Credit, Money, Interest, and Prices

Preliminary and Incomplete

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Abstract

We develop a monetary theory where monetary policy operates exclusively through the bank-lending channel. Credit demand and deposit creation are dynamically linked. Policy tools affect lending through the provision of reserves and their influence on interbank market rates. A credit crunch causes debt-deflation episode that sends agents to their borrowing constraints. Unemployment increases because firms reduce utilization to avoid the risk of violating borrowing limits. Standard monetary policy has power only if credit is extended. We study the cross-section and aggregate dynamics of credit, monetary aggregates, nominal interest, and prices after several policy experiments.

Keywords: Monetary Policy, Banking, Credit Channel

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1 Introduction

This paper provides a model of the credit-channel of monetary policy —a.k.a the bank-lending transmission mechanism of monetary policy.[1] The idea behind this mechanism is that monetary policy (MP) works because a central bank is endowed with policy tools that can influence its interbank market. In turn, the power to influence the interbank market has indirect effects on the supply of credit and deposits by banks. From that starting point, an expansive monetary policy can diffuse and have real effects on economic activity, relative prices, and the distribution of wealth.

Our framework provides is a laboratory to analyze classic questions in an environment where monetary policy operates exclusively through the credit channel. How does MP affect the supply of credit and deposits? How does this show on quantities and prices over time? Are the effects of movements in corridor rates different from open-market operations? How do these standard tools differ from direct monetary transfers to the public? What is *the* optimal monetary policy when it operates through the credit channel.

After we answer those questions, we also study the effects of a credit-crunch episode. In particular, we try to understand if this transmission mechanism can be clogged in the aftermath of a credit crunch. What happens if banks perceive higher credit risk? Can monetary policy face limits? How can the spending patterns of savers affect the credit-worthiness of borrowers? When is a zero-lower bound reached and what does it mean in this context?

The building block of our model is an environment akin to the pure currency model of Lucas (1973) and Bewley (1980) where a continuum of agents face idiosyncratic risk and use money as savings instruments to hedge liquidity risk. Different from Lucas, and closer to Bewley we allow for a credit market.[2] What is special about the environment is the instruments used in credit markets. Here, as occurs with most legal segments of the economy, savings are a mix between electronic deposits issued by banks and currency. This property is very important because it means that the stock of outstanding inside money is linked to the supply of credit: the stock of inside money can only be altered through the issuances of financial sector liabilities. Deposits, in turn, only created through the provision of credit or dividends payouts by the financial sector. The paper shows that feature has important implications for the transmissions of shocks from the banking sector to interest rates and prices, and furthermore, on the incentives to provide credit. In turn, the desire to hold more deposits can can generate a debt-deflation phenomenon.

Our model can speak to the connections between a "modern" monetary policy, prices, credit, and the distribution of wealth. By modern monetary policy we mean a world where monetary policy operates through credit-deposit creation by the banking system banking system and not via the transfer currency to the public of the government by monetary authority.[3] Most of the time,

[1] In Bernanke and Gertler's "Inside the Black box **?**" we find the description of the credit channel.

[2] Lucas (1973) imposes a transactions technology that satisfies the Clower's dictum that that money buys goods and goods buy money, but goods do not buy goods. Clower (1965)

[3] Traditionally, monetary economics has relied on models that are based on a pure currency world —outside money.

our economy will be essentially cash-less, and operation through an electronic system of credits and debit accounts. Instead, monetary policy works because the financial sector has a demand for reserves and this complements the provision of credit and the issuances of deposits. This is modeled as occur in practice, for clearing or regulatroy purposes. Open market operations and corridor rates affect the holdings of reserves and indirectly the provision of credit. Here, a zero-lower bound emerges explicitly because deposits and reserves can always be converted into currency.

