Cautious Hiring[*]

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Abstract

There have been significant changes in the cyclicality of US average labor productivity since the early 1990s. Previously, labor productivity was procyclical but beginning with the recession of the early 1990s labor productivity rises immediately following a recession before returning to pre-recession levels. In this paper we develop a dynamic general equilibrium model in which a change in the importance of firm specific human capital can explain the new pattern in labor productivity as well as partially account for the decrease in the rate of employment recovery (jobless recoveries) observed in the most recent three recessions. Additionally, we present empirical support that the importance of firm specific human capital has in fact increased for recent recessions.

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1 Introduction

It has taken nearly seven years for the unemployment rate to return to pre-recession levels following the 2008 recession. This slow recovery in the unemployment rate is a phenomenon shared with the two previous recessions (1990 and 2001) but contrasts sharply with recessions prior to these where recoveries in the unemployment rate were relatively faster. This emergent pattern is directly related to a conversation commonly referred to among policymakers, journalists as well as in the economic literature as *jobless recovery*.[1] Jobless recoveries refer to periods following recessions in which rebounds in aggregate output are accompanied by slower recoveries in aggregate employment.[2]

Closely related to the jobless recoveries is the cyclicality of average labor productivity. When employment recovery lags recovery in GDP, labor productivity rises. In fact, during the three most recent recessions, average labor productivity has been countercyclical, as opposed to the procyclical pattern observed in earlier recessions.[3]

We propose a novel mechanism that accounts for this change in the cyclicality of average labor productivity and also partially explains the decrease in the speed of recovery in the unemployment rate. Namely, the increase in the relative productivity of senior to junior workers.

Central to our model is the investment in firm specific human capital of workers over time.[4] The marginal productivity of newly hired junior workers is low compared with senior workers who have already been working on the job for some time. Firm specific skills, combined with a lack of commitment of firms, implies that wage rates will not fully reflect the marginal productivity of workers. In particular, when a junior worker turns senior, his productivity has improved due to an accrual of firm specific skills. However, this worker does not receive an equivalent gain in his wage. This is because the senior worker cannot credibly threaten to quit the firm. Departure from

[1]For examples refer to Gordon (1993), Bernanke (2003), Gali et al. (2012) and Jaimovich and Siu (2012).

[2]Definition taken from Jaimovich and Siu (2012)

[3]This has been previously documented by McGrattan and Prescott (2012), Gali and van Rens (2010) and Berger (2012) among others.

[4]We use a broad notion of specific investment in human capital as originally presented in Becker (1964). Forms of specific investment in workers can include costs of familiarizing new employees with the firm, hiring costs, employment agency fees, expenses incurred by new employees finding jobs, time spent interviewing and testing new employees, as well as reduction in productivity of new employees due to rotation among departments to learn about employee ability.

the firm results in the loss of all the firm specific investment. As a result, there is a compression in wage payments. Junior workers are paid more than their marginal productivity and senior workers are paid less.

This compressed wage structure implies that hiring junior workers is similar to an investment. A cost is incurred in the period junior workers are hired, and a profit is made later on when these junior workers turn senior. The stock of senior workers with a firm is valuable to the firm. Hiring new workers lowers the average profits of the firm and makes bankruptcy more likely. Bankruptcy is costly to a firm since it causes a separation from their stock of senior workers. Firms trade off a higher rate of growth with a higher probability of default. As a result, firms hire cautiously and grow slowly.

A key parameter in our model is the relative productivity of senior workers to junior workers. As this ratio increases, the cost of bankruptcy also increases and firms endogenously choose to grow more slowly. This translates to a slower rate of unemployment recovery after a recession.

When the economy is recovering from a recession, there are two opposing forces on average labor productivity. On one hand, output and consumption fall over the recession and gradually climb back. This increasing profile of consumption lowers the stochastic discount factor of households and firms. As a result, firms of all sizes cut back on hiring, preferring present period profits over growth. This reduces average firm size and due to the decreasing returns to scale technology, tends to drive up average labor productivity.

The second and opposing force is related to the composition of senior to junior workers employed in the economy. Following an unexpected shock, a large number of workers are separated from firms. In the ensuing recovery, the fraction of junior workers temporarily increases. This effect results in a reduciton of average labor productivity since junior workers are less productive than their senior counterparts. The faster firms elect to hire workers, the stronger this force since the composition of senior to junior workers is directly related to the speed of hiring.

When θ^S/θ^J is small (fast recovery), firms grow fast. Consequently, the composition effect outweighs the decreasing returns to scale effect. This results in a procyclical pattern for average labor productivity. When θ^S/θ^J is large (slow recovery), firms grow slowly. This weakens the composition effect and the decreasing returns to scale effect dominates. As a result, a countercyclical pattern of average labor productivity emerges.

We use the return to seniority to estimate the importance of firm specific human capital over time.[5] Our estimation indicates that firm specific human capital signficantly increased in importance following the mid 1980s. We believe that the cause for this change is linked to job polarization as documented in the literature which provides substantial support of the rise of polarization over the last couple of decades\footnote{Acemoglu (1999), Autor et al. (2003), Autor and Dorn (2013), Autor et al. (2006), Goos and Manning (2007), Goos et al. (2009), and Jaimovich and Siu (2012).}. Specifically, the rise of computers has been cited as a substitute for routine labor and a complement for non-routine cognitive tasks. The more technical and job specific the skills demanded by employers are, the greater the value of a senior worker relative to a junior one. In other words, polarization provides a plausible explanation for the increase in importance of firm specific human capital.

Increasing the importance of firm specific human capital leads to more cautious hiring at the firm level. This reduces the rate of recovery in unemployment at the aggregate level following a recession. Further, our benchmark model suggests that changes in the importance of firm specific human capital can account for the new pattern observed in labor productivity over the business cycle.

2 Related Literature

Ever since the recession of the early 1990s, a number of papers including Gali et al. (2012), Gordon (1993), Groshen and Potter (2003), and Bernanke (2003) have documented the slower rate of recovery in labor through recent business cycles. Our model explains a portion of this change in the speed of recovery through a change in the importance of firm specific human capital since the

[5] We follow methods used by both Altonji and Williams (2005) and Topel (1991) to estimate the returns to seniority over time.

mid 1980s.

Our theoretical explanation overlaps with several other theoretical explanations for slow recovery by the introduction of a mechanism which separates the marginal productivity of labor and the wage. For example, Hall (2014) and Midrigan et al. (2014) emphasize the role of discount rates. In their models, hiring a new worker has an upfront cost which is earned back from future differences between productivity and wage. If somehow the discount rate of firms fall during the recession (for example, due to tightening of borrowing constraints as in Midrigan et al. (2014)), hiring new workers becomes less profitable. Similar to their models, our model also features an upfront cost of hiring new workers. The difference is that in our model, this cost is due to the low productivity of newly hired workers relative to existing workers. In addition, our mechanism for generating the slower recoveries in recent recessions is an increasing gap between the relative productivity levels of new and existing workers, instead of an increase in the discount rate. An added benefit of our method is its ability to explain emergent patterns in labor productivity over the business cycle.

Our mechanism does not fully account for changes observed in the data. Complementary explanations of slow recovery in employment include Herkenhoff (2014) which shows that increased access to credit card borrowing of unemployed households prolongs unemployment duration. Rabinovich and Mitman (2012) show that the extension of countercyclical unemployment benefits also lead to jobless recoveries.

In addition to explaining slow reocveries, our work adds to the existing literature on the new patterns observed in average labor productivity.[6]

There is also a smaller literature which provides explanations for both the jobless recoveries and the change in the cyclicality of labor productivity. Sims and Pries (2011) suggest that the nature of aggregate shocks has grown more asymmetric across sectors. This leads to a reallocation of labor across sectors and a slow recovery. Our mechanism can also be motivated through sectoral reallocation in that workers reallocate from routine sectors into non-routine sectors. Our paper is also related to Berger (2012) which attributes changes in average labor productivity to firm

[6]Examples include McGrattan and Prescott (2012), Arellano et al. (2012), Aghion and Saint-Paul (1998), Gali and van Rens (2010), and Barnichon (2010).

restructuring and the removal of less productive employees during recoveries. In both papers, workers with a firm are a valuable asset and constitute a state variable. In Berger (2012), only certain new employees will be a good match and so hiring new employees is costly and the existing worker stock is valuable. In our model, new employees require firm specific training which is costly and results in a valuable existing worker stock.

In addition to changes in the importance of firm specific skills for a job, our mechanism relies on consideration of a firm's labor decision as a form of investment. In this manner, our paper builds on the seminal work of Becker (1964) in the sense that the costs and returns of firm-specific training are largely attributed to firms and that there is a difference between the marginal product of labor and the wage. There is also an existing empirical work on this topic including Frazis and Loewenstein (2006), Shaw and Lazear (2008) and Isen (2012).

The rest of the paper is organized as follows. In Section 3 we document the two main features which have changed over more recent recessions. Section 4 presents the model and definition of equilibrium. In Section 5 we present our steady state results. Section 6 calibrates our parameters to the data. In Section 7 we present the business cycle properties. Section 8 considers several extensions of the benchmark model and section 9 concludes.

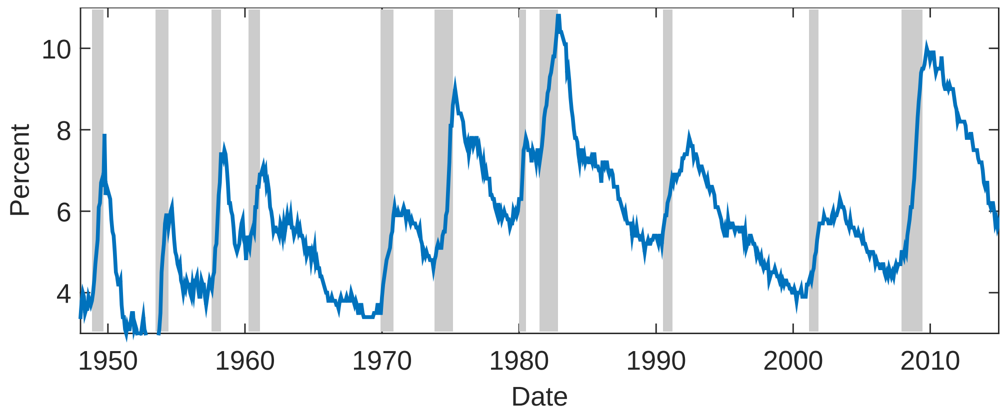
3 Data

The emergence of slower recoveries in employment beginning with the recession in the early 1990s has been well documented. Perhaps even more significant is that these recoveries in employment are not only becoming slower, but that they are also becoming slower *relative* to the recovery in GDP. This can be observed through the cyclicality of average labor productivity. In this section, we more formally present these emergent patterns over time and specifically over several recent recessions.

3.1 Unemployment Rates

The unemployment rate between January of 1967 through April of 2016 is displayed in Figure 3. As can be observed, the rate of recovery in unemployment has slowed over time.

Figure 1 – Civilian Unemployment Rate: Jan 1948- Apr 2016

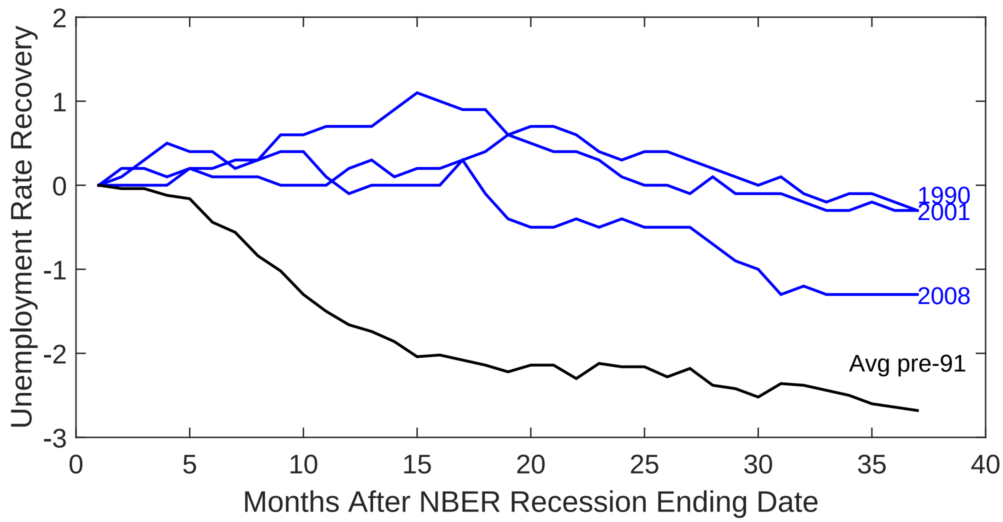


Source: Current Population Survey

To make the comparison of unemployment rate recoveries more clear, we also provide a normalized plot in Figure 2. In this figure, the unemployment rates at the NBER ending dates are normalized to zero for each recession. The top three blue lines are the recovery paths of unemployment rates following the recent three recessions. The bottom lines is the average recovery path for recessions prior to 1991.[7] This comparison demonstrates that the recent three recessions feature relatively slower recovery paths in their unemployment rates. Additional comparison in the speed of recovery in the unemployment rate is included in the appendix.

[7] Only recessions with more than 3 years of recovery before the onset of the following recession were used in the calculation of the average.

