Trade and Interdependence in International Networks*

François de Soyres†

Toulouse School of Economics

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Abstract

This paper studies the relationship between international trade and business cycle synchronization. Using data from OECD countries, I find substantive support for the role of trade in intermediate inputs and monopolistic pricing in synchronizing GDP fluctuations across countries. I then build a model of international trade in intermediates with heterogeneous firms and monopolistic competition. Quantitative explorations show that the model is able to replicate 85% of the empirical relationship between trade in inputs and GDP comovement, making a significant step toward solving the "trade comovement puzzle". Finally, I clarify the role of the ingredients and show that markups and extensive margin adjustments create a link between domestic productivity and foreign shocks.

Keywords: International Trade, International Business Cycle Comovement, Networks, Input-Output Linkages

JEL Classification Numbers: F12, F17, F4, F62, L22

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†Contact: Toulouse School of Economics, 21 Allée de Brienne, 31000 Toulouse, France. Email: francois.de.soyres@gmail.com.

1 Introduction

The "Trade Comovement Puzzle", uncovered by Kose and Yi (2001, 2006), refers to the inability of international business cycle models to quantitatively account for the high and robust empirical relationship between international trade and GDP comovement.[1] Using different versions of the workhorse international real business cycle (IRBC) model, several authors have succeeded to qualitatively replicate the positive link between trade and GDP comovement but fall short of the quantitative relationship by one order of magnitude.[2]

In this paper, I propose a model of international trade in inputs with monopolistic pricing and firm entry/exit that can account for 85% of the Trade-Comovement slope. My contribution is twofold. First, I use data from OECD countries to update the empirical relationship between trade and business cycle synchronization and emphasize the role of trade in intermediate inputs. Moreover, I provide evidence that countries with higher markups have a more negative correlation between their GDP and the relative price of their imports, suggesting that markups have the potential to create a link between foreign shocks and domestic GDP. Motivated by those results, I propose a general equilibrium dynamic model of trade in inputs with monopolistic pricing and firm entry/exit and assess its ability to replicate the empirical findings. In the benchmark calibration, the model is able to replicate 85% of the Trade-Comovement slope, making an significant step toward solving the "trade comovement puzzle". Moreover, the model features a quantitatively important link between foreign shocks and domestic productivity suggesting that countries who are engaged in trade in input should have correlated TFP, a prediction that I validate in the data.

Empirics Since the seminal paper by Frankel and Rose (1998), a large empirical literature has studied cross countries' GDP synchronization, showing that international trade is an important and robust determinant of GDP correlation. In this paper, I update those findings and find substantive support for the importance of trade in inputs and price distortions.

I refine previous analysis by constructing a panel dataset where each country pair appears four times, one for each of the four 10-years time window ranging from 1969Q1 to 2008Q4. I then use the panel dimension of this dataset and control for country pair fixed effects that can be correlated with trade proximity.[3] I show that the relationship between trade and comovement is high and statistically significant in all specification, keeping the "Trade Comovement Puzzle" alive.

Furthermore, I make use of disaggregated trade data to disentangle the effect of trade in final good from the trade in intermediate inputs on output synchronization. Regressing GDP comove-

[1] For empirical studies on this topic, among many others, see Frankel and Rose (1998), Clark and van Wincoop (2001), Imbs (2004), Baxter and Kouparitsas (2005), Kose and Yi (2006), Calderon, Chong, and Stein (2007), Inklaar, Jong-A-Pin, and Haan (2008), Di Giovanni and Levchenko (2010), Ng (2010), Liao and Santacreu (2015) or Duval et al (2015)

[2] For quantitative studies, see Kose and Yi (2001, 2006), Burstein, Kurz and Tesar (2008), Johnson (2014) or Liao and Santacreu (2015)

[3] In a fixed effect or first difference regression, identification comes from within country-pair variation rather than cross-country differences.

ment on indexes of trade proximity in final and intermediate goods, I show that in most empirical specification trade in intermediate inputs captures most of the explanatory power. This suggests that the rise in global supply chains plays a particular role in the synchronization of GDP across countries.

Finally, I study the role of monopolistic pricing in generating a link between terms of trade and GDP movements. Using the Price Cost Margin as a proxy for monopoly power, I find that countries with higher markups also experience a higher decrease in their GDP when the price of their import rises.

Theory Motivated by those facts, I propose dynamic general equilibrium a model of international trade that relies on three key assumptions: (i) input-output linkages across countries, (ii) heterogeneous firms and (iii) monopolistic competition. Production is performed by a continuum of heterogeneous firms combining in a Cobb-Douglas fashion labor, capital and a nested CES aggregate of intermediate inputs bought from other firms from their home country as well as from abroad. Based on their expected profit, firms choose the set of countries they serve (if any). In this context, a firm's marginal cost depends on the number and on the productivity of its suppliers, giving rise to a strong interdependency in pricing and revenues as well as in the export participation decisions. Moreover, monopolistic competition and fluctuations in the mass of producing firms are key elements in order to break the link between imports and production, thus allowing domestic GDP to be affected by foreign shocks.

By computing the elasticity of domestic Gross National Income (GNI) with respect to a technological shock abroad, I show that international propagation of TFP shocks depends on (1) the structure of the worldwide network of input-output linkages and (2) the endogenous change in the set of firms serving each markets due to a shock abroad.

The first point is related to Kose and Yi (2006) who argued that a two-country setting is inappropriate when studying international propagation because those models tend to grossly exaggerate the impact that a typical country has on its trading partners. In this paper, I show that in a world with input-output linkages, international propagation runs through the whole network of firms all around the world and that one must model the whole economy to take those effects into account. Hence, the extent to which a shock in country k can affect the GDP in country k' does not only depend on the strength of economic ties between those two countries, but also on their relative centrality in the worldwide network of input-output linkages.

The second element - the importance of extensive margin adjustments - suggests that classic international real business cycle (IRBC) models with representative firms tend to underestimate the extent to which technological shocks to one country can affect its trading partners. By introducing firm entry/exit of firms, the model features a quantitatively important transmission channel.

In the workhorse IRBC model, international trade quantitatively plays little role in propagating shocks across countries.[4] The key reason behind this inability has been studied by Kehoe and Ruhl (2008, henceforth KR). KR shows that in a perfectly competitive world with a fixed number of firms, shocks to the terms-of-trade have no impact on a country's productivity and affect the GDP only through a change in domestic factor supply. The intuition is as follow: GDP is the difference between final production and imported inputs, measured using a base price. When the price of imports rises, a country decides to import less and to produce less. Up to a first order approximation, the change in imports and the change in final good production exactly cancel out so that the difference between the two (the GDP) stays the same, up to the effect due to any change in the domestic factor of production. In other words, changes in domestic GDP after a foreign shock are only driven by changes in the domestic factors of production. The argument relies heavily on perfect competition and the absence of distortion at every step from imports to the production of final output.[5]

In the present paper, I expose two ways of getting around this negative result. First, when introducing a price distortion anywhere in the production chain between the firm that imports and the final good production, a change in the price of imported inputs can have a first order impact on GDP even with fixed factor supply. If prices are distorted upward (through the existence of markups for example), a rise in the price of imports is amplified along the supply chain and the reduction in final good production is larger than the reduction in imports, leaving room for a change in GDP. As noted previously in Gopinath and Neiman (2014) as well as in Kim (2014), price distortions can introduce a wedge between the marginal cost of imported input and their marginal product in final good production and open the room for a change in GDP.[6] In the present work, I focus on a particular price distortion: monopolistic pricing. The fact that firms do not take price as given is a key element in breaking the tight relationship between the fluctuation of imports and final good production and allows domestic productivity to be affected by foreign technological shocks.

Second, fluctuations in the number of producing firms is a second powerful way to get around KR's negative result. The reason is itself twofold. First, fluctuations in the mass of firms that do not take prices as given makes the price distortion described above time variant which amplifies the relationship between trade and GDP fluctuations. Second, if there is love for variety in the production of the final good, any change in the mass of input suppliers impacts final good production above and beyond the price changes. With love for variety, a firm that has access to more suppliers

[4] More precisely, trade plays little role in generating GDP movement, but it has an important role in generating movement in consumption, investment and welfare.

[5] When imports are used to produced intermediates, which are then used to produce final good, the price of intermediates reflects at the same time their marginal cost of production and their marginal productivity in final good production. A change in the price of imports leads to efficient adjustment at every step of the production chain so that the change in imports and in final production are the equal, implying that GDP stays constant. This "negative result" is at the heart of the Trade-Comovement Puzzle.

[6] Related to this point, Burstein and Cravino (2015) show that if all firms take prices as given, a change in trade cost can affect aggregate productivity only to the extend that it changes the production possibility frontier at constant prices. This can be interpreted as saying that shocks to the *foreign* trading technology has no impact on aggregate TFP if all firms take prices as given, so that any change in GDP is due to a change in the supply of domestic factors of production.

can produce a higher level of output for the same level of inputs. Hence, an increase in the quantity of imported inputs that is accompanied by a change in the mass of suppliers leads to a disproportionate increase in final good production, opening the room for a GDP change. A simple way to understand this result is to see that with love for variety, firms play the role of a productive input. Hence, even with fixed labor and capital, GDP can change solely due to a change in the mass of firms.

The combination of input-output linkages, monopolistic pricing and fluctuation in the mass of producing firms allow the model to reproduce the link observed in the data between international trade in inputs and GDP comovement across countries. Moreover, since domestic GDP can react to foreign shocks even when domestic factors of production are fixed, such a change would then be reflected by a change in measured TFP.[7]

Quantitative Analysis In order to assess the ability of the model to replicate the strong relationship between trade in inputs and GDP co-movement, I precisely calibrate the model to 14 OECD countries and a composite "rest-of-the-world". The model is calibrated to match the observed GDP, trade flows and the *level* of GDP comovement across all country pairs between 1989 and 2008. From this reference point, I vary technological parameters in order to generate a decrease and an increase of the trade flows from their observed value. In all configurations, I feed the model with the same sequence of TFP shocks, creating a panel dataset in which each country-pair appears three time and that allows me to identify the trade comovement *slope*. Fixed effect regressions on this simulated dataset shows that the model is able to replicate 85% of the trade-comovement *slope* observed in the data.

By shutting off one by one the key elements in the model, I also decompose the role of the ingredients. Both the markup and the adjustments in the mass of firms serving every market are quantitatively important for the results. The extensive margin adjustments can then be further decomposed into variations into (1) the productivity thresholds separating exporters and non exporters for every country and (2) the mass of producing firms. While both are found to have am impact on the model's performance, the fluctuation in the mass of producing firms appears to have a larger role.

Relationship to the literature

If the empirical association between bilateral trade and GDP comovement has long been known, the underlying economic mechanisms leading to this relationship are still unclear. Using the workhorse IRBC with three countries, Kose and Yi (2006) have shown that the model can explain at most 10% of the *slope* between trade and business cycle synchronization, leading to what they called the "Trade Comovement Puzzle". Since then, many papers have refined the puzzle,

[7]Indeed, using a "growth accounting" perspective and keeping factor of production constants, any change in GDP is accounted for in the Solow residual.

highlighting different ingredients that could bridge the gap between the data and the predictions of classic models.

