

# Why Are Real Interest Rates So Low?

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## **Abstract**

*Preliminary and very incomplete.*

We try to account for the persistently low real interest rates (the “secular stagnation” hypothesis) in a quantitative model that encompasses most of the likely factors (demographics, productivity, inequality, financial constraints). We find it difficult to account for both the level and the change over the past two decades, at least in a model without uncertainty, and turn to explore the role of attitudes toward risk.

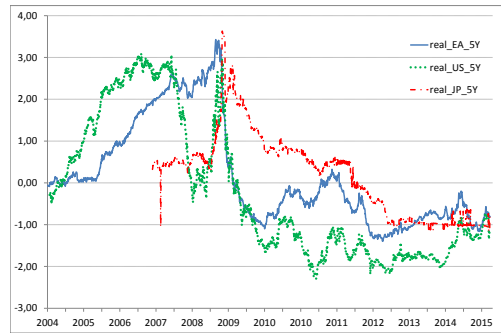


Figure 1: Real 5-year interest rates in the US, Japan, and the Euro-area.

## 1 Introduction

It has been close to 9 years since the “Global Financial Crisis” began.<sup>1</sup> Within two years of its inception, short-term nominal interest rates were driven to near-zero levels in the large advanced economies (U.S., Euro-area, UK, Japan) and have stayed there since. The Fed just recently “lifted off” while the UK hasn’t begun to consider when it would do so, and Japan and the Euro area have gone below zero. With low and (relatively) steady inflation, real rates have been negative for a while, and not just short-term rates but also rates at the 5-year and 10-year horizon (Hamilton et al., 2015).

Much of the macroeconomic research responding to the financial crisis has taken place within the paradigm of the DSGE. Understanding the reasons for reaching the lower bound (the reason for low interest rate) was less urgent than understanding the proper responses to the situation. Also, the methodology relies on some approximation around a steady state, whether linear or nonlinear (Fernandez-Villaverde et al., 2012; Gust, López-Salido, and M. E. Smith, 2012). Hence the low (real) interest rates are modeled as the result of an exogenous shock, for example to the discount rate or to a borrowing constraint (Eggertsson and Woodford, 2003; Eggertsson and Krugman, 2012), inducing deviations from a steady state whose dynamics (modified as needed by policy) are the core prediction of the model.

After a decade of low interest rates, the shock paradigm becomes less attractive because of the strains it places on the assumption of independent Gaussian shocks (Aruoba, Cuba-Borda, and Schorfheide, 2013). Instead, a growing concern has emerged with a hypothesis dubbed by Larry Summers “secular stagnation”: low interest rates may not be temporary deviations but a now-permanent state of affairs, Two recent policy-oriented publications (Teulings and Baldwin, 2014; Bean et al., 2015) have recently collected the possible explanations for such a permanent decline in interest rates.

So far there has been little quantitative evaluation of the competing explanations (Rachel and T. D. Smith, 2015, but see) and none of it model-based. Our question is simple: can we

<sup>1</sup>The beginning of the crisis is commonly dated to the closure of two Paribas funds in August 2007.

account for current low interest rates in a model that encompasses the most likely factors? Our answer, so far, is that it's darn difficult.

In our first step, the framework we use combines Coeurdacier, Guibaud, and Jin, 2015 and Eggertsson and Mehrotra, 2014 to encompass most of the current explanations for low interest rates.

## 2 Low interest rates in a model with no uncertainty

### 2.1 Introduction

Bean et al. (2015) list a number of possible explanation for low real interest rates: demography, income inequality, limited financial development or borrowing constraints, productivity growth, falling price of investment goods, demand and supply of safe assets.

All but the last can be accounted for in the simple OLG model we present here, which nests Eggertsson and Mehrotra (2014) and Coeurdacier, Guibaud, and Jin (2015). The determination of the interest in those models comes down to the Euler equation of one agent, within which the constraints faced by other agents and other determinants enter through market-clearing.

