} | \mathcal{F}_t^c] - \tilde{P}_t \right). \quad (\text{A.179})$$

where the second line follows by the law of iterated expectations.

A.5.3 Proof of Proposition 8

We can write

$$\mathbb{E}[\tilde{U}|\mathcal{F}_t^c] - \tilde{P}_t = \frac{1}{K_t^c} \sum_{j=0}^t \Phi \tau^2 S^2 \Omega_j^2 (\tilde{Q}_j - \tilde{P}_t) - \frac{H}{K_t^c} \tilde{P}_t, \quad (\text{A.180})$$

knowing that the precision of the econometrician equals $K_t^c = H + \sum_{j=0}^t \Phi \tau^2 S^2 \Omega_j^2$. The price \tilde{P}_t can be written recursively as follows:

$$\tilde{P}_t = \sum_{j=0}^t \frac{S \Omega_j + S^2 \Omega_j^2 \Phi \gamma^2}{K_t} \tilde{Q}_j \quad (\text{A.181})$$

$$= \frac{K_{t-1}}{K_t} \tilde{P}_{t-1} + \frac{K_t - K_{t-1}}{K_t} \tilde{Q}_t \quad (\text{A.182})$$

and thus

$$\tilde{Q}_t - \tilde{P}_t = \frac{K_{t-1}}{K_t - K_{t-1}} (\tilde{P}_t - \tilde{P}_{t-1}) \quad (\text{A.183})$$

Equation (A.183) can be replaced in the following recursive form for $\tilde{Q}_j - \tilde{P}_t$:

$$\tilde{Q}_j - \tilde{P}_t = (\tilde{Q}_j - \tilde{P}_j) - \sum_{k=j+1}^t (\tilde{P}_k - \tilde{P}_{k-1}) \quad (\text{A.184})$$

and thus (A.180) becomes

$$\mathbb{E}[\tilde{U}|\mathcal{F}_t^c] - \tilde{P}_t = \frac{1}{K_t^c} \sum_{j=0}^t \Phi \tau^2 S^2 \Omega_j^2 \left[(\tilde{Q}_j - \tilde{P}_j) - \sum_{k=j+1}^t (\tilde{P}_k - \tilde{P}_{k-1}) \right] - \frac{H}{K_t^c} \tilde{P}_t \quad (\text{A.185})$$

$$= \frac{1}{K_t^c} \sum_{j=0}^t \Phi \tau^2 S^2 \Omega_j^2 \left[\frac{K_{j-1}}{K_j - K_{j-1}} (\tilde{P}_j - \tilde{P}_{j-1}) - \sum_{k=j+1}^t (\tilde{P}_k - \tilde{P}_{k-1}) \right] - \frac{H}{K_t^c} \tilde{P}_t \quad (\text{A.186})$$

The price \tilde{P}_t can also be written recursively:

$$\tilde{P}_t = \sum_{j=0}^t (\tilde{P}_j - \tilde{P}_{j-1}) \quad (\text{A.187})$$

and then

$$\mathbb{E}[\tilde{U}|\mathcal{F}_t^c] - \tilde{P}_t = \frac{1}{K_t^c} \sum_{j=0}^t \left(\frac{\tau^2 S \Omega_j \Phi}{1 + \tau^2 S \Omega_j \Phi} K_j - K_j^c \right) (\tilde{P}_j - \tilde{P}_{j-1}) \quad (\text{A.188})$$

which can be alternatively written as

$$\mathbb{E}[\tilde{U}|\mathcal{F}_t^c] - \tilde{P}_t = \frac{1}{K_t^c} \sum_{l=1}^{t+1} \left(\frac{\tau^2 S \Omega_{t-l+1} \Phi}{1 + \tau^2 S \Omega_{t-l+1} \Phi} K_{t-l+1} - K_{t-l+1}^c \right) (\tilde{P}_{t-l+1} - \tilde{P}_{t-l}) \quad (\text{A.189})$$

and thus

$$\mathbb{E} \left[\tilde{D}_{t+1}^i - \tilde{D}_t^i | \mathcal{F}_t^c \right] = \tau(K_t - K_t^i) \sum_{l=1}^{t+1} \frac{(K_{t+1} - K_t)}{K_{t+1} K_t^c} m_{t-l} (\tilde{P}_{t-l+1} - \tilde{P}_{t-l}), \quad (\text{A.190})$$

with

$$m_{t-l} \equiv \frac{\tau^2 S \Omega_{t-l+1} \Phi}{1 + \tau^2 S \Omega_{t-l+1} \Phi} K_{t-l+1} - K_{t-l+1}^c \quad (\text{A.191})$$

A.6 Risk-Neutral Arbitrageur

In this appendix, we derive equilibrium solutions for prices and optimal demands in the presence of an unconstrained, uninformed, risk-neutral arbitrageur, which we summarize in Theorem 6 below.