Burstein, Kurz and Tesar (2008) show that allowing for production sharing among countries can deliver tighter business cycle synchronization if the elasticity of substitution between home and foreign intermediate inputs is extremely low[8]. Arkolakis & Ramanarayanan (2009) analyse the impact of vertical specialization on the relationship between trade and business cycle synchronization. In their Ricardian model with perfect competition, they do not generate significant dependence of business cycle synchronization on trade intensity, but show that the introduction of price distortions that react to foreign economic conditions allows their model to reach a trade-comovement slope of 0.03, about a third of the of the slope estimated in the data in Kose & Yi (2006). Incorporating trade in inputs in an otherwise standard IRBC, Johnson (2014) shows that the puzzle cannot be solved by adding the direct propagation due to the international segmentation of supply chains only. Calibrating his model with 22 countries using the WIOD database, he finds the puzzle "alive and well" with the aggregate trade-comovement correlations for real value added and gross output being at most 10-20% the size of the observed correlations. Compare to those papers, I add firm entry and exit as well as monopolistic competition and argue that those are key ingredients for the model to deliver quantitative results in line with the data. Liao and Santacreu (2015) build on Ghironi & Melitz (2005) and Alessandria & Choi (2007) to develop a two-country IRBC model with trade in differentiated intermediates. They show that trade in intermediate varieties leads to an endogenous correlation of measured TFP[9] across trading partners. Compare to this paper, I add price distortions within each countries' production chain[10] as well as a production link between importers and exporters which creates a strong interdependency in firms' pricing end export decisions. Furthermore, I extend the quantitative analysis to many countries and show the international propagation of shocks is affected by the whole network of input/output linkages.[11]

The idea that trade disruption can result in significant decrease in efficiency for firms relying heavily on foreign inputs has been studied by Gopinath and Neiman (2014). Focusing on the 2001-2002 Argentinian crisis, they show that trade disruption can cause a significant drop in aggregate productivity. They then build a model with monopolistic pricing an exogenous cost of changing the number of firm's suppliers and are able to replicate the data, showing the importance of within firms' dynamics to understand aggregate productivity. Finally, the role of firms heterogeneity in international business cycles has been pioneered by Ghironi & Melitz (2005) and the business cycle implication of firms' heterogeneity is studied in Fattal-Jaef & Lopez (2014).

The rest of the paper is organized as follow: the second section studies empirically the rela-

[8] In their benchmark simulations, the authors take the value of 0.05 for this elasticity.

[9] Defined as the Solow residual at the country's level

[10] In their model, there is no distrotion between the price of foreign input and their marginal productivity in final good production because the final good producer directly buys the imported inputs

[11] In their model, no firm is both an importer and an exporter. While simplfying the resolution, this assumption prevents any network effect.

tionship between trade and output synchronization and highlights the important role of trade in intermediate inputs and monopolistic pricing. The third section exposes a quantitative model of international trade in intermediate goods with heterogeneous firms and monopolistic competition. In the fourth section, I present the calibration strategy and the quantitative results are presented in section five. Section six presents a simplified static model of small open economy that provides interesting intuitions for the role of each ingredients. Section seven concludes.

2 Empirical Evidence

In this section, I use a sample of 20 OECD countries[12] and update the initial Frankel and Rose (1998) analysis on the relationship between trade and GDP comovement. I also provide empirical support for the role of the ingredients discussed above, namely the importance of trade in intermediate inputs as well as the role of monopolistic pricing.

There are three main findings. First, the empirical association between business cycle synchronization and international trade is robust to country-pair fixed effects. Second, trade in intermediate goods play a significant role in explaining GDP comovement, while the explanatory power of trade in final good is found not significant in most specifications. Third, economies where markups are high are economies that feature a larger decrease in GDP when experiencing an increase in their terms-of-trade. Starting by the first two findings, I first describe the data, then I explain the econometric strategy and finally I present the results in details. In the last subsection, I describe the data and results relative to the third finding.

2.1 GDP comovement: the role of trade in Inputs and the Extensive Margin of trade

I use quarterly data on real GDP from the OECD database which I detrend using a HP filter with smoothing parameter 1600 to capture the business cycle frequencies.[13] Trade data come from the NBER-UN world trade database provided by the Center for International Data (CID) as well as the COMTRADE database. It features bilateral trade flows at the 4-digit level of disaggregation (SITC Rev. 4). Such a high level of disaggregation allows me to deepen the analysis by disentangling the effect of trade in final good from the trade in intermediate inputs.

In a first set of regressions, I construct a symmetric measure of bilateral trade intensity between countries i and j using total trade flows as: $\text{Total}_{ij}{=}\max\left(\frac{Total\ Trade_{ij}}{GDP_i}, \frac{Total\ Trade_{ij}}{GDP_j}\right)$. This measure has the advantage to take a high value whenever one of the two countries depends heavily on the other for its imports or exports.[14]

[12] The list of countries is: Australia, Austria, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Mexico, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States

[13] In appendix, I provide the results when using the band pass filter of Baxter and King and keeping the fluctuations between 32 and 200 quarters. Results are virtually unchanged

[14] The index mostly used in the literature was the sum of total trade flows divided by the sum of GDPs. While the

Then, in order to disentangle the influence of trade flows in inputs from the final goods, I construct the indexes Final_{ij} and Intermediate_{ij} with the same formulation but taking into account only the trade flows in final and intermediate goods respectively. I follow Feenstra and Jensen (2012) to separate the trade flows into final and intermediates and transform the SITC code into End-Use categorization. The End-use codes are used by the Bureau of Economic Analysis (BEA) to allocate goods to their final use, and are similar to the Broad Economic Categories of the United Nations Statistics Division. This categorization allows me separate products between final and intermediate goods.

The extent to which countries have correlated output can be influenced by many factors, including international trade, correlated shocks, financial linkages, monetary policies, etc... Because those other factors can themselves be correlated with the index of trade proximity in the cross section, using cross-section identification could yield biased results. In order to separate the effect of trade linkages from other elements, I construct a panel dataset by creating four periods of ten years each. In every time window, I compute GDP correlation for all country pairs as well as trade indexes defined above. The index relative to a given time window is the average of all yearly indexes. Using panel data allows me to control for time invariant country-pair specific factors that are not observables.

I estimate the following equations:

$$\begin{aligned}(i) \qquad \text{corr}(Y_{it}^{HP}, Y_{jt}^{HP}) &= \alpha_i + \beta_T \log(\text{Total}_{ijt}) + \epsilon_{I,ijt} \\ (ii) \qquad \text{corr}(Y_{it}^{HP}, Y_{jt}^{HP}) &= \alpha_{ii} + \beta_I \log(\text{Intermediate}_{ijt}) + \beta_F \log(\text{Final}_{ijt}) + \epsilon_{II,ijt}\end{aligned}$$

In the rest of this section I present three facts that characterize the relationship between GDP synchronization and international trade. All results are gathered in tables 1 and **??**.

Finding 1: The trade-comovement slope is high and statistically significant

As in previous studies, I find that an increase in the index of trade proximity is associated with an increase in GDP correlation. In table 1, the point estimates in column (1), (3) and (5) show that a doubling of the median index is associated with an increase of output correlation between 0.065 (in column (2)) and 0.21 (in column (1)). Moreover, the estimated numbers imply that moving from the 25th to the 75th percentile of trade proximity in my sample is associated with an increase of GDP correlations between 0.20 and 0.67, which is very significant.

Finding 2: Trade in Intermediate inputs plays a strong role in GDP comovement

To investigate further the relationship between trade and GDP comovement, columns (2), (4) and (6) disentangle the effect of trade in intermediate input with the trade in final goods. The results

empirical and simulated results hold when I use this index, it has the disadvantage that a country-pair consisting in with a very big country and a very small country cannot have a high index, despite the fact the small country might depend exclusively on the big country's imports and exports.

highlight a specific role for trade in intermediate inputs. In all specifications the index of trade proximity in intermediate goods is high and significant[15] with a doubling of the intermediate trade index associated with GDP comovement increase between 0.04 (column (6)) and 0.16 (column (2)) depending on the specification. This result suggests that international supply chains are an important determinant of the degree of business cycle synchronization across countries.

	dependant variable: corr(GDP_i^{HP},GDP_j^{HP})					
	(1)	(2)	(3)	(4)	(5)	(6)
log(Total)	0.315***		0.094**		0.125***	
	(15.04)		(2.56)		(3.60)	
log(Intermediate)		0.231***		0.065**		0.055**
		(9.10)		(3.03)		(2.10)
log(Final)		0.026		0.012		0.076**
		(0.67)		(0.48)		(2.21)
Country-Pair FE	yes	yes	yes	yes	yes	yes
Time Trend	no	no	yes	yes	no	no
Time FE	no	no	no	no	yes	yes
N	760					

t stat. in parentheses, *** means $p < 0.01$, ** means $p < 0.05$ and * means $p < 0.10$

Table 1: Strong Influence of Trade in Intermediates

The results presented here used a fixed effect specification. One might also consider that the variation across country-pairs are assumed to be random and uncorrelated with trade proximity indexes, in which case a random effect treatment would be a better fit. To discriminate between fixed or random effects, I run a Hausman specification test where the null hypothesis is that the preferred model is random effects against the fixed effects. This tests whether the error terms ϵ_{ijt} are correlated with the regressors, with the null hypothesis being they are not. Results display a significant difference ($p < 0.001$), indicating that the two models are different enough to reject the null hypothesis, and hence to reject the random effects in favor of the fixed effect model.

[15]The index of trade proximity in final goods is found positive and significant in the sole case of Fixed Effect regression with time dummies. In appendix, I present the result of the same analysis when GDP is detrended using the Baxter and King filter and keeping fluctuations between 32 and 200 quarters. In this case, trade in final good is insignificant in all specification and trade in intermediate inputs is high and significant in all specifications.

2.2 Terms of Trade and GDP: the role of Markups

In this section, I use data from 22 countries from 1971 to 2010.[16] to assess the role of price distortions and in particular markups in generating a link between terms of trade and GDP fluctuations.

I focus on a specific form of upward price distortions (markups) and test the following hypothesis: countries where markups are high experience a larger decrease in GDP when experiencing an increase in their terms-of-trade. In order to test this hypothesis, I compute the correlation of filtered GDP with the terms of trade and regress this correlation on an estimate of markups in the country. Results show that using either cross-country variations or within country time variations to identify the effect lead to the same conclusion: markups have a significant impact on GDP-Terms of Trade correlation, with higher markups associated with lower correlation between GDP and the terms of trade.

Data on real GDP and terms of trade at the annual frequency are both taken from the OECD database and filtered according to two different procedure. I first apply the Hodrick and Prescott filter with a smoothing parameter of 6.25 which captures the business cycle frequencies. I also apply the Baxter and King band pass filter and keep fluctuations between 8 and 25 years in order to capture medium-term business cycles (Comin and Gertler (2006)). Using the de-trended series, I compute the correlation between filtered GDP and filtered terms-of-trade for two 20-years time windows from 1971 to 2010, hence creating a panel dataset where each country appears two times. In order to assess the determinants of the GDP-Terms of Trade comovement, I finally regress this correlation on a measure of markups.

In particular, I use Price Cost Margin (PCM) as an estimate of markups within each industry. Introduced by Collins and Preston (1969) and widely used in the literature, PCM is the difference between revenue and variable cost, i.e. the sum of labor and material expenditures, over revenue:

$$PCM = \frac{\text{Sales} - \text{Labor expenditure} - \text{Material expenditure}}{\text{Sales}} \tag{1}$$

Data at the industry level come from the OECD STAN database, an unbalanced panel covering 107 sectors for 34 countries between 1970 and 2010. Due to missing data for many countries in the earliest years, I restrict the analysis for 22 countries.[17] I compute PCM for each industry-country-year and then construct an average of PCM within each country-year by taking the sales weighted average of PCM over each industry. Finally, the average PCM for a given time window is simply the mean of country-year PCM over all time periods. Results are presented in table 2.

[16]The list of countries is: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Iceland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, Portugal, Spain, Sweden, the United-Kingdom and the United-States

[17]For Germany, data are available only from 1991 onward (after the reunification), which is why the total number of observation in the regressions is 43.

	dependant variable: corr($GDP_i^{filtered}$,$ToT_i^{filtered}$)			
	—— Pooled Cross-sections ——		—— FE Regressions ——	
	HP-filter	BK-filter	HP-filter	BK-filter
Average PCM	-1.507*** (-2.70)	-2.049*** (-3.11)	-2.650*** (-2.87)	-3.705*** (-4.10)
Time dummies	no	no	yes	yes
N	43			

Note: The dependent variable is the correlation of filtered GDP with ToT. t stat. in parentheses, *** means $p < 0.01$

Table 2: Markups and GDP-ToT correlation

Finding 3: Higher markups are associated with a stronger GDP reaction to terms of trade

The first two columns of table 2 show the results of pooled cross-section analysis where I do not use the panel dimension of the dataset. In the last two columns, I perform fixed effect regression and add time dummies to control for time specific factors that might affect the correlation of GDP and terms-of-trade. In each of those case, regressions are performed using the two filtering methods and are found to be statistically significant, implying that countries with higher markups also experience a larger decrease in their GDP when the relative price of their import rises.