Figure 2 – Civilian Unemployment Rate Recovery Speed



Source: Current Population Survey, Authors' calculations

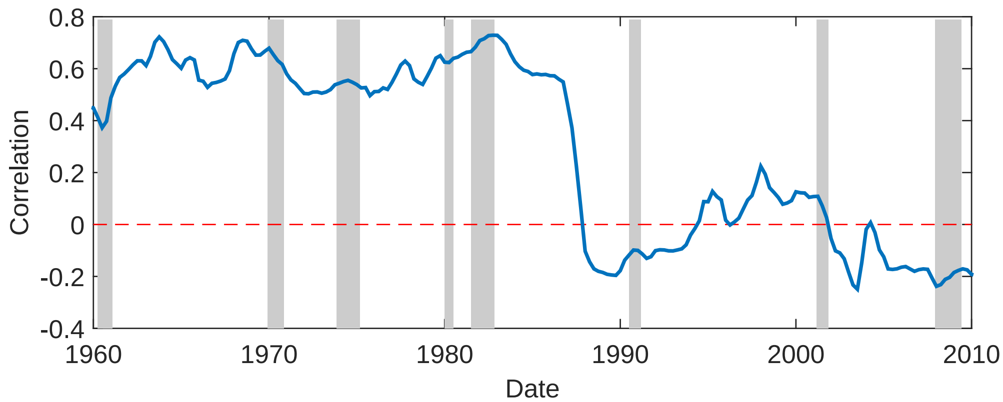
3.2 Average Labor Productivity

Not only have recoveries in unemployment slowed for recent recessions, they have slowed relative to the corresponding recoveries in output.[8] Define average labor productivity to be real output per hour worked. If GDP recovers more slowly than employment following a recession and changes in average hours worked are constant or relatively small, then average labor productivity will fall along the recovery path. Consequently, average labor productivity will be pro-cyclical. Alternatively, if GDP recovers faster than employment, average labor productivity will rise along the recovery path and will display a counter-cyclical pattern.

Figure 3 displays the 10 year centered moving average correlation between labor productivity and GDP. Each point in the series calculates the correlation between the 10 year seasonally adjusted HP filtered GDP and real output per hour surrounding the displayed date. Prior to the mid 1980s labor productivity was procyclical, however in the time following the mid 1980s labor productivity has been acyclical or even slightly counter cyclical

[8]This fact has been previously documented. See Gali and van Rens (2010); Barnichon (2010); Berger (2012); McGrattan and Prescott (2012) for examples.

Figure 3 – Ten Year Centered MA Correlation in Labor Productivity and GDP



Source: Bureau of Labor Statistics, Bureau of Economic Analysis, Authors' calculations

To investigate more closely the correlation between average labor productivity and GDP over the business cycle, we plot the two data series for each recession. In the three recessions immediately prior to 1985 (which we will label the "Fast Recoveries") the two series behaved similarly, dropping with or just before the onset of the recession and recovering fairly quickly following the recession. However, in the three most recent recessions, GDP patterns are similar to the earlier recessions with a slight slowing in the recovery rate. Patterns in labor productivity are not similar, in more recent recessions they remain fairly flat leading into the recession, increase rapidly beginning mid-recession, and fall a few years after the recession. These features can be observed in Figures 4 and 5.

Figure 4 – The "Fast Recovery" Recessions

Source: Bureau of Labor Statistics, Authors' calculations

Figure 5 – The "Slow Recovery" Recessions

Source: Bureau of Labor Statistics, Authors' calculations

4 Environment

Our economy features a simple model of firm growth. However, contrary to a standard model of firm growth with accumulation of physical capital, in our model, firms accumulate a human capital stock in the form of senior workers. Time is discreet. There are three types of agents: a continuum of representative households, a continuum of firms, and financial intermediaries. The problems of households and financial intermediaries are trivial. The primary focus is on firms.

4.1 Households

There is a measure L of households. Each period households supply labor inelastically to firms and consume wage and dividend income. Households own the firms and consume the dividends. Formally, the household's problem is given by

$$\max_{\{c_t\}_0^\infty} \sum_{t=0}^{\infty} E\left[\beta^t u(c_t)\right]$$

$$\text{s.t. } c_t \leq w_t + d_t, \text{ for all } t$$

where w_t is the wage income of the household and d_t is the dividends paid out by firms.[9]

[9]Our workforce is comprised of senior and junior workers which receive different wage rates. For simplicity, we assume that households are organized into families. Each family has a representative share of senior and junior

The stochastic discount factor of the households is given by:

$$\Lambda_t \equiv \frac{\beta E\left[u'(c_{t+1})\right]}{u'(c_t)}$$

In equilibrium, the resource constraint implies that consumption for each household is equal to the total output of the economy divided by the measure of households:

$$c_t = \frac{Y_t}{L}$$

4.2 Firms

There is a unit measure of firms. They produce consumption goods according to the production function $y = zn^{\alpha}$. $z \sim F$ is an idiosyncratic productivity shock which is iid across firms and time. n is the total effective labor units hired by the firm.

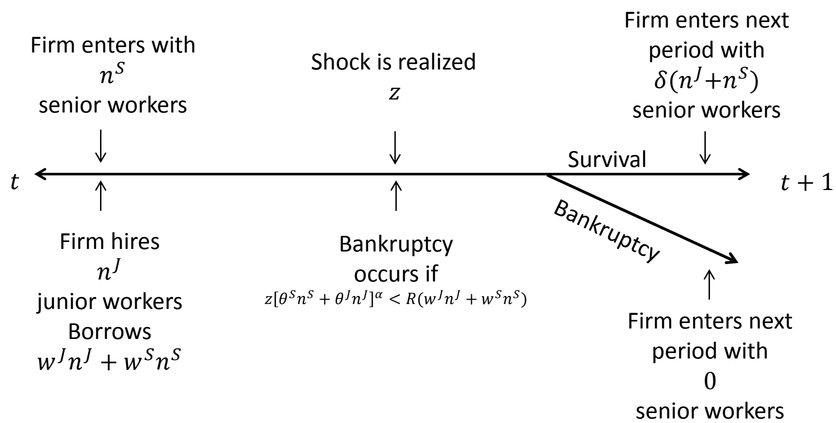
At each particular firm, workers differ in terms of their firm specific skills. In the first period a worker is employed with a firm, they are junior without any firm specific skills. Junior workers convert 1 unit of time into θ^J units of effective labor. After the initial period of employment, junior workers acquire firm specific skills automatically and become senior. Senior workers convert 1 unit of time into θ^S units of effective labor. Senior workers stay senior unless they separate from the firm. In the case of separation, senior workers lose all firm specific skills and start over as a junior worker in the labor market. Firms are unable to directly hire senior workers. Instead they must hire junior workers who will become senior in subsequent periods. A firm with n^S senior workers who decides to hire n^J junior workers has total effective labor units given by $n = \theta^J n^J + \theta^S n^S$.

4.2.1 Hiring Decision

Firms enter each period with n^S senior workers and decide how many junior workers n^J to hire. The wage bill for both types of workers has to be paid before production takes place. Firms

workers. Consequently, $w_t = w_t^J s_t^J + w_t^S s_t^S$, where s_t^J and s_t^S represent the share of junior and senior workers employed in the economy.

Figure 6 – Timing



finance their wage bill $\left(w^S n^S + w^J n^J\right)$ by taking out an intra-period working capital loan from the financial intermediaries. Next, productivity shock z is realized and production occurs. If the firm's revenue is sufficient to pay back the financial intermediary, firms pay back their debt, distribute the remaining profits as dividends to the households, and continue in the next period with $\delta\left(n^J + n^S\right)$ senior workers, where $1-\delta$ is the exogenous separation rate between firms and workers. If the proceeds from output sales are insufficient to pay back financial intermediaries, firms go bankrupt. When a firm declares bankruptcy, all revenue from the period is confiscated by the lenders and in the next period, firms start over with 0 senior workers. A visual depiction of timing is displayed in Figure 6.

We assume that at the end of each period, firms pay out all profits (if any) as dividends. In our model, there is no way for firms to retain earnings. There exists a large literature in finance which argues that there are substantial costs of maintaining a large buffer stock. For example Jensen (1986) argued that, in practice, if firms retain a large amount of their earnings in order to build up a buffer, managers use these funds in ways that benefit their private interests rather than shareholder interests. Since shareholders understand this, they give managers incentives to pay out funds immediately rather than retain them. We crudely model this effect by preventing firms from retaining any earnings. For brand new start-up firms this assumption makes no difference in the first period but does tighten the constraint in subsequent periods compared to the case where firms are able to retain earnings. Allowing retention of earnings weakens our result quantitatively but not qualitatively.

We normalize the price of final output to be $P = 1$ and measure wage rates w^J and w^S in units of real output.

The firm's decision solves the following Bellman equation:

$$V(n^S) = \max_{n^J} \left\{ \begin{array}{l} \int_{z^*(n^J,n^S)}^{1} \left[z \left[n^S \theta^S + n^J \theta^J \right]^{\alpha} - \left(w^S n^S + w^J n^J \right) R\left(n^J, n^S\right) \right] f(z) dz \\ +\beta \left[F\left(z^*\left(n^J,n^S\right)\right) V(0) + \left(1 - F\left(z^*\left(n^J,n^S\right)\right)\right) V\left[\delta\left(n^S + n^J\right)\right]\right] \end{array} \right\}$$

where $z^*\left(n^J, n^S\right)$ is the cutoff level of productivity for bankruptcy and $R\left(n^J, n^S\right)$ is the interest rate schedule on the working capital loan charged by financial intermediaries.

4.2.2 Financial Intermediaries

Competitive financial intermediaries make intra-period loans to the firms. Since productivity shocks are i.i.d. across firms, financial intermediaries are not subject to any aggregate revenue uncertainty and therefore behave as risk neutral lenders. They choose an interest rate schedule $R\left(n^J, n^S\right)$ based on the number of senior workers n^S and junior workers n^J at the firm. Interest rate schedule R and cutoff productivity level z^* for bankruptcy are jointly determined by:

$$z^* \left[n^S \theta^S + n^J \theta^J \right]^{\alpha} = R\left(w^S n^S + w^J n^J\right) \tag{1}$$

$$w^S n^S + w^J n^J = \left\{ \begin{array}{l} \left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right) R\left(w^S n^S + w^J n^J\right) \\ \quad + \int_{0}^{z^*(n^J,n^S)} z\left[n^S\theta^S + n^J\theta^J\right]^{\alpha} f(z) dz \end{array} \right\} \tag{2}$$

where the Equation 1 defines the cutoff level in productivity draws z^* below which firms are unable to repay the loan and Equation 2 is the break even condition for risk neutral lenders. The equation implicitly assumes that the intra-period risk-free rate is zero. We also assume that intra-period loans from financial intermediaries are state-uncontingent. Therefore, firms cannot use these loans to hedge against their productivity risks. This lack of insurance arises when there is some asymmetric information between the firm and the financial intermediaries. For example, if the productivity shocks are unobservable by the financial intermediaries.

4.2.3 Wage Bargaining

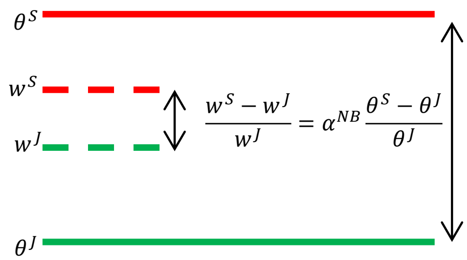
In our model, junior and senior workers have different levels of labor productivity due to differences in firm specific human capital.[10] A joint surplus is created in every period where a senior worker remains with a firm. If this match is destroyed, the surplus will be destroyed, and both the firm and the senior worker will become worse off: the firm has to replace the senior worker with some unskilled junior workers; and the senior worker has to start over at another firm as a junior worker. We assume that this match surplus between a firm and its senior workers is shared via bargaining.

Implicitly assumed here is a lack of commitment between firms and workers. When a junior worker turns senior, firms cannot commit to give this worker a raise in wage rates that is equal to the percentage gain in the worker's productivity. This is because the senior worker cannot credibly threaten to quit the firm as they are unable to bring any of their firm specific skills with them to another firm. As a result, ex-post (when a worker turns senior), the firm will pay this worker a lower wage than their marginal productivity. So ex-ante, a junior worker demands to be paid more than their productivity to make them indifferent between joining the firm or not. This lack of commitment leads to a compressed wage structure: junior workers are overpaid and senior workers underpaid relative to their marginal productivity. This idea is represented visually in Figure 7.

For simplicity, we assume that only a fraction ϕ of the gain in productivity from a junior to a

[10] We abstract from general human capital, assuming that wage rates fully compensate workers for any differences in marginal productivity which arise from differences which are transferable across firms.

Figure 7 – Wage Compression



senior worker is reflected in the gain in wage rates.[11]

$$\frac{w^S}{w^J} - 1 = \phi \left(\frac{\theta^S}{\theta^J} - 1 \right) \tag{3}$$

This pins down the relative wage rates of senior to junior workers. In the extreme case when $\phi = 0$, senior workers have no bargaining power. In this case firms make take it or leave it offers of $w^S = w^J$ and senior workers are paid the same as junior workers since their outside options are the same.

The market for junior workers is competitive and market clearing determines the wage rate of

[11]This particular form of the gain in wage rates can also be motivated via Nash Bargaining on the match surplus of the current period:

When a senior worker is matched with a firm, the worker gets paid w^S in the current period. If the worker quits the firm, they will start over as a junior worker at another firm where they would receive w^J. So the match surplus to the senior worker is $w^S - w^J$.

When a firm retains one of its senior workers, this worker will contribute θ^S units of effective labor in the current period and will be paid w^S. Alternatively, the firm could replace this quantity of effective labor by choosing to hire $\frac{\theta^S}{\theta^J}$ junior workers. The wage payment for these junior workers would be $\frac{\theta^S}{\theta^J} w^J$. We measure the match surplus to the firm as the total saving in the wage bill $\frac{\theta^S}{\theta^J} w^J - w^S$.

Assume a bargaining weight of ϕ for the senior worker, Nash Bargaining on the match surplus in the current period gives:

$$w^S = \arg\max_{w^S} \left(\frac{\theta^S}{\theta^J} \times w^J - w^S \right)^{1-\phi} \left(w^S - w^J \right)^{\phi}$$

which implies that

$$\frac{w^S}{w^J} - 1 = \phi \left(\frac{\theta^S}{\theta^J} - 1 \right)$$

junior workers w^J.[12]

There are two implications from Equation (3). First, the wage of senior workers relative to junior workers can be used to back out the differences in productivity across workers. Second, senior workers are more valuable to a firm relative to junior workers due to wage compression. In equilibrium, since firms compete for workers in the labor market, hiring a junior worker is a costly investment by firms which pays off when the worker becomes senior. In this sense, senior workers are valuable for the firm and should the firm face bankruptcy, the loss of senior workers represent a real cost. For this reason firms are cautious in their hiring in order to balance growth against the risk of losing their senior worker stock.

