The Implementation of Stabilization Policy

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June 10, 2015

Abstract: In a broad class of locally linearizable dynamic stochastic rational-expectations models, I consider various alternative observation sets for the policy maker, each of them made of the history of some endogenous variables or exogenous shocks until some current or past date. For each observation set, I characterize, within the set of *feasible* paths (paths that can be obtained as *one* local equilibrium under a policy-instrument rule consistent with this observation set), the subset of *implementable* paths (paths that can be obtained, in a minimally robust way, as *the unique* local equilibrium under such a rule). In two applications, I show that, for relevant observation sets, optimal feasible monetary policy may not be implementable in the basic New Keynesian model, even when the number of observed variables largely exceeds the number of unobserved shocks; while debt-stabilizing feasible tax policy is, contrary to conventional wisdom, implementable in the standard real-business-cycle model, even in the presence of policy-implementation lags of any length.

Keywords: stabilization policy, local-equilibrium determinacy, observation set, feasible path, implementable path.

JEL codes: E52, E61.

1 Introduction

Context. It is common practice nowadays to model macroeconomic stabilization policy by a policy-instrument rule ensuring local-equilibrium determinacy, i.e. local-equilibrium existence and uniqueness, in a locally linearizable dynamic stochastic rational-expectations model. The most prominent example is, of course, monetary policy, which is routinely modeled by an interest-rate rule ensuring local-equilibrium determinacy in a dynamic stochastic general-equilibrium (DSGE) model of the monetary transmission mechanism, as advocated by Woodford (2003). Such a rule enables the policy maker to preclude sunspot-driven macroeconomic fluctuations − the kind of fluctuations that, according to Clarida, Galí, and Gertler (2000), may have occurred in the U.S. before 1979. Despite the widespread nature of this practice, however, very little is known about what paths for the economy the policy maker can implement as the unique local equilibrium given her observation set, i.e. given the set of endogenous variables and exogenous shocks that she observes when she sets her policy instrument.[1] Consider, for instance, the path that maximizes some objective function subject to her observation-set constraint. This path is,

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[1] For convenience, throughout the paper I refer to the policy maker with the female pronoun "she."

by construction, consistent with her observation set, in the sense that the policy instrument can be expressed on this path as a function of only elements of that set. But we hardly know anything about the conditions − on the model, the policy instrument, the observation set, and the path itself − under which this path can be obtained as the unique local equilibrium under a policy-instrument rule consistent with that set.

Main Contribution. The main contribution of this paper is essentially to characterize these conditions in a general framework and to apply these general results to two specific contexts of interest. More precisely, in my general analysis, I consider a broad class of models, which includes in particular most existing DSGE models. In this class of models, I consider various alternative observation sets for the policy maker, each of them made of the history of some endogenous variables or exogenous shocks until some current or past date. For each observation set, I say that a local path for the endogenous variables is *feasible* (respectively *implementable*) when there exists a rule expressing the policy instrument as a function of only elements of the observation set and such that this path is *one* (respectively *the unique*, in a minimally robust way) local equilibrium under that rule.[2] The minimal-robustness requirement that I impose for implementability is that the addition of an exogenous policy shock of arbitrarily small variance to the policy-instrument rule in question (capturing, e.g., the policy maker's "trembling hand" or her round-off errors on the elements of the observation set) should still result in a unique local equilibrium, arbitrarily close to the path considered, rather than no local equilibrium at all. The twofold contribution of the paper can then be re-stated as follows. First, I characterize the subset of implementable paths, within the set of feasible paths, for each of these observation sets, in that broad class of models. Second, with two applications, I show that feasible-path (non-)implementability may be an issue in textbook models, for standard policy instruments, relevant observation sets, and interesting feasible paths. One application is about optimal monetary policy in the basic New Keynesian model, the other about debt-stabilizing tax policy in the standard real-business-cycle (RBC) model. The results obtained in these applications have important implications about the usefulness for central banks to track key unobserved exogenous rates of interest, the desirability of using labor-income or income taxes to stabilize public debt, and the choice of a benchmark path in normative analyses of stabilization policy.

General Analysis. Some results previously obtained in the literature can be interpreted as results about feasible-path implementability. Probably the main and best known of them is that no feasible path may be implementable when the policy maker observes only exogenous shocks, as rules expressing the policy instrument as a function of only exogenous shocks may lead to local-equilibrium multiplicity. This result was first obtained by Sargent and Wallace (1975) in the context of an ad hoc model of the monetary transmission mechanism. Today, it typically obtains in DSGE models when the policy instrument is the interest rate. However, the case in which the policy maker observes only exogenous shocks seems very restrictive. In most circumstances, she is more relevantly assumed to observe some − if not most − endogenous variables. Now, it is typically by making the policy instrument react out of equilibrium to endogenous variables that she can ensure local-equilibrium determinacy, as first shown by McCallum (1981). So which feasible paths are implementable when the policy maker observes some endogenous variables? May some feasible paths not be implementable only because they do not satisfy the minimal-robustness requirement for implementability? Are all feasible paths necessarily implementable when the

[2] This terminology should not be confused with the one used in the literature on optimal stabilization policy − by, e.g., Adão, Correia, and Teles (2003), Correia, Nicolini, and Teles (2008), and Correia, Farhi, Nicolini, and Teles (2013). In that literature, a feasible path refers to a path that satisfies the resource constraint(s), while an implementable path refers to a path that can be obtained as a decentralized equilibrium given the available policy instrument(s) (under the implicit assumption that the policy maker observes all current and past exogenous shocks).

number of observed endogenous variables exceeds the number of unobserved exogenous shocks? What if the policy maker observes endogenous variables with some lags, either because she plays before the private sector within each period (as in, e.g., Svensson and Woodford, 2005), or because of macroeconomic-data publication lags (as emphasized by, e.g., McCallum, 1999)? Or, equivalently, if policy-implementation lags compel her to set unconditionally in advance the policy instrument (as in, e.g., Schmitt-Grohé and Uribe, 1997)? These are the questions that I seek to answer in my general analysis.

Consider a broad class of locally linearizable dynamic stochastic discrete-time infinite-horizon rational-expectations models. Start with the benchmark (though admittedly restrictive) case in which the policy maker observes all exogenous shocks, in addition to some endogenous variables.[3] I show that, in this case, all feasible paths are implementable, however long the lag with which the endogenous variables are observed. So, although they put the policy maker behind the curve by preventing her from reacting out of equilibrium to current or recent endogenous variables, observation lags – or, equivalently, policy-implementation lags – are irrelevant for feasible-path implementability in this case. Now turn to the case in which some shocks are unobserved, but all the unobserved shocks can be inferred from the observed variables and shocks using only the structural equations (as, e.g., a productivity shock can be inferred from the observed input and output levels using only the production function). Then, all feasible paths remain implementable, even in the presence of observation or policy-implementation lags of any length. The reason is that using the structural equations to replace, in a policy-instrument rule, these unobserved shocks by functions of observed variables and shocks is neutral for robust local-equilibrium determinacy.

Finally, consider the case in which at least one unobserved shock cannot be inferred from the observed variables and shocks using only the structural equations (as, e.g., a discount-factor shock when the private sector's expectations are not observed). In this last case, there may exist some feasible paths that are not implementable, either because they cannot be obtained as the unique local equilibrium under a policy-instrument rule consistent with the observation set considered, or because they cannot be obtained as such in the minimally robust way required for implementability.[4] This non-implementability result may obtain even when the number of observed variables exceeds, possibly by far, the number of unobserved shocks. In the latter situation, there are many different policy-instrument rules consistent with the observation set and feasible path considered. However, there may exist a "canonical rule" from which all these rules can be derived by two operations that can either preserve or undo, but not restore, robust local-equilibrium determinacy: multiplication by polynomials in the lag operator, and addition to linear combinations, whose coefficients are polynomials in the lag operator, of the structural equations. Therefore, when this canonical rule does not ensure robust local-equilibrium determinacy, none of those rules does, and the feasible path considered is not implementable.

Applications. These general results can be applied to many different models, policy instruments, observation sets, and feasible paths. In this paper, for the sake of brevity, I consider only two applications.

In the first application, I show that optimal feasible monetary policy (a shortcut for the welfare-maximizing feasible path when the policy instrument is the interest rate) may not be implementable in the basic New Keynesian model for a reasonable observation set of the central bank, even when the

[3]Some examples of observable exogenous shocks include exogenous policy measures and foreign developments (considered as exogenous from the point of view of a small open economy), while some examples of unobservable endogenous variables include private agents' expectations and Lagrange multipliers of private agents' optimization problems.

[4]As I illustrate in Subsection 5.2, this non-implementability result obtains independently of whether the unobserved shocks are fundamental or non-fundamental for the observed variables on the feasible path considered, and is therefore largely unrelated to the well known non-identifiability result of Hansen and Sargent (1981, 1991).

number of observed endogenous variables largely exceeds the number of unobserved exogenous shocks. For some values of the structural parameters, the optimal feasible path is not implementable because it cannot be obtained as the unique local equilibrium under an interest-rate rule consistent with the central bank's observation set; and for other values, because it cannot be obtained as such in the minimally robust way required for implementability. This result has two important implications. First, it sounds a note of caution about one of the main lessons of the New Keynesian literature, namely the importance for central banks to track some key unobserved exogenous rates of interest such as, for instance, the counterfactual "natural rate of interest" (as emphasized by, e.g., Galí, 2008, Chapter 8, and Woodford, 2003, Chapter 4). From a normative perspective, the most important of these rates of interest is, ultimately, the exogenous value taken by the interest rate on the optimal feasible path. As my result shows, however, even when this value can be inferred in many alternative ways, on the optimal feasible path, from the variables and shocks observed by the central bank, there may be no way of setting the interest rate as a function of these variables and shocks that implements this path as the robustly unique local equilibrium. In this case, any attempt to track this rate of interest and implement the optimal feasible path will inevitably result in local-equilibrium multiplicity or, in the presence of exogenous policy shocks of arbitrarily small variance, non-existence of a local equilibrium. Second, as an illustration of the possibility that optimal feasible stabilization policy may not be implementable, this result carries a lesson about the choice of a benchmark path in normative analyses. The literature typically considers the optimal feasible path as the benchmark (and typically considers an observation set for the policy maker that makes her observation-set constraint slack in the optimization programme defining this path). However, even for a reasonable observation set of the policy maker, this benchmark may be too demanding, simply because the optimal feasible path may not be implementable. In this case, another natural benchmark to consider is the optimal implementable path or some proxy for this path.

The second application is about debt-stabilizing tax policy in the standard RBC model. Schmitt-Grohé and Uribe (1997) consider, in this model, a labor-income-tax-rate rule and an income-tax-rate rule that stabilize the current stock of public debt (in the absence of policy-implementation lags) or the expected future stock of public debt (in the presence of such lags), and find that these rules lead to local-equilibrium multiplicity for many empirically relevant values of the structural parameters. This finding has largely been interpreted as an argument against the use of labor-income or income taxes to stabilize the current or expected future stock of public debt. I show however that, in the same model, for the same alternative tax instruments, and for a reasonable observation set of the tax authority, all feasible paths along which the current or expected future stock of public debt is stabilized are implementable for all theoretically admissible values of the structural parameters, even in the presence of policy-implementation lags of any length. This result implies that Schmitt-Grohé and Uribe's (1997) finding should be interpreted not as an argument against debt-stabilizing (labor-)income-tax policy *per se*, but instead as an argument against one specific – though natural – way of implementing this policy. This result also illustrates the fact that focusing on specific parametric families of policy-instrument rules may provide a misleading picture of the implementability of a given feasible path.

Methodological Contribution. On the methodological front, for each model, policy instrument, observation set, and implementable path, I show how to design *arithmetically*, i.e. with a finite number of arithmetic operations (addition, subtraction, multiplication, and division), a policy-instrument rule that is consistent with this observation set and implements that path as the robustly unique local equilibrium in that model. More specifically, I show how to use Bézout's identity, the Euclidean division, and Cramer's rule, all of which involve a finite number of arithmetic operations, to transform the polynomials characterizing the structural equations and the implementable path into the polynomials characterizing the policy-instrument rule. This arithmetic-designability property implies that the coefficients of the policy-instrument rule can be explicitly expressed as *rational* functions of the structural and implementable-path parameters, i.e. as fractions of polynomial functions of these parameters. These functions are particularly easy to manipulate analytically. For instance, their derivatives can be easily computed to determine how the coefficients of the policy-instrument rule respond to an arbitrarily small change in the value of the structural or implementable-path parameters.

Discussion. Of course, unless the specific model, observation set, and implementable path considered are particularly simple, the policy-instrument rules that I design are typically more complex than the policy-instrument rules commonly considered in the literature, like Taylor's (1993) popular interest-rate rule, in the sense of involving a larger number of variables and shocks. This greater complexity does not matter at all in the rational-expectations paradigm. However, it might raise concerns about the relevance of this paradigm in that context. Such concerns should not be overstated. Actual decision procedures used by real-world policy makers amount to complex rules involving many inputs, yet this complexity does not seem to raise concerns of that kind. Svensson (2003, 2011) argues that, even within the realm of rational-expectations models, the policy maker's complex reaction function need not be communicated to the private sector. Similarly, it can be argued that all that the private sector needs to know, for such a rule to effectively implement a given path, is the path itself (which the policy maker can communicate through the publication of conditional economic forecasts) and the *existence* of such a rule, but not the rule itself. One of the main contributions of this paper is precisely to identify conditions under which there exists such a rule, and conditions under which there does not.

As in most of the literature on stabilization policy, I focus throughout the paper on local-equilibrium determinacy, that is to say that I abstract from the possible existence of non-local equilibria. In the context of interest-rate rules, as argued by Cochrane (2011), there is usually no solid economic reason to assume away the existence of non-local equilibria. The most common policy proposal to eliminate them, made initially by Christiano and Rostagno (2001) and Benhabib, Schmitt-Grohé, and Uribe (2002), discussed by Woodford (2003, Chapter 2), and used notably by Atkeson, Chari, and Kehoe (2010), consists in switching from an interest-rate rule ensuring local-equilibrium determinacy to a money-growth rule (possibly accompanied by a non-Ricardian fiscal policy) when the economy goes *outside* a specified neighborhood of the steady state considered. The interest-rate rules that I design (when the policy instrument is the interest rate) fit naturally into this proposal, insofar as they are followed by the central bank *inside* the specified neighborhood.

Related Papers. This paper is not the first one to analytically design policy-instrument rules implementing a given local path as the (robustly) unique local equilibrium in a given model. Some papers – e.g., Evans and Honkapohja (2003), Svensson and Woodford (2005), and Woodford (2003, Chapter 7) – do just that. However, these papers consider a specific model (typically a simple New Keynesian model), a specific stabilization-policy instrument (typically the interest rate), and a specific local path (typically the optimal, i.e. welfare-optimizing, local path). Moreover, they do not explicitly require the local path and the policy-instrument rule to be consistent with a given observation set of the policy maker, i.e., they implicitly consider an observation set that is large enough for the corresponding observation-set constraint to be slack in the optimization programme defining the optimal feasible path and for the rule that they design to be consistent with this observation set. My contribution is therefore to generalize their results along four dimensions − in terms of model, policy instrument, observation set, and feasible path − as well as to determine whether the feasible path considered is implementable in the first place. Making this contribution requires to develop a new method of designing policy-instrument rules, as the methods used in this literature can be applied only to simple models and local paths.[5]

Nor is this paper the first one to design, in a general framework, policy specifications that are consistent with a given local path and ensure local-equilibrium determinacy. Indeed, Giannoni and Woodford (2010) design, in a broad class of models, "target criteria" that are consistent with the optimal path (for a given objective function) and ensure local-equilibrium determinacy. However, these criteria are not required to be formulated as a policy-instrument rule consistent with a given observation set of the policy maker.[6] In fact, in many applications, they will not even be formulated as a policy-instrument rule (as they will not involve the current policy instrument).[7] They are not meant to describe how optimal stabilization policy could be *implemented*, but are instead proposed as, in Svensson and Woodford's (2005) terminology, a "higher-level policy specification." In this optimal-policy context, my contribution is therefore to determine whether and how these criteria can be operationally implemented given the policy maker's observation set, i.e. whether and how the policy maker can set the policy instrument as a function of only elements of her observation set to implement, as the robustly unique local equilibrium, the path that is optimal subject to her observation-set constraint − whether this constraint is slack, as implicitly assumed by Giannoni and Woodford (2010), or binding. In other words, I solve the problem, discussed by Ljungqvist and Sargent (2012, Chapter 19, Subsection 19.3.8), of the implementability and implementation of dynamic Stackelberg plans, conditionally on the Stackelberg leader's observation set.

Finally, this paper is also related to Bassetto (2002, 2004, 2005). Indeed, like the latter, it tackles implementation problems in macroeconomics by requiring that the policy maker's out-of-equilibrium behavior be feasible − i.e., in my case, consistent with her observation set. However, there are two important differences. First, this paper addresses a general local-equilibrium-determinacy issue, while Bassetto's papers address some specific global-equilibrium-determinacy issues. Second, the constraints faced out of equilibrium by the policy maker are only informational in this paper, while they may be either informational or physical in Bassetto's papers.

Outline. The rest of the paper is organized as follows. Section 2 defines feasible paths and implementable paths in a broad class of locally linearizable dynamic stochastic rational-expectations models. Section 3 provides, in this class of models, for various alternative observation sets of the policy maker, conditions for feasible paths to be (non-)implementable. Section 4 applies these general results to the (non-)implementability of optimal feasible monetary policy in the basic New Keynesian model and debt-stabilizing feasible tax policy in the standard RBC model. Section 5 further discusses the general results obtained in Section 3 and, in particular, highlights the methodological contribution of the paper. I then conclude and provide a technical appendix.

[5]These methods consist essentially in (i) considering a given parametric family (possibly a singleton) of policy-instrument rules consistent with the local path considered, and (ii) showing that, for each theoretically admissible value of the structural and local-path parameters, there exists, in this family, a rule ensuring (robust) local-equilibrium determinacy. In practice, the computations become analytically intractable as soon as the degree of the characteristic polynomial of the implied system (made of the structural equations and the rule considered) reaches the value four or five.

[6]Similarly, the policy-instrument rules that I design in a general framework in Loisel (2009), which not only are consistent with a given local path and ensure local-equilibrium determinacy, but also eliminate equilibrium trajectories gradually leaving the neighborhood of the steady state considered, are not required to be consistent with a given observation set of the policy maker.

[7]These criteria involve only the past, current, or expected future values of the endogenous variables and exogenous shocks featuring in the objective function considered when this function is purely quadratic.

2 Feasible Paths and Implementable Paths

This section introduces the concepts of feasible path and implementable path in a broad class of dynamic stochastic discrete-time infinite-horizon rational-expectations models. Each model of this class admits at least one steady state in the neighborhood of which its equilibrium conditions are log-linearizable. I restrict the analysis to the neighborhood of this steady state, log-linearize the equilibrium conditions in that neighborhood, and express all endogenous variables and exogenous shocks as log-deviations from their steady-state values. The agents are a policy maker ($\mathcal{PM}$) and a private sector ($\mathcal{PS}$).

2.1 Structural Equations

The behavior of $\mathcal{PS}$ consists in setting, at each date $t \in \mathbb{Z}$, an N-dimension vector of endogenous variables $\mathbf{Y}_t$ according to the following N structural equations:

$$\mathbb{E}_t \left\{ L^{-n} \left[\mathbf{A}(L) \mathbf{Y}_t + \mathbf{B}(L) i_t \right] \right\} + \mathbf{C}(L) \boldsymbol{\xi}_t = \mathbf{0}, \tag{1}$$

where i_t denotes the policy instrument set by $\mathcal{PM}$ at date t and $\boldsymbol{\xi}_t$ a M-dimension vector of exogenous disturbances, with $(M, N) \in \mathbb{N}^{*2}$ and $n \in \mathbb{N}$.[8] These equations are parametrized by $\mathbf{A}(X) \in \mathbb{R}^{N \times N}[X]$, $\mathbf{B}(X) \in \mathbb{R}^{N \times 1}[X]$, and $\mathbf{C}(X) \in \mathbb{R}^{N \times M}[X]$, where, for any $(m_1, m_2) \in \mathbb{N}^{*2}$, $\mathbb{R}^{m_1 \times m_2}[X]$ denotes the set of polynomials in X whose coefficients are $m_1 \times m_2$ matrices with real-number elements. Throughout the paper, $\mathbf{0}$ denotes a vector or a matrix whose elements are all equal to zero (and whose dimension depends on the specific context in which it is used). Moreover, L denotes the lag operator and $\mathbb{E}_t\{.\}$ the rational-expectations operator conditionally on the observation set of $\mathcal{PS}$ when it sets $\mathbf{Y}_t$. I assume for simplicity that this observation set is made of all the previous moves of $\mathcal{PM}$ and $\mathcal{PS}$ as well as all current and past exogenous disturbances, thus abstracting from any observation constraint of $\mathcal{PS}$, in order to focus on the implications of $\mathcal{PM}$'s observation constraints. Finally, each exogenous *disturbance*, i.e. each element of $\boldsymbol{\xi}_t$, follows a stationary VARMA process:

$$\mathbf{D}(L) \boldsymbol{\xi}_t = \mathbf{E}(L) \boldsymbol{\varepsilon}_t, \tag{2}$$

where $\mathbf{D}(X) \in \mathbb{R}^{M \times M}[X]$ is diagonal and such that $|\mathbf{D}(0)| \neq 0$ and all the roots of $|\mathbf{D}(X)|$ lie outside the unit circle of the complex plane; $\mathbf{E}(X) \in \mathbb{R}^{M \times M}[X]$ is diagonal and such that $|\mathbf{E}(X)| \neq 0$; and $\boldsymbol{\varepsilon}_t$ is a M-dimension vector of exogenous *shocks* such that $\mathbb{E}\{\boldsymbol{\varepsilon}_t\} = \mathbf{0}$, $\mathbb{E}\{\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_t\} = \mathbf{I}_M$, and $\mathbb{E}\{\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_{t-k}\} = \mathbf{0}$ for any $k \in \mathbb{Z}^*$. In turn, $|.|$ denotes the determinant operator (when applied to a matrix), $\mathbb{E}\{.\}$ the unconditional-mean operator, and $\mathbf{I}_m$, for any $m \in \mathbb{N}^*$, the $m \times m$ identity matrix.[9]

Two remarks are in order about the general specification (1) for the structural equations. First, this specification explicitly features an arbitrary finite number of lags and expected leads of the endogenous variables $\mathbf{Y}_t$ and i_t set at date t. As is well known, this system could be rewritten in an equivalent reduced form with no lags and only one expected lead, as is typically encountered in the literature (e.g., Svensson, 2011; Giannoni and Woodford, 2002; Woodford, 2003, Chapter 8). I depart from the literature in this respect because I want to distinguish endogenous variables according to the date at which they

[8] Throughout the paper, I use letters in bold to denote vectors and matrices that have (at least potentially) more than one element.

[9] The assumption that $\mathbf{D}(X)$ and $\mathbf{E}(X)$ are diagonal, which is made for convenience, is not restrictive at all. Indeed, if they were not diagonal, then, using Cramer's rule, the stochastic process of $\boldsymbol{\xi}_t$ could be rewritten in an equivalent form of type $|\mathbf{D}(L)|\boldsymbol{\xi}_t = \widetilde{\mathbf{E}}(L)\boldsymbol{\varepsilon}_t$, where $\widetilde{\mathbf{E}}(L) \in \mathbb{R}^{M \times M}[L]$, and the term $\mathbf{C}(L)\boldsymbol{\xi}_t$ in (1) could be replaced by $\widetilde{\mathbf{C}}(L)\widetilde{\boldsymbol{\xi}}_t$, where $\widetilde{\mathbf{C}}(L) \equiv \mathbf{C}(L)\widetilde{\mathbf{E}}(L)$ and $\widetilde{\boldsymbol{\xi}}_t \equiv |\mathbf{D}(L)|^{-1}\boldsymbol{\varepsilon}_t$ (so that $\widetilde{\boldsymbol{\xi}}_t$ follows a process of type (2) with diagonal matrices).

are set, in order to be able to specify explicitly which endogenous variables $\mathcal{PM}$ observes when she sets her policy instrument. This reduced form, which gathers endogenous variables set at different dates within the same vector, would therefore be ill-suited for my purpose.