Our approach is dramatically different with the cashless and bankless feature of canonical new-Keynesian models proposed by Woodford (1998). The new Keynesian model is the most prominent framework to evaluate monetary policy. In that framework, the central bank is assumed to control real interest rate thanks to two imperfections: sticky prices and some imperfection that makes money valuable. That environment is appealing to policy makers because it is consistent that policy rates, once converted into real rates, systematically move at the will of central banks. It also fits some empirical evidence that shows short-run non-neutralities and long-run neutralities.

The problem with this approach is that it leaves behinds many of the details that are part of the implementation and diffusion of monetary policy. Therefore, it is not useful to understand if policy is interrupted when credit markets are under stress and bypasses issues such as debt-deflation effects. This is a problem because, for starters, monetary policy is implemented through the banking system everywhere. If the banking system is not healthy, even if interbank rates fall, and the supply of high power money is vast, this may not translate into more credit to a segment of borrowers. Thus, at least a segment of borrowers may not respond to a stimulus. For savers, the lack of credit may also affect their spending, which in turn may put more stress onto borrowers.

The title of the paper is reminiscent of Patinkin's Money, Interest and Prices. Michael Woodford took money out of that title and, much of macroeconomic analysis that followed the advice of that book, of analyzing monetary policy models without money. The title of this paper is a counter reaction to that title. Like many now, not only do we think we need to add money back to monetary models, but we should also add banks.

1.1 Literature

There is a tradition in economics that dates at least Bagehot (1873) which stresses the importance of analyzing monetary policy together with financial institutions. A classic attempt to study policy in a model with a full description of households, firms and banks is Gurley and Shaw (1964). With few exceptions, modeling banks was a practice abandoned by macroeconomics for many years. Until the Great Recession, questions about the macroeconomic effects of monetary policy and how this policy is implemented through banks were treated independently.[4]

However, the implementation of monetary policy is far from that world —unless we have fiscal policy.

[4]This simplification seemed natural. In the US, banking didn't seem to matter for macroeconomic performance. For example, The banking industry was among the most stable industries in terms of solvency. More importantly, the pass-through from key policy rates and to lending terms seemed straight.

Schneider Piazzesi,

I-Theory

Silva

Greenwald

Schnabl

Lippi-Trachter

Kaplan-Violante-Moll,

New-Monetarist literature.

Uberto Ennis.

Andy Atkeson and other on Liquidity Effect.

Silvana Tenreyro and Vincent Sterk, The Transmission of Monetary Policy through Redistributions and Durable Purchases.

The closest modern dynamic macro model to ours is, Brunnermeier and Sannikov (2012). This paper also introduces inside money created by banks and outside money. Outside money plays the same role as inside money as a tool to materialize investment opportunities. We share the spirit of having money as an asset that allows transactions. We differ in that outside money here are central bank reserves which are not used for commercial transactions and play a role as an instrument to hedge illiquidity risks. This maturity mismatch problem explains why monetary policy affects bank lending. Another pair of closely related papers is Williamson (2012) and Rocheteau and Rodriguez-Lopez (2013). Williamson (2012) studies an environment where different assets, among which bank loans stand out, have different properties as mediums of payments. Rocheteau and Rodriguez-Lopez (2013) has a spillover from liquidity needs in a OTC market to the labor market where firms are issuing loans to hire workers. Like us, they use these frameworks to study the liquidity effects of different monetary policy tools.

Our model is also related to Stein (2012) and Stein et al. (2013) who study environments where there is and exogenous demand for safe short-term liquid assets with implications for policy. A common feature in all of these papers is that the classic Modigliani-Miller theorem for open-market operations (see Wallace (1981)) is broken.[5]

Some recent empirical papers documenting liquidity effects following open market operations include Krishnamurthy and Vissing-Jørgensen (2011, 2012). Our model also relates to the very known study of Kashyap and Stein (2000) which documents the effects of monetary policy via the lending channel. We contend that we provide a formalism to many of the arguments in that paper.

[5] Another distinction is that it has additional predictions about cross-sectional industry growth for banks. Along that dimension, a model related to this is one is Corbae and D'Erasmo (2013); **?** who also study the industry dynamics of the banking industry.

