Aggregate Consequences of Dynamic Credit Relationships*

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Abstract

Which financial frictions matter in the aggregate? This paper presents a general equilibrium model in which entrepreneurs finance a firm with a long-term contract. The contract is constrained efficient because firm revenue is costly to monitor and entrepreneurs may default. The cost of monitoring firms and the entrepreneurs' outside options determine the significance of moral hazard relative to limited enforcement for financial contracting. Calibrating the model to the U.S. economy, I find that the relative welfare loss from financial frictions is about 5 percent in terms of aggregate consumption with moral hazard, while it is 1 percent with limited enforcement. Reforms designed to strengthen contract enforcement increase aggregate consumption in the short-run, but their long-run effects are modest when monitoring costs are high. Weak contract enforcement contributes to aggregate fluctuations by amplifying the effect of aggregate technological shocks, but moral hazard does not.

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Introduction

Which financial frictions matter for the determination of macroeconomic outcomes, such as resource allocation, economic development, and business-cycle fluctuations? A large microeconomic literature on firm and industry dynamics has studied financial frictions that arise when information about firms is expensive to acquire, and those that arise when financial contracts are difficult to enforce. The cost of acquiring information and limits to contract enforceability create incentives that give rise to different types of financial contracts [Levine, 2005]. These financial contracts shape firms' input decisions, and, in the aggregate, the efficiency at which the economy transforms savings into investment. While there is broad consensus that financing frictions make it costly for firms to raise external finance (see Hubbard [1998] and Stein [2003] for surveys), there remains considerable debate surrounding the relative importance of alternative sources of financial frictions for the determination of macroeconomic outcomes.[1]

The goal of this paper is to make some progress by proposing a general equilibrium model nesting recent developments in the theory on dynamic financial contracting. In the model, entrepreneurs with uncertain lifetimes have each access to a blueprint to build and operate a long-lived firm. Starting a firm requires working capital every period and incurring a partially sunk fixed cost. New entrepreneurs do not have enough resources to finance their firms, and seek external financing by entering into a long-term contract with a financial intermediary. Once started, firms generate a random revenue stream that is a function of their resource input. The contracts are constrained efficient because firm revenues are costly to monitor, and entrepreneurs may default.

Two important primitives of the model are the cost of monitoring firms and the cost of repudiating contracts, which jointly determine the significance of moral hazard

[1] For example, Gilchrist, Sim, and Zakrajsek [2013] and Midrigan and Xu [2014] argue that the misallocation due to financial frictions is much less than previous estimates, and Carlstrom, Fuerst, and Paustian [2014] argue that financial frictions may not lead to a significant amplification mechanisms when financial contracts are optimal.

and limited enforcement for financial contracting. A higher monitoring cost means that private information is more significant for financial contracting. A lower cost of repudiating contract means that it is more difficult to enforce the contract. Specifically, a low repudiation cost means that defaulting entrepreneurs may find a new lender relatively easily, while a high repudiation cost means that defaulting entrepreneur are permanently shut off from capital markets. Importantly, the outside value option of default is endogenous and depends on both the repudiation cost and the relative quality of blueprints available in a given period. As a result of these financial frictions, young and small firms tend to operate below their efficiency level, and grow disproportionally faster than older and larger firms.

The model encapsulates recent developments in the theory of optimal long-term financial contracting under private information and limited contract enforcement proposed to account for empirical regularities on firm dynamics.[2] The special cases of the contract along the monitoring and enforcement dimension are closely related to the financial contracts studied by Albuquerque and Hopenhayn [2004], Quadrini [2004] and Clementi and Hopenhayn [2006].[3] While many properties of this class of financial contracts are well understood, their implications for the determination of macroeconomic outcome remains largely unexplored. This paper attempts to fill this gap by nesting these contracts in a general equilibrium framework.

Calibrating the four special case economies along the monitoring and enforcement dimensions to the U.S. economy, I discuss three main results. First, focusing on the steady state, I find that the welfare loss from moral hazard conditional on the level of contract enforcement is about 5 percent in terms of aggregate consumption, while the welfare loss from limited enforcement conditional on the monitoring cost is only about

[2] Empirical studies of firms have shown that smaller and younger firms pay fewer dividends, take on more debt, experience a more rapid but also more volatile growth, and that small and young firms' investment is more sensitive to cash flows (Cooley and Quadrini [2001], Cabral and Mata [2003], Oliveira and Fortunato [2006], Fagiolo and Luzzi [2006], and Lu and Wang [2010]).

[3] The contract is also related to Monnet and Quintin [2005], who consider the role of costly state verification and stochastic monitoring in a dynamic lending contract with fixed firm size.

1 percent. The larger distortion arising from moral hazard follows from the property that, when monitoring costs are high, incentive compatibility requires cutting credit to entrepreneurs reporting unverifiable poor performance, while increasing credit to entrepreneurs making positive transfers to the intermediary. In contrast, when monitoring costs are low, entrepreneurs that truthfully report poor performances can maintain their level of borrowing. In equilibrium, high monitoring costs imply that firm growth is more volatile, the firm distribution is wider and the average firm is smaller. Consequently, long-term contracting under moral hazard implies that aggregate resources must be divided up among a larger number of inefficient small firms, which leads to a greater misallocation.

Consistent with the empirical regularities reported by Arellano, Bai, and Zhang [2012], the model implies that, conditional on the monitoring cost, the growth of small and young firms relative to old and large firms is higher when contract enforcement is weaker. However, the model also predicts that conditional on the level of contract enforcement, the growth of small and young firms relative to old and large firms is higher when the monitoring technology is less efficient. Taken together, these results suggest that while the empirical regularities noted by Arellano et al. [2012] could be driven by cross-country differences in contract enforcement, they could also be driven by cross-country differences in the efficacy of monitoring technology, as discussed by Greenwood, Sanchez, and Wang [2010]. A contribution of this paper is measuring the relative importance of these two distinct frictions for the determination of aggregate resource allocations. Moreover, an innovation over most of the related literature is to allow agents to form long-term lending relationships. It is possible that the ability to contract over multiple periods could also be an important determinant of an aggregate resource allocation. For example, Moll, Townsend, and Zhorin [2013] find that moral hazard can lead to larger aggregate output losses relative to limited commitment when agents are restricted to using one-period debt contracts.

Second, I find that while a reform designed to increase contract enforcement leads to significant short-term welfare gains, its long-run effect is significantly lower in an economy with inefficient monitoring technology. In the model, a permanent increase in contract enforcement leads to aggregate consumption immediately rising by at least 1 percent, with high and low monitoring costs. However, while this growth is permanent with low monitoring costs, most of these initial gains vanish in the years following the reform when monitoring costs are high. The reason for the overshooting is that, at the time of the reform, greater contract enforcement allows a significant fraction of small, constrained firms immediately gain access to greater financing–since the outside option of default is lower. However, with high monitoring costs, a lower outside option of default also allows financial intermediaries to induce truth-telling by maintaining young and poor performing firms at a lower size. As a result, the distribution of firms widens over time, and is characterized by a greater fraction of smaller firms. In contrast, the cross section of firms in an economy with low monitoring costs converges almost immediately to the new stationary distribution, which is narrower and with most firms operating close to their efficient scale.

This result is consistent with empirical studies, such as Beck, Demirg-Kunt, and Maksimovic [2008] showing that small and medium sized firms across countries benefit disproportionally from higher levels of property rights protection by using significantly more external debt financing, and Djankov, McLiesh, and Shleifer [2007] showing that countries implementing reforms designed to increase contract enforcement and creditor rights tend to experience a greater level of external financing. Furthemore, recent quantitative studies have studied the long run effect of reforms designed to increase financial development by either increasing contract enforcement (Amaral and Quintin [2010], Buera, Kaboski, and Shin [2011]) or improving the efficacy of the monitoring technology (Greenwood, Sanchez, and Wang [2013]) on economic development. A contribution of this paper is to explore the nexus between these two notions of financial developments for both short-run and long-run dynamics. The exercise suggests that the long run benefit of financial development defined as greater contract enforceability can be greatly hampered by an inefficient monitoring technology.

Third, I find that weak contract enforcement leads to a substantial amplification of the impact of aggregate technological shocks. When entrepreneurs unexpectedly have access to more efficient blueprints, aggregate output falls sharply and subsequently surges before gradually returning to its steady-state level. When the outside option of default increases, preventing smaller and younger firms from defaulting and searching for higher quality projects at the time of the shock requires cutting credit to these firms, while promising them greater financing in the future. This mechanism reallocates resources away from the young and small incumbent firms to the new higher-productivity firms, setting these firms on a higher growth path. However, this amplification channel is not present when contract enforcement is strong, and the aggregate response of the economy is not significantly different from that of a frictionless economy.

This results is consistent with Cooley, Marimon, and Quadrini [2004], who show that weak contract enforcement can lead to an important amplification mechanism for technological shock in an environment without information friction. This paper extends Cooley et al. [2004] by considering the role of private information between lenders and borrowers, in addition to limited enforcement. A further contribution is to identify the type of financial frictions that render the financial system an amplifier of aggregate technological shocks. The results suggest that while financial frictions can amplify the effect of aggregate technological shocks, limited contract enforcement, rather than moral hazard, may be an important source of aggregate fluctuations.[4]

More broadly, this paper contributes to the literature on financial frictions and business cycle fluctuations, following the seminal work of Bernanke and Gertler [1989] and Kiyotaki and Moore [1997] (see, for example, the surveys by Quadrini [2011] and Brun-

[4] Interestingly, Veracierto [2014] finds that information frictions that affect consumers, rather than firms, in a real business cycle model also have no effects on business cycle fluctuations.

nermeier, Eisenbach, and Sannikov [2012]). A pervasive assumption in this literature is to restrict the horizon of all contracts to one period.[5] This assumption places collateral at the center stage of credit allocation, which may, under some conditions, amplify and propagate the effect of aggregate shocks.[6] However, when agents can contract over multiple periods, lending may occur in equilibrium with financial frictions even if entrepreneurs do not have sufficient collateral. The results suggest that the amplification and propagation of aggregate shocks is highly sensitive to the type of financial friction motivating the financial arrangement.

The rest of the paper is organized as follows: Section 2 and Section 3 presents the model; Section 4 defines the equilibrium; discusses the properties of the optimal contract; Section 5 parameterizes and calibrates the model; Section 6 discusses the misallocation in the steady state; Section 7 studies the transitional dynamics of the different economies following a permanent strengthening of contract enforcement; and Section 8 studies the impact of technological innovations on aggregate output dynamics. Proofs of propositions and numerical strategies are relegated to the Appendix.

2 Model

Time is discrete and infinite, and each period is indexed by t. The economy is populated by continuums of entrepreneurs and workers with uncertain lifetimes, and infinitely lived financial intermediaries. An agent's career path is an endowment, which cannot be altered. Long-lived firms are managed by entrepreneurs and use labor and capital to produce a numéraire good used for consumption and investment. Financial intermediaries offer financing to firms and insurance against mortality risk to individuals.

[5] Recent work by Dyrda [2014] considers the effect of aggregate uncertainty shocks in a related environment with long-term contracts under private information and full enforcement.

[6] In this class of models, entrepreneurs with low collateral, or low net-worth, are more financially constrained. Aggregate shocks that lower the value of the collateral can have a disproportionately large effect on the real economy by reducing aggregate investment. This in turn further depresses the value of collateral, creating a reinforcing cycle.

Financial frictions arise because risky firm revenues are only observed by entrepreneurs, and entrepreneurs operating under limited liability may default. Two important primitives of the model are the cost of monitoring firm revenue and the cost of repudiating the contract.

2.1 Entrepreneurs

Entrepreneurs are born without wealth, survive into the next period with probability $(1-\gamma_e)$, and are replaced by new ones upon death. Entrepreneurs are risk-neutral and discount the future at rate $\beta = (1-\gamma_e)\tilde{\beta}$. Entrepreneurs choose their consumption to maximize the value of their lifetime expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t \ . \tag{1}$$

Entrepreneurs do not participate in the labor market but may start a long-lived firm to generate consumption.

2.2 Technology

Following Cooley et al. [2004], long-lived firms can be built by entrepreneurs according to one of two blueprint technologies $z \in \{z_l, z_h\}$ with $z_l < z_h$. Blueprints are plans to set up firms with decreasing returns to scale technology $zf(k,n)$, where k and n are capital and labor input, respectively. The production function is strictly increasing in labor and capital, strictly concave, and satisfies $zf(k,0) = zf(0,n) = 0$.

The more efficient z_h blueprints may be in short supply. Let Γ_t be the measure of entrepreneurs searching for blueprints and N_t be the measure of high-productivity blueprints z_h available at time t. The probability that a searching entrepreneur finds a blueprint z_h in period t is defined as $q_t = \min\{N_t/\Gamma_t, 1\}$. The economy could be subject to shocks to q_t, which determines the opportunities available to entrepreneurs to set up firms.

Lastly, entrepreneurs can only manage one firm, starting a new firm requires entrepreneurs to abandon the current firm, and blueprints cannot be carried over into the next period unless a firm is started. These assumptions ensure that entrepreneurs are never idle.

