The Direction of Innovation

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Abstract

Research on the efficiency of innovation markets is usually concerned on whether the level of R&D firm investment is socially optimal. Instead, this paper studies whether R&D resources are employed optimally across research areas. Under weak assumptions, we find that competitive equilibrium innovative efforts are biased excessively into high returns areas. This form of market inefficiency is an entirely novel result, and it would take place even if innovators' profits coincided with the social value of innovations. The logic behind our result is simple: firms who engage in R&D races do not internalize the reduction in expected returns borne by competing innovators; and while this is true in all research lines, this negative externality is larger in high return R&D areas. We first demonstrate this finding in a simple, fundamental model. Then we embed our analysis in a canonical dynamic framework directly comparable with extant R&D models, and precisely identify the features of R&D competition that lead to the market failure we identify.

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1 Introduction

How do competing R&D firms and researchers choose their lines of research? Will they disproportionately engage in 'hot' R&D areas and research lines? Or will they shun away from congested R&D areas, each seeking its own special niche? Dating back to at least Schumpeter (1911), there is a large literature on innovation markets, welfare assessment of R&D races, and R&D policy design. Most research asks whether market R&D investment levels are socially optimal, and how to design policies that ensure optimal investment levels.[1] This paper asks a novel, important question: Are market R&D investments allocated optimally across R&D areas? Does R&D go in the 'right' direction? In a simple, general model, we find that R&D competition pushes firms to disproportionately engage in hot R&D lines, characterized by higher expected rates of return. The identification of this form of market failure is an entirely novel contribution to the study of innovation markets.

This result follows from these basic arguments. First, note that the function relating the number of researchers engaged in a R&D line with the probability of innovation discovery can be understood as a cumulative distribution function. Under a simple regularity assumption satisfied by many common statistical distributions, the marginal increment in the innovation probability increases less fast than the average probability of innovation of per-researcher, as researchers are added to the R&D line. Both the optimal and the competitive allocations of researchers across R&D lines are such that more engage in R&D lines with higher expected rates of returns. The optimal allocation equalizes across R&D lines the marginal probability to achieve innovations, multiplied by the marginal value of these innovations to consumers. Instead, competing firms invest in different R&D lines so as to equalize the average probability of innovation (again, multiplied by the innnovations' marginal consumers' value). Thus, the equilibrium allocation of researchers across R&D lines is suboptimal. Too many are engaged in the 'hot' R&D lines with higher prospects, whereas too few engage in less promising lines.[2]

[1] For a nice survey on the literature on IP policy design and enforcement, see Rockett (2010).

[2] Summarizing this line of reasoning: R&D firms who engage in R&D races do not internalize the reduction in expected returns borne on other potential innovators; and while this is true in all research lines, this negative externality is larger in high return R&D lines.

As anticipated earlier, this form of market failure is novel in the study of R&D.[3] Because our results are derived within models in which the social values of innovations coincide with the values appropriated by the innovators, the market inefficiency we identify is orthogonal to well understood inefficiencies that arise because of imperfect property rights of innovations returns.[4] Further, R&D firms are perfectly competitive in our model, and have access to perfect financial markets; so that the inefficiencies we identify cannot be ascribed to the Schumpeterian tradition. While our analysis covers also the case of sequential innovation vintages, our results are also orthogonal to the much studied inefficiencies caused by incomplete appropriability of the positive externalities borne by an innovation on follow-on innovations.[5] Likewise, and more subtly, it is important to underline that our findings hold also in models that abstract from the possibility of complementarities or substitutabilities among innovations or R&D approaches towards the same innovation.[6] Importantly, this clarifies the novelty of our findings relative to earlier work on the allocation of R&D investment across R&D approaches or R&D lines. Finally, we note that our analysis is staged in a framework with complete information. It is well known that asymmetric information may lead to herding, and hence we expect that it would make our results hold a fortiori.[7]

Once established our main result that hot R&D lines are overexploited within the frame of simple and fundamental model, we continue the analysis by enriching the framework, and by building a model with canonical assumptions about innovation discovery and R&D races.

[3]We discuss the state of the art on previously identified sources of market inefficiency in the R&D sector in the literature review (section 2).

[4]The social value of an innovation may exceed its private value because of innovation externalities, technological spillovers, and imitation or reverse engineering by competitors. On the other hand, the social value of an innovation can be lower than its private value, in the case it replaces a product previously on the market. It is not clear whether the difference between social and private value should be larger for hot innovations, or for less promising research. Of course, if the social value of an innovation is larger than its private value, it may attract more competing firms than it is socially optimal.

[5]Suppose that, without a first innovation, the idea for an improvement cannot exist. Then, there is a positive externality running from the first innovation to the second. This externality need not be internalized if the follow-on innovator is distinct from the first innovator.

[6]We extend our basic model to include this possibility in Appendix A, and show that our result that R&D firms overinvest in hot areas extend also to this case.

[7]Studies on herding and multi-armed bandits in information economics (e.g., Banerjee, 1992, Bikhchandani et al., 1992, Bolton and Harris, 1999, Smith and Sørensen, 2000, Keller et al., 2005, Rosenberg et al. 2007, Murto and Välimäki, 2009) do not directly address R&D races, as they abstract from competition, which is a key feature of R&D (exceptions are Moscarini and Squintani, 2010, and Halac, Kartik and Liu, 2015).

Our framework is sufficiently flexible and detailed to generate quantitative predictions, and can be taken to the data. Further, its canonical features allow an immediate comparison with many of the models in the literature on R&D. So, we can clearly identify the precise features of R&D competion that lead to our market inefficiency results.