3 A model of International Trade in Inputs

3.1 Setup

In this section, I build a quantitative model of international trade *in inputs* with *monopolistic competition* and *firm entry/exit* and assess its ability to replicate the strong relationship between trade and business cycle synchronization.[18] I consider an environment with N countries indexed by k. In each country, there is a representative agent with preferences over leisure and the set of available goods Ω_k described by

$$U_{k,0} = \mathbb{E}_0\left[\sum_{t=0}^{+\infty}\beta^t\left(\log\left(C_{k,t}\right) - \psi\frac{L_{k,t}^{1+\nu}}{1+\nu}\right)\right]$$

$$\text{with} \qquad C_t = \left(\int_{\Omega_k} q_{i,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

[18]In section 5, I present a decomposition of the role that each of the novel ingredients have on the quantitative results.

where ψ_k is a scaling parameter, ν is the inverse of the Frisch elasticity of labor supply and σ the elasticity of substitution between final goods. The agent chooses consumption, investment and labor in each period subject to the budget constraint

$$P_{k,t}\left(C_{k,t}+K_{k,t+1}-(1-\delta)K_{k,t}\right)=w_{k,t}L_{k,t}+r_{k,t}K_{k,t}+\Pi_{k,t}$$

where $\Pi_{k,t}$ is aggregate profit in country k at time t.

Production is performed by a continuum of heterogeneous firms combining in a Cobb-Douglas fashion labor ℓ_k, capital k_k and intermediate inputs I_k bought from other firms from their home country as well as from abroad. Firms' productivity is the product of an idiosyncratic part φ and a country specific part Z_k. In each country, the intermediate input index I_k takes a nested CES form allowing a distinction between the micro elasticity of substitution σ between firms from the same country and the macro (Armington) elasticity ρ that measures the substitutability of different country-specific bundles. The production function writes:

$$\begin{aligned}
Q_k(\varphi) &= Z_k.\varphi \ . \ I_k(\varphi)^{1-\eta-\chi} \ . \ \ell_k(\varphi)^{\chi} \ . \ k_k(\varphi)^{\eta} \\
\text{with} \qquad & I_k(\varphi)=\left(\sum_{k'}\omega_k(k')^{\frac{1}{\rho}}M_{k',k}^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \\
\text{and} \qquad & M_{k',k}=\left(\int_{\Omega_{k',k}} m_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}
\end{aligned}$$

where $M_{k',k}$ is a country-pair specific CES bundle of all varieties produced in country k' that are exported to country k and used for production, $\omega_k(k')$ is the share of country k' in the production process of country k with $\sum_{k'}\omega_k(k')=1$ and $\Omega_{k',k}$ is the endogenous set of firms based in k' and exporting to k. For later use, it is useful to define notations for the ideal price indexes dual to the two layers of the nested CES aggregation. I denote by $\mathcal{P}_{k,k'}$ the price of the country-pair specific bundle $M_{k',k}$ and IP_k the price if the intermediate input bundle I_k. The unit cost of the Cobb Douglas bundle aggregating I_k, k_k and ℓ_k (called the "input bundle") is PB_k and represents the price of the basic production factor in country k. The exact expressions of those objects are classic and can be found in the appendix.

The only stochastic elements of this model are the country specific technological shocks (Z_k) which follow an AR(1) process. In all countries, the distribution of productivity φ is Pareto with shape parameter γ and density $g(\varphi)=\gamma\varphi^{-\gamma-1}$. For simplicity and in line with the empirical results in section 2, I restrict trade to be only between firms and not with final consumers.

In order to be allowed to sell its variety to a country j, a firm from country i must pay a fixed cost f_{ij} (in unit of the "input bundle") as well as a variable (iceberg) cost τ_{ij}. Based on their expected profit, firms choose which countries they enter (if any), affecting both the level of competition and the marginal cost of all firms in the country. As will be clear below, profits are strictly increasing with productivity φ so that equilibrium export decisions are defined by country-pair specific thresholds $\varphi_{k,k'}$ above which firms from k find it profitable to pay the fixed cost $f_{kk'}$ and serve country k'. Finally, there is an exogenous mass of potential (but not actual) entrants M_k which I assume to be proportional to the size of labor force L_k.[19]

3.2 Equilibrium

In this section, I present the key conditions that characterize the equilibrium of the model, leaving to the appendix the precise derivations. Introducing X_k the aggregate consumers' revenue in k and S_k the total firms' spendings (including fixed costs payments) in country k respectively, the total demand faced by firm φ is given by

$$q(\varphi) = \left(\frac{p_{k,k}(\varphi)}{\mathcal{P}_k}\right)^{-\sigma} \frac{X_k}{\mathcal{P}_k} + \sum_{k'} \left(\frac{p_{k,k'}(\varphi)}{\mathcal{P}_{k,k'}}\right)^{-\sigma} \left(\frac{\mathcal{P}_{k,k'}}{IP_{k'}}\right)^{-\rho} \frac{\omega_{k'}(k)(1-\eta-\chi)S_{k'}}{IP_{k'}} \tag{2}$$

where $p_{k,k'}(\varphi)$ is the price charged by a firm from country k, with productivity φ, when selling in country k' and the summation is done over all markets that are served by a firm with productivity φ. I assume that firms are monopolists within their variety. Classically, they choose their price at a constant markup over marginal cost and the markup depends on the price elasticity of demand. In this case, the only elasticity that is relevant to firms' pricing is σ, capturing the fact that firms compete primarily with other firms coming from their home country since their individual pricing decision has no impact on their country-specific price index in every market.[20] The marginal cost of a firm with productivity φ in country k is $PB_k/(Z_k\varphi)$ and its optimal price is given by:

$$p_{k,k'} = \tau_{k,k'} \frac{\sigma}{\sigma-1} \frac{PB_k}{Z_k \varphi} \tag{3}$$

Unlike in the canonical Krugman (1980) or Melitz (2003) models of international trade, one cannot solve for prices independently for each firm. Through PB_k, the price charged by firm φ in country k depends on the prices charged by all firms supplying country k (both domestic and foreign) which in turn depend on the prices charged by their suppliers and so on and so forth. The input-output linkages across firms creates a link between the pricing strategies of all firms and one needs to solve for all those prices at once. Doing so requires solving for all country-pair specific price indexes $\mathcal{P}_{k,k'}$.

The definitions of price indexes give rise to a simple relationship between the price of the country

[19] This is the same assumption than in Chaney (2008) or di Giovanni and Levchenko (2012) for example.

[20] With a finite number of firms, both elasticities σ and ρ would appear in the pricing strategy. In such a case, every firm would take into account the fact that its own price has an impact on the unit cost of the corresponding country-specific bundle. Therefore, when decreasing its price a firm would attract more demand compare to firms from its own country but also increase the share of total demand that goes to every other firms from the its country.

k specific bundle at home, $\mathcal{P}_{k,k}$, and its counterpart in country k', $\mathcal{P}_{k',k}$:

$$\mathcal{P}_{k,k'} = \tau_{kk'} \left(\frac{\varphi_{k,k'}}{\varphi_{k,k}} \right)^{\frac{\sigma-\gamma-1}{1-\sigma}} \times \mathcal{P}_k \tag{4}$$

Intuitively, the ratio between the price of a country specific bundle in two different markets depends on the relative iceberg costs as well as the relative entry thresholds. Using this relation in the definition of price indexes in every country yields a system of N equations which jointly defines all price indexes:

$$\mathcal{P}_k^{1-\rho} = \mu_k \left(\sum_{k'} \omega_k(k') \left(\tau_{k'k} \left(\frac{\varphi_{k',k}}{\varphi_{k',k'}} \right)^{\frac{\sigma-\gamma-1}{1-\sigma}} \mathcal{P}_{k'} \right)^{1-\rho} \right)^{1-\eta-\chi} \quad , \; k = 1, ..., N \tag{5}$$

with μ_k depending on thresholds, mass of firms and parameters.[21] For given thresholds and mass of firms, this system admits a unique non negative solution.[22]

Turning now to the determination of export strategies, the thresholds above which firms from country k optimally decide to pay the fixed cost and serve market k' are simply given by:

$$\pi_{k,k'}(\varphi_{k,k'}) = f_{k,k'} \frac{PB_k}{Z_k} \quad \text{for all } k \text{ and } k' \tag{6}$$

where $\pi_{k,k'}(\varphi)$ is the variable profit earned by a firm with productivity φ in market k'. I assume that the fixed cost $f_{k,k'}$ is paid in unit of the basic production factor in country k deflated by the aggregate level of productivity, as is the case in Ghironi and Melitz (2005) for example.

Closing the model involves market clearing conditions for capital, labor and goods. Classic properties of Cobb-Douglas production functions imply that total labor and capital payments are equal to a fraction $\eta + \chi$ of firms' variable spendings, so that $w_k L_k + r_k K_k = (\eta + \chi) S_k$. Moreover, the investment Euler Equation (capital supply) is given by

$$\frac{1}{C_{k,t} - \psi \frac{L_{k,t}^{1+\nu}}{1+\nu}} = \beta \mathbb{E}_t \left[\frac{1}{C_{k,t+1} - \psi \frac{L_{k,t+1}^{1+\nu}}{1+\nu}} \times \left(\frac{r_{k,t+1}}{P_{k,t+1}} + (1-\delta) \right) \right] \tag{7}$$

while labor supply is:

$$\psi L_{k,t}^{\nu} = \frac{w_{k,t}}{P_{k,t}} \tag{8}$$

Finally, before solving for good market clearing, I define R_k the total sales of firms from country

[21] $\mu_k^{\frac{1-\sigma}{1-\rho}} = \frac{\gamma \varphi_{k,k}^{\sigma-\gamma-1}}{\gamma-(\sigma-1)} M_k \left(\frac{\sigma}{\sigma-1} \frac{w_k^{\chi} \times r_k^{\eta}}{\chi^{\chi} \times \eta^{\eta} \times (1-\eta-\chi)^{1-\eta-\chi}} \frac{1}{Z_k} \right)^{1-\sigma}$

[22] Following Kennan (2001) and denoting $G_k = \mathcal{P}_k^{1-\rho}$ and G the associated $N \times 1$ vector, it suffices to show that the system is of the form $G = f(G)$ with $f \; : \; \mathbb{R}^N \rightarrow \mathbb{R}^N$ a vector function which is strictly concave with respect to each argument, which is obvious as long as $0 < \eta + \chi < 1$. This argument stresses the importance of decreasing return to scale with respect to intermediate inputs in order to ensure unicity of the equilibrium.

k made on all markets. Trade being allowed only in intermediate goods, revenues in foreign countries come from other firms' spending while domestic revenues also include consumers' spendings. Then, total revenue of all firms from country k writes

$$R_k = X_k + \left[\sum_{k'} \left(\frac{\mathcal{P}_{k,k'}}{IP_{k'}} \right)^{1-\rho} \omega_{k'}(k)(1-\eta-\chi)S_{k'} \right] \tag{9}$$

This formula has a simple interpretation: firms in country k receive revenues from their final good sales to their home consumers (for a total amount of X_k) as well as from sales as intermediate goods on all markets. In every country k', firms allocate a constant fraction $1-\eta-\chi$ of all their spendings to intermediate inputs, which is then scaled by the weight $\omega_{k'}(k)$ representing the importance of country k in the production process of country k'. Finally, since country k specific bundle in k' is in competition with other country specific bundles in the input market, total revenues of k-firms when selling in k' also depend on the ratio of $\mathcal{P}_{k,k'}$ to $IP_{k'}$ to a power reflecting the relevant the price elasticity, in this case the macro (Armington) one ρ.

Computing revenues in all countries requires an expression for consumers' spendings. With a fixed mass of potential entrants, firms make profits which I assumed to be redistributed to consumers, so that $X_k = w_k L_k + r_k K_k + \Pi_k$. An expression of Π_k can be found using a property first noted by Eaton and Kortum (2005) according to which total profit in country k are proportional to total revenues.

Lemma 1 : Total profit in country k are proportional to total revenues:

$$\Pi_k = \frac{\sigma - 1}{\gamma\sigma} R_k \tag{10}$$

Proof: see Appendix.