2.3 Firms

Starting a a type-z firm requires paying an initial fixed cost I_0, which is partially sunk as firms can always be liquidated with scrap value $S < I_0$. Once started, a firm requires period resources R_t used to rent capital k_t and to hire labor n_t at wage w_t before production can take place. The capital used in production is fully depreciated at the end of a period. The maximum revenue a firm can generate is increasing in resources R_t, and given by

$$\begin{aligned} F(R_t, z) = \quad & \max_{k_t, n_t} z f(k_t, n_t) \\ \text{s.t.} \quad & k_t + w_t n_t \leq R_t \ . \end{aligned} \tag{2}$$

Firm revenue is subject to a sequence of independent and identically distributed idiosyncratic shocks $(\nu_t)_{t \geq 0}$, where $P(\nu_t = 1) = 1 - P(\nu_t = 0) = p$ so that realized revenue is given by $\nu_t F(R_t, z)$. A firm may also be terminated upon the death of its entrepreneur, which occurs with probability γ_e and is analogous to receiving a permanent zero-productivity shock.[7]

The previous assumptions imply there exists a unique level of resources $\tilde{R}(z)$ that maximizes expected profit, such that

$$\tilde{R}(z) = \operatorname*{argmax}_R pF(R, z) - R \tag{3}$$

[7] This assumption captures other sources of firm exit not modeled explicitly and allows me to pin down the steady state distribution of firms without keeping track of individual firms (see, for instance, Cooley and Quadrini [2001], Cooley et al. [2004], and Smith and Wang [2006]).

for a given interest rate r and wage rate w. The assumption of diminishing returns to scale with fixed startup cost implies that it can be optimal to organize production around large firms. It is profitable to start a firm only if the expected discounted lifetime profit of a firm operating at full scale is greater than the set-up cost I_0, such that

$$\frac{pF(\tilde{R}(z), z) - \tilde{R}(z)}{1-\beta} > I_0 \ . \tag{4}$$

2.4 Workers

Workers are born without wealth and survive into the next period with probability $(1-\gamma_w)$. Deceased workers are replaced by new ones in the following period. Workers are risk averse, are endowed with one unit of time each period that they allocate between work and leisure, and discount the future at rate $(1-\gamma_w)\hat{\beta}$.

Workers dislike working but enjoy consumption, and labor is paid a competitive wage w_t. Following Blanchard [1985] and Smith and Wang [2006], workers use their income to buy some of the numéraire consumption good c_t, and to purchase contingent claims d_{t+1} priced at p_t^a from a financial intermediary. These claims pay one unit of consumption in the next period if the worker is alive and zero otherwise. Workers choose consumption and labor to maximize the value of their lifetime expected utility

$$E_0 \sum_{t=0}^{\infty} ((1-\gamma_w)\hat{\beta})^t u(c_t, 1-h_t) \ , \tag{5}$$

subject to the period budget constraint

$$c_t + p_t^a d_{t+1} \leq (1+r_{t+1})d_t + w_t h_t \ , \tag{6}$$

and $d_{t+1} \geq -\epsilon$, where ϵ is a natural borrowing limit.

2.5 Financial intermediation

Infinitely lived financial intermediaries are risk neutral and discount the future at the same rate as entrepreneurs. Given perfect competition in financial markets, I concentrate on a representative financial intermediary. The representative financial intermediaries offer workers a saving technology in the form of actuarially fair one-period annuity contracts allowing them to smooth lifetime consumption and insure against mortality risk. I assume that workers can fully commit to repaying their debt. Entrepreneurs seek a financial intermediary to finance their firm, which requires an initial investment $I_0 + R_0$ and working capital $R_{t>0}$ in the subsequent periods. However, unlike workers, entrepreneurs cannot commit to repaying their debt. Entrepreneurs may repudiate the contract, and the outside option value for doing so could depend on the state of the economy, including the relative quality of blueprints available.

2.6 Information

The shock process subjecting firm revenues is known by everyone, but the actual realization of firm idiosyncratic revenue shocks is the entrepreneurs' private information. The representative financial intermediary has access to a costly monitoring technology allowing her to monitor the firm ex post by paying a fixed cost m. However, entrepreneurs consume any revenue they divert from falsely reporting a low revenue shock before any monitoring. That is, while the monitoring technology allows the intermediary to verify the truthfulness of a report, it does not let her recover any of the diverted period revenue. The entrepreneurs' outside value option is observable by the financial intermediary.

The combination of private information and limited enforcement under limited liability leads to financial frictions. Under these assumptions, an entrepreneur is incentivized to report a low shock after experiencing a high revenue shock and to default if her outside option value is large enough.

3 The optimal financial contract

The representative financial intermediary offers entrepreneurs a state-contingent financial contract that induces truthful reporting and debt repayment each period. Let $\mathbf{s}_t$ denote the vector of aggregate state variables. Denote the reporting strategy of an entrepreneur by the sequence of reports $\underline{\hat{\nu}} = \{\hat{\nu}_t(\nu^t)\}_{t\geq 0}$, where $\nu^t = (\nu_1, \ldots, \nu_t)$ is the true history of the revenue shocks experienced by the entrepreneur. The history of reports is denoted by $h^t = (\hat{\nu}_1, \ldots, \hat{\nu}_t)$. A financial contract $\sigma(z, \mathbf{s}_t) = \{\ell_t(h^{t-1}, z, \mathbf{s}_t), a_t(h^{t-1}, \hat{\nu}_t, z, \mathbf{s}_t), Q_t(h^{t-1}, z, \mathbf{s}_t), R_t(h^{t-1}, z, \mathbf{s}_t), \tau(h^{t-1}, \hat{\nu}_t, z, \mathbf{s}_t)\}$ offered to an entrepreneur in possession of a blueprint $z \in \{z_l, z_h\}$ is contingent on the state of the economy $\mathbf{s}_t$ and specifies: a liquidation rule $\ell_t(h^{t-1}, z, \mathbf{s}_t) \in \{0, 1\}$, transfer $Q_t(h^{t-1}, z, \mathbf{s}_t) \geq 0$ from the intermediary to the entrepreneur in the event of liquidation, a monitoring rule $a(h^{t-1}, \hat{\nu}_t, z, \mathbf{s}_t) \in \{0, 1\}$, period resources $R_t(h^{t-1}, z, \mathbf{s}_t)$, and transfers $\tau(h^{t-1}, \hat{\nu}_t, z, \mathbf{s}_t) \in \mathbb{R}$ between the entrepreneur and the financial intermediary conditional on the ex-post report $\hat{\nu}_t$.

The timing within a period is as follows: Once a firm is in operation, the intermediary decides whether the firm should be liquidated. If the firm is not liquidated, a monitoring rule is announced and resources are transferred to the firm to pay for labor and capital before production can take place. After revenue is realized, and before sending any report, the entrepreneur decides whether to continue with the contract or to default. If no default occurs, the entrepreneur sends her report, and the intermediary monitors according to the rule announced at the beginning of the period. At the end of the period, the firm either exits permanently with fixed probability or continues. Figure 1 summarizes the sequence of events within one period.

3.1 Recursive formulation

In what follows, I omit z to economize on notation, but it is understood that the contract terms are specific to a firm of type $z \in \{z_l, z_h\}$. After history h^{t-1}, and conditional on the aggregate state $\mathbf{s}_t$, the pair of contract and report strategy $(\sigma_z(\mathbf{s}_t), \underline{\hat{\nu}})$ implies an expected discounted cash flow $V_t(\sigma_z(\mathbf{s}_t), \underline{\hat{\nu}}, h^{t-1})$ and $B_t(\sigma_z(\mathbf{s}_t), \underline{\hat{\nu}}, h^{t-1})$ for the entrepreneur and the financial intermediary, respectively. Following Green [1988], Spear and Srivastava [1987] and others, the contract for a firm of type z can be solved recursively using V and $\mathbf{s}$ as state variables, and $V'(\mathbf{s}')$ as the continuation values awarded to the entrepreneur contingent on her revenue reports and the monitoring rule.

Following Quadrini [2004], Albuquerque and Hopenhayn [2004], and , Clementi and Hopenhayn [2006], a feasible and incentive compatible contract is optimal if it maximizes $B_t(V_t, \mathbf{s}_t)$ for every feasible $V_t(\mathbf{s}_t)$, conditional on $\mathbf{s}_t$. Thus, maximizing $B_t(V_t, \mathbf{s}_t)$ for every possible $V_t(\mathbf{s}_t)$ is analogous to maximizing the value of the joint surplus $W_t(V_t, \mathbf{s}_t) = B_t(V_t, \mathbf{s}_t) + V_t(\mathbf{s}_t)$, which can be interpreted as the value of the firm by interpreting $V_t(\mathbf{s}_t)$ and $B_t(V_t, \mathbf{s}_t)$ as debt and equity, respectively.

Let $V^{\hat{\nu}\nu}(\mathbf{s}_{t+1})$ be the continuation value promised, given a report $\hat{\nu} \in \{H, L\}$ was verified to be $\nu \in \{H, L\}$, and $V^{\hat{\nu}}(\mathbf{s}_{t+1})$ be the continuation value promised, given an *unverified* report $\hat{\nu} \in \{H, L\}$. To simplify the notation further, I assume that a firm uses all the resources it has access to, $R \leq \tilde{R}$, each period, so that $R - k - nw = 0$.[8] The continuation value must satisfy a promise-keeping constraint given that, at the beginning of the period, the intermediary announces it either commits monitor the

[8] Clementi and Hopenhayn [2006] have shown that this is in fact a condition for optimality of the contract.

firm, $a = 1$, or will not monitor the firm, $a = 0$:

$$V = p(F(R) - \tau^H) - (1-p)\tau^L + \beta\mathbf{E}[pV^H(\mathbf{s}') + (1-p)V^L(\mathbf{s}')|\mathbf{s}] \text{ if } a = 0 \tag{7}$$

$$V = p(F(R) - \tau^H) - (1-p)\tau^L + \beta\mathbf{E}[pV^{HH}(\mathbf{s}') + (1-p)V^{LL}(\mathbf{s}')|\mathbf{s}] \text{ if } a = 1 \ , \tag{8}$$

The promise-keeping constraint states that the entrepreneur's value at the beginning of the period, V, must be equal to the expected cash flow during the period plus the next period discounted expected continuation value.

The contract induces truth telling if it satisfies an incentive compatibility constraint:

$$F(R) - \tau^H + \beta V^H(\mathbf{s}') \geq F(R) - \tau^L + \beta V^L(\mathbf{s}') \qquad \text{if } a = 0 \tag{9}$$

$$F(R) - \tau^H + \beta V^{HH}(\mathbf{s}') \geq F(R) - \tau^L + \beta V^{LH}(\mathbf{s}') \qquad \text{if } a = 1 \ . \tag{10}$$

The incentive compatibility constraint requires that following the report of a high revenue shock, and regardless of the monitoring decision, payment to the entrepreneur plus the discounted expected continuation value associated with a truthful report must be no less than the value of diverting $F(R) - \tau^L$ and receiving a low continuation value.

The contract induces repayment of the debt if the value derived from repaying is greater than the entrepreneur's outside option value. An entrepreneur's outside option value is the value derived from starting a new firm next period $\bar{V}(\mathbf{s})$ minus any cost associated with repudiating the contract κ. The entrepreneur's outside option value is determined in general equilibrium but is treated as a constant for now. Conditional on no monitoring, enforcement requires that the contract satisfies the constraints

$$\left.\begin{array}{ll} F(R) - \tau^H + \beta V^H(\mathbf{s}') & \geq F(R) + \beta[\bar{V}(\mathbf{s}) - \kappa] \\ -\tau^H + \beta V^L(\mathbf{s}') & \geq \beta[\bar{V}(\mathbf{s}) - \kappa] \end{array}\right\} \text{ if } a = 0 \ . \tag{11}$$

The first constraint must be satisfied following the report of a high-revenue shock, while the second constraint must be satisfied following the report of a low-revenue shock.

Similarly, conditional on monitoring, enforcement requires that the contract satisfies

$$\left. \begin{array}{ll} F(R) - \tau^H + \beta V^{HH}(\mathbf{s}') & \geq F(R) + \beta[\bar{V}(\mathbf{s}) - \kappa] \\ -\tau^H + \beta V^{LL}(\mathbf{s}') & \geq \beta[\bar{V}(\mathbf{s}) - \kappa] \end{array} \right\} \text{ if } a = 1 \; . \tag{12}$$

Given an entrepreneur is alive at the beginning of the period, and given that the firm is not liquidated by the financial intermediary, the value of the joint surplus conditional on no monitoring is given by

$$\widehat{W}(V, \mathbf{s}; a = 0) = \max_{\tau^{\hat{\nu}}, R, V^{\hat{\nu}}(\mathbf{s}')} pF(R) - R + \beta \mathbf{E}\left[pW(V^H(\mathbf{s}'), \mathbf{s}') + (1-p)W(V^L(\mathbf{s}'), \mathbf{s}')|\mathbf{s}\right]$$

subject to (7), (9), (11), and

$$\tau^H \leq F(R) \text{ and } \tau^L \leq 0 \tag{13}$$

$$V^{\hat{\nu}}(\mathbf{s}') \geq 0 \;\; \forall \;\; \hat{\nu} \in \{H, L\} \tag{14}$$

$$\mathbf{s}' \sim \mathcal{H}(\mathbf{s}) \; , \tag{15}$$

and, the value of the contract conditional on monitoring the firm is given by

$$\widehat{W}(V, \mathbf{s}; a = 1) = \max_{\tau^{\hat{\nu}}, R, V^{\hat{\nu}\nu}(\mathbf{s}')} pF(R) - R + \beta \mathbf{E}\left[pW(V^{HH}(\mathbf{s}'), \mathbf{s}') + (1-p)W(V^{LL}(\mathbf{s}'), \mathbf{s}')|\mathbf{s}\right]$$

subject to: (8), (10), and (12)

$$\tau^H \leq F(R) \text{ and } \tau^L \leq 0 \tag{16}$$

$$V^{\hat{\nu}\nu}(\mathbf{s}') \geq 0 \;\; \forall \;\; \hat{\nu}\nu \in \{H, L\} \times \{H, L\} \tag{17}$$

$$\mathbf{s}' \sim \mathcal{H}(\mathbf{s}) \; . \tag{18}$$

The feasibility constraints (13) and (16) require that any positive transfers from the entrepreneur to the intermediary never exceed period revenue, since entrepreneurs do not have access to an alternative saving technology. Constraints (14) and (17) state that entrepreneurs operate their firm under limited liability. Lastly, the joint surplus maximization problem needs to be consistent with the aggregate state law of motion (15).

