# Equilibrium Default and the Unemployment Accelerator*

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**Abstract**

We provide evidence of a significant and persistent negative relation between a firm's workers and her probability to default. In contrast with most *"macro-finance"* models, this relation is robust to controlling for the several firm's variables, such as the firm's leverage and profitability. In particular, for a panel of most US publicly traded firms, we find that a 10% increase in a firm's workers is associated with a 3% decline in her probability to default. To account for this fact, we extend a standard search-friction labor-market model to incorporate firms default risk. This environment provides a micro-foundation where workers determine the firm's value, and consequently affecting her incentives to default. We argue that fluctuations in the value of a worker generate and significantly amplify business cycle fluctuations. In the context of our model, we find that fluctuations in the value of a worker explain more than 68% of credit spreads volatility, and almost 80% of default rate volatility.

**Keywords**: Unemployment, default risk, credit market frictions, search frictions, wage bargaining.

**JEL**: E24, E32, E44.

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# 1 Introduction

A large literature on *"macro-finance"* models, argues that a firm's capital is the relevant asset that determines her financial conditions.[1] We argue that this view is correct, but incomplete. We provide evidence of a significant and persistent negative relation between a firm's workers and her probability to default. This result is robust to controlling for variables typically used in the literature to account for default risk, such as the firm's leverage and profitability. In particular, we find that a 10% increase in a firm's workers is associated with a 3% decline in her probability to default.

To account for this fact, we provide a micro-founded model where workers act as an asset that affects a firm's financial conditions. In particular, we extend a standard model of the labor market as in Diamond (1982) and Mortensen and Pissarides (1994) (DMP henceforth) to incorporate firms' default risk. In this environment, firms' attach a value to being matched with a worker because it is costly to engage new ones in production. Consequently, changes in the value of a worker affect a firm's incentives to default. We find that fluctuations in the value of a worker explain more than 68% of credit spreads volatility, and almost 80% of default rate volatility.

The paper has two contributions. First, we empirically quantify the dynamic relation between a firm's worker and her probability to default. This is a new measure that had been neglected in previous work. To this end, we follow the seminal work by Merton (1974) and construct a measure of firms' "distance to default" for most publicly traded firms in the US; and merge it with firm's balance sheet, employment and business performance variables. The resulting panel allows us to isolate the relation that workers have on the firm's probability to default, relative to previous arguments used in the literature such as a the firm's leverage or profitability.

Our second contribution is to build-up a tractable model that can qualitatively account for the negative relation between a firm's workers and her probability to default, in line with the evidence. The main idea of our mechanism can be explained with a simple example. Think of a firm with $n$ workers, labor productivity $x$ and debt $\bar{b}$. If there is a search frictions in the labor market, wages $w$ paid to workers are lower than his productivity $x$.[2] Then, there is a value for the firm to be matched with a worker. Let $S$ denote this value, which satisfies a recursion as follows: $S = x - w + \beta S'$. Assume that a firm can decide to default on her debt and consecutively leave the market. Then, a firm will find it convenient to repay her debt only if

[1] By *"macro-finance"* models we refer to macroeconomic models that place allocate a more prominent role to the financial sector for understanding the dynamics of the business cycle. See Brunnermeier, Eisenbach, and Sannikov (2013) and Quadrini (2011) for recent surveys.

[2] The reason is that firms have to incur a cost in searching for workers, but will only be willing to do it if they make a profit after finding a match. See Pissarides (1985).

$Sn > \bar{b}$, because it is otherwise too expensive to keep the match. Note that, if there is a sudden drop in $S$, incentives to default increases. Thus, there is a feedback from the labor market in to the financial market. In our model, the increased incentives to default generate higher borrowing cost, which further reduces $S$. We think of this loop as an "unemployment accelerator". The larger the changes in the value of a worker, the larger the fluctuations of firms' funding costs that this mechanism will be able to explain. In our model, as in DMP, fluctuations in the value of a worker are associated with fluctuations in the unemployment rate and in the job finding probability. We show that our model replicates several key business cycle statistics of the labor market, and is also able to explain fluctuations of several financial variables like credits spreads, default rates and debt issuance.

Note in this simple example how employment serves as an asset. If the firm had no workers, it could not borrow since lenders would anticipate default on any positive amount of debt. We think of this as a "financial value of a worker" that affects interest rates and lending, in line with the evidence discussed above. The extent in which this mechanism will affect firms' financial conditions crucially depends on the labor market being volatile enough. However, at least since Shimer (2005), we know that this is indeed the case: unemployment and finding probabilities are too volatile as to be explained with standard models of the labor market.

## 1.1 Related Literature

Our paper is related to three lines of research: (i) empirical work on firms' financial conditions; (ii) macroeconomic models with financial frictions; and (iii) models of the labor market with search frictions. We briefly discuss how our paper relates to each topic and comment as well on other works in the intersection of these topics. More references are given later on.

In recent years, perhaps facilitated by eased accessibility to large databases, more empirical work has focused on firm-level financial and business performance data. Some of this work focuses on financial conditions and employment. For instance, the recent works by Giroud and Mueller (2014) and Chodorow-Reich (2014) provide evidence of a link between firm's employment and her balance sheet and credit accessibility conditions. We add to this discussion by arguing that a reverse link also holds: even after controlling for balance sheet and profitability conditions, more employees statistically lower a firm's probability to default. This is a new fact that hasn't been stressed before.

Closer to our work is the one by Belo, Lin, and Bazdresch (2014), who provide evidence of a link between employment growth and stock returns predictability, as well as a model with labor adjustment cost to account for this fact. We see our work as complementary: we focus on the predictive power that employment has on firms probability to default, and accordingly build-up a model with equilibrium default and labor market search-frictions to account for this fact. Although both of our models and empirical exercises are substantially different, we share the idea that workers affects a firm's value: in their set-up because of adjustment cost, and in our model is due to the interaction of equilibrium default and a search-friction.

Our paper also intersects with a large literature on frictional unemployment, as in Mortensen and Pissarides (1994) and Shimer (2005).[3] Closest to our work is Monacelli, Quadrini, and Trigari (2011) who also study the interaction between financial frictions and labor markets.[4] Although we see our work as complementary, there are a few differences between our papers. First, we focus on how employment affects firms' financial conditions, while they explore the reverse relation. Nevertheless, in line with their results, we show that financial conditions have significant effects on employment. A second difference is that our model equilibrium is efficient in the sense firms' debt and default policies maximizes the surplus of a match. A final, and more important difference, is that we provide micro-evidence of the effect that employment has on financial markets, a contribution that can also be useful for future work in this literature.

The rest of the paper is organized as follows. Section 2 presents the empirical results. Section 3 describes the model. Section 4 discusses the model equilibrium characterization, and Section 5 the quantitative results. Section 6 concludes.

## 2 Evidence

In this section, we provide evidence that firms' probability of default is statistically related to the workers she has. To this end, for all publicly traded firms in the US, we construct a panel with firms' financial and business performance information, as well as with a measure of her probability to default. We find that, even after controlling for standard determinants of default risk, firms with more workers are less likely to default. The relation is economically and statistically significant: on average, a 10% increase in a firm's workers is associated with a 3% decline in her probability to default.

We proceed in two steps. First, in Section 2.1 we follow the seminal work of Merton (1974) and construct a measure of firms' likelihood to default, the so called *"distance to default"*. This measure has recently been used in applied work, and has been shown to be a good predictor of firms' default riskiness.[5] In Section 2.2 we merge this measure of default risk with firms' with balance sheet, employment and investment data. We

[3] See Andolfatto (1996), Merz (1995) and Gertler and Trigari (2009) among others

[4] For further work in this intersection see: Buera, Fattal-Jaef, and Shin (2013), Garin (2011), Petrosky-Nadeau (2011), and the references therein.

[5] See Gilchrist and Zakrajsek (2012), Schaefer and Strebulaev (2008) and Duffie (2011).

use the resulting panel to evaluate the effects that employment has on firms' default risk.

## 2.1 A Measure of Default Risk

Following the seminar work by Merton (1974), and based on the Black-Scholes-Merton option-pricing model, this section we describe the construction of *"distance to default"* measure. The algorithm consists of two steps: first, inferring the firm's assets value given her market value; and second, use an option-price formula to compute the firm's probability to default. Because our computations closely follows the ones in Duffie (2011), we keep exposition brief.[6]

Let $S_{id}$ be the total value of the firm $i$ on day $d$, and assume that $S_{id}$ follows a geometric Brownian motion with instantaneous drift $\mu_{id}$ and volatility $\sigma_{id}$. It is also assumed that firms default if her value $S_{id}$ is below a liabilities based measure $L_{id}$.[7] The insight of Merton (1974) is that the firm's equity value $E_{id}$ can be viewed as an option on the underlying value of the firm $S_{id}$, with a strike price $L_{id}$, and maturity date $t+T$. Then, the value of equity $E_{it}$ is give by the Black-Scholes-Merton option-pricing formula as follows

$$E_{id} = S_{id}\Phi(d^1_{id}) - e^{-r_t T} L_{it}\Phi(d^2_{id}) \tag{1}$$

where $r_t$ denotes the instantaneous risk-free rate, and $d^1_{id}$ and $d^2_{id}$ are given as

$$d^1_{id} = \frac{\log(S_{id}/L_{id}) + (r_t + 0.5\sigma^2_{id})T}{\sigma^2_{id}T} \qquad d^2_{id} = d^1_{id} - \sigma_{id}\sqrt{T} \tag{2}$$

Operationally, we proceed as follows. The equity value $E_{id}$ is constructed using daily CRSP data on stock prices. The default threshold $L_{id}$ is computed as the firm's short-term debt plus one-half of her long-term debt. The source of liabilities data is Compustat at quarterly frequency, and the data is linearly interpolated to obtain daily observations. The risk-free interest rate $r_t$ is assumed to be the 10 years US treasury yields.[8] Finally, to obtain $\{\mu_{id}, \sigma_{id}\}$ we follow an iterative procedure: given a guess for $S_{id}$, we compute $\{\mu_{id}, \sigma_{id}\}$ from a 250-days rolling window, and then use equation (1) to obtain a new guess of $S_{id}$. We iterate until convergence.

[6]See Bharath and Shumway (2008) as well for a detailed explanation.
[7]Black-Scholes-Merton original formulation actually assumes that $L_{id}t, \mu_{id}$ and $\sigma_{id}$ are constant.
[8]See Appendix A for details.

Finally, we compute two variables: the distance to default $DF_{id}$, and the probability to default $\Phi_{id}$

$$
\begin{aligned}
DF_{id} &= \frac{\log(S_{id}/L_{id}) + \mu_{id}T - 0.5\sigma_{id}^2 T}{\sigma_{id}\sqrt{T}} \\
\Phi_{id} &= \Phi\left(-DF_{id}\right)
\end{aligned}
$$

Quarterly observation for each is taken as averages: $DF_{it} = \frac{1}{D_t}\sum_d DF_{id}$ and $\Phi_{it} = \frac{1}{D_t}\sum_d \Phi_{id}$ for all quarters $t$.

## 2.2 Employment and Default Risk

Next, we evaluate the effect that firms' workers have on the above computed probability of default. We argue that commonly used variables fall short to account for the variability in firms' default risk, and that employment contributes to explain the residual. In particular, we find three patterns in data. First, firms with employment higher than expected, have a persistently lower probability of default. Second, sectors with more volatile employment, also have more volatile default risk. Third, even after controlling for several aggregate and firm specific characteristics, a 10% increase in a firm's workers reduces her probability to default by 3%.

To perform these exercises, we construct a panel by complementing the *"distance to default"* measure, with Compustat data on firms' balance sheet, employment, profits and investment. The data has quarterly frequency, and covers 14,957 firms for the period 1965-2014. Details on data definitions and computations can be found in Appendix A.