Using this expression, together with Labor and capital demand, the good market clearing condition can be written in compact form as

$$\left\{ \underbrace{\left(\mathcal{I}_N - \left(W^T \circ P\right)\right)}_{=M} \begin{pmatrix} S_1 \\ \vdots \\ S_N \end{pmatrix} = 0_{\mathbb{R}^N} \right. \tag{11}$$

where W the weighting matrix defined as $W_{ij} = \omega_i(j)$, P a matrix defined by $P_{ij} = \left(\frac{\mathcal{P}_{i,j}}{IP_i}\right)^{1-\rho}$ and $\circ$ is the element-wise (Hadamard) product. To gain intuitions, one can note that the matrix P scales the weights $\omega_{k'}(k)$ in order to account for the competition across country-specific bundles. If the Armington elasticity ρ is above unity (country specific bundles are substitutes) then a country i which is able to charge low prices in some market j (a low $\mathcal{P}_{i,j}$) will attract a higher share of total expenditures from all firms in this country. Classically, this effect completely disappears in the case of a Cobb-Douglas aggregation of country specific bundles, because in such a case the spending shares are fixed.

The solutions of this system form a one dimensional vector space,[23] revealing that, as in any general equilibrium model, one needs to normalize one price in order to fully characterize the equilibrium. Setting $w_1 = 1$, implying $S_1 = L_1/\chi$, provides a unique solution for all variables by solving together the price system (5), the threshold system (6), the Spending system (11) as well as the labor and capital market equilibrium conditions.

GDP definition I define GDP in real terms using double deflation, consistent with BEA practices:[24]

$$GDP_k = \underbrace{\frac{X_k}{\mathcal{P}_k}}_{\text{Consumption + Investment}} + \underbrace{\sum_{k'} \frac{Trade_{k\to k'}}{\mathcal{P}_{k,k'}}}_{\text{Exports}} - \underbrace{\sum_{k'} \frac{Trade_{k'\to k}}{\mathcal{P}_{k',k}}}_{\text{Imports}} \tag{12}$$

Note that the first term is also equal to the Gross National Income (GNI$=w_k L_k + r_k K_k + \Pi_k$) since there is no trade in assets across countries.

3.3 GNI elasticity in a simplified case

In order to investigate the mechanisms driving the propagation of shocks across countries in the model, let us study a special case with $\rho = 1$ and fixed labor and capital supply.[25] The goal of this section is to compute the elasticity of GNI in country i with respect to a technology shock in country 1 for every country i:

$$\eta_{GNI_i,Z_1} = \frac{\partial \log(GNI_i)}{\partial \log(Z_1)}$$

In this simplified setup, I compute the elasticity of all endogenous variable with respect to technological shocks. This process leads to the closed-form formula in lemma 2.

Lemma 2 : In the Cobb-Douglas ($\rho = 1$) case and fixing both labor and capital supply, the elasticity of every GNI with respect to a technology shock in country 1 is given by:

$$\begin{pmatrix} \eta_{GNI_1,Z_1} \\ \vdots \\ \eta_{GNI_N,Z_1} \end{pmatrix} = (\mathcal{I}_N - (1-\eta-\chi)W - T)^{-1} \begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix} \tag{13}$$

[23] One can easily show that the matrix M is non invertible: summing all rows results in a column of zero.

[24] According to the BEA, real gross domestic product (GDP) is "*the value of the goods and services produced by the nation's economy less the value of the goods and services used up in production, adjusted for price changes*"

[25] Without capital supply, the model is completely static and without labor supply the mass of potential entrants is fixed in every country.

with W the weighting matrix defined above and T a "Transmission" matrix[26], function of γ and σ.

Proof: see Appendix.

This expression is reminiscent of what can be found in static Cobb-Douglas network models such as Acemoglu et al (2012) for example, with an additional effect coming from firm heterogeneity and the extensive margin adjustments captured by the matrix T. In this context, the international propagation pattern of country specific shocks runs through two channels. First, for fixed spending share, the matrix W records the input-output linkages if the export decision of firms are kept constant. Second, the change in prices and revenues in all markets triggers a change in the productivity thresholds $\varphi_{k,k'}$. This channel is characterized by the matrix T which is a function of σ and γ which govern the adjustments along the extensive margin.

The computations leading to the expressions of the GNI elasticity in this lemma are greatly simplified by the assumption that factors of production (labor and capital) are fixed. In the general model, however, this constitute an important amplification channel through two effects. First, as in many macro models, a positive productivity shock in any country contributes to the decrease of prices all over the world and hence an increase in real wage. This triggers an increase in labor supply that amplifies the benefits of the shock in terms of output.[27] In addition to this classic effect, there is a second channel going through the change in the mass of active firms in every country. With the assumption that the mass of potential entrepreneurs is proportional to the labor size, an increase in labor supply results in a proportional increase in the mass of potential entrants. Whether the mass of actual producing firms goes up or down in any country k will also be determined by the changes in the thresholds φ_{ik} for all i which in turns crucially depends on the value of the Armington elasticity ρ. In the Cobb-Douglas case where the expenditure shares are fixed, a positive technological shock will result in a decrease of all entry thresholds in every market. Putting pieces together, a positive shock triggers at the same time more potential entrepreneurs and a decrease of the entry threshold in every market. As a result, the new structure of input-output linkages amplifies the benefits of the shock.

4 Calibration

In the remaining of the paper, I perform a quantitative exercise and assess the model's ability to match the strong empirical relationship between trade proximity in intermediate input and output synchronization. The model is calibrated to 14 countries and a composite rest-of-the-world for the

[26] $T = \Lambda \mathcal{I}_N$, with $\Lambda = \frac{1}{\sigma + \frac{(\sigma-1)^2}{\gamma-(\sigma-1)}}$

[27] This increase in labor supply is tempered by the wealth effect. In models that use GHH preferences, the absence of wealth effect in the labor supply decision makes this channel of adjustments very strong.

time period 1989 to 2008. In what follows, I explain in detail my calibration strategy while the results are gathered in the next section.

For a simulation with N countries, there are $3 \times N^2 + N + 6$ parameters to set. For each country-pair (i, j), there are the weights $\omega_i(j)$, the iceberg trade cost τ_{ij} and the fixed cost $f_{i,j}$, then for every country i we have the scaling parameter ψ_i and finally the set of common parameters: χ and η for the labor and capital share respectively, ν for the (inverse) elasticity of labor supply, γ for the distribution of productivity draws, σ for the within country (micro) elasticity of substitution across varieties, ρ for the (macro) elasticity of substitution of country-specific bundles and the ratio of the mass of potential entrepreneurs to the labor size.

My calibration is a mixture of estimations from micro data (taken from the literature as well as re-estimated) and a matching of macro moments. The goal is to match exactly the relative GDP across all country pairs as well as the trade proximity in intermediate goods in order to give a reasonable account of the ability of the model to generate a strong link between trade and output synchronization despite the fact that typical trade flows between two given countries are very low compare to their GDPs.[28]

From micro data

The discount factor β is 0.99. The (inverse) elasticity of labor supply ν is 2/3 leading to a Frisch elasticity of 1.5.[29] The ratio of potential entrepreneurs M and the labor size L is taken to be 0.1 and is in line with the ratio of total number of firms divided by the population in the US, taking into account that not all potential entrepreneurs enter the economy in equilibrium. The variable (iceberg) trade costs are taken from the ESCAP World Bank: "International Trade Costs Database"[30]. This database features symmetric bilateral trade costs in its wider sense, including not only international transport costs and tariffs but also other trade cost components discussed in Anderson and van Wincoop (2004).

As in DiGiovanni and Levchenko (2012), fixed access costs are computed from the "Doing Business Indicators".[31] In particular, I measure the relative entry fixed costs in domestic markets by using the information on the amount of time required to set up a business in the country relative

[28] Using the same 14 countries as in the simulations (not including the rest-of-the-world), the average "trade proximity" in intermediate goods (that is: total trade flows in intermediate inputs divided by the sum of GDP) across all country pairs is about 0.01% of GDP for the time period 1999 to 2008, with a value of about half a percent for US-Canada and for France-Germany.

[29] The calibration of this parameter is subject to numerous debates between micro- and macro-economists. Estimates with micro data usually yields a low Frish elasticity of labor supply while the macro models traditionally take higher values. Sensibility analysis on this parameter are performed.

[30] See at `http://artnet.unescap.org/`

[31] The World Bank's Doing Business Initiative collected data on regulations regarding obtaining licenses, registering property, hiring workers, getting credit, and more. See `http://francais.doingbusiness.org/data/exploretopics/trading-across-borders` and `http://francais.doingbusiness.org/data/exploretopics/starting-a-business`

to the US.[32] If according to the Doing Business Indicators database, in country i it takes 10 times longer to register a business than in the U.S., then $f_{i,i} = 10 \times f_{US,US}$. I normalize the lowest entry fixed cost so that no entry threshold lies below the lower bound o the productivity distribution, which is taken to be one in every country. To measure the fixed costs associated with entry in a foreign market, I use the Trading Across Borders module of the Doing Business Indicators. I choose the number of days it takes to import to a specific country, using the same normalization as for the domestic entry cost.[33]

In the benchmark simulations, I choose the macro (Armington) elasticity ρ to be equal to unity while the micro elasticity σ is equal to 5. There are many papers estimating those elasticities for intermediate or final goods. Saito (2004) provides estimations from 0.24 to 3.5 for the Armington elasticity[34] and Anderson and van Wincoop (2004) report available estimates for the micro elasticity in the range of 3 to 10. Following Bernard, Eaton, Jensen, and Kortum (2003), Ghironi and Melitz (2005) choose a micro elasticity of 3.8. Recently, papers such as Barrot and Sauvagnat (2015) or Boehm, Flaaen and Pandalai-Nayar (2015) argue that firms' ability to substitute between their suppliers can be very low. The choice of a value of $\sigma = 5$ leads to markups of 25%. The aggregate profit rate, however, is only of 17.4% since firms have to pay fixed cost in order to access any market. There is also a theoretical convenience to use $\rho = 1$, as it allows the model to take the same form as other network models such as Acemoglu (2012), Bigio and La'O (2015) and many others.

The share of intermediate $(1-\eta-\chi)$ is fixed at 0.7 so that total of spending on intermediate over total revenues equals 0.56, the value reported in Acemoglu et al (2012). Note that this parameter is then tied to the choice of σ as total revenues are higher than spendings due ot the presence of markups. Finally, I set $\gamma = \sigma - 0.4$ as described in Fattal-Jaef and Lopez (2010) which allows me to match the standard deviation of plant size distribution in the US.

Matching of macro moments

I jointly set the country size parameters $(\psi_i)_{i=1,...,N}$ as well as the shares $\omega_i(j)$ (the matrix W) in order to match all countries relative GDP and all relative trade flows in real terms. More precisely, I normalize the real GDP of the composite rest-of-the-world to 100 and set all other real GDPs so that the ratio of their real GDP to the one of the rest-of-the-world in the simulated economy matches exactly its counterpart in the data for the time window 1989 to 2008. My targets are

[32] As argued in DiGiovanni and Levchenko (2012), using the time taken to open a business is a good indicator because it measures entry costs either in dollars or in units of per capita income, because in the model $f_{i,i}$ is a quantity of inputs rather than value.

[33] This approach means that the fixed cost associated with trade from France to the US is the same as the one from Germany to the US. One must keep in mind, however, that the iceberg variable cost will differ.

[34] Feenstra et al (2014) studies the macro and micro elasticities for final goods and reports estimates between -0.29 and 4.08 for the Armington elasticity. They find that for half of goods the macro elasticity is significantly lower than the micro elasticity, even when they are estimated at the same level of disaggregation.

Parameter	Value	Counterpart
β	0.99	Discount factor – Annual discount rate of 4%
ρ	1	Macro (Armington) Elasticity of substitution (from Literature)
σ	5	Micro Elasticity of substitution – Markup of 25%, Avrg Profit of 17.4%
ν	2/3	Labor Curvature – Frisch Elasticity of 1.5
M/L	0.1	Mass of plants over working population (targeting the US)
τ_{ij}	[1 - 3]	Iceberg trade cost – From ESCAP - World Bank
f_{ij}	[1 - 10]	Fixed trade cost – "Doing Business Indicators"
γ	4.6	Pareto shape – St. dev of plant size distrib. in US ($\sigma - 0.4$)
$1-\eta-\chi$	0.7	Input share – Intermediate spending over revenues: $(1-\eta-\chi)\frac{\sigma-1}{\sigma} = 0.56$
χ	0.09	Capital share – 30% of value added.