4.3 Recursive Stationary Equilibrium

Definition 1. A recursive stationary equilibrium consists of a value function $V\left(n^S\right)$, a policy function $n^J(n^S)$, prices $\left\{w^J, w^S, R(n^J, n^S)\right\}$, a distribution of firms sizes $g(n^S)$, and output Y such that:

1. Given $w^J, w^S, R\left(n^J, n^S\right)$: $V\left(n^S\right), n^J(n^S)$ solves the firm's Bellman equation:

$$V(n^S) = \max_{n^J} \left\{ \begin{array}{l} \int_{z^*\left(n^J, n^S\right)}^{1} \left[z\left[n^S\theta^S + n^J\theta^J\right]^\alpha - \left(w^S n^S + w^J n^J\right) R\left(n^J, n^S\right)\right] f(z)dz \\ +\beta\left[F\left(z^*\left(n^J, n^S\right)\right) V(0) + \left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right) V\left[\delta\left(n^S + n^J\right)\right]\right] \end{array} \right\}$$

where

$$z^*\left[n^S\theta^S + n^J\theta^J\right]^\alpha = R\left(n^J, n^S\right)\left(w^S n^S + w^J n^J\right)$$

[12] This isn't exactly right because firms of different sizes have different default probabilities. Therefore, the wage payment should also compensate workers for differences in default risk. In Section 8.1, we correct this. Here we implicitly assume junior workers are randomly matched to firms, or junior workers cannot observe the size of a firm when they decide which firm to join. Note that this simplification works against our result as it reduces the cost of junior workers for smaller firms which have higher default probabilities and therefore also reduces the effects of cautious hiring.

2. Financial intermediaries break even with $R\left(n^J, n^S\right)$:

$$\left(w^S n^S + w^J n^J\right) = \left\{ \begin{array}{l} \left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right) R\left(w^S n^S + w^J n^J\right) \\ + \int_0^{z^*\left(n^J, n^S\right)} z\left[n^S \theta^S + n^J \theta^J\right]^\alpha f(z) dz \end{array} \right\}$$

3. Wage rates satisfy:

$$\frac{w^S}{w^J} - 1 = \phi\left(\frac{\theta^S}{\theta^J} - 1\right)$$

4. The market for goods clears:

$$c = Y \quad \equiv \quad \int_z \int_{n^S} z\left[n^J(n^S)\theta^J + n^S\theta^S\right]^\alpha g(n^S) dn^S dF(z)$$

5. The labor market clears:

$$L = \int_{n^S} \left[n^J(n^S) + n^S\right] g(n^S) dn^S$$

6. The distribution of firm sizes $g\left(n^S\right)$ is consistent with the policy function $n^J\left(n^S\right)$:

$$g(n) = \begin{cases} \int_z \int_{n^S} \mathbf{1}\left\{\delta\left(n^S + n^J\left(n^S\right)\right) = n\right\} \mathbf{1}\left\{z \geq z^*\left(n^J\left(n^S\right), n^S\right)\right\} g\left(n^S\right) f(z) dn^S dz & \text{if } n > 0 \\ \int_z \int_{n^S} \mathbf{1}\left\{z < z^*\left(n^J\left(n^S\right), n^S\right)\right\} g\left(n^S\right) f(z) dn^S dz & \text{if } n = 0 \end{cases}$$

For a firm to reach size n, two things must occur. First the firm must hire junior workers so that the total number of senior workers after exogenous separation (at rate $1-\delta$) in the next period will be n. Second, the firm must not go bankrupt. All firms that go bankrupt start over at size 0 in the subsequent period.

5 Stationary Equilibrium

In this section, we provide some intuition about our mechanism. In particular, how the productivity gap θ^S/θ^J between senior and junior workers affects firm growth and the speed of recovery in employment following a recession.

In our model, senior workers are more productive than junior workers. As a junior worker transitions to a senior worker with a firm, they acquire firm specific skills which increase their productivity at that firm. However, since the improved productivity is due to firm specific skills, the senior worker cannot demand a wage raise that is equal to the increase in their productivity. This is because the outside option for a senior worker is no different than that for a junior worker: in case a senior worker threatens to quit the firm they are currently working for, they must start over as a junior worker at another firm due to the loss of their firm specific skills. As a result, senior workers are paid less than their marginal productivity and are a valuable resource to firms. Since junior workers are hired in a competitive labor market, the wage rate for junior workers has to be above their marginal productivity.[13] In this sense, hiring junior workers can be framed as making an investment in that they incur a negative expected return in the initial period and a positive expected return after they transition into a senior worker. Hiring junior workers is risky for a firm because junior workers lower the expected current period profit of the firm which increases the probability that the firm goes bankrupt.

A firm enters a period with a stock of senior workers which produce a positive expected cash flow for the firm. It uses these senior workers to compensate for the expected losses from hiring junior workers. Those junior workers which are retained transit into senior workers in the following period. However, the risky nature of hiring junior workers makes the firm cautious in hiring them too quickly. As firms gradually accumulate enough senior workers, they will reach a satiation point due to the decreasing returns to scale technology. At this point, firms hire just enough junior workers each period to compensate for the exogenous separation of senior workers.

Next we analyze how changes in θ^S/θ^J affect the rate of growth for firms. Equilibrium hiring decisions are only dependant on the ratio of θ^S/θ^J rather than the level of each.[14] So without loss

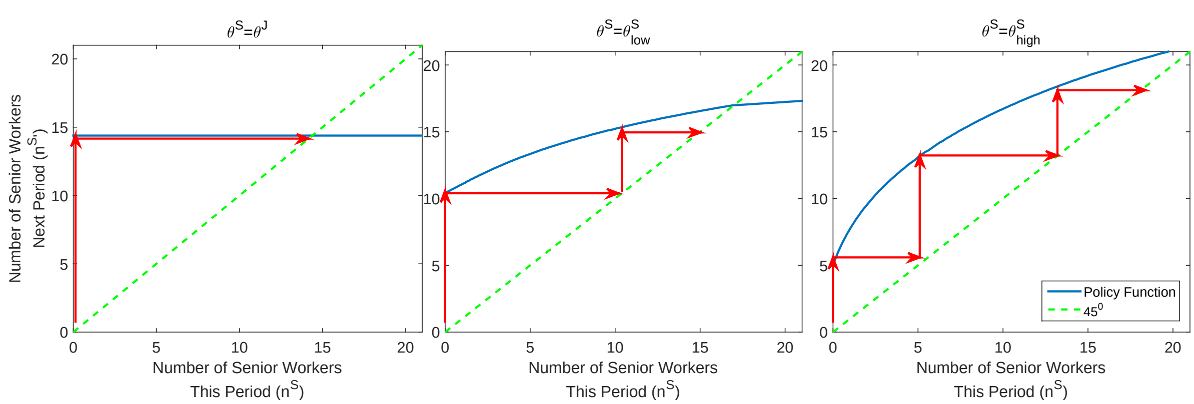
[13]If the wage rates for both senior and junior workers were below their productivities, there would be excess demand for labor and the market would not clear.

[14]For a general intuition of why this is the case, suppose θ^S and θ^J are both scaled by λ. In this case the equilibrium wage rises by λ^α and firms choose to hire the same quantities of both types of *****EFFECTIVE****** labor as before.

of generality we hold θ^J fixed at 1 and only analyze changes in θ^S throughout the remainder of this section. Consider a comparative statics exercise. We simulate three economies, one without firm specific human capital $(\theta^S = \theta^J)$, one with a smaller importance of firms pecific human capital, θ^S_{low}, and one with a larger importance of firm specific human capital, θ^S_{high}. Specifically $1 < \theta^S_{low} < \theta^S_{high}$. Then we plot and compare the policy functions and growth paths of each of the economies.

Figure 8 is a plot of the policy functions of the three economies. The horizontal axis plots the number of senior workers a firm enters the period with. The vertical axis is the number of senior workers the firm will have at the beginning of the subsequent period (specifically, it is equal to the number of senior workers plus the number of junior workers hired after accounting for those workers exogenously separated from the firm, $\delta\left(n^S + n^J\right)$). In the case where senior and junior workers are equally productive (left panel, $\theta^S/\theta^J = 1$), the optimal size of a firm is reached in one period no matter how many senior workers the firm starts with. This is because junior workers are as productive as senior workers and no longer incur an initial investment upon hiring (nor do senior worker offer a positive expected return in this setting). When we introduce a difference in the productivity levels between senior and junior workers (middle panel, θ^S_{low}), the optimal size of a firm is gradually reached as the firm balances growth against risk of bankruptcy. A further increase in the productivity gap between θ^J and θ^S (right panel, θ^S_{high}) causes start-up firms to take even smaller steps in terms of hiring. As a result, it takes more steps (a longer time) for firms to reach the optimal size.

Figure 8 – Firm Growth Paths for Different Productivity Gaps θ^S/θ^J



To understand how exactly an increase in θ^S/θ^J leads to slower growth of firms, we decompose the total effect into a partial equilibrium effect and a general equilibrium effect. Following an increase in θ^S, senior workers become more valuable to a firm. As a result, firms increase their hiring of junior workers who will become senior workers later on. This partial equilibrium effect tends to increase the growth rate of firms.[15] At the same time, an increase in the demand for junior workers drives up the wage rate of these workers. This general equilibrium effect tends to discourage firm hiring and slows down firm growth.

To separate these two effects, we conduct a counterfactual experiment. As we transit from one stationary equilibrium with θ^S_{low} to another with θ^S_{high} where $\theta^S_{low} < \theta^S_{high}$, we decompose the change in the policy functions into partial and general equilibrium effects. In Figure 9, we plot the respective policy functions. The blue curve is the policy function for the stationary equilibrium with θ^S_{low}, and the red curve is the policy function for the stationary equilibrium with θ^S_{high}. The green curve is a counterfactual policy function for an economy with θ^S_{high} but with wage rates for both senior and junior workers fixed at the levels in the equilibrium whith θ^S_{low}.[16]

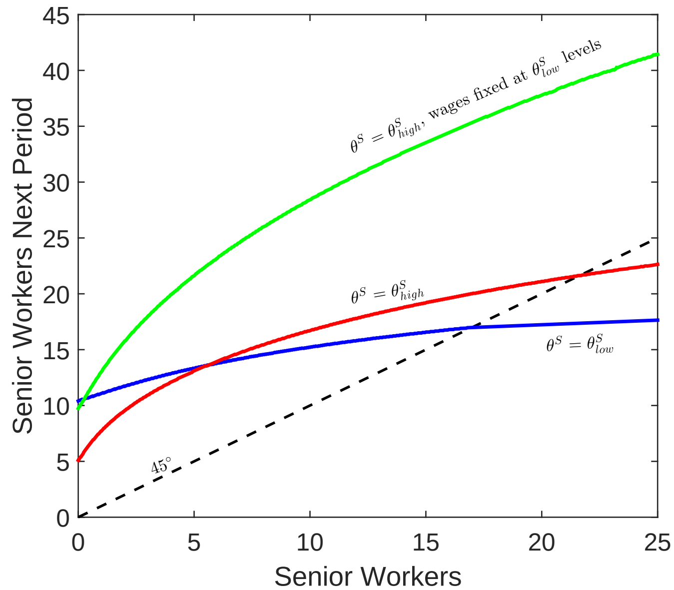
The partial equilibrium effect is represented by the change from the blue curve to the green curve. Following an increase in θ^S (holding wage rates fixed), senior workers become more valuable to firms. So firms increase their hiring of junior workers due to an increase in the continuation value of a worker. This partial equilibrium effect is heterogeneous across firms. Specifically it is weaker (or even negative) for small firms relative to large firms. As θ^S increases, the positive cash flow each senior worker brings into the firm increases. This lowers the probability of bankruptcy. Compared with large firms, small firms have few senior workers in stock, so the bankruptcy probability for small firms does not decline as much. As a result, small firms do not increase their hiring of junior workers as much as large firms.

Figure 10 plots the bankruptcy probabilities as a function of the number of junior workers hired. The blue, green and red solid curves correspond to firms of size 0, 5, and 15 respectively in an

[15] Note that there are multiple distinct forces which together comprise the complete partial equilibrium effect. This is explored further below.

[16] Since wages aren't permitted to adjust in this counterfactual experiment, the labor market does not clear for the green line.