We assume that there is a large number (ideally, a continuum) of R&D lines with a continuum of innovation values. Each researcher engaged in a R&D line is equally likely to discover the innovation, each innovation discovery time is an independent random event, with time-constant arrival rate. We first assume that arrival rates are constant across innovations, and that researchers cannot be moved across R&D lines over time —their allocation is chosen at time zero. We confirm that firms overinvest in hot R&D lines also within this 'canonical' model. To demonstrate that this framework easily leads to quantitative predictions we calculate the equilibrium welfare loss relative to first best, stipulating a specific functional form for the innovation value distribution. Specifically, when the innovation values are Pareto distributed of parameter η, the welfare loss can be significant when η is between 1.25 and 2.5.[8]

We then extend our canonical dynamic model in subsequent rounds, to incorporate several features of interest. We first allow for the possibility that the arrival rates of innovations differ across R&D lines. In this case, we show, our finding that R&D firms overinvest in hot areas still hold, with the simple and obvious modification that the attractiveness of a R&D line is not determined only by the expected market value of its innovations, but it depends also on the expected feasibility of these innovations.

Then, we turn to a more involved modification of our canonical model. We lift the assumption that researchers cannot ever be redeployed across R&D lines, and allow for their costly redeployment at any point in time. Due to the stationarity of the problem, we find that R&D firms never move researchers away from a R&D line, unless the line is exhausted as a consequence of innovation discoveries. So, we can approach again the problem using standard dynamic programming techniques and confirm our market inefficiency result that

[8]The welfare loss is negligible when η is close to its lower extreme, 1, but that it quickly increase as η grows, so as to reach its maximum of about 16%, for η close to 1.34, to then slowly decrease and disappear asymptotically as $\eta \to \infty$.

R&D firms overinvest in hot R&D lines (as long as the redeployment cost of researchers is not too small).

The formulation of this canonical dynamic model with redeployment of researchers allows also to identify a further source for the market inefficiency result that is the theme of this paper. We show that R&D firms over-invest in hot R&D lines even if researcher redeployment is costless, as long as there are duplicative efforts in R&D, modelled here by assuming that the aggregate arrival rate of one innovation discovery grows less than linearly with the number of engaged researchers.[9] Duplication costs are a well-established fact in the R&D literature, at least since Loury (1979) and Reinganum (1982). They have been identified as a force leading to socially excessive investment in experimentation, thus acting in the opposite direction of 'classical' sources of market inefficiency (e.g., imperfect property rights of innovations returns, and imperfect access to financing). Here, instead, we show that duplication costs reinforce this novel form of market inefficiency uncovered here, that firms overinvest in hot R&D areas.

We conclude the analysis by returning to the case without duplicative efforts (but with costly redeployment of researchers), to introduce the possibility of successive innovation vintages. We suppose that novel profitable R&D lines arise over time and replace exhausted R&D lines that already led to the discovery of profitable innovations. We focus on R&D line replacement that keep the economy in steady state. This formulation allows us to calculate in closed form the functions that describe the allocation of researchers across R&D lines in the market equilibrium, and in the first best. Again, we confirm that R&D firms overinvest in hot R&D lines also in the steady state version of our canonical dynamic model with costly redeployment of researchers. Furthermore, here, this result holds also when the redeployment cost is negligible. This is because both the equilibrium and the optimal welfare turn out to be independent of the redeployment cost.[10] These closed-form

[9]In the process of achieving a patentable innovation, competing firms often need to go through the same intermediate steps, and this occurs independently of each other firms intermediate results, which are jealously kept secret by the competing firms. As a result, the arrival rate of an innovation may increase non-linearly in the number of engaged researchers.

[10]As we will later show, this a consequence of the fact that both the equilibrium and optimal distribution of discovered innovation values need to match the same distribution of novel R&D lines in steady state.

expressions make welfare assessments simple and precise in 'mature' R&D industries for which it is appropriate to assume a stationary distribution of undiscovered innovations, through the steady emergence of innovation opportunities in successive vintages.

The implications of our study in terms of policy are transparent. Because competing firms overinvest in hot R&D lines in equilibrium, non-market based incentives that rebalance remuneration across R&D lines should be devised, so as to subsidize R&D lines with less profitable or less feasible innovations.[11] And, as argued below, both innovation subsidies and the upfront remuneration of ongoing research may be important, here. We discuss the literature on the subsidization of R&D in section 2, whereas in the conclusive section 5 we try to assess how existing funding mechanisms would plausibly fare vis-a-vis the form of market inefficiency identifid in this paper. Here, it suffices to mention that these mechanisms include fiscal incentives on innovations or ongoing research, research prizes, procurement (often for military purposes, but also for large civilian projects like public transport systems), and research grants, among these different mechanisms. While often State-funded, R&D subsidization can also be funded by private consortia or donors (especially, when taking the form of research prizes and grants). And the tenure system in academic institutions such as Universities and research Institutes also entails R&D subsidization.[12] Because subsidies can be at least partially funded with levies collected on patent monopoly profits, the policy intervention advocated here contain elements of cross-subsidization across R&D areas.

We conclude this introduction by underlining one important feature of the market inefficiency problem uncovered in this paper: that it cannot be solved by competition and

[11]Reasonably, subsidization should take place at an aggregate research area level. This because the overwhelming majority of individual patents have no market value whatsoever. For a humourous account of the phenomenon of trivial patents, see http://images.businessweek.com/ss/09/04/0408_ridiculous_patents/ According to the then director of public affairs for the U.S. Patent & Trademark Office, "There [were] around 1.5 million patents in effect [in the U.S. in 2005], and of those, maybe 3,000 were commercially viable," see http://www.businessweek.com/stories/2005-11-09/avoiding-the-inventors-lament

[12]To have a sense of the size of these mechanisms, we report the latest (2012) OECD figures on US R&D expenditure: business enterprise spending was aproximately 234,570 million dollars, equal to 59% of total R&D expediture and leading to a tax credit of 11,100 million dollars (4.7% of business expenditure), Government spending was 122,163 million dollars (31% of total R&D expenses), 11,813 million dollars came from academia, 13,092 from non-profit organizations, and 15,070 from foreign sources.

market forces alone. This is because of an institutionally unavoidable missing market constraint. As our welfare analysis makes explicit, the externalities exerted by researchers on competitors within the same R&D race would be internalized by R&D firms, only if markets were capable to fully remunerate R&D investments and effort upfront, instead of only rewarding innovation discovery through the award of patent monopoly profit.[13] But of course, this is impossible because of well understood impediments based on incomplete information, moral hazard, and R&D secrecy. The source of our finding that R&D firms overinvest in hot areas is that the negative externalities described above are larger for hot R&D lines. So, we conclude that the welfare inefficiency we identify here cannot be solved by competition and market forces alone.