Table 3: Parameters fixed using micro data

then N real GDP targets as well as N^2 directed trade flows (over GDP), with an equal number of parameters to match $(N^2 + N)$ if one considers the coefficients in the weighting matrix W and the scaling vector $(\psi_i)_{i=1,\dots,N}$.

Finally, I need to calibrate the variance-covariance matrix for the country-level TFP shocks $(Z_i)_{i=1,\dots,N}$. In order not to overestimate the impact of idiosyncratic shocks, I chose to set their volatility (the diagonal element of the variance-covariance matrix) so that the model can replicate the volatility of output (de-trended using HP filtering). This allows me to generate fluctuations in the simulated economy that are similar to those observed in the data. Similarly, I set the off diagonal elements (the covariance terms) so that the average correlation of GDP in the model match the one observed in the data. Recall that the goal of this exercise is not to explain the *level* of comovement across countries, but its *slope* when there is a change in trade. Hence, I set the *level* at the appropriate level and will vary parameters governing trade in order to evaluate the *slope*.

5 Quantitative results

Trade Comovement Slope

The goal of this section is to assess the ability of the model to replicate the strong empirical relationship between trade proximity in intermediate inputs and GDP synchronization. The calibration procedure presented in the previous section yields values for all parameters so that the model economy matches the data for the period 1989 to 2008. With those values, I simulate a sequence of 5,000 shocks and record the correlation of HP-filtered GDP as well as the average index of trade proximity. I then recalibrate the model with different targets for trade proximity across countries, decreasing and increasing the target by 5%. For each configuration, I feed the economy with the *exact same* sequence of 5,000 shocks and record the correlation of HP-filtered GDP as well as the average index of trade proximity. This gives rise to a panel data set in which I have $14 \times 13/2 = 91$ observations for each of the 3 configurations, hence a total of 273 observation. I then perform fixed effect regressions on the simulated dataset and find that the model is able to explain about 85% of the trade-comovement slope.

	dependant variable: corr(GDP_i^{HP},GDP_j^{HP})	
	Data	Model
log(Intermediate)	0.065***	0.056***

Decomposition - Role of the ingredients

In order to assess the role of each ingredient in the quantitative results, I then turn off one by one the different elements of the model. Results yield interesting insights. First, the sole addition of price distortions to an otherwise classic IRBC model with input-output linkages increases the trade comovement slope from 0.010 to 0.024. Moreover, allowing endogenous fluctuations in the productivity thresholds above which firms choose to access any market further increases the slope from 0.024 to 0.027, showing that the adjustment along the extensive margin contribute to the international propagation of shocks through trade. Finally, the amplification coming through the fluctuation in the mass of firms increases again the slope from 0.027 to 0.056, showing that fluctuations in the mass of producing plant is a powerful way to generate the empirical link between trade in inputs and GDP comovement.

	Trade-Comovement Slope
I/O linkages + Markups + Extensive Margin	0.056***
I/O linkages + Markups + Extensive Margin (Fixed M)	0.027***
I/O linkages + Markups	0.024***
I/O linkages	0.010***

Table 4: Decomposition of the result

Sensibility Analysis

In figure 1, I show the results of the quantitative exercise described above with different values for σ (the micro elasticity of substitution) and ν (the inverse of the Frisch elasticity of labor supply). We can see that both play an important role, with the trade comovement slope being larger whenever σ is lower and the Frisch elasticity is higher. The former reflects at the same time the importance of the markups and the love for variety in generating the result. The latter suggests that the adjustment along the labor margin as well as along the mass of potential entrepreneur both play a significant role.

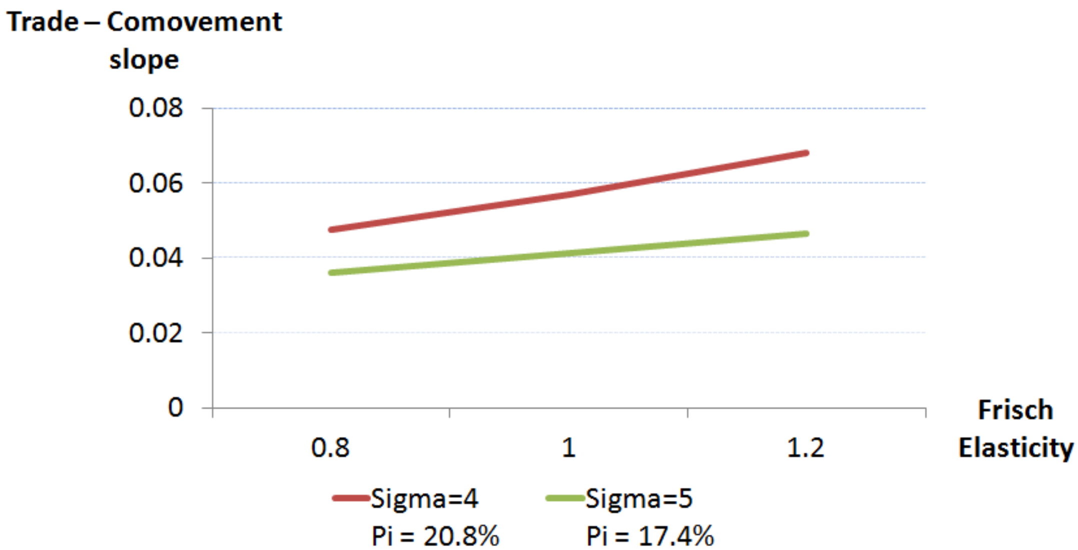

Figure 1: Sensibility Analysis

Impulse Response functions

In order to give a better sense of the mechanics behind the model, I consider a simplified version with two countries (Home and Foreign) that are symmetric in the steady state. Keeping the value of all technological parameter as described above[35], I generate impulse response function of Home GDP after a technological shock in Foreign. In order to have a sense of the trade comovement *slope*, I consider two calibrations of the W matrix: one that induces a low level of trade and the other with a high level of trade. By comparing the GDP response in those two cases, one can understand the effect of increasing trade on GDP synchronization. Figure 2 presents the result of this exercise for three versions of the model. In the benchmark case with no markups (perfect competition within each variety) and no extensive margin (no fixed cost to enter any market and a fixed mass of firms), the GDP hardly moves. When introducing monopolistic pricing for all varieties, increasing trade between the two countries leads to a significant increase in the Home GDP reaction after a foreign techonlogical shock. Finally, letting the mass of firms and entry decisions be as described in the quantitative models further amplify the trade comovement slope, with an increase in trade inducing a very high increase in GDP reaction.

Before describing the role of each of those ingredients in the context of a simplified model in section 6, I further decompose the GDP reactions described above by performing a "growth accounting" exercise in which I decompose GDP fluctuations into labor and capital movements as

[35]Except for the W matrix which is now symmetric and 2x2.

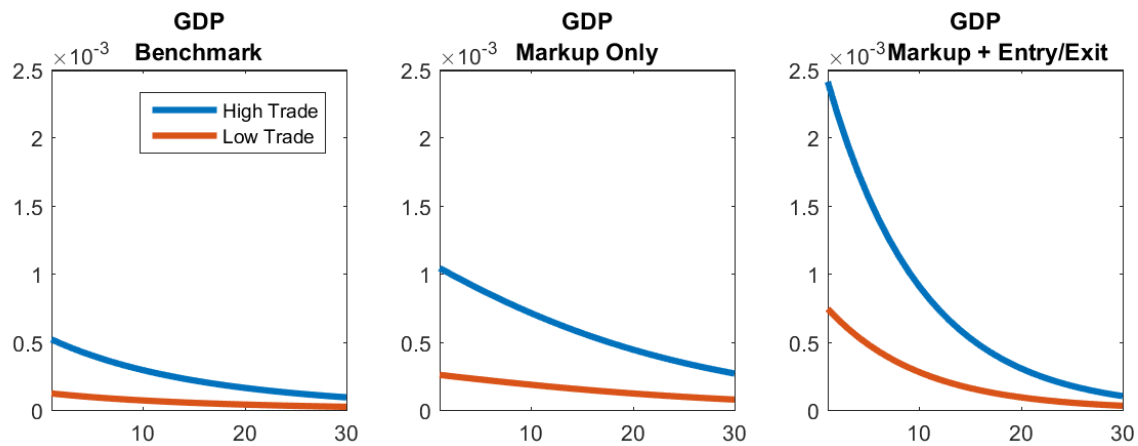

Figure 2: IRF of domestic GDP after a foreign shock

well as the Solow residual that is usually referred to as the aggregate TFP.[36] In the benchmark case with no markups and no extensive margin, one can see that GDP fluctuation is due almost only to fluctuations in factor supply with TFP playing a insignificant role. This result is consistent with Kehoe and Ruhl (2008) or Cravino and Burstein (2015) which argue that foreign technological shocks have no effect on domestic productivity up to a first order approximation.

Interestingly, this result does not hold anymore when markups are introduced and measured TFP is affected by a foreign shock. As described more precisely in section 6, the reason stems from fact that in the presence of markups, the increase in import due to the positive technological shock in the foreign country is smaller than the increase in final good production, hence opening the room for a GDP change even with fixed factor supply. As noted in Gopinath and Neiman (2014) as well as in Kim (2014), with positive markups, the opportunity cost of using up inputs is exceeded by the gains from their contribution to value added. Note also that the TFP change induces a larger reaction of domestic factors (labor and capital) which increases the GDP reaction after the foreign shock.

Finally, introducing fluctuations in the mass of firms serving all markets increases further the TFP movement. This effect is due to the love for variety encompassed in the Dixit-Stiglitz aggregation of inputs. With love for variety, one can think of the mass of firms as being an input for production since an economy with a higher number of firms has the ability to produce more final output with the same amount of inputs. As suggested by the decomposition in table 4, the most important part of this mechanism is not due to the fixed cost associated to the access of any market but rather to the fluctuation in the mass of potential entrants, that is assumed to be proportional to the labor force. Indeed, any fluctuation along the labor supply margin is associated with a change in the mass of potential entrants. With love for varieties, the production technology frontier is affected by such a change in the number of producer, so that the final output reacts more than

[36]Consistently with the theory, I used $\eta/(\eta+\chi)$ for the labor share and $\chi/(\eta+\chi)$ for the capital share to compute the solow residual

imported inputs. Moreover, since the Solow residual is computed using only Labor and Capital as domestic inputs and not controlling for the change in the mass of domestic firms, this increase in the production technology frontier is reflected in the TFP.

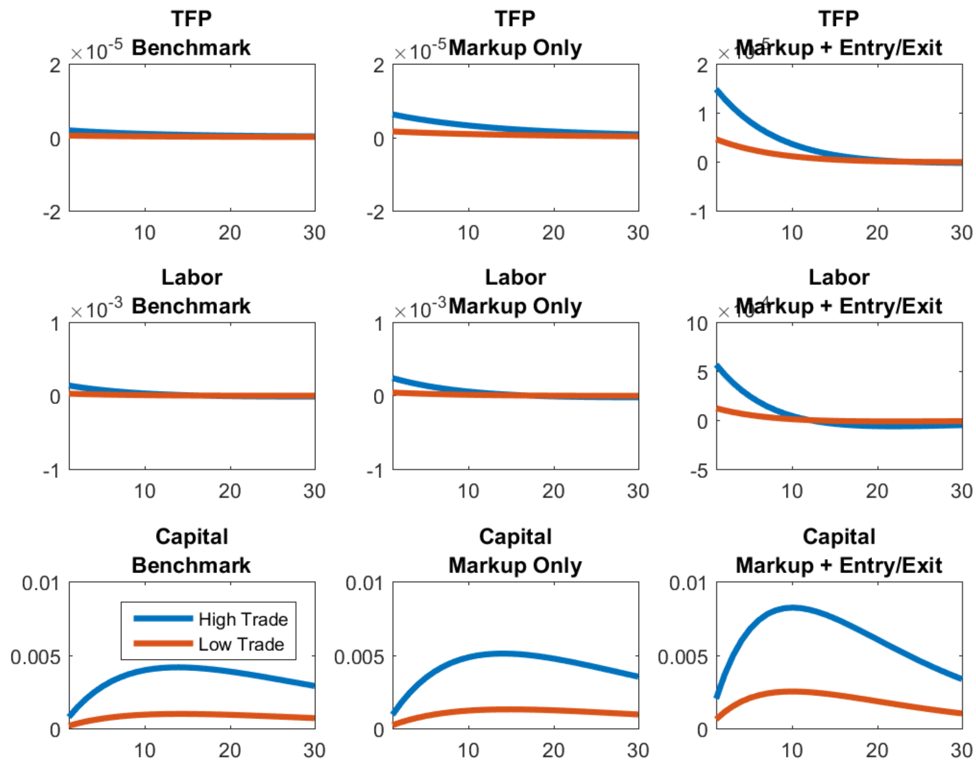

Figure 3: Growth Accounting Decomposition

6 A simple model

In this section, I build a very simple model to get intuition for the role of the two ingredients considered (monopolistic pricing and extensive margin adjustment) in generating a link between foreign shocks and domestic GDP.