The highest joint surplus can be achieved by randomizing the monitoring decision, such that

$$\begin{aligned} \widehat{W}(V,\mathbf{s}) = \max_{\delta\in[0,1],V_A,V_N} \quad & \delta\widehat{W}(V,\mathbf{s};a=1) + (1-\delta)\widehat{W}(V,\mathbf{s};a=0) \\ & \text{s.t. } \alpha V_A + (1-\alpha)V_N \geq V \ . \end{aligned} \tag{19}$$

where $\delta(V,\mathbf{s})$ is the probability that the firm will be audited following the report of a low revenue shock, and $V_A(\mathbf{s})$ and $V_N(\mathbf{s})$ are the continuation values awarded to the entrepreneur if the firm is audited and if the firm is not audited, respectively. Note that the decision to monitor the firm does not depend on the current period revenue realization but depends instead on the financial position, V, of the firm and the state of the economy $\mathbf{s}$ at the beginning of each period.

The firm can be liquidated at the beginning of each period, providing the salvage value S to the intermediary. It follows that the highest surplus can be achieved by randomizing the liquidation decision, such that

$$\begin{aligned} W(V,\mathbf{s}) = \max_{\alpha\in[0,1],Q,V_C} \quad & \alpha S + (1-\alpha)\widehat{W}(V_C,\mathbf{s}) \\ & \text{s.t. } \alpha Q + (1-\alpha)V_C \geq V \ , \end{aligned} \tag{20}$$

where $\alpha(V,\mathbf{s})$ is the probability that the firm is liquidated, and $Q(\mathbf{s})$ and $S-Q(\mathbf{s})$ are the transfer to the entrepreneur and the intermediary, respectively. In the event the firm is not liquidated with probability $1-\alpha(V,\mathbf{s})$, the entrepreneur is awarded the continuation $V_C(\mathbf{s})$.

4 Equilibrium

The assumptions about the workers imply a stationary demographic, which allows the representative financial intermediary to fully diversify worker mortality risk. Given the risk-free interest rate r and perfect competition in the financial intermediation sector, annuities are priced at the workers' survival rate $p^a = (1 - \gamma_w)$ and workers receive a gross return of $(1 + r)/(1 - \gamma_w)$ on their savings.

Setting the mass of workers to 1, let $d_j(\mathbf{s})$ and $h_j(\mathbf{s})$ be the deposits and hours worked of a j-year old worker. In every period t, γ_w new workers are born with zero wealth and therefore contribute $\gamma_w d_0(\mathbf{s}) = 0$ to aggregate deposits, and j-year old workers contribute $\gamma_w(1-\gamma_w)^j d_j(\mathbf{s})$ to aggregate deposits. It follows that aggregate net deposits and labor supply each period is given by

$$D_w(\mathbf{s}) = \gamma_w \sum_{j=1}^{\infty}(1 - \gamma_w)^j d_j(\mathbf{s}) \ \ \text{, and } H(\mathbf{s}) = \gamma_w \sum_{j=0}^{\infty}(1 - \gamma_w)^j h_j(\mathbf{s}) \ , \tag{21}$$

Perfect competition in the financial intermediation sector also implies that the representative financial intermediary breaks even on new contracts, so that

$$V_0(\mathbf{s}, z) = \sup_V \{B(V, \mathbf{s}, z) - I_0 = 0\} \ . \tag{22}$$

Equation (22) implies that the initial value to the entrepreneur $V_0(\mathbf{s}, z)$ is the greatest possible value such that the lender makes zero profit. It follows that the value of searching for a new project gross of repudiation cost is

$$\bar{V}(\mathbf{s}) = q(\mathbf{s})V_0(\mathbf{s}, z_h) + (1 - q(\mathbf{s}))V_0(\mathbf{s}, z_l) \ , \tag{23}$$

where $q(\mathbf{s})$ is the probability that a searching entrepreneur finds a z^h blueprint, and $V_0(\mathbf{s}, z)$ is the initial state of a contract signed to start a z-type firm. Thus, solving for the optimal contract requires solving iteratively for the value $\bar{V}(\mathbf{s})$, which is the fixed point of the following mapping

$$\bar{V}^{j+1}(\mathbf{s}) = T(\bar{V}^j)(\mathbf{s}). \tag{24}$$

The initial value $V_0(\mathbf{s}, z)$ determines the initial resources, $R(V_0(\mathbf{s}, z), \mathbf{s}, z)$, available to the firm, and total debt for a new firm is $R(V_0(\mathbf{s}), \mathbf{s}, z) + I_0$. In general, new and constrained entrepreneurs can increase their stake in their firm by making positive transfers to the intermediary to repay their debt and accumulate deposits. I discuss the dynamics of V and $R(V, \mathbf{s}, z)$ in the next section. Conditional on the aggregate state $\mathbf{s}$, an entrepreneurs is unconstrained if she has repaid all her debt and has accumulated enough deposit through her transfer to the intermediary to self-finance her firm. That is, conditional on $\mathbf{s}$, the value of the contract to the intermediary for an unconstrained entrepreneur is

$$- B(\tilde{V}(z), z) = \frac{\tilde{R}(z)}{1-\beta} = \frac{1+r}{r+\gamma_e}\tilde{R}(z) \tag{25}$$

which means that, conditional on $\mathbf{s}$, the return on an unconstrained entrepreneur's deposit is $r + \gamma_e$ and generates $\tilde{R}(z)$ each period. In effect, the contract provides insurance against the exogenous death shock by using deceased entrepreneurs' deposit to partially finance firms and workers.

Let $\mathbf{M}_z$ be the state space for type-z firm entrepreneurs' value, so that $V \in \mathbf{M}_z$. Let $\mathcal{M}_z(V)$ be the Borel $\sigma-$algebra generated by $\mathbf{M}_z$, and $\mu_z(\mathbf{s})$ the measure of type-z firms defined over $\mathcal{M}_z$ given the state of the economy $\mathbf{s}$. The representative intermediary holds a portfolio of contracts indexed by V and z so that the aggregate net deposit by entrepreneurs is:

$$D_e(\mathbf{s}) = - \sum_{z\in\{z_l, z_h\}} \int B(V, \mathbf{s}, z) d\mu_z(\mathbf{s}) \tag{26}$$

which could be positive or negative depending on the shape of the distribution of firms $\mu_z(\mathbf{s})$ implied by the financial contract. A positive $D_e(\mathbf{s})$ implies that the intermediary can use the deposits from entrepreneurs managing older and larger firms to finance younger and smaller firms, as well as workers. A negative $D_e(\mathbf{s})$ implies that the intermediary must raise additional deposits from workers to finance her portfolio of firms. Let $\Gamma(\mathbf{s})$ be the measure of new firms, which is equal to the measure of searching entrepreneurs given the assumptions in Section 2. The intermediary budget must be balanced each period, such that

$$D_e(\mathbf{s}') = (1+r(\mathbf{s}))D_e(\mathbf{s}) + \sum_{z\in\{z_l,z_h\}} \left(\int \tau(V,\mathbf{s},z)d\mu_z(\mathbf{s}) - \int R(V,\mathbf{s}',z)d\mu_z(\mathbf{s}') \right) + \Gamma(\mathbf{s}')I_0 - \Gamma(\mathbf{s})S \ . \tag{27}$$

With full capital depreciation, and given that the only saving technology available to entrepreneurs is the optimal contract, entrepreneurs do not hold any asset besides what is deposited with the representative intermediary. It follows that the capital market clears when the net supply of assets from workers and entrepreneurs is zero, that is,

$$D_w(\mathbf{s}) + D_e(\mathbf{s}) = 0 \ . \tag{28}$$

The labor market clears when labor demand from firms is equal to labor supplied by the workers, such that

$$H(\mathbf{s}) = \sum_{z\in\{z_l,z_h\}} \int n(V,\mathbf{s},z)d\mu_z(\mathbf{s}) \ . \tag{29}$$

Lastly, given the labor and capital market clear, the goods market also clears so that

$$Y(\mathbf{s}) = \sum_{z\in\{z_l,z_h\}} p \int F(R(V,\mathbf{s},z))d\mu_z(\mathbf{s}) \tag{30}$$

and aggregate output is divided between worker and entrepreneur consumption, and aggregate investment such that[9]

$$Y(\mathbf{s}) = C_w(\mathbf{s}) + C_e(\mathbf{s}) + K(\mathbf{s}) \ , \tag{31}$$

[9] See appendix for more details.

where $C_w(\mathbf{s})$ is workers' aggregate consumption,

$$C_e(\mathbf{s}) = Y(\mathbf{s}) - \sum_{z \in \{z_l, z_h\}} \int \tau(V, \mathbf{s}, z) d\mu_z(\mathbf{s}) \tag{32}$$

is aggregate consumption by entrepreneurs, and

$$K(\mathbf{s}) = \sum_{z \in \{z_l, z_h\}} \int k(V, \mathbf{s}, z) d\mu_z(\mathbf{s}) + \Gamma(\mathbf{s}') I_0 - \Gamma(\mathbf{s}) S \tag{33}$$

is aggregate investment. The definition of a general equilibrium follows.

Definition 1 *A general equilibrium consists of labor supply and consumption function* $h(d, \mathbf{s})$ *and* $c(d, \mathbf{s})$ *for workers, a contract* $\{R(V, \mathbf{s}, z)$, $\tau(V, \mathbf{s}, z)$, $V'(V, \mathbf{s}', z)$, $\alpha(V, \mathbf{s}, z)$, $\delta(V, \mathbf{s}, z)$, $Q(\mathbf{s}, z)$, $V_A(\mathbf{s}, z)$, $V_N(\mathbf{s}, z)$, $V_C(\mathbf{s}, z), \}$ *for each* $z \in \{z_l, z_h\}$*, an initial contract state* $V_0(\mathbf{s}, z)$ *for each* $z \in \{z_l, z_h\}$*, a wage rate* $w(\mathbf{s})$ *and interest rate* $r(\mathbf{s})$*; a mapping* T*, and a law of motion for the vector of aggregate state variables* $\mathbf{s}' \sim H(\mathbf{s})$*, such that*

1. *The labor and consumption functions maximize workers' utility,*
2. *The financial contract maximizes the value of the firm,*
3. *The initial state contract state is such that the intermediary breaks even on new contract,*
4. *The wage and interest rate clear the labor and capital market,*
5. *The value* $\bar{V}(\mathbf{s})$ *is the fixed point of* T*, and*
6. *The individual decisions are consistent with aggregate state's law of motion.*

The rest of the analysis focuses on the steady state property of the economy and on the economies' response to either permanent of temporary aggregate shocks. Proposition 1 and 2 establish that an steady state exists for this class of model economies when there are no aggregate shocks. Proposition 1 states that for a given set of prices r and w, and given there is only one type of blueprint, the distribution of firms converges to a stationary distributions in a finite number of periods.

Proposition 1 *For given prices r and w, and a fixed fraction of blueprint such that $q = q' = 0$, there exist a stationary distribution of firms that is ergodic.*

Proposition 2 states that when $q = q' = 0$, there exists an stationary equilibrium such that $\mathbf{s} = \mathbf{s}'$.

Proposition 2 *In the absence of aggregate shocks, there exists a stationary equilibrium.*

4.1 Characterization of the optimal contract

In this section, I assume that $q = q' = 0$ and the economy has reached a steady state such that $\mathbf{s} = \mathbf{s}'$ with only z_l-firms. For simplicity, I omit $\mathbf{s}$ and z from the notation, and it is understood that the properties discussed in this subsection are conditional on $\mathbf{s}$ and z.

After the contract is signed, the intermediary lends the setup cost I_0, and the initial resources R_0 to the entrepreneur. New entrepreneurs with zero net worth have an strong incentive to misreport revenues and default on their debt early on, so that new firms start with resources that are less than the efficiency level, such that $R_0 < \tilde{R}$. At this point, the entrepreneur's value is $V_0 < \tilde{V} = pF(\tilde{R})/(1-\beta)$.[10]

The evolution of firm resources $R(V)$ depends on the evolution of the entrepreneur stake V in her firm, which is determined by the optimal contract. If an entrepreneur's value reaches $\tilde{V}$, any agency and enforcement problem becomes irrelevant, as the entrepreneur is able to self-finance her firm at the efficient level $\tilde{R}$ in all subsequent periods. One way to reach this level is for the entrepreneur to accumulate enough deposits through her transfers to the intermediary to finance the firm using the one-period

[10] See Albuquerque and Hopenhayn [2004] and Clementi and Hopenhayn [2006] for a more detailed discussion.

returns on these deposits. However, if a firm experiences a long enough sequence of low-revenue shocks before reaching the unconstrained state, the firm's future expected value could fall below its scrap value S, prompting the intermediary to liquidate the firm to recoup $S - Q$.

Two important primitives of the contract are the monitoring cost m and the contract repudiation cost κ, which jointly determine the severity of moral hazard and limited enforcement for financial contracting. In this section, I characterize the contract under four assumptions:

(A1) **No monitoring and strong enforcement**: $m \to \infty$ and $\kappa \to \bar{V}$

(A2) **No monitoring and weak enforcement**: $m \to \infty$ and $\kappa \to \underline{\kappa}$

(A3) **Full monitoring and strong enforcement**: $m \to 0$ and $\kappa \to \bar{V}$

(A4) **Full monitoring and weak enforcement**: $m \to 0$ and $\kappa \to \underline{\kappa}$

where $\underline{\kappa}$ is the minimum repudiation cost for the contract to be feasible, which is not necessarily 0.

The previous assumptions imply that it is optimal to either always monitor the firm after a low revenue report or to never monitor the firm. Under assumptions (A1) and (A2), the cost of monitoring is so high that, if the contract is feasible, the intermediary never monitors the firm—-that is $\delta(V, \hat{\nu}) = 0$ for all V and $\hat{\nu} \in \{H, L\}$. Under assumptions (A3) and (A4), when the cost of monitoring is arbitrarily small, it is optimal for the intermediary to monitor the firm after the report of a low revenue shock—-that is $\delta(V, \hat{\nu} = L) = 1$ for all V and $\mathbf{s}$.