### 2.2 The Model

Time is discrete and there is no uncertainty. Each period  $t$  a generation is born that lives 3 periods  $y, m, o$ . The size of each generation is  $N_t = (1 + g_{N,t})N_{t-1}$ .

Each agent supplies one unit of labor inelastically when young and middle-aged, and consumes in all periods. Preferences are:

$$u(c_t^y) + \beta u(c_{t+1}^m) + \beta^2 u(c_{t+2}^o) \quad (1)$$

with the discount factor  $0 < \beta < 1$  and the utility function  $u(c) = (c^{1-1/\sigma})/(1 - 1/\sigma)$ .

The factors of production are capital, which depreciates at a rate  $\delta$ , and labor supplied by agents. The labor productivity of a member of generation  $t$  varies over the life cycle, being  $e_t^y$  when young and  $e_t^m$  ( $= 1$  for now) when middle-aged (**add heterogeneous productivities later**). The aggregate productivity of labor over time is  $A_t = (1 + g_{A,t})A_{t-1}$ . The neo-classical constant-returns production function combines capital (with share  $\alpha$ ) and labor (with share  $1 - \alpha$ ) to produce output, one unit of which can become either one unit of consumption or  $1/p_t^k$  units of investment; the relative price of investment goods is exogenous and follows  $p_t^k = (1 + g_{I,t})p_{t-1}^k$ . Markets are competitive and prices are flexible. Labor earns a wage  $w_t$  capital earns a return  $r_t^k$

Agents can purchase investment goods, and can also borrow from and lend to each other at a gross rate  $R_{t+1}$ , but they owe more (principal and interest) than a fraction  $\theta_t$  of next period's labor income.

The following equations summarize the above. Agents of generation  $t$  choose  $\{c_t^y, c_{t+1}^m, c_{t+2}^o, k_{t+2}, b_{t+1}^y, b_{t+2}^m\}$  to maximize (1) subject to three budget constraints and one borrowing constraint:

$$c_t^y = b_{t+1}^y + w_t e_t^y \quad (2)$$

$$c_{t+1}^m + p_{t+1}^k k_{t+2}^m + R_{t+1} b_{t+1}^y = w_{t+1} + b_{t+2}^m \quad (3)$$

$$c_{t+2}^o + R_{t+2} b_{t+2}^m = (p_{t+2}^k (1 - \delta) + r_{t+2}^k) k_{t+2}^m \quad (4)$$

$$R_{t+1} b_{t+1}^y \leq \theta_t w_{t+1}. \quad (5)$$

On the production side, the production function

$$(N_{t-2} K_t)^\alpha [A_t (e_t^y N_t + N_{t-1})]^{1-\alpha} \quad (6)$$

yields the wage rate and capital rental rate

$$w_t = (1 - \alpha) A_t k_t^\alpha \quad (7)$$

$$r_t^k = \alpha k_t^{\alpha-1} \quad (8)$$

with the capital/effective labor ratio, using clearing of capital and labor markets, written as

$$k_t \equiv \frac{N_{t-2} k_t^m}{A_t (e_t^y N_t + N_{t-1})}, \quad (9)$$

while the absence of arbitrage between capital and bonds imposes

$$R_{t+1} p_t^k = r_{t+1}^k + (1 - \delta) p_{t+1}^k. \quad (10)$$

The final condition requires clearing of the bond market at time  $t$ :

$$(1 + g_{L,t}) b_{t+1}^y + b_{t+1}^m = 0. \quad (11)$$

The solution proceeds as follows. The constraint (5), assumed to bind for the young agent, yields the loan demand  $b^y$  as a function of the wage, i.e.,  $k_{t+1}$ . The middle-aged agent's Euler equation yields the loan supply  $b^m$  as a function of  $k_{t+1}$  and  $k_{t+2}$ . Substituting these expressions in (11) gives a first-order difference equation in  $k$ , which can be solved for the steady-state level of capital, giving the interest rate  $R_t$  by (10) as a function of preferences, technology, productivity, demographics, credit constraints, and income inequality.