For the sake of exposition, I consider a static small open economy. In such a world, KR showed that a change in the price of imported inputs has no impact, up to a first order approximation, on measured productivity. This means that any change in GDP is due to a change in the supply of domestic factors of production.

6.1 The Kehoe and Ruhl (2008) curse

The economy produces two goods. The consumption good y is used for consumption and exports and is produced by combining the intermediate good m and domestic factors of production ℓ (possibly a vector) according to:

$$y = F(\ell, m) \tag{14}$$

where $F(.,.)$ has constant returns to scale and is concave with respect to each of its argument. Normalizing the price of y to one, the final good producer chooses intermediate and domestic inputs to maximize its profit taking as given the price of production factors. Optimality requires that factors are paid their marginal product:

$$F_\ell(\ell, m) = w \qquad \text{and} \qquad F_m(\ell, m) = p^m \tag{15}$$

with p^m the price of intermediate good m and w the price of domestic factors. The intermediate good is produced linearly with imports (x) according to

$$m = x \tag{16}$$

Perfect competition in this sector insures that the price p^m equates the marginal cost of production such that $p^m = p^x$ where p^x is the price of imports. Finally, real GDP Y is the sum of value added in the country. Using a double deflation method and base year pricing, it is defined as:

$$Y = F(\ell, m) - p_b.x \tag{17}$$

with p_b the import price in a "*base year*". Let us now compute the first order change in GDP when the Terms-of-Trade ($\equiv p_x$) rise. Assuming fixed factor supply for simplicity, we get

$$\begin{aligned} \frac{dY}{dp_x} &= F_m(\ell, m)\frac{\partial m}{\partial p^x} - p_b\frac{\partial x}{\partial p^x} \\ &= p^m\frac{\partial m}{\partial p^x} - p_b\frac{\partial m}{\partial p^x} \\ &= \frac{\partial m}{\partial p^m}(p^m - p_b) \end{aligned}$$

With a chain weighted construction of GDP, the base price is close to the actual price of imports ($p^b \approx p^x$).[37] Because we assumed perfect competition in the production of m, the price p^m satisfies

[37] With Laspeyres Indexes, the price used to value the goods is the price before the shock and it is indeed exactly p^x. If we use instead the Paasche Index, the price used to value the imports is the one that applies after the shock, so in our case $p^x + \epsilon$ because computing the derivative means that we are interested in small deviations. In many cases, statistical agencies use the Fisher chain-weighted price index which is the geometric average of the Paasche and the Laspeyres index.

the zero profit condition: $p^m = p^x$. As a consequence, we have :

$$\frac{dY}{dp_x} = m'(p^x)(p^m - p_b) \approx m'(p^x) \times (p^m - p^x) = 0 \tag{18}$$

In such a model with fixed factor supply, a change in the terms of trade has no impact on GDP. This is the negative result in KR. An increase in p^x leads to a decrease in imports x, a decrease in intermediate production m and hence a decrease in production y. With perfect competition and no price distortion, the first order decrease in final production is exactly the same as the decrease in imports, implying that GDP is not affected.

6.2 Markups and Love for variety

Consider now a variant of the economy described above with two new elements: (1) a *price wedge* $\mu \neq 1$ in the pricing of the intermediate m so that $p^m = \mu \times p^x$, and (2) love for variety in the final good production technology. In particular, I use the CES functional form proposed by Benassy (1996) that disentangle the role of the elasticity of substitution between varieties[38] from the sheer love of variety. The production function in the final good sector is

$$y = F(\mathcal{I}, \ell) \qquad \text{with} \;\; \mathcal{I} = \mathcal{M}^{\alpha - \frac{1}{\sigma-1}} \left(\int_0^{\mathcal{M}} m_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{19}$$

This production function displays product differentiation between the intermediate inputs (for $\sigma \neq +\infty$) as well as love for variety (for $\alpha > 0$) in the following sense: for a given amount of total imports, the larger the mass of input suppliers $\mathcal{M}$, the higher the amount of final production obtainable. If $\alpha = 1/(\sigma - 1)$, one recovers the usual Dixit-Stiglitz aggregation where the parameter σ governs at the same time the substitutability across intermediate goods and the love for variety.

For each variety m_i, there is a producer with a linear technology

$$\forall \; i \;\in [0, \mathcal{M}], \quad m_i = x_i \tag{20}$$

Since all intermediate producers have the same technology and are thus completely symmetric, I denote by m their (common) production and by x their (common) import levels. The bundle $\mathcal{I}$ can then be simply expressed as $\mathcal{I} = \mathcal{M}^{1+\alpha} m$ and the price index dual to the definition of the bundle is $\mathcal{P} = \mathcal{M}^{-\alpha} p^m$, which is also equal to $F_{\mathcal{I}}(\mathcal{I}, \ell)$, the marginal productivity of the input bundle in final good production. Finally, taking the derivative of Y with respect to p^x and using the same approximation that the base price used to value imports is close to the actual price, the first order

[38] In many models, the elasticity of substitution governs the degree of market power of monopolistic competitors and ultimately their markup. To keep the model general, I assume here that the markup μ can take any value, including $\mu = \sigma/(\sigma - 1)$.

change in GDP when the import price changes is given by

$$\frac{dY}{dp^x} = \left(\mathcal{M}\frac{\partial m}{\partial p^x} + \frac{\partial \mathcal{M}}{\partial p^x}m\right).(p^m - p^x) + \alpha p^m m \frac{\partial \mathcal{M}}{\partial p^x} \tag{21}$$

In the above expression, both terms play a role in getting GDP movement following a foreign shock. First, the very existence of a price wedge $\mu \neq 1$ means that $p^m - p^x > 0$ and the first term does not vanish. If $m'(p^x) < 0$[39] and prices are distorted upward due to monopolistic pricing, an increase in the price of imported inputs leads to a decrease in GDP. Intuitively, when p^x increases, the reduction in final good production y is larger than the reduction in imports x, which leaves room for a change in GDP. The key reason that explains that the decrease in final good production exceeds the decrease in imports is the presence of the markup: because the price of intermediate m rises more than one-for-one with the change in its cost of production, there is an amplification. Moreover, any change in the mass of firms $\mathcal{M}$ also plays a role the GDP reaction to p^x. A change in the number of firms that distort prices gives a time varying element of the aggregate price wedge, triggering a greater reaction of GDP after a foreign shock. Note that this effect is not governed by the love for variety which is captured by the parameter α. One can model many reasons why the mass of producing firms would change, including a free entry condition with initial sunk cost or any reason that changes the supply of potential entrepreneurs.[40] Overall, the key idea can governing this first term can be expressed as follow: if we introduce a wedge between the price of imports and the marginal product of imports in final good production, then terms of trade shocks have a direct and first order impact on GDP even with fixed factor supply.

Second, when $\alpha \neq 0$, another effect arises. When the production function exhibits love for varieties ($\alpha > 0$), any change in the mass of firms causes the input bundle $\mathcal{I}$ to be more reactive.[41] If the increase of p^x is accompanied by a decrease in the mass of producing firm,[42] the bundle $\mathcal{I}$ decreases not only because each intermediate producer will tend to produce less, but also because a decrease in the mass of firms mechanically decreases the bundle even for a given amount of inputs.

With love for variety, a producer that has access to more suppliers can produce more output for the same level of input. Another way of saying this is that the set of feasible combinations of output $\mathcal{I}$, and inputs $\int_0^{\mathcal{M}} m_i di = X$ is not independent of the mass of producers $\mathcal{M}$: a change of $\mathcal{M}$ has an effect on the production possibility frontier.

When $\partial\mathcal{M}/\partial p^x < 0$, an increase in the price of imports decreases the mass of intermediate firms

[39]This can be easily proved if assuming that $F(.)$ is a Cobb Douglas aggregation of domestic factors and intermediates.

[40]In additional appendix available upon request, I have modeled the free condition and showed that it indeed leads to a decrease in the mass of firms after an increase of import prices.

[41]It is informative to observe that the only difference between the case when $\alpha = 0$ and $\alpha \neq 0$ is the extra term in the derivative of $\mathcal{I}$ with respect to p^x.

[42]If the mass of firms is pinned down by a free entry condition, the decrease in profits of each intermediate producer when the price of imported input goes up leads to a decrease in the mass of firms.

and leads to a stronger reaction in final good production than in the intermediate good sector, leaving the door open a GDP change. Interestingly, this channel is at work independently of the price distortion channel discussed previously. Even in the absence of monopoly pricing, the sheer fluctuation in the mass of producing firms coupled with a love for variety in final good production creates a link between import price and GDP fluctuation even with fixed factor supply. Using a "growth accounting" perspective, those changes would then be accounted for in the Solow residual, meaning that markups and love for varieties coupled with entry/exit of firms are two ingredients that have the potential to create a link between domestic TFP and foreign shocks.

7 Conclusion

In this paper, I show that there exist a strong empirical link between GDP comovement and tradein intermediate inputs and that the relationship between domestic GDP and the terms of trade fluctuation is strongly impacted by the size of markups within each country.

Motivated by those findings, I propose a way to solve the "Trade Comovement Puzzle" by the combination of three ingredients: (1) trade in inputs, (2) monopolistic pricing and (3) firm entry/exit. With round-about production, the model features international supply chains which give rise to a strong interdependency across all firms in the economy for both their pricing and their export strategies. In this context, the propagation of technological shocks across countries depends on the worldwide network of input-output linkages and is strongly impacted by the adjustments along the extensive margin. The presence of price distortions as well as fluctuation in the mass of producing firms generate a strong link between import prices and GDP fluctuations.

Finally, I calibrate this model to match precisely the relative GDPs, trade proximities and standard deviation of output for 14 OECD countries and assess its ability to replicate the empirical findings. Overall, the quantitative exercise suggests that the model is able to replicate 85% of the trade comovement slope and makes an important step toward solving the "Trade Comovement Puzzle".

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A Empirical Appendix

A.1 Data description

I focus the empirical analysis on a subset of all OECD countries. For the evolution of GDP correlation, I use 24 countries for which I could gather quarterly data on real GDP, which gives me $N(N-1)/2 = 276$ country pairs. Those are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Luxembourg, Mexico, Netherlands, New-Zealand, Norway, Portugal, South-Africa, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States. For the study of the trade-comovement, I took out four countries from the sample. I get rid of Belgium and Luxembourg because they were pooled together in the trade data until the year 2000, preventing me from using 4 time windows and perform the fixed effect regression properly. I also took out New Zealand and South Africa because their trade data contained many zeros for some time periods,[43] resulting in some country-pairs being presents only for some time windows and hence reducing the effectiveness of the fixed effect regression. This leaves me in the end with 20 countries, so 190 country pairs for each of the four time window.

I use data from the OECD database "VPVOBARSA" which features quarterly GDP. In this database, numbers are expressed as constant 2005 prices converted with 2005 PPPs. As for the trade flows, I use two datasets. From 1968 to 1999, I use the NBER-UN world trade data updated on the 30th of January, 2009.[44] From 2000 onward, I use the revision of those data. In both datasets, trade flows are categorized using SITC4, which represents the first 4 digits of the SITC Rev 2 categorization. I used two different ways to separate final from intermediate goods. In the first method, I translate the SITC4 codes into END USE codes using the concordance table available on the CID website.[45] The end-use codes are used by the Bureau of Economic Analysis to allocate goods to their final use, within the National Income and Product Accounts. I then label as "intermediate goods" the products which end-use codes correspond to the list put together by Feenstra and Jensen. As a robustness check, I also use a second method to separate final and intermediate goods, which results are available upon request and are virtually the same. In this method, I translate the initial SITC4 codes into BEC using a table mappingone into the other from the World Bank website.[46] However, this table is not complete at all and, for example, almost 10% of the total flows for the year 1962 cannot be allocated to BEC codes. In order to improve the quality of the match, I took the list of all SITC4 codes that were still unmatched in the 1962

[43] To fix ideas, in 1969, while Spain was importing from 134 countries and exporting to 140 countries, South Africa was importing from only 65 and exporting to 77 countries. As for New Zealand, the numbers were 105 (imports) and 127 (exports).