Figure 9 – Partial and General Equilibrium Effects



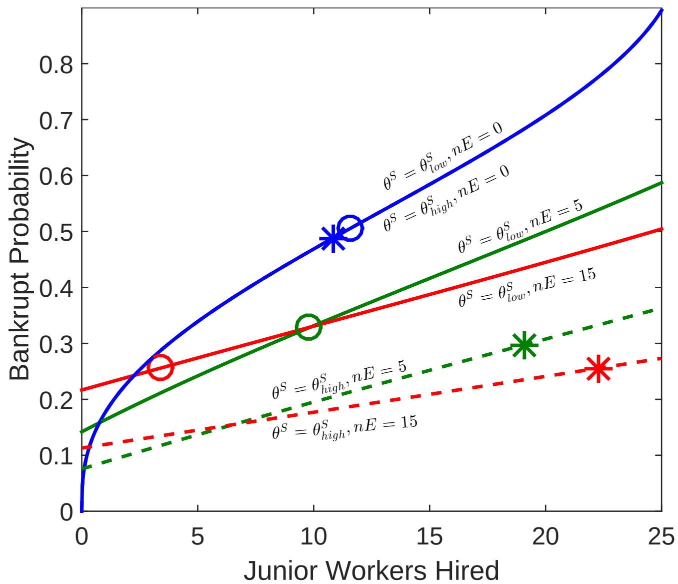
economy with $\theta^S = \theta^S_{low}$. The circles on each curve indicate the optimal hiring choices. Regardless of firm sizes, the more junior workers a firm hires, the higher the probability of bankruptcy. The dashed lines represent default probabilities using $\theta^S = \theta^S_{high}$ while holding wage rates fixed at the equilibrium amount when $\theta^S = \theta^S_{low}$. Note that there are significant reductions in the bankruptcy probability for firms of sizes 5 (solid green to dashed green) and 15 (solid red to dashed red). In other words, an increase in θ^S mitigates the bankruptcy risk and makes junior workers less risky to hire. So firms of sizes 5 and 15 increase the hiring of junior workers significantly (green circle to green asterisk, red circle to red asterisk). On the other hand, for firms of size 0, the bankruptcy probability curve under $\theta^S = \theta^S_{high}$ holding wages constant coincides with the case $\theta^S = \theta^S_{low}$. since these firms have no senior workers there is no effect on their default probabilities. Consequently, these start-up firms do not increase their hiring of junior workers.[17]

[17] In fact, firms of size 0 reduce the hiring of junior workers. The Bellman equation for a firm of size 0 is:

$$V(0) = \max_{n^J} E(z)\left[n^J \theta^J\right]^\alpha - w^J n^J + \beta V\left(\delta n^J\right) - \beta F\left(z^*\left(n^J, 0\right)\right)\left[V\left(\delta n^J\right) - V(0)\right]$$

As θ^S increases, the value function shifts upwards and becomes steeper. This has two opposing forces on the hiring decision. On the one hand, increased continuation value $V\left(\delta n^J\right)$ encourages firms to hire more junior workers who will become senior workers in the following period, given that firms survive. On the other hand, increased continuation value $V\left(\delta n^J\right)$ makes bankruptcy more painful (this is represented by a higher difference $\left[V\left(\delta n^J\right) - V(0)\right]$). Since hiring junior workers increases the probability of bankruptcy $F\left(z^*\left(n^J, 0\right)\right)$, this second force discourages firms from

Figure 10 – Default Probability Comparison



The general equilibrium effect is represented in Figure 9 by the change from the green curve to the red curve. An increase in the wage rate for junior workers is needed to clear the market as we move from the $\theta^S = \theta^S_{low}$ economy to the economy where $\theta^S = \theta^S_{high}$. This wage increase discourages firms from hiring junior workers. As a result, the red curve lies below the green one. This general equilibrium negatively affects the hiring of firms across all sizes.

Aggregating across the partial and general equilibrium effects (from the blue curve to the red curve), smaller firms with size close to 0 reduce their hiring, while larger firms increase their hiring. So as θ^S increases, a start-up firm of size 0 will grow more slowly initially, followed by more rapid growth later on.

In our simulation of a recession, a fraction of firms in the economy go bankrupt following some exogenous shock to the idiosyncratic productivity level. The recovery phase is mainly driven by these bankrupt firms starting over and growing back to the mature size. A higher level of θ^S/θ^J

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hiring junior workers.

For firms with a positive stock of senior workers, the Bellman equation is:

$$V\left(n^S\right) = \max_{n^J} E\left(z\right)\left[n^S\theta^S + n^J\theta^J\right]^{\alpha} - \left(w^S n^S + w^J n^J\right) + \beta V\left[\delta\left(n^S + n^J\right)\right] - \beta F\left(z^*\left(n^J, n^S\right)\right)\left\{V\left[\delta\left(n^S + n^J\right)\right] - V\left(0\right)\right\}$$

The two forces above are still present. However, the second force (bankruptcy concern) is mitigated by an increase in θ^S for large firms ($F\left(z^*\left(n^J, n^S\right)\right)$ shifts down as θ^S increases). Therefore, the first force dominates and it encourages firms to hire more junior workers.

slows down the growth rate of start-up firms and leads to a slower recovery in unemployment rate.

6 Quantitative Analysis

The relative productivity of senior to junior workers is the key parameter in our model. In this section, we provide an estimate of this parameter and analyze how it evolves over time.

In practice, disentangling the marginal contribution of a specific worker is quite difficult as production from most firms involves the coordinated effort of multiple individuals completing various tasks both separately and in groups. Further, hiring decisions by firms are endogenous with output. However, wage rates for workers of various seniorities are observable. In Section 4, we hypothesize that only a fraction α_{NB} of the gains in productivity are reflected in the wage. This gives rise to our calibration strategy for θ^S/θ^J: we estimate the wage return to seniority from the data, and then back out θ^S/θ^J through the wage bargaining equation:

$$\frac{w^S}{w^J} - 1 = \alpha_{NB}\left(\frac{\theta^S}{\theta^J} - 1\right).$$

6.1 Determining the relationship between θ^S and θ^J

Of crucial importance to our model is the contribution of firm specific human capital to the effectiveness of labor. More specifically that portion of firm specific skills which are not reflected in the wage. In practice, apart from the wage, disentangling the marginal contribution of a specific worker is quite difficult as production from most firms involves the coordinated effort of multiple individuals completing various tasks both separately and in groups. Still, there do exist some attempts in the literature to identify the gains in worker productivity from experience without relying on the wage data.

One attempt at quantifying the gains to experience is Shaw and Lazear (2008). They study a firm which installs windshields where output can be quantified and directly linked to individual workers. A main finding is that there is a very steep learning curve over the first 8 months on the job (53%). Further, their data show that these output gains with tenure are not reflected in equal percentage pay gains: pay profiles are much flatter than output profiles in the first year and a half on the job. Installing windshields has a relatively easy learning curve. Still, there are substantial gains to tenure which are not associated with an equivalent gain in wage providing support that workers are a form of investment.

6.1.1 Determining the relation between θ^S and θ^J from CPS

We estimate the return to seniority following the methodology of Topel (1991). We employ data from the Current Population Survey using the Displaced Workers, Occupational Mobility and Job Tenure supplements. In this survey we are able to identify workers who were displaced from jobs as a result of economic reasons (layoffs and plant closings). We use the loss in wages for workers who were displaced to discipline our estimation of the return to seniority.

Using data from the displaced worker supplement of the CPS data is advantageous for a number of reasons. First, experience and tenure move together, making it difficult to identify the contribution of each. Since separation does not alter a worker's experience but does change tenure, we attribute changes in the wage prior to displacement and post displacement to the return to tenure.[18] Second, the CPS data permits us to limit the selection effects of separation. Workers were displaced from their jobs for exogenous reasons instead of endogenous ones such as incapability of workers or bad match quality between workers and jobs.

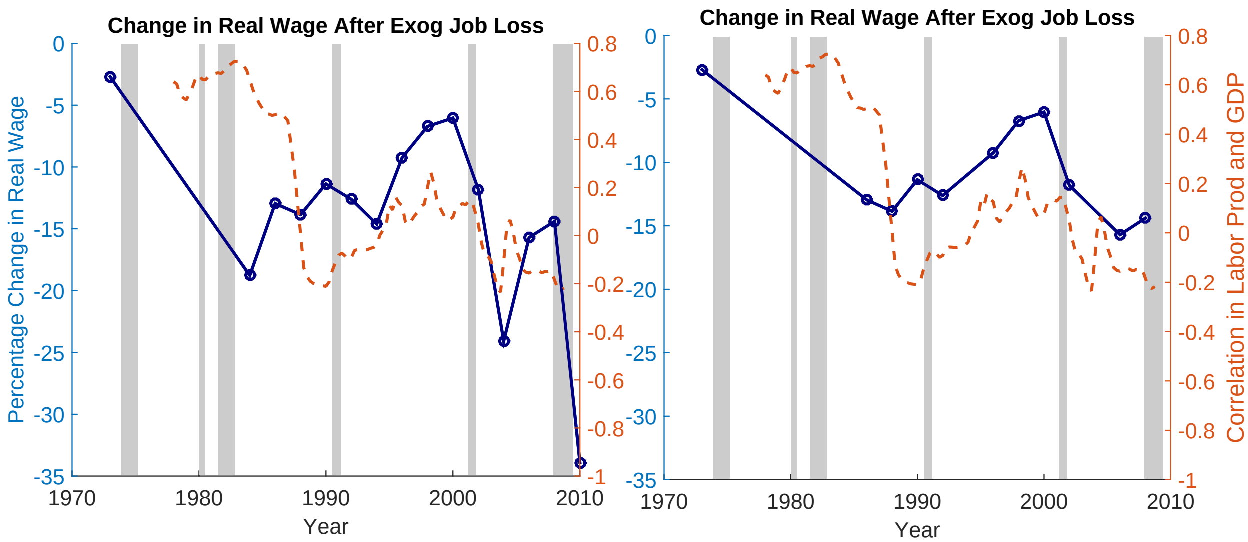
The average change in real wages after an exogenous job loss across all employees is displayed in Figure 11. The drop in wages following an exogenous separation is affected by the business cycle. Specifically, the first survey conducted immediately following a recession gives a drop in wages larger than the average drop experienced in the immediately following or preceding observation.[19] Panel A of Figure 11 includes all data points whereas Panel B of Figure 11 excludes the first point following each NBER recession. The series are overlayed with the data from Figure 3

[18] Depending on the year of the survey, those interviewed are asked if they were displaced in the preceding 1 to 5 years. Consequently, we are unaware of the exact year of displacement. The fact that general experience tends to increase wages causes our estimate of θ^S/θ^J to be biased downwards both for older and more recent recessions.

[19] This has also been documented by Davis and Wachter (2011)

which demonstrates a strong comovement between changes in the real wage and patterns in labor productivity over the business cycle providing suggestive evidence linking our mechanism and result.

Figure 11 – Evidence for Changes in θ^S/θ^J



The solid blue line is the percentage wage loss following a displacement. The dashed orange line is the average labor productivity.

Source: Displaced Worker's Supplement (CPS), Authors' calculations

The observed fall in wages for the single data point in 1973 was 2.75%. The average fall in wages across all subsequent points is 17.25%. If the points immediately following each recession are dropped the fall in wages is 12.95%.

Using cross-sectional U.S. data, Christofides and Oswald (1992), Blanchflower et al. (1996) and Hildreth and Oswald (1997) estimated that a one percentage point increase in a firm's profitability leads to an increase in wages of between 0.01% and 0.08%. Therefore, we choose a bargaining weight of $\alpha_{NB} = 5\%$ for senior workers.

Using the drop in wages for displaced workers from the CPS supplement for periods before and after the mid 1980s and a bargaining weight for senior workers of 5% the implied relative productivity between senior and junior workers is $\theta^S/\theta^J = 1.55$ for the recessions prior to the mid 1980s and $\theta^S/\theta^J = 4.45$ for those recessions after 1983 ($\theta^S/\theta^J = 3.59$ if points immediately following recessions are excluded.)

Unfortunately, use of the displaced worker supplement of CPS to estimate the return to tenure raises several potential problems. First, the loss in wage following a displacement may be due to reasons other than the loss in seniority (i.e. scarring). This leads to an upward bias in our estimation of returns to tenure. Second, since the loss in wage can only be calculated for workers who found a new job after displacement, we are missing the wage loss for workers who fail to find a new job. This selection may lead to a downward bias in our estimation of the percentage loss in wage. Third, the CPS Displaced Workers Survey contains data only for years 1973, 1984, 1986, ..., up to 2010. So 1973 is the only data point that can discipline our choice of θ^S/θ^J for recessions prior to 1983.

6.1.2 Determining the relation between θ^J and θ^S from PSID

As a robsutness check, we replicate the methods of Altonji and Williams (2005) with updated data and estimate the returns to seniority from PSID data.

Altonji and Williams (2005) use an instrumental variable approach to estimate the returns to seniority from 1975 to 1991. Unlike the CPS Displaced Worker Survey, job changes in PSID were not entirely due to exogenous reasons. This introduces some endogeneity problems when estimating the return to seniority. Calculating the increase in wage following a one year increase in tenure is potentially an upward biased measure of the return to seniority due to selection. This is because worker specific factors or match specific factors affect both wage and tenure. For example, a good worker or a good match between worker and job leads to both longer tenure and higher wage payment.

Altonji and Williams (2005) used the demeaned tenure over job matches as the instrument for tenure. Their IV1 estimator reported a 10 year of return to seniority of 5.42% for the period of 1975 to 1982 and 13.91% for the period of 1983 to 1991. Using a bargaining weight of $\alpha_{NB} = 5\%$, these imply $\theta^S/\theta^J = 2.15$ for the 1975 to 1982 period, or the period corresponding to the fast recovery recessions, and $\theta^S/\theta^J = 4.23$ for the 1983 to 1991 period, or the period corresponding to the slow recovery recessions.

We extended the methodology of Altonji and Williams (2005) to more recent waves of PSID.

Additionally, instead of using the 10 year return we calculate the return from tenure using the average tenure of the sample period. Using our updated data and methodology, average return to tenure from 1972 to 1982 is 4.84% and from 1983 to 2013 is 13.02%. Combining this number with a bargaining weight of 5% implies a $\theta^S/\theta^J = 1.97$ for older recessions and $\theta^S/\theta^J = 3.60$ for newer recessions.

For our subsequent simulations, we use $\theta^S/\theta^J = 2$ for recessions before 1985 and $\theta^S/\theta^J = 4$ for recessions after 1985.[20]

6.2 Parametrization

Our model includes nine parameters in the stationary equilibrium and an additional parameter to determine the magnitude of the shock. The parameters in our model are calibrated using exogenous moments in the data or are selected according to levels standard in the literature. We apply a constant relative risk aversion utility function for households of the form:

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}.$$

Each period in our model represents a year and we select a household discount rate of $\beta = 0.96$ to match an annual interest rate of 4%. We choose a household relative risk aversion of $\gamma = 5$ in line with work in the asset pricing literature and with Midrigan et al. (2014).

Labor share in the production function is chosen to be $\alpha = 0.65$.[21] The exogenous separation rate between firms and workers is chosen to be $1-\delta = 0.1$ to match an average employment spell of 2.5 years as in Shimer (2005) . We normalize the measure of firms to $M = 1$ and select a total measure of workers in the economy of $L = 16.5$ to match an average establishment size of 16.5

[20] Our calibration of the ratio $\theta^S/\theta^J = 2$ for pre-1985 recessions is consistent with findings of Frazis and Loewenstein (2006) who use 1982 EOPP data to measure growth in productivity over the first two years of employment. They find that productivity of workers increases by 80% over the first two years, and that productivity growth is only partially reflected in wage growth.