As will be clear in the following subsection, these limiting cases encapsulate well-known models of firm dynamics motivated by financial frictions and long-term contracts. The contract under assumption (A1) corresponds to the contract studied by Quadrini [2004] and Clementi and Hopenhayn [2006], as the no-default constraint never binds under limited liability.[11] Assumption (A2) imposes an additional constraint to prevent the entrepreneur from repudiating the contract in some states. An interpretation of this special case is an extension of Clementi and Hopenhayn [2006] in which the entrepreneur of a liquidated firm is not in autarky. The contracts under assumptions (A3) and (A4) are related to the contract studied by Albuquerque and Hopenhayn [2004] and Cooley et al. [2004], but differ in that firms experience an idiosyncratic revenue shock after the loan is advanced and production takes place.

4.1.1 No monitoring

Under (A1), the contract is identical to the one studied by Clementi and Hopenhayn [2006], except for the exogenous firm exit shock.[12] Feasibility implies that $\tau^L = \tau(\hat{\nu} = 0) = 0$. Given that the entrepreneurs and the financial intermediary are risk-neutral, it is optimal to set repayment such that $\tau^H = F(R)$ whenever $V^H(V) < \widetilde{V}$, as it allows for the fastest accumulation of equity toward the unconstrained level. To see this, note that setting $\tau = F(R)$ implies that $V/\beta = pV^H + (1-p)V^L$ from the participation constraint, so that the entrepreneur's value grows at the maximum feasible rate. It follows that the incentive compatibility constraint simplifies to

$$\beta V^H \geq F(R) + \beta V^L \ . \tag{34}$$

It is clear that the enforcement constraint is never binding under assumption (A1), as it requires that $\beta V^H(V) \geq F(R)$, and limited liability requires that $V^L(V)$ and $V^H(V)$ be non-negative. It follows that, when contract enforcement is the strongest and the monitoring cost is high, the optimal contract only needs to disciplines the moral hazard.

Combining the incentive compatibility constraint with the promise-keeping constraint implies that the promised continuation values evolve according to $\beta V^L(V) =$

[11] Unlike Quadrini [2004], I do not impose renegotiation-proof-ness of the contract.
[12] Results established in Clementi and Hopenhayn [2006] are stated without proof.

$V - pF(R(V))$ when $V \leq \tilde{V}$, and $\beta V^H(V) = V + (1-p)F(R(V))$ when $V^H(V) \leq \tilde{V}$. As a result, the entrepreneur's value decreases after a low-revenue report and no transfer, and increases after a high-revenue report and a transfer $F(R)$ to the intermediary. Moreover, these continuation values imply that, conditional on not exiting, firms grow on average when $V^H(V) \leq \tilde{V}$, since from the above $\mathrm{E}(V'|V) = pV^H + (1-p)V^L = \frac{V}{\beta} > V$ for all $V^H(V) \leq \tilde{V}$.

Feasibility and incentive compatibility imply that period resources, $R(V)$, are determined by $F(R(V)) \leq \beta[V^L(V) - V^L(V)]$. Clementi and Hopenhayn [2006] have shown that $R(V)$ is generally increasing in V for $V < \tilde{V}$ except in the neighborhood of the liquidation region. Moreover, a constrained entrepreneur always receives less than the efficient level of resources, $R(V) < \tilde{R}$ for all $V < \tilde{V}$. In other words, the contract stipulates that only those entrepreneurs that have repaid their debt and accumulated enough deposits can self-finance their firm at full scale.

Given the firm does not exogenously exit, the firm either reaches the unconstrained level $\widetilde{V}$ after experiencing a sufficiently long but finite sequence of high-revenue shocks, or face liquidation with a positive probability whenever V falls below a threshold V_C. In the event that the firm is liquidated, it is optimal for the intermediary to transfer $Q = 0$ to the entrepreneur.

Under (A2), contract enforcement requires that the continuation values be no lower than $F(R) + \beta(\bar{V} - \kappa)$ and $\beta(\bar{V} - \kappa)$, following the reports of a high- and low-revenue shock, respectively. In contrast to the strong enforcement case under (A1), the enforcement constraint is binding for small V. A binding enforcement constraint restricts the feasible set of values for $R(V)$, $V^H(V)$ and $V^L(V)$, yielding a lower surplus $\hat{W}$, and lower resources $R(V)$ and continuation value $V^H(V)$ and $V^L(V)$ for all $V < \tilde{V}$. In other words, a higher outside option value requires the entrepreneur to build a greater stake in her firm to obtain the same level of financing.

Furthermore, a lower joint surplus for all $V < \tilde{V}$ implies that the liquidation threshold V_C is higher. To see ther, note that the liquidation cutoff V_C is such that $\hat{W}'(V_C) = (\hat{W}(V_C) - S)/V_C$. Thus, a lower joint surplus $\hat{W}'(V)$ for all $V < \tilde{V}$ implies that V_C is larger as it reduces the slope of the line tangent to $\hat{W}(V)$ extending from S. However, whether the increase in the liquidation threshold lead to an increase in equilibrium liquidation depends on the size of the outside value option. For instance, there is no liquidation in equilibrium if $V_C < \beta(\bar{V} - \kappa)$.

Panel (a) of Figure 2 summarizes the main properties of the optimal rule for period loan and continuation values as a function of V under assumption (A1) and (A2).

4.1.2 Full monitoring

Under (A3), the monitoring cost is arbitrarily small, $m \to 0$, and it is always optimal for the financial intermediary to monitor a firm reporting a low revenue shock. If the entrepreneur is found to be misreporting, the financial intermediary optimally sets $V^{LH}(V) = 0$ for all $V < \widetilde{V}$ as the surplus cannot be improved with any positive continuation value. In this case, the firm is immediately liquidated and the financial intermediary recovers S. As before, feasibility implies that $\tau^L(V) = 0$, and $\tau^H(V) = F(R(V))$ whenever $V^{HH}(V) < \widetilde{V}$. Under (A3), enforcement requires that

$$\beta V^{HH} \geq F(R) \text{ and } V^{LL} \geq 0 \ , \tag{35}$$

so that the incentive compatibility and limited liability constraints are redundant, and limited enforcement is the dominant friction. Taking the first-order condition of $\hat{W}(V, a = 1)$ with respect to V^{LL}, and using the envelope condition yields $V^{LL}(V) = V$. Furthermore, the participation constraint implies that $V^{HH}(V) = cV$ for $V^{HH}(V) < \widetilde{V}$, where $c = \frac{1-\beta p}{\beta(1-p)} > 1$ when $\beta < 1$. It follows that the entrepreneur's value increases after the report of high revenue shock, but does not decrease following the truthful report of a low revenue shock. Consequently, other things being equal, a firm may never face liquidation if the entrepreneur's initial stake is greater than the liquidation threshold, such that $V_0 > V_C$. As with the no monitoring cases, firms grow on average since $\mathrm{E}(V'|V) = pcV + (1-p)V > V$ for all $V^{HH}(V) \leq \tilde{V}$.

Unlike the no monitoring cases, firms that can be monitored cheaply may start operating at full scale before the entrepreneur is financially unconstrained. Note that the enforcement constraint implies that there exist a value V_u such that

$$R(V) = \begin{cases} F^{-1}(\beta c V) & \text{if } V < V_u < \tilde{V} \\ \tilde{R} & \text{if } V_u \leq V \leq \tilde{V} \end{cases} \tag{36}$$

where V_u is such that $F^{-1}(\beta c V_u) = \tilde{R}$. Given the assumption about the production function $zf(k,n)$, the maximum revenue function $F(R)$ is strictly increasing and concave, which implies that $R(V)$ is also strictly increasing for $V \leq V^u$ and is equal to $\tilde{R}$ for $V > V^u$. This result is in line with Albuquerque and Hopenhayn [2004].

Under assumption (A4), defaulting entrepreneurs can easily start a new firm, making it more difficult to enforce the contract. As previously discussed, the inability to fully exclude misreporting or defaulting entrepreneurs from financial markets restricts the feasible set of values for R, V^{HH} and V^{LL} available to the intermediary to implement the contract, which reduces the joint surplus for all $V < \tilde{V}$. Combining the participation constraint with the enforcement constraint yields

$$\beta V^{HH} \geq F(R) + \beta(\bar{V} - \kappa) \ , \tag{37}$$

$$\text{and } \beta V^{LL} \geq \beta(\bar{V} - \kappa) \tag{38}$$

and feasibility requires that the entrepreneur value V be at least $\beta(\bar{V} - \kappa)$. Weak enforcement implies that period resources are determined by

$$R(V) = \begin{cases} F^{-1}(cV - \beta(\bar{V} - \kappa)) & \text{if } V < V_u' < \tilde{V} \\ \tilde{R} & \text{if } V_u' \leq V \leq \tilde{V} \ , \end{cases} \tag{39}$$

where V_u' is such that $F^{-1}(cV_u' - \beta(\bar{V}(\mathbf{s}) - \kappa)) = \widetilde{R}$. Consequently, $R(V)$ is lower for any given $V < V_u'$ when enforcement is weak ($\kappa = \underline{\kappa}$), and an entrepreneur needs to accumulate more equity to obtain the unconstrained level of financing since $V_u' > V_u$ given the concavity of $F(R)$.

Panel (b) of Figure 2 summarizes the main properties of the optimal rule for period loan and continuation values as a function of V under (A3) and (A4).

5 Parameterization and calibration

Let the instantaneous utility function for the workers be[13]

$$u(c, l) = \ln(c) + \eta \ln(1 - l) \ , \tag{40}$$

and let the production function be

$$zf(k, n) = z\zeta(k^{\xi} l^{1-\xi})^{\theta} \ . \tag{41}$$

Given the parameterization, it remains to assign values to the workers' inter temporal discount rate β, the elasticity of leisure η, workers' death rate γ_w, the probability of high revenue shocks p, the production parameters z, ζ, ξ, and θ, the firms' exogenous exit rate γ_e, the setup cost I_0, the salvage value S, and the repudiation cost κ.

A period in the model is one year. The mass of workers and entrepreneurs are each normalized to 1. Workers' mortality rate γ_w is set to 2 percent, which implies an average working life of 50 years. I follow Cooley et al. [2004] and set $\theta = 0.85$. The parameter ζ is set to normalize the period resources used by unconstrained firms $\widetilde{R}$ under (A1). I assume that 20 percent of the initial investment is sunk, so that $S = 0.8 \times I_0$, and that firms have a 50 percent chance of experiencing a high- or low-revenue shock, so that

[13] This functional form implies closed-form solutions for the aggregate supply of labor and aggregate deposits given the workers' demographic assumption—-see Smith and Wang [2006] for more details.

$p = 0.5$.

The remaining parameters I_0, γ_e, ξ, η and κ jointly affect the stochastic process for V, and in turn the distribution of firms in the economy. The parameters η and ξ determine the demand and supply of labor, and the workers' discount rate $\hat{\beta}$ determines the interest rate r. A higher I_0 reduces the maximum level of initial debt an intermediary can optimally commit to, which reduces the starting value V_0 and tightens the credit constraint of young firms. When monitoring is not feasible under (A1) and (A2), a lower V_0 also implies a higher hazard rate of liquidation for all constrained firms. Moreover, with weak contract enforcement under (A2) and (A4), a higher repudiation cost κ has the opposite effect of a higher initial cost I_0. A higher κ reduces entrepreneurs' outside option value, letting the representative intermediary start new firms at a higher V_0.

I find a set of parameters that jointly match the following steady state moments in the simulated economies: 1/3 of aggregate hours are spent working, the risk-free rate r is 0.04, the labor income share is 60 percent, the firm exit and entry rate is 6.2 percent consistent with Cooper and Haltiwanger [2006], and new firms under (A1) operate with 30 percent of the unconstrained level of capital. While this last target is somewhat arbitrary, it allows me to pin down I_0 and provides a benchmark to measure the effect of different types of financial friction on new firm size. Lastly, the lowest repudiation cost $\underline{\kappa}$ is set to 0.45, which corresponds to the lowest value to ensure that the contracts are feasible when defaulting entrepreneurs are not excluded from financial markets.

6 Misallocation

This section discusses the effect of moral hazard and limited contract enforcement on aggregate resource allocation. Table 2 reports the main aggregate statistics of the economy under (A1) through (A4). Table 2 shows that moral hazard is considerably more costly in terms of resource misallocation than limitedq enforcement. The aggregate output loss from high monitoring cost is 4.3 percent and 5.2 percent conditional on a weak and strong level of contract enforcement, respectively. Conversely, the aggregate output loss from weak enforcement is 0.5 percent and 1.5 percent conditional on high and low monitoring cost, respectively.

The significant difference in resource misallocation attributed to moral hazard and limited enforcement stems from differences in how the financial sector prices new contracts, and the firm dynamics implied by these optimal contracts. First, Table 2 shows new firms are larger when monitoring cost are the lowest. Second, recall from the discussion in Section 4.1 that, when monitoring costs are the highest, incentive compatibility requires cutting credit to entrepreneurs reporting unverifiable poor performance. In contrast, entrepreneurs truthfully reporting poor performances can maintain their level of borrowing when monitoring costs are low.

To quantify the significance of these effects, define firm investment growth rate as $\ln(k_t) - \ln(k_{t-1})$, where k_t is the fixed fraction of the period loan R_t used as working capital in period t. Figure 3 plots the mean firm investment growth rate (top panel) and the standard deviation of firm investment growth rate (bottom panel) conditional on firm age in years under (A1) through to (A4).

Consistent with empirical regularities, firm growth and the volatility of firm growth generally decrease as firm become older. Figure 3 shows that when monitoring costs are low, all firms aged 15 years or more are operating at full scale. By contrast, a significant fraction of 50-years-old and older firms continue to be financially constrained when monitoring costs are high. Moreover, Figure 3 shows that the growth of young firms is roughly three times more volatile when monitoring is not feasible.