[44] This data is constructed from United Nations trade data by Robert Feenstra and Robert Lipsey. It can be downloaded, as well as the associated documentation, from the Center for International Data (CID)'s website at http://cid.econ.ucdavis.edu/nberus.html

[45] See at http://cid.econ.ucdavis.edu/usix.html, under the directory "1972-2006 U.S. import data - SAS and STATA".

[46] See http://wits.worldbank.org/product_concordance.html

data (the first year available in the CID data) and used the SITC to BEC table provided on the EUROSTAT website[47]. This table is in PDF format and one needs to manually go through file in order to identify which of those unmatched SITC codes (at the 4 digits level) can be allocated uniquely and without ambiguity to any of my two categories. As an illustration, this method allows me to take care of 97.64% of all trade flows in the year 1962 and 99.48% of all trade flows in 1990. Finally, from BEC codes, one can categorize goods into Intermediate, Capital and Final goods with a concordance table available on the UNSTAT website.[48]

A.2 Additional Results

Trade-Comovement slope with Baxter and King Filtering

	dependant variable: corr(GDP_i^{BK},GDP_j^{BK})					
	(1)	(2)	(3)	(4)	(5)	(6)
log(Total)	0.330***		0.100*		0.126**	
	(12.52)		(1.97)		(2.52)	
log(Intermediate)		0.289***		0.135***		0.121***
		(9.45)		(3.41)		(3.12)
log(Final)		-0.044		-0.054		-0.000
		(-0.95)		(-1.20)		(0.01)
Country-Pair FE	yes	yes	yes	yes	yes	yes
Time Trend	no	no	yes	yes	no	no
Time FE	no	no	no	no	yes	yes
N	760					

t stat. in parentheses, *** means $p < 0.01$, ** means $p < 0.05$ and * means $p < 0.10$

Trade-TFP slope with HP Filtering

	dependant variable: corr(TFP_i^{HP},TFP_j^{HP})					
	(1)	(2)	(3)	(4)	(5)	(6)
log(Total)	0.272***		0.099**		0.123***	
	(11.05)		(2.78)		(3.52)	
log(Intermediate)		0.205***		0.049		0.035
		(7.53)		(1.45)		(1.07)
log(Final)		0.018		0.044		0.095**
		(0.44)		(1.11)		(2.50)
Country-Pair FE	yes	yes	yes	yes	yes	yes
Time Trend	no	no	yes	yes	no	no
Time FE	no	no	no	no	yes	yes
N	612					

[47] See at http://unstats.un.org/unsd/tradekb/Knowledgebase/Intermediate-Goods-in-Trade-Statistics
[48] See at http://unstats.un.org/unsd/tradekb/Knowledgebase/Intermediate-Goods-in-Trade-Statistics

Trade-TFP slope with Baxter and King Filtering

	dependant variable: corr(TFP_i^{BK},TFP_j^{BK})					
	(1)	(2)	(3)	(4)	(5)	(6)
log(Total)	0.296***		0.079		0.101**	
	(9.58)		(1.63)		(2.06)	
log(Intermediate)		0.290***		0.126**		0.113**
		(8.55)		(2.56)		(2.34)
log(Final)		-0.081		-0.054		-0.012
		(-1.48)		(-1.00)		(-0.22)
Country-Pair FE	yes	yes	yes	yes	yes	yes
Time Trend	no	no	yes	yes	no	no
Time FE	no	no	no	no	yes	yes
N	612					

B Theoretical Appendix

B.1 Equilibrium Conditions in the general CES case

Price Indexes and Pricing System

$$\mathcal{P}_{k,k'} = \left(\int\limits_{\Omega_{k,k'}} p_{k,k'}(\varphi)^{1-\sigma} g(\varphi) d\varphi \right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad IP_k = \left(\sum_{k'=1,...,N} \alpha_k(k') \mathcal{P}_{k',k}^{1-\rho} \right)^{\frac{1}{1-\rho}}$$

$$PB_k = \chi^{-\chi} \times \eta^{-\eta} \times (1-\eta-\chi)^{(\eta+\chi-1)} \times IP_k^{1-\eta-\chi} \times w_k^{\chi} \times r_k^{\eta}$$

Using the optimal pricing strategy $p_{k,k'} = \tau_{k,k'} \frac{\sigma}{\sigma-1} \frac{PB_k}{Z_k \varphi}$ with the definition of the price index relative to each country specific bundle, we have the pricing system:

$$\mathcal{P}_k^{1-\rho} = \mu_k \left(\sum_{k'} \alpha_k(k') \left(\tau_{k'k} \left(\frac{\varphi_{k',k}}{\varphi_{k',k'}} \right)^{\frac{\sigma-\gamma-1}{1-\sigma}} \mathcal{P}_{k'} \right)^{1-\rho} \right)^{1-\eta-\chi} \quad , \ k = 1, ..., N$$

with $\mu_k^{\frac{1-\sigma}{1-\rho}} = \frac{\gamma \varphi_{k,k}^{\sigma-\gamma-1}}{\gamma-(\sigma-1)} M_k \left(\frac{\sigma}{\sigma-1} \frac{w_k^{\chi} \times r_k^{\eta}}{\chi^{\chi} \times \eta^{\eta} \times (1-\eta-\chi)^{1-\eta-\chi}} \frac{1}{Z_k} \right)^{1-\sigma}$.
I denote $G_k = \mathcal{P}_k^{1-\rho}$ and the system becomes simply :

$$G_k = \mu_k \left(\sum_{k'} \alpha_k(k') \left(\tau_{k'k} \left(\frac{\varphi_{k',k}}{\varphi_{k,k}} \right)^{\frac{\sigma-\gamma-1}{1-\sigma}} \right)^{1-\rho} G_{k'} \right)^{1-\eta-\chi} \quad , \ k = 1, ..., N$$

Entry Thresholds

In very market, entry occurs until the profit of the least productive firms is equal to the fixed cost of accessing the market. Denoting by X_k total final good spending by consumers ($X_k = P_k(C_k+I_k) = w_kL_k + r_kK_k + \Pi_k$), we get

- At Home

$$\pi_{k,k}(\varphi_{k,k}) = f_{k,k}\frac{PB_k}{Z_k}$$

$$\Leftrightarrow \quad \varphi_{k,k} = \left(\frac{\sigma}{\sigma-1}\frac{PB_k}{Z_k}\frac{1}{\mathcal{P}_k}\right) \times \left(\frac{\sigma f_{k,k}\frac{PB_k}{Z_k}}{X_k + \left(\frac{\mathcal{P}_k}{IP_k}\right)^{1-\rho}\alpha_k(k)(1-\eta-\chi)S_k}\right)^{\frac{1}{\sigma-1}}$$

- Abroad

$$\pi_{k,k'}(\varphi_{k,k'}) = f_{k,k'}\frac{PB_k}{Z_k}$$

$$\Leftrightarrow \quad \varphi_{k,k'} = \left(\tau_{kk'}\frac{\sigma}{\sigma-1}\frac{PB_k}{Z_k}\frac{1}{\mathcal{P}_{k,k'}}\right) \times \left(\frac{\sigma f_{k,k}\frac{PB_k}{Z_k}}{\left(\frac{\mathcal{P}_{k,k'}}{IP_{k'}}\right)^{1-\rho}\alpha_{k'}(k)(1-\eta-\chi)S_{k'}}\right)^{\frac{1}{\sigma-1}}$$

Replacing $\mathcal{P}_{k,k'}$ by its expression using $\mathcal{P}_k$, we also get

$$\varphi_{k,k'}^{1+\frac{(\gamma-(\sigma-1)).(\sigma-\rho)}{(\sigma-1)^2}} = \left(\tau_{kk'}^{\frac{\rho-1}{\sigma-1}}\frac{\sigma}{\sigma-1}\frac{PB_k}{Z_k}IP_{k'}^{\frac{1-\rho}{\sigma-1}}\left(\varphi_{k,k}^{\frac{\sigma-\gamma-1}{\sigma-1}}.\mathcal{P}_k\right)^{\frac{\rho-\sigma}{\sigma-1}}\right) \times \left(\frac{\sigma f_{k,k'}\frac{PB_k}{Z_k}}{\alpha_{k'}(k)(1-\eta-\chi)S_{k'}}\right)^{\frac{1}{\sigma-1}}$$

Spending System

In the Cobb-Douglas case, total revenue of all firms from country k can be written as

$$R_k = X_k + \left[\sum_{k'}\left(\frac{\mathcal{P}_{k,k'}}{IP_{k'}}\right)^{1-\rho}\alpha_{k'}(k)(1-\eta-\chi)S_{k'}\right]$$

Using lemma 1, we have : $\Pi_k = \frac{\sigma-1}{\gamma\sigma}R_k$. Moreover, capital and labor demand impose $r_kK_k+w_kL_k = (\eta+\chi)S_k$. finally, we get:

$$X_k = (\eta+\chi)S_k + \frac{\sigma-1}{\gamma\sigma}R_k$$

Then using lemma 1 and the labor demand equation the whole spending system can be simply written as

$$\left\{ \underbrace{\left(\mathcal{I}_N - \left(W^T \circ P\right)\right)}_{=M}\begin{pmatrix} S_1 \\ \vdots \\ S_N \end{pmatrix} = 0_{\mathbb{R}^N} \right.$$

where W the weighting matrix defined as $W_{ij} = \alpha_i(j)$, P a matrix defined by $P_{ij} = \left(\frac{\mathcal{P}_{i,j}}{IP_i}\right)^{1-\rho}$ and $\circ$ is the element-wise (Hadamard) product. One can easily show that the matrix M is non invertible[49]

[49] One can easily see that summing all rows results in a column of zero.

and is of rank exactly $N-1$, meaning that the solutions of the system is a one dimensional space. This is reassuring because it means we can normalize one price to one. I then normalize $w_1 = 1$ and with the labor demand equation this results is

$$S_1 = \frac{L_1}{\chi}$$

Labor and Capital Market Equilibrium
Using the labor supply equation, L_k is simply

$$L_k^{\nu} = \frac{1}{\psi}\frac{w_k}{\mathcal{P}_k}$$

Equipped with S_k the total spending of all firms in k, wages w_k and rental rate r_k are defined simply by

$$w_k = \chi \frac{S_k}{L_k} \qquad \text{and} \qquad r_k = \eta \frac{S_k}{K_k}$$

B.2 Proof of Lemma 1

Reminder of Lemma 1 : Total profit in country k are proportional to total revenues:

$$\Pi_k = \frac{\sigma - 1}{\gamma\sigma} R_k$$

Proof
First, since firms charge a constant markup $\sigma/(\sigma-1)$ over marginal cost, we know that variable profits are a fraction $1/\sigma$ of total revenues. Hence, total profits net of fixed costs for all firms in k are simply

$$\Pi_k = \frac{R_k}{\sigma} - \sum_{k'} FC_{k \to k'}$$

where $FC_{k\to k'}$ is the sum of fixed cost payment from all firms from country k serving market k'. Then, note that total fixed cost payment for all firms in country k is

$$\begin{aligned} FC_{k\to k'} &= M_k \int_{\varphi_{k,k'}}^{+\infty} f_{kk'} \times \frac{PB_k}{Z_k} \times \gamma \varphi^{-\gamma-1} \times d\varphi \\ &= M_k f_{kk'} \frac{PB_k}{Z_k} \times \varphi_{k,k'}^{-\gamma} \end{aligned}$$

