On the Direction of Innovation

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- This paper: "Does it go in the right direction?"
- Basic theory: heterogenous patent races: *hotter and cooler*
- Equilibrium allocates scarce researchers to different patent races.
- Equilibrium and optimal allocation will rarely coincide.
- Under plausible assumptions, too many researchers in hot areas

Related Literature

- "Public good" nature of knowledge: Arrow (1962)
- Patent races and entry (Kamien & Schwartz (1976), Loury (1979), Lee and Wilde (1980),Dasgupta and Stiglitz (1980), Reinganum (1982), Judd (1985)
- Correlation of R&D programs with competing innovations: Dasgupta and Maskin (1987), Bhattacharya and Mookherjee (1986), JH Cardon, D Sasaki (1998)
- Divergence between social and private returns: Brian and Lemus (2013)
- Optimal and equilibrium product variety

A simple model

- Two research areas (patent races), one potential discovery each.
- Social *and* private value of discovery $z_1 < z_2$.
- Total M homogenous researchers.
- Discovery with probability $p(m_j)$.

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- Compare competitive and optimal allocations

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- In both cases $m_2 > m_1$

Comparison

- Key ratio (wedge):

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Proposition

Assume concave $p\left(m\right)$. Then $m_2 > \tilde{m}_2$ iff $\frac{(p(m)/m)}{p'(m)}$ increases with m.

Generalization

- Set of innovations/goods indexed by z with distribution F
- Welfare/utility: $\int_0^1 zp\left(m\left(z\right)\right) dF\left(z\right)$
- Resource constraint: $\int_0^1 m\left(z\right) dF\left(z\right) = M$

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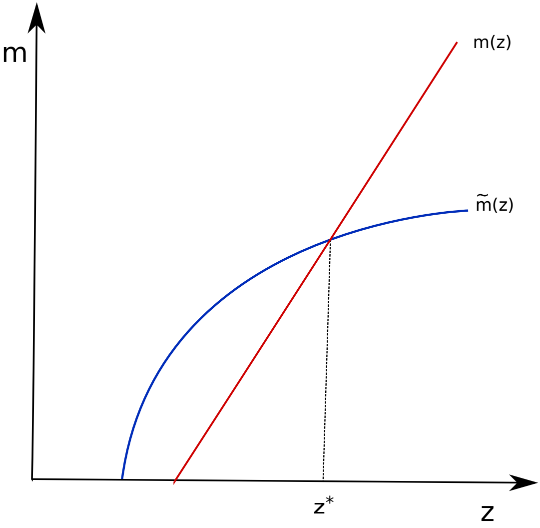
- Welfare/utility: $\int_0^1 zp(m(z))\, dF(z)$
- Resource constraint: $\int_0^1 m(z)\, dF(z) = M$
- Equilibrium and optimal conditions same as before.

Bias to hot areas



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Proposition

The competitive equilibrium is biased to hot (cold) areas iff the wedge $\frac{p(m)/m}{p'(m)}$ is increasing (decreasing) in m.

Canonical model - no redeployment

- $m(z)$ chosen at time zero
- line ends with discovery
- no redeployment after discovery
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- Can redefine problem letting $P(m) = \int \exp(-rt)\, p(t, m)\, dt$ and same propositions apply.

Example: Poisson case

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P(m) &= m\lambda \int \exp\left(-\left(r + m\lambda\right) t\right) dt \\
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- Key ratio for proposition:

$$
\begin{aligned}
\frac{P(m)/m}{P'(m)} &= \frac{\lambda}{r+m\lambda} \Big/ \frac{\lambda r}{\left(r+m\lambda\right)^2} \\
&= \frac{r+m\lambda}{r}
\end{aligned}
$$

- Increasing in m.

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- Poisson model
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- Closed form, very simple formulas:

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- Maximal wedge U^0/U approximately 1.2 ($\eta = 1.34$) and $\underline{z}^o/\underline{z} = 0.3$

Dynamic allocation with redeployment

- Poisson model (r, λ) with 2 research lines $z_1 < z_2$
- cost of redeployment c
- Researchers can redeploy immediately after first arrival.
- Atomistic researchers.
- Allocation (m_1, m_2) and $m_i = M$ immediately after arrival in other area

Competitive equilibrium

- innovator's expected value from second arrival

$$\hat{v}_i = \left(\frac{M\lambda}{r + M\lambda} \right) z_i / M = \frac{\lambda z_i}{r + M\lambda}$$

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- Value functions for initial research lines:

$$\begin{aligned} v_1(m_1) &= \frac{m_1\lambda}{r + M\lambda} (z_1/m_1 + \hat{v}_2 - c) + \frac{m_2\lambda}{r + M\lambda} (\hat{v}_1) \\ v_2(m_2) &= \frac{m_1\lambda}{r + M\lambda} (\hat{v}_2) + \frac{m_2\lambda}{r + M\lambda} (z_2/m_2 + \hat{v}_1 - c) \end{aligned}$$

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- At interior solution $v_1(m_1) = v_2(m_2)$. Subtracting both equations:

$$(m_2 - m_1)c = (z_2 - z_1)$$

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- If $c = 0$, then extreme solution: $m_2 = M$

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$$W(\tilde{m}_1, \tilde{m}_2) = \frac{\tilde{m}_1 \lambda (z_1 + \hat{w}_2 - \tilde{m}_1 c) + \tilde{m}_2 \lambda (z_2 + \hat{w}_1 - \tilde{m}_2 c)}{r + M\lambda}$$

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- Marginal values:

$$W_{\tilde{m}_i} = \frac{\lambda \left(z_i + \hat{w}_{-i}\right) - 2\lambda \tilde{m}_i c}{r + M\lambda}$$

- Solution

$$\left(z_2 - z_1\right)\left(1 - \frac{\lambda M}{r + \lambda M}\right) = 2c\left(\tilde{m}_2 - \tilde{m}_1\right)$$

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- If $c = 0$, corner solution $m_2 = M$. Same as equilibrium!

Costly redeployment

- Conditions derived before:

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- Excessive entry to hot areas for two reasons:
  1. Option value effect
  2. Negative switching cost externality (mg cost of redeployment $>$ average cost)

Continuum of research lines

- Competitive allocation $m(t,z)$ optimal $\tilde{m}(t,z)$
- Increasing sequences
- Resource constraint binds up to a point and

$$\begin{aligned} \tilde{m}'(z) &= \left(\frac{1 - v_z(t,z)}{2c}\right) \\ m'(z) &= 1/c \end{aligned}$$

- The same 2 effects

Steady state analysis

- Exogenous arrival rate of problems to solve α with distribution F and density f
- Stationary allocation $m(z)$ for $z \geq z_0$
- Invariant distribution $G(z)$ with density $g(z)$ on $z \geq z_0$

Comparison: allocation

- Both support in $[z_0, z_{max}]$
- $m(z_0) = \tilde{m}(z_0)$
- $m(z) > \tilde{m}(z)$ for $z > z_0$
- $g(z) = \frac{\alpha}{\lambda} f(z) / m(z)$ for all z
- Social planner "leaves" more good opportunities for future exploitation

Welfare compared

- Limiting result:

$$\lim_{c \downarrow 0} \frac{V^{opt}}{V^{comp}} = \frac{E\left(z|z \geq z_0\right)}{z_0}$$

- Quantitative:
  - F lognormal with mean 7 and SD 1.5
  - Consistent with (Pakes)/ Schankerman's estimates
- Ratio for $c = 1$ million $= 3.6$
- Ratio as $c \downarrow 0$ approximately 6!

Final remarks

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