To understand the importance of employment on default risk, we first compute the effect that variables *other than employment* have on firms' probability to default. We then show that the unexplained component is very responsive to the firm's number of worker. In particular, we estimate the following regression

$$
\ln \Phi_{it} = \alpha_i^{\Phi} + \gamma_t^{\Phi} + \beta^{\Phi} X_{it} + \varepsilon_{it}^{\Phi} \tag{3}
$$

where $\Phi_{it}$ is firm's $i$ probability to default in quarter $t$, as computed in Section 2.1, and $X_{it}$ is a set firm specific controls. These controls include firm's total assets, total liabilities, the profits-to-assets ratio and the investment-to-assets ratio.[9] While total assets and liabilities are meant to capture firms' financial conditions, the inclusion of profits and investment is to reflect firms' performance (a proxy for productivity). Importantly,

[9] Total assets corresponds to ATQ in Compustat, and total liabilities to LTQ. We compute investment as CAPXY and profits as OIBDPQ. Assets and liabilities enter in logs. Profits-to-Assets and Investment-to-Assets are included normalized by the mean -by firm- of these variables. See Appendix A for details on sources and definitions.

we do not include firms' market value data, since it was used when computing the probability to default $\Phi_{it}$. We also allow for firms' fixed effects ($\alpha_i^\Phi$) to represent idiosyncratic differences, as well as time dummies ($\gamma_t^\Phi$) to capture aggregate macroeconomic conditions.

Equation (3) has the typical variables that affect firms default in most *"macro-finance models"*. Thus, we think of the residuals $\varepsilon_{it}^\Phi$ as the component of the firm's default risk that cannot be accounted for standard arguments in the literature. We are interested in understanding how these residuals depend on firms' employment. To this end, we estimate an analogous equation for employment. In particular, we estimate

$$\ln E_{it} \quad = \quad \alpha_i^E + \gamma_t^E + \beta^E X_{it} + \varepsilon_{it}^E \tag{4}$$

As before, equation (4) regresses employment on typically used measures of the firm $X_{it}$. Thus, we think of a positive $\varepsilon_{it}^E$ as a firm with employment higher than expected in that quarter.

If employment is not a significant determinant of default risk, other than because of it's correlation to the firms' variables $X_{it}$, the residuals $\varepsilon_{it}^\Phi$ and $\varepsilon_{it}^E$ should be uncorrelated.[10] We show that this is not the case: both residuals are positively correlated in their levels and variance.

### 2.2.1 Employment Level and Default Risk

A firm with employment higher than expected, as captured by a positive $\varepsilon_{it}^E$, induces a persistent decline in default probabilities. Figure 1 shows this by plotting, for different horizons $h$, the estimate $\delta_h$ of the following regression

$$\varepsilon_{i,t+h}^\Phi = \delta_0 + \delta_h \varepsilon_{it}^E + u_{i,t+h}$$

An increase in the firms' workers declines default risk for almost two years. This clearly indicates how workers act as an asset to the firm, even after we control for several firm specific variables.

### 2.2.2 Employment Volatility and Default Risk

We argue next that sectors with more volatile employment also exhibit more volatile default risk. As before, we compute volatility on the residuals $\varepsilon_{it}^\Phi$ and $\varepsilon_{it}^E$. If this residuals were to be unrelated, there should be no reason to expect a positive (nor negative) relation between these variances.

[10] For this statement, we are implicitly assuming that $X_{it}$ contains all relevant information to determined default, other than employment.

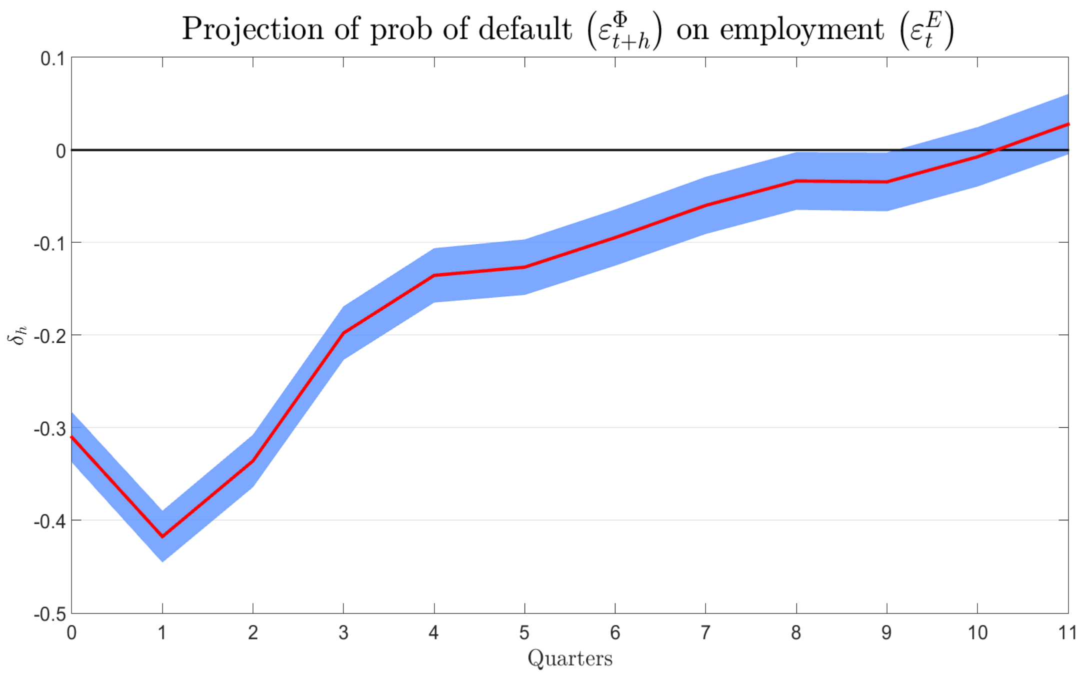


Figure 1: OLS estimate of $\varepsilon^{\Phi}_{i,t+h} = \delta_0 + \delta_h \varepsilon^{E}_{it} + u_{i,t+h}$

***Notes:*** Shaded areas represent 95% confidence intervals.

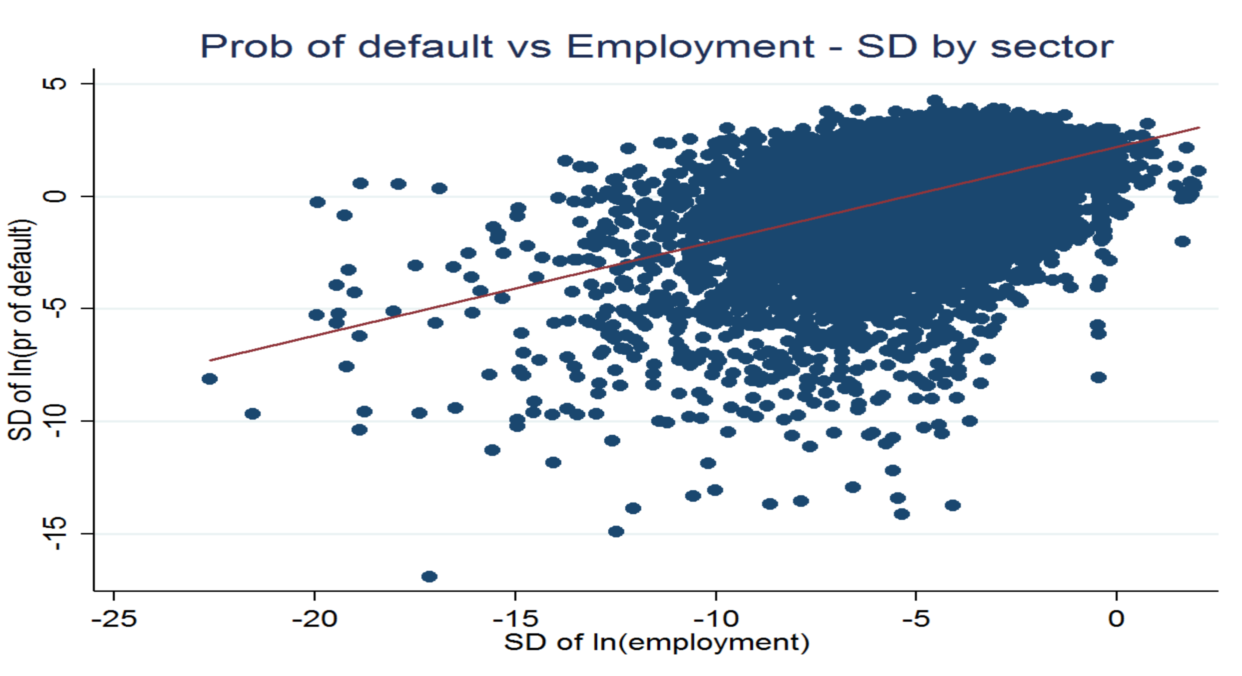


Figure 2: Variance o employment and default risk per 3 digit SIC sector

***Notes:*** The red line is the linear fit, with a slope of 0.42 with a 95% confidence interval of $[0.41, 0.43]$.

We divide firms into groups $s \in \mathbb{S}$ by 3 digits SIC code, and then compute variances per sector as follows

$$\mathbb{V}_s^{\Phi} = \frac{1}{TN_s} \sum_{t,i \in s} \omega_{ist} \left(\varepsilon_{it}^{\Phi} - \bar{\varepsilon}_s^{\Phi}\right)^2$$
$$\mathbb{V}_s^{E} = \frac{1}{TN_s} \sum_{t,i \in s} \omega_{ist} \left(\varepsilon_{it}^{E} - \bar{\varepsilon}_s^{E}\right)^2$$

where $\omega_{ist}$ is the assets-value weight of firm $i$, with assets $A_{it}$, in group $s$ at period $t$: $\omega_{ist} = A_{it} / \sum_{j \in s} A_{jt}$; and $\bar{\varepsilon}_s^{\Phi} = \sum_{i \in s} \omega_{ist} \varepsilon_{it}^{\Phi}$ and $\bar{\varepsilon}_s^{E} = \sum_{i \in s} \omega_{ist} \varepsilon_{it}^{E}$ are the residuals weighted means.

Figure 2 shows a statistically positive relation between sectoral employment and default risk volatility. Again, notice that this variance is beyond what is accounted by firms' controls $X_{it}$. In other words, if two sectors have equally volatile profits, assets and liabilities, but the later has a more labor market, we can expect more volatile default risk. This is the relation that our model captures in Section 3.

Table 1: Firms' Employment and Default Risk

| | Coefficient | 95% Confidence Interval | |
|---|---|---|---|
| *ln* Employment | −0.31 | −0.34 | −0.28 |
| *ln* Assets | −2.13 | −2.16 | −2.10 |
| *ln* Liabilities | 2.19 | 2.17 | 2.21 |
| 100*Profits-to-Assets | −0.046 | −0.05 | −0.045 |
| 100*Investment-to-Assets | −0.023 | −0.05 | 0.005 |
| overall $R^2$ | 23% | | |

#### 2.2.3 Effect of Employment on Default Risk

Finally, we provide an estimate of the marginal effect of employment on default risk, once all other idiosyncratic components of the firm have been taken into account. To achieve this, we re-estimate equation (3), but we include firms' employment in the right hand side. In particular, we estimate the following equation

$$\ln \Phi_{it} \quad = \quad \alpha_i^{\Phi} + \gamma_t^{\Phi} + \beta^{\Phi} X_{it} + \eta \ln E_{it} + \zeta_{it}^{\Phi} \tag{5}$$

We interpret the coefficient $\eta$ in equation (5) as the marginal effect that employment has on probability of default.[11]

As Table 1 shows, a 10% increase in the firm's workers induces a 3% decline in her probability to default.