2 A General Environment

We begin with a description of a general environment and then we specialize to study different monetary policy and credit market regimes. Time is continuous and the horizon infinity. The economy is populated by three groups of agents: the public, banks, and the government.

The Public. There is a measure-one continuum of households that belong to the non-financial sector. Households have preferecences described by present discounted utility:

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} U\left(c_t\right) dt\right]$$

where the instantaneous utility is:

$$U\left(c_t\right) \equiv \frac{\left(c_t - \bar{c}\right)^{1-\gamma} - 1}{1-\gamma}.$$

Here $\bar{c}$ is a bliss-point and γ is the coefficient of risk-aversion when $\bar{c} = 0$. The role of the bliss point is to generate non-linear Engle curves.

Each household has two groups of members, an entrepreneur and a continuum mass $\bar{n}$ of workers. This division allows us to talk about unemployment and to describe a demand externality that the central bank may want to correct. Liken in Hansend and Rogerson (1984), labor is indivisible so each of the workers in the household either works or does not.

The individual state of a household is given by his (nominal) financial wealth s. When credit is avialable, s can be negative. Household's are heterogeneous because their wealth evolves over time. Therefore, at all points in time, there's a distribution $f_t\left(s\right)$ among households. We describe the evolution of $f_t\left(s\right)$ below.

Banks. Banks are intermediaries between borrowers and lenders. There is free entry in the banking industry and a bank can operate without capital.[6] The details of financial intermediation are explained below. Linearity in the profits of banks implies the presence of a representative bank. So in what follows we provide the description for a single bank.

Technology. Each entrepreneur manages a production unit. The entrepreneur can choose between two possible technologies. The choice of technologies is given by $u \in \{L, H\}$ where u defines an utilization intensity. Each technology requires a specific amount of workers to be operational. In particular,

- If $u = L$, the entrepreneur needs to hire n_L workers.

- If $u = H$, the entrepreneur needs to hire n_H workers.

[6] Adding a bank equity would rerquire and additional state variable. Rescrtictions such as capital requirements or limited participation —entry barriers— would generate bank profits.

Given the choice of u —and n— output is produced at a rate $y(u)$. The technology with high utilization produces output at a higher rate because: $y(H) > y(L)$. By assumption, $\bar{n} = n_H$ so if all entrepreneurs where to operate with the high intensity technology, all workers would be employed.

The only reason why the entrepreneur may choose a low utilization technology is because the high technology is riskier. As before, if $u = 1$, the entrepreneur faces an idiosyncratic cost that can be also interpreted as output loss or even as demand risk.[7] In particular, his output faces a risk equivalent to $\sigma(u)\,dZ_t$ where dZ_t are the white noise associated with a Brownian motion process. Each household faces it's own idiosyncratic risk given by Z_t. By assumption, $\sigma(H) > \sigma(L)$. We will describe when and why the entrepreneur chooses one over the other technology. For now, it is just worth saying that this technology choice will be summarized by a stochastic process u_t. This idiosyncratic risk is born by the entrepreneur and cannot be diversified.

The Labor Market. The labor market suffers from a particualr imperfection: there is a hold-up problem. Once an entrepreneur hires a worker output becomes specific to the worker as in Caballero and Hammour (1995). Thus, after being hired, workers are in a position to bargain over the total output produced in a time interval.[8] Thus, the split of output among entrepreneurs is η and $1-\eta$, where η can be interpreted as a bargaining parameter. More interestingly, η will be a parameter that describes the extent of a demand externality. It will be akin to a parameter that governs the extent of real-wage rigidity. Taking η we would recover the outcomes of an efficient Walrasian labor market.

The choice of technology affects the demand the amount of workers hired —there is potential unemployment. Consequently, there could be idiosyncratic labor income risk but we assume that labor-income risk is diversified within the household.