The paper is presented as follows. The related literature is discussed in the nest section. The following one, section 3 establishes our main results within the frame of simple and fundamental model. We continue the analysis in section 4 by enriching the framework so as to build a dynamic model with canonical assumptions that we extend in several directions, so as to identify the precise features of R&D competion that lead to our market inefficiency results. Section 5 concludes, and the least informative part of the formal analysis is in appendix.

2 Literature Review

The form of market inefficiency identified in this paper is novel. We now briefly discuss sources of market failures previously singled out in the R&D literature and policy remedies proposed to alleviate them. Later in the section, literature on the existing mechanisms for the funding of R&D is briefly reviewed.

Early literature (e.g., Schumpeter, 1911; Arrow, 1962; and Nelson, 1959) pointed at limited appropriability of innovation social value by innovators and at limited access to

[13] By the Coase Theorem (Coase, 1960) first best would be achieved by market competitive equilibrium, if markets were completed so that researchers could frictionlessly trade their R&D efforts on these markets, as they exert efforts and achieve intermediate findings.

finances as the main distorting forces in R&D markets. Both these features lead to the implication that market investment in R&D is insufficient relative to first best, and appear to be of first order importance, empirically. Since the classical work of Mansfield et al. (1977) estimates of the social return of innovations calculate that they may be twice as large as the private return to the innovator; whereas evidence for a "funding gap" for investment innovation has been documented, for example, by Hall and Lerner (2010), especially in countries where public equity markets for venture capitalist exit are not highly developed.

A large academic literature has developed to provide policy remedies. The usual prescriptions advocate strong innovation protection rights,[14] and the subsidization of R&D. Wright (1983) compares patents, prizes, and procurement as three alternative mechanisms to support R&D. Patents have the advantage that they delegate R&D investment decisions to the 'informed parties,' R&D firms, and provide incentives so that they conduct R&D efficiently. But prizes and subsidies could sometimes dominate patents, as they do not induce a monopoly welfare loss. Instead of awarding monopolistic IP through patents, the state could award a prize equal to monopoly profit, place the innovation in the public domain and obtain a welfare gain. To obviate the obvious drawback that the State would hardly know the value of monopoly profit at the time of invention, Kremer (1998) suggests an ingenious mechanism based on the idea of patent buyout.[15]

In a similar vein, Cornelli and Schankerman (1999) show that the optimal direct mechanism to reward R&D firms whose productivity is private information can be implemented by using either an upfront menu of patent lengths and fees or a renewal fee scheme. Scotchmer (1999) shows that a similar mechanism would also be optimal when the economy has a single firm innovating once and where the cost and value of the innovation are privately known by the firm. Hopenhayn and Mitchell (2001) show that if innovations differ both

[14]However, dissenting opinions such as Boldrin and Levine (2007) underline the negative effects of patents on social welfare through monopoly pricing, and on the incentives for future innovations, even provocatively challenging the views that patents are needed to remunerate R&D activity.

[15]After the innovator exercised full patent rights for a short period of time, the patent is sold with a second price auction. With small probability, the winner of the auction is awarded the patent; else, the innovation is placed in the public domain. In either case, the innovator is paid the highest bid in the auction as a reward for the innovation. The idea is that, at the time of the auction, the industry has had the time to form an assessment of the market value of the innovation, and this assessment can be elicited by the planner through the auction mechanism.

in terms of expected returns and 'fertility' (more fertile innovations generate follow-ons—developed by other firms—that replace the original innovation in a short time span), then the optimal contract involves a menu of length and breadth offered to different types of innovation.

One source of market inefficiency that has received much attention in the literature is due to the sequential, cumulative nature of innovations. As identified by Horstman et al. (1985) and Scotchmer (1991) the problem arises when, without a 'first' innovation, the idea for 'follow-on' innovations cannot exist. There is a positive externality running from the first innovation to the second, and this innovation need not be internalized if the follow-on innovators are distinct from the first innovator. This problem is especially significant when the first innovation is a 'basic research' finding with little market value, and the follow-on innovations are highly-profitable applications. And as well as creating this externality problem, the cumulative nature of innovations can also induce distortions on the timing of innovation disclosure (see, for example, Matutes et al., 1996, and Hopenhayn and Squintani, 2015).

A large academic literature has developed to study optimal patent length and breadth for this 'cumulative innovation case' (e.g., Green and Scotchmer, 1995, Scotchmer, 1996, O'Donoghue et al., 1998, O'Donoghue, 1998, Denicoló, 2000). Summarizing their conclusions, there appears to be a strong argument for protection from literal imitation (large lagging breadth), if licensing is fully flexible and efficient, then a strong argument for leading breadth can also be made, whereas strong patentability requirements receive some support when licensing does not function well.