Theorem 6. *There exists a partially revealing rational expectations equilibrium in the 4 trading session economy in which the price signal, \tilde{Q}_t , for $t = 0, \dots, 3$, satisfies*

$$\tilde{Q}_t = \tilde{U} - \frac{1}{r S \Omega_t} \tilde{X}_t \quad (\text{A.192})$$

and in which the arbitrageur's demand \tilde{x}_t satisfies

$$\tilde{x}_t = \frac{1}{2\lambda_t} \left(E \left[\tilde{P}_{t+1} | \{\tilde{Q}_j\}_{j=0}^t \right] - \varphi_t \tilde{Q}_t - \sum_{j=0}^{t-1} \xi_{j,t} \tilde{Q}_j \right). \quad (\text{A.193})$$

The price coefficients satisfy

$$\varphi_t = \frac{A_t - r S \sum_{j=0}^{t-1} \Omega_j}{D_t}, \quad \xi_{j,t} = \frac{B_{j,t} + r S \Omega_j}{D_t}, \quad \gamma_t = \frac{C_t + 1}{D_t}, \quad \lambda_t = \frac{1}{D_t} \quad (\text{A.194})$$

where A , B_j , C , and D correspond to the coefficients of the aggregate demand of informed traders

$$\int_{\omega_t^i} \tilde{D}_t^i = A_t \tilde{U} + \sum_{j=0}^{t-1} B_{j,t} \tilde{Q}_j - C_t \tilde{X}_t - D_t \tilde{P}_t. \quad (\text{A.195})$$

Proof. We provide the proof for a two trading session economy. The model is solved backwards, starting from date 1 and then going back to date 0. First, conjecture that prices in period 0 and period 1 are

$$\tilde{P}_0 = \beta_0 \tilde{U} - \gamma_{0,0} \tilde{X}_0 + \lambda_0 \tilde{x}_0 \quad (\text{A.196})$$

$$\tilde{P}_1 = \beta_1 \tilde{U} - \gamma_{1,0} \tilde{X}_0 - \gamma_{1,1} \tilde{X}_1 + \lambda_1 \tilde{x}_1 \quad (\text{A.197})$$

Consider the normalized price signal in period zero (which is informationally equivalent to \tilde{P}_0):

$$\tilde{Q}_0 = \frac{1}{\beta_0} (\tilde{P}_0 - \lambda_0 \tilde{x}_0) = \tilde{U} - \frac{\gamma_{0,0}}{\beta_0} \tilde{X}_0 \quad (\text{A.198})$$

where the demand of the risk-neutral trader is observable because she only trades on public information, i.e., prices. Replace \tilde{X}_0 from (A.198) into (A.196) to obtain

$$\tilde{P}_1 = \varphi_1 \tilde{U} + \xi_1 \tilde{Q}_0 - \gamma_{1,1} \tilde{X}_1 + \lambda_1 \tilde{x}_1 \quad (\text{A.199})$$

where $\varphi_1 = \beta_1 - \gamma_{1,0} \frac{\beta_0}{\gamma_{0,0}}$ and $\xi_1 = \gamma_{1,0} \frac{\beta_0}{\gamma_{0,0}}$. We normalize the price signal in period $t = 1$ and obtain \tilde{Q}_1 :

$$\tilde{Q}_1 = \frac{1}{\varphi_1} \left(\tilde{P}_1 - \xi_1 \tilde{Q}_0 - \lambda_1 \tilde{x}_1 \right) = \tilde{U} - \frac{\gamma_{1,1}}{\varphi_1} \tilde{X}_1 \quad (\text{A.200})$$