Second, despite appearances, this general specification allows for the presence of any *past* expectation of $\mathcal{PS}$ in the structural equations, simply because these equations may feature any $\mathbf{Y}_{t-k}$ with $k \in \mathbb{N}^*$ and $\mathbf{Y}_t$ may contain any expectation formed by $\mathcal{PS}$ at date t.[10] Therefore, this specification encompasses models with policy-transmission lags of the kind considered by, e.g., Rotemberg and Woodford (1999), Woodford (2003, Chapter 5), Christiano, Eichenbaum, and Evans (2005), and Svensson and Woodford (2005). In these models, $\mathcal{PS}$ decides unconditionally in advance on (at least some of) its actions, so that the structural equations make (at least some of) its current actions depend on its past expectations.

2.2 Technical Assumptions

I make five non-restrictive assumptions on $\mathbf{A}(X)$ and $\mathbf{B}(X)$. To state these assumptions, let me introduce the following notations. For each $j \in \{1, ..., N\}$, let $\mathbf{u}_j$ denote the N-element vector whose j^{th} element is equal to one and whose other elements are equal to zero. Let $\mathcal{S}_A \equiv \{j \in \{1, ..., N\} | \mathbf{u}_j' \mathbf{A}(X) \neq \mathbf{0}\}$ denote the set of integers j such that the j^{th} structural equation features at least one endogenous variable set by $\mathcal{PS}$, and $\mathcal{S}_B \equiv \{j \in \{1, ..., N\} | \mathbf{u}_j' \mathbf{B}(X) \neq 0\}$ the set of integers j such that the j^{th} structural equation features the policy instrument. Throughout the paper, for any polynomial $\boldsymbol{\mathcal{P}}(X)$ whose coefficients are either real numbers or matrices with real-number elements, let $d_{\boldsymbol{\mathcal{P}}}$ denote the degree of $\boldsymbol{\mathcal{P}}(X)$, and $\boldsymbol{\mathcal{P}}_k$ the coefficient of X^k in $\boldsymbol{\mathcal{P}}(X)$ for each $k \in \{0, ..., d_{\boldsymbol{\mathcal{P}}}\}$. For each $j \in \mathcal{S}_A$, let $n_j^a \equiv n - \min[k \in \{0, ..., d_{\mathbf{A}}\} | \mathbf{u}_j' \mathbf{A}_k \neq \mathbf{0}]$ denote the maximum lead of the endogenous variables set by $\mathcal{PS}$ featuring in the j^{th} structural equation. When $\mathcal{S}_A = \{1, ..., N\}$, let $\widehat{\mathbf{A}}(X) \equiv [\; L^{n_1^a} \mathbf{u}_1 \quad \cdots \quad L^{n_N^a} \mathbf{u}_N \;] X^{-n} \mathbf{A}(X) \in \mathbb{R}^{N \times N}[X]$. Lastly, let $\mathbb{R}[X]$ denote the set of polynomials in X with real-number coefficients, and $\Delta_j(X) \in \mathbb{R}[X]$, for each $j \in \{1, ..., N+1\}$, the determinant of the $N \times N$ matrix obtained by removing, from the $N \times (N+1)$ matrix $X^{\max\{d_{\mathbf{A}}, d_{\mathbf{B}}\}} [\; \mathbf{A}(X^{-1}) \;|\; \mathbf{B}(X^{-1}) \;]$, its j^{th} column. The five assumptions on $\mathbf{A}(X)$ and $\mathbf{B}(X)$ can then be stated as follows:

Assumption 1: $\mathcal{S}_A = \{1, ..., N\}$.

Assumption 2: $\forall j \in \{1, ..., N\}$, $n_j^a \geq 0$.

Assumption 3: $\mathcal{S}_B \neq \varnothing$.

Assumption 4: $|\widehat{\mathbf{A}}(0)| \neq 0$.

Assumption 5: $\forall j \in \{1, ..., N\}$, $\Delta_j(X) \neq 0$.

These assumptions are not restrictive at all. Indeed, any relevant model should satisfy Assumptions 1 and 2 for the structural equations to represent $\mathcal{PS}$'s choice of $\mathbf{Y}_t$, and Assumption 3 for the policy instrument to have an effect on this choice. Then, given Assumptions 1, 2, and 3, Assumption 4 is made without any loss in generality, since any system of type (1) satisfying Assumptions 1 to 3 but not Assumption 4 can straightforwardly be rewritten − using the stochastic process of the exogenous disturbances (2) − in an equivalent form of type (1) either satisfying Assumptions 1 to 4, or not satisfying Assumption 1. Finally, if Assumption 5 were not satisfied, i.e. if there existed $j \in \{1, ..., N\}$ such that $\Delta_j(X) = 0$, then there would exist a linear combination of the structural equations that would involve

[10] To introduce a given expectational variable z_t into $\mathbf{Y}_t$, one simply needs to consider the definition of this variable (of type $z_t \equiv \mathbb{E}_t\{i_{t+k}\}$ or $z_t \equiv \mathbb{E}_t\{\mathbf{u}_j' \mathbf{Y}_{t+k}\}$, where $j \in \{1, ..., N\}$ and $k \in \mathbb{N}^*$) as an additional structural equation.

only elements of $\{\mathbb{E}_t\{\mathbf{u}_j'\mathbf{Y}_{t+k}\}|k \in \mathbb{Z}\}$ and $\{\mathbb{E}_t\{\boldsymbol{\xi}_{t+k}\}|k \in \mathbb{Z}\}$, so that the variable $\mathbf{u}_j'\mathbf{Y}_t$ should then be considered as exogenous, not endogenous.

In addition, I make for convenience the following assumption:

Assumption 6 ("weak Sargent-Wallace property"): *the system of structural equations (1) has at least one stationary solution for $(\mathbf{Y}_t)_{t\in\mathbb{Z}}$ whatever the exogenous stationary process for $(i_t)_{t\in\mathbb{Z}}$.*

This assumption is sufficient, but not necessary for the results that I obtain. I make it nonetheless because it can be stated in a simple and transparent way, unlike weaker assumptions, and because it seems little restrictive anyway, at least in a monetary-policy context.[11] Indeed, it is well known that existing DSGE models of the monetary transmission mechanism typically have what Giannoni and Woodford (2002) and Woodford (2003, Chapter 8) call the "Sargent-Wallace property," after Sargent and Wallace (1975), that is to say that interest-rate rules expressing the interest rate as a function of only exogenous shocks typically lead to local-equilibrium multiplicity in these models. Therefore, these models typically have the weaker Sargent-Wallace property stated in Assumption 6, i.e. they typically satisfy this assumption.

2.3 Policy-Instrument Rule

The behavior of $\mathcal{PM}$ is described by a rule that expresses i_t as a function of the elements of the observation set of $\mathcal{PM}$ when she sets i_t. Let O_t denote this observation set. All the alternative sets O_t that I will consider are included in the set made of the current and past endogenous variables set by $\mathcal{PS}$, the past policy instruments, and the current and past exogenous shocks: $O_t \subseteq \{\mathbf{Y}^t, i^{t-1}, \boldsymbol{\varepsilon}^t\}$, where, for any variable z and any date t, z^t denotes the history of variable z until date t included, i.e. $(z_{t-j})_{j\in\mathbb{N}}$. This focus on subsets of $\{\mathbf{Y}^t, i^{t-1}, \boldsymbol{\varepsilon}^t\}$ is not restrictive since $\mathbf{Y}_t$ may contain any endogenous variable set by $\mathcal{PS}$ at date t, in particular any expectation formed by $\mathcal{PS}$ at date t. I consider the class of rules that express i_t as a time-invariant function of a finite number of elements of O_t and, therefore, of $\{\mathbf{Y}^t, i^{t-1}, \boldsymbol{\varepsilon}^t\}$; i.e., the class of rules of type

$$\mathbf{F}(L)\,\mathbf{Y}_t + G(L)\,i_t + \mathbf{H}(L)\,\boldsymbol{\varepsilon}_t = 0, \tag{3}$$

where $\mathbf{F}(X) \in \mathbb{R}^{1\times N}[X]$, $G(X) \in \mathbb{R}[X]$ is such that $G(0) \neq 0$, and $\mathbf{H}(X) \in \mathbb{R}^{1\times M}[X]$.

2.4 Feasible Paths and Implementable Paths

I consider the class of state-contingent paths for the endogenous variables that can be written as stationary VARMA processes driven by the vector of exogenous shocks $\boldsymbol{\varepsilon}_t$. In other words, I consider the class of paths that can be written in the form

$$\mathbf{S}(L)\begin{bmatrix} \mathbf{Y}_t' & i_t \end{bmatrix}' = \mathbf{T}(L)\,\boldsymbol{\varepsilon}_t, \tag{4}$$

where $\mathbf{S}(X) \in \mathbb{R}^{(N+1)\times(N+1)}[X]$ is such that $|\mathbf{S}(0)| \neq 0$ and all the roots of $|\mathbf{S}(X)|$ lie outside the unit circle, and $\mathbf{T}(X) \in \mathbb{R}^{(N+1)\times M}[X]$. I then define feasible paths and implementable paths as follows:

Definition 1 (Feasible Path): *given an observation set $O_t \subseteq \{\mathbf{Y}^t, i^{t-1}, \boldsymbol{\varepsilon}^t\}$, a path P of type (4) is said to be feasible when there exists a rule R of type (3) (i) expressing i_t as a function of only elements of O_t and (ii) such that P is one stationary solution of the system made of (1) and R.*

[11] Lemma 2 states the weaker but more complex condition that is actually used to establish the results.

Definition 2 (Implementable Path): *given an observation set* $O_t \subseteq \{\mathbf{Y}^t, i^{t-1}, \boldsymbol{\varepsilon}^t\}$*, a path* P *of type (4) is said to be implementable when there exists a rule* R *of type (3) (i) expressing* i_t *as a function of only elements of* O_t*, (ii) such that* P *is the unique stationary solution of the system made of (1) and* R*, and (iii) such that the system made of (1) and the rule obtained by adding an exogenous policy shock of arbitrarily small variance to* R *has a unique stationary solution, which is arbitrarily close to* P*.*

A feasible path is therefore a path that can be obtained as *one* local equilibrium under a policy-instrument rule consistent with $\mathcal{PM}$'s observation set, while an implementable path is a path that can be obtained, in a minimally robust way, as *the unique* local equilibrium under such a rule. An implementable path is, of course, necessarily feasible. But a feasible path may not be implementable. Section 3 of the paper is precisely devoted to the study of feasible-path implementability.

The minimal-robustness requirement for implementability – i.e., Condition (iii) of Definition 2 – is imposed to avoid knife-edge results. The exogenous policy shock of arbitrarily small variance mentioned in this condition can be interpreted as, for instance, the policy maker's "trembling hand" or her round-off errors on the elements of O_t.[12] As I illustrate in the next subsection, the case in which some rules satisfy Conditions (i) and (ii) but not Condition (iii) may naturally arise because of a stochastic singularity.

2.5 An Illustration

This subsection illustrates, in the simplest possible setup (with $N = M = 1$), two different ways in which a feasible path may not be implementable. Consider a log-linearized model in which at each date $t \in \mathbb{Z}$, $\mathcal{PS}$ sets a single endogenous variable, y_t, according to the following structural equation:

$$y_t = \alpha \mathbb{E}_t \{y_{t+1}\} + i_t + \xi_t, \tag{5}$$

with $\xi_t = \varepsilon_t + \beta \varepsilon_{t-1}$, where $\alpha \in \mathbb{R}^*$, $\beta \in \mathbb{R}$, and ε_t is an i.i.d. exogenous shock realized at date t. Assume that the observation set of $\mathcal{PM}$ when she sets i_t is $O_t = \{y^t, i^{t-1}\}$. For any $\theta \in \mathbb{R}^*$, consider the path $[\begin{array}{cc} y_t & i_t \end{array}]' = [\begin{array}{cc} \theta\varepsilon_t & (\theta - 1)\varepsilon_t - \beta\varepsilon_{t-1} \end{array}]'$, which satisfies (5). On this path, noted P_θ, i_t can be expressed as a function of only elements of O_t:

$$i_t = (\theta - 1)\theta^{-1} y_t - \beta\theta^{-1} y_{t-1}, \tag{6}$$

so that P_θ is feasible. To study the implementability of P_θ, let me first note that all the rules of type (3) consistent with O_t and P_θ can be written in a form of type

$$\mathcal{P}(L) \left[i_t - (\theta - 1)\theta^{-1} y_t + \beta\theta^{-1} y_{t-1} \right] = 0 \tag{7}$$

with $\mathcal{P}(X) \in \mathbb{R}[X]$ and $\mathcal{P}(0) \neq 0$. Since (6) implies (7), any local equilibrium under rule (6) is also a local equilibrium under any rule of type (7). Now, under rule (6), the dynamics of y_t is governed by the following equation:

$$\mathbb{E}_t \{y_{t+1} - \theta\varepsilon_{t+1}\} - (\alpha\theta)^{-1}(y_t - \theta\varepsilon_t) - \beta(\alpha\theta)^{-1}(y_{t-1} - \theta\varepsilon_{t-1}) = 0, \tag{8}$$

obtained by replacing i_t in (5) by the right-hand side of (6). Solving (8) for y_t requires to distinguish between three alternative cases regarding the values of the structural parameters α and β and the feasible-path parameter θ.

[12] I abstract from round-off errors on the structural parameters because, unlike round-off errors on variables and shocks, they do not raise any discontinuity problem of the kind described in the next subsection.

First, α, β, and θ may be such that both roots of the polynomial $\mathcal{Q}(X) \equiv X^2 - (\alpha\theta)^{-1}X - \beta(\alpha\theta)^{-1}$ lie *inside* the unit circle of the complex plane. In this case, there is an infinity of local equilibria under rule (6), as follows from Blanchard and Kahn's (1980) analysis, and therefore an infinity of local equilibria under any rule of type (7). As a consequence, P_θ cannot be obtained as the unique local equilibrium under a policy-instrument rule consistent with O_t, so that it is not implementable.

Second, α, β, and θ may be such that both roots of $\mathcal{Q}(X)$ lie *outside* the unit circle. In that case, the system made of (5) and (6) does not meet Blanchard and Kahn's (1980) order condition, but it has nonetheless a unique stationary solution, i.e. there is a unique local equilibrium under rule (6), which is P_θ. Given the absence of initial conditions, this local-equilibrium-determinacy result is due to a stochastic singularity: the equation (8) is expressed in terms of the variable $y_t - \theta\varepsilon_t$ without any exogenous residual term. This result is, however, not robust to the introduction of an exogenous policy shock e_t of arbitrarily small variance. Indeed, if rule (6) is replaced by $i_t = (\theta-1)\theta^{-1}y_t - \beta\theta^{-1}y_{t-1} + e_t$, then the equation (8) becomes $\mathbb{E}_t\{y_{t+1} - \theta\varepsilon_{t+1}\} - (\alpha\theta)^{-1}(y_t - \theta\varepsilon_t) - \beta(\alpha\theta)^{-1}(y_{t-1} - \theta\varepsilon_{t-1}) + \alpha^{-1}e_t = 0$, so that the stochastic singularity disappears, and there is no local equilibrium. Similarly, for any $\mathcal{P}(X) \in \mathbb{R}[X]$ such that $\mathcal{P}(0) \neq 0$, there is no local equilibrium if the policy maker follows the rule $\mathcal{P}(L)[i_t - (\theta-1)\theta^{-1}y_t + \beta\theta^{-1}y_{t-1}] = e_t$, instead of rule (7). Therefore, P_θ cannot be obtained, in the minimally robust way described in Condition (iii) of Definition 2, as the unique local equilibrium under a policy-instrument rule consistent with O_t, so that it is not implementable.

Third, α, β, and θ may be such that one root of $\mathcal{Q}(X)$ lies inside the unit circle, and the other outside. In this last case, it is straightforward to check that P_θ can be obtained, in the minimally robust way just mentioned, as the unique local equilibrium under a policy-instrument rule consistent with O_t, so that it is implementable.

3 The (Non-)Implementability of Feasible Paths

This section derives conditions for feasible-path (non-)implementability in the class of models presented in the previous section. I first describe the class of alternative observation sets that I consider for $\mathcal{PM}$, and then examine three different cases in turn, depending on whether there are some unobserved shocks, and, if there are some, whether they can be inferred from the observation set using only (1) and (2).

3.1 Class of $\mathcal{PM}$'s Observation Sets

I consider the class of alternative observation sets O_t of type

$$O_t = \{\mathbf{u}_j'\mathbf{Y}^{t-\ell_Y}, i^{t-1}, \mathbf{v}_k'\boldsymbol{\varepsilon}^{t-\ell_\varepsilon} | j \in \mathcal{J}, k \in \mathcal{K}\}, \tag{9}$$

where $(\ell_Y, \ell_\varepsilon) \in \mathbb{N}^2$, $\mathcal{J}$ and $\mathcal{K}$ are two sets such that $\mathcal{J}^* \subseteq \mathcal{J} \subseteq \{1, ..., N\}$ and $\mathcal{K} \subseteq \{1, ..., M\}$, and, for each $k \in \{1, ..., M\}$, $\mathbf{v}_k$ denotes the M-element vector whose k^{th} element is equal to one and whose other elements are equal to zero. In turn, $\mathcal{J}^* \equiv \{j \in \{1, ..., N\} | n - \min[k \in \{0, ..., d_\mathbf{A}\} | \mathbf{A}_k\mathbf{u}_j \neq \mathbf{0}] \geq 1\}$ denotes the set of integers j such that the structural equations (1) feature a term of type $\mathbb{E}_t\{\mathbf{u}_j'\mathbf{Y}_{t+k}\}$ with $k \in \mathbb{N}^*$.

Four main remarks are in order about this class of observation sets. First, it allows for the possibility that some endogenous variables are never observed by $\mathcal{PM}$ (when $\mathcal{J} \subsetneq \{1, ..., N\}$). These unobserved variables may be, for instance, Lagrange multipliers of $\mathcal{PS}$'s optimization problems, expectations of $\mathcal{PS}$

(when $\mathbf{Y}_t$ includes some expectations of $\mathcal{PS}$, as in, e.g., models with policy-transmission lags of the kind briefly discussed in Subsection 2.1), or variables about which $\mathcal{PM}$ receives a noisy signal. Second, the assumption $\mathcal{J}^* \subseteq \mathcal{J}$ is made for convenience, essentially to overcome difficulties raised by Blanchard and Kahn's (1980) rank condition. This assumption is typically not restrictive when the unobserved variables are expectations of $\mathcal{PS}$ or variables about which $\mathcal{PM}$ receives a contemporaneous noisy signal. Indeed, the structural equations can then typically be written in a form of type (1) satisfying Assumptions 1 to 6 and not featuring any expected future value of such variables. Third, this class of observation sets allows for the possibility that some exogenous shocks are observed by $\mathcal{PM}$ (when $\mathcal{K} \neq \varnothing$). In practice, most kinds of shocks, like, e.g., preference shocks, productivity shocks, and measurement errors, cannot reasonably be assumed to be (directly) observed by $\mathcal{PM}$, but certain kinds of shocks can, like exogenous policy measures or foreign developments (considered as exogenous from the point of view of a small open economy).

Fourth, this class of observation sets allows for the absence (when $\ell_Y = \ell_\varepsilon = 0$) or the presence (when $\ell_Y \geq 1$ or $\ell_\varepsilon \geq 1$) of observation lags.[13] This flexibility enables me to capture a variety of alternative assumptions explicitly or implicitly made in the literature. Consider, for instance, monetary policy. Many interest-rate rules encountered in DSGE models of the monetary transmission mechanism are extensions of Taylor's (1993) popular rule that require the interest rate to react out of equilibrium to current endogenous variables, and therefore implicitly require the central bank to observe the value taken by these current endogenous variables when setting the interest rate. However, many studies addressing the issue of monetary-policy implementation or a closely related issue, from Poole (1970) and Sargent and Wallace (1975) to King and Wolman (2004), Svensson and Woodford (2005), and Atkeson, Chari, and Kehoe (2010), explicitly assume on the contrary that the central bank plays before the private sector at each date and therefore does not observe current endogenous variables when setting the interest rate.[14] I capture these two alternative possibilities by allowing for $\ell_Y = 0$ and for $\ell_Y = 1$. The presence of longer observation lags on endogenous variables (when $\ell_Y \geq 2$) can be justified by the existence of macroeconomic-data-publication lags (as emphasized by, e.g., McCallum, 1999, and Orphanides, 2001).[15] Moreover, these observation lags may apply not only to endogenous variables, but also to exogenous shocks, for instance to exogenous foreign macroeconomic developments.

3.2 Case 1: Observed Shocks

I start with the benchmark case in which (i) $\mathcal{PM}$ observes all exogenous shocks, i.e. $\mathcal{K} = \{1, ..., M\}$, and (ii) observation lags on shocks are not strictly longer than observation lags on variables, i.e. $\ell_\varepsilon \leq \ell_Y$. Condition (i) makes this case admittedly restrictive, but the results obtained will also serve, in the next subsection, as a useful starting point to investigate an alternative case in which not all exogenous shocks are observed. Condition (ii) is imposed mostly for convenience and could be relaxed at the cost of greater complexity. In this benchmark case, the rules of type (3) that are consistent with O_t are those

[13] Observation lags of length $\ell_\varepsilon = \ell_Y \equiv \ell$ are, of course, equivalent to policy-implementation lags of length ℓ, which compel $\mathcal{PM}$ to choose unconditionally the value of her policy instrument ℓ periods in advance (as in, e.g., Schmitt-Grohé and Uribe, 1997).

[14] This alternative within-period-timing assumption is arguably better suited to capture the fact that, due to information-collecting, information-processing, and decision-making frictions, the policy maker takes her decisions at a lower frequency than the private sector considered as a whole (though not necessarily than each individual private agent).