There is also a vast literature advocating subsidization of 'basic' research, starting at least as early as Nelson (1959) and Arrow (1962), by means of various mechanisms including research prizes or grants, and academic research funding.[16] In terms of mechanism design, Hopenhayn et al. (2006) study a quality ladder model of cumulative innovations and find the optimal mechanism to be a mandatory buyout system. The buyout takes the form

[16] Aghion, Dewatripont, Stein (2008) provide a different account of why it can be socially optimal to have basic research be subsidized in academia. By serving as a precommitment mechanism that allows scientists to freely pursue their own interests, academia can be indispensable for early-stage research.

of a transfer fee, function of innovation quality and paid to the granting authority, and a payment to the current market leader for its displacement, as well as a specified buyout amount that the new leader would accept to be displaced by another.

The issue of innovation spillovers may complicate policy design not only among sequential innovations, but because of 'horizontal' market value complementarities or substitutabilities among innovations, as pointed out by Cardon and Sasaki (1998) and Lemley and Shapiro (2007), for example. This possibility is entirely distinct from the market inefficiency we identify in this paper: our results hold also in the case of innovations whose market values are independent of each other.[17]

A possible policy remedy to inefficiencies caused by horizontal spillovers are patent pools: agreements among patent owners to license a set of their patents to one another or to third parties. These arrangements have existed since, at least, the 1856 sewing machine pool. A recent legal doctrine is that only 'essential patents' be included in pools: first, the patents included in the pool must be complements; second, patents in the pool must not have close substitutes outside the pool, so as not to foreclose competing patents outside the pools.[18] Lerner and Tirole (2004) build a tractable model of a patent pool, and identify a simple condition to establish whether patent pools are welfare enhancing.[19]

We devote the second part of this section to briefly reviewing research on existing mechanisms for the funding of R&D.

The main form of State intervention to increase the level of innovation is funding through grants and direct subsidies. Interestingly, grants are a relative modern invention: for most

[17]Even further distantly related to our work, there is also a literature studying how complementarities and substitutabilities among different research approaches to achieve the same innovation influence welfare in R&D races (e.g., Bhattacharya and Mookherjee, 1986; Dasgupta and Maskin, 1987). Of course, this is very different from the analysis of this paper, which considers several innovations, without distinguishing different approaches to achieve any of them.

[18]Indeed, Shapiro (2001) finds patent pools raise welfare when patents are perfect complements and harm welfare when they are perfect substitutes.

[19]They note that a member of the pool who considers whether to individually license its innovation may be constrained by either a 'competition margin' or a 'demand margin.' The demand margin binds if the licensor could individually raise its license price without triggering an exclusion from the patents selected by the licensees. They find that patent pools always increase welfare when the demand margin binds in the absence of pool, and that, as patents become more substitutable, the competition margin more likely binds and the pool more likely decreases welfare.

of history, publicly sponsored research was in-house.[20] Before WWII, R&D funding by the U.S. federal government was a small percentage of total R&D. The federal government became a primary source of R&D funds during the war, of which 70% is currently given out as extramural funding. The percentage of total R&D paid for by the federal government has decreased since 1953, to about 31% in 2012, but it is still larger than 1930's figures, which was between 12% and 20%.[21]

The other important form of R&D subsidization are fiscal incentives. The U.S. Research Credit can be used to offset current, prior, and future corporate income tax liability, for 20% of R&D expenditures exceeding a 'base amount', or 14% of the excess of the qualified research expenditures over 50% of the average of the three prior years' expenditures (Alternative Simplified Credit).[22] Similar fiscal incentives are in place in Japan and France, whereas China has an even more generous incentives, including a tax deduction equal to 150% of qualifying R&D expenses, a reduced 15% corporate tax rate for companies granted High and New Technology Enterprise (HNTE) status, and qualified Technology Advanced Service Enterprises in designated cities with over 50% revenue derived from providing qualified technology advanced services outsourced by foreign entities; as well as VAT exemption/Zero-rated treatment for certain R&D services performed for foreign entities.[23] Instead, the German tax system does not include relief for R&D expenditure, and R&D funded almost entirely through grants.

While less quantitatively relevant nowadays, research prizes also play a role in stimulat-

[20]As reported by Maurer and Scotchmer (2004), this dates back at least to ancient Egypt, where the engineer Imhotep was hired for the building of pyramids, and among the ranks of scientists working at courts over the centuries, one can list, for example, Archimedes, Kepler, Brahe, Euler, and Lagrange.

[21]Precise latest (2012) OECD figures for US R&D expenditure are reported in footnote 12. The OECD reported percentage of total R&D paid for by the State is similar in other leading economies such as China (21%, in 2013), Japan (17%, in 2013), Germany (29%, in 2012), France (35%, in 2012), and the UK (27%, in 2012).

[22]There are also special credits for basic research (e.g., research conducted in universities), payments to energy research consortium, and research relating to orphan drug.

[23]Likewise, the UK offers the 225% super deduction for R&D expenses for companies with less than 500 employees, with cash credits available for companies in loss position (for up to 24.75% of the qualifying expenditure), and 130% super deduction for larger companies, with 10% taxable credit, for larger companies. These figures are reported in the "2014 Global Survey of R&D Tax Incentives", published by Deloitte T.T. Ltd.

ing R&D activity.[24] For example, aerospatial research has been fostered partly by the 'X Prize Foundation' established in 1996 with a $10 million prize for the first private firm to carry three passengers to a suborbital height of 100 km twice within a fortnight. The development of CFC-free refridgerators was fostered by the 'Super Efficient Refrigerator Prize' of $30m announced in 1992, the first initiative of the "Golden Carrot" awards, sponsored by the U.S. Environmental Protection Agency, in partnership with non-profit companies, utilities and environmental groups.[25] One further form of R&D State funding is contract research, or procurement. This mechanism is most common for military innovations, and often takes the form of prototype competition.[26]

One of the most common type of research paid upfront takes place in the universities.[27] Academia's distinctive advantage as a R&D institution is that it motivates scientists by allowing them to freely pursue their own research interests. These motivations have been recently complemented with market-based incentives, through legislation such as the Bayh-Dole and Stevenson-Wydler Acts of 1980, which authorize the patenting of innovations developed with federal funds in universities and national laboratories, and the Federal Technology Transfer Act of 1986, that lets State universities and national laboratories create partnerships with private entities, called CRADAs, and thus spin off their innovations into the private sector. These reforms led to substantial expansion of patents for university performed research (see, for example, Lach and Schankerman, 2004), and to the relative increase of industry support for university research.[28]

[24]Earlier R&D prizes include the award offered by the French Empire in 1795 for a means to preserve food to feed Napoleon's armies and navy, and awarded in 1810 to Nicolas Appert. (His technique, based on heat-sterilization of food packed in bottles, is still in use). Other targeted prizes led to improvements of the steam engine, to the first water turbine, and to the precise determination of longitude on ships.