Accordingly, the loan demand of the young is

$$b_t^y = \frac{\theta_t w_{t+1}}{R_{t+1}} \quad (12)$$

while the loan supply of the middle-aged, using the agent's middle-aged Euler equation and (10), is given by<sup>2</sup>

$$(1 + \beta^{-\frac{1}{\sigma}} R_{t+1}^{1-\frac{1}{\sigma}}) b_{t+1}^m = p_t^k k_{t+1}^m - (1 - \theta_{t-1}) \frac{w_t}{1 + \beta^{-\frac{1}{\sigma}} R_{t+1}^{1-\frac{1}{\sigma}}} \quad (13)$$

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<sup>2</sup>See Eq. (48) of Eggertsson and Mehrotra (2014), which is slightly different because in their model capital earns returns in the same period as it is produced.

so that

$$p_t^k k_{t+1}^m = -\theta_t(1 + g_{L,t}) \frac{w_{t+1}}{R_{t+1}} + (1 - \theta_{t-1}) \frac{w_t}{1 + \beta^{-\frac{1}{\sigma}} R_{t+1}^{1-\frac{1}{\sigma}}}. \quad (14)$$

Replacing

$$\begin{aligned} k_{t+1}^m &= \frac{K_{t+1}}{N_{t-1}} = \frac{1}{N_{t-1}} k_{t+1} A_{t+1} (e_t^y N_{t+1} + N_t) \\ &= (1 + g_{A,t+1})(1 + g_{L,t})(e_t^y(1 + g_{L,t+1}) + 1) \end{aligned}$$

and using (7), and simplifying, we have

$$\begin{aligned} (1 + g_{A,t+1})(1 + g_{L,t}) &\left[ p_t^k k_{t+1} (e_t^y(1 + g_{L,t+1}) + 1) + \theta_t \frac{(1 - \alpha)k_{t+1}^\alpha}{R_{t+1}} \right] \\ &= (1 - \theta_{t-1}) \frac{(1 - \alpha)k_t^\alpha}{1 + \beta^{-\frac{1}{\sigma}} R_{t+1}^{1-\frac{1}{\sigma}}} \end{aligned} \quad (15)$$

with

$$R_{t+1} = \frac{p_{t+1}^k}{p_t^k} (\alpha k_{t+1}^{\alpha-1} + 1 - \delta) \quad (16)$$

a function of  $k_{t+1}$ .

To express (15) as a law of motion of capital, write:

$$\begin{aligned} (1 - \theta_{t-1})(1 - \alpha) \frac{k_t^{\alpha-1}}{p_t^k} k_t &= (1 + g_{A,t+1})(1 + g_{L,t}) \\ \left[ 1 + \beta^{-\frac{1}{\sigma}} \left( \alpha \frac{k_{t+1}^{\alpha-1}}{p_{t+1}^k} + 1 - \delta \right) \frac{p_{t+1}^k}{p_t^k} \right]^{1-\frac{1}{\sigma}} &\left[ 1 + e_t^y(1 + g_{L,t+1}) + \frac{\theta_t(1 - \alpha)k_{t+1}^{\alpha-1}/p_{t+1}^k}{\alpha k_{t+1}^{\alpha-1}/p_{t+1}^k + 1 - \delta} \right] k_{t+1} \end{aligned} \quad (17)$$

Note that, when  $\delta = 1$ ,  $\sigma = 1$ , and  $p_t^k = 1$  (17) reduces to Eq. (12) of Coeurdacier, Guibaud, and Jin (2015).