All of the exercises performed in this section suggests that workers act as an asset to the firm, in the same manner that capital does. Importantly, we find this relation to be even after we control for several firm's variable as well as aggregate conditions. This is the new fact in the paper, and the model we present below is capable of replicating this relation.

## 3 Model

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. The economy is populated by a continuum of firms, a continuum of workers and a representative household. Firms have access to a production technology given

[11] The log-log nature of equation (5) allows to interpret $\eta$ as an elasticity.

as

$$y = axn \tag{6}$$

where $n$ stand for labor. Her productivity is given by an idiosyncratic component $a$ and an aggregate one $x$. In order to hire new workers, firms must post vacancies in a labor market with search frictions in the spirit of Diamond-Mortensen-Pissarides (DMP) with two ingredients: exogenous separation and multi-workers firms.[12] Finally, with probability $s$ every period, a match between a firm and worker is exogenously terminated.

Household preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{7}$$

where $U(\cdot)$ satisfies standard assumptions and $\beta$ is his discount factor. The household is composed of a continuum of workers who individually search for labor opportunities and bargain over wages with firms. Although at any point in time they may be either employed or unemployed, we follow Andolfatto (1996) and Merz (1995) and assume that at the end of the period workers pool all their resources together and have perfect consumption insurance among them.

As typical in the literature with search frictions, we assume the existence of a matching function $m(V, U)$, where $V = \int \bar{v}_i di$ is the total of vacancies posted by firms and $U$ is the unemployment level. The probability of a firm filling a vacancy is given by $q = \frac{m(V,U)}{V}$, and the probability of a worker finding a job is $f = \frac{m(V,U)}{U}$. We further assume that $m(V, U)$ is homogeneous of degree one and satisfies standard assumptions.

In order to fund its activities, firms can issue debt or equity.[13] However, since firms may decide to default, debt is risky. The only financial assets traded in this economy are the bonds issued by each one of these firms. We further assume that debt has long maturity: every period, a fraction $\lambda$ of the firm's outstanding liabilities must be repaid in order to avoid default.[14] Upon default the firm disappear and losses all of her workers. Thus, there is an endogenous (default induced) separation every period. Finally, we assume that the household owns all of these firms.

To gain tractability, we make the following assumptions

**Assumption 1** *The idiosyncratic productivity component $a$ is i.i.d. across firms and time with distribution $H$. The aggregate productivity component $x$ follows Markov processes $x \sim P_x(x', x)$, that satisfies standard*

[12]See Mortensen and Pissarides (2000), chapter 6.
[13]In our setup, we understand equity injections as negative dividend payments.
[14]Similar assumptions are used in Arellano and Ramanarayanan (2012) and Gomes, Jermann, and Schmid (2013). As in those papers, we think of $1/\lambda$ as the average maturity of the debt.

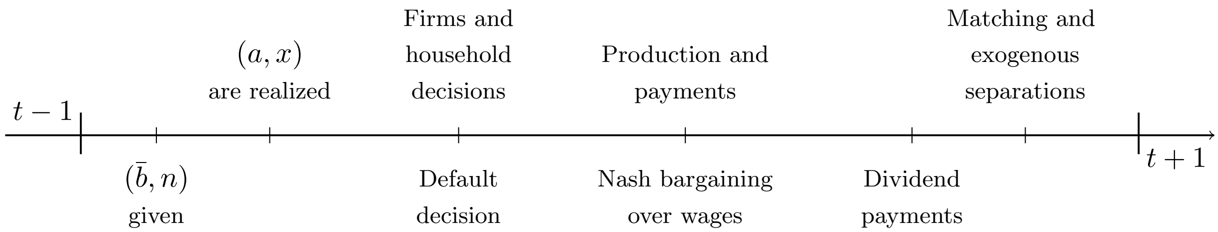


Figure 3: Timing in the model in Period $t$

*properties.*

Assuming that the idiosyncratic productivity component is *i.i.d.* allows us to reduce the dimensionality of the state space. This type of assumption has extensively been used in the literature of macroeconomic models with financial frictions.[15] Regarding default punishments, we follow previous work and assume that defaulting firms disappear.[16] As we will see later, the firm's value affects default decisions breaking the link between leverage and the credit spread endogenously: the state of the economy will also matter.

In order to keep the model simple, we assume that there is no entry of new firms. However, since firms have access to a constant returns to scale technology, this assumption is innocuous and does not affect any interesting content of the model.

**Timing** within a period is as follows. At the beginning of period $t$, every firm starts with a level of debt $\bar{b}$, workers $n$ and a realization of the exogenous shocks $x$ and $a$. After this, each firm decides weather to default or not. In case of no default, she can issue new debt, post vacancies to find new workers, and produce output with the workers she is currently matched with. Simultaneously, households make consumption and saving decisions, wages are paid and workers not engaged in production receive unemployment benefits. The firm can pay dividends after bargaining with the workers. Finally, labor market opens: exogenous separations and new matches are realized. Period $t+1$ begins. Figure 3 shows the timing just described.

Two comments are worth making about our timing. First, the model has two type of separations: an exogenous one at the end of the period, and an endogenous one due to default at the beginning of the period. Second, we restrict firm's dividend payments to be made at the end of the period. This implies that, when bargaining with the worker, the firm shares all of the costs and benefits of issuing debt. We show in Section 4 that this implies firm's policies that maximizes the value of a match. For the effects of an alternative timing assumption, see Monacelli, Quadrini, and Trigari (2011).

[15]See for instance Gertler and Kiyotaki (2009), Kiyotaki and Moore (2012) and Bigio (2012) where they assume this for investment opportunities.

[16]See for instance Arellano, Bai, and Kehoe (2011), Arellano (2008) and Gomes and Schmid (2010).

We will next explain the firm's, worker's and household's problems, define a recursive equilibrium and characterize it. For the moment we will let $z$ denote the aggregate state of the economy and later define it explicitly.[17]

**Notation 1** *Let $z$ denote the aggregate state of the economy and $\Gamma(z)$ its law of motion: $z' = \Gamma(z)$.*

We will refer to employment (and unemployment) as the number of workers engaged in production, which is determined after the default decision. Thus, if $\bar{N}$ is the number of workers with a match at the beginning of the period and $H$ is the default rate, the employment rate is given by $N = (1-H)\bar{N}$ and unemployment is $U = 1-(1-H)\bar{N}$.

## 3.1 Firms

After observing the realization of the aggregate productivity $x$ and her idiosyncratic productivity $a$, the firm must decide weather to default or not. If she decides not to default, she can choose dividend payments $d$, vacancies posting $\bar{v}$ and next period total debt $\bar{b}'$. Let $E(a,\bar{b},n,z)$ be the value of a firm before the default decision, with idiosyncratic productivity $a$, debt $\bar{b}$ and $n$ workers when the aggregate state of the economy is $z$. Then

$$\begin{aligned} E(a,\bar{b},n,z) &= \max_{d,nd}\left\{0, \max_{d,\bar{v},\bar{b}'}\left\{d + \beta \mathbb{E}_{z',a'}\left[\Lambda(z,z')E(a',\bar{b}',n',z')|z\right]\right\}\right\} \qquad (8)\\ \text{subject to} & \\ y &= axn - w(a,b,z,\mu)n - (1-\tau)\lambda\bar{b} \\ d + \kappa\bar{v} &\leq y + p(b',z)\left[\bar{b}' - (1-\lambda)\bar{b}\right] \\ n' &= (1-s)n + \bar{v}q(z) \\ z' &= \Gamma(z) \end{aligned}$$

where $\mu = \{v, b'\}$, with $b = \bar{b}/n$ and $v = \bar{v}/n$.

The first line in the feasible set is the firm's current income: it accounts for production, minus wage and debt payments. Notice that only a fraction $\lambda$ of current debt $\bar{b}$ is paid. Also, we assume that there is a tax benefit $\tau$ to debt, so that actual payments are $(1-\tau)\lambda\bar{b}$. This assumption induce firms to issue debt in equilibrium.[18] The second line in the feasible set is the firm's budget constraint: her expenditures are

[17] "Finding the state is an art".

[18] Defaults by itself breaks Modigliani-Miller irrelevance result. In particular, the firm would strictly prefer to use equity only. Thus, the corporate tax is necessary for having debt issuance in equilibrium. See Gourio (2013) for similar assumptions.

dividends $d$ and the cost vacancy posting $\kappa$ times the number of vacancies $\bar{v}$. Her available income is given by $y$ plus new debt issuance $\bar{b}' - (1-\lambda)\bar{b}$ times the price of new debt $p(b', z)$. The third line is the law of motion for workers: there is an exogenous separation rate $s$, and a new amount of matches $\bar{v}q(z)$ are met, where $q(z)$ is the probability of filling a vacancy.

The firm's problem is then to maximize the present discount value of her dividend payments. In doing so, it takes into account the option to default at the beginning of the period. Furthermore, it internalizes the impact that its portfolio decision will have on prices. Also, as is typical in the literature (for instance Jermann and Quadrini, 2012), since households own the firms, their stochastic discount factor $\Lambda(z, z')$ is used in order to discount flows. We will show later that this problem has a tractable solution.

A departure of this model from a neoclassical environment is the dependence of both debt prices and wages on the individual portfolio decision. The reason why debt prices depend on debt per worker $b$ is simply because default probabilities depend on this, and prices reflect the likelihood of this scenario. This same logic is found in default models like Arellano (2008), Eaton and Gersovitz (1981) or Gomes and Schmid (2010).[19] Note as well that the firm may desire to issue new debt ($\bar{b}' > (1-\lambda)\bar{b}$) or decrease her liabilities ($\bar{b}' < (1-\lambda)\bar{b}$), in which case we assume that the firm is repurchasing old debt. Finally, since wages are the result of a bargaining process, the portfolio composition will affect wages because it affects the joint surplus of a match between a firm and a worker. We will discuss the details of this in Section 4.

Notice that the asset structure of this economy is extremely large. In particular, the firm is choosing to finance its activities from a continuum of bonds indexed by $b'$ with corresponding price schedule $p(b', z)$. Since her optimal decision will depend on this price, we need to compute an (equilibrium) value for each one of these bonds. This will be obtained from the household problem below.

The next lemma characterizes the firm's optimal policy and shows that the marginal value of a worker to a firm is independent of the number of workers. This will become useful in solving the Nash bargaining problem.

**Lemma 1** ***[Firms Value Function]*** *The value function of a firm is linear in workers $n$*

$$E(a, \bar{b}, n, z) = e(a, b, z)n$$

[19]One difference in our set up is the persistence of the idiosyncratic shock. Given our *i.i.d.* distribution assumption, the likelihood of default next period does not depend on the current *idiosyncratic* productivity. This is why the price of debt does not depend on the firm's productivity, although it does on the aggregate state of the economy, which includes the aggregate productivity $x$.