[25]The contest was won by Whirlpool. But the prize was not paid because sales fell about 30% short of target required in the SERP call.

[26]For example, in the 1970's the US Air Force adopted a system, which led to the F-16 and F-18 fighter jets, where two rival companies received contracts to build prototypes followed by a flight competition to demonstrate quality. Earlier, the Air Force usually acquired prototypes from a single vendor after a contest to choose the best written proposal; that procedure was abandoned as it led to large cost overruns in the 1960's (see, Maurer and Scotchmer, 2004).

[27]Scholarly research by tenured academics has a long history, dating back to the Library of Alexandria, which supported resident scholars such as Archimedes, Hipparchos, Eratosthenes, Euclid, and Hero of Alexandria. Inventors as famous as Galileo, Newton, Dalton, Volta, Ampere and Maxwell all worked as university residents, instead of in response to project-specific incentives.

[28]Jaffe (1989) finds a significant effect of university research on corporate patents through 'geographical spillovers', particularly in the areas of Drugs and Medical Technology, and Electronics, Optics, and Nuclear

The economic assessment of the mechanisms of R&D funding we have described depends on whether they succeed in stimulating R&D that would have not otherwise taken place, or whether they crowd out private investment in innovation markets. The evidence is not always favorable. For example, Wallsten (2000) estimates a simultaneous model of expenditure and funding for a sample of U.S. firms recipient of grants through the Small Business Innovation Research (SBIR) program. Controlling for the endogeneity of grants, he finds no evidence of R&D investment increase and evidence of full crowding-out of private investment.

In contrast, Lach (2002) estimates the relative increase in R&D expenditures of subsidized versus nonsubsidized firms using panel data on a sample of Israeli companies, and finds that this effect is present for small firms, whereas it fades in the larger firms. Likewise, González, Jaumandreu and Pazó (2005) structurally estimate the effects of R&D subsidies on the firms' (extensive) decisions about whether or not to perform R&D in a dataset of Spanish manufacturing firms. They find no evidence of crowding out of private funds, and evidence that subsidies stimulate R&D, and that some firms would stop R&D activity in their absence. Similarly, Bloom, Griffith and Van Reenen (2000) estimate a model of R&D investment using a panel of data on tax changes and R&D spending in nine OECD countries over a 19-year period (1979–1997). They find evidence that tax incentives are effective in increasing R&D intensity, even after allowing for permanent country-specific characteristics, world macro shocks and other policy influences. Specifically, estimate that a 10% fall in the cost of R&D stimulates slightly over a 1% rise in the level of R&D in the short-run, and just under a 10% rise in R&D in the long-run.

To conclude this brief review, we mention recent work by Takalo, Tanayama and Toivanen (2013).[29] To assess welfare effects of targeted R&D subsidies, they formulate a model

Technology. A study by Mansfield (1995) on data from 66 firms in 7 manufacturing industries and from over 200 academic researchers finds that a substantial proportion of innovations in high-technology industries have been based directly on recent academic research. The extent to which a university is credited by firms with making major contributions to these firms' innovations is related to the quality of the relevant university's academics, to the size of its R&D expenditures in relevant fields, and to the proportion of the industry's members located nearby.

[29]The literature on R&D subsidization is very large; for reasons of space, our review is by necessity incomplete.

that includes the application and R&D investment decisions of firms and the subsidy-granting decision of the public granting agency. Their structural estimation, based on unusually detailed project-level data from Finland, assesses the social rate of return on targeted subsidies between 30% to 50%.[30]

3 A Simple Model

There are two research lines $j = 1, 2$, with one potential discovery each. A continuum of $M > 0$ researchers needs to be allocated to either research line. Each research line j delivers a value z_j to the successful innovator, upon discovery of an innovation. To isolate the findings of this paper from other effects well-known in the literature, we assume that the social values of innovations coincide with the private values z_1 and z_2. Without loss of generality, we assume that $z_2 > z_1 (> 0)$; so, the 'hot R&D line' is line 2.

The discovery technology is as follows. When a mass of researchers m_j participates in the patent race for innovation j, the discovery of j occurs with probability $P(m_j)$, increasing in m_j. We assume that the function P is twice jdifferentiable, strictly increasing and concave, i.e., that $P' > 0$ and $P'' < 0$; with $P(0) = 0$. When a larger number of researchers are engaged in an R&D line, the overall probability of discovery increases, but there are diminishing returns. Note that these assumptions imply that $P(m)/m$ strictly decreases in m.[31]

For each R&D line $j = 1, 2$, each individual researcher's expected private value of engaging in the R&D line j is $z_j P(m_j)/m_j$, when m_j researchers are engaged in line j, because each one of them is equally likely to win the R&D race. To simplify the exposition, we assume that the two innovations $j = 1, 2$ enter additively in the social welfare function W, so that $W(m_1, m_2) = z_1 P(m_1) + z_2 P(m_2)$.[32]

[30] Further, they find that large firms generate a larger spillover rate, as do technically more challenging projects. Firms with higher-value-added current production have higher marginal returns to R&D and higher application costs. Profitability and application cost shocks are positively related, implying that firms do not apply for subsidies for the privately most profitable projects.