To express (15) as a difference equation in interest rates, rewrite (16) as

$$\alpha \frac{k_{t+1}^{\alpha-1}}{p_{t+1}^k} = \frac{R_{t+1}}{1 + g_{I,t+1}} - 1 + \delta \equiv z_t$$

and (15) as:

$$\begin{aligned} (1 - \theta_{t-1}) \frac{1 - \alpha}{\alpha} z_t k_t &= (1 + g_{A,t+1})(1 + g_{L,t}) \\ \left[ 1 + \beta^{-\frac{1}{\sigma}} R_{t+1}^{1-\frac{1}{\sigma}} \right] &\left[ 1 + e_t^y(1 + g_{L,t+1}) + \theta_t \frac{1 - \alpha}{\alpha} \frac{z_{t+1}}{1 - \delta + z_{t+1}} \right] k_{t+1}. \end{aligned} \quad (18)$$

In steady state, this becomes

$$\begin{aligned} (1 - \theta) \frac{1 - \alpha}{\alpha} \left( \frac{R}{1 + g_I} - 1 + \delta \right) &= \\ (1 + g_A)(1 + g_L) \left[ 1 + \beta^{-\frac{1}{\sigma}} R^{1-\frac{1}{\sigma}} \right] &\left[ 1 + e^y(1 + g_L) + \theta \frac{1 - \alpha}{\alpha} (1 - (1 - \delta) \frac{1 + g_I}{R}) \right]. \end{aligned} \quad (19)$$

The factors listed by Bean et al. (2015) can be mapped into the model's parameters as follows. Demography is  $g_L$ , intergenerational inequality is  $e^y$ , **intra income inequality is to be added**, limited financial development is  $\theta$ , productivity growth is  $g_A$ , falling price of investment goods is  $g_I$ . In addition we have preference parameters  $\beta$  and  $\sigma$ , technology parameter  $\alpha$  and  $\delta$ .

## Comparing steady states

Rewriting (19) as

$$\frac{(1+g_A)(1+g_L)(1+g_I)}{1-\theta} [1/R + (\beta R)^{-\sigma}] \left[ \theta + \frac{\alpha(1+e^y(1+g_L))}{(1-\alpha)[1-(1-\delta)(1+g_I)/R]} \right] = 1$$

shows that

1.  $R$  is increasing in  $g_A, g_L, g_I, \theta$
2. While  $\beta R \leq 1$ ,  $R$  is increasing in  $\sigma$ .

Second, we can linearize (19) around any given collection of parameters  $\{g_L, e^y, \theta, g_A, g_I\}$  to see the impact of various changes in these parameters on the steady-state interest rate. In particular:

$$r^a = \frac{\sigma(1-\beta^a)}{1+\sigma} + \frac{2}{N(1+\sigma)} \left[ \frac{3\alpha-1}{1-\alpha} + \frac{2\alpha e^y}{1-\alpha} \right] + \frac{2}{(1+\sigma)} \left[ g_I^a + g_A^a + g_L^a + \theta \frac{1+\alpha e^y}{N(\alpha+\alpha e^y)} + (1-N\delta^a) \right]$$

## Comparing transition paths

We can compare steady states and, since we have the law of motion, we can also study transition paths due to temporary changes in any of these parameters from any initial  $k_0$  by mechanically computing forward.

### 2.3 More general preferences

In case we use more general preferences, we should instead replace  $b_{t+1}^m$ , using (11), into the Euler equation:

$$\begin{aligned} u'(c_t^m) &= \beta u'(c_{t+1}^o) R_{t+1} \\ u'((1-\theta)w_t - p_t^k k_{t+1}^m + b_{t+1}^m) &= \beta R_{t+1} u'(p_{t+1}^k (1-\delta) + r_{t+1}^k) k_{t+1}^m - R_{t+1} b_{t+1}^m \end{aligned}$$

### 2.4 Adding intra-generational inequality

At first sight it is not obvious how to incorporate intra-generational inequality. The way Eggertsson and Mehrotra (2014) do it is in the section with exogenous outcomes. Some middle-aged agents have low endowment, but expect high old-age endowment, hence want to borrow. They justify this old-age endowment as a social security-style transfer; but such a transfer would have to be financed by a tax on the rich middle-aged, which would essentially undo the borrowing constraint on the poor middle-aged.