Considering this diversification workers of each household receive a common labor income flow of:

$$w_t^l = (1-\eta)\int_{\mathcal{S}} y(u(s,t))\, f_t(s)\, ds$$

where $f_t(s)$ is the distribution of wealth and $u(s,t)$ is the choice of technology of household s at time period t. In turn, we can compute the unemployment rate via:

$$\Upsilon(t) = \int_{\mathcal{S}} \left[1 - (u(s,t) - 1)\frac{n_2}{\bar{n}}\right] f_t(s)\, ds.$$

[7]Demand risk can be introduced easily by assuming that products are heterogeneous and aggregated via an Armington Aggregator.

[8]This construction can be approximated by a limit. Suppos that technologies are fixed over specific time intervals $\Delta t, 2\Delta t$, ... For every interval, assume that once the technology is chosen and workers are hired, contracts are negotiated on the spot and according to a bargaining problem. In particular, workers may threaten the entrepreneur not to work in which case they receive no output. Pressumably, this hold-up problem leads to an output split according to some Nash-bargaining problem —also a la Rubinstein. In that case, output is divided in η and $(1-\eta)$ shares to entrepreneurs and workers correspondingly.

Outside Money. There's an exogenous supply of monetary base M_t which can be thought of fiat currency issued by a central bank or commodity money such as gold. Commodity money can be introduced via lump-sup transerfers to the public. In particular, transfers can depend on the household's wealth s_t. The individual holdings of currency by a household are denoted by m_t. We refer to the holdings of currency as currency held by the public:

$$M0_t \equiv \int_{\mathcal{S}} m(t,s) f_t(s) ds.$$

In addition to households, banks can also hold part of the money supply. We call the holdings of money by banks reserves and we denote reserves by m_t^{bank}. Naturally, the following accounting identity holds:

$$M0_t + m_t^{bank} = M_t.$$

The Credit Market. Banks can issue deposits to purchase currency, to make a loans, or to pay for interest expenses. By convention, deposits are a claim on currency so currency and deposits are exchanged at par. There's an endogenous spread between borrowing and lending, r_t^b and r_t^d. In particular, r_t^b is instantaneous nominal interest rate charged to borrowers —a borrower is any household with $s_t < 0$. In turn, r_t^d is the interest on savings accounts at banks. A saver household can choose to save his wealth in deposits or currency.

Credit to households are limited by a pair of borrowing limits. There are two lower bounds, a maximal debt capacity $\bar{s}$ and a time-varying borrowing limit $\tilde{s}_t$. The maximal debt capacity implies that:

$$s_t \geq \bar{s}$$

at all points in time. The borrowing limit is introduced to model a credit crunch in a simple way and is novel in the literature. In particular, in the region $s \in [\bar{s}, \tilde{s}_t]$, we assume that the household can increas its debt, but only by the accumulation of capital. In other words, in that regions of savings, households cannot obtain more principal loans, although they are allowed to increase their debt through the accruing of interest rates. Formally, this constraint is:

$$ds_t \geq r_t s_t dt.$$

The economic motivation of this constraint is that during a credit crunch, a bank may allow agents to refinance their debt, possibly accumulating more debt through accruing of interest payments, but not extend more of the principal. Forcing a borrower to repay immediately may be unfeasible or may lead to a potential inefficient defualt. Thus, it may be in the interest of the bank to let the agent accumulate interset debt.[9]

If we combine this constraint with the household's budget constraint we obtain:

$$c_t dt \geq dw_t \text{ in } s \in [\bar{s}, \tilde{s}_t],$$

but since w_t is random unless $u_t = 1$. Thus, the constrain also forces $u_t = 1$. Thus, it can be sumarized by forcing:

$$u_t = 1 \text{ and } ds_t \geq r_t s_t dt \text{ in } s \in [\bar{s}, \tilde{s}_t].$$

When we model a credit crunch we allow for changes in $\tilde{s}_t$. This is convenient for technical reasons, because $\tilde{s}$ can be allowed to jump unexpectedly, and we don't face the problem of agents violating their constraints. Instead, it is not possible to tighten a borrowing limit $\bar{s}$ as in Guerrieri and Lorenzoni (in continuous time).