- If $k \neq k'$, we can also express total revenues (sales) from k to k' as

$$\begin{aligned} R_{k,k'} &= M_k \int_{\varphi_{k,k'}}^{+\infty} \left(\tau_{kk'} \frac{\sigma}{\sigma - 1} \frac{PB_k}{Z_k} \frac{1}{\mathcal{P}_{k,k'}} \right)^{1-\sigma} \times \omega_{k'}(k) S_{k'} \varphi^{\sigma-1} g(\varphi) d\varphi \\ &= \frac{\gamma M_k}{\gamma - (\sigma - 1)} \times \left(\tau_{kk'} \frac{\sigma}{\sigma - 1} \frac{PB_k}{Z_k} \frac{1}{\mathcal{P}_{k,k'}} \right)^{1-\sigma} \times \omega_{k'}(k) S_{k'} \varphi_{k,k'}^{\sigma-\gamma-1} \end{aligned}$$

  Next, using the expression for the threshold $\varphi_{k,k'}^{\sigma-1}$ derived above (as a function of $\mathcal{P}_{k,k'}$), we get

$$R_{k,k'} = \frac{\gamma M_k}{\gamma - (\sigma - 1)} \times \sigma f_{k,k'} \frac{PB_k}{Z_k} \varphi_{k,k'}^{-\gamma}$$

  And we recognize finally that

$$R_{k,k'} = \frac{\gamma}{\gamma - (\sigma - 1)} \times \sigma FC_{k \to k'}$$

- For domestic revenues, we can show using the same steps that

$$X_k + R_{k,k} = \frac{\gamma}{\gamma - (\sigma - 1)} \times \sigma FC_{k \to k}$$

Combining those expressions, we get

$$\begin{aligned} \sum_{k'} FC_{k \to k'} &= \frac{\gamma - (\sigma - 1)}{\gamma \sigma} \times \left(X_k + \sum_{k'} R_{k,k'} \right) \\ &= \frac{\gamma - (\sigma - 1)}{\gamma \sigma} \times R_k \end{aligned}$$

Using this expression of $\sum_{k'} FC_{k \to k'}$ in the definition of profits completes the proof.

B.3 Proof of Lemma 2

Reminder of Lemma 2 : In the Cobb-Douglas ($\rho = 1$) and fixed labor supply case, the elasticity of every GNI with respect to a technology shock in country 1 is given by:

$$\begin{pmatrix} \eta_{GNI_1,Z_1} \\ \vdots \\ \eta_{GNI_N,Z_1} \end{pmatrix} = (\mathcal{I}_N - (1 - \eta - \chi) W - T)^{-1} \begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix}$$

with W the weighting matrix defined above and T a "Transmission" matrix function of γ and σ.

Proof:

In this simplified case ($\rho = 1$ and fixed labor and capital supply), the labor and capital demand schedules $w_k L_k = \chi S_k$ and $r_k K_k = \eta S_k$ provide a one to one mapping between total spendings S_k and the wages w_k and the interest rate r_k. Moreover, inspecting the spending system (11) when $\rho = 1$ reveals that once a choice of numeraire is done (that is, taking $w_1 = 1$ and hence fixing $S_1 = L_1/\chi$), the vector of spendings $(S_i)_{i=1,\ldots,N}$ is independent of the technology level. Using lemma 1 and the fact that labor and capital supply are fixed, we can then show that total consumers' spending X_i also independent of technology level. Thus, since $GNI_k = X_k/\mathcal{P}_k$ the GNI elasticity is simply the opposite of the elasticity of the country's consumers price index. Moreover, with fixed labor supply and the assumption that the mass of potential entrepreneurs is proportional to labor size, the mass of firms M_i is fixed for every country i. In the next sections, I compute elasticities of all endogenous variables step by step until I can solve for the price index elasticities.

B.3.1 Price Indexes

Home Price Index at home $\mathcal{P}_k$

Using the definitions of price indexes, we can easily show that

$$\frac{\partial \log(\mathcal{P}_k)}{\partial \log(Z_k)} = -1 + \frac{\partial \log(PB_k)}{\partial \log(Z_k)} + \left(\frac{\gamma - (\sigma - 1)}{\sigma - 1}\right) \frac{\partial \log(\varphi_{k,k})}{\partial \log(Z_k)}$$

We can see in this formula the direct effect of lowering all prices in country k plus two other indirect effects : the propagation going through the input-output linkages in the PB_k term as well as the extensive margin of entry of new firms through the $\varphi_{k,k}$ term.

Foreign Price Index "at their home" $\mathcal{P}_{k'}$

$$\frac{\partial \log(\mathcal{P}_{k'})}{\partial \log(Z_k)} = \frac{\partial \log(PB_{k'})}{\partial \log(Z_k)} + \left(\frac{\gamma - (\sigma - 1)}{\sigma - 1}\right) \frac{\partial \log(\varphi_{k',k'})}{\partial \log(Z_k)}$$

The foreign price index "at their home" is not affected directly but only through the effects going through the input output linkages as well as the entry of new firms.

Export Price indexes $\mathcal{P}_{i,j}$

The price index relative to varieties from i sold on j's market is affected by the shock according to:

$$\frac{\partial \log(\mathcal{P}_{i,j})}{\partial \log(Z_k)} = \frac{\partial \log(\mathcal{P}_i)}{\partial \log(Z_k)} + \left(\frac{\gamma - (\sigma - 1)}{\sigma - 1}\right) \left(\frac{\partial \log(\varphi_{i,j})}{\partial \log(Z_k)} - \frac{\partial \log(\varphi_{i,i})}{\partial \log(Z_k)}\right)$$

We can see that the effect of a technology shock on exporting price indexes depends on the widening in the range of exported goods, as measured by the second term, in the brackets.

Input Bundle Price $PB_{k'}$ Abroad

Using the fact that wages are not affected by technology shocks, I can compute the elasticity of the input bundle price with respect to a technology shock at home as follow:

$$\frac{\partial \log(PB_{k'})}{\partial \log(Z_k)} = (1-\eta-\chi)\sum_j \omega_{k'}(j)\left[\frac{\partial \log(\mathcal{P}_j)}{\partial \log(Z_k)} + \left(\frac{\gamma-(\sigma-1)}{\sigma-1}\right)\left(\frac{\partial \log(\varphi_{j,k'})}{\partial \log(Z_k)} - \frac{\partial \log(\varphi_{j,j})}{\partial \log(Z_k)}\right)\right]$$

B.3.2 Thresholds

Home Entry Threshold $\varphi_{k,k}$ at Home

Using the definition of the thresholds from above and replacing $\frac{\partial \log(PB_k)}{\partial \log(Z_k)} - 1$ by its expression in the expression of the elasticity of the Home price index at home, we get

$$\frac{\partial \log(\varphi_{k,k})}{\partial \log(Z_k)} = \frac{1}{\sigma-1+\kappa\sigma} \times \frac{\partial \log(\mathcal{P}_k)}{\partial \log(Z_k)}$$

The scaling factor $(\frac{1}{\sigma-1+\kappa\sigma})$ is positive while the second term is negative, meaning that a positive technology shock trigers the entry of more firms in the country, which amplifies the effect of the shock.

Export Entry Threshold $\varphi_{k,k'}$ for Home firms exporting to k'

Using the second definition of the export thresholds from above, we get

$$(\frac{\gamma}{\sigma-1})\frac{\partial \log(\varphi_{k,k'})}{\partial \log(Z_k)} = (1+\frac{1}{\sigma-1}) \times \left(\frac{\partial \log(PB_k)}{\partial \log(Z_k)} - 1\right) + \kappa\frac{\partial \log(\varphi_{k,k})}{\partial \log(Z_k)} - \frac{\partial \log(\mathcal{P}_k)}{\partial \log(Z_k)}$$

Moreover, replacing $\frac{\partial \log(PB_k)}{\partial \log(Z_k)} - 1$ by its expression we get and using the fact that $1+\kappa = \frac{\gamma}{\sigma-1}$, we get

$$\frac{\partial \log(\varphi_{k,k'})}{\partial \log(Z_k)} = \frac{1}{\sigma-1+\kappa\sigma} \times \frac{\partial \log(\mathcal{P}_k)}{\partial \log(Z_k)}$$

Home Entry Threshold $\varphi_{k',k'}$ Abroad

Using the definition of the thresholds from above and replacing $\frac{\partial \log(PB_{k'})}{\partial \log(Z_k)}$ by its expression, we get

$$\frac{\partial \log(\varphi_{k',k'})}{\partial \log(Z_k)} = \frac{1}{\sigma-1+\kappa\sigma} \times \frac{\partial \log(\mathcal{P}_{k'})}{\partial \log(Z_k)}$$

Export Entry Threshold $\varphi_{k',j}$ for Foreign firms exporting to j

With the "second" definition of the threshold and using the expression of $\eta_{\varphi_{k',k'},Z_k}$, one can show that the elasticity of the exporting threshold is proportional to the elasticity of the domestic entry threshold and that the scaling factor do not depend on the export market considered:

$$\frac{\partial \log(\varphi_{k',j})}{\partial \log(Z_k)} = \frac{1+\kappa}{\frac{\gamma}{\sigma-1}} \times \frac{\partial \log(\varphi_{k',k'})}{\partial \log(Z_k)}$$

Finally, using the expression for $1+\kappa$, we get

$$\frac{\partial \log(\varphi_{k',j})}{\partial \log(Z_k)} = \frac{\partial \log(\varphi_{k',k'})}{\partial \log(Z_k)} = \frac{1}{\sigma - 1 + \kappa\sigma} \times \frac{\partial \log(\mathcal{P}_{k'})}{\partial \log(Z_k)}$$

B.3.3 Final Expression

Now that I have an expression for the elasticity of all thresholds as functions of the elasticities of price indexes, I can gather the results. Introducing $\Lambda = \frac{1}{\sigma + \frac{(\sigma-1)^2}{\gamma-(\sigma-1)}}$, I define a matrix T (for Transmission) as $T = \text{diag}(\Lambda, ..., \Lambda)$. This matrix characterizes the additional propagation mechanism due to the change in the mass of firms in all markets. Then, the price index elasticities are defined by

$$\begin{pmatrix} \eta_{\mathcal{P}_1,Z_1} \\ \vdots \\ \eta_{\mathcal{P}_N,Z_1} \end{pmatrix} = (\mathcal{I}_N - (1-\eta-\chi)W - T)^{-1} \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

Finally, noting that for all i, $\eta_{\mathcal{P}_i,Z_1} = -\eta_{GNI_i,Z_1}$ concludes the demonstration.
In order to gain intuition on this formula, a few comments are in order. First, note that in the case of complete autarcy of all countries we have $W = \mathcal{I}_N$ to that the elasticity for country 1 is simply $\eta_{GNI_1,Z_1} = 1/(\eta + \chi - \Lambda)$ whereas all other elasticities are zero. This result is reminiscent of what is found in Jones (2011) with the additional propagation mechanism due to the adjustment along the extensive margin captured by Λ. Interestingly, this special case highlights the fact that we need $(1-\eta-\chi)+\Lambda < 1$ in order to get a positive own-country elasticity. This condition is necessary for the validity of (13), since it corresponds to imposing that the reason of the geometric sequence is below one.[50] Secondly, noting that $\Lambda = \frac{1}{\sigma + \frac{(\sigma-1)^2}{\gamma-(\sigma-1)}}$, one can see that for a fixed σ, $\Lambda(\gamma)$ is a strictly increasing function. When $\gamma \to \sigma - 1$, $\Lambda \to 0$ and when $\gamma \to +\infty$, $\Lambda \to 1/\sigma$. For a labor and capital share so that $\eta + \chi = 0.7$ we can see that any value $\sigma > 1.5$ is sufficient to insure the validity of the condition $(1-\eta-\chi)+\Lambda < 1$ for any value of γ within the range of admissible values ($\gamma > \sigma - 1$).

[50] The formula (13) is the matrix analogue of summing an infinite geometric sequence. $(1-\eta-\chi)W+T$ corresponds to the first order effect of the shock, $((1-\eta-\chi)W+T)^2$ is the second order effect, etc... The total effect can then be described by the matrix $(\mathcal{I}_N - (1-\eta-\chi)W - T)^{-1}$ if and only if the eigenvalues of the matrix $(1-\eta-\chi)W+T$ all lie within the unit circle. In the autarcy case, this condition is insured by $(1-\eta-\chi)+\Lambda < 1$.