[15] Some variables, most notably some financial-market prices, can clearly be observed with negligible lags by the policy maker. However, as argued by Leeper, Sims and Zha (1996, p. 40) and Rotemberg and Woodford (1999, pp. 93-94), the policy maker may still react with non-negligible lags to these variables because of data-processing and decision-making frictions.

that satisfy

$$\text{if } \mathcal{J} \subsetneq \{1, ..., N\}, \quad \text{then} \quad \forall j \in \{1, ..., N\} \setminus \mathcal{J}, \mathbf{F}(X)\mathbf{u}_j = 0, \tag{10}$$

$$\text{if } \ell_Y \geq 1, \quad \text{then} \quad \forall k \in \{0, ..., \ell_Y - 1\} \cap \{0, ..., d_{\mathbf{F}}\}, \mathbf{F}_k = \mathbf{0}, \tag{11}$$

$$\text{and if } \ell_\varepsilon \geq 1, \quad \text{then} \quad \forall k \in \{0, ..., \ell_\varepsilon - 1\} \cap \{0, ..., d_{\mathbf{H}}\}, \mathbf{H}_k = \mathbf{0}. \tag{12}$$

To study feasible-path implementability in this case, let me first establish two useful preliminary results. To that aim, let me note $n_j^b \equiv n - \min[k \in \{0, ..., d_{\mathbf{B}}\} | \mathbf{u}_j' \mathbf{B}_k \neq 0]$, for each $j \in \mathcal{S}_B$, the maximum lead of the policy instrument featuring in the j^{th} structural equation, and let me call "eigenvalues" of a rational-expectations system that can be written in Blanchard and Kahn's (1980) form the eigenvalues of the matrix that characterizes the deterministic part of this form.[16] The first preliminary result can be stated as follows:

Lemma 1: *for any rule of type (3) such that*

$$\text{if } \max_{j \in \mathcal{S}_B} \left(n_j^b - n_j^a\right) \geq 0, \text{ then } \forall k \in \left\{0, ..., \max_{j \in \mathcal{S}_B} \left(n_j^b - n_j^a\right)\right\} \cap \{0, ..., d_{\mathbf{F}}\}, \mathbf{F}_k = \mathbf{0}, \tag{13}$$

the system made of the structural equations (1) and this rule can be written in Blanchard and Kahn's (1980) form with $n^a \equiv \sum_{j=1}^N n_j^a$ *non-predetermined variables, and its non-zero eigenvalues are (and have the same multiplicity as) the non-zero roots of the polynomial*

$$X^{\max(d_{\mathbf{F}}, d_G)} \left[\sum_{j=1}^N (-1)^{N+1-j} \mathbf{F}\left(X^{-1}\right) \mathbf{u}_j \Delta_j(X) + G\left(X^{-1}\right) \Delta_{N+1}(X)\right]. \tag{14}$$

Proof: see Appendix A.1. ■

In short, Appendix A.1 essentially uses (i) Assumption 4 and Condition (13) to show that the system mentioned in Lemma 1 has exactly n^a non-predetermined variables, and (ii) a standard result in time-series analysis (see, e.g., Hamilton, 1994, Chapter 10, Proposition 10.1) to show that the non-zero eigenvalues of this system are the non-zero roots of (14).[17] Now, for any set $\mathcal{J} \subseteq \{1, ..., N\}$, let $\mathcal{D}_{\mathcal{J}}(X) \in \mathbb{R}[X]$ denote the greatest common divisor, defined up to a non-zero real-number multiplicative scalar, of all the polynomials $\Delta_j(X)$ for $j \in \mathcal{J} \cup \{N+1\}$ (none of which is zero, given Assumptions 4 and 5). The second preliminary result is the following:

Lemma 2: *for any set* $\mathcal{J} \subseteq \{1, ..., N\}$, $\mathcal{D}_{\mathcal{J}}(X)$ *has at most* n^a *roots outside the unit circle (taking into account their multiplicity).*

Proof: see Appendix A.2. ■

[16] In other words, the eigenvalues of a rational-expectations system that can be written in a form of type $\mathbb{E}_t\{\mathbf{X}_{t+1}\} = \mathbf{M}\mathbf{X}_t + \boldsymbol{\eta}_t$, where $\mathbf{X}_t$ is a vector of endogenous variables set at date t or earlier and $\boldsymbol{\eta}_t$ a vector of stochastic exogenous terms realized at date t or earlier, are the eigenvalues of the matrix $\mathbf{M}$. In particular, the eigenvalues of a path of type (4) are the roots of $|X^{d_{\mathbf{S}}}\mathbf{S}(X^{-1})|$, as follows from a standard result in time-series analysis (see, e.g., Hamilton, 1994, Chapter 10, Proposition 10.1).

[17] Many DSGE models are such that $\max\{n_j^b - n_j^a | j \in \mathcal{S}_B\} < 0$ and therefore such that all rules of type (3) meet Condition (13). However, DSGE models with policy-transmission lags of the kind briefly discussed in Subsection 2.1 are often such that $\max\{n_j^b - n_j^a | j \in \mathcal{S}_B\} \geq 0$, because their system of structural equations, when written in a form of type (1) satisfying Assumptions 1 to 6, often includes an equation of type $z_t \equiv \mathbb{E}_t\{i_{t+d}\}$ (and other equations involving z_{t-d}), where $d \in \mathbb{N}^*$ denotes the length of the policy-transmission lag.

Appendix A.2 simply uses Assumption 6 and Lemma 1, together with Blanchard and Kahn's (1980) order condition, to prove Lemma 2. These two preliminary results, Lemmas 1 and 2, enable me to establish the following proposition, which states that all feasible paths are implementable in this benchmark case:

Proposition 1 (Feasible-Path Implementability in Case 1): *for any set $\mathcal{J}$ such that $\mathcal{J}^* \subseteq \mathcal{J} \subseteq \{1, ..., N\}$ and any $(\ell_Y, \ell_\varepsilon) \in \mathbb{N}^2$ such that $\ell_\varepsilon \leq \ell_Y$, all feasible paths are implementable when $O_t = \{\mathbf{u}_j' \mathbf{Y}^{t-\ell_Y}, i^{t-1}, \boldsymbol{\varepsilon}^{t-\ell_\varepsilon} | j \in \mathcal{J}\}$.*

Proof: see Appendix A.3. ■

Appendix A.3 proves Proposition 1 by designing, for each observation set and feasible path, a policy-instrument rule that is consistent with this observation set and implements that path as the robustly unique local equilibrium. It derives the coefficients of this rule from the structural and feasible-path parameters, i.e. $\mathbf{F}(X)$, $G(X)$, and $\mathbf{H}(X)$ from $\mathbf{A}(X)$, $\mathbf{B}(X)$, $\mathbf{C}(X)$, $\mathbf{D}(X)$, $\mathbf{E}(X)$, $\mathbf{S}(X)$, and $\mathbf{T}(X)$, using a new method that rests on Bézout's identity, the Euclidean division, and Cramer's rule. More specifically, it first uses Bézout's identity to design $(\mathcal{U}_j(X))_{i \in \mathcal{J} \cup \{N+1\}} \in \mathbb{R}[X]^{|\mathcal{J}|+1}$ such that $\sum_{j \in \mathcal{J} \cup \{N+1\}} \mathcal{U}_j(X)$ $\Delta_j(X) = \mathcal{D}_{\mathcal{J}}(X)$, where $|.|$ denotes the cardinality operator (when applied to a set).[18] It then uses the Euclidian division and Lemma 2 to design, from $(\mathcal{U}_j(X))_{j \in \mathcal{J} \cup \{N+1\}}$, some $\mathbf{F}(X)$ and $G(X)$ such that (i) $\mathbf{F}(X)$ satisfies (10), (11), and (13), (ii) (14) has exactly n^a roots outside the unit circle (taking into account their multiplicity), and (iii) the roots of (14) inside the unit circle are the eigenvalues of the feasible path considered. At this stage, given Lemma 1, any rule of type (3) with these $\mathbf{F}(X)$ and $G(X)$ is such that the system made of the structural equations (1) and this rule has (i) a unique stationary solution (even when an exogenous policy shock is added to this rule), and (ii) the same eigenvalues inside the unit circle as the feasible path considered. Finally, Appendix A.3 uses Cramer's rule to residually design some $\mathbf{H}(X)$ satisfying (12) and such that the feasible path considered is one, and hence the unique, stationary solution of this system. The fact that this system has the same eigenvalues inside the unit circle as the feasible path considered ensures that this $\mathbf{H}(X)$ is a polynomial, i.e. a power series of *finite* degree.

Interestingly, Proposition 1 implies that observation lags − or, equivalently, policy-implementation lags − are irrelevant for feasible-path implementability in Case 1. This irrelevance result may be viewed as surprising. Indeed, most of the policy-instrument rules considered in the literature, at least in the context of DSGE models of the monetary transmission mechanism, manage to ensure (robust) local-equilibrium determinacy by requiring the policy instrument to react out of equilibrium to *current* endogenous variables (as already mentioned in the previous subsection). If anything, the literature suggests that, by preventing $\mathcal{PM}$ from reacting out of equilibrium to current or even recent endogenous variables, observation lags reduce her ability to ensure local-equilibrium determinacy.[19] I show however that, in fact, they do not − provided that $\mathcal{PM}$'s choice is not arbitrarily restricted to a specific parametric family of policy-instrument rules. In essence, the reason is that, in models that raise non-trivial local-equilibrium-indeterminacy issues, $\mathcal{PS}$'s current actions depend directly on its expectation of its future

[18] Bézout's identity is named after Bézout (1767), who extended to polynomials a result first obtained for integers by Bachet de Méziriac (1624, Proposition 18). It is sometimes unnamed and presented as a corollary of the Euclidean algorithm (as in, e.g., Prasolov, 2004, Chapter 2, Theorem 2.1.1).

[19] For instance, Benhabib (2004, Subsection 3.4) considers, in the context of a specific model, a parametric family of simple interest-rate rules that are consistent with observation lags of length $\ell \in \mathbb{N}^*$. Holding constant the values of the model's other parameters, he finds numerically that local-equilibrium multiplicity arises for sufficiently large values of ℓ.

actions and, therefore, indirectly on its expectation of the future policy instrument. Now, by imposing an out-of-equilibrium reaction of the current policy instrument to $\mathcal{PS}$'s past actions, a rule also imposes an out-of-equilibrium reaction of $\mathcal{PS}$'s expectation of the future policy instrument to its current actions. The latter reaction can be viewed as the feedback mechanism that ensures (robust) local-equilibrium determinacy.

3.3 Case 2: Unobserved but Inferable Shocks

I now turn to the case in which (i) $\mathcal{PM}$ does not observe all exogenous shocks, i.e. $\mathcal{K} \subsetneq \{1, ..., M\}$, (ii) $\ell_\varepsilon = \ell_Y \equiv \ell$, and (iii) the unobserved shocks $\{\mathbf{v}_k' \boldsymbol{\varepsilon}^{t-\ell} | k \in \{1, ..., M\} \setminus \mathcal{K}\}$ can be inferred from $O_t = \{\mathbf{u}_j' \mathbf{Y}^{t-\ell}, i^{t-1}, \mathbf{v}_k' \boldsymbol{\varepsilon}^{t-\ell} | j \in \mathcal{J}, k \in \mathcal{K}\}$ using *only* the structural equations (1) and the stochastic process (2), i.e., without using the feasible path considered.[20] By the latter condition, I mean more specifically that (1) and (2) imply a relationship of the form

$$\boldsymbol{\Gamma}(L)\boldsymbol{\varepsilon}_t = \boldsymbol{\Lambda}(L) \left[\begin{array}{ccc} \mathbf{Y}_t' \left[\begin{array}{ccc} \mathbb{1}_{1 \in \mathcal{J}} \mathbf{u}_1 & \dots & \mathbb{1}_{N \in \mathcal{J}} \mathbf{u}_N \end{array} \right] & i_{t-1} & \boldsymbol{\varepsilon}_t' \left[\begin{array}{ccc} \mathbb{1}_{1 \in \mathcal{K}} \mathbf{v}_1 & \dots & \mathbb{1}_{M \in \mathcal{K}} \mathbf{v}_M \end{array} \right] \end{array} \right]', \tag{15}$$

where $\boldsymbol{\Gamma}(X) \in \mathbb{R}^{M \times M}[X]$ is such that $|\boldsymbol{\Gamma}(0)| \neq 0$ and all the roots of $|\boldsymbol{\Gamma}(X)|$ lie outside the unit circle, $\boldsymbol{\Lambda}(X) \in \mathbb{R}^{M \times (N+1+M)}[X]$, and, for any statement s, $\mathbb{1}_s \equiv 1$ if s is true and $\mathbb{1}_s \equiv 0$ if s is wrong.

For any path P that would be feasible if the observation set were $O_t = \{\mathbf{u}_j' \mathbf{Y}^{t-\ell}, i^{t-1}, \boldsymbol{\varepsilon}^{t-\ell} | j \in \mathcal{J}\}$, Proposition 1 can be used to establish the existence of a rule R of type (3) satisfying (10), (11), and (12) with $\ell_\varepsilon = \ell_Y$, consistent with P, and robustly ensuring local-equilibrium determinacy. Write R as $\mathbf{F}_R(L)\mathbf{Y}_t + G_R(L)i_t + \mathbf{H}_R(L)\boldsymbol{\varepsilon}_t = 0$ and multiply its left- and right-hand sides by $|\boldsymbol{\Gamma}(L)|$ to get the policy-instrument rule $|\boldsymbol{\Gamma}(L)|[\mathbf{F}_R(L)\mathbf{Y}_t + G_R(L)i_t + \mathbf{H}_R(L)\boldsymbol{\varepsilon}_t] = 0$, which I note R'. Since $|\boldsymbol{\Gamma}(X)|$ is a scalar polynomial, one can replace $|\boldsymbol{\Gamma}(L)|\mathbf{H}_R(L)\boldsymbol{\varepsilon}_t$ in R' by $\mathbf{H}_R(L)|\boldsymbol{\Gamma}(L)|\boldsymbol{\varepsilon}_t$ and then use (15) − rewritten in terms of $|\boldsymbol{\Gamma}(L)|\boldsymbol{\varepsilon}_t$ using Cramer's rule − to get a rule R'' of type (3) satisfying (10), (11), and (12) with $\ell_\varepsilon = \ell_Y$, such that $\mathbf{H}(X)\mathbf{v}_k = 0$ for each $k \in \{1, ..., M\} \setminus \mathcal{K}$, and consistent with P. This transformation of R into R'' shows that P is feasible also when $O_t = \{\mathbf{u}_j' \mathbf{Y}^{t-\ell}, i^{t-1}, \mathbf{v}_k' \boldsymbol{\varepsilon}^{t-\ell} | j \in \mathcal{J}, k \in \mathcal{K}\}$. Moreover, this transformation is neutral for local-equilibrium determinacy. Indeed, the system made of (1), (2), and R'' is equivalent to, and hence has the same stationary solution(s) for $(\mathbf{Y}_t, i_t)_{t \in \mathbb{Z}}$ as, the system made of (1), (2), and R'. In turn, the latter system has the same stationary solution(s) for $(\mathbf{Y}_t, i_t)_{t \in \mathbb{Z}}$ as the system made of (1), (2), and R, since whatever $\boldsymbol{\Gamma}(X)$, R' has a unique stationary solution for $\mathbf{F}_R(L)\mathbf{Y}_t + G_R(L)i_t + \mathbf{H}_R(L)\boldsymbol{\varepsilon}_t$, namely the solution $\mathbf{F}_R(L)\mathbf{Y}_t + G_R(L)i_t + \mathbf{H}_R(L)\boldsymbol{\varepsilon}_t = 0$, which corresponds to R. Given that P is the unique local equilibrium under R, it is also the unique local equilibrium under R' and under R''. Finally, since adding an exogenous policy shock to R is neutral for local-equilibrium determinacy and all the roots of $|\boldsymbol{\Gamma}(X)|$ lie outside the unit circle, adding an exogenous policy shock to R'' is also neutral for local-equilibrium determinacy, so that P is also implementable when $O_t = \{\mathbf{u}_j' \mathbf{Y}^{t-\ell}, i^{t-1}, \mathbf{v}_k' \boldsymbol{\varepsilon}^{t-\ell} | j \in \mathcal{J}, k \in \mathcal{K}\}$. As a consequence, I obtain the following result:

Proposition 2 (Feasible-Path Implementability in Case 2): *for any set $\mathcal{J}$ such that $\mathcal{J}^* \subseteq \mathcal{J} \subseteq \{1, ..., N\}$, any set $\mathcal{K} \subsetneq \{1, ..., M\}$, and any $\ell \in \mathbb{N}$, if (1) and (2) imply a relationship of type (15), then, when $O_t = \{\mathbf{u}_j' \mathbf{Y}^{t-\ell}, i^{t-1}, \mathbf{v}_k' \boldsymbol{\varepsilon}^{t-\ell} | j \in \mathcal{J}, k \in \mathcal{K}\}$, (i) the set of feasible paths is the same as when $O_t = \{\mathbf{u}_j' \mathbf{Y}^{t-\ell}, i^{t-1}, \boldsymbol{\varepsilon}^{t-\ell} | j \in \mathcal{J}\}$, and (ii) all feasible paths are implementable.*

[20] When $\ell_\varepsilon \leq \ell_Y$ (as in Case 1), the condition $\ell_\varepsilon = \ell_Y$ is necessary for the unobserved shocks $\{\mathbf{v}_k' \boldsymbol{\varepsilon}^{t-\ell_\varepsilon} | k \in \{1, ..., M\} \setminus \mathcal{K}\}$ to be inferable from $O_t = \{\mathbf{u}_j' \mathbf{Y}^{t-\ell_Y}, i^{t-1}, \mathbf{v}_k' \boldsymbol{\varepsilon}^{t-\ell_\varepsilon} | j \in \mathcal{J}, k \in \mathcal{K}\}$, because $\mathbf{Y}^{t-\ell_Y}$ cannot depend on $\boldsymbol{\varepsilon}_{t-\ell_\varepsilon}$ when $\ell_\varepsilon < \ell_Y$.

Two remarks are worth being made about Case 2 and Proposition 2. First, Proposition 2 implies that observation and policy-implementation lags do not matter for feasible-path implementability in Case 2 – like in Case 1 and naturally so, given how the proof of Proposition 2 builds on Proposition 1. Second, as apparent from (15) (given that $|\mathbf{\Gamma}(0)| \neq 0$), Case 2 requires that the number of unobserved shocks be lower than or equal to the number of observed variables set by $\mathcal{PS}$ ($M - |\mathcal{K}| \leq |\mathcal{J}|$), or, equivalently, strictly lower than the total number of observed variables including the policy instrument ($M - |\mathcal{K}| < |\mathcal{J}| + 1$). With this requirement in mind, Proposition 2 might not come that much as a surprise. Indeed, when $M - |\mathcal{K}| < |\mathcal{J}| + 1$, there are some degrees of freedom in the choice of a policy-instrument rule consistent with the observation set O_t and the feasible path considered, given that O_t includes, for each past date until $t - \max(1, \ell)$, $|\mathcal{J}| + 1$ variables whose values on this path depend on only $M - |\mathcal{K}|$ unobserved shocks. Isn't it always possible to exploit these degrees of freedom to find a policy-instrument rule consistent with the observation set and feasible path considered and robustly ensuring local-equilibrium determinacy, thus establishing the implementability of this path? The next subsection provides a negative answer to this question.

3.4 Case 3: Unobserved and Non-Inferable Shocks

The last case that I consider is one in which (i) $\mathcal{PM}$ does not observe all exogenous shocks, i.e. $\mathcal{K} \subsetneq \{1, ..., M\}$, and (ii) a certain condition is met, which requires that, in models raising non-trivial local-equilibrium-indeterminacy issues (i.e., models such that $n^a \geq 1$), at least one unobserved shock cannot be inferred from O_t using only (1) and (2). To state this condition, let me note $\mathbb{R}(X)$ the set of rational expressions – i.e. fractions of polynomials – in X with real-number coefficients, and $\text{rank}_{\mathbb{R}(X)}$ the rank operator over $\mathbb{R}(X)$.[21] Consider a given feasible path, note it P, and write it as (4). The set $\mathcal{R}_P$ of rules of type (3) that are consistent with O_t and P, i.e. that express i_t as a function of only elements of O_t and are satisfied on P, is strictly included in the set $\overline{\mathcal{R}}_P$ of equations that involve only elements of $\cup_{t\in\mathbb{Z}} O_t$ and can be written as linear combinations, with coefficients in $\mathbb{R}(L)$, of the $N+1$ equations of (4). The latter set is a vector space over $\mathbb{R}(L)$, of dimension

$$\dim(\overline{\mathcal{R}}_P) = N + 1 - \text{rank}_{\mathbb{R}(X)}[\ \mathbf{S}_{\mathcal{J}}(X) \mid \mathbf{T}_{\mathcal{K}}(X)\ ]$$

with $\mathbf{S}_{\mathcal{J}}(X) \equiv \mathbf{S}(X)[\ \mathbb{1}_{1\notin\mathcal{J}}\mathbf{w}_1 \ \ ... \ \ \mathbb{1}_{N\notin\mathcal{J}}\mathbf{w}_N\ ]$ and $\mathbf{T}_{\mathcal{K}}(X) \equiv \mathbf{T}(X)[\ \mathbb{1}_{1\notin\mathcal{K}}\mathbf{v}_1 \ \ ... \ \ \mathbb{1}_{M\notin\mathcal{K}}\mathbf{v}_M\ ]$, where, for each $j \in \{1, ..., N+1\}$, $\mathbf{w}_j$ denotes the $(N+1)$-element vector whose j^{th} element is equal to one and whose other elements are equal to zero. Now, let $\overline{\mathcal{E}}$ denote the set of equations that involve only elements of $\cup_{t\in\mathbb{Z}} O_t$ and can be written as linear combinations, with coefficients in $\mathbb{R}(L)$, of the N equations of (1) and M equations of (2). This set is also a vector space over $\mathbb{R}(L)$. Given Assumption 4, its dimension is

$$\dim(\overline{\mathcal{E}}) = |\mathcal{S}| - \text{rank}_{\mathbb{R}(X)}[\ \mathbf{A}_{\mathcal{S},\mathcal{J}}(X) \mid \mathbf{C}_{\mathcal{S},\mathcal{K}}(X)\ ]$$

with $\mathbf{A}_{\mathcal{S},\mathcal{J}}(X) \equiv [\ \mathbb{1}_{1\in\mathcal{S}}\mathbf{u}_1 \ \ ... \ \ \mathbb{1}_{N\in\mathcal{S}}\mathbf{u}_N\ ]' X^{-n}\mathbf{A}(X)[\ \mathbb{1}_{1\notin\mathcal{J}}\mathbf{u}_1 \ \ ... \ \ \mathbb{1}_{N\notin\mathcal{J}}\mathbf{u}_N\ ]$ and $\mathbf{C}_{\mathcal{S},\mathcal{K}}(X) \equiv [\mathbb{1}_{1\in\mathcal{S}}\mathbf{u}_1 \ \ ... \ \ \mathbb{1}_{N\in\mathcal{S}}\mathbf{u}_N\ ]'\mathbf{C}(X)[\ \mathbb{1}_{1\notin\mathcal{K}}\mathbf{v}_1 \ \ ... \ \ \mathbb{1}_{M\notin\mathcal{K}}\mathbf{v}_M\ ]$, where $\mathcal{S} \equiv \{j \in \{1, ..., N\} | n^a_j = 0, n^b_j \leq 0\}$ denotes the set of integers j such that the j^{th} structural equation is not forward-looking. Since the structural equations are satisfied on P, $\overline{\mathcal{E}}$ is a subspace of $\overline{\mathcal{R}}_P$, so that $\dim(\overline{\mathcal{R}}_P) \geq \dim(\overline{\mathcal{E}})$. The condition characterizing Case 3 is

$$\dim(\overline{\mathcal{R}}_P) = \dim(\overline{\mathcal{E}}) + 1. \tag{16}$$

[21] For instance, a matrix $\mathbf{M}(X) \in \mathbb{R}^{N\times N}[X]$ is of rank N over $\mathbb{R}(X)$ when $\forall (P_1(X), ..., P_N(X)) \in \mathbb{R}(X)^N \smallsetminus (0, ..., 0)$, $\sum_{j=0}^{N} P_j(X)\mathbf{M}(X)\mathbf{u}_j \neq \mathbf{0}$. I use the *field* of rational expressions $\mathbb{R}(X)$, rather than the *ring* of polynomials $\mathbb{R}[X]$, because the rank operator and vector spaces are defined over fields, not rings.

To derive the implications of Condition (16) for the inferability of unobserved shocks, suppose for a moment that all the unobserved shocks can be inferred from O_t using only (1) and (2). Then $\dim(\overline{\mathcal{R}}_P) = |\mathcal{J}| + 1 - (M - |\mathcal{K}|)$, so that Condition (16) implies that $\dim(\overline{\mathcal{E}}) = |\mathcal{J}| - (M - |\mathcal{K}|)$. In turn, the latter equality implies that none of the structural equations is forward-looking (i.e. $\mathcal{S} = \{1, ..., N\}$, or equivalently $n^a = 0$), given Assumption 4 and the assumption $\mathcal{J}^* \subseteq \mathcal{J}$. Therefore, provided that at least one structural equation is forward-looking (i.e. $\mathcal{S} \subsetneq \{1, ..., N\}$, or equivalently $n^a \geq 1$), Condition (16) requires that at least one unobserved shock cannot be inferred from O_t using only (1) and (2).