*where* $b = \bar{b}/n$. *The function* $e(a,b,z)$ *is given as follows*

$$\begin{aligned} e(a,b,z) &= \max_{d,nd}\left\{0, \max_{v,b'}\left\{\tilde{e}(a,b,z,v,b')_{w(a,b,z,\mu)}\right\}\right\} \qquad (9)\\ \textit{where} & \\ \tilde{e}(a,b,z,v,b')_w &= -\kappa v - (1-\tau)\lambda b + ax - w + p(b',z)\left[b'\left[(1-s)+vq(z)\right] - (1-\lambda)b\right] \\ &+ \left[(1-s)+vq(z)\right]\beta\mathbb{E}_{a'z,'}\left[\Lambda(z,z')e(a',b',z')|z\right] \end{aligned}$$

*Policies for vacancy posting and debt are also linear in labor*

$$\begin{aligned} \bar{v}(a,\bar{b},n,z) &= \boldsymbol{v}(b,z)n \\ \bar{b}'(a,\bar{b},n,z) &= \boldsymbol{b}'(b,z)\left[(1-s)+\boldsymbol{v}(b,z)q(z)\right]n \end{aligned}$$

*where* $\boldsymbol{v}(b,z)$ *and* $\boldsymbol{b}'(b,z)$ *are a function of firm's debt per worker* $b$ *and the aggregate state* $z$. *Finally, under certain conditions, a firm defaults if and only if her idiosyncratic productivity is below a threshold* $\underline{\boldsymbol{a}}(b,z)$.[20]

(All proofs are in Appendix B.) A few things are important about Lemma 1. First, the value function $e(\cdot)$ in equation (9) is the value to a firm of having a worker at the beginning of the period. The function $\tilde{e}(\cdot)$ is the value of having a worker conditional on not defaulting. Second, if all firms start with the same debt per worker $b$, all firms will follow the same policies $\boldsymbol{v}(b,z)$ and $\boldsymbol{b}'(b,z)$ in any state $z$, which means that all firms will start with the same debt per worker next period. Thus, if the initial distribution of debt per worker is degenerate, so it will be at any future point time. To keep the model tractable, we assume that this is the case. Third, the fact that, per worker, every firm follows the same policy allows for easy aggregation and help us to keep the dimensionality of the state space small. Finally, the default decision follows a threshold as a function of the firms financial position per worker. We will characterize this threshold in Section 4 and show that is tightly related to the value of a worker.

Although identical along the equilibrium path, it will be useful to distinguish between debt per worker at a particular firm and at the aggregate level.[21]

**Notation 2** *Let* $b$ *denote a firm debt per worker and* $B$ *the average debt per worker over firms.*

[20] The condition reads as follows: $x - \frac{\partial w(a,b,z,\mu)}{\partial a} > 0 \quad \forall(a,b,z,\mu)$. We show in the Appendix that this condition is satisfied.

[21] Our model falls in the "big $K$, little $k$" type of model. See, for instance, Ljungqvist and Sargent (2004), chapter 7.

### 3.2 Workers

At the beginning of the period, a worker can be either employed or unemployed. Let $\mathcal{J}(a,b,z)$ be the value to a worker of being matched with a firm with productivity $a$ and debt per worker $b$ when the aggregate state of the economy is $z$, and let $\mathcal{U}(z)$ the value of being unemployed. Let $W(a,b,z) = w(a,b,z,\mu(b,z))$ be the equilibrium wage paid by a firm with individual state $(b,z)$. Then

$$\begin{aligned}
\mathcal{J}(a,b,z) &= \begin{cases} W(a,b,z) + \beta \mathbb{E}_{a',z'}\left[\Lambda(z,z')\left\{(1-s)\mathcal{J}(a',\boldsymbol{b}'(b,z),z') + s\mathcal{U}(z')\right\}|z\right] & \text{if } a > \underline{\boldsymbol{a}}(b,z) \\ \mathcal{U}(z) & \text{otherwise} \end{cases} \\
\mathcal{U}(z) &= \bar{u} + \beta \mathbb{E}_{a',z'}\left[\Lambda(z,z')\left\{f(z)\mathcal{J}(a',B(z'),z') + (1-f(z))\mathcal{U}(z')\right\}|z\right]
\end{aligned}$$

where $B(z')$ is the average debt per worker firms will have next period. Notice that, in the expression for $\mathcal{U}(z)$, we already used that the distribution of debt per worker is degenerate.

The value of being employed is the sum of the wage paid today plus the expected continuation value of being matched. This continuation value takes into account the probability of separation $s$, the debt per worker $\boldsymbol{b}'(b,z)$ that the firm will start next period with, as well as the firm's probability of default. Similarly, the value of being unemployed includes unemployment benefits $\bar{u}$ plus the continuation value: with probability $f(z)$, the worker will find a match and start producing next period with a firm with debt per worker $B(z')$, and with probability $1-f(z)$ he will remain unemployed. Notice that the value of being unemployed depends on the state of the economy $z$ only.

Let $g(a,b,z) = \mathcal{J}(a,b,z) - \mathcal{U}(z)$ be the surplus to a worker of being matched with a firm with productivity $a$ and debt per worker $b$ when the aggregate state of the economy is $z$. Then

$$g(a,b,z) = \begin{cases} \tilde{g}(\boldsymbol{b}'(b,z),z)_{W(a,b,z)} & \text{if } a > \underline{\boldsymbol{a}}(b,z) \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where

$$\tilde{g}(b',z)_w = w - \bar{u} + \beta \mathbb{E}_{a',z'}\left[\Lambda(z,z')\left\{(1-s)g(a',b',z') - f(z)g(a',B(z'),z')\right\}|z\right]$$

The function $\tilde{g}(\cdot)_w$ is the surplus for a worker of being matched with a firm that doesn't default, issues new debt $b'$ and pays wages $w$ this period.

### 3.3 Nash Bargaining

Wages are the solution of a Nash bargaining procedure. At the moment of bargaining, the value for a firm of having a worker is $\tilde{e}(\cdot)$, and the value to a worker of being in a match is $\tilde{g}(\cdot)$. Then, wages are given by

$$w(a,b,z,v,b') \quad = \quad \arg\max_{\vec{w}} \left\{ \tilde{e}(a,b,z,v,b')_w^{\gamma} \tilde{g}(a,b,z,b')_w^{1-\gamma} \right\} \tag{11}$$

Notice that the wage solution in equation (11) is computed for a given policy of the firm $\{v, b'\}$, since firm's policies affect the total value of the match. Importantly, all of the firm's inflows and outflows from debt and vacancy posting are shared with the worker. Notice that, since dividend payments are done at the end of the period, the firm cannot affect the flows at the bargaining step with her dividend policy.[22] As we show in Section 4, this generates that the firm's policies maximizes the value of a match.

The outcome of the bargaining must determine wages as a function of two set of variables: idiosyncratic state variables $(b, a)$ as well as the aggregate state $z$. In this sense, we are allowing for wages to be as flexible as possible. Alternatively, we could impose some form of "stickiness" in the wage by not allowing it to be a function of certain set of variables. The benefit of the former assumption is that we can focus in our mechanism. The cost is that, as remarked before (for instance Shimer, 2005), wages may end up being more volatile than its empirical counterpart.[23]

### 3.4 Household

The household can choose consumption and bond holdings. In particular, he has to decide how much to save in each type of bond indexed by $b$. His income has three components: first, wage earnings and unemployment benefits made by workers; second, dividends paid by firms; and third, his financial wealth coming from last period portfolio returns. Let $V(\omega, z)$ denote the value of a family with financial wealth $\omega$ when the aggregate

[22] In other words, all firms resources are "on the table" when bargaining with a worker.

[23] Notice that, in the extreme case of fixed wages, profits volatility would be much higher, amplifying the effect of the labor market into the financial market.

state of the economy is $z$. Then

$$
\begin{aligned}
V(\omega, z) &= \max_{\{C, b'_h(b'), \omega'\}} \{U(C) + \beta \mathbb{E}_{z'}[V(\omega', z')|z]\} \qquad (12)\\
&\text{subject to} \\
C &+ \int p(b', z) b'_h(b') db' + \mathbb{T}(z) \leq \omega + d(z) + \bar{u}U(z) + \mathbb{E}_a \left[\mathbb{I}\{a \geq \underline{\boldsymbol{a}}(B(z), z)\} W(a, B(z), z)\right] \bar{N} \\
U(z) &= (1 - \bar{N}) + H(\underline{\boldsymbol{a}}(B(z), z)) \bar{N} \\
\omega' &= \int [1 - H(\underline{a}(b', z'))] \left[\lambda + (1 - \lambda) p(\boldsymbol{b'}(b', z'), z')\right] db' \\
z' &= \Gamma(z)
\end{aligned}
$$

The first line in the feasible set is the household's budget constraint. His available income is his financial wealth $\omega$, plus the dividends $d(z)$ paid by the firm that period, and the labor earnings made by the workers. His expenditures are given by consumption, plus bond purchases as well as tax payments $\mathbb{T}(z)$. The second line states that unemployment level is the fraction of workers that didn't start the period with a match, plus workers that lost their match due to default. The third line describe financial wealth evolution. Next period, he will receive returns on his portfolios decision which are given by the non-defaulted bonds: a fraction $\lambda$ matures, and the remaining $1 - \lambda$ has a market value $p(\boldsymbol{b'}(b, z), z)$.

The state variable $z$ contains the initial level of employed workers $\bar{N}$, and the law of motion of employment is captured by $\Gamma(z)$.

### 3.5 Government, Aggregate Feasibility and Dynamics

We assume that the government runs a balanced budget every period: it raises taxes from households $\mathbb{T}(z)$ in order to pay unemployment benefits $\bar{u}U(z)$ and debt subsidies $\tau$ to non-defaulting firms. Government's budget constraint reads

$$\mathbb{T}(z) = \bar{u}U(z) + \tau \int \lambda \bar{b}_i \mathbb{I}\{a_i \geq \underline{\boldsymbol{a}}(B(z), z)\} di$$

Finally, let $Y(z)$ and $I(z)$ be the total output and investment in the economy when the aggregate state of the economy is $z$. Then

$$
\begin{aligned}
Y(z) &= \int a_i x n_i \mathbb{I}\{a_i \geq \underline{\boldsymbol{a}}(B(z), z)\} di \\
I(z) &= \int \kappa \bar{v}_i \mathbb{I}\{a_i \geq \underline{\boldsymbol{a}}(B(z), z)\} di
\end{aligned}
$$

Aggregate feasibility for the economy the is

$$Y(z) = C(z) + I(z)$$

As most of the literature in labor market search, we follow Mortensen and Pissarides (1994) in computing finding probabilities. In particular, let $\theta$ be the market tightness which is given by the ratio of vacancies to unemployment

$$\theta = \frac{\int \bar{v}_i di}{U}$$

We will assume $q(z) = q^m(\theta)$ and $f(z) = f^m(\theta)$ with $\partial q^m/\partial\theta < 0$ and $\partial f^m/\partial\theta > 0$.

Finally, the state of the economy is given by: $z = (x, B, \bar{N})$, where $B$ is the average debt per worker across firms and $\bar{N}$ is the number of workers that started the period matched with a firm.

## 3.6 Equilibrium Definition

**Definition 1** *A recursive competitive equilibrium for this economy is given by value functions* $\{E(a,b,n,z), \mathcal{J}(a,b,z), \mathcal{U}(z), V($ *policies functions for the firm* $\{d(a,\bar{b},n,z), \bar{v}(a,\bar{b},n,z), \bar{b}'(a,\bar{b},n,z)\}$*, policies for the household* $\left\{C(\omega,z), \{b_h(\omega,z,b')\}_{\{b'\}}\right\}$*, finding probabilities* $\{q(z), f(z)\}$*, prices* $\{p(b',z), w(a,b,z,\mu)\}$*; such that, given prices, finding probabilities and an aggregate law of motion* $\Gamma(z)$*: (i) Firm's policies solve its problem and achieve value* $E(a,b,n,z)$*, (ii) Household's policies solve his problem and achieve value* $V(\omega,z)$*, (iii) Wages* $w(a,b,z,\mu)$ *are given as the Nash bargaining solution, (iv) Finding probabilities are consistent with individual policies, (v) Bonds market clears:* $\int \bar{b}'(a,b,n,z) = b_h(\omega,z,b')\ \forall b'$*, (vi) Goods market clears:* $Y(z) = C(z) + I(z)$*, (vii) Law of motion for the state: the mapping* $z' = \Gamma(z)$ *is consistent with individual policies and markets clearing.*

# 4 Characterization

In this section we characterize many equilibrium outcomes of the model. Two results are of particular importance. First, firm's optimal decisions can be obtained from a "fictional" planner problem for the match, which allow us to characterize firm's optimal policies without computing wages. As a corollary, this "fictional" planner problem implies that firm's policies are efficient in the sense that it maximizes her joint surplus with a worker. Second, and as in many models with search frictions, market tightness satisfies a forward looking equation. However, in our model default probabilities affect this equation, as well as labor condition affect default probabilities.

