

# Inflation, Debt, and Default

Sewon Hur  
U. of Pittsburgh

Illenin Kondo  
Board of Governors

Fabrizio Perri  
Minneapolis Fed

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# Motivation

- The majority of sovereign debt in advanced economies is held domestically.
- In these countries, the co-movement between inflation and domestic consumption growth also varies considerably over time.

# Domestic Share of Government Debt

| Country        | Year |      |      |      |
|----------------|------|------|------|------|
|                | 2004 | 2008 | 2012 | Mean |
| Australia      | 83.3 | 85.6 | 61.9 | 76.9 |
| Belgium        | 50.7 | 41.0 | 58.9 | 50.2 |
| Canada         | 77.6 | 83.8 | 72.1 | 77.8 |
| Denmark        | 74.5 | 75.2 | 70.9 | 73.5 |
| Finland        | 23.1 | 38.1 | 25.9 | 29.0 |
| France         | 57.9 | 57.8 | 51.5 | 55.7 |
| Germany        | 68.6 | 53.5 | 41.4 | 54.5 |
| Greece         | 40.8 | 41.2 | 22.2 | 34.7 |
| Italy          | 59.9 | 60.9 | 66.1 | 62.3 |
| Japan          | 95.7 | 91.9 | 92.1 | 93.3 |
| Korea*         | 92.2 | 90.8 | 83.9 | 89.0 |
| Netherlands    | 44.4 | 45.2 | 55.8 | 48.5 |
| Norway         | 43.5 | 50.6 | 71.5 | 55.2 |
| Portugal       | 24.0 | 27.3 | 35.9 | 29.0 |
| Spain          | 55.7 | 62.6 | 78.1 | 65.5 |
| Sweden         | 64.4 | 75.5 | 61.4 | 67.1 |
| United Kingdom | 81.9 | 78.1 | 72.4 | 77.5 |
| United States  | 80.8 | 78.0 | 73.3 | 77.3 |
| Mean           | 63.3 | 63.9 | 60.4 | 62.5 |

Sources: BIS, Haver

# Motivation

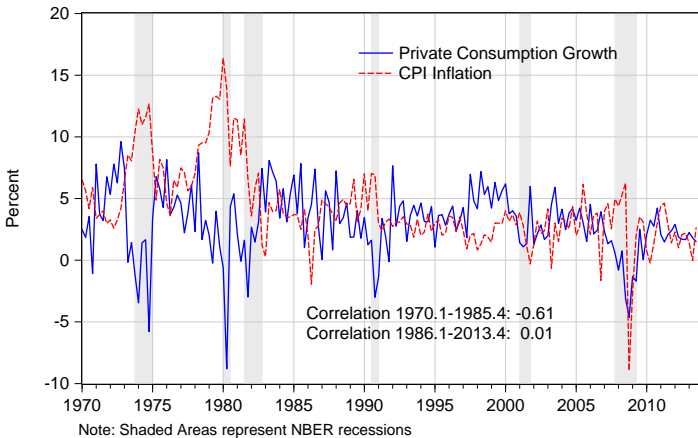


Figure: Inflation and consumption growth co-movement in the U.S.

# The question

How does the inflation process - in particular the co-movement of inflation and consumption growth - jointly affect interest rates, debt dynamics, and debt crises?

# Why does inflation matter?

- The co-movement of inflation and consumption growth affects the risk of nominal debt
- ⇒ Pro-cyclical inflation makes nominal debt
- + less risky to lender: receives more in bad times
  - more risky to borrower: pays out more bad times