To study feasible-path implementability in Case 3, let me first define the canonical rules for a feasible path as follows:

Definition 3 (Canonical Rule for a Feasible Path): *given an observation set $O_t \subseteq \{\mathbf{Y}^t, i^{t-1}, \boldsymbol{\varepsilon}^t\}$, a canonical rule for a feasible path is a rule of type (3) that (i) is satisfied on this path, (ii) expresses i_t as a function of only elements of O_t, (iii) does not belong to $\overline{\mathcal{E}}$, and (iv) has a polynomial $[\ \mathbf{F}(X) \mid G(X)\ ]$ of minimal degree.*

Consider a given canonical rule for a given feasible path P satisfying (16), note it $\mathcal{C}_P$, and write it as (3).[22] Given that $\mathcal{C}_P \in \overline{\mathcal{R}}_P$ and $\mathcal{C}_P \notin \overline{\mathcal{E}}$, Condition (16) implies that $\overline{\mathcal{R}}_P = \overline{\mathcal{E}} \oplus \overline{\mathcal{C}}_P$, where $\oplus$ denotes the direct-sum operator and $\overline{\mathcal{C}}_P$ the one-dimension subspace spanned by $\mathcal{C}_P$, i.e. $\overline{\mathcal{C}}_P \equiv \{\mathcal{Q}(L)[\mathbf{F}(L)\mathbf{Y}_t + G(L)i_t + \mathbf{H}(L)\boldsymbol{\varepsilon}_t] = 0 | \mathcal{Q}(X) \in \mathbb{R}(X)\}$. Now consider a given policy-instrument rule consistent with O_t and P, i.e. a given element of $\mathcal{R}_P$, and note it R. Since $\mathcal{R}_P \subseteq \overline{\mathcal{R}}_P$, R is also an element of $\overline{\mathcal{R}}_P$. As such, it can be written as the sum of an element of $\overline{\mathcal{E}}$ and an element of $\overline{\mathcal{C}}_P$. Let R' denote the latter element. Since $[\ \mathbf{F}(X) \mid G(X)\ ]$ is of minimal degree, R' can be written as $\mathcal{Q}(L)[\mathbf{F}(L)\mathbf{Y}_t + G(L)i_t + \mathbf{H}(L)\boldsymbol{\varepsilon}_t] = 0$ where the rational expression $\mathcal{Q}(X)$ is a polynomial: $\mathcal{Q}(X) \in \mathbb{R}[X]$. The system made of (1) and R is equivalent to the system made of (1) and R', so that R implements P (as the robustly unique local equilibrium) if and only if R' does. In turn, the system made of (1) and R' has the same stationary solution(s) for $(\mathbf{Y}_t, i_t)_{t\in\mathbb{Z}}$ as the system made of (1) and $\mathcal{C}_P$, since whatever $\mathcal{Q}(X) \in \mathbb{R}[X] \setminus \{0\}$, $\mathcal{Q}(L)[\mathbf{F}(L)\mathbf{Y}_t + G(L)i_t + \mathbf{H}(L)\boldsymbol{\varepsilon}_t] = 0$ has a unique stationary solution for $\mathbf{F}(L)\mathbf{Y}_t + G(L)i_t + \mathbf{H}(L)\boldsymbol{\varepsilon}_t$, namely the solution $\mathbf{F}(L)\mathbf{Y}_t + G(L)i_t + \mathbf{H}(L)\boldsymbol{\varepsilon}_t = 0$, which corresponds to $\mathcal{C}_P$. Therefore, R' implements P if and only if $\mathcal{C}_P$ does and all the roots of $\mathcal{Q}(X)$ lie outside the unit circle, and hence only if $\mathcal{C}_P$ does. Finally, $\mathcal{C}_P$ implements P if and only if $\mathcal{C}_P$ robustly ensures local-equilibrium determinacy. This result can be stated in the following way:

Proposition 3 (Feasible-Path Implementability in Case 3): *for any set $\mathcal{J}$ such that $\mathcal{J}^* \subseteq \mathcal{J} \subseteq \{1, ..., N\}$, any set $\mathcal{K} \subsetneq \{1, ..., M\}$, and any $(\ell_Y, \ell_\varepsilon) \in \mathbb{N}^2$, when $O_t = \{\mathbf{u}_j'\mathbf{Y}^{t-\ell_Y}, i^{t-1}, \mathbf{v}_k'\boldsymbol{\varepsilon}^{t-\ell_\varepsilon} | j \in \mathcal{J}, k \in \mathcal{K}\}$, any feasible path P satisfying (16) is implementable if and only if an arbitrarily given canonical rule for P robustly ensures local-equilibrium determinacy.*

While $\mathrm{rank}_{\mathbb{R}(X)}[\ \mathbf{S}_{\mathcal{J}}(X) \mid \mathbf{T}_{\mathcal{K}}(X)\ ] \leq N - |\mathcal{J}| + M - |\mathcal{K}|$ in general, consider for simplicity the case in which $\mathrm{rank}_{\mathbb{R}(X)}[\ \mathbf{S}_{\mathcal{J}}(X) \mid \mathbf{T}_{\mathcal{K}}(X)\ ] = N - |\mathcal{J}| + M - |\mathcal{K}|$, so that $\dim(\overline{\mathcal{R}}_P) = |\mathcal{J}| + 1 - (M - |\mathcal{K}|)$. In this case, Condition (16) can be rewritten as $|\mathcal{J}| + 1 - (M - |\mathcal{K}|) = \dim(\overline{\mathcal{E}}) + 1$, so that there are $\dim(\overline{\mathcal{E}}) + 1$ more observed variables than unobserved shocks, and therefore $\dim(\overline{\mathcal{E}}) + 1$ degrees of freedom in the choice of a policy-instrument rule consistent with the observation set and feasible path considered.

[22] The existence of a canonical rule for P is implied by Definition 1 and Condition (16).

However, as Proposition 3 says, these degrees of freedom cannot be exploited to find, among all these rules, one that ensures robust local-equilibrium determinacy. The reason is that all these rules can be derived from a canonical rule by two operations that can either preserve or undo, but not restore, robust local-equilibrium determinacy: multiplication by elements of $\mathbb{R}[L]$ (which corresponds to one degree of freedom), and addition to linear combinations, with coefficients in $\mathbb{R}(L)$, of the structural equations (which corresponds to $\dim(\overline{\mathcal{E}})$ degrees of freedom). Thus, a feasible path may not be implementable even when the number of observed variables exceeds, possibly by far, the number of unobserved shocks. I illustrate this possibility in the next section in the context of optimal monetary policy in the basic New Keynesian model.

4 Two Applications

The general results established in the previous section can be applied to many different models, policy instruments, observation sets, and feasible paths. In this section, for the sake of brevity, I consider only two applications, which show that feasible-path (non-)implementability may be an issue in textbook models, for standard policy instruments, relevant observation sets, and interesting feasible paths. The results that I obtain can be summarized as follows. First, optimal feasible monetary policy may not be implementable in the basic New Keynesian model for reasonable observation sets of the central bank, even when the number of observed endogenous variables largely exceeds the number of unobserved exogenous shocks. Second, contrary to conventional wisdom, debt-stabilizing feasible tax policy is implementable in the standard RBC model for all theoretically admissible values of its structural parameters, for reasonable observation sets of the tax authority, even in the presence of policy-implementation lags of any length.

4.1 Optimal Monetary Policy in the Basic New Keynesian Model

In this subsection, I study the implementability of optimal feasible monetary policy in the well known basic New Keynesian model. Some fifteen years ago, Rotemberg and Woodford (1999, p. 103) wrote that "the construction of a feedback rule for the funds rate that implements the optimal allocation − that is not only consistent with it but also renders it the unique stationary equilibrium consistent with the proposed policy rule − remains a nontrivial problem." In essence, this statement is still valid today when the optimal allocation and the feedback rule are required to be consistent with a given observation set of the central bank. In fact, as I will show in this subsection in the context of the basic New Keynesian model, such a feedback rule may simply not exist for reasonable observation sets of the central bank.

I refer the reader to Woodford (2003, Chapters 2, 4, and 6) and Galí (2008, Chapter 3) for a detailed presentation of the basic New Keynesian model. I restrict the analysis to the neighborhood of the zero-inflation steady state, log-linearize the equilibrium conditions in this neighborhood, and express all the endogenous variables and exogenous disturbances as log-deviations from their values at that steady state. In most of the subsection, I focus on the case in which the economy is hit by only two exogenous disturbances, one affecting the discount factor and the other the elasticity of substitution between differentiated goods. In this case, at each date $t \in \mathbb{Z}$, $\mathcal{PS}$ sets the consumption level c_t, the inflation rate π_t, the output level y_t, hours worked n_t, and the real wage w_t, according to the following structural equations:

$$
\begin{aligned}
c_t &= \mathbb{E}_t\{c_{t+1}\} - \gamma\left(i_t - \mathbb{E}_t\{\pi_{t+1}\}\right) + \eta_t, && (17)\\
\pi_t &= \beta\mathbb{E}_t\{\pi_{t+1}\} + \kappa y_t + u_t, && (18)\\
y_t &= (1-s)c_t, && (19)\\
n_t &= \alpha^{-1}y_t, && (20)\\
w_t &= \gamma^{-1}c_t + \chi n_t, && (21)
\end{aligned}
$$

where i_t denotes the interest rate set by the central bank ($\mathcal{CB}$) at date t. These structural equations are, respectively, the Euler equation, the Phillips curve, the goods-market-clearing condition, the production function, and the consumption vs. leisure trade-off condition. I assume that the exogenous disturbances η_t and u_t follow stationary ARMA(1,1) processes:

$$
\begin{aligned}
\eta_t &= \rho_\eta \eta_{t-1} + \varepsilon_t^\eta + \theta_\eta \varepsilon_{t-1}^\eta, && (22)\\
u_t &= \rho_u u_{t-1} + \varepsilon_t^u + \theta_u \varepsilon_{t-1}^u, && (23)
\end{aligned}
$$

where ε_t^η and ε_t^u are i.i.d. exogenous shocks such that $\mathbb{E}\{\varepsilon_t^\eta \varepsilon_{t-k}^u\} = 0$ for any $k \in \mathbb{Z}$. The structural parameters are the labor elasticity of output α, the discount factor β, the intertemporal elasticity of substitution γ, the reduced-form parameter κ, the steady-state ratio of government purchases to output s, the inverse of the Frisch labor-supply elasticity χ, the auto-regressive parameters ρ_η and ρ_u, and the moving-average parameters θ_η and θ_u. They are such that $0 < \alpha < 1$, $0 < \beta < 1$, $\gamma > 0$, $\kappa > 0$, $0 \leq s < 1$, $\chi > 0$, $-1 < \rho_\eta < 1$, and $-1 < \rho_u < 1$ (while θ_η and θ_u may take any real-number value).[23]

The observation set that I consider for $\mathcal{CB}$ is $O_t = \{c^{t-1}, \pi^{t-1}, y^{t-1}, n^{t-1}, w^{t-1}, i^{t-1}\}$. This observation set has three notable features. First, it contains no exogenous shock, which seems reasonable given the nature of the two shocks considered. Second, it contains no current endogenous variable. This feature is necessary for the existence of a well defined optimal feasible path.[24] Moreover, as mentioned in Subsection 3.1, it can be viewed as a consequence of the timing − considered in many studies of monetary-policy implementation − in which $\mathcal{CB}$ plays before $\mathcal{PS}$ within each period, and this timing is arguably better suited than the reverse timing to capture the fact that central banks take their decisions at a lower frequency than the private sector considered as a whole. Third, this observation set contains no expectation of $\mathcal{PS}$. This last feature captures the fact that the available measures of the private sector's expectations (measures that can be inferred from survey responses or financial-market prices) are often viewed as less reliable than the data about the private sector's actions (e.g., data about consumption, output, or inflation).[25]

The goal of this subsection is to study the implementability of the timeless-perspective optimal feasible path, i.e., the path that maximizes welfare from Woodford's (1999b) timeless perspective subject to the structural equations and $\mathcal{CB}$'s observation-set constraint.[26] I assume for simplicity that the steady

[23] I allow for $s > 0$ to prepare the ground for the introduction of government-purchases disturbances at the end of the subsection. And I allow for $|\theta_\eta| \geq 1$ and $|\theta_u| \geq 1$ because there is no economic reason to rule out non-fundamental ARMA disturbances, as stressed by, e.g., Lippi and Reichlin (1993). In particular, the limit case in which θ_η and θ_u tend towards infinity can be interpreted as a situation in which news shocks perfectly inform $\mathcal{PS}$ about one-period-ahead disturbances, as I elaborate below.

[24] Indeed, if $\mathcal{CB}$'s observation set were instead $O_t = \{c^t, \pi^t, y^t, n^t, w^t, i^{t-1}\}$, then all the interest-rate rules consistent with O_t and the optimal feasible path would have *infinite* coefficients − a possibility that my framework does not allow for − because i_t would be the only endogenous variable whose value on this path depends on ε_t^η.

[25] Similarly, in the literature on structural-shock identification initiated by Hansen and Sargent (1981, 1991) and mentioned in Subsection 5.2, the econometrician is assumed to observe the private sector's actions but not its expectations.

[26] This path can be defined as the limit of the date-t_0 Ramsey-optimal feasible path as $t_0 \to -\infty$. The date-t_0 Ramsey-

state considered is efficient (due to an employment or production subsidy offsetting the monopolistic-competition distortion), so that the second-order approximation of the date-t welfare loss function in the neighborhood of this steady state is of the form $L_t = \mathbb{E}_t\{\sum_{k=0}^{+\infty} \beta^k[(\pi_{t+k})^2 + \lambda(y_{t+k})^2]\}$, where $\lambda > 0$. Noting $\mu \equiv (2\beta\lambda)^{-1}[\lambda + \beta\lambda + \kappa^2 - \sqrt{(\lambda + \beta\lambda + \kappa^2)^2 - 4\beta\lambda^2}]$, I characterize the timeless-perspective optimal feasible path by the following proposition:

Proposition 4 (Timeless-Perspective Optimal Feasible Path, in the Basic New Keynesian Model, When $\mathcal{CB}$ Observes Only Past Variables): *if $\rho_u \neq \mu$, then the timeless-perspective optimal feasible path when $O_t = \{c^{t-1}, \pi^{t-1}, y^{t-1}, n^{t-1}, w^{t-1}, i^{t-1}\}$ can be written as*

$$\begin{bmatrix} c_t \\ \pi_t \\ y_t \\ n_t \\ w_t \\ i_t \end{bmatrix} = \sum_{j=1}^{9} \left[\begin{array}{ccccc|c} & & \omega_j \mathbf{I}_5 & & & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ \hline 0 & f_j^{\pi} & f_j^{y} & 0 & 0 & g_j \end{array}\right] \begin{bmatrix} c_{t-j} \\ \pi_{t-j} \\ y_{t-j} \\ n_{t-j} \\ w_{t-j} \\ i_{t-j} \end{bmatrix} + \sum_{j=0}^{3} \begin{bmatrix} \mathbf{\Omega}_j \\ 0 \quad 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{t-j}^{\eta} \\ \varepsilon_{t-j}^{u} \end{bmatrix}, \qquad (24)$$

where $(\omega_1, \omega_2) \equiv (\mu + \rho_u, -\mu\rho_u)$; $\forall j \in \{3, ..., 9\}$, $\omega_j \equiv 0$; $\forall j \in \{1, ..., 9\}$, $(f_j^{\pi}, f_j^{y}, g_j) \in \mathbb{R}^3$; $\forall j \in \{0, ..., 3\}$, $\mathbf{\Omega}_j \in \mathbb{R}^{5\times 2}$, with $\text{rank}(\mathbf{\Omega}_0) = 2$; and the last line of (24) is a canonical rule for this path.

Proof: see Appendix A.4. ■

Interestingly, the timeless-perspective optimal feasible path (24) is inertial in response to both η and u disturbances, i.e., it makes $(c_t, \pi_t, y_t, n_t, w_t, i_t)$ depend on both η^{t-1} and u^{t-1}. By contrast, as is well known since Clarida, Galí, and Gertler (1999) and Woodford (1999b), when $\{\eta^t, u^t\} \subseteq O_t$ the timeless-perspective optimal feasible path is inertial *only* in response to u disturbances, i.e., it makes $(c_t, \pi_t, y_t, n_t, w_t, i_t)$ depend on u^{t-1} but not on η^{t-1}. To understand why the optimal responses to η disturbances are inertial when $\eta_t \notin O_t$, consider for simplicity the case in which η disturbances are i.i.d. (i.e., $\rho_\eta = \theta_\eta = 0$) and there are no u disturbances. In this case, the first-best path $(c_t, \pi_t, y_t, n_t, w_t, i_t) = (0, 0, 0, 0, 0, \gamma^{-1}\eta_t)$ is not feasible, because i_t cannot depend on η_t, and the optimal non-inertial feasible path is $(c_t, \pi_t, y_t, n_t, w_t, i_t) = (\eta_t, \kappa(1-s)\eta_t, (1-s)\eta_t, \alpha^{-1}(1-s)\eta_t, [\gamma^{-1} + \alpha^{-1}\chi(1-s)]\eta_t, 0)$. Inertia then enables $\mathcal{CB}$ to make $\mathbb{E}_t\{c_{t+1}\} + \gamma\mathbb{E}_t\{\pi_{t+1}\}$ depend negatively on η_t, and thus to relax the constraint imposed by the Euler equation (17), in order to bring c_t closer to zero − and therefore, via the Phillips curve (18) and the goods-market-clearing condition (19), to bring also (π_t, y_t) closer to zero. The optimal degree of inertia is the one that equalizes the marginal gain from bringing (y_t, π_t) closer to $(0, 0)$ and the marginal cost of moving $(\mathbb{E}_t\{y_{t+1}\}, \mathbb{E}_t\{\pi_{t+1}\})$ further away from $(0, 0)$.[27]

To determine the implementability of the timeless-perspective optimal feasible path (24), I cannot use Proposition 2, since equations (17) to (23) do not imply any relationship of type (15): the history of the unobserved shocks $\varepsilon^{\eta,t-1}$ and $\varepsilon^{u,t-1}$ cannot be inferred from O_t using only equations (17) to (23), because the only structural equations involving the corresponding disturbances, (17) and (18), also involve unobserved expectations. Instead, I use Proposition 3. The system made of equations (17) to (23) is straightforwardly shown to be written in a form of type (1) and (2) with $N = 5$, $\mathbf{Y}_t = [\ c_t \quad \pi_t \quad y_t \quad n_t \quad w_t\ ]'$,

optimal feasible path is, in turn, defined as the state-contingent path for the endogenous variables that maximizes welfare at date t_0 subject to the structural equations and $\mathcal{CB}$'s observation-set constraint. I consider the timeless-perspective optimal feasible path, rather than the date-t_0 Ramsey-optimal feasible path, simply for consistency with the general analysis of the previous sections, in which feasible paths were specified as VARMA processes over $t \in \mathbb{Z}$ without initial conditions.

[27] This optimal-inertia result was first obtained by Aoki (2006), in the context of the same model (except for the stochastic process of the exogenous disturbances), for an observation set made of past endogenous variables and noisy signals about current endogenous variables. In the case of infinitely noisy signals, this observation set boils down to the one I consider.

$M = 2$, $\boldsymbol{\xi}_t = [\ \eta_t \quad u_t\ ]'$, $\boldsymbol{\varepsilon}_t = [\ \varepsilon_t^\eta \quad \varepsilon_t^u\ ]'$, and $n = 1$, that satisfies Assumptions 1 to 6 for all theoretically admissible values of the structural parameters.[28] Moreover, the observation set considered is of type (9) with $\mathcal{J} = \{1, 2, 3, 4, 5\}$, $\mathcal{K} = \varnothing$, and $\ell_Y = 1$. Finally, the feasible path (24) – noted P for the occasion – is such that $\dim(\overline{\mathcal{R}}_P) = 4$, since $\text{rank}(\boldsymbol{\Omega}_0) = 2$, while the equations (17) to (23) are such that $\dim(\overline{\mathcal{E}}) = 3$. Therefore, $\dim(\overline{\mathcal{R}}_P) = \dim(\overline{\mathcal{E}}) + 1$: we are in Case 3, and Propositions 3 and 4 imply that the feasible path (24) is implementable if and only if the last line of (24) robustly ensures local-equilibrium determinacy.

The latter result can be explained as follows. There are $\dim(\overline{\mathcal{R}}_P) = 4$ degrees of freedom in the choice of a policy-instrument rule consistent with the observation set and feasible path considered, essentially because this observation set includes, for each past date, six endogenous variables whose values on that feasible path depend on only two unobserved exogenous shocks. However, these degrees of freedom cannot be exploited to find a rule robustly ensuring local-equilibrium determinacy, because the vector space $\overline{\mathcal{R}}_P$ is spanned by the $\dim(\overline{\mathcal{E}}) = 3$ intra-temporal structural equations (19) to (21) and the last line of (24). In other words, any policy-instrument rule consistent with the observation set and feasible path considered can be written in a form of type $\alpha(L)[i_t - \sum_{j=1}^{9}(f_j^\pi \pi_{t-j} + f_j^y y_{t-j} + g_j i_{t-j})] + \beta(L)[y_t - (1 - s)c_t] + \gamma(L)[n_t - \alpha^{-1}y_t] + \delta(L)[w_t - \gamma^{-1}c_t - \chi n_t] = 0$, where $(\alpha(X), \beta(X), \gamma(X), \delta(X)) \in \mathbb{R}[X]^4$ is such that $\alpha(0) \neq 0$ and $\beta(0) = \gamma(0) = \delta(0) = 0$. Therefore, any such rule robustly ensures local-equilibrium determinacy only if the last line of (24) does.

For any calibration of the model's structural parameters, I can thus determine numerically, using the analytical expression of $(f_j^\pi, f_j^y, g_j)_{1 \leq j \leq 9}$ provided in Appendix A.4, whether the timeless-perspective optimal feasible path is implementable when $\mathcal{CB}$ observes only past actions. As can be readily checked, whether it is implementable or not depends only on the parameters β, $\gamma(1-s)$, κ, λ, ρ_η, ρ_u, θ_η, and θ_u. So, for instance, let me consider Galí's (2008, Chapter 3) and Woodford's (2003, Chapter 4) calibrations of the basic New Keynesian model, respectively characterized by $(\beta, \gamma(1-s), \kappa, \lambda) = (0.99, 1.00, 0.125, 0.021)$ and $(\beta, \gamma(1-s), \kappa, \lambda) = (0.99, 6.25, 0.022, 0.003)$, and let me focus on values of ρ_η, ρ_u, θ_η, and θ_u such that $\rho_\eta = \rho_u \equiv \rho$ and $\theta_\eta = \theta_u \equiv \theta$ (so that the two disturbances follow identical stochastic processes). As shown in Figure 1, I then obtain that the timeless-perspective optimal feasible path is not implementable for many admissible values of ρ and θ – broadly the same values under both calibrations.