Let $S(a,b,z) = e(a,b,z) + g(a,b,z)$ be the joint surplus of a match. The following proposition characterizes firm's policies and surplus evolution as a result of a "fictional" planner problem.

**Proposition 1 (*A "Fictional" Planner Problem*)** *Optimal firm's policies for default, debt issuance and vacancy posting are given by the following problem*

$$S(a,b,z) = \max_{d,nd}\Bigg\{0, \max_{v,b'}\Bigg\{ -\kappa v - (1-\tau)\lambda b + ax - \bar{u} + p(b',z)\left[b'[(1-s)+vq(z)] - (1-\lambda)b\right] \tag{13}$$
$$+\beta\mathbb{E}_{a',z'}\left[\Lambda(z,z')\left\{[(1-s)+vq(z)\gamma]S(a',b',z') - (1-\gamma)f(z)S(a',B(z'),z')\right\}|z\right]\Bigg\}\Bigg\}$$

The importance of Proposition 1 is that, given a bond price function $p(b',z)$, it allows to compute all of the firm's policies. In the following propositions we will use the expression for surplus in equation (12) to provide insights about the firm's policies.

The value $S(a,b,z)$ is the joint surplus of a match at the firm's optimal policies. Proposition 1 tells us that this surplus coincides with the maximum of the expression on the right hand side of equation (12). If we can show that the right hand side of equation (12) is the joint surplus of a match for a given firm's policies, we can conclude that the firm's optimal policies actually maximizes the value of a match. Proposition 2 shows that this is the case. The intuition for this result is simple: Nash bargaining impose a proportionality between the value of the firm and the value of the surplus, $e(a,b,z) = \gamma S(a,b,z)$. Therefore, the incentives of the firm are align with the ones of the match.

In the Appendix we formally characterize the Pareto frontier for a match and show that firm's policies for debt and default are Pareto efficient. The next proposition simply states this result.

**Proposition 2 (*Pareto Efficiency*)** *Assume that the value of a worker to be in a match is proportional to the value of the match. Then, firm's optimal policies for vacancies* $\boldsymbol{v}(b,z)$*, debt* $\boldsymbol{b}'(b,z)$ *and default* $\underline{\boldsymbol{a}}(b,z)$ *are Pareto efficient for the match.*

A few things are worth noting of the previous proposition. First of all, the default decision can be efficient for the match, but is never efficient for the economy. The reason is that debt payments are transfers across agents, but default destroys a match which is costly to rebuild. Second, the bargaining parameter $\gamma$ affects not only the sharing rule for a match, but also the outside value that firms and workers have. Thus, the Pareto frontier depends on the sharing rule. Finally, there can still be inefficiencies associated with the search frictions in the labor market.[24]

[24]See Hosios (1990).

We now turn to the incentives involved in the default decision. As stated in Lemma 1, a firm defaults if its idiosyncratic productivity is below a certain threshold. Equation (12) allows for a simple computation of the default threshold and the following proposition characterizes this value.

**Proposition 3 (*Default Threshold*)** *The default threshold* $\underline{\boldsymbol{a}}(b,z)$ *is given by*

$$\begin{aligned}\underline{\boldsymbol{a}}(b,z) &= \frac{1}{x}\Big[\pi(b,\boldsymbol{b}'(b,z),z)+\bar{u}+(1-\gamma)f(z)\beta\mathbb{E}_{a',z'}\left[\Lambda(z,z')S(a',B(z'),z')|z\right]\dots \\ &- (1-s)\beta\mathbb{E}_{a',z'}\left[\Lambda(z,z')S(a',\boldsymbol{b}'(b,z),z')|z\right]\Big]\end{aligned} \tag{14}$$

*where* $\pi(b,b',z)$ *is the debt outflow given by*

$$\pi(b,b',z)=(1-\tau)\lambda b-p(b',z)\left[b'(1-s)-(1-\lambda)b\right]$$

The expression for the default threshold in (14) is rather intuitive and one of the core results in the paper. It is simultaneously capturing both static and dynamic forces in the model. Default responds to two static components. First, the overall productivity in the economy $x$ reduces the default threshold: the higher the productivity, the larger the benefits of producing and the smaller the incentives to default. Second, the larger the debt outflow $\pi$ is, the higher the default threshold. This is also intuitive: On one hand, the lager are the debt payments $(1-\tau)\lambda b$, the more incentives the firm has to default. On the other hand, the higher is the price of the debt that the firm issues, the more incentives it has to procrastinate default and take advantage of the favorable financial market conditions.

The default decision is also affected by a forward component captured in the expected future value of a match $\mathbb{E}_{a',z'}\left[\Lambda(z,z')S(a',\boldsymbol{b}'(b,z),z')|z\right]$. This is also reasonable result: if a firm is expecting to obtain large returns from being match with a worker, she has less incentives to default since she would otherwise loose her workers. This makes clear how workers act as an asset for the firm: as in most standard models of default, the firm's incentives to default are lower the higher are the expected future returns of its assets. Unlike other models, the value of this asset is given by her workers and tightly related to the labor market. Notice as well that the outside value of a worker, given by $\bar{u}+f(z)(1-\gamma)\mathbb{E}_{a',z'}\left[\Lambda(z,z')S(a',B'(z'),z')|z\right]$, also affects the default decision. Intuitively, the larger the outside value is, the less attractive the match is for the worker, and the more incentives the match has to default.

Notice how important were the assumptions made for the model. On one hand, if default was modeled as a static decision, we could never capture how expectations of future labor market conditions affect financial market. In the same line, if the labor market were Walrasian, there would be no value of being matched to a worker since it is inexpensive to find a new one. These two key assumptions allows us to establish a clear connection between the two markets. It becomes clear from the expression for default threshold (14) how a worsening in the labor market affects financial conditions: a decline in the future value of a match will induce higher default rates, which will turn in lower bond prices and borrowing.

Last proposition shows how a worsening in the labor market affects financial conditions. In the next proposition, we show that the economy also exhibits a feedback in the opposite direction: a worsening in financial markets affects labor market conditions.

**Proposition 4** *In equilibrium, the value of a match for a non-defaulting firm ($a > \underline{\boldsymbol{a}}(b,z)$) is given by*

$$\begin{aligned} S(a,b,z) &= ax - \bar{u} + \beta \mathbb{E}_{a',z'} \left[ \Lambda(z,z') \left\{ (1-s) S(a', \boldsymbol{b}'(b,z), z') - (1-\gamma) f(z) S(a', B(z'), z') \right\} |z \right] \\ &- \pi(b, \boldsymbol{b}'(b,z), z) \end{aligned} \quad (15)$$

*Note that equation (15) implies an equilibrium average value of a match as*

$$\mathbb{E}_a \left[ S(a, B(z), z) \right] = x \int_{\underline{\boldsymbol{a}}(B(z),z)}^{\infty} \left[ a - \underline{\boldsymbol{a}}(B(z),z) \right] dH(a)$$

*Assuming that the matching function is given by $m(V,U) = V^{\nu} U^{1-\nu}$. Then, the finding probability is given as follows*

$$\kappa f(z)^{\frac{1-\nu}{\nu}} = p(\boldsymbol{b}'(B(z),z),z) \boldsymbol{b}'(B(z),z) + \gamma \beta \mathbb{E}_{a',z'} \left[ \Lambda(z,z') S(a', \boldsymbol{b}'(B(z),z),z') | z \right] \quad (16)$$

*where we used that $q(z) = f(z)^{-\frac{1-\nu}{\nu}}$.*

The value of a match $S(a,b,z)$ in equation (15) has two components. The one in the first line is the standard DMP component: it includes production today plus the continuation value of match, minus the outside values of the worker (unemployment benefits and value of finding a new firm). The second line is not standard and highlights the financial value of a worker: the value of the match is lower the higher is the debt outflow $\pi$. The reason why higher debt decreases the value of a match is because it implies a larger outflow. Similarly, having workers is partly attractive to the firm because it allows her to issue debt which payments are subsidies. Lower prices of debt makes this activity less profitable, lowering the value of a match.

Equation (16) comes from the firm's first order condition with respect to vacancies in equation (12). This is an indifference condition that changes vacancies per worker $v$ *without* changing debt per worker tomorrow $b'$, and thus changes total debt tomorrow *without* affecting the price of debt today. The cost of the additional vacancy posting is the left hand side of (16). The benefit, on the right hand side of (16), includes the fraction $\gamma$ of the surplus that the firm obtains from the new worker plus the debt inflow from the total increase of debt.[25] This equation shows the two values of a worker: the standard "production" value which is encoded in the surplus, as well as the "financial value" of the worker which is the inflow of debt that the firm issues at a constant price. In other words, *by posting vacancies and increasing the number of workers, the firm can issue more debt without affecting its price.*

We turn now to bond prices and quantities.

**Proposition 5** *The price of any bond type $b'$ when the aggregate state of the economy is $z$ is given by*

$$p(b', z) = \beta \mathbb{E}_{z'} \left[ \Lambda(z, z') \left[1 - H(\underline{\boldsymbol{a}}(b', z'))\right] \left[\lambda + (1-\lambda) p(\boldsymbol{b'}(b', z'), z')\right] |z \right] \ \forall b' \tag{17}$$

*The optimal quantity of debt per worker issued by a firm $\boldsymbol{b'}(b, z)$ satisfies*

$$\begin{aligned} p(b', z) \quad &+ \quad \frac{\partial p(b', z)}{\partial b'} \left[ b' - \frac{(1-\lambda)b}{1-s} \right] \\ &= \quad \beta \mathbb{E}_{z'} \left[ \Lambda(z, z')[1 - H(\underline{a}(b', z'))] \left[(1-\tau)\lambda + (1-\lambda) p(\boldsymbol{b'}(b', z'), z')\right] |z \right] \end{aligned} \tag{18}$$

Equation (17) comes from household first order condition and is a standard asset pricing equation: the value of the bond is the expectation of its payoff weighted by the household's (buyer) stochastic discount factor. Notice that this payoff includes, conditional on non-default, the fraction that matures next period $\lambda$, plus the value tomorrow $p(\boldsymbol{b'}(b', z'), z')$ of the remaining bond $1 - \lambda$. This proposition allows us to price any bond in the economy, even those that are not actually traded in equilibrium. Consequently, we can make sense of objects like risk-free rates and credit spreads while still keeping the model highly tractable. This price function (and not only the price of the bond actually traded) is internalized by the firms when making her portfolio decision: they understand that they can affect the cost of funding by changing their debt holdings.[26]

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[25]Equation (16) can be nicely understood with a perturbation argument. Imagine that we want to increase the number of workers tomorrow by a fraction of $\Delta n' = \phi n'$ without changing the amount of debt per worker tomorrow. Then, we need an increase in total vacancies of $\Delta \bar{v} = \frac{\phi n'}{q(z)}$ and an increase of total debt of $\Delta \bar{b}' = \phi \bar{b}'$. The cost of this perturbation is $\kappa \frac{\phi n'}{q(z)}$. The benefit is the new debt $p(b', z)\phi \bar{b}'$ plus the fraction $\gamma$ of the surplus that the firm obtains from the new workers: $\phi n' \gamma \beta \mathbb{E}_{a', z'} \left[S(a', b', z')|z\right]$. An optimal perturbation makes these cost and benefits equal

$$\frac{\phi n'}{q(z)} = p(b', z)\phi \bar{b}' + \phi n' \gamma \beta \mathbb{E}_{a', z'} \left[S(a', b', z')|z\right]$$

Canceling terms, we have equation (16).