Central-Bank Overnight Operations. Adding to his holdings of reserves, a bank can borrow reserves or lend reserves to the central bank. In particular, the central bank can make overnight loans or borrow reserves overnight from commercial banks. The total overnight loans to a commercial bank are,

$$m_t^{on} = d_t^{FED} - b_t^{FED},$$

where d_t^{FED} are overnight loans to the central bank and b_t^{FED} are deposits by banks at the central bank. The balance sheet of a commercial bank at every point in time thuse satisfies:

Assets	**Liabilities**
$m_t + m_t^{on}$	d_t
b_t	d_t^{FED}
b_t^{FED}	

The central bank sets a borrowing r_t^{er} and lending rates r_t^{dw}. For the entire paper we assume that $r_t^{er} < r_t^{dw}$. There is a constraint that must be satisfied at all times:

$$m_t + m_t^{on} \geq \varsigma_t d_t$$

[9]Notice that when a bank extends the principal, it issues deposits. Instead, when debt increases through interest rates it only accumulates assets and equity increases. Furthermore, if forcing repayments induces default, the bank may be better of rolling-over debt, rather than writing down their debts. I think this is in a nutshell the idea behind zombie lending (Caballero, Kashyap and Stein AER). This phenomenon is called evergreening.

Here, $\varsigma_t \in [0,1]$ where term ς_t can can be thought of as a combination of reserve requirement rates imposed by the central bank, or a variable that follows from the liquidity management of banks as in Bianchi and Bigio (2016).

The profits of an individual bank are given by the sum of the profits in credit markets and in its transactions with the central bank:

$$\pi_t = r_t^b b_t + r_t^{er} b_t^{FED} - \left(r_t^d d_t + r_t^{dw} d_t^{FED} \right).$$

Accounting Identities. The consolidated balance sheet of a commercial banks has the following features. There's an identity for the issuances of loans and liabilities between banks and the public. Subtract from the balance sheet identity of a given bank the borrowing and lending from the central bank. In that case we obtain:

$$m_t^{bank} + b_t = d_t. \tag{1}$$

Since savings by the public can only be held either in currency or deposits we have the following identity:

$$M0_t + d_t = \int_0^\infty s f_t\left(s\right) ds. \tag{2}$$

In turn, all loans are held by banks:

$$b_t = -\int_{-\infty}^0 s f_t\left(s\right) ds. \tag{3}$$

Substituting in (2) and (3) into (1), we obtain:

$$m_t^{bank} = \int_{-\infty}^\infty s f_t\left(s\right) ds - M0_t.$$

Thus, we obtain the following identity:

$$M_t = \int_0^\infty s f_t\left(s\right) ds,$$

that is, the net savings in this economy equal the money supply.

2.1 Monetary Policy Regimes

An important property of the model is that its solution depends on the monetary authority's ability to tax.

2.2 Agent Problems

Household's Problem. Combining a household's labor income and his expected entrepreneurial income yields an expected flow of nominal income of

$$h\left(u_t, t\right) = \eta y\left(u_t\right) + w_t^l$$

where $\eta y_t\left(u\right)$ is the household's entrepreneurial wealth and w_t^l the wage described earlier.[10] Now, considering the household's total income also depends on the shock, the household receives a total income flow of:

$$dw_t = h\left(u, t\right) dt + \sigma(u) dZ_t$$

depending on his technology choice. The law of motion of the household's wealth is given by:

$$ds_t = \left(r_t\left(s_t\right)\left(s_t - m_t\right) - p_t c_t\right) dt + dw_t$$

where $r_t\left(s_t\right)$ is, as we noted before:

$$r_t = \begin{cases} r^d \text{ if } s_t > 0 \\ r^b \text{ if } s_t \leq 0 \end{cases}.$$