# Why does inflation matter?

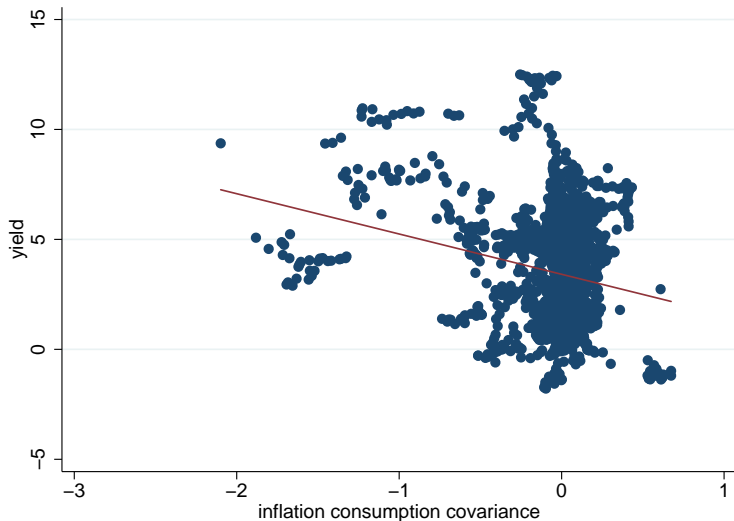
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  - more risky to borrower: pays out more bad times
- Inflation affects debt pricing
- Debt pricing → debt dynamics → debt crises → ...

## Related literature

- **Sovereign default**  
Eaton and Gersovitz (1981), Arellano (2008), Aguiar and Gopinath (2007), Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2009)
- **Domestic/Selective default**  
Reinhart and Rogoff (2011), D'Erasmus and Mendoza (2013), Mallucci (2015)
- **Default and inflation**  
Aguiar, Amador, Farhi and Gopinath (2012), Sunder-Plassman (2013)
- **Cyclicity of inflation**  
Boudoukh (1993), Ang, Bekaert, and Wei (2008), Campbell, Pflueger, and Viceira (2014), Song (2014)
- **Monetary unions**  
Neumeyer (1998), Aguiar et al. (2013), Corsetti and Dedola (2013)



# Inflation Consumption Covariance and Real Interest Rates



# Evidence

- Inflation and connection to real yields on government debt
- Compute co-movement of innovations to inflation and to consumption growth
- OECD 1970-2012 using overlapping windows
- Less pro-cyclical inflation is associated with higher yield

# Conditional co-movement between inflation and consumption growth

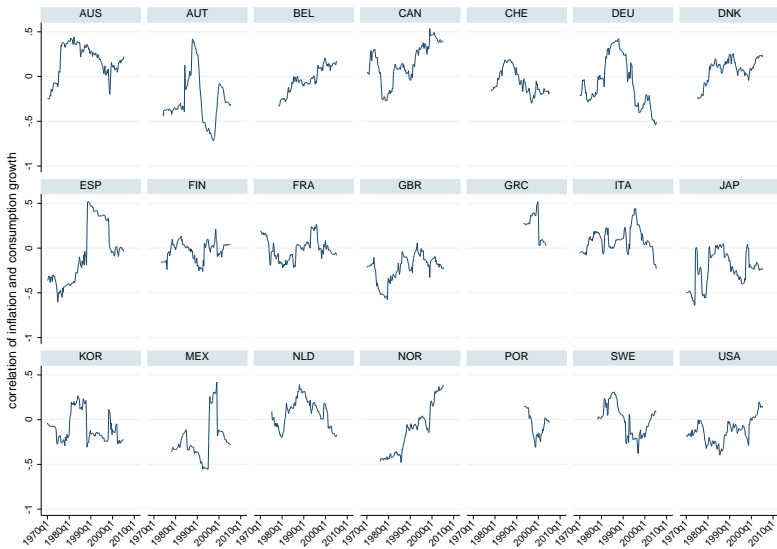
- Follow Boudoukh (1993)
- VAR country by country on quarterly data

$$\begin{bmatrix} \pi_{it} \\ g_{it}^c \end{bmatrix} = A_i \begin{bmatrix} \pi_{i,t-1} \\ g_{i,t-1}^c \end{bmatrix} + \begin{bmatrix} \varepsilon_{\pi it} \\ \varepsilon_{git} \end{bmatrix}$$

- Compute conditional co-movement between  $\varepsilon_{\pi it}$  and  $\varepsilon_{git}$  using overlapping five-year windows