Observing $\{\tilde{Q}_0, \tilde{Q}_1\}$ is equivalent with observing $\{\tilde{P}_0, \tilde{P}_1\}$. As in the setup of Section 2.2, we conjecture the following relationships:

$$\tilde{Q}_0 = \tilde{U} - \frac{1}{\tau S \Omega_0} \tilde{X}_0 \quad (\text{A.201})$$

$$\tilde{Q}_1 = \tilde{U} - \frac{1}{\tau S \Omega_1} \tilde{X}_1 \quad (\text{A.202})$$

Period 1 At time $t = 1$, both the precision and the posterior mean of an investor i remain identical to those of Section 2.2 in (A.34) along with her demand in (A.35). Integrating informed agents' demand again yields (A.38). The risk-neutral agent solves

$$\max_{\tilde{x}_1} \tilde{x}_1 E[\tilde{U} - \tilde{P}_1 | \tilde{Q}_0, \tilde{Q}_1] = \max_{\tilde{x}_1} \tilde{x}_1 E[\tilde{U} - \varphi_1 \tilde{Q}_1 - \xi_1 \tilde{Q}_0 - \lambda_1 \tilde{x}_1 | \tilde{Q}_0, \tilde{Q}_1]. \quad (\text{A.203})$$

and her optimal demand satisfies

$$\tilde{x}_1 = \frac{1}{2\lambda_1} \left(E[\tilde{U} | \tilde{Q}_0, \tilde{Q}_1] - \varphi_1 \tilde{Q}_1 - \xi_1 \tilde{Q}_0 \right)$$

where

$$E[\tilde{U} | \tilde{Q}_0, \tilde{Q}_1] = \frac{\tau^2 S^2 \Phi \left(\Omega_0^2 \tilde{Q}_0 + \Omega_1^2 \tilde{Q}_1 \right)}{H + \tau^2 S^2 \Phi (\Omega_0^2 + \Omega_1^2)}. \quad (\text{A.204})$$

The market clearing condition is $\tilde{D}_1 + \tilde{x}_1 = \tilde{X}_0 + \tilde{X}_1$. Once we impose market clearing, we can use the conjectured equation (A.199) to get the undetermined coefficients φ_1 , ξ_1 , $\gamma_{1,1}$, and λ_1 :

$$\begin{aligned} \varphi_1 &= \frac{S \Omega_1 (1 + \tau^2 S \Phi \Omega_1)}{K_1}, \\ \xi_1 &= \frac{S \Omega_0 (1 + \tau^2 S \Phi \Omega_0)}{K_1}, \\ \gamma_{1,1} &= \frac{1 + \tau^2 S \Phi \Omega_1}{\tau K_1}, \\ \lambda_1 &= \frac{1}{\tau K_1} \end{aligned} \quad (\text{A.205})$$

From these solutions, we can verify that, indeed, $\frac{\gamma_{1,1}}{\varphi_1} = \frac{1}{\tau S \Omega_1}$. Hence, (A.26) is also verified in the presence of the risk-neutral agent.

Period 0 The problem of investor i at time $t = 0$ is, as in Section 2.2,

$$\max_{\tilde{D}_0^i} \mathbb{E} \left[-e^{-\frac{1}{\tau} \tilde{W}_2^i} \middle| \tilde{Z}_0^i, \tilde{Q}_0 \right] \quad (\text{A.206})$$

where the expectation now takes into account the new price function in (A.199). Importantly, Lemma 2 still holds and informed agents' portfolio remains independent of the expected number of signals they will get in the future. The risk-neutral agent solves

$$\max_{\tilde{x}_0} \tilde{x}_0 E[\tilde{P}_1 - \tilde{P}_0 | \tilde{Q}_0] = \max_{\tilde{x}_0} \tilde{x}_0 E[\tilde{P}_1 - \beta_0 \tilde{Q}_0 - \lambda_0 \tilde{x}_0 | \tilde{Q}_0]. \quad (\text{A.207})$$

The optimization problem in (A.207) only involves the profits of period 0 because we assume that the risk-neutral agent does not take into account that a deviation from her strategy will affect current and future price signals for informed agents who cannot detect a deviation in her strategy; in that sense, the risk-neutral agent is myopic. As a result, her optimal demand satisfies

$$\tilde{x}_0 = \frac{1}{\lambda_0} \left(E[\tilde{P}_1 | \tilde{Q}_0] - \beta_0 \tilde{Q}_0 \right) \quad (\text{A.208})$$

where

$$E[\tilde{P}_1 | \tilde{Q}_0] = \frac{S \left(2\tau^2 S \Phi + \frac{H}{H + S(\Omega_0 + \Omega_1 + \tau^2 S \Phi (\Omega_0^2 + \Omega_1^2))} \right)}{2(H + \tau^2 S^2 \Phi \Omega_0^2)} \quad (\text{A.209})$$