[31] In fact, the derivative of $P(m)/m$ is proportional to $P'(m)m - P(m)$ which equals $P'(m)m - [P(m) - P(0)]$, strictly smaller than zero because P is strictly concave and $P(0) = 0$.

[32] This assumption is not needed to show our results here. In appendix A, we generalize Proposition 1,

We focus on interior equilibria (m_1, m_2), so that $0 < \{m_1, m_2\} < M$. It is immediate that an interior allocation (m_1, m_2) is an equilibrium if and only if $m_1 + m_2 = M$ and the following 'no-arbitrage condition' holds:

$$z_1 P(m_1)/m_1 = z_2 P(m_2)/m_2. \tag{1}$$

In an interior equilibrium (m_1, m_2), the expected private value of researching line j is the same across $j = 1, 2$.[33] Further, because $P(m)/m$ decreases in m, it must be the case that $m_1 < m_2$ in equilibrium.

In order to assess the equilibrium welfare properties, we now determine the optimal allocation. A social planner chooses $\tilde{m}_1$ and $\tilde{m}_2$ so as to maximize the welfare function $W(\tilde{m}_1, \tilde{m}_2) = z_1 P(\tilde{m}_1) + z_2 P(\tilde{m}_2)$ subject to the condition $\tilde{m}_1 + \tilde{m}_2 = M$. Equating the first-order conditions, at an interior solution:

$$z_1 P'(\tilde{m}_1) = z_2 P'(\tilde{m}_2). \tag{2}$$

Because P is strictly concave, it follows that $\tilde{m}_1 < \tilde{m}_2$.

The comparison of the equilibrium and social planner solutions hinges on the following regularity condition on the success probability function $P(\cdot)$. First, note that, because $P(m)$ increases in m and equals to zero for $m = 0$, it can be interpreted as a cumulative distribution function, and $P'(m)$ can be interpreted as the associated density function. Hence, the function $\Gamma(m) = mP'(m)/P(m)$ is the *generalized reverse hazard rate* of the distribution P.

The key regularity condition we impose is that $\Gamma(m)$ strictly decreases in m.[34] As shown by the online appendix of Che, Dessein and Kartik (2013), this is a weak assumption. It is satisfied by many common statistical distributions with support in the positive reals (e.g., the exponential distribution, the Pareto distribution, the power function distribution, the Weibull distribution, and the Gamma distribution of parameter bigger than 1).

stated below, to any utility and welfare specification.

[33] An interior equilibrium exists as long as z_1/z_2 is not too small relative to $P(M)/M$. A sufficient condition is that $\lim_{m \downarrow 0} P(m)/m = \infty$.

[34] Note that the earlier assumption that $P(m)/m$ decreases in m implies that $\partial P(m)/\partial m \propto mP'(m) - P(m) \leq 0$, so that $\Gamma(m) = mP'(m)/P(m) \leq 1$.

Under this condition, we show that firms over-invest in the hot R&D line 2, relative to the optimal solution $(\tilde{m}_1, \tilde{m}_2)$. In other terms, we prove that $m_2 > \tilde{m}_2 > \tilde{m}_1 > m_1$.

Proposition 1 *Suppose that the innovation's success probability $P(m)$ is increasing and concave, and the associated generalized reverse hazard rate $\Gamma(m) = mP'(m)/P(m)$ strictly decreases in m. In equilibrium, R&D firms overinvest in the hot R&D line 2, relative to the optimal allocation of researchers; i.e., $m_2 > \tilde{m}_2 > \tilde{m}_1 > m_1$.*

Proof. Dividing the equilibrium no-arbitrage condition (1) by the optimal solution condition (2), we obtain:

$$\frac{P(m_1)/m_1}{P'(\tilde{m}_1)} = \frac{P(m_2)/m_2}{P'(\tilde{m}_2)}. \tag{3}$$

Now suppose by contradiction that $m_2 \leq \tilde{m}_2$, so that $m_1 \geq \tilde{m}_1$. Then, using the concavity of P and the assumption that the associated generalized reverse hazard rate $\Gamma(m) = mP'(m)/P(m)$ strictly decreases in m, we obtain:

$$\frac{P(m_2)/m_2}{P'(\tilde{m}_2)} > \frac{P(m_2)/m_2}{P'(m_2)} > \frac{P(m_1)/m_1}{P'(m_1)} > \frac{P(m_1)/m_1}{P'(\tilde{m}_1)}, \tag{4}$$

contradicting the equality (3) above. ■

Proposition 1 states the novel market inefficiency identified in this paper: under reasonable conditions on the probability of success function P, the equilibrium direction of innovation is inefficient, as R&D firms over-invest in hot R&D lines. For clarity, this result is shown here in a simple model that can be easily exposited and explained. The next section of this paper shows how to use this simple model to build a fully fledged dynamic model, with canonical assumptions about innovation discovery and R&D races.

4 Extended Analysis

For clarity of exposition, the model presented in the previous section is very streamlined. We now enrich that framework, and build a model with canonical assumptions about innovation discovery and R&D races. We then extend the model in subsequent rounds, to incorporate further and further features of interest. Our framework is sufficiently flexible and detailed to generate quantitative predictions, and can be easily taken to the data. Further, its canonical features allow an immediate comparison with many of the models in the literature on R&D.

A Canonical Dynamic Model We begin by extending the model of section 3 to allow for a large number (in fact, a continuuum) of R&D lines, with innovation values $z \geq 0$. We say that the innovation value is distributed according to the cumulative distribution function F, which we assume to be twice differentiable. Again, there is a mass $M > 0$ of researchers or R&D firms, who allocate to the different R&D lines according to a measurable function m. Specifically, for each innovation of value z, we denote by $m(z)$ the mass of researchers competing for the discovery of that innovation. Hence, the resource constraint $\int_0^\infty m(z)\, dF(z) \leq M$ needs to be satisfied. The probability that an innovation of value z is discovered is denoted by $P(m(z))$, again, assumed increasing in $m(z)$.