The corresponding Hamilton-Jacobi-Bellman equation of the household problem is given by:

$$\rho V\left(s, t\right) = \max_{c,u,m} \; U\left(c\right) + V_s'\left(r_t\left(s - m\right) - p_t c + h\left(u, t\right)\right) + \frac{1}{2} V_s'' \sigma^2(u) + V_t$$

subject to:

$$u = 1 \text{ and } c \in \left[0, h\left(u, t\right)\right] \text{ in } s \in \left[\bar{s}, \tilde{s}_t\right].$$

[10] Observe that η controls the labor-income externality. The technology choice of agents u_t reduces the income of other agent, but this effect cannot be internalized because of the hold-up problem.

The Bank's Problem. The problem of an individual bank is static. Thus, it solves:

$$\begin{aligned}\pi_t &= \max_{\left\{d_t, b_t, b_t^{FED}\right\} \geq \mathbb{R}^+} r_t^b b_t + r_t^{er} b_t^{FED} - \left(r_t^d d_t + r_t^{dw} d_t^{FED}\right) \\ \text{subject to } d_t &= m_t + m_t^{on} + b_t \\ m_t^{on} &= d_t^{FED} - b_t^{FED} \\ m_t + m_t^{on} &\geq \varsigma_t d_t.\end{aligned}$$

The first constraint is the balance-sheet identity for the bank. The second are his overnight borrowing and the last constraint are the reserve requirements. The possibility of storage and arbitrage in rates by banks imposes constraints on the policy instruments that banks face, namely, it introduces ar zero lower bound.

2.3 Law of Motion for Wealth

Let $c(s,t), u(s,t)$ and $m(s,t)$ be the solutions to the household's problem. Then, the drift of the household's nominal wealth evolves according to:

$$\mu(s,t) \equiv (r_t(s)(s - m(s,t)) - p_t c_t) + \eta y(u(s,t)) + w_t^l.$$

The volatility for the household is:

$$\sigma_s^2(s,t) \equiv \sigma^2(u(s,t)).$$

These objects are all we need to characterize the evolution of wealth. In particular, the law of motion of wealth satisfies the following Kolmogorov-Forward partial-differential equation:

$$\frac{\partial}{\partial t} f_t(s) = -\frac{\partial}{\partial s}\left[\mu(s,t) f_t(s)\right] + -\frac{\partial}{\partial s}\left[\sigma_s^2(s,t) f_t(s)\right].$$

The solution to this equation yields a law of motion for wealth.

2.4 Equilibrium Definition

Next, we define an equilibrium.

Definition. Given an initial condition for the distribution of household wealth $f_0(s)$ and a policy rule, an equlibrium are price functions $p(t), r^d(t), r^b(t)$ such that:

1. Optimality of households is satisfied.

2. Optimality of banks are satisfied.

3. Government satisfies its budget constraint.

4. The law of motion for $f_t(s)$

5. Markets clear.

2.5 Analysis

Let's analyze the behavior of banks. We have a proposition:

Proposition 1 *Consider a point in time where* $\left[r_t^{er}, r_t^{dw}\right] > 0$. *Borrowing and lending rates are related via the following formula:*

$$r_t^d = (1 - \varsigma_t)\, r^b$$

and

$$r_t^b = \begin{cases} r_t^{dw} \text{ if } \rho M_t > d_t \\ r_t^{er} \text{ if } \rho M_t < d_t \\ r^b \in \left[r_t^{er}, r_t^{dw}\right] \text{ if } \rho M_t = d_t \end{cases}.$$

3 Equilibrium in a pure Currency Economy

To study an economy in a pure currency world, we shut down all forms of borrowing and lending. We use this example to study some fundamental mechanisms that are present in the paper that are independent of credit markets.

To move to a pure currency world, we set $\bar{s} = \tilde{s}_t = 0$. This is a world where banks can take deposits but only for currency. Banks just provide the role of safe keeping. Since reserve requirements are not binding, then there are no central bank loans. Furthermore, there are no interest rate payments.