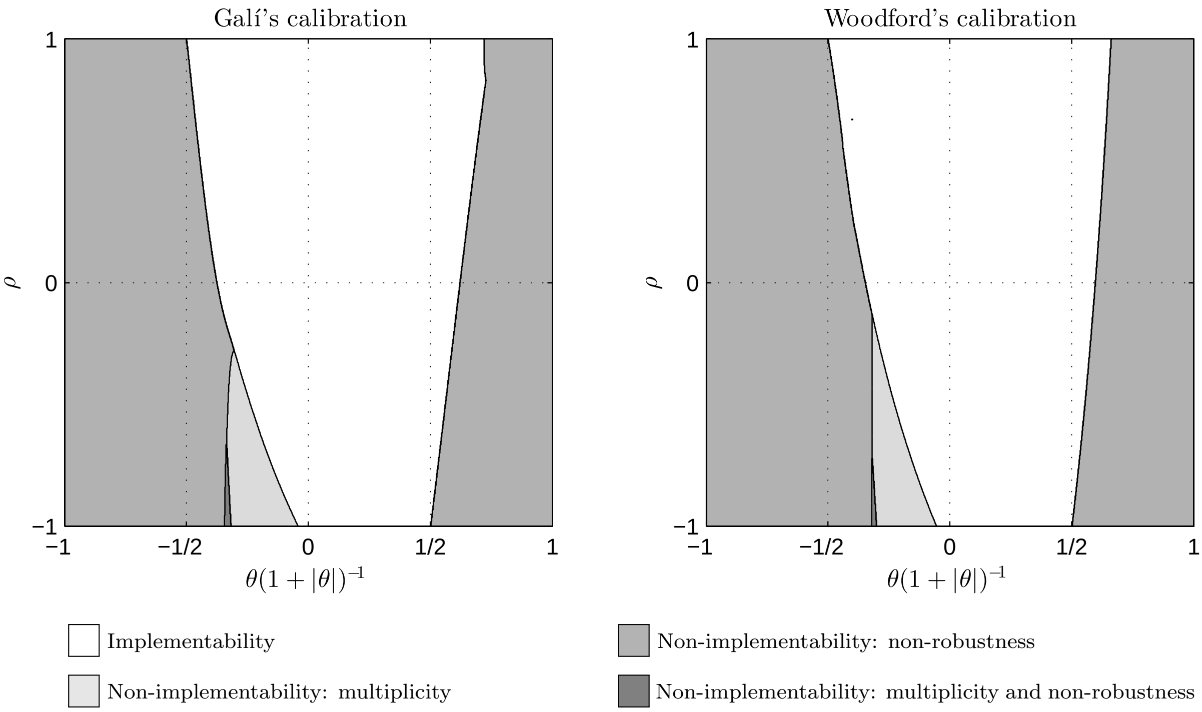
For some values of ρ and θ (light-gray areas in Figure 1), this path is not implementable because all the interest-rate rules consistent with $\mathcal{CB}$'s observation set and this path lead to local-equilibrium multiplicity. For other values of ρ and θ (dark-gray areas in Figure 1), it is not implementable because, even though all the interest-rate rules consistent with $\mathcal{CB}$'s observation set and this path may ensure local-equilibrium determinacy, none does so in the minimally robust way required for feasible-path implementability: adding an exogenous policy shock of arbitrarily small variance to any of these rules leads to non-existence of a local equilibrium. The latter case arises notably for positive values of ρ and θ provided that θ is sufficiently large, i.e. provided that the responses of the disturbances to the shocks are sufficiently hump-shaped, the value $\theta = 3$ (respectively $\theta = 2$) being enough to make the path non-implementable for all values of ρ under Galí's (respectively Woodford's) calibration. In particular, this case arises for an infinite value of θ, i.e. for *news* shocks.[29] Finally, for still other values of ρ and θ (very-dark-gray areas in Figure

[28] A proof that Assumption 6 is satisfied can be found in, e.g., Woodford (2003, Chapter 4).

[29] As θ tends towards infinity, and the variance of ε_t^η and ε_t^u towards zero at speed θ^2 (so that the variances of $\widetilde{\varepsilon}_t^\eta \equiv \theta \varepsilon_t^\eta$ and $\widetilde{\varepsilon}_t^u \equiv \theta \varepsilon_t^u$ are constant), the stochastic processes of η_t and u_t converge respectively towards $\eta_t = \rho \eta_{t-1} + \widetilde{\varepsilon}_{t-1}^\eta$ and $u_t = \rho u_{t-1} + \widetilde{\varepsilon}_{t-1}^u$, so that $\widetilde{\varepsilon}_t^\eta$ and $\widetilde{\varepsilon}_t^u$ can be interpreted as news shocks perfectly informing $\mathcal{PS}$ about one-period-ahead disturbances.

1), the timeless-perspective optimal feasible path is not implementable because all the interest-rate rules consistent with $\mathcal{CB}$'s observation set and this path lead to local-equilibrium multiplicity in the absence of exogenous policy shocks and to non-existence of a local equilibrium in the presence of such shocks.[30]

Figure 1: Implementability of the timeless-perspective optimal feasible path, in the basic New Keynesian model, when $\mathcal{CB}$ observes only past variables



This non-implementability result sounds a note of caution about one of the main lessons of the New Keynesian literature, namely the importance for central banks to track some key unobserved exogenous rates of interest such as, for instance, the counterfactual "natural rate of interest."[31] From a normative perspective, the most important of these rates of interest is, ultimately, the exogenous value taken by the interest rate on the optimal feasible path.[32] As my result shows, however, even when this value can be inferred in many alternative ways, on the optimal feasible path, from the endogenous variables observed by the central bank, there may be no way of setting the interest rate as a function of these variables that implements this path as the robustly unique local equilibrium. In this case, any attempt to track this rate of interest and implement the optimal feasible path will inevitably result in local-equilibrium multiplicity or, in the presence of exogenous policy shocks of arbitrarily small variance, non-existence of a local equilibrium.

This non-implementability result has been obtained under the assumption that the economy is hit by only two exogenous disturbances (affecting the discount factor and the elasticity of substitution between differentiated goods). In Appendix A.5, I show that it is robust to the relaxation of this assumption.

[30] In the first two cases, the system made of the structural equations (17) to (21) and the last line of (24) does not meet Blanchard and Kahn's (1980) order condition because it has strictly fewer (in the first case) or strictly more (in the second case) eigenvalues outside the unit circle than non-predetermined variables. In the third case, this system meets Blanchard and Kahn's (1980) order condition but not their rank condition.

[31] In Galí's (2008, Chapter 8) words: "these new models identify tracking the natural equilibrium of the economy, which is not directly observable, as an important challenge for central banks." And in Woodford's (2003, Chapter 4): "keeping track of its current value would be an important (and far from trivial) task of central-bank staff."

[32] The optimal path in question has to be the optimal *feasible* path for the central bank to be able to track this exogenous rate of interest successfully, i.e. to be able to infer it from her observation set on this path.

More specifically, I show that the set of structural-parameter values for which the timeless-perspective optimal feasible path is implementable is unaffected by the introduction of three additional disturbances into the model (affecting government purchases, productivity, and consumption utility or labor disutility, and following stationary fundamental ARMA processes), so that in particular Figure 1 is still valid in the presence of these additional disturbances. This robustness is essentially due to the facts that (i) the timeless-perspective optimal feasible path still meets Condition (16) in the presence of these additional disturbances, so that Proposition 3 can still be used to determine its implementability, and (ii) these additional disturbances appear in the intra-temporal structural equations, so that their realizations can be inferred from $\mathcal{CB}$'s observation set in a way that is neutral for robust local-equilibrium determinacy.

4.2 Debt-Stabilizing Tax Policy in the Standard RBC Model

Schmitt-Grohé and Uribe (1997) consider, in a standard RBC model, a labor-income-tax-rate rule and an income-tax-rate rule that stabilize, both in and out of equilibrium, the current stock of public debt (in the absence of policy-implementation lags) or the expected future stock of public debt (in the presence of such lags).[33] They find that these rules lead to local-equilibrium multiplicity for many empirically relevant values of the structural parameters. Their finding has largely been interpreted as an argument against the use of labor-income or income taxes to stabilize the current or expected future stock of public debt. However, the fact that *these* (labor-)income-tax-rate rules fail to ensure local-equilibrium determinacy does not imply that *all* the (labor-)income-tax-rate rules that stabilize the current or expected future stock of public debt in equilibrium – but not necessarily out of equilibrium – fail to ensure local-equilibrium determinacy. In this subsection, I challenge the interpretation commonly made of their finding by using the general results of the previous section to show that, in the same model, for the same alternative tax instruments, and for a reasonable observation set of the tax authority, all feasible paths along which the current or expected future stock of public debt is stabilized are implementable for all theoretically admissible values of the structural parameters, even in the presence of policy-implementation lags of any length.[34]

In Schmitt-Grohé and Uribe's (1997) model, at each date $t \in \mathbb{Z}$, $\mathcal{PS}$ sets the output level y_t, the capital stock k_t, investment x_t, hours worked h_t, the consumption level c_t, the (after-tax) rental price of capital u_t, and the (after-tax) wage w_t, according to the following structural equations:

$$y_t = a_t + (1 - s_h)k_t + s_h h_t, \tag{25}$$

$$k_t = (1 - \delta)k_{t-1} + x_{t-1}, \tag{26}$$

$$x_t = \delta s_i^{-1} y_t - \delta s_c s_i^{-1} c_t - \delta(1 - s_c - s_i)s_i^{-1} g_t \tag{27}$$

$$h_t = \gamma^{-1} w_t - \gamma^{-1} c_t, \tag{28}$$

$$c_t = \mathbb{E}_t\{c_{t+1}\} - \chi \mathbb{E}_t\{u_{t+1}\}, \tag{29}$$

$$u_t = a_t - s_h(k_t - h_t) - \omega\tau(1 - \tau)^{-1}\tau_{t-d}, \tag{30}$$

$$w_t = a_t + (1 - s_h)(k_t - h_t) - \tau(1 - \tau)^{-1}\tau_{t-d}, \tag{31}$$

[33] Most of Schmitt-Grohé and Uribe's (1997) analysis is conducted in continuous time. I refer here to the discrete-time analysis conducted at the end of Section 3, in Section 4, and in the appendix of their paper.

[34] Since Schmitt-Grohé and Uribe (1997), several papers have shown that policy instruments other than labor-income or income taxes (e.g., government purchases in Guo and Harrison, 2004) can be used in the same model to stabilize the current or expected future stock of public debt without generating local-equilibrium multiplicity. My point is that this can be done even with labor-income or income taxes, provided that the current or expected future stock of public debt is not required to be stabilized also out of equilibrium.

where τ_t denotes the labor-income-tax rate (when $\omega = 0$) or income-tax rate (when $\omega = 1$) set by the tax authority ($\mathcal{TA}$) at date t, and $d \in \mathbb{N}$ the length of the tax-policy-implementation lags. The exogenous productivity and government-purchases disturbances a_t and g_t are assumed to follow stationary fundamental ARMA processes:

$$\rho_a(L)a_t = \theta_a(L)\varepsilon_t^a, \quad (32)$$

$$\rho_g(L)g_t = \theta_g(L)\varepsilon_t^g, \quad (33)$$

where ε_t^a and ε_t^g are i.i.d. exogenous shocks such that $\mathbb{E}\{\varepsilon_t^a \varepsilon_{t-k}^g\} = 0$ for any $k \in \mathbb{Z}$.[35] The structural parameters are γ, δ, s_c, s_h, s_i, τ, χ, ω, and the coefficients of the polynomials $\rho_a(X)$, $\rho_g(X)$, $\theta_a(X)$, and $\theta_g(X)$. They are such that $\gamma > 0$, $0 < \delta < 1$, $0 < s_c < 1$, $0 < s_i < 1$, $0 < s_c + s_i < 1$, $0 < s_h < 1$, $0 < \tau < 1$, $\chi > 0$, $\omega \in \{0, 1\}$, $\rho_a(0) \neq 0$, $\rho_g(0) \neq 0$, $\theta_a(X) \neq 0$, $\theta_g(X) \neq 0$, and all the roots of $\rho_a(X)$, $\rho_g(X)$, $\theta_a(X)$, and $\theta_g(X)$ lie outside the unit circle. The system made of equations (25) to (33) can straightforwardly be written in a form of type (1) and (2) with $N = 7$, $\mathbf{Y}_t = [\ y_t \quad k_t \quad x_t \quad h_t \quad c_t \quad u_t \quad w_t\ ]'$, $i_t = \tau_t$, $M = 2$, $\boldsymbol{\xi}_t = [\ a_t \quad g_t\ ]'$, $\boldsymbol{\varepsilon}_t = [\ \varepsilon_t^a \quad \varepsilon_t^g\ ]'$, and $n = 1$, that satisfies Assumptions 1 to 6 for all theoretically admissible values of the structural parameters.

The observation set that I consider for $\mathcal{TA}$ is $O_t = \{y^{t-\ell}, x^{t-\ell}, h^{t-\ell}, c^{t-\ell}, u^{t-\ell}, w^{t-\ell}, \tau^{t-1}, \varepsilon^{g,t-\ell}\}$, where $\ell \in \mathbb{N}$.[36] This observation set is of type (9) with $\mathcal{J} = \{1, 3, 4, 5, 6, 7\}$, $\mathcal{K} = \{2\}$, and $\ell_\varepsilon = \ell_Y$. Now, the history of the unobserved shock $\varepsilon^{a,t-\ell}$ can be inferred from O_t using only the structural equations (25) and (26) and the stochastic process (32), since these equations and process imply the following relationship of type (15): $[1-(1-\delta)L]\theta_a(L)\varepsilon_t^a = \rho_a(L)\{[1-(1-\delta)L](y_t - s_h h_t) - (1-s_h)x_{t-1}\}$. Therefore, we are in Case 2, and Proposition 2 implies that all feasible paths are implementable, in particular all feasible paths along which the current or expected future stock of public debt is stabilized, for all theoretically admissible values of the structural parameters.[37]

So, in the context of these model, alternative tax instruments, and observation set, a tax authority can always conduct a tax policy that stabilizes the current or expected future stock of public debt in equilibrium without generating local-equilibrium multiplicity, in particular even in the presence of observation or policy-implementation lags of any length, i.e. for any $(\ell, d) \in \mathbb{N}^2$. This result implies that Schmitt-Grohé and Uribe's (1997) finding should be interpreted not as an argument against debt-stabilizing (labor-)income-tax policy *per se*, but instead as an argument against one specific (though natural) way of implementing this policy: one that achieves the policy's goal not only in, but also out of equilibrium.[38] Debt-stabilizing tax policy may well inherently amplify business cycles, as recalled by Schmitt-Grohé and Uribe (1997). But, at least in the context of their model, their alternative tax instruments, and this observation set, and as long as it is not required to achieve its goal also out of equilibrium, this policy does not *inherently* generate local-equilibrium multiplicity.

[35] This model amounts to the one detailed in the appendix of Schmitt-Grohé and Uribe's (1997) paper, augmented with policy-implementation lags and technology and government-purchases disturbances following stationary fundamental ARMA processes. They use this model with lags but without disturbances at the end of Section 3, and with stationary AR(1) disturbances but without lags in Section 4.

[36] The results would be identical if $\mathcal{TA}$ were instead assumed to observe the capital stock or not to observe the government-purchases shock, i.e. for $O_t = \{\mathbf{Y}^{t-\ell}, \tau^{t-1}, \varepsilon^{g,t-\ell}\}$, $O_t = \{y^{t-\ell}, x^{t-\ell}, h^{t-\ell}, c^{t-\ell}, u^{t-\ell}, w^{t-\ell}, \tau^{t-1}\}$, or $O_t = \{\mathbf{Y}^{t-\ell}, \tau^{t-1}\}$.

[37] It is easy to check that all feasible paths remain implementable when some additional exogenous disturbances (like consumption-utility or labor-disutility disturbances) are introduced into the model.

[38] The tax-rate rules considered by Schmitt-Grohé and Uribe (1997) may involve, for instance, y_t and g_t (for the income-tax-rate rule stabilizing the current stock of public debt when $d = 0$), or y_t, g_t, u_t, b_{t-1}, $(\mathbb{E}_t\{y_{t+k}\})_{1 \leq k \leq d}$, $(\mathbb{E}_t\{u_{t+k}\})_{1 \leq k \leq d}$, and $(\mathbb{E}_t\{\tau_{t+k}\})_{1 \leq k \leq d}$ (for the income-tax-rate rule stabilizing the expected future stock of public debt when $d \geq 1$), where b_{t-1} denotes the stock of public debt at date $t-1$.

5 Discussion

In this section, I make three sets of additional remarks about the general results obtained in Section 3. First, I highlight the methodological contribution of the paper, namely the arithmetic design of policy-instrument rules. Second, I show the absence of direct relationship between the issues of feasible-path implementability and exogenous-shock identifiability. Third, I briefly discuss the extension of the results to models with several policy instruments.

5.1 Arithmetic Design of Policy-Instrument Rules

On the methodological front, for each observation set and implementable path, conditionally on a given known number of roots of $\mathcal{D}_{\mathcal{J}}(X)$ outside the unit circle, I show in Appendix A.3 how to design *arithmetically*, i.e. with a finite number of arithmetic operations (addition, subtraction, multiplication, and division), a policy-instrument rule that is consistent with this observation set and implements that path as the robustly unique local equilibrium.[39] This method of designing policy-instrument rules does not require, in particular, the determination of any polynomial roots (except trivially roots of polynomials of degree one). Instead, it uses Bézout's identity, the Euclidean division, and Cramer's rule, all of which involve a finite number of arithmetic operations, to directly transform the polynomials $\mathbf{A}(X)$, $\mathbf{B}(X)$, $\mathbf{C}(X)$, $\mathbf{D}(X)$, $\mathbf{E}(X)$, $\mathbf{S}(X)$ and $\mathbf{T}(X)$ characterizing the structural equations, the exogenous disturbances, and the implementable path considered into the polynomials $\mathbf{F}(X)$, $G(X)$, and $\mathbf{H}(X)$ characterizing the policy-instrument rule.

This conditional arithmetic-designability property implies that, for each model, policy instrument, observation set, and implementable path, the coefficients of the corresponding policy-instrument rule can be explicitly expressed as *rational* functions of the structural and implementable-path parameters, i.e. as fractions of polynomial functions of these parameters, in any given region of the structural-parameter space within which the number of roots of $\mathcal{D}_{\mathcal{J}}(X)$ outside the unit circle is constant. Of course, unless the specific model and implementable path considered are particularly simple, these rational functions will involve, by most standards, an unusually large number of terms in the numerator and the denominator. However, their analytical manipulation with a symbolic-computation software should raise no practical difficulty. For instance, their derivatives could be easily computed to determine how the coefficients of the policy-instrument rule respond to an arbitrarily small change in the value of the structural or implementable-path parameters.

Moreover, in some cases, the number of roots of $\mathcal{D}_{\mathcal{J}}(X)$ outside the unit circle can easily be shown to be constant over the whole structural-parameter space, so that the coefficients of the policy-instrument rule mentioned in the previous paragraph can be *globally* (instead of *locally*) expressed as rational functions of the structural and implementable-path parameters. Consider, for instance, a typical medium-scale DSGE model of the monetary transmission mechanism, such as Smets and Wouters' (2007), with the interest rate as the policy instrument. Its structural equations and exogenous disturbances can easily be written in a form of type (1) and (2) with $N = 13$ and $M = 6$ (ignoring the monetary-policy disturbance) that satisfies Assumptions 1 to 5 for all theoretically admissible values of its structural parameters, except

[39] This rule is designed only in Case 1. However, starting from this rule, it is easy to arithmetically design a similar rule in Case 2, following the procedure described in Subsection 3.3. Finally, the arithmetic design of such a rule (when it exists) is trivial in Case 3. Of course, none of these rules is unique.

a set of values of measure zero.[40] Suppose that the central bank observes (possibly with lags) all the endogenous variables set by the private sector, i.e. $\mathcal{J} = \{1, ..., N\}$.[41] The roots of $\mathcal{D}_{\{1,...,N\}}(X)$, if any, are the solutions in X of the system made of the equations $\Delta_j(X) = 0$ for $j \in \{1, ..., N+1\}$. This system is non-linear in the unknown X, but linear in the $D \equiv max\{d_{\Delta_j} | j \in \{1, ..., N+1\}\}$ unknowns X^d for $d \in \{1, ..., D\}$. It is easy to check analytically that this linear system has no solution, as $D < N+1$, and therefore that $\mathcal{D}_{\{1,...,N\}}(X)$ has no root, and in particular no root outside the unit circle, for all theoretically admissible values of the structural parameters (except possibly a set of values of measure zero).

5.2 Feasible-Path Implementability and Exogenous-Shock Identifiability

It might be thought at first sight that the issue of feasible-path implementability that I raise and study in this paper could be related to the well known issue of exogenous-shock identifiability first raised by Hansen and Sargent (1981, 1991). Hansen and Sargent (1981, 1991) provide concrete examples of dynamic stochastic rational-expectations models whose exogenous shocks cannot be identified by an econometrician because they are not fundamental for the observed variables on the equilibrium path, i.e. they do not belong to the space spanned by square-summable linear combinations of current and past observed variables on the equilibrium path. Similarly, when $O_t = \{\mathbf{Y}^t, i^{t-1}\}$ for instance, if $\boldsymbol{\varepsilon}_t$ is not fundamental for $\mathbf{Y}_t$ – and therefore cannot be recovered from $\mathbf{Y}^t$ – on a given feasible path, and if i_t depends on $\boldsymbol{\varepsilon}^t$ on this path, then isn't this path necessarily non-implementable, on the ground that the policy maker's only original source of information about $\boldsymbol{\varepsilon}_t$ is $\mathbf{Y}^t$?

The answer is no. For instance, in the context of Subsection 4.1's model and observation set, the shocks ε_t^η and ε_t^u happen to be non-fundamental for the variables c_t, π_t, y_t, n_t, and w_t on the timeless-perspective optimal feasible path for the whole grid of values of ρ and θ that I consider under both calibrations. Yet, this path is implementable for many values of ρ and θ, as shown in Figure 1. So non-fundamentalness of the unobserved shocks for the observed variables on a given feasible path is not a sufficient condition for the non-implementability of this path.

Neither is it a necessary condition for that matter. For instance, in the context of Subsection 2.5's model and observation set, consider, for any $(\theta, \phi) \in \mathbb{R}^* \times \mathbb{R}$, the path $[\ y_t \quad i_t\ ]' = [\ \theta\varepsilon_t + \phi\varepsilon_{t-1} \quad (\theta - \alpha\phi - 1)\varepsilon_t + (\phi - \beta)\varepsilon_{t-1}]'$, which satisfies (5). This path is feasible, since i_t can be expressed on this path as a function of only elements of the policy maker's observation set: $i_t = -\phi\theta^{-1}i_{t-1} + (\theta - \alpha\phi - 1)\theta^{-1}y_t + (\phi - \beta)\theta^{-1}y_{t-1}$. It is straightforward to write the system made of (5) and the latter equation in Blanchard and Kahn's (1980) form $\mathbb{E}_t\{[\ y_{t+1} \quad y_t \quad i_t\ ]'\} = \mathbf{M}[\ y_t \quad y_{t-1} \quad i_{t-1}\ ]' + [\ -\alpha^{-1}\varepsilon_t - \beta\alpha^{-1}\varepsilon_{t-1} \quad 0 \quad 0\ ]'$ and to check that the eigenvalues of the 3×3 matrix $\mathbf{M}$ are 0 and the two roots of $\mathcal{Q}(X)$, which do not depend on ϕ. Therefore, as follows from Subsection 2.5's analysis, the feasible path considered may be implementable or non-implementable whether ε_t is fundamental or not for y_t on this path, i.e. whether $|\phi| < |\theta|$ or $|\phi| \geq |\theta|$. The only consequence of ε_t being non-fundamental for y_t on this path (i.e. $|\phi| \geq |\theta|$) is that all the policy-instrument rules consistent with the policy maker's observation set and this path are then "superinertial," in the sense that the polynomial in L characterizing the i^t term in each of these rules

[40] This set of values of measure zero is the set characterized by $(1-c_1)[z_1 z_y - \alpha\phi_p z_1 - (1-\alpha)\phi_p] + c_2 c_y = 0$, where c_1, c_2, c_y, z_1, z_y, α, and ϕ_p are reduced-form parameters. I do not require Assumption 6 to be met because (i) this assumption is used only as a sufficient condition for $\mathcal{D}_{\mathcal{J}}(X)$ to have at most n^a roots outside the unit circle, and (ii) as I will show, in the context of these model, policy instrument, and observation set, $\mathcal{D}_{\mathcal{J}}(X)$ always has no root outside the unit circle.

[41] In this model, the endogenous variables set by the private sector are the output level, the consumption level, the investment flow, the installed capital stock, the capital utilization rate, the utilized capital stock, the rental rate on capital, the capital-stock value, hours worked, the real wage, the wage mark-up, the inflation rate, and the price mark-up.

has at least one root inside the unit circle (namely, the root $-\theta\phi^{-1}$).[42] However, as is well known since Rotemberg and Woodford (1999) and Woodford (1999a), superinertial policy-instrument rules may very well ensure (robust) local-equilibrium determinacy.

5.3 Extension to Models With Several Policy Intruments

Throughout the paper, I have focused on models with a single policy instrument. However, some of the results obtained can easily be extended to models with several policy instruments. Indeed, in such models, one can (i) set all the policy instruments, except one, according to their expressions in the VARMA specification of the feasible path considered, (ii) treat these policy-instrument rules in the same way as structural equations (if they involve some endogenous variables) or exogenous disturbances (if they do not), and (iii) apply Proposition 1 or 2 to the remaining policy instrument. As an illustration, in a previous version of this paper (Loisel, 2013), I proceed this way to establish, in one of the models considered by Correia, Farhi, Nicolini, and Teles (2013), the implementability of optimal unconventional fiscal policy with two tax instruments when the interest rate is at the zero lower bound.