High monitoring costs imply that the distribution of firms in the economy is much wider, and that firms are on average much smaller. Figure 4 shows that conditional on the level of enforcement, new firms operate further away from their efficiency level when monitoring cost are high. Moreover, 15-year-old firms in economies with high monitoring costs operate at about 60 percent of their efficient scale, on average, while most same-age firms in economies with low monitoring costs operate at full scale.

In sum, long-term contracting under private information implies that aggregate resources must be divided up among a larger number of inefficiently small firms, leading to a more severe misallocation of resources relative to an economy in which limited enforcement is the dominant friction.

7 Financial development

This section discusses the effects of a reform designed to permanently strengthen contract enforcement. This reform could reflect, for example, the establishment of a credit bureau that accumulates and disseminates creditor information to lenders, or a strengthening of the legal system that protect creditors' right [Djankov et al., 2007]. In the model, the reform consists of an unanticipated and permanent increase in the repudiation cost κ, such that entrepreneurs' outside option of default is reduced to the firm period revenue $\nu F(R)$. The experiment considers the effect of this reform in an economy under (A2) with high monitoring costs and under (A4) with low monitoring costs. Solving for the economies' transition dynamics is computationally intensive as it requires solving for the dynamics of the distribution of firms, which is an infinite object, and iterate over the trajectory of prices until all markets clear in every period—see Appendix for computational details.

The main result is summarized by Figure 5, plotting the transition of aggregate output in the economy with high and low monitoring cost during the 40 years following the reform. At the time of the reform, aggregate output immediately rises by 1.3 percentage point and 1.5 percentage point in the economies with low and high monitoring cost, respectively. However, about 50 percent of these initial gains disappear 10 years after the reform with high monitoring cost, and 80 percent of these initial gains disappear in the long run.

The intuition for the large overshooting of aggregate output with high monitoring costs is as follows. From the discussion in the previous section, weak contract enforcement, $\kappa = \underline{\kappa}$, is associated with smaller average firm size conditional on firm age. From the discussion in Section 4.1, a higher κ implies that firms can access more resources for a given V. It follows that the immediate effect of a strengthening of contract enforcement is that a large fraction of small constrained firms gain access to greater financing and expand.

With high monitoring costs, stronger contract enforcement implies that the financial intermediaries can induce truth telling by maintaining young and poor performing firm at a lower size than before the reform, as seen in Panel (a) of Figure 2. In other words, the truth inducing contract can be implemented using a larger set of equity value, V. Since it is optimal to cut credit to firms that report low revenue shock, poor-performing firms may be sustain at a smaller size than before the reform. Over time, the distribution of firm becomes wider with a greater mass of small firms. As a result, aggregate resources must be divided up among a greater number of smaller inefficient firms, offsetting the initial gains. In contrast, the cross section of firms converges almost immediately to the new stationary distribution with low monitoring costs, which is much narrower with most firm operating close to or at their efficient level.

8 Technological shocks

This section discusses the effect of financial frictions on the transmission of aggregate technological shocks. Following Cooley et al. [2004], I consider the effect of a measure-zero aggregate shock that affects the composition of firms by temporarily changing the quality of blueprints available in the four different economies. A shock in period

t increases the probability q of finding a z_h blueprint to 1 in this period only, where $z_h > z_l$. The technology of incumbent firms remains unchanged at z_l, but higher productivity firms may be started. I assume that the difference in productivity between high and low productivity firms is such that unconstrained type z_h firms are 30% larger in terms of the period resource intensity than type z firms, which is achieved by setting $z_l = 1$ and $z_h = 1.04$. As with the previous experiment, solving for the transition dynamics of the economies requires solving for the equilibrium trajectory of prices such that markets clear in every period given the dynamics of the firm size distribution—see Appendix for details.

Figure 6 and Figure 7 summarize the main result, plotting the response of aggregate output in economies with weak and strong enforcement, respectively, conditional on high and low monitoring costs. The aggregate response of the frictionless economy serves as the benchmark in the two figures. Figure 6 shows that when enforcement is weak, the economy initially contracts at the time of the shock and then expands to a level about 50 percent higher than the frictionless economy in the subsequent periods. The initial drop in output is about half as large when monitoring cost is high. Aggregate output subsequently declines but remains higher than the frictionless economy for about 8 years after impact. When enforcement is strong, Figure 7 shows that the response of aggregate output in the economy with financial friction is not significantly different from the frictionless economy. Importantly, this result holds conditional on high and low monitoring cost, suggesting that weak contract enforcement, rather than moral hazard, is an important source of aggregate fluctuations.

To understand the mechanics driving these results, it is useful to start with the frictionless economy. In a frictionless economy, aggregate output immediately increases by about 0.4 percentage point, and then geometrically decreases in the subsequent period until the economy reaches its steady state. Before the shock, all z_l firms operate at full scale with $\widetilde{R}(z_l)$. At the time of the shock, 6.2 percent of the z_l firms exogenously exit and are replaced by new z_h firms, which are about 30 percent larger. With log-utility and full capital depreciation, the risk-free rate and the aggregate hours spent in the labor market remain constant.[14] As the pool of type z_h firms is replaced by new type-z_l firms at rate $\gamma_e = 6.2$, aggregate output gradually returns to its steady state level.

When enforcement is weak, the arrival of type-z_h blueprints increases the expected value of starting a firm $\bar{V}(\mathbf{s})$, which increases the incentive of young and smal type-z_l incumbents to default. The contract prevents default by cutting credit to these firms at the time of the shock, while maintaining credit to older and larger firms. With high monitoring costs under (A2), the participation and incentive compatibility constraints require that

$$F(R(V, \mathbf{s}, z_l), z_l) \leq \beta(V^H(V, \mathbf{s}, z_l) - V^L(V, \mathbf{s}, z_l)) \ , \tag{42}$$

and the increase in $\bar{V}(\mathbf{s})$ implies that the continuation values $V^H(V, \mathbf{s}, z_l)$ and $V^L(V, \mathbf{s}, z_l)$ are lower for a given V. Since the maximum revenue function F is concave, a smaller difference between these continuation values implies that all constrained entrepreneur receive less resources $R(V)$ for the same equity level V. Similarly, with low monitoring costs under (A4), the participation and enforcement constraints require that

$$F(R(V, \mathbf{s}, z_l), z_l) \geq \beta(V^{HH}(V, \mathbf{s}, z_l) - \bar{V}(\mathbf{s}) + \kappa), \tag{43}$$

and since $V^{HH}(V, \mathbf{s}, z_l) = cV$ for $V < \widetilde{V}(\mathbf{s}, z_l)$, $R(V, \mathbf{s}, z_l)$ is also lower for all firms with equity such that $V < V_u(\mathbf{s}, z_l)$.

Figures 6 shows that the initial drop in aggregate output is about half the size with high monitoring cost. With high monitoring costs under (A2), firms are on average

[14] That is, the increase in average technology raises the demand for labor and working capital, which leads to two opposing effects. The increase in average technology raises the demand for deposits used to fund new and larger firms, and the demand for labor. Aggregate output and the wage rate rise, inducing workers to save more. However, the increase in saving decreases the return on deposits, and the higher wage rate reduces the incentive to work.

smaller and the initial credit contraction is on average less severe than in an economy under (A4) with low monitoring costs. In particular, the increase in $\bar{V}(\mathbf{s})$ at $t = 0$ with low monitoring cost implies that $V_u(\mathbf{s}_0, z_l) > V_u(\mathbf{s}_{-1}, z_l)$, so that a mass of entrepreneurs previously operating at full scale temporarily operate below their efficiency level.

Figure 7 shows that the economies' aggregate output response are not significantly different from the frictionless economy with strong contract enforcement. With high monitoring costs, the continuation values are not affected by the increase in $\bar{V}$. With low monitoring costs, the enforcement constraint is always satisfied because of limited liability. Put differently, when enforcement is strong, the enforcement constraint is redundant and the amplification mechanism is absent. Interestingly, while the dynamics of aggregate output in these economies with strong enforcement are not significantly different from the frictionless economy, the firm size distribution and its evolution are completely different.

This result is broadly consistent with Cooley et al. [2004]. The environment of Cooley et al. [2004] is closely related to the low-monitoring costs economy under (A3) and (A4), but is extended to consider the role of firm idiosyncratic revenue shocks.[15] Taken together, these results suggest that while weak enforcement is an important source of aggregate fluctuation, moral hazard is not. In fact, the results suggest that moral hazard somewhat dampens the economy's response to technological shocks.

9 Conclusion

There remains considerable uncertainty about how frictions in financial markets affect the aggregate allocation of resources and the transmission of aggregate shocks. In this paper, I study a general equilibrium model in which agents can form long-term lending

[15] In Cooley et al. [2004], the borrowing constraint of small incumbent is relaxed at the time of the shock because, in contrast to this model, capital advanced in one period can only be used in the next period. Under this assumption, the contract prevents small firm from defaulting by promising a higher value next period, which is achieved by increasing the resources available next period.

relationships under moral hazard and limited enforcement. I show that moral hazard leads to a significant larger misallocation relative to limited enforcement. Strengthening contract enforcement increases welfare, but most of these gains are only temporary in economies with inefficient monitoring technology. Weak contract enforcement, rather than moral hazard, is found to amplify the impact of aggregate technological shocks.

Taken together, this analysis suggests that while long-term lending contracts are effective in overcoming financial frictions, their aggregate effects crucially depends on both the nature of the agency problems and the structure of financial markets in which these contracts are implemented. Since input financing frictions are captured by standard measure of firm productivity, it is possible that the noted sectorial and cross-country differences in firm access to credit attributed to heterogeneous productivity (Castro, Clementi, and Macdonald [2009], Buera et al. [2011]) could also reflect differences in monitoring efficiency arising from, for example, different management practices (Bloom, Eifert, Mahajan, McKenzie, and Roberts [2013]), as well as differences in legal institutions (La Porta, de Silanes, Shleifer, and Vishny [1997], La Porta, de Silanes, Shleifer, and Vishny [1998]).

Lastly, this paper is a first pass at mapping the effects of long-term financial contracts that affect firm financing under two broad types of financial frictions to macroeconomic outcomes in a coherent framework. Other issues not considered in this paper, such as partial commitment as in Kovrijnykh [2013], and asymmetric information about outside option of default as in Hopenhayn and Werning [2008] could also have important aggregate implications. Progress toward identifying the significance of different types of financial frictions across industries and regions is crucial to better assess their relevance for the determination of aggregate outcomes.