Let's present some results about "helicopter drops". We turn off the demand feedback mechanism and set $\eta = 1$.

3.1 Demand Feed-Back Mechanism

In this model, the only reason to activate the safe technology is to avoid hitting a borrowing constraint. Since if every entrepreneur chooses the safe technology is equivalent to choosing the total number of workers in the economy,

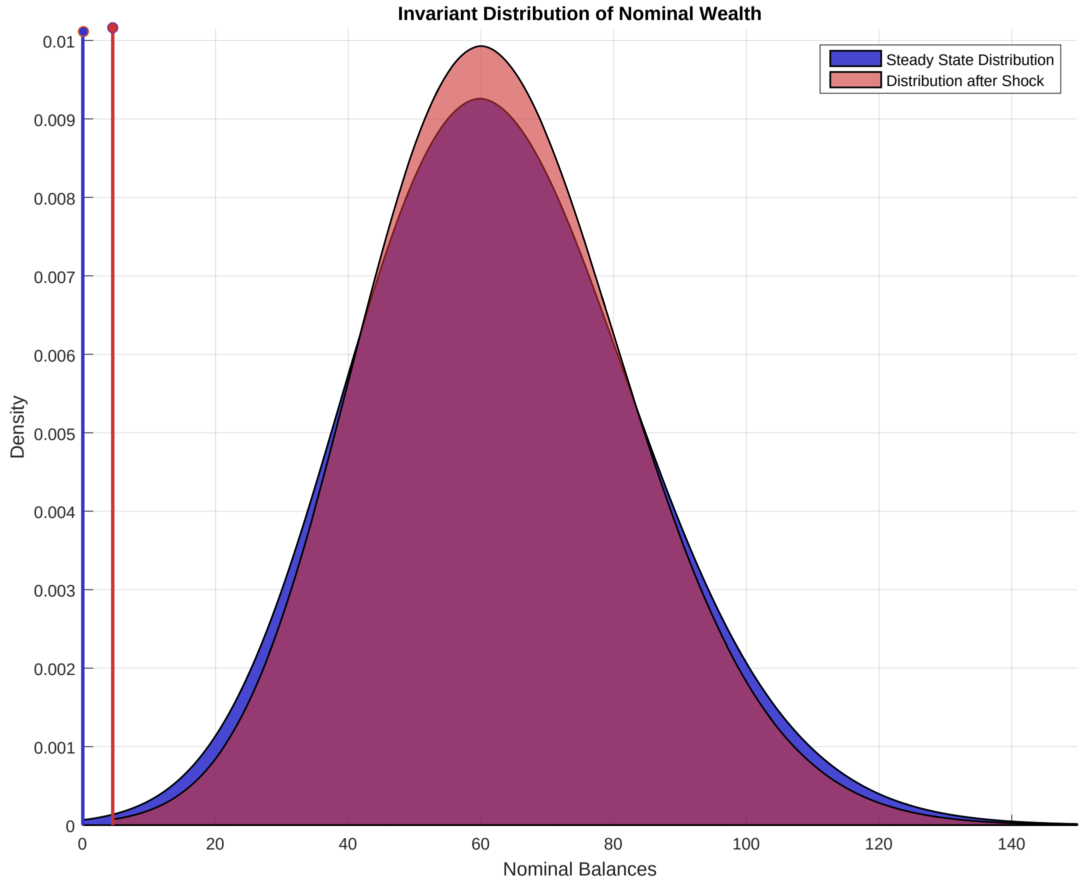

Figure 1: Invariant Distribution and Distribution after Helicopter Shock

3.2 Expected and Unexpected Monetary Transfers

- Helicopter drops: uniform shock. - Experiment, one time and for all unexpected monetary injection.

- Shock shits distribution to the right.
- Begins from a distribution, which is more compressed distribution relative to steady state (additive plus scalar translation.