6 Conclusion

Overall, one important message of the paper is that the policy maker's observation set is a key feature of the environment that should be explicitly stated alongside other features such as preferences, technologies, and markets. Not only do the set of feasible paths and its subset of implementable paths depend on this observation set, but they do so in partly unrelated ways (as a change in the observation set may affect either both of them, or only the former, or only the latter, or none of them). Given the non-trivial nature of these links, the first main contribution of the paper has been to characterize − at least partially − the subset of implementable paths, within the set of feasible paths, for various alternative observation sets of the policy maker, in a broad class of models that includes many, if not most of the models currently used for macroeconomic-stabilization-policy analysis. As a side methodological result, the paper has also shown, for each observation set and implementable path, how to design arithmetically a policy-instrument rule that is consistent with this observation set and implements that path as the robustly unique local equilibrium, so that the coefficients of this rule can be explicitly expressed as rational functions of the structural and implementable-path parameters.

These general results can be readily applied to many different models, policy instruments, observation sets, and feasible paths. The second main contribution of the paper has precisely been to apply them to two specific contexts, concerning two different stabilization policies in two different textbook models. The first application has shown that optimal feasible monetary policy may not be implementable in the basic New Keynesian model for reasonable observation sets of the central bank. One implication of this result is that central banks should make a cautious use of the key unobserved rates of interest that the New Keynesian literature recommends them to track. Another one is that normative analyses of stabilization policy should consider as a benchmark not only the optimal feasible path, but also the optimal implementable path or some proxy for the latter path.[43] The second application has shown that,

[42] The term "superinertial" was coined by Woodford (1999a).

[43] The optimal implementable path can typically not be determined when it does not coincide with the optimal feasible path, because its determination then requires to optimize over a set of policy-instrument rules with a finite but unbounded number of coefficients and an unbounded range of values for each coefficient, subject to the robust-local-equilibrium-determinacy constraint. A proxy for this path can then be obtained by focusing on rules with a large but bounded number

contrary to conventional wisdom, debt-stabilizing feasible tax policy is implementable in the standard RBC model for reasonable observation sets of the tax authority. In so doing, it has illustrated the fact that focusing on specific parametric families of policy-instrument rules may provide a misleading picture of the implementability of a given feasible path. These two applications are, of course, specific in many ways, but the door is left open to a wide range of other applications.

Appendix

In this appendix, I prove Lemmas 1 and 2 and Propositions 1 and 4, and I study the robustness of Subsection 4.1's non-implementability result.

A.1 Proof of Lemma 1

Consider a system of structural equations of type (1) and a rule of type (3) satisfying (13). Note them, respectively, $\mathbf{S}$ and R (with the understanding that any operator applied to the system $\mathbf{S}$ or the rule R is applied to both the left- and right-hand sides of this system or rule). I proceed in two steps: first, I show that the system made of $\mathbf{S}$ and R can be written in Blanchard and Kahn's (1980) form with n^a non-predetermined variables; second, I show that its non-zero eigenvalues are the non-zero roots of (14).

Step 1: for each $j \in \mathcal{S}_B$, replace sequentially, for $k = 0, ..., n_j^b + n^a$, the term $\mathbb{E}_t\{i_{t+n_j^b-k}\}$ (if it appears) in $\mathbf{u}_j'\mathbf{S}$ by its expression in $\mathbb{E}_t\{L^{k-n_j^b}R\}$. Let $\widetilde{\mathbf{S}}$ denote the resulting system, and let $(\widetilde{n}_j^a)_{1\leq j\leq N}$ and $\widehat{\widehat{\mathbf{A}}}(X)$ denote the counterparts of $(n_j^a)_{1\leq j\leq N}$ and $\widehat{\mathbf{A}}(X)$ for $\widetilde{\mathbf{S}}$. Note that $\widehat{\widehat{\mathbf{A}}}(0) = \widehat{\mathbf{A}}(0)$, since R satisfies (13). The rest of Step 1 is essentially devoted to the rewriting of $\widetilde{\mathbf{S}}$. For simplicity, I will keep the same notations $\widetilde{\mathbf{S}}$, $(\widetilde{n}_j^a)_{1\leq j\leq N}$, and $\widehat{\widehat{\mathbf{A}}}(X)$ throughout the rewriting process. The operation conducted at the beginning of Step 1 ensures that none of the rewritten systems $\widetilde{\mathbf{S}}$ will include any term of type $\mathbb{E}_t\{i_{t+k}\}$ with $k \in \mathbb{N}$. Now re-order the lines of $\widetilde{\mathbf{S}}$ so that $\widetilde{n}_1^a \geq ... \geq \widetilde{n}_N^a$. Let $K \in \{1, ..., N\}$ and $(j_1, ..., j_K) \in \{1, ..., N\}^K$ be such that $\widetilde{n}_1^a = ... = \widetilde{n}_{j_1}^a > \widetilde{n}_{j_1+1}^a = ... = \widetilde{n}_{j_2}^a > ... > \widetilde{n}_{j_{K-1}+1}^a = ... = \widetilde{n}_{j_K}^a = \widetilde{n}_N^a$. Re-order the columns of $\widetilde{\mathbf{A}}(X)$ so that $\forall j \in \{1, ..., N-1\}$, the $(N-j) \times (N-j)$ matrix noted $\mathbf{M}_j$ obtained by removing the first j lines and the first j columns from $\widehat{\widehat{\mathbf{A}}}(0)$ is invertible, this re-ordering being made possible by Assumption 4. Re-order the elements of $\mathbf{Y}_t$ accordingly, and note $\widetilde{\mathbf{Y}}_t$ the resulting vector. For each $j \in \{1, ..., j_1\}$, replace $\mathbf{u}_j'\widetilde{\mathbf{S}}$ by

$$\mathbf{u}_j'\widehat{\widehat{\mathbf{A}}}(0)^{-1}\mathbb{E}_t\left\{\begin{bmatrix} \mathbf{u}_1' & L^{\widetilde{n}_2^a-\widetilde{n}_1^a}\mathbf{u}_2' & \cdots & L^{\widetilde{n}_N^a-\widetilde{n}_1^a}\mathbf{u}_N' \end{bmatrix}\widetilde{\mathbf{S}}\right\}.$$

This operation makes the first j_1 lines of $\widehat{\widehat{\mathbf{A}}}(0)$ identical to the first j_1 lines of $\mathbf{I}_N$. If $K = 1$, then the system made of $\widetilde{\mathbf{S}}$ and R, which is equivalent to the system made of $\mathbf{S}$ and R, can easily be written in Blanchard and Kahn's (1980) form with $j_1\widetilde{n}_{j_1}^a = \sum_{j=1}^N n_j^a = n^a$ non-predetermined variables. Alternatively, if $K \geq 2$, then for each $i \in \{j_1+1, ..., N\}$ and each $j \in \{1, ..., j_1\}$, replace sequentially, for $k = 1, ..., \widetilde{n}_{j_1}^a - \widetilde{n}_{j_2}^a$, the term $\mathbb{E}_t\{\mathbf{u}_i'\widetilde{\mathbf{Y}}_{t+\widetilde{n}_{j_1}^a-k}\}$ (if it appears) in $\mathbf{u}_j'\widetilde{\mathbf{S}}$ by its expression in

$$\mathbf{M}_{j_1}^{-1}\mathbb{E}_t\left\{\begin{bmatrix} L^{\widetilde{n}_{j_1+1}^a-\widetilde{n}_{j_1}^a+k}\mathbf{u}_{j_1+1}' & \cdots & L^{\widetilde{n}_N^a-\widetilde{n}_{j_1}^a+k}\mathbf{u}_N' \end{bmatrix}\widetilde{\mathbf{S}}\right\}.$$

This operation does not affect the first j_1 lines of $\widehat{\widehat{\mathbf{A}}}(0)$, and ensures in particular that for each $i \in \{j_1+1, ..., j_2\}$, (i) $\widetilde{\mathbf{S}}$ does not include any term of type $\mathbb{E}_t\{\mathbf{u}_i'\widetilde{\mathbf{Y}}_{t+k}\}$ with $\widetilde{n}_{j_2}^a < k < \widetilde{n}_{j_1}^a$, and (ii) $\mathbf{u}_{j_1+1}'\widetilde{\mathbf{S}}$,

of coefficients and a large but bounded range of values for each coefficient.

..., $\mathbf{u}'_{j_2}\widetilde{\mathbf{S}}$ are the only lines of $\widetilde{\mathbf{S}}$ in which $\mathbb{E}_t\{\mathbf{u}'_i\widetilde{\mathbf{Y}}_{t+\widetilde{n}^a_{j_2}}\}$ may appear. Proceed in a similar way as previously to transform $\mathbf{u}'_j\widetilde{\mathbf{S}}$ for $j \in \{j_1+1, ..., j_2\}$, then (if $K \geq 3$) $\mathbf{u}'_j\widetilde{\mathbf{S}}$ for $j \in \{j_2+1, ..., j_3\}$, and so on up to $\mathbf{u}'_j\widetilde{\mathbf{S}}$ for $j \in \{j_{K-1}+1, ..., j_K\}$. Then, for each $i \in \{1, ..., N\}$, (i) the only term of type $\mathbb{E}_t\{\mathbf{u}'_i\widetilde{\mathbf{Y}}_{t+k}\}$ with $k \geq \min\{\widetilde{n}^a_j | 1 \leq j \leq N\}$ that appears in $\widetilde{\mathbf{S}}$ is $\mathbb{E}_t\{\mathbf{u}'_i\widetilde{\mathbf{Y}}_{t+\widetilde{n}^a_i}\}$, and (ii) this term appears only in the line $\mathbf{u}'_i\widetilde{\mathbf{S}}$ and its coefficient is one (as $\widehat{\widehat{\mathbf{A}}}(0) = \mathbf{I}_N$). The system made of $\widetilde{\mathbf{S}}$ and R, which is equivalent to the system made of $\mathbf{S}$ and R, can then easily be written in Blanchard and Kahn's (1980) form with $\sum_{k=1}^{K} j_k \widetilde{n}^a_{j_k} = \sum_{j=1}^{N} n^a_j = n^a$ non-predetermined variables.

Step 2: let $\mathcal{S}'_B \equiv \{j \in \mathcal{S}_B | n^b_j \geq n^a_j\}$. For each $j \in \mathcal{S}'_B$, $\exists K_j(X) \in \mathbb{R}[X]$ such that $\Phi(X) \equiv X^{n^a_j - n}\mathbf{u}'_j\mathbf{B}(X) - X^{n^a_j - n^b_j}K_j(X)G(X)$ is a polynomial, i.e. $\Phi(X) \in \mathbb{R}[X]$, with $\Phi(0) = 0$. Now, the non-zero eigenvalues of the system made of $\mathbf{S}$ and R are those of the corresponding perfect-foresight deterministic system

$$\left[\begin{array}{c|c} L^{-n}\mathbf{A}(L) & L^{-n}\mathbf{B}(L) \\ \hline \mathbf{F}(L) & G(L) \end{array}\right] \left[\begin{array}{c} \mathbf{Y}_t \\ i_t \end{array}\right] = \mathbf{0},$$

which are in turn those of $\mathbf{\Psi}(L)[\begin{array}{cc} \mathbf{Y}'_t & i_t \end{array}]' = \mathbf{0}$ with

$$\mathbf{\Psi}(X) \equiv \left[\begin{array}{c|c} & X^{n^a_1 - n}\mathbf{u}'_1\mathbf{B}(X) \\ \widehat{\mathbf{A}}(X) & \vdots \\ & X^{n^a_N - n}\mathbf{u}'_N\mathbf{B}(X) \\ \hline \mathbf{F}(X) & G(X) \end{array}\right],$$

or equivalently those of $\mathbf{\Omega}(L)[\begin{array}{cc} \mathbf{Y}'_t & i_t \end{array}]' = \mathbf{0}$ with $\mathbf{\Omega}(X) \equiv$

$$\left[\begin{array}{c|c} \mathbf{u}'_1\widehat{\mathbf{A}}(X) - \mathbb{1}_{1\in\mathcal{S}'_B}X^{n^a_1 - n^b_1}K_1(X)\mathbf{F}(X) & X^{n^a_1 - n}\mathbf{u}'_1\mathbf{B}(X) - \mathbb{1}_{1\in\mathcal{S}'_B}X^{n^a_1 - n^b_1}K_1(X)G(X) \\ \vdots & \vdots \\ \mathbf{u}'_N\widehat{\mathbf{A}}(X) - \mathbb{1}_{N\in\mathcal{S}'_B}X^{n^a_N - n^b_N}K_N(X)\mathbf{F}(X) & X^{n^a_N - n}\mathbf{u}'_N\mathbf{B}(X) - \mathbb{1}_{N\in\mathcal{S}'_B}X^{n^a_N - n^b_N}K_N(X)G(X) \\ \hline \mathbf{F}(X) & G(X) \end{array}\right],$$

where, for any statement s, $\mathbb{1}_s \equiv 1$ if s is true and $\mathbb{1}_s \equiv 0$ if s is wrong. Given that R satisfies (13), and by construction of the polynomials $K_j(X)$ for $j \in \mathcal{S}'_B$, I get that $\mathbf{\Omega}(X)$ is a polynomial, i.e. $\mathbf{\Omega}(X) \in \mathbb{R}^{(N+1)\times(N+1)}[X]$, with

$$\mathbf{\Omega}(0) = \left[\begin{array}{c|c} \widehat{\mathbf{A}}(0) & \mathbf{0} \\ \hline \mathbf{F}(0) & G(0) \end{array}\right].$$

Since $|\widehat{\mathbf{A}}(0)| \neq 0$ and $G(0) \neq 0$, $\mathbf{\Omega}(0)$ is invertible, so that according to a standard result in time-series analysis (see, e.g., Hamilton, 1994, Chapter 10, Proposition 10.1), the non-zero eigenvalues of the system made of $\mathbf{S}$ and R are the non-zero roots of $|X^{d_{\mathbf{\Omega}}}\mathbf{\Omega}(X^{-1})|$. Note that $\mathbf{\Omega}(X)$ has been obtained from $\mathbf{\Psi}(X)$ by adding $-X^{n^a_j - n^b_j}K_j(X)$ times the $(N+1)^{th}$ line of $\mathbf{\Psi}(X)$ to the j^{th} line of $\mathbf{\Psi}(X)$ for each $j \in \mathcal{S}'_B$. Now, adding a scalar multiple of one row to another row leaves the determinant of a matrix unchanged. Therefore, $|X^{d_{\mathbf{\Omega}}}\mathbf{\Omega}(X^{-1})| = |X^{d_{\mathbf{\Omega}}}\mathbf{\Psi}(X^{-1})|$. Finally, $|X^{d_{\mathbf{\Omega}}}\mathbf{\Psi}(X^{-1})|$ is equal to (14) up to a multiplicative factor of type X^z with $z \in \mathbb{Z}$. Therefore, the non-zero roots of $|X^{d_{\mathbf{\Omega}}}\mathbf{\Psi}(X^{-1})|$ are those of (14). Lemma 1 follows.

A.2 Proof of Lemma 2

Consider the policy-instrument rule $i_t = \widetilde{\mathbf{H}}(L)\boldsymbol{\varepsilon}_t$, where $\widetilde{\mathbf{H}}(X)$ is an arbitrary element of $\mathbb{R}[X]$. This rule is of type (3) with $\mathbf{F}(X) = \mathbf{0}$ and $G(X) = 1$, and satisfies (13). Therefore, Lemma 1 implies that (i) the system made of (1) and this rule can be written in Blanchard and Kahn's (1980) form with n^a non-predetermined variables, and (ii) the non-zero eigenvalues of this system are (and have the same multiplicity as) the non-zero roots of (14), i.e. the non-zero roots of $\Delta_{N+1}(X)$. Now, given Assumption 6, this system has at least one stationary solution for $(\mathbf{Y}_t)_{t\in\mathbb{Z}}$. Therefore, Blanchard and Kahn's (1980) order condition implies that $\Delta_{N+1}(X)$ has at most n^a roots outside the unit circle (taking into account their multiplicity), and hence so does $\mathcal{D}_{\mathcal{J}}(X)$ for any set $\mathcal{J} \subseteq \{1,...,N\}$. Lemma 2 follows.

A.3 Proof of Proposition 1

Consider a given set $\mathcal{J}$ such that $\mathcal{J}^* \subseteq \mathcal{J} \subseteq \{1,...,N\}$ and a given $(\ell_Y, \ell_\varepsilon) \in \mathbb{N}^2$ such that $\ell_\varepsilon \leq \ell_Y$. Consider also a given feasible path for $O_t = \{\mathbf{u}_j'\mathbf{Y}^{t-\ell_Y}, i^{t-1}, \boldsymbol{\varepsilon}^{t-\ell_\varepsilon} | j \in \mathcal{J}\}$, note it P, and write it as (4). I proceed in two steps: first, I design some $\mathbf{F}(X)$ satisfying (10) and (11) and some $G(X)$ such that, whatever $\mathbf{H}(X)$, the system made of the structural equations (1) and the corresponding rule (3) has a unique stationary solution (even when an exogenous policy shock is added to this rule); second, I design $\mathbf{H}(X)$ satisfying (12) such that this solution is P.

Step 1: Bézout's identity implies that there exists $(\mathcal{U}_j(X))_{j\in\mathcal{J}\cup\{N+1\}} \in \mathbb{R}[X]^{|\mathcal{J}|+1}$ such that

$$\sum\nolimits_{j\in\mathcal{J}\cup\{N+1\}} \mathcal{U}_j(X)\,\Delta_j(X) = \mathcal{D}_{\mathcal{J}}(X). \tag{A.1}$$

Let $\Theta(X) \equiv |X^{d_\mathbf{S}}\mathbf{S}(X^{-1})| \in \mathbb{R}[X]$. Let $\mathcal{Z}(X)$ be a given polynomial such that (i) $\mathcal{Z}(X)\mathcal{D}_{\mathcal{J}}(X)$ has exactly n^a roots outside the unit circle (taking into account their multiplicity), and (ii) $\Theta(X)$ is a divisor of $\mathcal{Z}(X)\mathcal{D}_{\mathcal{J}}(X)$. The existence of $\mathcal{Z}(X)$ is a consequence of (i) Assumptions 4 and 5, which imply that $\forall j \in \{1,...,N+1\}$, $\Delta_j(X) \neq 0$, and therefore that $\mathcal{D}_{\mathcal{J}}(X) \neq 0$, (ii) Lemma 2, which states that $\mathcal{D}_{\mathcal{J}}(X)$ has at most n^a roots outside the unit circle, and (iii) the restriction to local paths, which requires that all the roots of $\Theta(X)$ lie inside the unit circle. Let $m \in \mathbb{N}$ be such that

$$m \geq 2d_{\Delta_{N+1}} - d_{\mathcal{D}_{\mathcal{J}}} - d_{\mathcal{Z}} + \max\left\{d_{\mathcal{U}_{N+1}} + 1, \max_{j\in\mathcal{J}}\left(d_{\mathcal{U}_j}\right) + \max\left[\ell_Y, \max_{j\in\mathcal{S}_B}\left(n_j^b - n_j^a\right) + 1\right]\right\} - 1.$$

Finally, given that $\Delta_{N+1}(X) \neq 0$, let $\mathcal{Q}(X) \in \mathbb{R}[X]$ and $\mathcal{R}(X) \in \mathbb{R}[X]$ denote respectively the quotient and the remainder of the Euclidean division of $X^m\mathcal{Z}(X)$ by $\Delta_{N+1}(X)$, i.e. the unique polynomials such that $X^m\mathcal{Z}(X) = \Delta_{N+1}(X)\mathcal{Q}(X) + \mathcal{R}(X)$ and $d_{\mathcal{R}} < d_{\Delta_{N+1}}$. Multiplying the left- and right-hand sides of (A.1) by $\mathcal{R}(X)$, I obtain $\mathcal{R}(X)\sum_{j\in\mathcal{J}\cup\{N+1\}}\mathcal{U}_j(X)\Delta_j(X) = \mathcal{R}(X)\mathcal{D}_{\mathcal{J}}(X)$ and therefore

$$\sum\nolimits_{j\in\mathcal{J}} \left[\mathcal{R}(X)\mathcal{U}_j(X)\right]\Delta_j(X) + \left[\mathcal{R}(X)\mathcal{U}_{N+1}(X) + \mathcal{Q}(X)\mathcal{D}_{\mathcal{J}}(X)\right]\Delta_{N+1}(X) = X^m\mathcal{Z}(X)\mathcal{D}_{\mathcal{J}}(X). \tag{A.2}$$

Let $\mathcal{F}_j(X) \equiv \mathcal{R}(X)\mathcal{U}_j(X)$ for $j \in \mathcal{J}$, $\mathcal{F}_j(X) \equiv 0$ for $j \in \{1,...,N\} \setminus \mathcal{J}$, and $\mathcal{G}(X) \equiv \mathcal{R}(X)\mathcal{U}_{N+1}(X) + \mathcal{Q}(X)\mathcal{D}_{\mathcal{J}}(X)$. The choice of $G(X) \equiv |\mathbf{D}(X)|X^{d_\mathcal{G}}\mathcal{G}(X^{-1}) \in \mathbb{R}[X]$ is admissible since it implies that $G(0) \neq 0$. Moreover, given that

$$\begin{cases} m \geq 2d_{\Delta_{N+1}} - d_{\mathcal{D}_{\mathcal{J}}} - d_{\mathcal{Z}} + \max\left\{d_{\mathcal{U}_{N+1}} + 1, \max_{j\in\mathcal{J}}\left(d_{\mathcal{U}_j}\right) + \max\left[\ell_Y, \max_{j\in\mathcal{S}_B}\left(n_j^b - n_j^a\right) + 1\right]\right\} - 1 \\ m = d_{\Delta_{N+1}} + d_{\mathcal{Q}} - d_{\mathcal{Z}} \\ d_{\Delta_{N+1}} > d_{\mathcal{R}} \end{cases}$$

$$\Longrightarrow d_{\mathcal{Q}} + d_{\mathcal{D}_{\mathcal{J}}} \geq d_{\mathcal{R}} + \max\left\{d_{\mathcal{U}_{N+1}} + 1, \max_{j\in\mathcal{J}}\left(d_{\mathcal{U}_j}\right) + \max\left[\ell_Y, \max_{j\in\mathcal{S}_B}\left(n_j^b - n_j^a\right) + 1\right]\right\}$$

$$\Longrightarrow \begin{cases} d_{\mathcal{Q}} + d_{\mathcal{D}_{\mathcal{J}}} > d_{\mathcal{R}} + d_{\mathcal{U}_{N+1}} \\ d_{\mathcal{Q}} + d_{\mathcal{D}_{\mathcal{J}}} \geq d_{\mathcal{R}} + \max_{j\in\mathcal{J}}\left(d_{\mathcal{U}_j}\right) + \max\left[\ell_Y, \max_{j\in\mathcal{S}_B}\left(n_j^b - n_j^a\right) + 1\right] \end{cases}$$

$$\Longrightarrow d_{\mathcal{G}} = d_{\mathcal{Q}} + d_{\mathcal{D}_{\mathcal{J}}} \geq \max_{j\in\mathcal{J}}\left(d_{\mathcal{F}_j}\right) + \max\left[\ell_Y, \max_{j\in\mathcal{S}_B}\left(n_j^b - n_j^a\right) + 1\right],$$

the choice of $\mathbf{F}(X) \equiv |\mathbf{D}(X)|X^{d_{\mathcal{G}}} \sum_{j=1}^{N}(-1)^{N+1-i}\mathcal{F}_j(X^{-1})\mathbf{u}_j' \in \mathbb{R}^{1\times N}[X]$ is admissible too since it implies that $\mathbf{F}(X)$ satisfies (10) and (11). Finally, this choice is such that $\mathbf{F}(X)$ also satisfies (13). Lemma 1 then implies that any rule of type (3) with these $\mathbf{F}(X)$ and $G(X)$ is such that (i) the system made of the structural equations (1) and this rule can be written in Blanchard and Kahn's (1980) form with n^a non-predetermined variables, and (ii) the non-zero eigenvalues of this system are the non-zero roots of (14) and hence, given (A.2), the non-zero roots of $X^{d_{\mathcal{W}}}\mathcal{W}(X^{-1})\mathcal{Z}(X)\mathcal{D}_{\mathcal{J}}(X)$, where $\mathcal{W}(X) \equiv |\mathbf{D}(X)|$. Now, by construction of $\mathcal{Z}(X)$, $X^{d_{\mathcal{W}}}\mathcal{W}(X^{-1})\mathcal{Z}(X)\mathcal{D}_{\mathcal{J}}(X)$ has exactly n^a roots outside the unit circle (taking into account their multiplicity). Therefore, the system made of the structural equations (1) and any rule of type (3) with these $\mathbf{F}(X)$ and $G(X)$ meets Blanchard and Kahn's (1980) *order* condition. Moreover, except possibly for a zero-measure subset of the set of admissible polynomials $(\mathcal{U}_j(X))_{j\in\mathcal{J}\cup\{N+1\}}$ and $\mathcal{Z}(X)$, this system meets Blanchard and Kahn's (1980) *rank* condition too, given the assumption $\mathcal{J}^* \subseteq \mathcal{J}$. As a consequence, this sytem has a unique stationary solution for $(\mathbf{Y}_t, i_t)_{t\in\mathbb{Z}}$, even when an exogenous policy shock is added to the rule.