Graph

# Conditional correlation between inflation and consumption growth



# Inflation cyclicalty and real interest rates

|                                  | Real yield on government debt |                      |                      |
|----------------------------------|-------------------------------|----------------------|----------------------|
|                                  | (1)                           | (2)                  | (3)                  |
| Inflation consumption covariance | -2.518***<br>(0.484)          | -2.636***<br>(0.518) | -2.357***<br>(0.405) |
| Inflation                        |                               | 0.080<br>(0.409)     | -0.089<br>(0.431)    |
| Consumption growth               |                               | -0.018<br>(0.726)    | -0.042<br>(0.704)    |
| Lagged government debt           |                               | -0.006<br>(0.008)    | -0.005<br>(0.009)    |
| Inflation Variance               |                               |                      | 0.310<br>(0.180)     |
| Consumption Variance             |                               |                      | 0.008<br>(0.185)     |
| adj. $R^2$                       | 0.769                         | 0.782                | 0.787                |
| $N$                              | 2772                          | 2682                 | 2682                 |

Standard errors in parentheses. All regressions include country and time fixed effects.

# Inflation cyclicalty and debt accumulation

|                                  | Change in government debt |                      |                      |
|----------------------------------|---------------------------|----------------------|----------------------|
|                                  | (1)                       | (2)                  | (3)                  |
| Inflation consumption covariance | 0.027<br>(0.091)          | 0.477***<br>(0.136)  | 0.494***<br>(0.121)  |
| Inflation                        | 0.013<br>(0.029)          | 0.025<br>(0.155)     | 0.002<br>(0.145)     |
| Consumption growth               | -1.366***<br>(0.209)      | -1.347***<br>(0.219) | -1.370***<br>(0.262) |
| Lagged government debt           | -0.000<br>(0.003)         | 0.002<br>(0.003)     | 0.002<br>(0.003)     |
| Inflation Variance               |                           |                      | 0.046<br>(0.067)     |
| Consumption Variance             |                           |                      | -0.021<br>(0.077)    |
| Real yield on government debt    |                           | 0.127**<br>(0.047)   | 0.124**<br>(0.051)   |
| adj. $R^2$                       | 0.492                     | 0.571                | 0.572                |
| $N$                              | 2870                      | 2682                 | 2682                 |

Standard errors in parentheses. All regressions include country and time fixed effects.

## Additional evidence

- Less pro-cyclical inflation is associated with lower CDS spreads suggesting reduced default probability
- Less pro-cyclical inflation is associated with higher volatility of spreads

# This paper

- Understand effects of the inflation process on borrowing costs, debt dynamics and debt crises
- Model of sovereign debt, builds on standard model (Arellano 2008)
- New ingredients:
  - **Inflation**, exogenous  
(e.g. changes monetary independence, changes in nature of shocks supply/demand in the economy)
  - **risk-averse, domestic** lenders
- Long term debt



# Model

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- Both lenders and borrowers are *risk-averse*

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- Both lenders and borrowers are *risk-averse*
- Nominal bonds have stochastic return (exogenous *inflation* process)

# Model

- Closed economy, discrete time  $t = 0, 1, 2, \dots$ , one good
- Endowments  $y$  and inflation  $\pi$  follow a **joint** Markov Process
- Agents
  - unit measure of hand-to-mouth households (poor, impatient)
  - $\mu$  measure of lenders (rich, patient)
  - government
- Government
  - borrows from and defaults on lenders, on behalf of poor households, using nominal bonds
  - maximizes welfare of poor households

# Households

- Household types:
  - hand-mouth households (poor, impatient)
  - rich, patient lenders
- Both households have preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta_i^t u(c_{it})$$

where  $0 < \beta_h < \beta_\ell < 1$

- Lenders receive  $\alpha y$

## Long-term debt

- We follow Hatchondo and Martinez (2009) in modeling long-term debt
- A bond issued in period  $t$  promises an infinite stream of coupons, decreasing at rate  $\delta$
- Law of motion for coupon payment obligations:

$$B' = (1 - \delta)B - x$$

where  $x$  is new issuance

# Government

- Given the option to default, the government chooses

$$V^o(B, s) = \max_{c,d} \{V^c(B, s), V^d(B, s)\}$$

where  $B$  is incoming assets and  $s = (\pi, y)$

## Value of default

- The value of default is given by

$$V^d(B, s) = u(y - \phi^d(y)) + \beta_h \mathbf{E}_{s|s'} \left[ \theta V^o \left( \frac{\lambda B}{1 + \pi'}, s' \right) + (1 - \theta) V^d \left( \frac{\lambda B}{1 + \pi'}, s' \right) \right]$$

where  $0 < \theta < 1$  is the probability of regaining access to credit markets,  $0 < \lambda < 1$  is the recovery rate, and

$$\phi^d(y) = \max \left\{ 0, \frac{d_1}{d_0} y + \left( d_1 - \frac{d_1}{d_0} \right) y^2 \right\}$$

- The default cost at mean is  $\phi^d(1) = d_1$
- Default costs matter when  $\phi^d(y) > 0$ , when  $y < 1 + d_0$

## Value of repayment

- The value, conditional on not defaulting is given by

$$V^c(B, s) = \max_{B'} \left\{ u \left( y - q(B, s, B') (B' - (1 - \delta)B) + B \right) + \beta_h \mathbf{E}_{s'|s} \left[ V^o \left( \frac{B'}{1 + \pi'}, s' \right) \right] \right\}$$

where  $q(B, s, B')$  is the bond price

- Real return on government debt is stochastic (even in absence of default)



## Bond price

In this environment, the bond price satisfies

$$q(B, s, B') = \beta_\ell \mathbf{E}_{s'|s} \left[ \frac{1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right)}{1 + \pi'} \times \left( 1 + (1 - \delta) q^* \left( \frac{B'}{1 + \pi'}, s' \right) \right) \frac{u'(c'_\ell)}{u'(c_\ell)} \right] + \beta_\ell \mathbf{E}_{s'|s} \left[ \frac{d^* \left( \frac{B'}{1 + \pi'}, s' \right)}{1 + \pi'} q^d \left( \frac{B'}{1 + \pi'}, s' \right) \frac{u'(\alpha y^d(y'))}{u'(c_\ell)} \right]$$

where  $q^d$  is the price of a bond in default

## Bond price in default

The price of a bond in default satisfies

$$q^d(B, s) = \beta_\ell \lambda \theta \mathbf{E}_{s'|s} \left[ \frac{1 - d^* \left( \frac{\lambda B}{1 + \pi'}, s' \right)}{1 + \pi'} \times \right. \\ \left. \left( 1 + (1 - \delta) q^* \left( \frac{\lambda B}{1 + \pi'}, s' \right) \right) \frac{u'(c'_\ell)}{u'(\alpha y^d(y))} \right] \\ + \beta_\ell \lambda \mathbf{E}_{s'|s} \left[ \frac{1 - \theta + \theta d^* \left( \frac{B'}{1 + \pi'}, s' \right)}{1 + \pi'} q^d \left( \frac{\lambda B}{1 + \pi'}, s' \right) \frac{u'(\alpha y^d(y'))}{u'(\alpha y^d(y))} \right]$$

## Cyclicity of inflation and borrowing costs

- When  $\delta = 1$  and  $\lambda = 0$ , the bond price can be written as

$$q(B, s, B') = \beta_\ell \mathbf{E}_{s'|s} \left[ \frac{1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right)}{1 + \pi'} \right] \mathbf{E}_{s'|s} \left[ \frac{u'(c'_\ell)}{u'(c_\ell)} \right] \\ + \beta_\ell \mathbf{cov}_s \left[ \frac{1 - d^* \left( \frac{B'}{1 + \pi'}, s' \right)}{1 + \pi'}, \frac{u'(c'_\ell)}{u'(c_\ell)} \right]$$

- Default and inflation **increase** borrowing costs; so does countercyclical default (standard effects)
- Pro-cyclical inflation **reduces** borrowing costs (new channel).