Integrating informed investors' optimal demand and imposing market clearing $D_0 + \tilde{x}_0 = \tilde{X}_0$, we obtain β_0 , $\gamma_{0,0}$, and λ_0 . We can then verify that, indeed, $\frac{\gamma_{0,0}}{\beta_0} = \frac{1}{\tau S \Omega_0}$. By induction, the solution of the equilibrium for the for trading dates takes the form in Theorem 6. \square

Arbitrageur's Profits In this section, we show that the arbitrageur can only make profits if she allows momentum to persist. She therefore optimally forgoes profits. To illustrate this, we compute the unconditional profits Π she expects to make between time t and $t + 1$. In particular, simple computations show that

$$\Pi = E \left[\tilde{x}_t \left(\tilde{P}_{t+1} - \tilde{P}_t \right) \right] = \frac{1}{\lambda_t} E \left[\left(E \left[\tilde{P}_{t+1} - \tilde{P}_t | \{\tilde{Q}_j\}_{j=0}^t \right] \right)^2 \right] = \lambda_t E \left[\tilde{x}_t^2 \right].$$

We then compare these profits to those of an econometrician, who is not strategic and ignores price impact ($\lambda_t \equiv 1$). The profits that the arbitrageur optimally forgoes to keep momentum in the model are therefore given by

$$\Pi' = E \left[\left(E \left[\tilde{P}_{t+1} - \tilde{P}_t | \{\tilde{Q}_j\}_{j=0}^t \right] \right)^2 \right] - \Pi = \left(1 - \frac{1}{\lambda_t} \right) E \left[\left(E \left[\tilde{P}_{t+1} - \tilde{P}_t | \{\tilde{Q}_j\}_{j=0}^t \right] \right)^2 \right]. \quad (\text{A.210})$$

We plot the profits she makes and the profits she forgoes at time $t = 1$ in Figure 8. For low meeting intensities, Figure 8 shows that the arbitrageur extracts half of the momentum rents, consistent with the behavior of a monopolist. As the meeting intensity increases, however, the momentum profits she forgoes significantly increase. The reason is that she now trades against agents who are better informed on average; accordingly, she has a larger price impact and therefore trades less aggressively on momentum. We conclude that it is difficult to arbitrage away momentum in a market characterized by fast diffusion of information among investors.

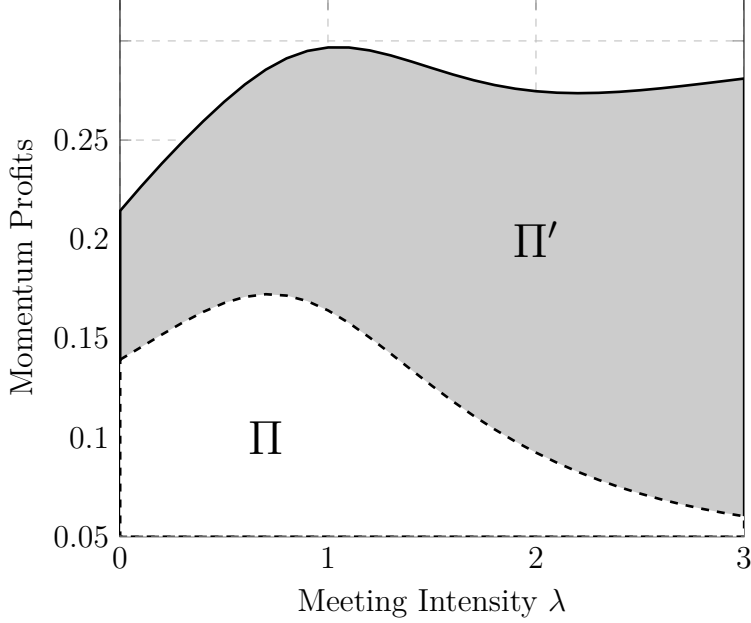


Figure 8: Profits of the Arbitrageur as a Function of λ

The solid line represents the total profits on momentum in the model, as a function of the meeting intensity λ . The dashed line represents the profits made by the arbitrageur. The shaded area represents the profits that arbitrageur forgoes. The calibration is $H = S = \Phi = 1$ and $\tau = \frac{1}{3}$.