[21] In our model we abstract from physical capital. Over the business cycle capital stock is much less cyclical than employment. Using a decreasing returns to scale production function simulates a larger quantity of capital per employee during recessions.

employees over the past 10 years.[22] The firm specific productivity z is i.i.d. over time following the distribution $f(z) \sim U[0,1]$.[23] Selection of θ^S/θ^J and the bargaining weight α^{NB} were discussed in the previous section.

To model a recession we introduce a one period, unexpected volatility shock. The shock is a mean preserving spread on the productivity draws of all firms which increases the weight of the tails and causes a larger than expected number of firms to fail. For both the fast recovery recessions and slow recovery recessions we select a shock which matches the observed drop in output for the modeled recession. For additional information on the nature of our volatility shock refer to the Appendix.

Table 1 – Parametrization Table

Parameter	Value	Description	Source
β	0.96	Discount Rate	Annual Interest Rate 4%
γ	5	CRRA Parameter	Literature*
α	0.65	Labor Share	60% Labor Share Income
$1-\delta$	0.1	Exogenous Separation Rate	Avg Emp Spell of 2.5 Years
α_{NB}	0.05	NB Weight for Workers	Literature**
M	1	Total Measure of Firms	Normalization
L	16.5	Total Measure of Workers	Average Size of Firm: BLS
θ^J	1	Prod of New Hired Workers	Normalization
θ^S_{old}	2	Prod of Exp Workers	CPS Displaced Worker Survey & PSID
θ^S_{new}	4	Prod of Exp Workers	CPS Displaced Worker Survey & PSID

* Asset Pricing Literature and Midrigan et al. (2014)

** Christofides and Oswald (1992), Blanchflower et al. (1996) and Hildreth and Oswald (1997)

7 Business Cycle Properties

Using the parameters from section 6, we now compare the business cycle properties of the unemployment rate and average labor productivity generated by our model to those in the data. Specifically, we compare our model predictions to the data for the largest recession in our sample period

[22] Data from Bureau of Labor Statistics, Business Employment Dynamics tables on employment by establishment and number of establishments

[23] Note, we assume z follows a uniform distribution for analytic tractability. Our model result is invariant to the mean of the distribution.

before and after the mid 1980s: the 1973 recession and the 2008 recession.

Our model generates recovery in unemployment rates that are of similar speed with those in the data. It also qualitatively matches the procyclical pattern of average labor productivity for the 1973 recession and the countercyclical pattern in average labor productivity for the 2008 recession observed in the data.

7.1 Model Transition Problem

Our model focuses on the recovery period following a recession. For simplicity we model a recession as a one-time, unexpected increase to the cross-sectional variance of productivity shock z.[24] Then we look at how the economy recovers from the recession. We assume that the real wage is fixed at the level in the stationary equilibrium during the transition as observed in the data.[25]

Let the shock happen in period $t = 0$. We assume that it takes T periods for the economy to get back to the stationary equilibrium after the shock. The Bellman equation for individual firms is given by:

$$V_t(n^S) = \max_{n^J} \left\{ \begin{array}{c} \int_{z^*(n^J,n^S)}^{1} \left\{ z \left[n^J \theta^J + n^S \theta^S \right]^{\alpha} - \left[w^S n^S + w^J n^J \right] R\left(n^J, n^S\right) \right\} f(z)\, dz \\ + \Lambda_{t,t+1} \left[F\left(z^*\left(n^J, n^S\right)\right) V_{t+1}(0) + \left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right) V_{t+1}\left(\delta\left(n^S + n^J\right)\right) \right] \end{array} \right\}$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor for the household between periods t and $t+1$:

$$\Lambda_{t,t+1} \equiv \beta \frac{u'(c_{t+1})}{u'(c_t)}$$

Value functions during the transition are indexed by time t. This is because total output/consumption

[24] For our mechanism to work, we only need more firms to go bankrupt during the recession than in the stationary equilibrium. The cause of the recession is unimportant for purposes of our analysis.

[25] Since our model does not include a search element, there would be no unemployment in our model without the use of some form of wage rigidity. In the data wages are only weakly procyclical. This is similar to Shimer (2012) who assumed real wage rigidities in order to explain jobless recoveries after a one-time stock that reduces the capital stock.

is changing over the transition which will affect the stochastic discount factors $\Lambda_{t,t+1}$. The algorithm for solving the transition dynamics is relegated to the Appendix.

7.2 Transition Intuition

7.2.1 Slower Recovery in the Unemployment Rate

Following a recession, a larger fraction of firms go bankrupt relative to the stationary equilibrium. Consequently, there are more start-up firms and less mature firms. Since small firms employ fewer workers than larger firms, and wage rates are assumed to be sticky, there is unemployment in the economy. When the economy is recovering from the variance shock, small firms hire junior workers and gradually grow back to the optimal size. Recovery in the unemployment rate follows.

When θ^S/θ^J is low, the difference between the productivity levels of senior and junior workers is small. Therefore, start-up firms grow fast, leading to a fast recovery in the unemployment rate. When θ^S/θ^J is high, start-up firms grow slowly, which leads to a slower unemployment recovery. Sample policy functions are compared in Figure 8 of Section 5.

7.2.2 Change in the Cyclicality of Average Labor Productivity

The average labor productivity in our model is calculated as output per worker. Since senior and junior workers have different productivity levels, we can decompose the average labor productivity into two parts: output per effective unit of labor supplied, and the average effective labor per worker:

$$\begin{aligned}\text{Average Labor Productivity} &= \frac{Y}{\text{Total Labor}}\\ &= \frac{Y}{\text{Total Labor}} \times \frac{\text{Effective Labor Units}}{\text{Effective Labor Units}}\\ &= \underbrace{\frac{Y}{\text{Effective Labor Units}}}_{\text{Decreasing Returns to Scale Factor}} \times \underbrace{\frac{\text{Effective Labor Units}}{\text{Total Labor}}}_{\text{Worker Composition Factor}} \qquad (4)\end{aligned}$$

The first term in Equation 4 (henceforth, decreasing returns to scale factor), $\frac{Y}{\text{Effective Labor Units}}$, measures how efficiently output can be produced with effective labor units. Due to the decreasing returns to scale technology, this efficiency is higher when total effective labor is smaller. The second term (henceforth, worker composition factor), $\frac{\text{Effective Labor Units}}{\text{Total Labor}}$, measures the average amount of effective labor units provided per worker. It is a weighted average of θ^S and θ^J, where the weights are given by the shares of senior and junior workers in the economy. The more senior workers there are, the larger this term will be.[26]

During the transition, the decreasing returns to scale effect factor and the worker composition factor have opposing forces on average labor productivity. On one hand, after the recession, start-up firms desire to grow which results in a larger influx of junior workers. These junior workers provide a smaller amount of effective labor per worker than senior workers which drives down average labor productivity through the worker composition factor. On the other hand, during the recovery, the average size of firms in the economy is smaller than in the stationary equilibrium. This effect comes through the stochastic discount factor. During a recession, output is small relative to future periods. This increases the importance of present consumption relative to future growth and results in a smaller average size of firms. Due to the decreasing returns to scale factor, this tends to increase average labor productivity.[27]

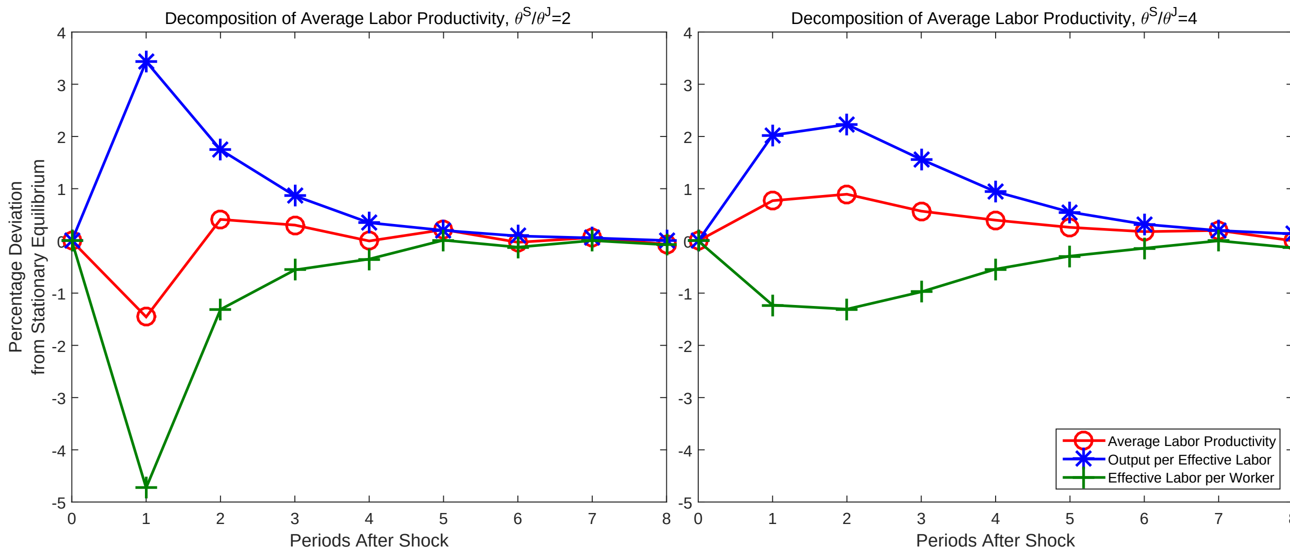
When θ^S/θ^J is low, start-up firms grow fast by hiring a lot of junior workers. The lower productivity of junior workers outweighs the decreasing returns to scale force, causing average labor productivity to be procyclical. When θ^S/θ^J is high, recovery is slower. Start-up firms are more cautious in their hiring of junior workers. The downward pressure of unproductive junior workers is diluted across time which causes the decreasing returns to scale force to dominate. In this case, average labor productivity becomes countercyclical.

Figure 12 plots this decomposition of average labor productivity. The left panel corresponds to the fast recovery economy ($\theta^S/\theta^J = 2$) and the right panel corresponds to the slow recovery economy ($\theta^S/\theta^J = 4$). In each panel, the red line plots average labor productivity during the transition which is decomposed into output per effective labor unit (decreasing returns to scale

[26]To be more precise, ALP $= \frac{\int_{n^S}\left[\theta^S n^S+\theta^J n^J\right]^{\alpha} g\left(n^S\right) dn^S}{\int_{n^S}\left[\theta^S n^S+\theta^J n^J\right] g\left(n^S\right) dn^S} \times \frac{\int_{n^S}\left[\theta^S n^S+\theta^J n^J\right] g\left(n^S\right) dn^S}{\int_{n^S}\left[n^S+n^J\right] g\left(n^S\right) dn^S}$

[27]Our model abstracts from capital. Empirically, over the business cycle, the fluctuations in employment are not fully matched by changes in capital and consequently there is more capital available per worker during recessions. This serves as our motivation for modeling production using a decreasing returns to scale technology in labor.

Figure 12 – Decomposition of Average Labor Productivity



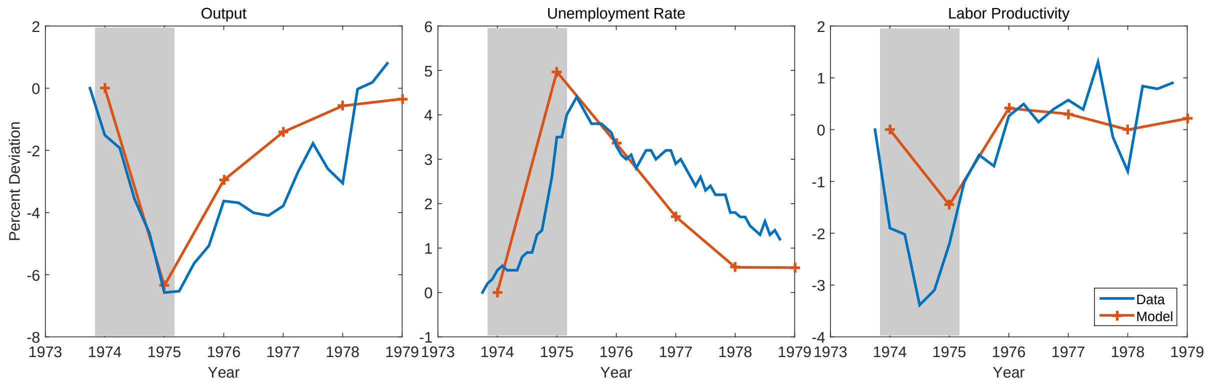
effect, blue line) and effective labor per worker (worker composition effect, green line). Comparing the worker composition effect of the slow recovery economy (right panel) with the fast recovery one (left panel) we observe that the effective labor per worker (green line) falls by significantly less. As outlined above, this is because firms are hiring junior workers more slowly and cautiously in the slow recovery economy. This slower hiring also results in a slower recovery of GDP growth. A more steady consumption path translates to less disruption of the stochastic discount factor and firm size. All of this translates to a smaller decreasing returns to scale effect. In aggregate we observe that worker compensation effect outweighs the decreasing returns to scale effect in the fast recovery economy and vice versa in the slow recovery economy.

7.3 Comparing Model Predictions to the Data

Using the parameters a and θ^S calibrated in the last section, we compute the evolution of output, the unemployment rate, and average labor productivity during the transition. For the 1973 recession we use $\theta^S = 2$, and for the 2008 recession we use $\theta^S = 4$. a is selected to match the percentage drop in output in the data.[28]

[28] For older recessions ($\theta^S = 2$) we use $a = 12$, and for more recent recessions ($\theta^S = 4$) we use $a = 4.5$.

Figure 13 – 1973 Recession: Output, Unemp. Rate, and Avg. Labor Prod.