[26]Note that the price formula would change if there were non-zero recovery rates upon default. An extension including recovery rates, as well as its quantitative importance, is left for future research.

Equation (18) comes from firm's first order conditions and determines her optimal debt decision. The right hand side is the expected cost of issuing debt: with a certain probability, the firm will repay a fraction $\lambda$ of her liabilities next period (minus subsidies), and the remaining fraction $1-\lambda$, which is still a liability to the firm and has a market value of $p(\boldsymbol{b}'(b',z'),z')$. The left hand side is the marginal benefit of debt: it includes the price per unit of debt $p(b',z)$ plus the change in price for increasing debt $\frac{\partial p(b',z)}{\partial b'}$ times the amount of new debt (tomorrow, per worker) $b' - \frac{(1-\lambda)b}{1-s}$. The optimal firm's policy equalizes marginal benefits and costs.

To understand the effects of long term debt, we can use equations (17) and (18) to compute the policy of debt as follows

$$\boldsymbol{b}'(b,z) = \tau\lambda \frac{\mathbb{E}_{z'}\left[\Lambda(z,z')[1-H(\underline{\boldsymbol{a}}(\boldsymbol{b}'(b,z),z'))]|z\right]}{-\partial p(\boldsymbol{b}'(b,z),z)/\partial b'} + \frac{(1-\lambda)b}{1-s} \tag{19}$$

Equation (19) shows how long term debt generates higher persistence of debt. If $\lambda \approx 1$, debt would respond only to expected default probabilities. In this case, debt adjust almost one to one with the arrival of shocks, making it very volatile. In contrast, when $\lambda \approx 0$, debt today has a higher loading on past debt and is less responsive to the arrival of shocks. This makes leverage highly persistent.[27] Furthermore, since debt will be less responsive to the state of the economy, prices of debt will be more volatile in an economy with long maturity, inducing more cyclicality in credit spreads.

Equation (17) imposes a restriction on our parameters. In particular, for the model to have stable dynamics, we need $\frac{1-\lambda}{1-s} < 1$, which imposes an upper on the maturity of debt $\lambda < s$.

We finalize the section by discussing how equilibrium affects labor productivity. In equilibrium, output is given as follows

$$Y(z) = x\mathcal{P}(z)N \tag{20}$$

where $N = [1 - H(\underline{\boldsymbol{a}}(B,z)]\bar{N}$ is the number of workers actually engaged in production, and $\mathcal{P}(z) = \mathbb{E}_a\left[a|a \geq \underline{\boldsymbol{a}}(B,z)\right]$ is the average idiosyncratic productivity conditional on no default.

The term $\mathcal{P}(z)$ is an expectation truncated by the default threshold, and thus increasing in the default threshold. Since default in our model is countercyclical, $\mathcal{P}(z)$ is countercyclical as well. Intuitively, in periods of high default only productive firms survive. Consequently, fluctuations in aggregate productivity $x$ are dampen by fluctuations in $\mathcal{P}(z)$. This sharply contrast with predictions of models about misallocation,

[27] Put differently, the policy functions in the model can be indexed by parameters. Then $\frac{\partial \boldsymbol{b}'(b,z;\lambda)}{\partial b}$ is a (positive and) decreasing function of $\lambda$. A graph of the policy for different $\lambda$'s is available upon request.

where the endogenous component of TFP is procyclical.[28] In our model, an econometrician who doesn't take into account fluctuations in $\mathcal{P}(z)$ will underestimate the contribution of aggregate productivity $x$ to output fluctuations. In a recent paper, McGrattan and Prescott (2012) provide evidence that, during the last three decades, labor productivity has become significantly less correlated with the stance of the business cycle. We think that the endogenous exit induced by default is a potential explanation of this fact, which we leave for future research.

# 5 Quantitative Evaluation

## 5.1 Calibration

We calibrate our model to perform the quantitative evaluation. Some of our model parameters are standard and we borrow values from previous literature. Other parameters are calibrated within the model in order to match certain moments. We explain our calibration next.

A period in our model is a month. We choose $\beta = 0.96^{1/12}$ that implies a risk-free annual interest rate of 4%. We use an utility function $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$ and set $\sigma = 0$ to use risk-neutral preferences. We use a Cobb-Douglas matching function technology $m(V, U) = V^{\nu} U^{1-\nu}$, and set the elasticity of the matching technology to $\nu = 0.5$. The bargaining parameter is set to $\gamma = 0.5$. These values for $\gamma$ and $\nu$ are standard in the literature.[29] We assume a cost of vacancy posting $\kappa = 17$ to match a monthly mean of the finding probability of 45%. The unemployment benefit is set to $\bar{u} = 0.6$, a middle value between Shimer (2005) and Hagedorn and Manovskii (2008).[30] The exogenous separation rate is $s = 0.033$ so that the total separation, including the endogenous one due to default, is 0.035 per month. The average maturity of debt is $1/\lambda$, we set $\lambda = 0.035$ to match an average maturity of debt of 2.5 years. While average maturity of corporate debt is closer to 4 years (see Gomes, Jermann, and Schmid, 2013), our model imposes an upper bound on $\lambda > s$ as discussed in (19). The tax benefit on debt is set to $\tau = 0.13$ as described by the U.S. Government Accountability Office (GAO, 2013). We assume that idiosyncratic productivity is log-normally distributed $\ln a \sim \mathcal{N}(\mu_a, \sigma_a^2)$. We choose $\sigma_a = 0.17$ to match an average default rate of 2% a year, and set $\mu_a$ so that

[28] See Buera and Moll (2013) and Khan and Thomas (2013).

[29] See Gertler and Trigari (2009) for a discussion.

[30] Shimer (2005) uses a value of $\bar{u} = 0.4$ while Hagedorn and Manovskii (2008) uses $\bar{u} = 0.95$. While Shimer (2005) argues that the standard Mortensen-Pissarides model fails to explain the volatility of vacancies and unemployment, Hagedorn and Manovskii (2008) argues that a high value for unemployment benefits improves the model quantitative performance. We choose a value for unemployment benefits closer to Shimer (2005) so that we can focus in the novelties of our paper.

Table 2: Parameter values

| $\beta$ | $\sigma$ | $\nu$ | $\gamma$ | $\kappa$ | $s$ | $\lambda$ | $\tau$ | $\bar{u}$ | $(\mu_a, \sigma_a)$ | $(\rho_x, \sigma_x)$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $0.96^{1/12}$ | 0 | 0.5 | 0.5 | 17 | 0.033 | 0.035 | 0.13 | 0.6 | (-0.007,0.2) | (0.98,0.005) |

$\mathbb{E}(a) = 1$. Finally, we assume an AR(1) process for productivity

$$\ln x_t = \rho_x \ln x_{t-1} + \sigma_x \varepsilon_t$$

We set $\rho_x = 0.98$ and $\sigma_x = 0.005$ to match output per worker in the US. Table 2 summarizes all of the parameter values.

We solved the model globally using piecewise-linear function approximation. See the Appendix for details.

## 5.2 Business Cycle Statistics

We analyze the model's quantitative performance by evaluating its impulse response and business cycle statistics. In non-linear model there are different ways to construct the associated impulse-response. Our initial interest is in understanding which is the *average* effect of a negative productivity innovation over the model's *ergodic distribution*. To answer this, we proceed as follows: we independently simulate 50,000 economies for a long period of time; then, at $t = 0$, we feed all of these economies with the same negative productivity innovation; and from then onward we kept on independently simulating each economy. The impulse response function is the average over the 50,000 economies for each period.

Figure 4 shows the impulse response to a negative productivity innovation. After a decrease in productivity, there is an increase in the separation rate. This is due to the increase in default rates: with lower productivity, today's production and future surpluses of a match are lower as well. Thus, firms have less incentives to repay their debt which trigger default (see equation (14)). In response to the higher default rates, credit spreads are higher.[31] In turn, firms respond by "deleveraging" and achieving lower debt per worker ratios. These dynamics imply that both the production and financial values of a worker are depressed. As argued in equation (16), this induces a sharp decrease in the finding probability and an increase of unemployment.

Recently, many papers have used environments with frictional financial markets to analyze the effects

[31] Credit spreads on firm's bind prices are computed with respect to risks-free bond of the same maturity.

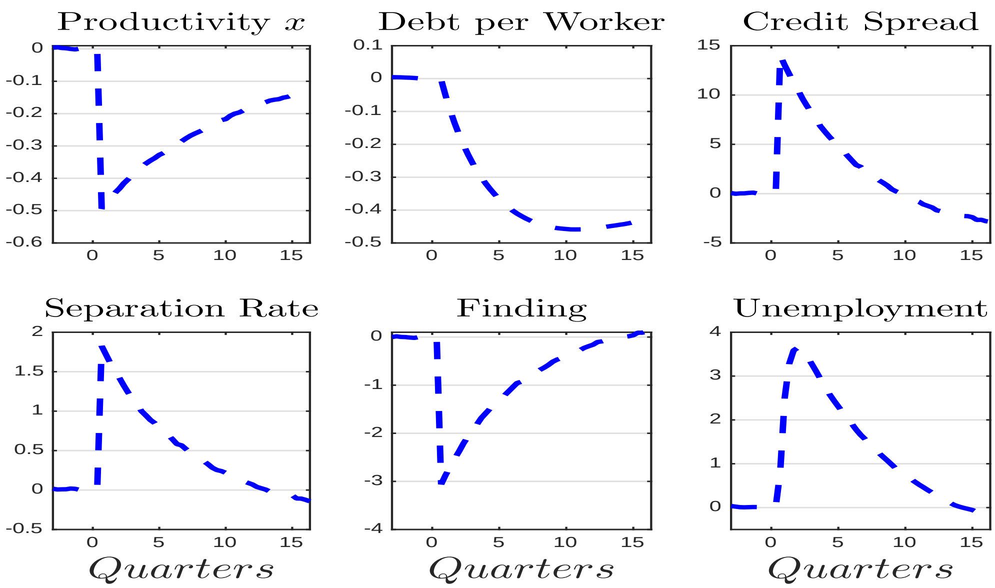


Figure 4: Impulse response function to a negative productivity innovation.

of a capital depreciation shock.[32] This shock mechanically decreases output, but also increases leverage at the firm level which may have further consequences. We proceed by analyzing an analogous shock in our economy. In particular, we study the effect of an unexpected one period increase in the separation rate where every firm looses 2% of their workers.[33,34] Total debt for each firm remains constant.

Figure 5 shows the impulse response to the separation shock.[35] By construction, the amount of debt per worker increases at the moment of the shock. This induce an increase in default rates (see equation (14)) that leads to an increase in the separation rate.[36] In turn, credit spreads rises. This depresses the financial value of a worker, which leads to a decrease and slow recovery in the finding probability (see equation (16)).

[32]See Brunnermeier and Sannikov (2012), Gertler and Kiyotaki (2009) and Moreira and Savov (2013) among others.

[33]At any given period the law of motion for labor is

$$\bar{N}' = (1-s)\left[1 - H(\underline{\boldsymbol{a}}(B(z),z))\right]\bar{N} + f(z)\left[1 - H(\underline{\boldsymbol{a}}(B(z),z))\bar{N}\right]$$

At the period of the shock, it is

$$\bar{N}' = 0.98\left\{(1-s)\left[1 - H(\underline{\boldsymbol{a}}(B(z),z))\right]\bar{N} + f(z)\left[1 - H(\underline{\boldsymbol{a}}(B(z),z))\bar{N}\right]\right\}$$

[34]No agent in the economy expected the sudden increase in separation, and they believe that it will not happen again in the future. Thus, this exercise can be thought as unexpectedly changing the state of the economy and analyzing the dynamics thereafter.

[35]This impulse response is also computed as an average over 50,000 economies.