Once we make endowments endogenous as factor incomes, then old-age income is the return in saving, and it would make no sense for middle-age agents to want to borrow against their own savings.

### 3 Empirical evaluation

Part of the parameters are readily observable (or at least have straightforward empirical counterparts). Such are the technological parameters  $g_A$ ,  $g_I$  and demographic parameters  $g_L$ . The labor share  $\alpha$  is also relatively uncontroversial. Preference parameters are more difficult to calibrate; most difficult is probably the  $\theta$  parameter describing financial constraints.

The empirical strategy relies on a linearized form of (19). We start from the period 1995–2004 and find a combination of parameters that (a) seem plausible and (b) match the **level** of the interest rate. We then evaluate the same expression for the period 2005–14, with known values of the parameters (keeping preference parameters constant) so as to decompose the contribution of each factor to the fall in interest rates. We can also see if the factors add up to the full extent of the actual fall in interest rates. Finally we can use projections for 2015–24 (some factors, like demographics, are readily forecastable, others like productivity are known within error bands) to see if the model predicts sustained low interest rates as the secular stagnation hypothesis does.

Our results are too preliminary but they suggest that only values of  $\theta$  very close to zero (implausibly tight borrowing constraints) are compatible with the known parameter values. This leads us to the second step of our agenda, namely, incorporating attitudes toward risk.

### 4 Safe Assets

Can we introduce “shortage” of safe assets into this framework? Caballero, Farhi, and Gourinchas (2008) present a continuous time model which can be readily mapped into a discrete-time OLG model.

For now, consider agents living  $k$  periods and consuming only in the last period of their life. In the first period they are endowed with a fraction  $1 - \alpha$  of the perishable, exogenous output  $X_t$  which grows at rate  $1 + g$ . They sell the output to buy  $s_t^t$  shares in trees. Then, every period but the last they collect a dividend  $d_{t+i}$  per share and use it to buy more shares. In the last period they collect the final dividend, sell their shares, and consume.

The budget constraints are

$$\begin{aligned} p_t s_t^t &= (1 - \alpha) X_t \\ p_{t+i} s_{t+i}^t &= (p_{t+i} + d_{t+i}) s_{t+i-1}^t, \quad i = 1, \dots, k - 1 \\ c_{t+k} &= (p_{t+k} + d_{t+k}) s_{t+k-1}^t. \end{aligned}$$

Market clearing in the goods market implies  $d_t = \alpha X_t$ . Market clearing in the shares market implies

$$\sum_{i=0}^{k-1} s_t^{t-i} = \sum_{i=0}^{k-1} s_{t-1}^{t-1-i} \quad (20)$$

hence this sum is a constant independent of  $t$  and  $s_{t+i}^t = s^i$  for all  $t$ . The budget constraints translate into

$$\begin{aligned} p_t &= \frac{1-\alpha}{s^0} X_t \\ s^i &= \left(1 + s^0 \frac{\alpha}{1-\alpha}\right) s^{i-1} \end{aligned}$$

which, substituted into (20), yields

$$\sum_{i=0}^{k-1} s^i = \frac{(1 + s^0 \alpha / (1 - \alpha))^k - 1}{s^0 \alpha / (1 - \alpha)}$$

or

$$s^0 = \frac{1-\alpha}{\alpha} \left( (1-\alpha)^{-1/k} - 1 \right).$$

The interest rate is then

$$r_t = \frac{p_t + d_t}{p_{t-1}} = \frac{X_t(1-\alpha)/s^0 + \alpha X_t}{X_{t-1}(1-\alpha)/s^0} = (1+g)(1-\alpha)^{-1/k} \simeq 1 + g + \alpha/k$$

which is the discrete-time analog of the formula  $r = g + \delta/\theta$  in Caballero, Farhi, and Gourinchas (2008), since their  $\delta$  corresponds to  $\alpha$  here, and  $\theta$  the fraction of newborns corresponds to  $1/k$ .

## 5 Risk and ambiguity

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