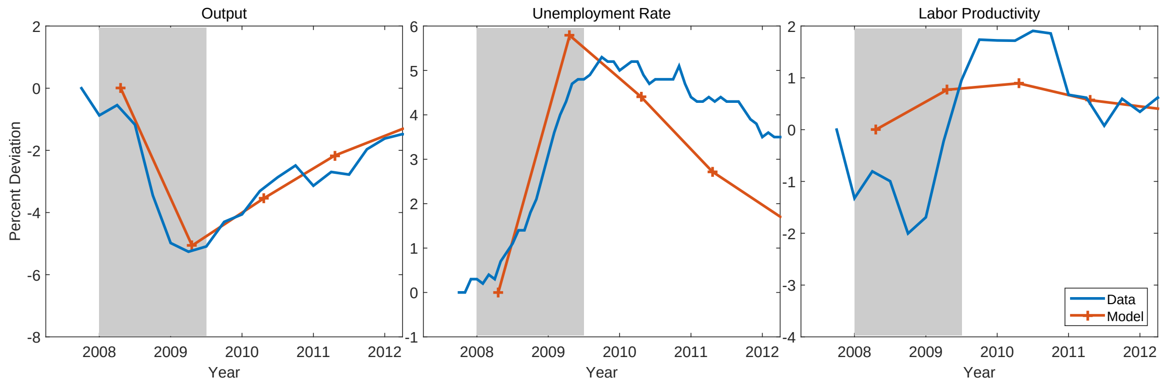
Source: Bureau of Labor Statistics, Authors' calculations

In Figure 13 we compare the time series of output, the unemployment rate, and average labor productivity between our model and the data throughout the recession of 1973. Note that our model only uses a single parameter, a, to match the data to this particular recession.

Our model does a reasonable job at approximating the rate of recovery in output given that the only moment we target is the initial drop in output. As for the pattern in unemployment, we can see from the middle panel of Figure 13 that the unemployment rate recovers by 1.6% (from 9% to 7.4%) within the first year of recovery in the data. In our simulation, the unemployment rate recovers by 1.7%.

From the right panel of Figure 13, we see that following the 1973 recession, average labor productivity dropped with output in the data. Our model delivers this procyclicality of labor productivity. The intuition for this is as follows. In the 1973 recession, the difference between the productivity levels of senior and junior workers was relatively small. Therefore, start-up firms grew quickly. This fast growth in employment results in firms spending little time on the more productive portion of the decreasing returns to scale technology. However, our model suggests that average labor productivity in the economy was reduced following the recessions by the large influx of junior workers with lower productivity.

Figure 14 – 2008 Recession: Output, Unemp. Rate, and Avg. Labor Prod.



Source: Bureau of Labor Statistics, Authors' calculations

Figure 14 compares our model to the data for the 2008 recession. The only difference in the parametrization between the 1973 recession and the 2008 recession is a change in θ^S and the severity of the shock to idiosyncratic productivity draws a. Again the model does a fairly good job at predicting the rate of recovery in output. In the unemployment path following the 2008 recession, the unemployment rate went down by 0.5% within the first year of recovery in the data. Our model predicts a 1.5% drop (middle panel of Figure 14). Our model predicts a slower rate of recovery in unemployment as a result of the change in θ^S/θ^J and though it only explains a portion of the reduction in the rate of unemployment recovery, we are not using any parameters to target either the rise in unemployment or the rate of recovery. Of course there are other effects such as financial frictions, increased specificity of labor, and changes to unemployment insurance which are discussed in the literature and are also contributing to the reduction in the rate of unemployment recovery. These effects are outside the scope of this paper. Note that our model is relatively simple and it is mathematically impossible for it to perfectly match all three graphs as any single graph can be calculated directly as a function of the other two graphs.[29] The data is not constrained in the way our model is due to entrance and exit from the labor force and changes on the intensive margin for workers.

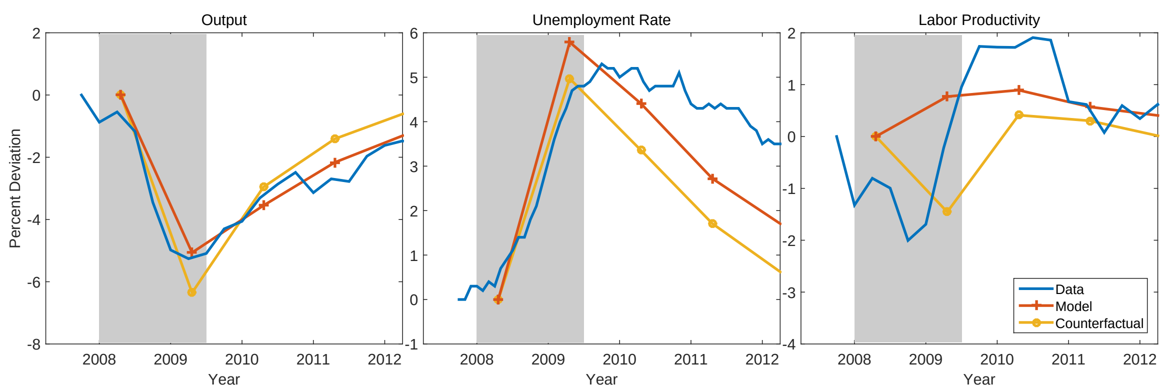
In the 2008 recession, average labor productivity went up while output dropped in the data.

[29] For example, labor productivity is calculated as $\frac{Y}{N}$. Our model does not consider labor force participation nor changes to employment on the intensive margin. Therefore, labor productivity is a direct function of output and unemployment.

Our model also delivers this counter cyclical pattern in labor productivity (right panel of Figure 14). Intuitively, in 2008, the difference between the productivity of senior and junior workers is relatively large. Therefore, start-up firms grow slowly due to fear of losing their senior workers upon bankruptcy. This slow growth in employment causes the reduction in the average effective labor per worker to fall by less than in the 1973 recession and allows firms to take advantage of the highly productive portion of the decreasing returns to scale technology. As a result, average labor productivity rises over the recovery path.

Figure 15 is a counterfactual exercise of what would have happened to the 2008 recession if θ^S/θ^J had remained at 2. The blue line is the data, the green line is the benchmark model result using $\theta^S/\theta^J = 4$, and the red line is the counterfactual model result if $\theta^S/\theta^J = 2$. If θ^S/θ^J remained at 2, our model would predict a faster recovery in both output and the unemployment rate (measuring recovery as a percent of the original increase.) Additionally, average labor productivity would be procyclical just like the 1973 recession.

Figure 15 – 2008 Recession: Output, Unemp. Rate, and Avg. Labor Prod.



Source: Bureau of Labor Statistics, Authors' calculations

8 Extensions

8.1 Risk Compensation in the Wage Rates

Throughout our previous analysis, we have been assuming firms are wage takers when they choose how many junior workers to hire. Junior workers are randomly assigned to firms who want to hire them.

If junior workers could observe the size of firms before deciding which one to join, they would demand a higher wage rate for firms which have a higher probability of going bankrupt. This is because all firm specific skills of the worker are lost upon bankruptcy. In equilibrium, smaller firms endogenously choose a higher bankruptcy risk as a result of their desire to grow. In this case, small firms would have to pay a higher wage for junior workers than mature firms in order to compensate junior workers for the risk of losing the skills they are about to accumulate.

In this extension we incorporate risk compensation into our model. It does not affect our result qualitatively for two reasons. Firstly, small firms are more prone to bankruptcy and have to pay a higher wage to attract junior workers. This further slows the growth of small firms and qualitatively strengthens our cautious hiring mechanism. Second, under our parametrization, the bargaining weight for senior workers is $\alpha_{NB} = 5\%$. This implies that the wage difference between senior and junior workers is small relative to the difference in their productivity levels. In other words, there is not much risk for firms to compensate.

8.1.1 Wage Determination

Our approach to determining the appropriate risk compensation in wages is to compare each firm to a hypothetical risk free firm. The risk free firm provides us with two benchmark prices:

1. It provides a benchmark risk-free wage for junior workers w^J_{rf}. This price is guessed initially and market clearing informs whether the guess is correct.

2. Using wage bargaining, it pins down the risk-free wage for senior workers:

$$\frac{w^S_{rf} - w^J_{rf}}{w^J_{rf}} = \alpha_{NB} \frac{\theta^S - \theta^J}{\theta^J}.$$

Other firms take the wage rate of the risk-free firm as given and set offer wage rates which include an appropriate risk compensation mark-up over the risk-free wage. In particular, a firm with probability p of going bankrupt pays junior and senior workers wage rates:

$$w^J(p) = w_{rf}^J + \beta\delta p\left(w_{rf}^S - w_{rf}^J\right)$$
$$w^S(p) = w_{rf}^S + \beta\delta p\left(w_{rf}^S - w_{rf}^J\right)$$

These wage rates compensate the workers for their expected wage loss in the event of bankruptcy $\beta\delta p\left(w_{rf}^S - w_{rf}^J\right)$.[30]

8.1.2 Firm's Problem

Firms solve a similar problem as before, expect that now, wage rates are endogenously dependent on their hiring decision. The Firm's Bellman equation is given by:

$$V\left(n^S\right) = \max_{n^J} \int_{z^*}^{\bar{z}} \left\{z\left[\theta^J n^J + \theta^S n^S\right]^\alpha - \left(w^S\left(F\left(z^*\right)\right)n^S + w^J\left(F\left(z^*\right)\right)n^J\right)R\left(n^J, n^S\right) + \beta V\left[\delta\left(n^S + n^J\right)\right]\right\} f(z)dz + \int_{\underline{z}}^{z^*} \left\{0 + \beta V(0)\right\} f(z)\,dz$$
$$\text{s.t.}\, w^J\left(F\left(z^*\right)\right) = w_{rf}^J + \beta\delta F\left(z^*\right)\left(w_{rf}^S - w_{rf}^J\right)$$
$$w^S\left(F\left(z^*\right)\right) = w_{rf}^S + \beta\delta F\left(z^*\right)\left(w_{rf}^S - w_{rf}^J\right)$$
$$w^S\left(F\left(z^*\right)\right)n^S + w^J\left(F\left(z^*\right)\right)n^J = \left(w^S\left(F\left(z^*\right)\right)n^S + w^J\left(F\left(z^*\right)\right)n^J\right)R\left(n^J, n^S\right)\left(1 - F\left(z^*\right)\right) + \int_{\underline{z}}^{z^*} z\left[\theta^J n^J + \theta^S n^S\right]^\alpha f(z)\,dz$$
$$z^*\left[\theta^J n^J + \theta^S n^S\right]^\alpha = \left(w^S\left(F\left(z^*\right)\right)n^S + w^J\left(F\left(z^*\right)\right)n^J\right)R\left(n^J, n^S\right)$$

8.1.3 Competitive Equilibrium

Definition 2. A recursive stationary equilibrium consists of a value function $V\left(n^S\right)$, a policy function $n^J(n^S)$, prices $\left\{w_{rf}^J, w_{rf}^S, R(n^J, n^S)\right\}$, a distribution of firms sizes $g(n^S)$, and output Y such that:

[30] Due to lack of commitment by firms, wage rates have been front-loaded so that junior workers are overpaid and senior workers underpaid. However, we assume that the risk compensation $\beta\delta p\left(w_{rf}^S - w_{rf}^J\right)$ isn't front-loaded. This is somewhat inconsistent in that, when a worker turns senior, the firm could always refuse to pay the risk compensation since the senior worker cannot credibly commit to quit. Front-loading risk compensation significantly increases the complexity by making the wage determination a fixed point problem. However, failing to model risk compensation entirely through front-loading works against cautious hiring as a larger front-loading of wages would only strengthen our cautious hiring mechanism.

1. Given $w^J_{rf}, w^S_{rf}, R\left(n^J, n^S\right)$: $V\left(n^S\right), n^J\left(n^S\right)$ solves firm's Bellman equation:

$$V\left(n^S\right) = \max_{n^J} \int_{z^*}^{\bar{z}} \left\{ z\left[\theta^J n^J + \theta^S n^S\right]^\alpha - \left(w^S\left(F\left(z^*\right)\right)n^S + w^J\left(F\left(z^*\right)\right)n^J\right)R\left(n^J, n^S\right) + \beta V\left[\delta\left(n^S + n^J\right)\right]\right\} f(z)dz + \int_{\underline{z}}^{z^*} \left\{0 + \beta V(0)\right\} f(z)\,dz$$

$$\text{s.t.} w^J\left(F\left(z^*\right)\right) = w^J_{rf} + \beta\delta F\left(z^*\right)\left(w^S_{rf} - w^J_{rf}\right)$$

$$w^S\left(F\left(z^*\right)\right) = w^S_{rf} + \beta\delta F\left(z^*\right)\left(w^S_{rf} - w^J_{rf}\right)$$

$$w^S\left(F\left(z^*\right)\right)n^S + w^J\left(F\left(z^*\right)\right)n^J = \left(w^S\left(F\left(z^*\right)\right)n^S + w^J\left(F\left(z^*\right)\right)n^J\right)R\left(n^J, n^S\right)\left(1 - F\left(z^*\right)\right) + \int_{\underline{z}}^{z^*} z\left[\theta^J n^J + \theta^S n^S\right]^\alpha f(z)\,dz$$

$$z^*\left[\theta^J n^J + \theta^S n^S\right]^\alpha = \left(w^S\left(F\left(z^*\right)\right)n^S + w^J\left(F\left(z^*\right)\right)n^J\right)R\left(n^J, n^S\right)$$

2. Wage rates satisfy:

$$\frac{w^S_{rf}}{w^J_{rf}} - 1 = \alpha_{NB}\left(\frac{\theta^S}{\theta^J} - 1\right)$$