[36]The separation rate we plot does not include the initial 2% separation shock. Thus, all of the change in separation rates is endogenous and due to the change in default rates

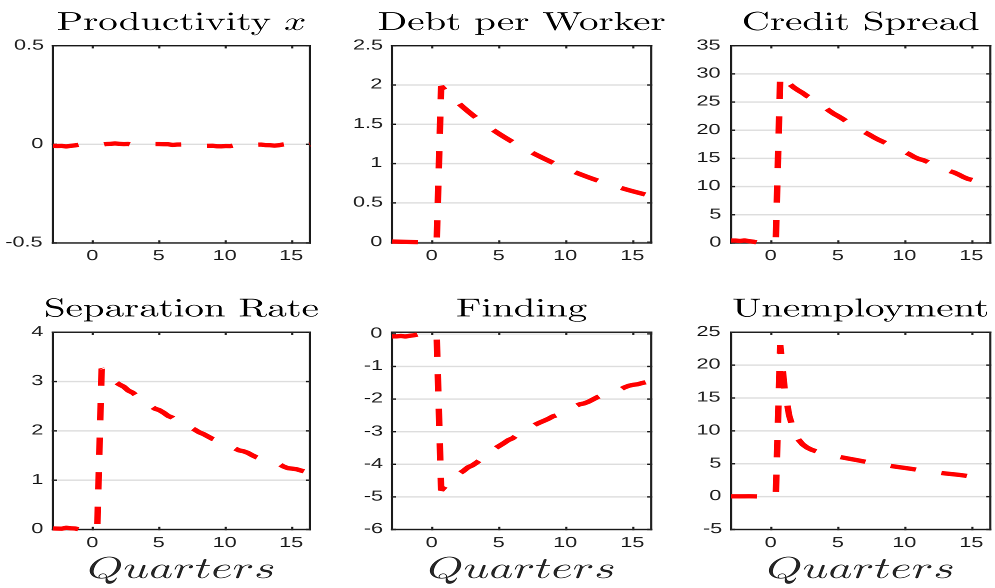


Figure 5: Impulse response function to a positive separation shock.

As a result, although the initial shock last for one period only, unemployment remains high for more than four years.

Table 3 shows a set of business cycle statistics in the model and compares them with their empirical counterpart for the years 1951-21012.[37] Overall, the model generates business cycle moments that compare reasonable well with data, specially for a one shock model. For instance, correlation with output and persistence of each variable are in line with the data. More importantly, standard deviation of finding probability and unemployment in the model are very close to the empirical ones. This shows that the inclusion of frictional financial markets helps to solve the "Shimer Puzzle" (Shimer, 2005), a similar result to the one found by Petrosky-Nadeau (2011).

A natural next question is: how much of these business cycle statistics are explained by the "unemployment accelerator" explored in Section 4? Next section provides an answer to this question.

[37] As usual in the literature, we applied the Hodrick-Prescott filter to the data. Regarding the model statistics, we simulated our model 10,000 times, each simulation of 61 years length (1951-2012), and filtered the simulated data in the same manner than the actual data. We computed statistics for each of the 10,000 simulations, and the numbers in Table 3 are the means of these statistics across simulations.

Table 3: Business Cycle Moments

| | Labor Productivity | Finding | Unemployment | Total Debt | Credit Spread |
|---|---|---|---|---|---|
| | US economy, 1951:2012 | | | | |
| Std. dev. | 0.02 | 0.13 | 0.19 | 0.06 | 0.62 |
| Autocorrelation | 0.89 | 0.88 | 0.94 | 0.98 | 0.74 |
| Corr. with output | 0.56 | 0.87 | −0.88 | 0.13 | −0.46 |
| | Model | | | | |
| Std. dev. | 0.02 | 0.08 | 0.11 | 0.03 | 0.42 |
| Autocorrelation | 0.88 | 0.84 | 0.89 | 0.99 | 0.84 |
| Corr. with output | 0.66 | 0.63 | −0.63 | 0.57 | −0.54 |

**Note:** Result from model simulation and data. All variables are reported in logs as deviation from an HP trend with smoothing parameter $10^5$. Data covers the period 1951-2012. Model results are averages over 10,000 simulations of 61 years length.

### 5.3 Evaluating the Mechanism

As discussed in Section 4, the key contribution of our model is the two sided interaction between the labor and the financial markets. In this section, we quantify the importance of this interaction. In particular, we compute volatilities of financial market variables absent any fluctuations in the labor market. We interpret this exercise as measuring the effects that the labor market has on financial markets. Similarly we compute volatilities of labor market variables absent any fluctuation in financial markets and interpret this as the contribution that financial markets have on labor market fluctuations.

As can be seen from the default threshold in equation (14), the impact that the labor market has on the financial market is transmitted through changes in the value of a match $S(a, b, z)$ and by changes in the finding probability $f(z)$. In order to assess the importance of labor market fluctuations, we want to shut down movements in these two variables $S(a, b, z)$ and $f(z)$. Similar logic applies for the importance of the financial market: to asses the effects of financial fluctuations on the labor marker, we want to shut down movements in the default rate. To do these computations, we endow the government with three policy instruments: a transfer to firms per worker $T^S(z)$; a transfer to firms in case of no default $T^a(z)$; and a subsidy to vacancy posting per workers $T^v(z)$. The transfer $T^S(z)$ is uncontingent on default and every firm receives it.[38]

The three policy instruments change some of the equilibrium conditions. In particular, they affect the default threshold, the average value of a match and the finding probability. We describe the modified system

[38] We are implicitly assuming that the government has in some sense a better enforcement technology than the private sector. In particular, if $T^S(z) < 0$, defaulting firms must still pay for this negative transfer.

of equations below

$$\underline{\boldsymbol{a}}(b,z) = \frac{1}{x}\Big[\pi(b,\boldsymbol{b}'(b,z),z)+\bar{u}+(1-\gamma)f(z)\beta\mathbb{E}_{a',z'}\left[\Lambda(z,z')S(a',B(z'),z')|z\right]\dots \tag{21}$$
$$- \; T^a(z)-(1-s)\beta\mathbb{E}_{z'}\left[S(a',\boldsymbol{b}'(b,z),z')|z\right]\Big]$$
$$\mathbb{E}_a\left[S(a,b,z)\right] = T^S(z)+x\int_{\underline{\boldsymbol{a}}(b,z)}^{\infty}(a-\underline{\boldsymbol{a}}(b,z))dH(a) \tag{22}$$
$$\kappa f(z)^{\frac{1-\nu}{\nu}} = T^v(z)+p(B'(z),z)B'(z)+\gamma\beta\mathbb{E}_{a',z'}\left[S(a',B(z'),z')|z\right] \tag{23}$$

These three equations are a modified version of equations (14), (15) and (16) that account for the new government policy instruments.

Next proposition characterize how to set the system of government transfers to eliminate different fluctuations in the model.

**Proposition 6 (Transfers and Fluctuations)** *Assume that* $T^a(z)=0$ *and*

$$T^S(z) = S^*-x\int_{\underline{\boldsymbol{a}}(B(z),z)}^{\infty}(a-\underline{\boldsymbol{a}}(B(z),z))dH(a) \tag{24}$$
$$T^v(z) = \kappa f^{*\frac{1-\nu}{\nu}}-p(\boldsymbol{b}'(B(z),z),z)\boldsymbol{b}'(B(z),z)-\gamma\beta\mathbb{E}_{z'}\left[S(a',\boldsymbol{b}'(B(z),z),z')|z\right] \tag{25}$$

*Then, the average value of a match and the finding probability are constant and independent of the aggregate state of the economy*

$$\mathbb{E}_a\left[S(a,B,z)\right]=S^*, \qquad f(z)=f^* \qquad \textit{for all } z.$$

*Next, assume that* $T^S(z)=T^v(z)=0$ *and*

$$T^a(z) = -x\underline{\boldsymbol{a}}^*+\bar{u}+(1-\tau)\lambda B(z)-p(\boldsymbol{b}'(B(z),z),z)\left[(1-s)\boldsymbol{b}'(B(z),z)-(1-\lambda)B(z)\right] \tag{26}$$
$$- \; (1-s-f(z)(1-\gamma))\beta\mathbb{E}_{z'}\left[S(a',\boldsymbol{b}'(B(z),z),z')|z\right]$$

*Then, the default threshold is constant and independent of the aggregate state of the economy*

$$\underline{\boldsymbol{a}}(B,z)=\underline{\boldsymbol{a}}^* \textit{ for all } z.$$

Proposition 6 tells us exactly how to compute each one of the policy instruments in order to shut down fluctuations in the labor or in the financial market. In particular, if we set $T^S(z)$ and $T^v(z)$ as in equations (24) and (25), the finding probability and the average value of a match are constant and there is no fluctuations

Table 4: Quantifying the Mechanism

| | Standard Deviations | | | Mean | | |
|---|---|---|---|---|---|---|
| | Full Model | Fixed $S$ and $f$ | Fixed $a$ | Full Model | Fixed $S$ and $f$ | Fixed $a$ |
| Finding | 0.08 | – | 0.004 | 45% | 45% | 14% |
| Unemployment | 0.11 | 0.004 | 0.003 | 7.2% | 6.9% | 19% |
| Total Debt | 0.03 | 0.001 | 0.014 | 9.88 | 8.2 | 2.82 |
| Credit Spread | 0.41 | 0.13 | 0.01 | 9% | 5.4% | 6.4% |
| Default Rate | 1.67 | 0.36 | – | 1.6% | 0.34% | 1.7% |

in the labor market. Similarly, if we set $T^a(z)$ as in (26), the default rate is constant and fluctuations in financial markets are absent.

The second column of Table 4 computes the volatility of several variables of the model when $T^S(z)$ and $T^v(z)$ are set according to equations (24) and (25) so that there are no fluctuations in the finding probability and in the average value of a match.[39] By construction, the standard deviation of finding is zero, and the variance of unemployment significantly decreases. More interestingly, relative to the full model, the standard deviation of credit spreads decrease from 0.40 to 0.17, and the one of default rate decreases from 1.44 to 0.30. Thus, in the context of our model, fluctuations in the labor market explain 68% of the variations in credits spreads and almost 80% of the variations in default rates.

The third column of Table 4 performs the same exercise by setting $T^a(z)$ according to equations (26) so that there is no fluctuations in the default rate. The standard deviation of the finding probability decreases by a factor of 20, and the one of unemployment decreases by a factor larger than 36. We conclude that financial markets is an important determinant of labor market fluctuations.

## 6 Conclusions

In most *"macro-finance"* models, asset, debt and profits are sufficient statistics to determine a firm's employment and probability to default. Importantly, there should not be any relationship between default probability and employment after controlling for these statistics. We find that this is not the case. In the cross-section of US publicly traded firms, after controlling for these standard statistics, there is still a significant and statistically large negative relation between a firm's workers and her probability to default. To account for this fact, we provide a micro-founded model where workers determine both, the firm's value

[39]The values $f^*$ and $S^*$ in equations (24) and (25) are set to the ergodic means of the finding probability and the value of a match in the full model respectively.

and price of her debt. In line with the evidence, our model provides an endogenous relationship between the labor market and the financial market. We show that this two side interaction between labor market and financial market, greatly amplifies business cycle fluctuations.

There are several dimension for future work that this paper leaves open. First of all, our model focused only on the effect that employment has on default, and neglects other standard channels, such as the firm's physical capital.[40] Although this allowed us to center the analysis on our new mechanism, we think it could be insightful to include capital in the model and revisit the quantitative results. A second venue for future work is the design of optimal unemployment benefits in our model: more generous benefits can provide more insurance to households, but would also have averse effects on firms' borrowing conditions.