# Key mechanisms

Co-movement of inflation and consumption growth affects

- Interest rates on debt  
significant and uniform across states
- Debt dynamics  
especially during crisis times  
⇒ risky debt and precautionary motives
- Default incentives  
especially during bad times

# Quantitative experiment

- Calibrate the model to match default frequency, mean and volatility of spreads in advanced economies
- Assess impact of different inflation processes on interest rates, debt dynamics, and crises

# Functional forms

- Preferences

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

- Stochastic Process

$$\begin{bmatrix} \log y' \\ \pi' \end{bmatrix} = \begin{bmatrix} \rho_y & \rho_{\pi,y} \\ \rho_{y,\pi} & \rho_\pi \end{bmatrix} \begin{bmatrix} \log y \\ \pi \end{bmatrix} + \begin{bmatrix} \varepsilon_y \\ \varepsilon_\pi \end{bmatrix}$$

where

$$\begin{bmatrix} \varepsilon_y \\ \varepsilon_\pi \end{bmatrix} = N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_y^2 & \sigma_{\pi,y} \\ \sigma_{\pi,y} & \sigma_\pi^2 \end{bmatrix} \right)$$

# Parameters

| Parameters                          | Values               | Targets   |
|-------------------------------------|----------------------|---|
| Gov't discount factor $\beta_h$     | 0.80                 | annual default probability: 1.8 percent*  |
| default penalty $d_0$               | -0.082               | average spread: 2.5 percent   |
| $d_1$                               | 0.095                | st dev of spread: 3.0 percent   |
| Lender discount factor $\beta_\ell$ | 0.99                 | risk-free rate: 1 percent   |
| Risk aversion $\gamma$              | 2                    |   |
| Stochastic process                  |                      | VAR estimates   |
| $\rho_y = \rho_\pi$                 | 0.90                 |   |
| $\rho_{\pi,y} = \rho_{y,\pi}$       | 0.00                 |   |
| $\sigma_y$                          | 0.010                |   |
| $\sigma_\pi$                        | 0.015                |   |
| $\sigma_{\pi,y}$                    | {0, $\pm 0.000034$ } |   |
| Probability of re-entry $\theta$    | 0.10                 | average exclusion: 10 quarters <sup>†</sup>                                     |
| Recovery parameter $\lambda$        | 0.96                 | average recovery rate, discounted to period of default: 50 percent <sup>‡</sup> |
| Relative lender endowment $\alpha$  | 7                    | income of top 43 percent / income of bottom 57 percent (US)                     |
| Measure of lenders $\mu$            | 0.75                 | 0.43/0.57   |
| Debt maturity $\delta$              | {1, 0.04}            | short-term debt and 5-year maturity   |

\*: Reinhart and Rogoff (2009), †: Richmond and Dias (2008), ‡: Benjamin and Wright (2009)

## Model with default

- Borrowing costs are lower with pro-cyclical inflation
- Yet, debt is also lower with pro-cyclical inflation
- So are default rates

Table: Debt and Default

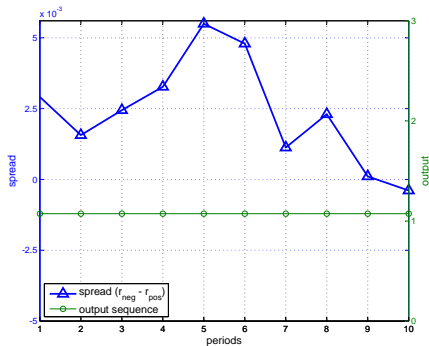
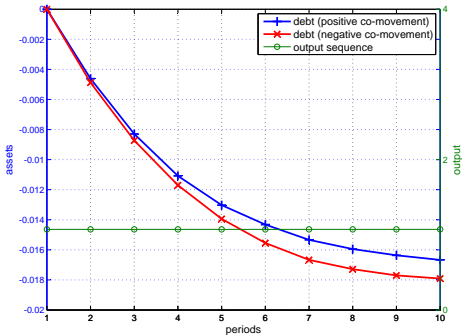
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|                         | Positive<br>co-movement | Negative<br>co-movement |
|-------------------------|-------------------------|-------------------------|
| Default prob. (percent) | 2.15                    | 1.84                    |
| Spreads (percent)       | 0.86                    | 0.91                    |
| Debt (percent)          | 24.41                   | 22.88                   |