A.7 Dynamic Setup

This Appendix mainly follows [Andrei \(2013\)](#). Consider the following processes for dividends and noisy supply:

$$D_t = \kappa_d D_{t-1} + \varepsilon_t^d \quad (\text{A.211})$$

$$X_t = \kappa_x X_{t-1} + \varepsilon_t^x \quad (\text{A.212})$$

where $0 \leq \kappa_d \leq 1$ and $0 \leq \kappa_x \leq 1$. The dividend and supply innovations are *i.i.d.* with normal distributions: $\varepsilon_t^d \sim \mathcal{N}(0, 1/H)$ and $\varepsilon_t^x \sim \mathcal{N}(0, 1/\Phi)$. There is one riskless bond assumed to have an infinitely elastic supply at positive constant gross interest rate R .

The economy is populated by a continuum of rational agents, indexed by i , with CARA utilities and common risk aversion $1/\tau$. Each agent lives for two periods, while the economy goes on forever (overlapping generations). All investors observe the past and current realizations of dividends and of the stock prices. Additionally, each investor observe an information signal about the dividend innovation 3-steps ahead:

$$\tilde{z}_t^i = \varepsilon_{t+3}^d + \tilde{\varepsilon}_t^i \quad (\text{A.213})$$

As time goes by, investors share their private information at random meetings. The information structure and the probability density function over the number of private signals is described in [Andrei \(2013\)](#). As usual in noisy rational expectations, we conjecture a linear function of model

innovations for the equilibrium price:

$$P_t = \alpha D_t + \beta X_{t-3} + (a_3 \ a_2 \ a_1) \epsilon_t^d + (b_3 \ b_2 \ b_1) \epsilon_t^x \quad (\text{A.214})$$

Proposition 1 in [Andrei \(2013\)](#) describes the rational expectations equilibrium, which is found by solving a fixed point problem provided by the market clearing condition. Infinite horizon models with overlapping generations have multiple equilibria (there are 2^N equilibria for a model with N assets). The model studied here has 2 equilibria, one low volatility equilibrium and one high volatility equilibrium. We focus on the low volatility equilibrium, which is the limit of the unique equilibrium in the finite version of the model.

To understand how the two equilibria arise, let's assume that there is no private information. In this case, the equilibrium price has a closed form solution:

$$P_t = \frac{\kappa_d}{R - \kappa_d} D_t - \frac{\Sigma}{\tau} \frac{\kappa_x^3}{R - \kappa_x} X_{t-3} - \frac{\Sigma}{\tau} \frac{\kappa_x^2}{R - \kappa_x} \epsilon_{t-2}^x - \frac{\Sigma}{\tau} \frac{\kappa_x}{R - \kappa_x} \epsilon_{t-1}^x - \frac{\Sigma}{\tau} \frac{1}{R - \kappa_x} \epsilon_t^x \quad (\text{A.215})$$

where $\Sigma \equiv (\alpha + 1)^2 \sigma_d^2 + b_1^2 \sigma_x^2$. Thus, the coefficient b_1 has to solve a quadratic equation:

$$b_1 = -\frac{\tau}{R - \kappa_x} \left[\left(\frac{R}{R - \kappa_d} \right)^2 \sigma_d^2 + b_1^2 \sigma_x^2 \right] \quad (\text{A.216})$$

For different parameter values, the above quadratic equation can have two solutions, one solution, or none. In this particular example (no private information), the autocovariance of stock returns, $\text{Cov}(P_{t+1} - P_t, P_{t+2} - P_{t+1})$, is

$$\begin{aligned} \text{Cov}(P_{t+1} - P_t, P_{t+2} - P_{t+1}) = & -\alpha^2 \sigma_d^2 \frac{1 - \kappa_d}{1 + \kappa_d} + \beta^2 (\kappa_x - 1)^2 \kappa_x \frac{\sigma_x^2}{1 - \kappa_x^2} + \\ & \begin{pmatrix} \beta - b_3 & b_3 - b_2 & b_2 - b_1 & b_1 \end{pmatrix} \begin{pmatrix} -\beta(1 - \kappa_x) \\ \beta - b_3 \\ b_3 - b_2 \\ b_2 - b_1 \end{pmatrix} \end{aligned} \quad (\text{A.217})$$

It can be shown numerically that this covariance is generally negative when $\kappa_d < 1$ and $\kappa_x < 1$. In the random walk specification [\(A.211\)](#) - [\(A.212\)](#), the covariance is zero.