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10 Figures and tables

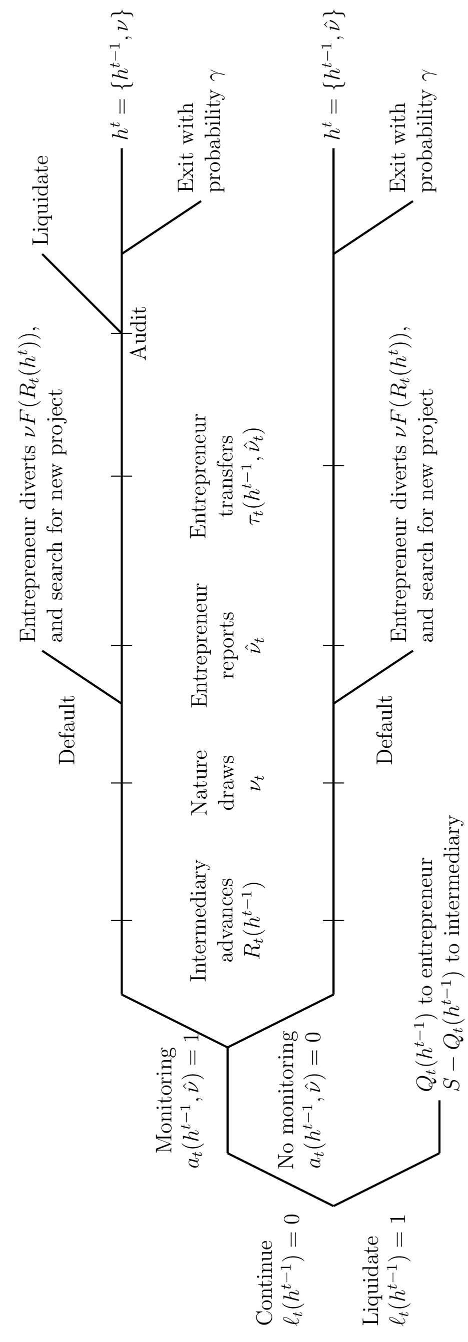

Figure 1: Timing within a period

Optimal loan size $R(V)$ and continuation values $V'(V)$

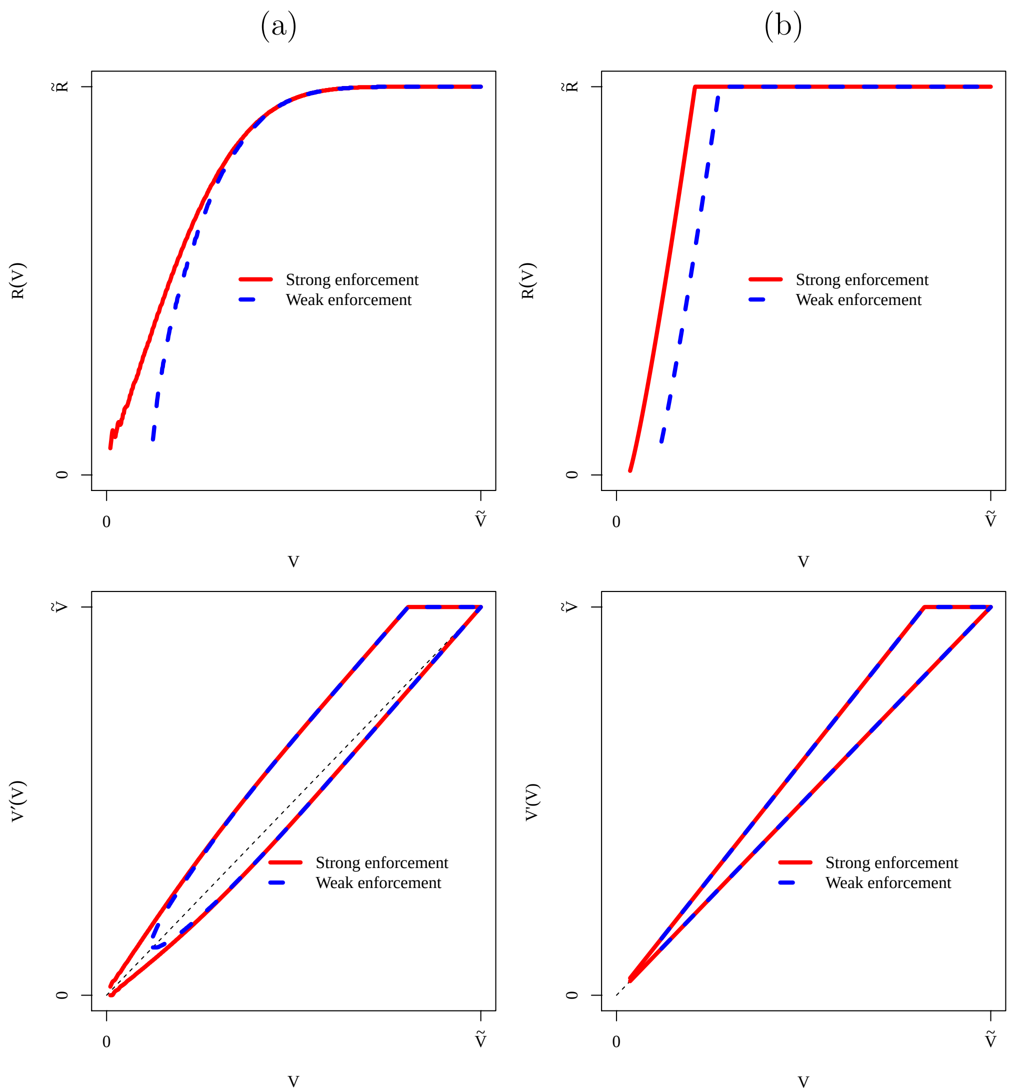

Figure 2: This figure plots the optimal loan size (top) and continuation values (bottom) as function of firm equity V. The left hand side plots the decision rules with high monitoring costs, and the right hand sides plots the decision rule with low monitoring costs. In the bottom panel, the curves below or on the 45 degree dashed lines correspond to the low continuation values $V^L(V)$ and $V^{LL}(V)$, and the curves above the 45 degree dashed line correspond to the high continuation values $V^H(V)$ and $V^{HH}(V)$.

Firm dynamics

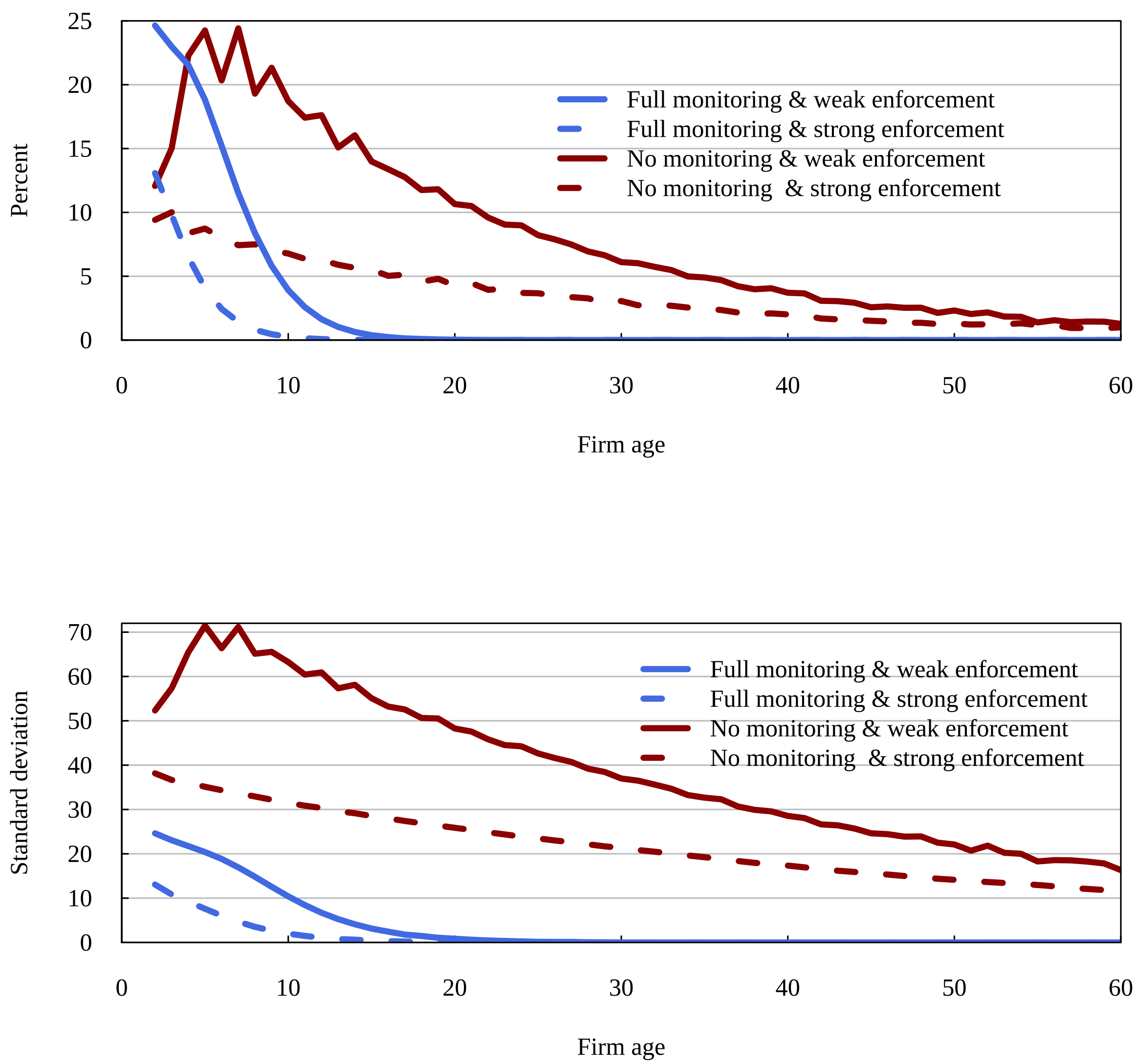

Figure 3: The top panel plots the firm investment growth rate conditional on firm age in years, and the bottom panel plots the standard deviation of firm investment growth rate conditional on firm age in years.

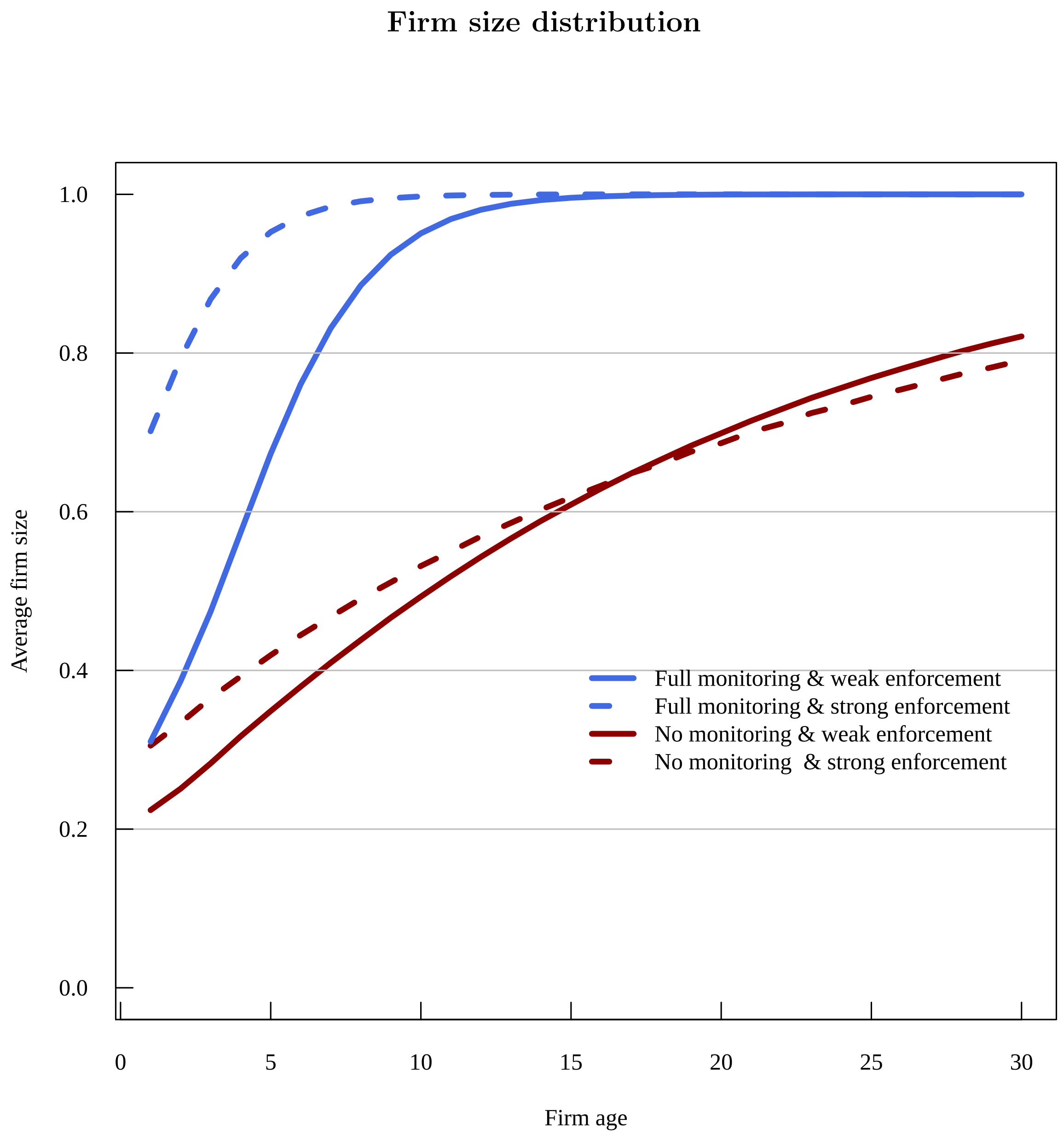

Figure 4: This figure plots the average firm size in terms of capital conditional on firm age in years. For each economy, the efficient level of capital has been normalized to 1.

The effect of financial development

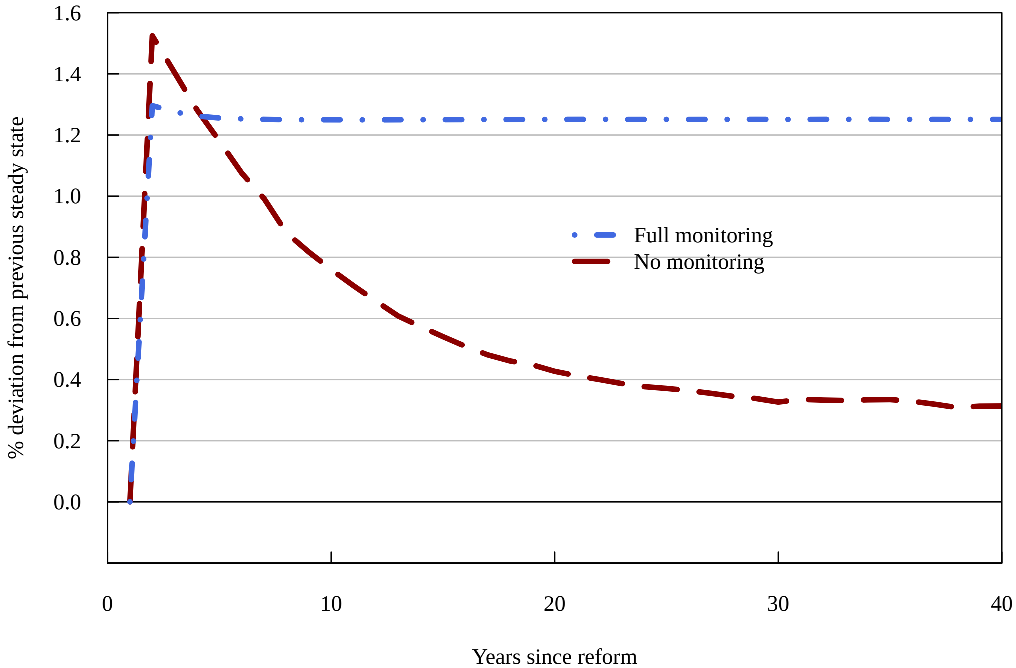

Figure 5: This figure plots the response of aggregate output to a permanent strengthening of contract enforcement in economies with high and low monitoring costs. The response of aggregate output is measured as the percentage deviation from the pre-reform steady state.