3. Labor market clears:

$$L = \int_{n^S}\left[n^J(n^S) + n^S\right] g(n^S)dn^S$$

4. Final good market clears:

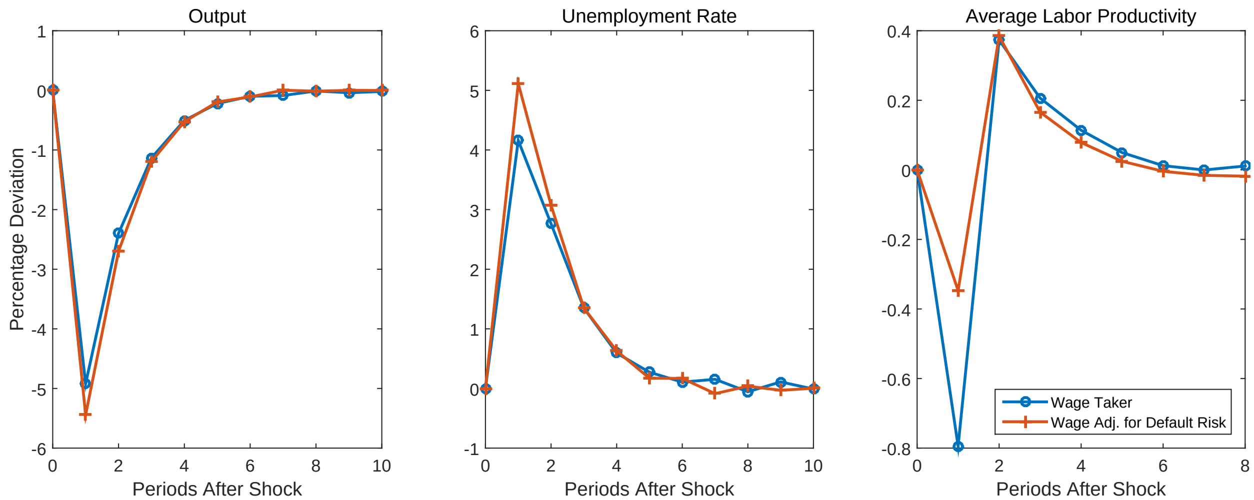
$$Y = \int_z \int_{n^S} z\left[n^J(n^S)\theta^J + n^S\theta^S\right]^\alpha g(n^S)dn^S dF(z)$$

5. The distribution of firm sizes $g\left(n^S\right)$ is consistent with the policy function $n^J\left(n^S\right)$:

$$g(n) = \begin{cases} \int_z \int_{n^S} \mathbf{1}\left\{\delta\left(n^S + n^J\left(n^S\right)\right) = n\right\} \mathbf{1}\left\{z \geq z^*\left(n^J\left(n^S\right), n^S\right)\right\} g\left(n^S\right) f(z)\,dn^S dz & \text{if } n > 0 \\ \int_z \int_{n^S} \mathbf{1}\left\{z < z^*\left(n^J\left(n^S\right), n^S\right)\right\} g\left(n^S\right) f(z)\,dn^S dz & \text{if } n = 0 \end{cases}$$

For a firm to reach size n, two things must occur. First the firm must hire junior workers so that the total number of senior workers after exogenous separation (at rate $1-\delta$) in the next period will be n. Second, the firm must not go bankrupt. All firms that go bankrupt start over at size 0 in the subsequent period.

Figure 16 – Transition Path Comparison $\theta^S/\theta^J = 2$

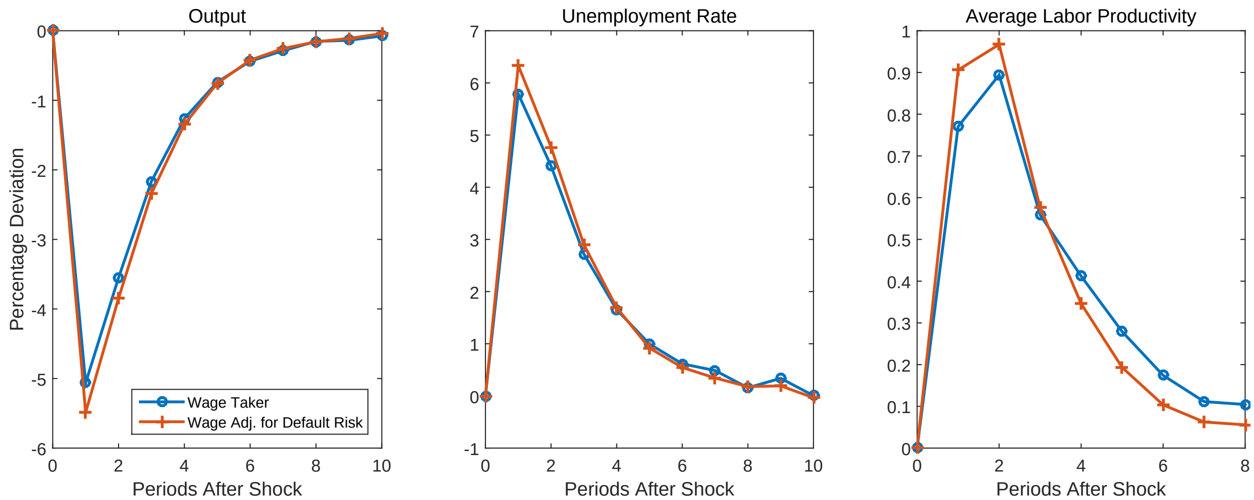


8.1.4 Transition Dynamics Results

Figure 16 compares the transition paths of output, unemployment rates, and average labor productivity between the benchmark model where firms are wage takers (blue curves) and the case where firms compensate workers for the bankruptcy risk (green curves). For this Figure, we use $\theta^S/\theta^J = 2$ which corresponds to the older recessions. The transition dynamics of the two cases look very similar. This is partially due to the bargaining weight we use which is $\alpha_{NB} = 5\%$. So the wage difference between junior and senior workers is small and there is no much wage loss if a senior worker separates from a firm upon bankruptcy. As a result, the risk compensation is small and does not signicantly affect outcomes.

Figure 17 is a similar comparison for newer recessions, where $\theta^S/\theta^J = 4$. Again our models generates a similar result whether risk compensation is included in the wage rates or not. One difference is, when risk compensation is included, the level of average labor productivity is slightly higher. This is because small firms are more susceptible to bankruptcy risk hence have to pay more to compensate their workers. This makes small firms grow even more slowly. The influx of unproductive junior workers are diluted over time and firms spend more time on the more productive portion of the decreasing returns to scale technology. Therefore, the average labor productivity is higher. This force works in the same direction for all ratios of θ^S/θ^J.

Figure 17 – Transition Path Comparison θ^S/θ^J



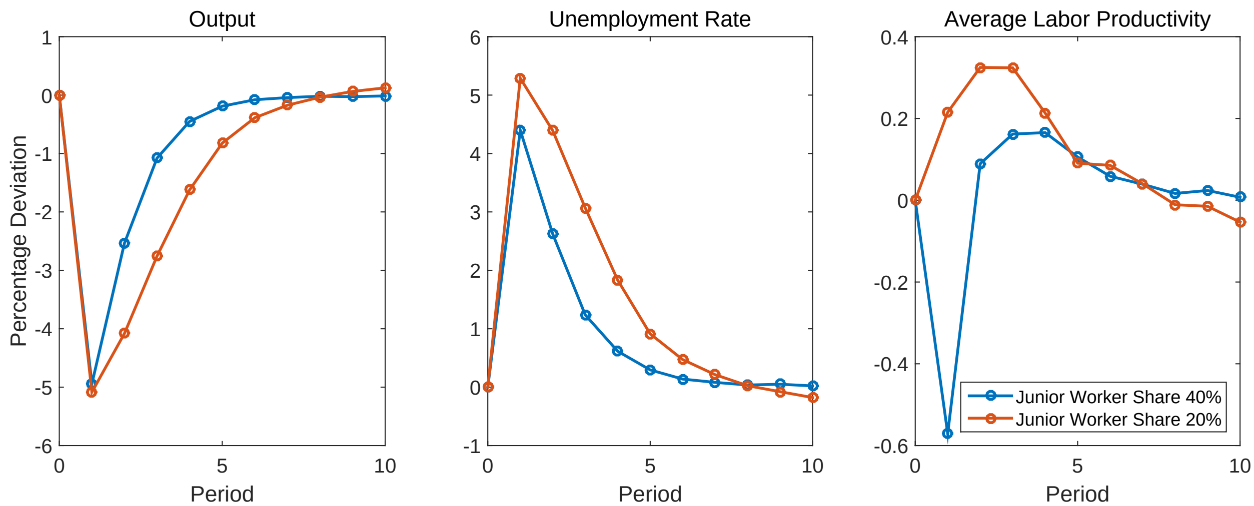
8.2 General CES Production Function

Throughout our previous analysis, we have been assuming the effective labor inputs of junior and senior workers are perfect substitutes. In this subsection we consider a more general constant elasticity of substitution (CES) production function:

$$y = \left\{ \Upsilon \left(\theta^J n^J\right)^{\varepsilon} + (1-\Upsilon)\left(\theta^S n^S\right)^{\varepsilon} \right\}^{\alpha/\varepsilon}$$

With complementarity between the effective labor input of junior and senior workers, small firms grow even slower. This is because the labor input of senior workers complement that of junior workers. Small firms have few senior workers so their junior workers are less productive. A similar result to the benchmark model can be achieved through changes either to the relative productivity of senior to junior workers (as before) or to the share parameter, Υ. In this setup, senior workers and junior workers are to some extent complentary. A change in their relative importance continues to affect the cyclicality of labor productivity over the business cycle in the same manner as our benchmark model. For example, suppose some of the work previously completed by junior workers is replaced by computer programs. In this case, if the share parameter, Υ, for junior workers drops from 40% to 20%, then recovery is slower and the cyclicality of average labor productivity

Figure 18 – CES Production Function



is flipped. A graphical example is provided in Figure 18.[31] This provides an additional potential explanation for emergent patterns in labor productivity and unemployment recovery, namely that an increased relative importance of experienced workers through the share parameter can achieve a similar empirical result.[32]

9 Conclusion

In this paper we document two changes in the pattern of business cycles starting from the 1990 recession. First, the speed of the recovery in unemployment rates has slowed significantly. Second, labor productivity has switched from being procyclical to acyclical. We present a model which can contribute towards the explanation of both facts through the variation of a single parameter, namely the relative productivity of senior to junior workers.

In our model, when a recession occurs, many firms go bankrupt and need to start up again. This causes two opposing forces on average labor productivity. First, start-up firms employ a disproportionately large number of junior workers resulting in a decrease of average labor productivity. The

[31] The graph was generated using $\varepsilon = 0.7$ (elasticity of substitution of 3.33), $\Upsilon = 0.4$ for older recession and $\Upsilon = 0.2$ for newer recession. $\theta^S = 2$ for both cases.

[32] It is important to note that this production function has an unrealistic implication. Namely, if a firm has a large ratio of senior employees, it is possible that the marginal productivity of junior employees is larger than their senior counterparts.

second and countering force is that, during the recovery, the average size of firms in the economy is smaller than in the stationary equilibrium. This effect comes through the stochastic discount factor. During a recession, output is small relative to future periods. This increases the importance of present consumption relative to future growth and results in a smaller average size of firms. Due to the decreasing returns to scale technology, this tends to increase average labor productivity. These two forces result in a non-linear relationship between the cyclicality of labor productivity and the relative productivity of senior to junior workers.

In older recessions the difference between the productivity of senior and junior workers is low so that new firms tend to grow quickly. This causes an influx of junior workers during periods immediately following a recession. In total, the downward forces on average labor productivity dominate the upward force and results in the procyclicality of labor productivity observed in older recessions.

As the productivity of senior relative to junior workers increases, firms become more cautious (slower) in hiring in order to avoid losing their stock of senior workers upon bankruptcy. This is because the stock of senior workers becomes more valuable to a firm (and the resulting investment required to develop a junior worker into a senior worker increases). This slow hiring reduces the strength of the downward effect on labor productivity and results in firms spending more time on the relatively productive portion of their decreasing returns to scale technology. In this case, the upward force on average labor productivity dominates the downward force and results in the countercyclicality of labor productivity observed in more recent recessions.

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A Comparing recovery in Unemployment Across recessions

In this section an alternative method of comparing the speed of recovery in the unemployment rate is provided. Table 2 displays the difference in unemployment rates between the end of each of the last six recessions and each of the following five years. In recessions prior to the mid 1980s, recovery in unemployment following the low point in GDP growth began immediately and continued in the following five years. Contrast that with the most recent three recessions where the unemployment rate has remained fairly level or increased in the first two to three years following the recession and even five years after the recession the unemployment rate recovery lags by approximately two percentage points behind the earlier recessions.[33]

Table 2 – Unemployment Recovery

NBER Recession End Date	UR	Dif. in UR Between Recession End and Select Dates: 1 Year	2 Years	3 Years	4 Years	5 Years
Fast Recovery Recessions						
11/1970	5.9	-0.1	0.6	1.1	NA	NA
03/1975	8.6	1	1.2	2.3	2.8	2.3
11/1982	10.8	2.3	3.6	3.8	3.9	5
Average		**1.07**	**1.80**	**2.40**	**3.35**	**3.65**
Slow Recovery Recessions						
03/1991	6.8	-0.9	-0.9	-0.2	0.8	0.8
11/2001	5.5	-0.4	-0.7	-0.2	0.3	0.9
06/2009	9.5	0.1	0.4	1.3	2	3.4
Average		**-0.40**	**-0.40**	**0.30**	**1.03**	**1.70**

Source: Current Population Survey and NBER recession dates. Author's calculations

B Selecting θ^S/θ^J

As outlined in Section 6, we use data from the supplemental questions for the CPS to infer the relative productivity of a senior to a junior worker. Unfortunately these supplements are not administered every year and the questions have varied somewhat over time. The first time these supplemental questions were asked was in 1973, questions inquiring about displacement as well as prior and current wages were not asked again until 1984 and have continued to be included bi-annually through present.

Following the method used in Topel (1991) we restrict attention to male respondents between the ages of 20 and 60 whose jobs end exogenously. We then deflate nominal wages by the GNP price deflator for consumption expenditure. For these workers we calculate the average change in log weekly wages for the prior and current jobs and use equation 3 to calculate the ratio of θ^S/θ^J implied by the data through the lens of our model.

[33] The 1980 recession is omitted due to the proximity of the following recession. For the same reason certain dates following the 1971 recession are not included.

The drop in wages following an exogenous separation is affected by the business cycle. Specifically, the first survey conducted immediately following a recession gives a drop in wages larger than the average drop experienced immediately following or preceeding the survey. We therefore calculate the implied θ^S/θ^J both including and excluding these points. For the supplemental survey administered prior to the double dip recession we find an average change in log weekly wages of -2.7% which implies a value of θ^S/θ^J of 1.55.[34] For those surveys administered after the double dip recession we find an average change in log weekly wages of -14.7% which implies a value of θ^S/θ^J of 4.45. When we exclude the post recession data points we observe an average change in log weekly wages of -11.5% which implies a value of θ^S/θ^J of 3.59. In our benchmark model we select an θ^S/θ^J of 2 for the "fast recovery" recessions and 4 for the "slow recovery" recessions.