Step 2: consider first the case in which $\max\{n_j^b - n_j^a | j \in \mathcal{S}_B\} < 0$. There exists a unique power series $\mathbf{\Xi}(X) \equiv \sum_{k=0}^{+\infty} \mathbf{\Xi}_k X^k$, whose coefficients $\mathbf{\Xi}_k$ are $1 \times M$ matrices with real-number elements, such that, along the path P,

$$\mathbf{F}(L)\,\mathbf{Y}_t + G(L)\,i_t + \mathbf{\Xi}(L)\,\boldsymbol{\varepsilon}_t = \mathbf{0}, \tag{A.3}$$

where $\mathbf{F}(X)$ and $G(X)$ are the polynomials designed in Step 1. If $\ell_\varepsilon \geq 1$, then this series is such that $\forall k \in \{0, ..., \ell_\varepsilon - 1\}$, $\mathbf{\Xi}_k = 0$, since $\ell_\varepsilon \leq \ell_Y$. Therefore, $\mathbf{H}(X) \equiv \mathbf{\Xi}(X)$ satisfies (12), and it is an admissible choice if and only if the number of non-zero coefficients $\mathbf{\Xi}_k$ is finite, i.e. if and only if the power series $\mathbf{\Xi}(X)$ is a polynomial: $\mathbf{\Xi}(X) \in \mathbb{R}^{1\times M}[X]$. Let me now show that the latter condition is met. There exists a unique $\mathbf{\Pi}(X) \in \mathbb{R}^{N\times M}[X]$, with $d_{\mathbf{u}_j'\mathbf{\Pi}} \leq n_j^a - 1$ for each $j \in \{0, ..., N\}$, such that, along the path P,

$$\left[\begin{array}{c|c} \widehat{\mathbf{A}}(L) & \begin{array}{c} L^{n_1^a - n}\mathbf{u}_1'\mathbf{B}(L) \\ \vdots \\ L^{n_N^a - n}\mathbf{u}_N'\mathbf{B}(L) \end{array} \end{array}\right] \left[\begin{array}{c} \mathbf{Y}_t \\ i_t \end{array}\right] + \left[\begin{array}{c} L^{n_1^a}\mathbf{u}_1'\mathbf{C}(L) \\ \vdots \\ L^{n_N^a}\mathbf{u}_N'\mathbf{C}(L) \end{array}\right] \boldsymbol{\xi}_t + \mathbf{\Pi}(L)\,\boldsymbol{\varepsilon}_t = \mathbf{0}. \tag{A.4}$$

Since, as a scalar polynomial, $|\mathbf{D}(X)|$ is such that $|\mathbf{D}(X)|\mathbf{K} = \mathbf{K}|\mathbf{D}(X)|$ for any matrix $\mathbf{K}$, multiplying the left- and right-hand sides of (A.4) by $|\mathbf{D}(L)|$ leads to

$$|\mathbf{D}(L)| \left[\begin{array}{c|c} \widehat{\mathbf{A}}(L) & \begin{array}{c} L^{n_1^a - n}\mathbf{u}_1'\mathbf{B}(L) \\ \vdots \\ L^{n_N^a - n}\mathbf{u}_N'\mathbf{B}(L) \end{array} \end{array}\right] \left[\begin{array}{c} \mathbf{Y}_t \\ i_t \end{array}\right] + \left[\begin{array}{c} L^{n_1^a}\mathbf{u}_1'\mathbf{C}(L) \\ \vdots \\ L^{n_N^a}\mathbf{u}_N'\mathbf{C}(L) \end{array}\right] \widetilde{\mathbf{E}}(L)\boldsymbol{\varepsilon}_t + |\mathbf{D}(L)|\mathbf{\Pi}(L)\,\boldsymbol{\varepsilon}_t = \mathbf{0}, \tag{A.5}$$

where $\widetilde{\mathbf{E}}(X)$ is the unique element of $\mathbb{R}^{M\times M}[X]$ such that (2) can be equivalently rewritten as $|\mathbf{D}(L)|\boldsymbol{\xi}_t = \widetilde{\mathbf{E}}(L)\boldsymbol{\varepsilon}_t$. The system made of (A.3) and (A.5) is of type $\mathbf{\Lambda}_1(L)[\ \mathbf{Y}_t' \quad i_t\ ]' = \mathbf{\Lambda}_2(L)\boldsymbol{\varepsilon}_t$ where (i) $\mathbf{\Lambda}_1(X) \in \mathbb{R}^{(N+1)\times(N+1)}[X]$ is such that $|\mathbf{\Lambda}_1(0)| \neq 0$, since $|\mathbf{D}(0)| \neq 0$, $|\widehat{\mathbf{A}}(0)| \neq 0$, $\max\{n_j^b - n_j^a | j \in \mathcal{S}_B\} < 0$, and $G(0) \neq 0$, and (ii) $\mathbf{\Lambda}_2(X)$ is a power series whose coefficients are $(N+1) \times M$ matrices with real-number elements. Cramer's rule then implies that there exist $\mathbf{\Gamma}(X) \in \mathbb{R}^{(N+1)\times M}[X]$ and $(m_1, ..., m_{N+1}) \in \mathbb{N}^{N+1}$ with $m_j \geq d_{\Delta_j}$ for $j \in \{1, ..., N+1\}$ such that this system can be rewritten as

$$\begin{gathered} |\mathbf{D}(L)|^{N+1} L^{d_{\mathcal{Z}} + d_{\mathcal{D}_{\mathcal{J}}}} \mathcal{Z}\left(L^{-1}\right) \mathcal{D}_{\mathcal{J}}\left(L^{-1}\right) [\ \mathbf{Y}_t' \quad i_t\ ]' = \mathbf{\Gamma}(L)\,\boldsymbol{\varepsilon}_t + \\ [\ (-1)^{N+1} L^{m_1} \Delta_1\left(L^{-1}\right) \quad \cdots \quad (-1)^{2N+1} L^{m_{N+1}} \Delta_{N+1}\left(L^{-1}\right)\ ]' |\mathbf{D}(L)|^N \mathbf{\Xi}(L)\,\boldsymbol{\varepsilon}_t, \end{gathered} \tag{A.6}$$

given Step 1. But Cramer's rule also implies that there exists $\mathbf{\Upsilon}(X) \in \mathbb{R}^{(N+1)\times M}[X]$ such that the path

P can be rewritten as $L^{d_\Theta}\Theta(L^{-1})[\ \mathbf{Y}_t' \quad i_t\ ]' = \mathbf{\Upsilon}(L)\boldsymbol{\varepsilon}_t$, which implies

$$\begin{aligned} &|\mathbf{D}(L)|^{N+1}L^{d_{\mathcal{Z}}+d_{\mathcal{D}_{\mathcal{J}}}}\mathcal{Z}\left(L^{-1}\right)\mathcal{D}_{\mathcal{J}}\left(L^{-1}\right)[\ \mathbf{Y}_t' \quad i_t\ ]' = \\ &|\mathbf{D}(L)|^{N+1}[L^{d_\Theta}\Theta\left(L^{-1}\right)]^{-1}L^{d_{\mathcal{Z}}+d_{\mathcal{D}_{\mathcal{J}}}}\mathcal{Z}\left(L^{-1}\right)\mathcal{D}_{\mathcal{J}}\left(L^{-1}\right)\mathbf{\Upsilon}\left(L\right)\boldsymbol{\varepsilon}_t, \end{aligned} \tag{A.7}$$

where $[X^{d_\Theta}\Theta\left(X^{-1}\right)]^{-1}X^{d_{\mathcal{Z}}+d_{\mathcal{D}_{\mathcal{J}}}}\mathcal{Z}\left(X^{-1}\right)\mathcal{D}_{\mathcal{J}}\left(X^{-1}\right) \in \mathbb{R}[X]$ by construction of $\mathcal{Z}(X)$. Given Assumption 5, the identification of (A.6) with (A.7) shows that $\mathbf{\Xi}(X) \in \mathbb{R}^{1\times M}[X]$, so that the choice of $\mathbf{H}(X) \equiv \mathbf{\Xi}(X)$ is admissible. In the alternative case in which $\max\{n_j^b - n_j^a | j \in \mathcal{S}_B\} \geq 0$, an admissible $\mathbf{H}(X)$ can be designed in a similar way by using the fact that the $\mathbf{F}(X)$ designed in Step 1 satisfies (13). Therefore, the rule of type (3) with the $\mathbf{F}(X)$, $G(X)$, and $\mathbf{H}(X)$ designed in Steps 1 and 2 satisfies (10), (11), and (12), and is such that (i) the system made of (1) and this rule has a unique stationary solution (even when an exogenous policy shock is added to this rule), and (ii) the path P is one stationary solution of this system. Proposition 1 follows.

A.4 Proof of Proposition 4

Assume that $\rho_u \neq \mu$. Given that the timeless-perspective optimal feasible path is defined as the limit of the date-t_0 Ramsey-optimal feasible path as $t_0 \to -\infty$, I proceed in four steps: in the first step, I write the optimization problem that defines the date-t_0 Ramsey-optimal feasible path when $O_t = \{\varepsilon^{\eta,t-1}, \varepsilon^{u,t-1}\}$; in the second and third steps, I solve this problem and determine the date-t_0 Ramsey-optimal feasible path when $O_t = \{\varepsilon^{\eta,t-1}, \varepsilon^{u,t-1}\}$; in the fourth step, I deduce from the latter path the timeless-perspective optimal feasible path when $O_t = \{\varepsilon^{\eta,t-1}, \varepsilon^{u,t-1}\}$, show that it can be written as (24), conclude that it is also the timeless-perspective optimal feasible path when $O_t = \{c^{t-1}, \pi^{t-1}, y^{t-1}, n^{t-1}, w^{t-1}, i^{t-1}\}$, and finally show that the last line of (24) is a canonical rule for this path.

Step 1: the date-t_0 Ramsey-optimal feasible path is defined as the state-contingent path for the endogenous variables that minimizes the welfare loss function at date t_0 subject to the structural equations and $\mathcal{CB}$'s observation-set constraint. To determine this path when $O_t = \{\varepsilon^{\eta,t-1}, \varepsilon^{u,t-1}\}$, I follow the undetermined-coefficients method and specify the inflation rate and the output level in the following general linear way: $\pi_{t_0+k} = \sum_{j=0}^{k} a_{j,k}^{\pi}\varepsilon_{t_0+k-j}^{\eta} + \sum_{j=0}^{k} b_{j,k}^{\pi}\varepsilon_{t_0+k-j}^{u}$ and $y_{t_0+k} = \sum_{j=0}^{k} a_{j,k}^{y}\varepsilon_{t_0+k-j}^{\eta} + \sum_{j=0}^{k} b_{j,k}^{y}\varepsilon_{t_0+k-j}^{u}$ for $k \geq 0$.[44] I look for the values of coefficients $(a_{j,k}^{\pi}, b_{j,k}^{\pi}, a_{j,k}^{y}, b_{j,k}^{y})_{k\geq 0, 0\leq j\leq k}$ that minimize

$$L_{t_0} = \mathbb{E}_{t_0}\left\{\sum_{k=0}^{+\infty}\beta^k\left[\left(\sum_{j=0}^{k} a_{j,k}^{\pi}\varepsilon_{t_0+k-j}^{\eta} + \sum_{j=0}^{k} b_{j,k}^{\pi}\varepsilon_{t_0+k-j}^{u}\right)^2 + \lambda\left(\sum_{j=0}^{k} a_{j,k}^{y}\varepsilon_{t_0+k-j}^{\eta} + \sum_{j=0}^{k} b_{j,k}^{y}\varepsilon_{t_0+k-j}^{u}\right)^2\right]\right\}$$

subject to the following constraints:

$$a_{0,k}^{y} - a_{1,k+1}^{y} - \sigma a_{1,k+1}^{\pi} - (1-s) = 0 \text{ for } k \geq 0, \tag{A.8}$$

$$a_{j,k}^{\pi} - \beta a_{j+1,k+1}^{\pi} - \kappa a_{j,k}^{y} = 0 \text{ for } k \geq 0 \text{ and } j \in \{0, ..., k\}, \tag{A.9}$$

$$b_{0,k}^{y} - b_{1,k+1}^{y} - \sigma b_{1,k+1}^{\pi} = 0 \text{ for } k \geq 0, \tag{A.10}$$

$$b_{0,k}^{\pi} - \beta b_{1,k+1}^{\pi} - \kappa b_{0,k}^{y} - 1 = 0 \text{ for } k \geq 0, \tag{A.11}$$

$$b_{j,k}^{\pi} - \beta b_{j+1,k+1}^{\pi} - \kappa b_{j,k}^{y} - (\rho_u + \theta_u)\rho_u^{j-1} = 0 \text{ for } k \geq 1 \text{ and } j \in \{1, ..., k\}, \tag{A.12}$$

[44] I do not consider a deterministic term c_k^{π} (respectively c_k^{y}) in the expression of π_{t_0+k} (respectively y_{t_0+k}) because this term is clearly zero on the date-t_0 Ramsey-optimal feasible path.

where $\sigma \equiv \gamma(1-s)$. These constraints are derived from (i) the structural equations (17), (18), and (19), (ii) the stochastic processes (22) and (23), and (iii) the observation set O_t considered, which implies that i_t cannot depend on $(\varepsilon_t^\eta, \varepsilon_t^u)$. I note respectively $(\Omega_k^{IS,a})_{k\geq 0}$, $(\Omega_{j,k}^{PC,a})_{k\geq 0, 0\leq j\leq k}$, $(\Omega_k^{IS,b})_{k\geq 0}$, $(\Omega_{0,k}^{PC,b})_{k\geq 0}$, and $(\Omega_{j,k}^{PC,b})_{k\geq 1, 1\leq j\leq k}$ the Lagrange multipliers associated with these constraints. Since $\mathbb{E}\{\varepsilon_t^\eta \varepsilon_{t-k}^u\} = 0$ for any $k \in \mathbb{Z}$, coefficients $(a_{j,k}^\pi, a_{j,k}^y)_{k\geq 0, 0\leq j\leq k}$ are determined separately from coefficients $(b_{j,k}^\pi, b_{j,k}^y)_{k\geq 0, 0\leq j\leq k}$.

Step 2: let me first focus on the determination of coefficients $(a_{j,k}^\pi, a_{j,k}^y)_{k\geq 0, 0\leq j\leq k}$. The first-order conditions of the Lagrangian's minimization with respect to these coefficients are

$$\begin{aligned}
2V_\eta \beta^k a_{0,k}^\pi - \Omega_{0,k}^{PC,a} &= 0 \text{ for } k \geq 0, \\
2V_\eta \beta^k a_{1,k}^\pi + \sigma \Omega_{k-1}^{IS,a} - \Omega_{1,k}^{PC,a} + \beta \Omega_{0,k-1}^{PC,a} &= 0 \text{ for } k \geq 1, \\
2V_\eta \beta^k a_{j,k}^\pi - \Omega_{j,k}^{PC,a} + \beta \Omega_{j-1,k-1}^{PC,a} &= 0 \text{ for } k \geq 2 \text{ and } j \in \{2, ..., k\}, \\
2V_\eta \beta^k \lambda a_{0,k}^y - \Omega_k^{IS,a} + \kappa \Omega_{0,k}^{PC,a} &= 0 \text{ for } k \geq 0, \\
2V_\eta \beta^k \lambda a_{1,k}^y + \Omega_{k-1}^{IS,a} + \kappa \Omega_{1,k}^{PC,a} &= 0 \text{ for } k \geq 1, \\
2V_\eta \beta^k \lambda a_{j,k}^y + \kappa \Omega_{j,k}^{PC,a} &= 0 \text{ for } k \geq 2 \text{ and } j \in \{2, ..., k\},
\end{aligned}$$

where V_η denotes the variance of ε_t^η. After some algebra to get rid of the Lagrange multipliers, I obtain that these first-order conditions amount to the following conditions on coefficients $(a_{j,k}^\pi, a_{j,k}^y)_{k\geq 0, 0\leq j\leq k}$:

$$\beta\lambda a_{1,k}^y + (1+\kappa\sigma)\lambda a_{0,k-1}^y + \beta\kappa a_{1,k}^\pi + (1+\beta+\kappa\sigma)\kappa a_{0,k-1}^\pi = 0 \text{ for } k \geq 1, \tag{A.13}$$

$$\beta\kappa a_{2,k}^\pi + \beta\lambda a_{2,k}^y + \beta\kappa a_{1,k-1}^\pi + \kappa\lambda\sigma a_{0,k-2}^y + (\beta+\kappa\sigma)\kappa a_{0,k-2}^\pi = 0 \text{ for } k \geq 2, \tag{A.14}$$

$$\kappa a_{j,k}^\pi + \lambda a_{j,k}^y - \lambda a_{j-1,k-1}^y = 0 \text{ for } k \geq 3 \text{ and } j \in \{3, ..., k\}. \tag{A.15}$$

Noting $v \equiv k - j$, $A_{j,v}^\pi \equiv a_{j,k}^\pi$, and $A_{j,v}^y \equiv a_{j,k}^y$ for $k \geq 0$ and $0 \leq j \leq k$, I can rewrite (A.8), (A.9), (A.13), (A.14), and (A.15) respectively as

$$\begin{aligned}
A_{0,v}^y - A_{1,v}^y - \sigma A_{1,v}^\pi - (1-s) &= 0 \text{ for } v \geq 0, &(A.16)\\
A_{j,v}^\pi - \beta A_{j+1,v}^\pi - \kappa A_{j,v}^y &= 0 \text{ for } j \geq 0 \text{ and } v \geq 0, &(A.17)\\
\beta\lambda A_{1,v}^y + (1+\kappa\sigma)\lambda A_{0,v}^y + \beta\kappa A_{1,v}^\pi + (1+\beta+\kappa\sigma)\kappa A_{0,v}^\pi &= 0 \text{ for } v \geq 0, &(A.18)\\
\beta\kappa A_{2,v}^\pi + \beta\lambda A_{2,v}^y + \beta\kappa A_{1,v}^\pi + \kappa\lambda\sigma A_{0,v}^y + (\beta+\kappa\sigma)\kappa A_{0,v}^\pi &= 0 \text{ for } v \geq 0, &(A.19)\\
\kappa A_{j,v}^\pi + \lambda A_{j,v}^y - \lambda A_{j-1,v}^y &= 0 \text{ for } j \geq 3 \text{ and } v \geq 0, &(A.20)
\end{aligned}$$

which implies that $\forall j \geq 0$, $A_{j,v}^\pi$ and $A_{j,v}^y$ do not depend on v, so that I can note them a_j^π and a_j^y respectively. Equations (A.17) and (A.20) imply the recurrence equation $\beta\lambda a_{j+2}^\pi - (\beta\lambda + \kappa^2 + \lambda)a_{j+1}^\pi + \lambda a_j^\pi = 0$ for $j \geq 2$. The roots of the corresponding characteristic polynomial are μ (defined in the main text) and $\mu' \equiv (2\beta\lambda)^{-1}[\lambda + \beta\lambda + \kappa^2 + \sqrt{(\lambda+\beta\lambda+\kappa^2)^2 - 4\beta\lambda^2}]$. Since $0 < \mu < 1$ and $\beta\mu'^2 \geq 1$, as can be readily checked, the solution of the recurrence equation that minimizes L_{t_0} is of the form $a_j^\pi = a_2^\pi \mu^{j-2}$ for $j \geq 2$. Equation (A.17) then implies that $a_j^y = (1-\beta\mu)\kappa^{-1}a_2^\pi\mu^{j-2}$ for $j \geq 2$. Coefficients a_0^π, a_1^π, a_2^π, a_0^y, and a_1^y are then determined by the linear system made of (A.16), (A.17) for $j \in \{0, 1\}$, (A.18), (A.19), and $a_2^y = (1-\beta\mu)\kappa^{-1}a_2^\pi$. I thus eventually obtain $a_0^\pi = a_0$, $a_1^\pi = a_1$, $a_j^\pi = a_2\mu^{j-2}$ for $j \geq 2$, $a_0^y = \kappa^{-1}(a_0 - \beta a_1)$, $a_1^y = \kappa^{-1}(a_1 - \beta a_2)$, and $a_j^y = (1-\beta\mu)\kappa^{-1}a_2\mu^{j-2}$ for $j \geq 2$, with $\begin{bmatrix} a_0 & a_1 & a_2 \end{bmatrix}' \equiv \mathbf{M}^{-1}\begin{bmatrix} 0 & 0 & \kappa(1-s) \end{bmatrix}'$, where

$$\mathbf{M} \equiv \begin{bmatrix} (\beta\kappa^2 + \kappa\lambda\sigma + \kappa^2 + \kappa^3\sigma + \lambda) & \beta\kappa(\kappa - \lambda\sigma) & -\beta^2\lambda \\ (\beta\kappa + \kappa^2\sigma + \lambda\sigma)\kappa & \beta\kappa(\kappa - \lambda\sigma) & \beta(-\beta\lambda\mu + \kappa^2 + \lambda) \\ 1 & -(1+\beta+\kappa\sigma) & \beta \end{bmatrix}.$$