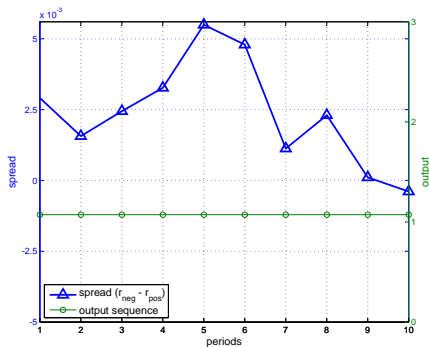
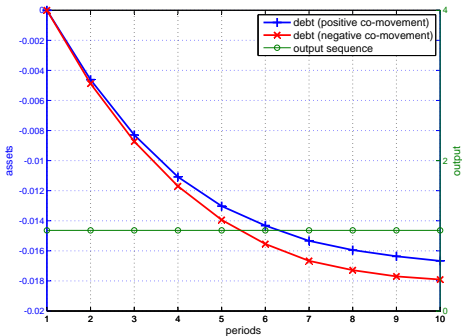
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# Inflation and debt dynamics: precautionary motives

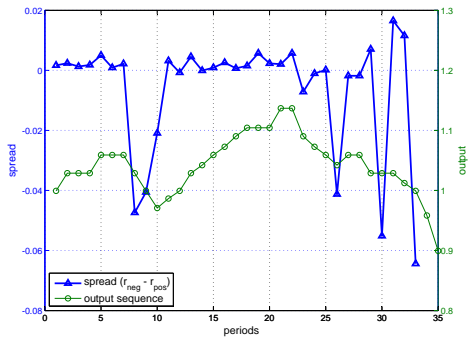
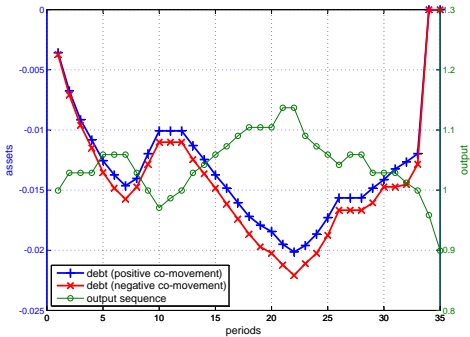


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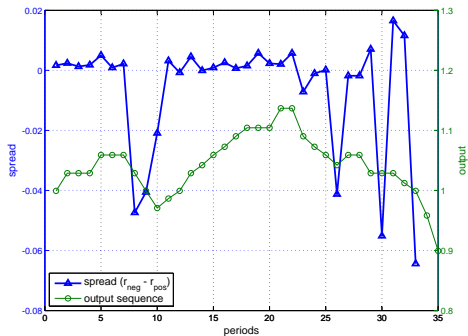
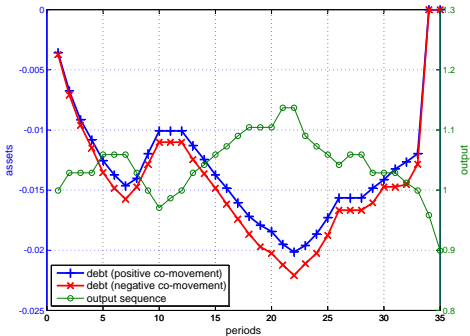


- Precautionary motives from pro-cyclical inflation increase with debt (i.e. as the borrower gets poorer)
- Meanwhile, lenders demand lower yield

# Inflation and debt dynamics

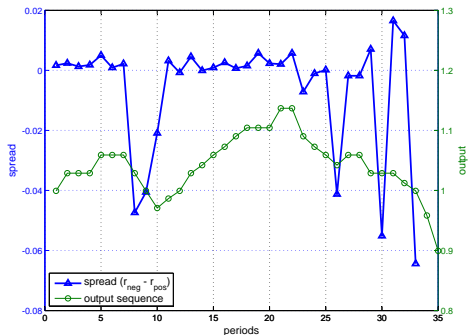
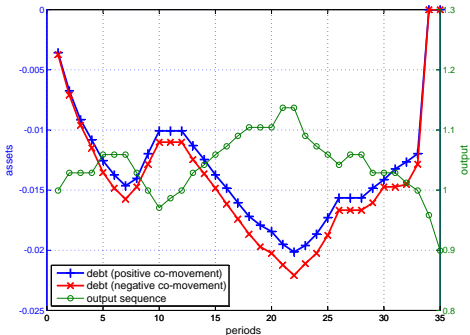