- along transition:

4 Equilibrium in Credit Economy

Zero-Lower Bound.

4.1 Connection to Classic Monetary Theory

Quantity Theory and Endogenous Velocity.

Endogenous Money Multiplier.

4.2 A Credit Crunch

4.3 Debt-Deflation

5 Monetary Policy through the Credit Channel

5.1 Corridor Rates

5.2 Open-Market Operations

6 Solvency Risk

Now the agent chooses transitions between default (d) and being part of credit markets (m) regime.

$$s \to s - \bar{s} \quad \text{from } m \text{ to } d$$
$$s \to s - f \quad \text{from } d \text{ to } m$$

- Value functions (with switching points s^m, s^d and smooth-pasting conditions)

$$V_d : [0, s^m] \to \mathbb{R}, \quad V_d(s^m) = V_s(s^m - f),\ V_d'^m) = V_s'^m - f)$$

$$V_s : [s^d, \infty) \to \mathbb{R}, \quad V_m(s^d) = V_d(s^d - \bar{s}),\ V_m'^d) = V_d'^d - \bar{s})$$

HJB equations

$$\rho V_d = \max_{c,u} \; U(c) + V_d' \left(r_t s - c + \bar{w}(u)\right) + \frac{1}{2} V_d''^2 (u)$$

- I think in both of these equations, $u = 2$ will be chosen, except for a sticky reflecting boundary at $s = 0$ for V_d
- For V_m, we need a transversality condition at infinity (to make sure the solution is non-explosive)
- First-order condition for c,

$$U'(c) = V_m' \quad U'(c) = V_d'$$

- Boundary condition at $s = 0$ for V_d

$$\rho V_d = U(c) + V_d' (\bar{w}(1) - c)$$

- We can search for this boundary condition numerically by picking V_d' : the higher V_d', the higher V_d

7 Conclusion

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8 Appendix

8.1 Bank's Problem

Let's consider the bank's problem. Substituting the definition of the bank's loan on his budget constraint we obtain:

$$d_t = m_t + m_t^{on} + b_t$$

$$\begin{aligned} \pi_t &= \max_{\{d_t, b_t, b_t^{FED}\} \geq \mathbb{R}^+} r_t^b b_t + r_t^{er} b_t^{FED} - \left(r_t^d d_t + r_t^{dw} d_t^{FED} \right) \\ \text{subject to } d_t &= m_t + d_t^{FED} - b_t^{FED} + b_t \\ m_t + d_t^{FED} - b_t^{FED} &\geq \varsigma_t d_t. \end{aligned}$$

Let's first observe that

8.2 Solution to Household Problem

An upper Bound on the Household's Value Function

What value function does the agent get if he has savings s and (1) a sure income of $\bar{w}(2)$ and (2) he can borrow at rate r^s as well? In this case, the agent's wealth is $n = s + \bar{w}(2)/r^s$. His optimal consumption rate given the rate of savings is

$$\frac{c}{n} = r^s + \frac{\rho - r^s}{\gamma}.$$

This has to be positive in order to guarantee that the agent's value function is finite.

The value function is given by

$$\bar{V}(n) = \frac{nc^{-\gamma}}{1-\gamma}$$

The derivative of the value function with respect to n is

$$\bar{V}(n)' = \left(r^s + \frac{\rho - r^s}{\gamma} \right) n^{-\gamma}$$

We can try solving the market equation from the right according to (**??**), starting with $V_m(s) = \bar{V}(s + \bar{w}(2)/r^s)$. We play with $V_m'(s)$ - the maximum allowable is $\bar{V}(n)'$, and the minimum is such that

$$\rho \bar{V}(s + \bar{w}(2)/r^s) = \frac{cV_m'}{1-\gamma} + V_m'^s s - c + \bar{w}(1)) = \frac{\gamma}{1-\gamma}(V_m'^{-1/\gamma} V_m' + V_m'^s s + \bar{w}(1))$$

since $c^{-\gamma} = V_m'$.