The value of participating in the R&D race for each innovation of value z is now $U_z(m(z)) = zP(m(z))$. Maintaining the earlier additivity assumption, the welfare associated with allocation m is expressed as $W(m) = \int_0^\infty zP(m(z))\, dF(z)$. As in the simpler the model of section 3, the equilibrium and optimality conditions imply that the quantity $\frac{P(m(z))}{m(z)} / P'(\tilde{m}(z))$ is constant over z. And, again, when the generalized reverse hazard rate $\Gamma(m) = mP'(m)/P(m)$ strictly decreases in m, competing firms overinvest in hot R&D lines; i.e., formally, there exists a threshold $\bar{z}$ such that $m(z) < \tilde{m}(z)$ for $z < \bar{z}$ and $m(z) > \tilde{m}(z)$ for $z > \bar{z}$.[35] Further, because the allocation functions m and $\tilde{m}$ cross only once and need to satisfy the same resource contraint, it is also the case that the smallest active R&D line innovation value is higher in equilibrium than in the first best; i.e., $\tilde{z}_0 = \inf_z\{\tilde{m}(z) > 0\} \leq z_0 = \inf_z\{m(z) > 0\}$.

We now introduce explicit dynamics in the model, momentarily assuming that the allocation of researchers is chosen at time zero, and they cannot be redeployed across R&D lines afterwards. Given the R&D allocation function m, we express the expected value for

[35] The proof of this result is entirely analogous to the proof of Proposition 1, and so it is omitted.

engaging in R&D lines of innovation value z as:

$$U(z;m) = \int_0^\infty z e^{-rt} p(t, m(z))\, dt, \tag{5}$$

where r is the discount rate, and $p(t, m(z))$ is the density of the discovery of any innovation of value z at time t, given that a mass $m(z)$ of researchers investigates on each innovation of value z. We assume that the cumulative distribution function associated with the density $p(t, m(z))$ increases in $m(z)$ in first-order stochastic sense. The formulation (5) can be subsumed into our earlier extended model, simply by redefining the function P as $P(m(z)) = \int_0^\infty e^{-rt} p(t, m(z))\, dt,$ for all $m(z) \geq 0$.[36] As a result, we can again express the value of each R&D line of innovation value z as $U(z; m(z)) = zP(m(z))$.

Following the same arguments that led to Proposition 1, we obtain again that R&D firms overinvest in the hot R&D lines, in equilibrium, when $P(\hat{m})$ is concave, and the generalized reverse hazard rate $\Gamma(\hat{m}) = \hat{m}P'(\hat{m})/P(\hat{m})$ strictly decreases in $\hat{m}$.

Most importantly, these conditions on $P(\hat{m})$ and $\Gamma(\hat{m})$ hold whenever the density $p(t, \hat{m})$ takes the canonical exponential form $p(t, \hat{m}) = \hat{m}\lambda e^{-\hat{m}\lambda t}$.[37] This expression represents instances in which each innovation's random discovery time is independent across innovations, and the arrival rate of each innovation discovery to each researcher engaged in every R&D line is time-invariant and equal to λ.

The following result summarize the analysis conducted so far in this section.

Corollary 2 *Suppose that there is a continuum of R&D lines, whose innovation discoveries are independent events, and that the arrival rate of each innovation discovery to each*

[36] The expression $P(m(z))$ is not a probability of success, here. Rather it represents the expected discount factor of each innovation of value z. Evidently, as $m(z)$ increases, the discount factor increases.

[37] With simple manipulations, in fact, we see that

$$P(\hat{m}) = \int_0^\infty e^{-rt} \hat{m}\lambda e^{-\hat{m}\lambda t} dt = \frac{\hat{m}\lambda}{r + \hat{m}\lambda},$$

which is increasing and concave in $\hat{m}$, and the generalized reverse hazard rate takes the form

$$\Gamma(\hat{m}) = \frac{\hat{m}P'(\hat{m})}{P(\hat{m})} = \frac{\lambda r}{(r + \hat{m}\lambda)^2} / \frac{\lambda}{r + \hat{m}\lambda} = \frac{r}{r + \hat{m}\lambda},$$

which strictly decreases in $\hat{m}$.

researcher is constant in time and equal to λ *in every R&D line. Then, in equilibrium, R&D firms overinvest in the hot R&D lines relative to the optimal allocation of researchers: there exists a threshold* $\bar{z}$ *such that* $m(z) < \tilde{m}(z)$ *for* $z < \bar{z}$ *and* $m(z) > \tilde{m}(z)$ *for* $z > \bar{z}$*.*

Importantly, this result demonstrates the novel market inefficiency we identify in this paper (that competing firms overinvest in hot R&D lines), within a canonical dynamic model directly comparable with the many R&D models since Loury (1979) and Reinganum (1981), that are also built on the assumption of exponential arrivals of innovations to researchers.

In the remainder of the section, we work with this canonical formulation to both provide quantitative predictions and to perform a number of variations and robustness exercises. This allows us to clearly assess the quantitative relevance of the market inefficiency result presented in Corollary 2, and to single out the different features of R&D competition that lead to this finding. By doing this, we complement the general condition on the generalized reverse hazard rate Γ, uncovered in the simpler model of section 3.