# Inflation and debt dynamics



- On average lower rates and debt with pro-cyclicality
- But more volatile rates: riskier debt precisely in bad times

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- But more volatile rates: riskier debt precisely in bad times

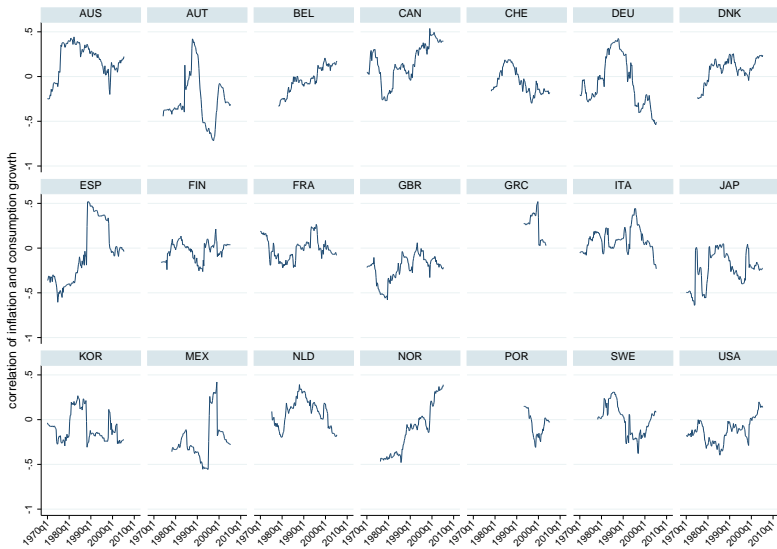
... a tale of periphery EMU accession?

# Conclusion

- Model of sovereign debt with risk averse domestic lenders and borrowers
- Inflation pro-cyclicality can be important in explaining the observed cross section of government debt, interest rates, and debt crises
- Next steps: robustness on default evidence + long term debt

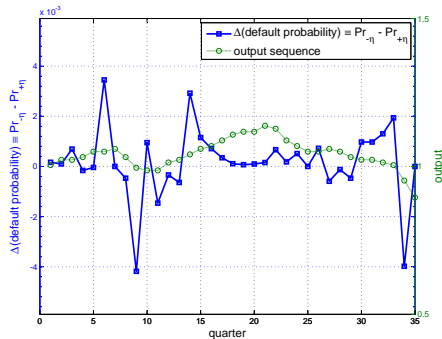
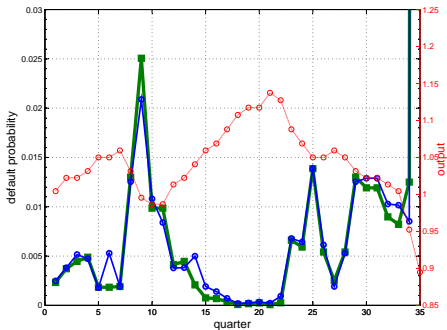
# appendix

# Conditional correlation between inflation and consumption growth





# Default probabilities



Back

## Default costs

- Consider the quadratic default costs

$$h^+(y) = \max \left\{ 0, \underbrace{d_0 d_1 y + (d_1 - d_0 d_1) y^2}_{h(y)} \right\}$$

with  $d_1 > 0$  and  $d_0 < 0$

- The default cost at mean is  $h(1) = d_1$
- Default costs matter when  $h(y) > 0 \Rightarrow y > y^*$  with

$$y^* = \left( 1 - \frac{d_1}{d_0 d_1} \right)^{-1} \sim 1 + \frac{1}{d_0} < 0$$

- So we aim for  $d_1 \sim .10$  and  $\frac{1}{d_0} \sim -.05$