C Simplifying the Firm's Bellman Equation with a Uniformly Distributed Shock

The firm's Bellman equation is:

$$V(n^S) = \max_{n^J} \left\{ \begin{array}{c} \int_{z^*\left(n^J,n^S\right)}^{1} \left[z\left[n^S\theta^S + n^J\theta^J\right]^{\alpha} - \left(w^S n^S + w^J n^J\right) R\left(n^J,n^S\right)\right] f(z)dz \\ +\beta\left[F\left(z^*\left(n^J,n^S\right)\right)V(0) + \left(1-F\left(z^*\left(n^J,n^S\right)\right)\right)V\left[\delta\left(n^S+n^J\right)\right]\right] \end{array} \right\}$$

where z^* and interest rate schedule R solves:

$$z^*\left[n^S\theta^S + n^J\theta^J\right]^{\alpha} = R\left(w^S n^S + w^J n^J\right)$$

$$w^S n^S + w^J n^J = \left\{ \begin{array}{c} \left(1-F\left(z^*\left(n^J,n^S\right)\right)\right) R\left(w^S n^S + w^J n^J\right) \\ + \int_{0}^{z^*\left(n^J,n^S\right)} z\left[n^S\theta^S + n^J\theta^J\right]^{\alpha} f(z)dz \end{array} \right\}$$

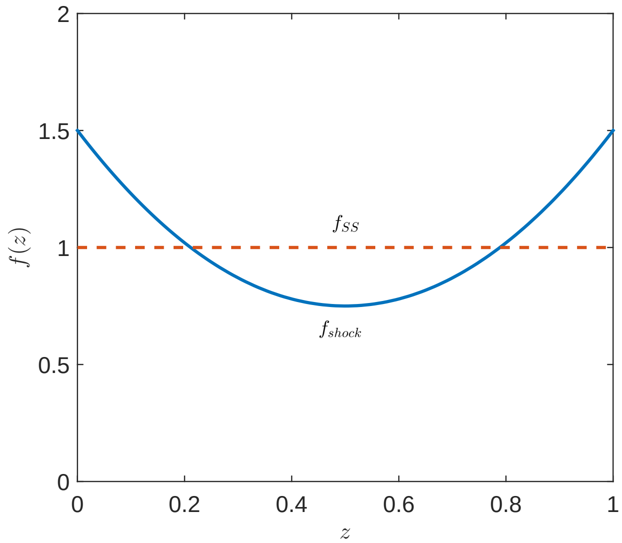
Using uniformly distributed $z \sim U[0,1]$, we can simplify firm's Bellman equation into:

$$V\left(n^S\right) = \max_{n^J} \frac{1}{2}\left[n^S\theta^S + n^J\theta^J\right]^{\alpha} - \left(w^S n^S + w^J n^J\right) + \beta\left\{(1-z^*)V\left[\delta\left(n^S+n^J\right)\right] + z^* V(0)\right\}$$

where $z^* \equiv 1 - \sqrt{1 - \frac{2\left(w^S n^S + w^J n^J\right)}{\left[n^S\theta^S + n^J\theta^J\right]^{\alpha}}}$.

[34] Supplemental survey questions which include data regarding wages of the prior and current job were only included one time before the double dip recession (in 1973). Therefore the value of θ^S/θ^J for the "fast recoveries" is based on the data from this single survey.

Figure 19 – Aggregate Uncertainty Shock $(a = 3)$



D Simplifying the Firm's Bellman Equation with Uniformly Distributed Shock (Non-random Match Case)

Using uniform distribution $z \sim U[0,1]$, the firm's Bellman equation becomes:

$$V\left(n^S\right) = \max_{n^J} \frac{1}{2}\left[n^S\theta^S + n^J\theta^J\right]^\alpha - \left(w^S_{rf}n^S + w^J_{rf}n^J\right) - \left(n^S + n^J\right)\beta\delta z^*\left(w^S_{rf} - w^J_{rf}\right) + \beta\left(1 - z^*\right)V\left[\delta\left(n^S + n^J\right)\right] + z^*\beta V(0)$$

where

$$z^* = \frac{2\left(w^S_{rf}n^S + w^J_{rf}n^J\right)}{\left[y\left(n^J,n^S\right) - \beta\delta\left(w^S_{rf} - w^J_{rf}\right)\left(n^S + n^J\right)\right] + \sqrt{\left[y\left(n^J,n^S\right) - \beta\delta\left(w^S_{rf} - w^J_{rf}\right)\left(n^S + n^J\right)\right]^2 - 2\left(w^S_{rf}n^S + w^J_{rf}n^J\right)y\left(n^J,n^S\right)}}$$

E Aggregate Uncertainty Shocks

In order to analyze the transition path implied by our model we apply a one period exogenous aggregate uncertainty shock applied to the distribution of the productivity draws. In the steady state, the distribution of shocks is represented by the uniform distribution $f(z) = 1$ for $z \in [0,1]$. To induce a recession we apply a one period shock which is a mean preserving spread on the original productivity draws for firms and is given according to $f(z) = a(z - .5)^2 + (1 - \frac{a}{12})$. This maintains the average productivity across firms while increasing the weight in the tails of the distribution. A visual representation of the uncertainty shock is displayed in Figure 19. Note that the focus of our paper is on the recovery following recession. The actual method of implementing a shock is not central to our results.

F Algorithm for Solving Transition Dynamics

Our transition problem features heterogeneous firms. Aggregation doesn't hold because of the decreasing returns to scale production technology. At each point in time, we must keep track of the distribution of firms because it affects the total output in the economy, which in turn affects the stochastic discount factor used by firms to value payoffs in different states.

We solve for the transition using the following algorithm. Instead of iterating on a sequence of value functions, we guess and iterate on a sequence of discounted value functions.

1. Guess a sequence of discounted value function $\{DV_t : t = 1, ..., T\}$ defined by

$$DV_{t+1}\left(n^S\right) \equiv \frac{u'\left(C_{t+1}\right)}{u'\left(C_t\right)} V_{t+1}\left(n^S\right). \tag{F5}$$

   With DV defined in Equation F5, we can rewrite the firm's Bellman equation as follows:

$$V_t(n^S) = \max_{n^J} \left\{ \begin{array}{c} \int_{z^*(n^J,n^S)}^{1} \left\{ z\left[n^J\theta^J + n^S\theta^S\right]^{\alpha} - \right. \\ \left. \left[w^S n^S + w^J n^J\right] R\left(n^S, n^J\right)\right\} f(z)\, dz \\ +\beta\left[F\left(z^*\left(n^J, n^S\right)\right) DV_{t+1}(0) + \right. \\ \left.\left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right) DV_{t+1}\left(\delta\left(n^S + n^J\right)\right)\right] \end{array} \right\}. \tag{F6}$$

   Note that DV_{t+1} is relevant for the period t hiring decision of firms.

2. For each t, given DV_{t+1}, we solve (F6). Let $n_t^J\left(n^S\right)$ be the hiring policy function, and $\tilde{V}_t$ the updated value function. The policy function $n_t^J\left(n^S\right)$ allows us to calculate the distribution of firms in period $t+1$:

$$g_{t+1}(n) = \int_{n^S} \mathbf{1}\left\{\delta\left(n^S + n_t^J\left(n^S\right)\right) = n\right\} g_t\left(n^S\right)\left(1 - F\left(z^*\left(n^S\right)\right)\right) dn^S.$$

   In order to update the discounted value function, we also need to calculate the stochastic discount factor, hence output in the economy:

$$c_t = Y_t = \int_{n^S} \int_{z^*(n^J,n^S)}^{1} z\left[n^J h_t^J\left(n^S\right) + n^S\theta^S\right]^{\alpha} f(z)\, dz g_t\left(n^S\right) dn^S$$

3. Based on the updated value function and the consumption level calculated in Step 2, we update the sequence of discounted value functions

$$\tilde{DV}_{t+1} = \frac{u'\left(C_{t+1}\right)}{u'\left(C_t\right)} \tilde{V}_{t+1}\left(n^S\right).$$

4. Repeat Steps 1-3 using the updated discounted value functions.

5. Repeat until the discounted value functions all converge

$$\sup_{1\leq t\leq T}\ \sup_{n^S}|DV_t - \tilde{DV}_t| < \text{tolerence}.$$

G Cross-Sectional Intuition for How θ^S/θ^J Affects Average Labor Productivity

In the paper, we provide intution of how θ^S/θ^J affects average labor productivity by looking at forces across time. In this section, we conduct a similar analysis on the cross-section. During the recovery phase following a recession, there are two opposing forces on average labor productivity. First, there is a compositional effect. The recession leads to the bankruptcy of a fraction of firms. These firms start over with no senior workers and are less productive than the average firm in the stationary equilibrium as a result of their high proportion of junior workers. This tends to drag down the average labor productivity. Second, there is a decreasing returns to scale effect. During the recovery, output and consumption recover from the initial drop and gradually climb back to the stationary equilibrium level. This increasing path of consumption levels lowers the stochastic discount factor of households and firms. As a result, firms become more present-oriented and are less willing to invest in junior workers. They produce with fewer workers and a higher ratio of senior to junior workers. This leads to higher average labor productivity within each firm due to the decreasing returns to scale technology.

Whether average labor productivity will be pro or counter cyclical depends on the relative strength of the compositional effect across firms and the decreasing returns to scale effect within firms. When θ^S/θ^J is low, firm specific skills are less important. Start-up firms grow fast. This leads to a strong compositional effect. Further, when start-up firms grow fast, their low labor productivity contributes to the economy-wide average labor productivity with a larger weight further increasing the strength of the compositional effect.[35] As a result, the average labor productivity is procyclical. When θ^S/θ^J is high, firm specific skills are more important. Start-up firms grow slowly. This leads to a weaker compositional effect and hence a countercyclical average labor productivity.

In Figure 20 we provide two cross sectional snapshots (one in the steady state and one during the transition) for a fast recovery economy (θ^S_{low}/θ^J). In the left panel, we compare the size distribution of firms in the stationary equilibrium (blue bar) and during the first period of recovery (red bar). As can be observed in the figure, there are more start-up firms and less firms at the mature size in the recovery relative to the stationary equilibrium. Over time, these bankrupt firms grow back to the optimal size. The right panel displays the average labor productivity by firm size. As can be observed in this figure, smaller firms are substantially less productive than larger firms as a result of their higher ratio of junior workers. Additionally, all firms of a specific size are slightly

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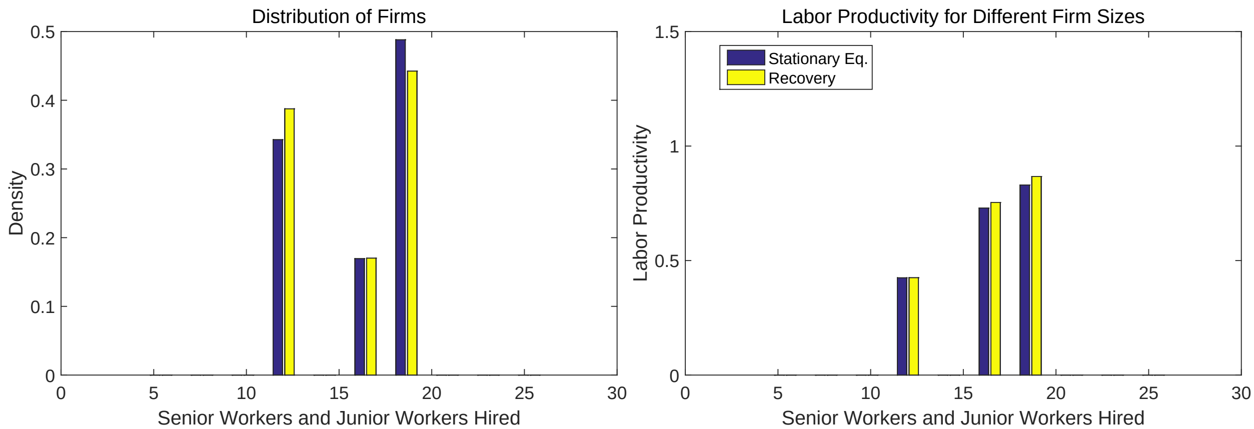
$$\text{Average Labor Productivity} \equiv \frac{Y}{L} = \sum_{n^S} \frac{g\left(n^S\right)L\left(n^S\right)}{L}\frac{Y\left(n^S\right)}{L\left(n^S\right)}$$

, where $g\left(n^S\right)$ is the measure of firms of size n^S, $L\left(n^S\right)$ is the total labor used in a firm of size n^S, and $Y\left(n^S\right)$ is the expected output of a firm of size n^S.

more productive in recovery relative to their stationary counterpart. This is due to the stochastic discount factor causing firms emphasize present period profits over future growth in the recovery. In total, the composition effect of an increase in start-up firms outweighs the decreasing returns to scale effect for. This change in the distribution of firm sizes negatively affects the average labor productivity of the economy.

Figure 20 – Distribution of Firm Sizes and Labor Productivities by Firm Sizes ($\theta^S = \theta^S_{low}$)



In Figure 21 we present the cross sectional snapshots for a slow recovery economy (θ^S_{high}/θ^J). As before, the left panel compares the size distribution of firms in the stationary equilibrium (blue bar) and during the first period of recovery (red bar). Again as can be observed in the figure, there are more start-up firms and less firms at the mature size in the recovery relative to the stationary equilibrium. However, small firms are much smaller than they were in the fast recovery economy. As a result their total contribution to average labor productivity is sigficantly reduced. The graph of average labor productivity is similar to the fast recovery figure for the same reasons as before. In this case, the composition effect of an increase in start-up firms is muted by the smaller average size of start-up firms and the decreasing returns to scale effect dominates.

Figure 21 – Distribution of Firm Sizes and Labor Productivities by Firm Sizes ($\theta^S = \theta^S_{high}$)