If agents receive private information, the model has to be solved numerically using the methodology described in [Andrei \(2013\)](#). More precisely, α , β , a , and b solve the following equations:

$$(\alpha + 1)\kappa_d - R\alpha = 0 \quad (\text{A.218})$$

$$\bar{K}_t \beta \kappa_x - \bar{K}_t R \beta - \frac{1}{\tau} \kappa_x^3 = 0 \quad (\text{A.219})$$

$$\bar{K}_t b^* \mathbb{B}^{-1} \mathbb{A} + \bar{L}_t \mathbb{H} - \bar{K}_t R a = \mathbf{0}_{1 \times 3} \quad (\text{A.220})$$

$$\bar{K}_t b^* + \bar{L}_t \mathbb{B}^* - \bar{K}_t R b - \frac{1}{\tau} \begin{pmatrix} \kappa_x^2 & \kappa_x & 1 \end{pmatrix} = \mathbf{0}_{1 \times 3} \quad (\text{A.221})$$

where \bar{K}_t , b^* , \mathbb{B} , \mathbb{A} , \bar{L}_t , \mathbb{H} , and \mathbb{B}^* are defined in Appendix A.3 of [Andrei \(2013\)](#).

A.8 Rumors

To prove Theorem 3, we adapt the expression for the price \tilde{P} in Brennan and Cao (1997) and write

$$\tilde{P}_t = \beta_t \tilde{U} + \alpha_t \tilde{V} + \sum_{j=0}^{t-1} \xi_{j,t} \tilde{Q}_j - \gamma_t \tilde{X}_t. \quad (\text{A.222})$$

The price is informationally equivalent to

$$\tilde{Q}_t = \frac{1}{\beta_t} \tilde{P}_t - \sum_{j=0}^{t-1} \xi_{j,t} \tilde{Q}_j = \tilde{U} + \frac{\alpha_t}{\beta_t} \tilde{V} - \frac{\gamma_t}{\beta_t} \tilde{X}_t. \quad (\text{A.223})$$

Furthermore, we can write agent i 's individual demand as

$$\tilde{D}_t^i = \omega_t^i \tilde{P}_t + \sum_{k=0}^t \lambda_k^i \tilde{Q}_k + \sum_{k=0}^t \theta_k^i \tilde{Z}_k^i. \quad (\text{A.224})$$

By the law of large numbers, we have that $\int_{i \in [0,1]} \tilde{Z}_k^i d\mu(i) = \tilde{U} + \tilde{V}$. As a result, when we aggregate individual demands, we obtain

$$\int_{i \in [0,1]} \tilde{D}_t^i d\mu(i) = \bar{\omega}_t \tilde{P}_t + \sum_{k=0}^t \bar{\lambda}_k \tilde{Q}_k + \sum_{k=0}^t \bar{\theta}_k \tilde{U} + \sum_{k=0}^t \bar{\theta}_k \tilde{V}$$

where $\bar{\omega}_t = \sum_{\mathbb{N}} \pi_t(k) \omega_t^i(k)$, $\bar{\lambda}_t = \sum_{\mathbb{N}} \pi_t(k) \lambda_t^i(k)$, and $\bar{\theta}_t = \sum_{\mathbb{N}} \pi_t(k) \theta_t^i(k)$.