The effect of technological shocks under weak enforcement

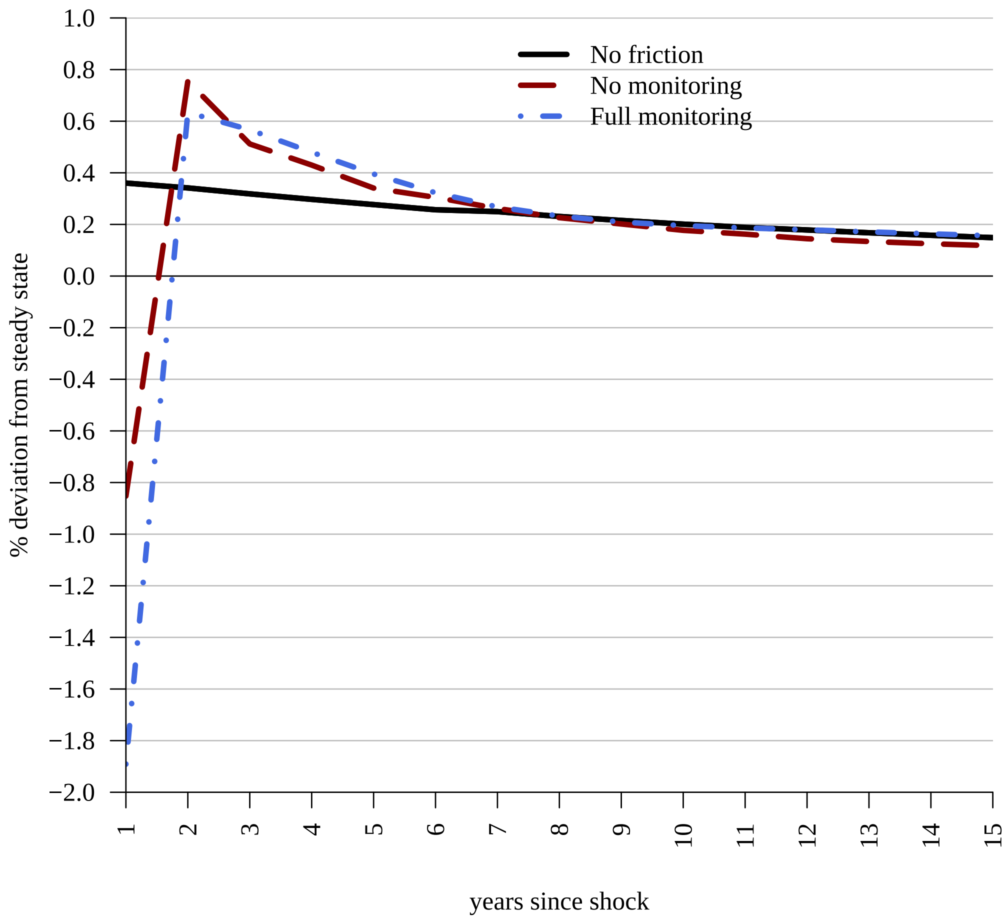

Figure 6: This figure plots the response of aggregate output to one-period improvement in the blueprint technology with weak contract enforcement in economies with high and low monitoring costs, and in the frictionless economy. The response of aggregate output is measured as the percentage deviation from the pre-reform steady state.

The effect of technological shocks under strong enforcement

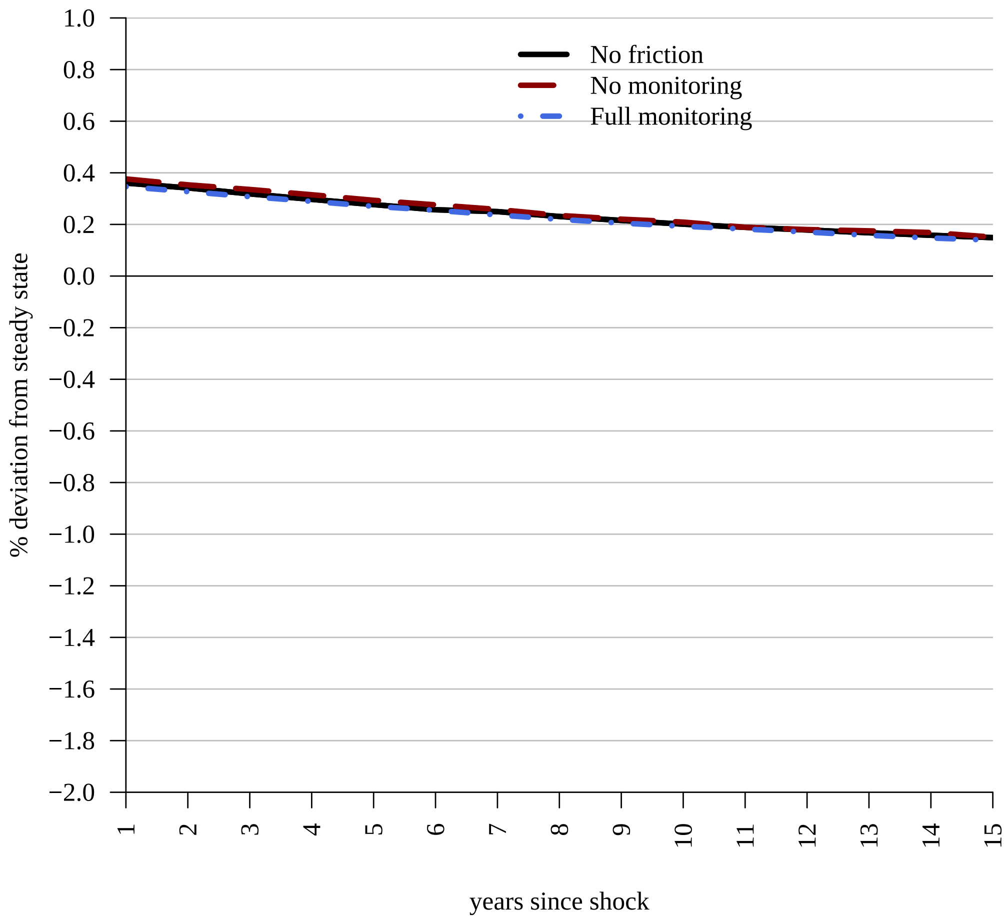

Figure 7: This figure plots the response of aggregate output to one-period improvment in the blueprint technology with strong contract enforcement in economies with high and low monitoring costs, and in the frictionless economy. The response of aggregate output is measured as the percentage deviation from the pre-reform steady state.

Table 1: Parameter values

		(A2)	(A1)	(A4)	(A3)	
		No monitoring: $m \to \infty$		**Full monitoring:** $m \to 0$		
		Weak enforcement $\kappa \to \underline{\kappa}$	**Strong enforcement** $\kappa \to \bar{V}$	**Weak enforcement** $\kappa \to \underline{\kappa}$	**Strong enforcement** $\kappa \to \bar{V}$	**No friction**
Workers' discount rate	$\hat{\beta}$	0.9591	0.9591	0.96225	0.9623	0.9620
Elasticity of leisure	η	2.251	2.242	1.8962	1.891	1.946
Firm exogenous exit rate	γ_e	0.062	0.061	0.062	0.062	0.062
Capital intensity	ξ	0.2175	0.2205	0.249	0.260	0.272
				Common parameters		
Worker mortality rate	γ_w			0.02		
Span of control	θ			0.85		
Firm technology	z_l			1		
	z_h			1.04		
Setup investment	I_0			0.13		
Salvage value	S			$0.8 \times I_0$		
Revenue shock	p			0.5		
Minimum repudiation cost	$\underline{\kappa}$			0.45		

Table 2: Steady state allocations and prices

	(A2)	(A1)	(A4)	(A3)	
	No monitoring: $m \to \infty$		**Full monitoring:** $m \to 0$		
	Weak enforcement $\kappa \to \underline{\kappa}$	**Strong enforcement** $\kappa \to \bar{V}$	**Weak enforcement** $\kappa \to \underline{\kappa}$	**Strong enforcement** $\kappa \to \bar{V}$	**No friction**
Targeted:					
Interest rate	0.040	0.040	0.040	0.040	0.040
Hour worked	0.333	0.333	0.333	0.333	0.333
Labor income share	0.603	0.602	0.600	0.600	0.600
Entry/exit rate	0.062	0.062	0.062	0.062	0.062
Relative size of entrants	-	0.305	-	-	-
Untargeted:					
Relative size of entrants	0.224	-	0.310	0.701	1
Average size	0.548	0.573	0.826	0.953	1
Fraction unconstrained	0.192	0.197	0.256	0.255	1
Wage rate	1.334	1.340	1.387	1.409	1.406
Output	0.737	0.741	0.770	0.782	0.787
Consumption/Output	0.821	0.819	0.790	0.779	0.767
Investment/Output	0.179	0.181	0.210	0.221	0.233

A Appendix

A.1 Existence of a general equilibrium

I only prove the existence of a stationary equilibrium for an economy with one type of firm–i.e., $q' = q$. I concentrate on the economy under Assumption (A1) for which monitoring is not feasible and enforcement is the strongest. That is, $m \to \infty$ and $\kappa \to \bar{V}$. The proof is similar for the other cases.

Given interest rate r and wage w, perfect competition in the financial market implies that new firms start with equity V_0. Consider the sequence $(X_t)_{t \geq 0}$ of equity levels from a single firm indefinitely replaced by a new one upon liquidation or exogenous exit, with $X_0 := V_0$. It is clear $(X_t)_{t \geq 0}$ is a sequence of random variables, and its evolution depends on the properties of the optimal contracts and on the sequence of shocks – liquidation lottery, revenue shock, and exogenous exit.

The proof consists of four parts. The first part show that $\mathbf{X} = (X_t)_{t \geq 0}$ is a time-homogeneous Markov chain such that

$$X_{t+1} = T_\omega(X_t, \epsilon_t), (\epsilon_t)_{t \geq 0} \sim \phi_\omega \in \mathscr{P}(Z), X_0 = V_0 \in S \tag{44}$$

where $T_\omega : S \times Z \to S$ is a collection of measurable functions indexed by $\omega \in \Omega$ the parameter space, $(\epsilon_t)_{t=1}^{\infty}$ is a sequence of independent random shocks with joint distribution ϕ_ω, and S and Z are the state space and the probability space, respectively. The second part of the proof establishes that this Markov chain admits a unique stationary distribution, which can be attained in a finite number of periods starting from any initial distribution. The third part of the proof establishes that the stationary distribution is continuous in ω. The last part defines a continuous mapping of Ω on itself and applies the Schauder Fixed-Point Theorem, which, together with the first three results and the condition that Ω is a compact and convex set, implies that this mapping admits at least one fixed point.

Proposition 3 (Part 1) **X** *is a time-homogeneous Markov chain on a general state space.*

Equip the state space S with a boundedly compact, separable, metrizable topology $\mathscr{B}(S)$. Let $(Z, \mathscr{Z})$ be the measure space for the shocks. Let A be any subset of $\mathscr{B}(S)$. It follows for any $x \in [V_C, \widetilde{V})$

$$P(x, A) = \begin{cases} (1-\gamma_e)(1-p)\alpha(V^L(x)) + \gamma_e & \text{if } A = \{V_0\} \text{ and } V^L(x) < V_C \\ (1-\gamma_e)(1-p)(1-\alpha(V^L(x)) & \text{if } A = \{V_C\} \\ (1-\gamma_e)(1-p) & \text{if } A = \{V^L(x)\} \text{ and } V^L(x) \geq V_C \\ (1-\gamma_e)p & \text{if } A = \{V^H(x)\} \\ 0 & \text{otherwise} \end{cases} \tag{45}$$

And for $x = \{\widetilde{V}\}$

$$P(x, A) = \begin{cases} \gamma_e & \text{if } A = \{V_0\} \\ (1-\gamma_e) & \text{if } A = \{\widetilde{V}\} \\ 0 & \text{otherwise} \end{cases} \tag{46}$$

For each $A \in \mathscr{B}(S)$, $P(\cdot, A)$ is a non-negative function on $\mathscr{B}(S)$, and for each $x \in S$, $P(x, \cdot)$ is a probability measure on $\mathscr{B}(S)$. Therefore, for any initial distribution ψ, the stochastic process **X** defined on S^∞ is a time-homogeneous Markov chain. ∎

Proposition 4 (Part 2) **X** *is globally stable.*

Let **M** denote the corresponding Markov operator, and let $\mathscr{P}(S)$ denote the collection of firms distribution generated by **M** for a given initial distribution.[16]

Write the stochastic kernel P with the density representation p so that $P(x, dy) = p(x, y)dy$ for all $x \in S$. The Dobrushin coefficient $\vartheta(p)$ of a stochastic kernel p is defined

[16]Note that Stokey, Lucas, and Prescott [1989, Theorem 12.12] fails to apply in this case because the stochastic kernel is not monotone on $[V_C, a]$ where a is such that $V^L(a) = V_C$. For instance, consider

by

$$\vartheta(p) := \min\left\{\int p(x,y) \wedge p(x',y)dy : (x,x') \in S \times S\right\} . \tag{47}$$

$(\mathscr{P}(S), \mathbf{M})$ is globally stable if $(\psi\mathbf{M}^t)_{t\geq 0} \to \psi^*\mathbf{M}$ where $\psi^* \in \mathscr{P}(S)$ is the unique fixed point of $(\mathscr{P}(S), \mathbf{M})$, which occurs if the Markov operator is a uniform contraction of modulus $1-\vartheta(p)$ on $\mathscr{P}(S)$ whenever $\vartheta(p) > 0$. Since a firm dies with a fixed, exogenous and independent probability γ_e each period, and is instantaneously replaced by a new one of size V_0, it follows that

$$P(x, \{V_0\}) \geq 0 \; \forall \; x \in S \; . \tag{48}$$

Equation (11.15) and Exercise (11.2.24) in Stachurski [2009] yield $\vartheta(p) > \gamma_e$. By Stachurski [2009, Th. 11.2.21], it follows that

$$||\psi\mathbf{M} - \psi'\mathbf{M}||_{TV} \leq (1-\gamma_e)||\psi - \psi'|| \tag{49}$$

for every pair ψ, ψ' in $\mathscr{P}(Z)$, and where $_{TV}$ indicate the total variation norm. ∎

Proposition 5 (Part 3) *ψ^* is continuous in ω.*

The result follows if the conditions of LeVan and Stachurski [2007, Proposition 2] are satisfied.[17] Consider again the state space S equipped with a boundedly compact, separable, metrizable topology, $(Z, \mathscr{Z})$ a measure space, and $\mathscr{P}(Z)$ the collection of

the function $f(x) = x$. From the above,

$$\begin{aligned}
&\int_{[V_C,a]} P(x,dy)y \\
&= (1-\gamma_e)\{(1-p)[\alpha(V^L(x))V_0 + (1-\alpha(V^L(x)))V_C] + pV^H(x)\} + \gamma_e V_0 \\
&= (1-\gamma_e)[(1-p)V_0(1-V^L(x)/V_C) + (1-p)V^L(x) + pV^H(x)] + \gamma_e V_0 \\
&= (1-\gamma_e)[(1-p)V_0\alpha(V^L(x)) + x/\beta] + \gamma_e V_0
\end{aligned}$$

which is generally not increasing. The intuition is that when x falls below a, the probability of liquidation $\alpha(x)$ becomes non-zero in case of a low revenue shock. However, liquidation sends x to state V_0, which can be larger than V_C and $V^H(x)$, so that the lower the x, the higher $\mathbf{E}(x'|x)$.