Step 3: let me now turn to the determination of coefficients $(b^{\pi}_{j,k}, b^{y}_{j,k})_{k\geq 0, 0\leq j\leq k}$. The first-order conditions of the Lagrangian's minimization with respect to these coefficients are the same as those with respect to coefficients $(a^{\pi}_{j,k}, a^{y}_{j,k})_{k\geq 0, 0\leq j\leq k}$ except that $\forall k \geq 0$, $\forall j \in \{0, ..., k\}$, $a^{\pi}_{j,k}$, $a^{y}_{j,k}$, $\Omega^{IS,a}_{k}$, and $\Omega^{PC,a}_{j,k}$ should be respectively replaced by $b^{\pi}_{j,k}$, $b^{y}_{j,k}$, $\Omega^{IS,b}_{k}$, and $\Omega^{PC,b}_{j,k}$, and V_{η} by V_u, where V_u denotes the variance of ε^{u}_{t}. Therefore, after some algebra to get rid of the Lagrange multipliers, I obtain that these first-order conditions amount to three conditions on coefficients $(b^{\pi}_{j,k}, b^{y}_{j,k})_{k\geq 0, 0\leq j\leq k}$ that are the same as Equations (A.13), (A.14), and (A.15), except that $\forall k \geq 0$, $\forall j \in \{0, ..., k\}$, $a^{\pi}_{j,k}$ and $a^{y}_{j,k}$ should be respectively replaced by $b^{\pi}_{j,k}$ and $b^{y}_{j,k}$. Noting $v \equiv k - j$, $B^{\pi}_{j,v} \equiv b^{\pi}_{j,k}$, and $B^{y}_{j,v} \equiv b^{y}_{j,k}$ for $k \geq 0$ and $0 \leq j \leq k$, I can rewrite (A.10), (A.11), (A.12), and these three conditions respectively as

$$B^{y}_{0,v} - B^{y}_{1,v} - \sigma B^{\pi}_{1,v} = 0 \text{ for } v \geq 0, \quad \text{(A.21)}$$

$$B^{\pi}_{0,v} - \beta B^{\pi}_{1,v} - \kappa B^{y}_{0,v} - 1 = 0 \text{ for } v \geq 0, \quad \text{(A.22)}$$

$$B^{\pi}_{j,v} - \beta B^{\pi}_{j+1,v} - \kappa B^{y}_{j,v} - (\rho_u + \theta_u)\rho_u^{j-1} = 0 \text{ for } j \geq 1 \text{ and } v \geq 0, \quad \text{(A.23)}$$

$$\beta\lambda B^{y}_{1,v} + (1+\kappa\sigma)\lambda B^{y}_{0,v} + \beta\kappa B^{\pi}_{1,v} + (1+\beta+\kappa\sigma)\kappa B^{\pi}_{0,v} = 0 \text{ for } v \geq 0, \quad \text{(A.24)}$$

$$\beta\kappa B^{\pi}_{2,v} + \beta\lambda B^{y}_{2,v} + \beta\kappa B^{\pi}_{1,v} + \kappa\lambda\sigma B^{y}_{0,v} + (\beta+\kappa\sigma)\kappa B^{\pi}_{0,v} = 0 \text{ for } v \geq 0, \quad \text{(A.25)}$$

$$\kappa B^{\pi}_{j,v} + \lambda B^{y}_{j,v} - \lambda B^{y}_{j-1,v} = 0 \text{ for } j \geq 3 \text{ and } v \geq 0, \quad \text{(A.26)}$$

which implies that $\forall j \geq 0$, $B^{\pi}_{j,v}$ and $B^{y}_{j,v}$ do not depend on v, so that I can note them b^{π}_{j} and b^{y}_{j} respectively. Equations (A.23) and (A.26) imply the recurrence equation $\beta\lambda b^{\pi}_{j+2} - (\beta\lambda + \kappa^2 + \lambda)b^{\pi}_{j+1} + \lambda b^{\pi}_{j} = \lambda(1-\rho_u)(\rho_u+\theta_u)\rho_u^{j-1}$ for $j \geq 2$, which is identical to the recurrence equation obtained above for $(a^{\pi}_{j})_{j\geq 2}$ except for the term on the right-hand side. Therefore, the roots of the corresponding characteristic polynomial are μ, μ', and ρ_u, and, given that $0 < \mu < 1$, $\beta\mu'^2 \geq 1$, and $\rho_u \neq \mu$, the solution of the recurrence equation that minimizes L_{t_0} is of the form $b^{\pi}_{j} = (b^{\pi}_{2} - \varphi)\mu^{j-2} + \varphi\rho_u^{j-2}$ for $j \geq 2$, where $\varphi \in \mathbb{R}$. The recurrence equation for $j = 2$ implies that $\varphi = [\beta\lambda\rho_u^2 - (\beta\lambda + \kappa^2 + \lambda)\rho_u + \lambda]^{-1}\lambda(1-\rho_u)(\rho_u+\theta_u)\rho_u$. Equation (A.23) for $j \geq 2$ then implies that $b^{y}_{j} = (1-\beta\mu)\kappa^{-1}(b^{\pi}_{2} - \varphi)\mu^{j-2} + [(1-\beta\rho_u)\varphi - (\rho_u+\theta_u)\rho_u]\kappa^{-1}\rho_u^{j-2}$ for $j \geq 2$. Coefficients b^{π}_{0}, b^{π}_{1}, b^{π}_{2}, b^{y}_{0}, and b^{y}_{1} are then determined by the linear system made of (A.21), (A.22), (A.23) for $j = 1$, (A.24), (A.25), and $b^{y}_{2} = \kappa^{-1}[(1-\beta\mu)b^{\pi}_{2} + \beta(\mu-\rho_u)\varphi - (\rho_u+\theta_u)\rho_u]$. I thus eventually obtain $b^{\pi}_{0} = b_0$, $b^{\pi}_{1} = b_1$, $b^{\pi}_{j} = (b_2 - \varphi)\mu^{j-2} + \varphi\rho_u^{j-2}$ for $j \geq 2$, $b^{y}_{0} = \kappa^{-1}(b_0 - \beta b_1 - 1)$, $b^{y}_{1} = \kappa^{-1}[b_1 - \beta b_2 - (\rho_u+\theta_u)]$, and $b^{y}_{j} = (1-\beta\mu)\kappa^{-1}(b_2-\varphi)\mu^{j-2} + [(1-\beta\rho_u)\varphi - (\rho_u+\theta_u)\rho_u]\kappa^{-1}\rho_u^{j-2}$ for $j \geq 2$, where $[\ b_0 \quad b_1 \quad b_2\ ]' \equiv \mathbf{M}^{-1}[\ \lambda[1+\beta(\rho_u+\theta_u)+\kappa\sigma] \quad \lambda[\beta(\rho_u+\theta_u)\rho_u - \beta^2(\mu-\rho_u)\varphi + \kappa\sigma] \quad 1-(\rho_u+\theta_u)\ ]'$.

Step 4: the coefficients $(a^{\pi}_{j,k}, b^{\pi}_{j,k}, a^{y}_{j,k}, b^{y}_{j,k})_{k\geq 0, 0\leq j\leq k}$ that I have obtained in Steps 2 and 3 give me the inflation rate and the output level on the date-t_0 Ramsey-optimal feasible path when $O_t = \{\varepsilon^{\eta,t-1}, \varepsilon^{u,t-1}\}$, as functions of shocks having occurred since date t_0. By making t_0 tend towards $-\infty$, I straightforwardly get these two variables on the timeless-perspective optimal feasible path when $O_t = \{\varepsilon^{\eta,t-1}, \varepsilon^{u,t-1}\}$, as functions of all current and past shocks:

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \mathbf{P}_0 \begin{bmatrix} \varepsilon^{\eta}_{t} \\ \varepsilon^{u}_{t} \end{bmatrix} + \mathbf{P}_1 \begin{bmatrix} \varepsilon^{\eta}_{t-1} \\ \varepsilon^{u}_{t-1} \end{bmatrix} + \sum_{j=2}^{+\infty} \left(\mu^{j-2}\mathbf{P}_{\mu} + \rho_u^{j-2}\mathbf{P}_u\right) \begin{bmatrix} \varepsilon^{\eta}_{t-j} \\ \varepsilon^{u}_{t-j} \end{bmatrix}, \quad \text{(A.27)}$$

where $\mathbf{P}_0 \equiv \begin{bmatrix} a_0 & b_0 \\ \frac{a_0 - \beta a_1}{\kappa} & \frac{b_0 - \beta b_1 - 1}{\kappa} \end{bmatrix}$, $\mathbf{P}_1 \equiv \begin{bmatrix} a_1 & b_1 \\ \frac{a_1 - \beta a_2}{\kappa} & \frac{b_1 - \beta b_2 - (\rho_u+\theta_u)}{\kappa} \end{bmatrix}$,

$\mathbf{P}_{\mu} \equiv \begin{bmatrix} a_2 & b_2 - \varphi \\ \frac{(1-\beta\mu)a_2}{\kappa} & \frac{(1-\beta\mu)(b_2-\varphi)}{\kappa} \end{bmatrix}$, and $\mathbf{P}_u \equiv \begin{bmatrix} 0 & \varphi \\ 0 & \frac{(1-\beta\rho_u)\varphi - (\rho_u+\theta_u)\rho_u}{\kappa} \end{bmatrix}$.

Equations (19) to (21) and (A.27) can be equivalently rewritten in the VARMA form corresponding to the first five lines of (24) with $\mathbf{\Omega}_0 \equiv \mathbf{J}\mathbf{P}_0$, $\mathbf{\Omega}_1 \equiv \mathbf{J}[-(\mu+\rho_u)\mathbf{P}_0+\mathbf{P}_1]$, $\mathbf{\Omega}_2 \equiv \mathbf{J}[\mu\rho_u\mathbf{P}_0-(\mu+\rho_u)\mathbf{P}_1+\mathbf{P}_\mu+\mathbf{P}_u]$, and $\mathbf{\Omega}_3 \equiv \mathbf{J}[\mu\rho_u\mathbf{P}_1 - \rho_u\mathbf{P}_\mu - \mu\mathbf{P}_u]$, where

$$\mathbf{J} \equiv \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ (1-s)^{-1} & 0 & 1 & \alpha^{-1} & \gamma^{-1}(1-s)^{-1}+\alpha^{-1}\chi \end{bmatrix}'.$$

Moreover, rank$(\mathbf{\Omega}_0) = 2$, since rank$(\mathbf{P}_0) = 2$. Then, using (17), (19), and (A.27), I residually obtain the interest rate on the timeless-perspective optimal feasible path when $O_t = \{\varepsilon^{\eta,t-1}, \varepsilon^{u,t-1}\}$, as a function of all past shocks:

$$i_t = \mathbf{Q}_1 \begin{bmatrix} \varepsilon^\eta_{t-1} \\ \varepsilon^u_{t-1} \end{bmatrix} + \sum_{j=2}^{+\infty} \left(\mu^{j-2}\mathbf{Q}_\mu + \rho_\eta^{j-2}\mathbf{Q}_\eta + \rho_u^{j-2}\mathbf{Q}_u\right) \begin{bmatrix} \varepsilon^\eta_{t-j} \\ \varepsilon^u_{t-j} \end{bmatrix}, \tag{A.28}$$

where $\mathbf{Q}_1 \equiv \begin{bmatrix} \frac{(1+\beta-\beta\mu+\kappa\sigma)a_2-a_1+(1-s)\kappa(\rho_\eta+\theta_\eta)}{\kappa\sigma} & \frac{(1+\beta-\beta\mu+\kappa\sigma)b_2-b_1+\beta(\mu-\rho_u)\varphi+(\rho_u+\theta_u)(1-\rho_u)}{\kappa\sigma} \end{bmatrix}$,

$\mathbf{Q}_\mu \equiv \begin{bmatrix} \frac{-(\kappa-\lambda\sigma)a_2\mu}{\lambda\sigma} & \frac{-(\kappa-\lambda\sigma)(b_2-\varphi)\mu}{\lambda\sigma} \end{bmatrix}$, $\mathbf{Q}_\eta \equiv \begin{bmatrix} \frac{(1-s)(\rho_\eta+\theta_\eta)\rho_\eta}{\sigma} & 0 \end{bmatrix}$,

and $\mathbf{Q}_u \equiv \begin{bmatrix} 0 & \varphi\rho_u - \frac{(1-\rho_u)[(1-\beta\rho_u)\varphi-(\rho_u+\theta_u)\rho_u]}{\kappa\sigma} \end{bmatrix}$.

Multiplying the left- and right-hand sides of (A.28) by $(1-\rho_\eta L)$ leads to

$$i_t - \rho_\eta i_{t-1} = \mathbf{R}_{1,1} \begin{bmatrix} \varepsilon^\eta_{t-1} \\ \varepsilon^u_{t-1} \end{bmatrix} + \mathbf{R}_{2,1} \begin{bmatrix} \varepsilon^\eta_{t-2} \\ \varepsilon^u_{t-2} \end{bmatrix} + \sum_{j=3}^{+\infty} \left(\mu^{j-3}\mathbf{R}_{\mu,1} + \rho_u^{j-3}\mathbf{R}_{u,1}\right) \begin{bmatrix} \varepsilon^\eta_{t-j} \\ \varepsilon^u_{t-j} \end{bmatrix}, \tag{A.29}$$

where $\mathbf{R}_{1,1} \equiv \mathbf{Q}_1$, $\mathbf{R}_{2,1} \equiv -\rho_\eta\mathbf{Q}_1 + \mathbf{Q}_\mu + \mathbf{Q}_\eta + \mathbf{Q}_u$, $\mathbf{R}_{\mu,1} \equiv (\mu-\rho_\eta)\mathbf{Q}_\mu$, and $\mathbf{R}_{u,1} \equiv (\rho_u-\rho_\eta)\mathbf{Q}_u$. By recurrence on $k \in \mathbb{N}^*$, using (A.27) and (A.29) at date $t-k$, I easily get that, for any $k \in \mathbb{N}^*$ and any $(\delta_j)_{j\in\{1,...,k\}} \in \mathbb{R}^k$,

$$\begin{aligned} i_t - \rho_\eta i_{t-1} = \sum_{j=1}^{k} \left\{\mathbf{R}_{1,j}\mathbf{P}_0^{-1} \begin{bmatrix} \pi_{t-j} \\ y_{t-j} \end{bmatrix} + \delta_j(i_{t-j} - \rho_\eta i_{t-j-1})\right\} + \mathbf{R}_{1,k+1} \begin{bmatrix} \varepsilon^\eta_{t-k-1} \\ \varepsilon^u_{t-k-1} \end{bmatrix} \\ +\mathbf{R}_{2,k+1} \begin{bmatrix} \varepsilon^\eta_{t-k-2} \\ \varepsilon^u_{t-k-2} \end{bmatrix} + \sum_{j=k+3}^{+\infty} \left(\mu^{j-k-3}\mathbf{R}_{\mu,k+1} + \rho_u^{j-k-3}\mathbf{R}_{u,k+1}\right) \begin{bmatrix} \varepsilon^\eta_{t-j} \\ \varepsilon^u_{t-j} \end{bmatrix}, \end{aligned}$$

where the matrices noted $\mathbf{R}$ with a subscript are recursively defined by $\mathbf{R}_j \equiv \begin{bmatrix} \mathbf{R}_{1,j} & \mathbf{R}_{2,j} & \mathbf{R}_{\mu,j} & \mathbf{R}_{u,j} \end{bmatrix}$ and $\mathbf{R}_{j+1} = \mathbf{R}_j\mathbf{P} - \delta_j\mathbf{R}_1$ for all $j \in \mathbb{N}^*$, with

$$\mathbf{P} \equiv \begin{bmatrix} -\mathbf{P}_0^{-1}\mathbf{P}_1 & -\mathbf{P}_0^{-1}(\mathbf{P}_\mu+\mathbf{P}_u) & -\mu\mathbf{P}_0^{-1}\mathbf{P}_\mu & -\rho_u\mathbf{P}_0^{-1}\mathbf{P}_u \\ \mathbf{I}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{I}_2 & \mu\mathbf{I}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{I}_2 & \mathbf{0}_2 & \rho_u\mathbf{I}_2 \end{bmatrix},$$

where in turn, for any $m \in \mathbb{N}$, $\mathbf{0}_m$ denotes the $m \times m$ zero matrix. Let d denote the degree of the minimal polynomial of the restriction of $\mathbf{P}'$ to $\{\mathbf{R}_1'\}$, i.e. the unique element of $\{1, ..., 8\}$ such that

$$\text{rank} \begin{bmatrix} \mathbf{R}_1\mathbf{P}^d \\ \vdots \\ \mathbf{R}_1\mathbf{P}^0 \end{bmatrix} = \text{rank} \begin{bmatrix} \mathbf{R}_1\mathbf{P}^{d-1} \\ \vdots \\ \mathbf{R}_1\mathbf{P}^0 \end{bmatrix} = d.$$

Since $\mathbf{R}_{d+1} = \mathbf{R}_1(\mathbf{P}^d - \sum_{j=1}^d \delta_j\mathbf{P}^{d-j})$, the choice of $\begin{bmatrix} \delta_1 & ... & \delta_d \end{bmatrix}$ such that

$$\begin{bmatrix} \delta_1 & ... & \delta_d \end{bmatrix} \begin{bmatrix} \mathbf{R}_1\mathbf{P}^{d-1} \\ \vdots \\ \mathbf{R}_1\mathbf{P}^0 \end{bmatrix} = \mathbf{R}_1\mathbf{P}^d$$

implies $\mathbf{R}_{d+1} = \mathbf{0}$. Therefore, the timeless-perspective optimal feasible path when $O_t = \{\varepsilon^{\eta,t-1}, \varepsilon^{u,t-1}\}$ is such that the equation

$$i_t = \sum_{j=1}^{9} (f_j^\pi \pi_{t-j} + f_j^y y_{t-j} + g_j i_{t-j}) \tag{A.30}$$

is satisfied on this path, where $[\ f_j^\pi \quad f_j^y\ ] \equiv \mathbf{R}_{1,j} \mathbf{P}_0^{-1}$ for $j \in \{1, ..., d\}$, $[\ f_j^\pi \quad f_j^y\ ] \equiv [\ 0 \quad 0\ ]$ for $j \in \{d+1, ..., 9\}$, $g_1 \equiv \delta_1 + \rho_\eta$, $g_j \equiv \delta_j - \rho_\eta \delta_{j-1}$ for $j \in \{2, ..., d\}$ if $d \geq 2$, $g_{d+1} \equiv -\rho_\eta \delta_d$, and $g_j = 0$ for $j \in \{d+2, ..., 9\}$ if $d \leq 7$. As a consequence, this path coincides with the timeless-perspective optimal feasible path when $O_t = \{c^{t-1}, \pi^{t-1}, y^{t-1}, n^{t-1}, w^{t-1}, i^{t-1}\}$. Finally, by construction of d, (A.30) is a canonical rule for the latter path. Proposition 4 follows.

A.5 Robustness of Subsection 4.1's Non-Implementability Result

Suppose that the economy is also hit by three additional disturbances, a government-purchases disturbance g_t, a productivity disturbance a_t, and a consumption-utility or labor-disutility disturbance ν_t, so that the Euler equation (17) is left unchanged and the other structural equations (18) to (21) become respectively

$$\pi_t = \beta \mathbb{E}_t \{\pi_{t+1}\} + \kappa (y_t - \phi_g g_t - \phi_a a_t - \phi_\nu \nu_t) + u_t, \tag{A.31}$$

$$y_t = (1-s) c_t + s g_t, \tag{A.32}$$

$$n_t = \alpha^{-1} y_t - \alpha^{-1} a_t, \tag{A.33}$$

$$w_t = \gamma^{-1} c_t + \chi n_t + \nu_t, \tag{A.34}$$

where $\phi_g g_t + \phi_a a_t + \phi_\nu \nu_t$ represents the flexible-price output level, with $(\phi_g, \phi_a, \phi_\nu) \in \mathbb{R}^3$. The date-$t$ welfare loss function then becomes $L_t = \mathbb{E}_t \{\sum_{k=0}^{+\infty} \beta^k [(\pi_{t+k})^2 + \lambda (y_{t+k} - \phi_g g_{t+k} - \phi_a a_{t+k} - \phi_\nu \nu_{t+k})^2]\}$. Assume moreover that the three additional disturbances follow stationary fundamental ARMA processes:

$$\rho_g(L) g_t = \theta_g(L) \varepsilon_t^g, \tag{A.35}$$

$$\rho_a(L) a_t = \theta_a(L) \varepsilon_t^a, \tag{A.36}$$

$$\rho_\nu(L) \nu_t = \theta_\nu(L) \varepsilon_t^\nu, \tag{A.37}$$

where $\rho_j(X)$ and $\theta_j(X)$ for any $j \in \{g, a, \nu\}$ are such that $(\rho_j(X), \theta_j(X)) \in \mathbb{R}[X]^2$, $\rho_j(0) \neq 0$, $\theta_j(X) \neq 0$, and all the roots of $\rho_j(X)$ and $\theta_j(X)$ lie outside the unit circle, while ε_t^g, ε_t^a, and ε_t^ν are i.i.d. exogenous shocks such that $\mathbb{E}\{\varepsilon_t^i \varepsilon_{t-k}^j\} = 0$ for any $k \in \mathbb{Z}$ and any $(i,j) \in \{\eta, u, g, a, \nu\}^2$ such that $i \neq j$. The system made of equations (17), (22), (23), and (A.31) to (A.37) is straightforwardly shown to be written in a form of type (1) and (2) with $N = 5$, $\mathbf{Y}_t = [\ c_t \quad \pi_t \quad y_t \quad n_t \quad w_t\ ]'$, $M = 5$, $\boldsymbol{\xi}_t = [\ \eta_t \quad u_t \quad g_t \quad a_t \quad \nu_t\ ]'$, $\boldsymbol{\varepsilon}_t = [\ \varepsilon_t^\eta \quad \varepsilon_t^u \quad \varepsilon_t^g \quad \varepsilon_t^a \quad \varepsilon_t^\nu\ ]'$, and $n = 1$, that satisfies Assumptions 1 to 6 for all theoretically admissible values of the structural parameters.

Assume finally that $\mathcal{CB}$ observes ε_t^g with a one-period lag (say, because it plays before the fiscal authority within each period), but never observes ε_t^a nor ε_t^ν, so that $O_t = \{c^{t-1}, \pi^{t-1}, y^{t-1}, n^{t-1}, w^{t-1}, i^{t-1}, \varepsilon^{g,t-1}\}$.[45] This observation set is of type (9) with $\mathcal{J} = \{1,2,3,4,5\}$, $\mathcal{K} = \{3\}$, and $\ell_Y = \ell_\varepsilon = 1$. The equations (17), (22), (23), and (A.31) to (A.37) are such that $\dim(\overline{\mathcal{E}}) = 1$, while the timeless-perspective optimal feasible path is easily shown – using (24) with $\text{rank}(\boldsymbol{\Omega}_0) = 2$, (A.33), and (A.34) – to be such that $\dim(\overline{\mathcal{R}}_P) = 2$. Therefore, this feasible path meets Condition (16), and Proposition 3

[45] The results would be identical if $\mathcal{CB}$ were instead assumed never to observe ε_t^g, i.e. for $O_t = \{c^{t-1}, \pi^{t-1}, y^{t-1}, n^{t-1}, w^{t-1}, i^{t-1}\}$.

implies that it is implementable if and only if an arbitrarily given canonical rule for this path robustly ensures local-equilibrium determinacy.

Now, a canonical rule for this path can be obtained as follows: (i) add, to the right-hand side of the last line of (24), an exogenous term involving $\varepsilon^{g,t-1}$, $\varepsilon^{a,t-1}$, and $\varepsilon^{\nu,t-1}$, such that the resulting equation is satisfied on this path; (ii) multiply both the left- and the right-hand sides of the latter equation by the divisor of $(1-\mu L)\theta_a(L)\theta_\nu(L)$ of minimal degree for the resulting equation to involve $\varepsilon^{g,t-1}$, $\varepsilon^{a,t-1}$, and $\varepsilon^{\nu,t-1}$ only through a finite number of terms of type g_{t-k}, a_{t-k}, ν_{t-k}, ε^g_{t-k}, $\theta_a(L)\varepsilon^a_{t-k}$, and $\theta_\nu(L)\varepsilon^\nu_{t-k}$ with $k \in \mathbb{N}^*$; and (iii) replace, in the latter equation, for each $k \in \mathbb{N}^*$, the terms g_{t-k}, a_{t-k}, ν_{t-k}, $\theta_a(L)\varepsilon^a_{t-k}$, and $\theta_\nu(L)\varepsilon^\nu_{t-k}$ by their respective expressions $s^{-1}y_{t-k} - s^{-1}(1-s)c_{t-k}$, $y_{t-k} - \alpha n_{t-k}$, $w_{t-k} - \gamma^{-1}c_{t-k} - \chi n_{t-k}$, $\rho_a(L)(y_{t-k} - \alpha n_{t-k})$, and $\rho_\nu(L)(w_{t-k} - \gamma^{-1}c_{t-k} - \chi n_{t-k})$ implied by (A.32), (A.33), (A.34), (A.36), and (A.37). The resulting equation robustly ensures local-equilibrium determinacy if and only if the last line of (24) does, because all the operations used to transform the latter into the former (addition of an exogenous term; multiplication by a polynomial in L whose roots lie all outside the unit circle; addition of linear combinations, whose coefficients are polynomials in L, of the structural equations) are neutral for robust local-equilibrium determinacy. Therefore, the timeless-perspective optimal feasible path is implementable in the presence of the three additional disturbances g_t, a_t, and ν_t for exactly the same values of the structural parameters as in their absence. In particular, Figure 1 is still valid in the presence of these additional disturbances.

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