Importantly, we have abstracted, both at the empirical and the model level, from firm heterogeneity. Arguably, some of the mechanisms discussed in this paper can vary across firms depending, for instance, on the size, age or industry. Understanding how these frictions may vary (or not) with firms' characteristics is top in our future research priorities.

[40] We did however control for many firms' characteristics, including capital, on the empirical section. See Section 2.

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# A Data Sources and Definitions

TO BE ADDED

# B Proofs

## B.1 Lineary of value Functions

**Lemma 1** ***[Firms Value Function]*** *The value function of a firm is linear in workers n*

$$E(a, B, n, z) = e(a, b, z)n$$

*Policies for vacancy posting, capital and debt are also linear in labor*

$$\begin{aligned} v'(a, B, n, z) &= \boldsymbol{v}(b, z)n \\ B'(a, B, n, z) &= \boldsymbol{b}'(b, z)\left[(1-s) + \boldsymbol{v}(z)q(z)\right]n \end{aligned}$$

*Assume that*

$$x' - \frac{\partial w(a, b, z, m)}{\partial a} > 0 \qquad \forall (a, b, z, m) \tag{1}$$

*Firms default if and only if their idiosyncratic productivity is below a threshold $\underline{a}(\tilde{b}, z)$, where $\tilde{b}$ and is the amount of debt per worker.*

**Proof.** Define $\tilde{b}' = \frac{B'}{n'}$, we can rewrite the problem as

$$E(a, b, n, z) = n \max_{m} \left\{ \max_{d,nd} \left\{ 0, ax - w(\cdot) - \kappa v - ((1-\tau)\lambda + (1-\lambda)p(b', z))b + \dots \right. \right. \tag{2}$$

$$\left. \left. \dots + (1 - s + vq(z))[p(b', z)b' + \beta \mathbb{E}\left[\Lambda(z, z')\frac{E(a', b', n', z')}{n'}\right] \right\} \right\} \tag{3}$$

with $z' = \Gamma(z)$. We guess and verify that $E(\omega, n, z) = w + e(z)n$. Using the guess and verify, and chaneing the control variable in per worker, we can check that

$$e(a, b, z) = n \max_{m} \left\{ \max_{d,nd} \left\{ 0, ax - w(\cdot) - c(v, 1) - ((1-\tau)\lambda + (1-\lambda)p(b', z))b + \dots \right. \right. \tag{4}$$

$$\left. \left. \dots + (1 - s + vq(z))[p(b', z)b' + \beta \mathbb{E}\left[\Lambda(z, z')\frac{e(a', b', z')}{n'}\right] \right\} \right\} \tag{5}$$

■

**Proposition 1** *Optimal firm's policies for default, debt issuance and vacancy posting are given by the following* *problem*

$$S(a,b,z) = \max_{d,nd}\left\{0, \max_{v,b'}\left\{ -\kappa v - (1-\tau)\lambda b + ax - \bar{u} + p(b',z)\left[b'[(1-s)+vq(z)] - (1-\lambda)b\right]\right.\right. \tag{6}$$
$$\left.\left. + \beta\mathbb{E}_{a',z'}\left[\Lambda(z,z')\left\{[(1-s)+vq(z)\gamma]S(a',b',z') - (1-\gamma)f(z)S(a',B(z'),z')\right\}|z\right]\right\}\right\}$$

**Proof.** The equilibrium condition for Nash Bargaining are given by

$$\begin{aligned}
w(a,b,z,m) &= \arg\max\left\{\tilde{e}(a,b,z,v,b')_w^{\gamma}\tilde{e}(a,b,z,v,b')_w^{1-\gamma}\right\} \\
\tilde{e}(a,b,z,v,b')_w &= -\kappa v - (1-\tau)\lambda b + ax - w + p(b',z)\left[b'\left[(1-s)+vq(z)\right] - (1-\lambda)b\right] + \dots \\
&\dots + \left[(1-s)+vq(z)\right]\beta\mathbb{E}_{a'z,'}\left[\Lambda(z,z')e(a',b',z')|z\right] \\
\tilde{g}(a,b,z,b')_w &= w - \bar{u} + \beta\mathbb{E}_{a',z'}\left[\Lambda(z,z')\left\{(1-s)g(a',b',z') - f(z)g(a',B(z'),z')\right\}|z\right]
\end{aligned}$$

Let define $\tilde{S}(a,b,z,v,b') = \tilde{e}(a,b,z,v,b')_{w^*} + \tilde{g}(a,b,z,b')_{w^*}$. Then from the first order condition of Nash Bargaining

$$\tilde{e}(a,b,z,v,b')_{w^*} = \gamma\tilde{S}(a,b,z,v,b') \qquad \tilde{g}(a,b,z,b')_{w^*} = (1-\gamma)\tilde{S}(a,b,z,v,b') \tag{7}$$

For all $a,b,z,v,b'$, where $\tilde{S}(a,b,z,v,b')$ satisfies

$$\tilde{S}(a,b,z,v,b') = -\kappa v - (1-\tau)\lambda b + ax - u + p(b',z)\left[b'\left[(1-s)+vq(z)\right] - (1-\lambda)b\right] + \dots \tag{8}$$
$$\dots + \beta\mathbb{E}_{a',z'}\left[\Lambda(z,z')\left\{(1-s)S(a',b',z') - f(z)(1-\gamma)S(a',B(z'),z')\right\}|z\right] \tag{9}$$

and $S(a',b',z') = \tilde{S}(a,b,z,v^*,b^{*\prime})_{w^*}$. Using the value function of the firm

$$e(a,b,z) \quad = \max\nolimits_{d,nd}\{0, \max\nolimits_{v,b'}\tilde{e}(a,b,z,v,b')_{w^*}\} \tag{10}$$
$$= \max\nolimits_{d,nd}\{0, \max\nolimits_{v,b'}\gamma\tilde{S}(a,b,z,v,b')_{w^*}\} \tag{11}$$

Given that $e(a,b,z) = \gamma S(a,b,z)$ we have that

$$S(a,b,z) = \max_{d,nd}\left\{0, \max_{v,b'}\left\{ -\kappa v - (1-\tau)\lambda b + ax - \bar{u} + p(b',z)\left[b'[(1-s)+vq(z)] - (1-\lambda)b\right]\right.\right. \tag{12}$$
$$\left.\left. + \beta\mathbb{E}_{a',z'}\left[\Lambda(z,z')\left\{[(1-s)+vq(z)\gamma]S(a',b',z') - (1-\gamma)f(z)S(a',B(z'),z')\right\}|z\right]\right\}\right\}$$

■

**Proposition 2 (Efficiency)** *Assume that a worker always obtains a fix fraction of the value of a match. Then, firm's policies for debt* $\boldsymbol{b}'(b,z)$ *and default* $\underline{\boldsymbol{a}}(b,z)$ *are efficient for the match.*

**Proof.** Let $e^P(b, z, \Delta)$ be the value of a match with debt $b$ and promise utility to the worker $\Delta$ when the aggregate state of the economy is $z$. Then

$$\begin{aligned} e^P(b,z,\Delta) &= \max_{b',v,\{w(a)\},\chi,\{\Delta(z')\}_{z'}} \mathbb{E}_a \Bigg\{ \mathbb{I}\{\chi = \mathrm{d}\}\{0\} + \mathbb{I}\{\chi = \mathrm{nd}\} \Bigg\{ ax - w(a) + \ldots \\ &- \kappa v - \left[(1-\tau)\lambda + (1-\lambda)p(b',z)\right] b + \left[(1-s) + vq(z)\right] p(b',z)b' \\ &+ (1-s)\beta \mathbb{E}_{z'} \left[\Lambda(z,z')e^P(b',z',\Delta')|z\right] + vq(z)\beta\mathbb{E}_{z'}\left[\Lambda(z,z')e^P(b',z',\tilde{\Delta}(b',z'))|z\right] \Bigg\} \Bigg\} \end{aligned} \tag{13}$$

subject to

$$\Delta \leq \mathbb{E}_a \left[\mathbb{I}\{\chi = \mathrm{d}\}\{0\} + \mathbb{I}\{\chi = \mathrm{nd}\}\left\{w - \bar{u} + \beta\mathbb{E}_{z'}\left[\Lambda(z,z')\left\{(1-s)\Delta(z') - f(z)g(z')\right\}|z\right]\right\}\right] \tag{14}$$

where $\tilde{\Delta}(b, z)$ is the value to a worker of a new match with a firm with debt $b$ when the aggregate state of the economy is $z$, and $g(z) = \tilde{\Delta}(B(z), z)$. We proceede to prove the proposition in thre steps.

**Lemma 2** *The sum of the values in a match is independent of* $\Delta$

$$S^P(b,z) = e^P(b,z,\Delta) + \Delta$$

**Proof.** Let $\mu(b, z, \Delta)$ be the largrange multiplier on (14). First order condition with respect to wages in (13) imply $\mu(b, z, \Delta) = 1$. Aplying Benveniste and Scheinkman to (13) we obtan $\frac{\partial e^P(b,z,\Delta)}{\partial \Delta} = -\mu(b, z, \Delta)$. Then, $\frac{\partial\{e^P(b,z,\Delta)+\Delta\}}{\partial\Delta} = 0$. ∎

**Lemma 3** *The function* $S^P(b, z)$ *satisfies the following recursion*

$$\begin{aligned} S^P(b,z) &= \max_{b',v,\chi,} \mathbb{E}_a \Bigg\{ \mathbb{I}\{\chi = d\}\{0\} + \mathbb{I}\{\chi = nd\} \Bigg\{ ax - \bar{u} + \ldots \\ &- \kappa v - \left[(1-\tau)\lambda + (1-\lambda)p(b',z)\right] b + \left[(1-s) + vq(z)\right] p(b',z)b' \\ &+ (1-s)\beta\mathbb{E}_{z'}\left[\Lambda(z,z')S^P(b',z')|z\right] + \\ &+ \beta\mathbb{E}_{z'}\left[\Lambda(z,z')\left\{vq(z)e^P(b',z',\tilde{\Delta}(b',z')) - f(z)g(z')\right\}|z\right] \Bigg\} \Bigg\} \end{aligned} \tag{15}$$

*where* $e^P(b', z', \Delta)$ *satisifes equation* (13).

**Proof.** The recursion in (15) comes from summing (13) and (14) and using that $S^P(b, z) = e^P(b, z, \Delta) + \Delta$. ∎

**Lemma 4** *Assume that* $\tilde{\Delta}(b, z) = (1 - \gamma)S^P(b, z)$ *for all* $\{b, z\}$. *Then* $S^P(b, z)$ *satisifes the same recursion than* $S(b, z)$ *and the efficient policies for the match coincide with the ones of the firm.*

**Proof.** If $\tilde{\Delta}(b,z) = (1-\gamma)S^P(b,z)$, then $e^P(b,z,\tilde{\Delta}(b,z)) = \gamma S^P(b,z)$. Then, the recursion in (15) can be written as

$$
\begin{aligned}
S^P(b,z) \quad &= \quad \max_{b',v,\chi,} \mathbb{E}_a \Bigg\{ \mathbb{I}\{\chi = \mathrm{d}\}\{0\} + \mathbb{I}\{\chi = \mathrm{nd}\} \Bigg\{ ax - \bar{u} + \dots \\
&- \quad \kappa v - \big[(1-\tau)\lambda + (1-\lambda)p(b',z)\big]\, b + \left[(1-s) + vq(z)\right] p(b',z)b' \\
&+ \quad \beta \mathbb{E}_{z'} \Big[ \Lambda(z,z') \Big\{ \left[(1-s) + vq(z)\gamma\right] S^P(b',z') - f(z)(1-\gamma)S^P(B(z'),z') \Big\} |z \Big]
\end{aligned} \tag{16}
$$

where we used that $g(z) = (1-\gamma)S^P(B(z),z)$. ■ ■