[17] LeVan and Stachurski [2007, Proposition 2] is an application of LeVan and Stachurski [2007, Theorem 1] of which Stokey et al. [1989, Theorem 12.13] is a special case.

probabilities on $(Z, \mathscr{Z})$. From the above, the model can be written as

$$X_{t+1} = T_\omega(X_t, \epsilon_t), \text{ where } \epsilon_t \sim \psi_\omega \in \mathscr{P}(Z), \ \ \forall t \in \mathbb{N} \tag{50}$$

where $(\epsilon_t)_{t=1}^\infty = (\{D_{1,t}, D_{2,t}, D_{2,t}\})_{t=1}^\infty$ is the vector of independently distributed binary random variable corresponding to the liquidation, revenue and death shock realization, and $T_\omega : S \times Z \to S$ is measurable. Given the price vector ω, the stochastic kernel can be written as $P_\omega(x, B) := \psi_\omega\{z \in Z : T_\omega(x, z) \in B\}$, and given the parameter space Ω, the family of stochastic kernel is $\{P_\omega : \omega \in \Omega\}$. Let N be any subspace of Ω, and define $\Lambda(\omega) := \{\mu \in \mathscr{P}(S) : \mu = \mu P_\omega\}$ the collection of invariant distribution corresponding indexed by ω.

Lemma 1 (LeVan and Stachurski [2007]) *If $\Lambda(\omega) = \{\mu_\omega\}$, then $\omega \mapsto \mu_\omega$ is continuous on N if the following four conditions are satisfied:*

1. *the map $N \ni \omega \mapsto T_\omega(x, z) \in S$ is continuous for each pair $(x, z) \in S \times Z$*
2. *for each compact $C \subset S$, there is a $K < \infty$ with*

$$\int d(T_\omega(x, z), T_\omega(x', z))\psi_\omega(dz) \le K d(x, x'), \forall x, x' \in C, \forall \omega \in N \tag{51}$$

3. *$\exists$ a Lyapunov function $\mathcal{V} \in \mathscr{L}(S)$, $\lambda \in (0, 1)$, and $L \in [0, \infty)$ s.t. $\forall \omega \in N$*

$$P_\omega \mathcal{V}(x) := \int \mathcal{V}(T_\omega(x, z))\psi_\omega(dz) \le \lambda \mathcal{V}(x) + L \quad \forall x \in S \tag{52}$$

4. *$\omega \mapsto \psi_\omega$ is continuous in total variation norm.*

That $\Lambda(\omega)$ is nonempty, and $\Lambda(\omega) = \{\mu_\omega\}$ for each $\omega \in N$ follows from Proposition 1. Condition (1) requires the optimal value function $W(x)$ to be continuous in ω which follows from Berge's theorem (see, for example, Ausubel and Deneckere [1993]).

Condition (4) holds as the shocks are independent and the probability of liquidation is $\alpha = (x - V_C)/V_C$, which is continuous in ω since V_C is continuous in ω from condition (1). To show that condition (2) holds, define again $a \ni V^{LL}(a) = V_C$, and $b \ni V^{HH}(b) \geq \widetilde{V}$. Pick any $x, x' \in C \subset [V_C, a)$. Without loss of generality assume $x > x'$ so that $\alpha(V^L(x')) > \alpha(V^L(x))$. By noting that $\alpha(V^L(x')) - \alpha(V^L(x)) = (V^L(x') - V^L(x))/V_C$, and $x = \beta[pV^H(x) + (1-p)V^L(x)]$ at optimum, it follows that

$$\begin{aligned}
& \int d(T_\omega(x,z), T_\omega(x',z))\psi_\omega(dz) \\
< \;& |pV^H(x) + (1-p)V^L(x) - pV^H(x') - (1-p)V^L(x')| \\
= \;& \frac{1}{\beta}|x - x'| = \frac{1}{\beta}d(x,x').
\end{aligned}$$

The above inequality also holds for any $x, x' \in C \subset [a, b)$. Last, recall that $V^L(x) = (x - pF(R(x)))/\beta$. So, for any $x, x' \in C \subset [b, \widetilde{V})$

$$\begin{aligned}
& \int d(T_\omega(x,z), T_\omega(x',z))\psi_\omega(dz) \\
< \;& (1-p)|V^L(x) - V^L(x')| \\
= \;& \frac{(1-p)}{\beta}|x - x'| = \frac{(1-p)}{\beta}d(x,x').
\end{aligned}$$

It remains to show condition (3) holds. Pick $\mathcal{V}(x) = x$, which is a Lyapunov function since S is boundedly compact. Then,

$$P_\omega x = \begin{cases} (1-\gamma_e)\{(1-p)[\alpha(V^L(x))V_0 + (1-\alpha(V^L(x)))V_C] + pV^H(x)\}\mathbf{1}_{[V_C,a)}(x) \\ + [pV^H(x) + (1-p)V^L(x)]\mathbf{1}_{[a,b)}(x) \\ + \widetilde{V}\mathbf{1}_{(x=\widetilde{V})}(x)\} + \gamma_e V_0 \end{cases}. \tag{53}$$

Pick any $x \in [V_C, a)$ so that $V_C \leq x \leq V^H(x)$. Then,

$$\begin{aligned} P_\omega x &< (1-p)(1-\alpha(V^L(x)))V_C + pV^H(x) + [(1-p)\alpha(V^L(x)) + \gamma_e]V_0 \\ &\leq (1-p)V_C + pV^H(x) + V_0 \\ &\leq \lambda x + \sup_{\omega \in N} V_0 = \lambda \mathcal{V}(x) + L \end{aligned}$$

The same inequality holds for any $x \in [a, b)$, since $V^L(x) < x < V^H(x)$. Last, when $x = \widetilde{V}$,

$$\begin{aligned} P_\omega x &= (1-\gamma_e)\widetilde{V} + \gamma_e V_0 \\ &\leq \lambda x + \sup_{\omega \in N} V_0 \end{aligned}$$

$\blacksquare$

Proposition 6 (Part 4) *There exists an equilibrium.*

Using the capital and labor market clearing conditions, define the mapping

$$\mathbf{f}(\omega) = \begin{bmatrix} D_w(\omega) + D_e(\omega) \\ \int n(V,\omega) d\mu(\omega) - H \end{bmatrix}$$

such that $\mathbf{f} : \Omega \mapsto \mathbb{R}^2$. Prices r and w must each be positive and greater than zero. Without loss of generality, assume that r and w are bounded above by arbitrarily large but finite numbers $\bar{r}$ and $\bar{w}$. It follows that the set Ω is compact and convex. Define the mapping $\Phi : \Omega \mapsto \Omega$ such that

$$\Phi(\omega) = \underset{\omega \in \Omega}{\operatorname{argmax}} -||\mathbf{f}(\omega)||^2 \tag{54}$$

From Proposition 4 and Proposition 6, the maximand is continuous in ω so that the correspondence Φ is also continuous. Applying the Schauder Fixed-Point [Stokey et al., 1989, Theorem 17.4] yields the results. ∎

A.2 Worker's problem

The worker's problem can be written recursively as

Problem 1

$$\begin{aligned} U(d,\mathbf{s}) = \max_{d',c,h} \quad & u(c,h) + (1-\gamma_w)\hat{\beta}\mathbb{E}U(d',\mathbf{s}') \\ s.t. \quad & c + (1-\gamma_w)d' = (1+r)d + wh \\ & d' \geq \epsilon \\ & \\ & \mathbf{s}' \sim \mathcal{H}(\mathbf{s}) \end{aligned} \tag{P0}$$

where $\mathcal{H}(\mathbf{s})$ is the law of motion for $\mathbf{s}$, and ϵ is a finite limit on borrowing, which is the maximum amount of debt a worker can service by working full time.

A.3 Clearing of the goods market

Consider the steady state case with only one type of firm. A stationary distribution of firms implies that $\mathbf{s} = \mathbf{s}'$, which I omit from the notation. The definition of aggregate output and entrepreneurs' consumption implies that

$$\begin{aligned} Y &= p \textstyle\int F(R(V))d\mu \\ &= C_e + p \textstyle\int \tau(V)d\mu \ . \end{aligned} \tag{55}$$

In a steady state where $\mathbf{s}' = \mathbf{s}$, the balance budget condition implies that

$$rD_e = p \int \tau(V)d\mu - \int R(V)d\mu + \Gamma(\mathbf{s})(I_0 - S) \tag{56}$$

Multiplying the capital market clearing condition by r and substituting for rD_e in the intermediary balance budget condition yields

$$-rD_w = p\int \tau(V)d\mu - \int R(V)d\mu + \Gamma(\mathbf{s})(I_0 - S)$$

which implies that

$$Y = C_e + \int R(V)d\mu + rD_w + \Gamma(I_0 - S)\ . \qquad (57)$$

Using the fact that

$$\int R(V)d\mu = \int k(V)d\mu + w\int n(V)d\mu \qquad (58)$$

yields

$$Y = C_e + \int k(V)d\mu + \Gamma(I_0 - S) + w\int n(V)d\mu + rD_w\ , \qquad (59)$$

It follows that

$$Y = C_e + K + w\int n(V)d\mu + rD\ , \qquad (60)$$

where

$$K = \int k(V)d\mu + \Gamma(I_0 - S)\ . \qquad (61)$$

Finally, the labor market clearing condition and the aggregate budget constraint for workers

$$C_w + D = wH + (1+r)D \qquad (62)$$

yields

$$Y = C_e + C_w + K\ . \qquad (63)$$

A.4 Numerical implementation

All computations were done using the **R** language.[18]

A.4.1 Solving for the steady state

(1) Guess a value for r,w from a compact set

(2) Guess a value for $\bar{V}$ from a compact set

(3) Solve the contract using value iteration on a grid, and solve for the initial value from $V_0 = \sup_V\{W(V) - V = I_0\}$. If $V_0 \neq \bar{V}$ go to (2), if equal go to (4)

(4) Estimate the invariant distribution. When firms are never monitored, use the Look-Ahead Estimator as described in Stachurski and Martin [2008]. When firms are always monitored, $V^{HH}(V)$ is a linear function of V and $V^{LL}(V) = V$, which implies that the process for $(V)_{t\geq 0}$ is discrete Markov on a finite state space. Stachurski [2009, Excercise 4.3.7] shows that for a finite Markov chain with N state and a stochastic kernel P, the invariant distribution ψ is such that $\mathbf{1}_{1\times N} = \psi(\mathbf{I}_N - P + \mathbf{1}_{N\times N})$, where $\mathbf{1}_{N\times M}$ is a $N \times M$ matrix of ones, and $\mathbf{I}_N$ is an identity matrix of size N.

  (1') Check labor market clearing: if does not clear, guess a new w using bisection and go to (2'), if clears go to (1")

  (2') Guess a new value for $\bar{V}$ using bisection

  (3') Solve the contract using value iteration on a grid, and solve for the initial value from $V_0 = \sup_V\{W(V) - V = I_0\}$. If $V_0 \neq \bar{V}$ go to (2'), if $V_0 = \bar{V}$ go to (4'). Note that the program is now using the old distribution as an approximation since it should not be too different around a particular value of w, which increases the efficiency of the algorithm substantially

[18]See http://www.r-project.org.

(4') Go to (1')

(1") Check capital market clearing: if does not clear, guess a new r using bisection go to (2"), if clears go to (4)

(2") Guess a new value for $\bar{V}$ using bisection

(3") Solve the contract using value iteration on a grid, and get the initial values from $V_0 = \sup_V\{W(V) - V = I_0\}$. If $V_0 \neq \bar{V}$ go to (2'), if $V_0 = \bar{V}$ go to (4")

(4") go to (1')

(4) Stop when the maximum absolute difference between supply and demand in two markets falls below the desired tolerance level

A.4.2 Computing transition dynamics

This algorithm is a brute force method to finds the exact sequence of prices and state-contingent optimal decision rules such that all markets in every period. This method can be interpreted as a non-parametric estimation of the law of motion for the distribution of firm along the equilibrium price path. While the algorithms converge and allow for an explicit aggregation of the decisions of a very large number of agents, it is very computationally intensive. Nevertheless, it was found that this brute force method was necessary to obtain an accurate aggregation given the non-linearity of the decision rules in some case.

(1) Simulate the economy with a large number of firms (I use 6 million) for a large number of period T until the economy reaches its steady state, and use it as the $t = 0$ firm distribution

(2) Generate the sequences of idiosyncratic shocks

(3) At $t = 1$, either permanently increase κ (strengthening of contract enforcement) or replace all deceased firms with new z_h-firms (improvement in blueprint technology)

(4) Guess a sequence of prices

  (1') Guess a value for $\bar{V}$ from an interval

  (2') Solve the contract using value iteration on a grid, and get the initial value from $V_0 = \sup_V \{W(V) - V = I_0\}$. If $V_0 \neq \bar{V}$ go to (1'), if $V_0 = \bar{V}$ go to (3')

  (3') Update the distribution of firms given the realization of idiosyncratic shocks

  (3') Estimate the marginal distribution of firms (Stachurski and Martin [2008]), and check the market clearing conditions. If markets do not clear in all periods, find the prices that would have cleared the markets each period using the estimate of the firm distribution. Finding these prices requires solving the system of non-linear equations from the market clearing conditions using the estimated of the firm distribution to integrate

  (4') Update the sequence of prices, and go to (2)

(4) Iterate until the maximum discrepancy from the markets clearing conditions falls below the desired threshold, and the sequence of prices no longer changes.