Imposing market clearing, we have

$$\sum_{k=0}^t \tilde{X}_k - \sum_{k=0}^t \bar{\theta}_k \tilde{U} - \sum_{k=0}^t \bar{\theta}_k \tilde{V} = \bar{\omega}_t \tilde{P}_t + \sum_{k=0}^t \bar{\lambda}_k \tilde{Q}_k. \quad (\text{A.225})$$

Substituting

$$\tilde{P}_t = \beta_t \tilde{Q}_t + \sum_{j=0}^{t-1} \xi_{j,t} \tilde{Q}_j \quad (\text{A.226})$$

into the above equation, we obtain

$$\sum_{k=0}^t \tilde{X}_k - \sum_{k=0}^t \bar{\theta}_k \tilde{U} - \sum_{k=0}^t \bar{\theta}_k \tilde{V} = \bar{\omega}_t (\beta_t \tilde{Q}_t + \sum_{j=0}^{t-1} \xi_{j,t} \tilde{Q}_j) + \sum_{k=0}^t \bar{\lambda}_k \tilde{Q}_k. \quad (\text{A.227})$$

Furthermore, notice that

$$\tilde{X}_k = \frac{\beta_k}{\gamma_k} \left(\tilde{U} + \frac{\alpha_k}{\beta_k} \tilde{V} - \tilde{Q}_k \right). \quad (\text{A.228})$$

Substituting and regrouping, we obtain

$$\tilde{X}_t + \sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} \left(\tilde{U} + \frac{\alpha_k}{\beta_k} \tilde{V} - \tilde{Q}_k \right) - \sum_{k=0}^t \bar{\theta}_k \tilde{U} - \sum_{k=0}^t \bar{\theta}_k \tilde{V} = (\bar{\omega}_t \beta_t + \bar{\lambda}_t) \tilde{Q}_t + \sum_{j=0}^{t-1} (\xi_{j,t} \bar{\omega}_t + \bar{\lambda}_j) \tilde{Q}_j,$$

or, equivalently,

$$\tilde{X}_t + \left(\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} - \bar{\theta}_k \right) \tilde{U} + \left(\sum_{k=0}^{t-1} \frac{\beta_k \alpha_k}{\gamma_k \beta_k} - \sum_{k=0}^t \bar{\theta}_k \right) \tilde{V} - \sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} \tilde{Q}_k \equiv f(\{\tilde{Q}_j\}_{j=0}^t). \quad (\text{A.229})$$

The right-hand side of this equation is only a function of $\{\tilde{Q}_j\}_{j=0}^t$. By separation of variables, the left-hand side must also be a function of $\{\tilde{Q}_j\}_{j=0}^t$ only. Hence, it must be that

$$-\left(\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} - \bar{\theta}_k \right) \left(\tilde{U} + \frac{\sum_{k=0}^{t-1} \frac{\beta_k \alpha_k}{\gamma_k \beta_k} - \sum_{k=0}^t \bar{\theta}_k}{\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} - \bar{\theta}_k} \tilde{V} - \frac{1}{\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} - \bar{\theta}_k} \tilde{X}_t \right) \equiv \tilde{Q}_t. \quad (\text{A.230})$$

This equality holds if and only if

$$\frac{\sum_{k=0}^{t-1} \frac{\beta_k \alpha_k}{\gamma_k \beta_k} - \sum_{k=0}^t \bar{\theta}_k}{\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} - \bar{\theta}_k} = \frac{\alpha_t}{\beta_t} \quad (\text{A.231})$$

and

$$\frac{1}{\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} - \bar{\theta}_k} = \frac{\gamma_t}{\beta_t}. \quad (\text{A.232})$$

Without loss of generality, we set

$$\frac{\gamma_t}{\beta_t} = \frac{1}{rS\Omega_t} \quad (\text{A.233})$$

so that

$$\sum_{k=0}^t (\bar{\theta}_k) = \sum_{k=0}^t \frac{\beta_k}{\gamma_k} = rS \sum_{k=0}^t \Omega_k. \quad (\text{A.234})$$

and

$$\frac{\alpha_t}{\beta_t} = \frac{\eta \Lambda_t}{rS\Omega_t} \quad (\text{A.235})$$

so that:

$$\eta \sum_{k=0}^t \bar{\theta}_k = \sum_{k=0}^t \frac{\beta_k \alpha_k}{\gamma_k \beta_k} = \sum_{k=0}^t \Lambda_k. \quad (\text{A.236})$$

The system of equations in (43) follows. This system of equations is a fixed point: to solve it, we solve the problem recursively (as in Appendix A.2, except accounting for the rumor) over 4 periods. We then start with guess values for $\{\Omega_j\}_{j=0}^3$ and $\{\Lambda_j\}_{j=0}^3$ and get, through the fixed point in (43), new values for these coefficients. Iterating and invoking the Contraction-Mapping Theorem, we obtain the equilibrium coefficients.