Quantitative Welfare Predictions As well as being directly related with many existing R&D models, the canonical dynamic model we developed above yields easily computable quantitative predictions that can be taken directly to the data. To demonstrate this feature of our model, we posit a specific statistical law for the innovation value distribution F, and calculate the 'welfare wedge' $W(m)/W(\tilde{m})$, a measure of welfare loss that consists of the equilibrium welfare relative to the first best. Specifically, we suppose that innovation values are distributed according to a Pareto distribution of parameter $\eta > 1$, so that $F(z) = 1 - z^{-\eta}$ for $z \geq 1$. With significant algebraic manipulations reported in appendix B, we derive the following result.

Proposition 3 *Consider the canonical dynamic model developed earlier in this section, and suppose that the innovation values* z *are distributed according to a Pareto distribution* *of parameter $\eta > 1$. Then the welfare wedge is:*

$$\frac{W(m)}{W(\tilde{m})} = \frac{\eta - 1}{\eta} \left(\frac{2\eta - 1}{\eta - 1} \right)^{1/\eta}. \tag{6}$$

The significance of this result lies in the determination of a simple expression for welfare loss. Plotting the welfare-wedge expression (6), we see that the welfare loss is negligible for η close to 1, but that it quickly increase as η grows, so that the welfare loss $1 - W(m)/W(\tilde{m})$ reaches its maximum of about 16% for η close to 1.34 to then slowly decrease and disappear asymptotically as $\eta \to \infty$.dtbpFUX4.4998in3.0004in0ptPlot of the welfare wedge $W(m)/W(\tilde{m})$ calculated in Proposition 3.weweFigure 1

The next part few subsections present variations of the canonical dynamic we developed in the first part of this section, to assess the robustness of our main result that firms overinvest in hot R&D lines (Corollary 2).

Heterogenous Arrival Rates The first robustness exercise we perform on our canonical dynamic model is simple and intuitive: we allow for the possibility that the arrival rates of innovations differ across R&D lines. Letting $\lambda(z)$ be the arrival rate of innovations of value z, we see that, now,

$$P(z, m(z)) = \frac{m(z)\lambda(z)}{r + m(z)\lambda(z)}.$$

The same arguments that lead to Corollary 2 imply that again, R&D firms overinvest in the hot, most attractive, research lines, in equilibrium. Here, however, the evaluation of the attractiveness of an R&D line is not determined by its innovation value z alone, but rather by the expected flow value $z\lambda(z)$ of engaging in the R&D line. As a result, we can reformulate and extend Corollary 2 as follows.

Proposition 4 *Consider the canonical dynamic model in which the discovery arrival rate of innovations of value z is time-constant and equal to $\lambda(z)$. In equilibrium, firms overinvest in the R&D lines with the highest expected flow value $z\lambda(z)$: there exists a threshold ζ such that $m(z) < \tilde{m}(z)$ for $z\lambda(z) < \zeta$ and $m(z) > \tilde{m}(z)$ for $z\lambda(z) > \zeta$.*

This result provides a useful generalization of our finding that competing firms overinvest in hot R&D lines. In most applications, R&D lines differ both in terms of the expected rate of returns and the expected feasibility of innovations. Because of the canonical nature of exponential arrivals, this generalized result can be easily taken to industry datasets. Proposition 4 can be further generalized to broader classes of arrival densities $p(t, z, m(z))$, beyond the canonical exponential class in which $p(t, z, m(z)) = m(z)\lambda(z)e^{-m(z)\lambda(z)t}$, whenever an appropriate parametrization is suitable.

Redeployment of Researchers We now reconsider our canonical dynamic model (with the homogeneous innovation discovery rate λ), and relax the assumption that, after researchers are engaged across R&D lines at time zero, they cannot be redeployed across R&D lines as time goes by. Suppose that, at any point in time, each researcher can be moved across research lines by paying a switching cost $c \geq 0$. To make the exposition more intuitive, we momentarily return to the simple environment of section 3 and consider only two R&D lines, of values z_1 and z_2 with $z_1 < z_2$. We further simplify the exposition by assuming that $c \leq z_1 P'(M)$, and note that the latter term is strictly smaller than $z_1 P(M)/M = z_1\lambda/(r + M\lambda)$ because $P'' < 0$. In words, we assume that the redeployment cost c is smaller than the marginal expected profit for participating in the least valuable race, 1, when all other R&D firms engage in that race.[38]

Due to the stationarity of the problem, it is never the case that researchers move across R&D lines in equilibrium, nor that it is optimal to redeploy them, unless one of the two R&D lines $j = 1, 2$ is exhausted as a consequence of innovation discovery. So, we can approach again the problem using standard dynamic programming techniques. Proceeding backwards, we first suppose that one innovation j has been discovered. At the time of discovery, of course, it is optimal and sequentially rational that all researchers are redeployed to the other R&D line, $k \neq j$. So, we can solve the problem backwards, by first calculating the value $\bar{v}_k$ of researching the innovation line k when all other M researchers

[38]This assumption guarantees that it is always socially optimal (and a fortiori profitable) to redeploy the researchers employed in either of the two R&Ds line $j = 1, 2$, when the innovation j is discovered, to the other R&D line, until both innovations have been discovered.

are also engaged in line k. The value $\bar{v}_k$ is expressed as follows:

$$\bar{v}_k = \int_0^{\infty} e^{-rt}\frac{z_k}{M}\left(M\lambda e^{-M\lambda t}\right)dt = \frac{\lambda}{r+M\lambda}z_k, \tag{7}$$

because the innovation k arrives with aggregate hazard rate $M\lambda$ to one among the mass of M researchers employed, and each of them is the winner of the race with chance $1/M$, thus earning the value z_k.

Turning to the first stage of the game, in which both innovations are yet to be discovered, and denoting again by m_j the mass of researchers engaged in R&D line j, for $j = 1, 2$, we express the equilibrium value $v_j(m_j)$ of engaging in R&D line j through the Belman equation:

$$rv_j(m_j) = m_j\lambda\left[\frac{z_j}{m_j} + \max\{0, \bar{v}_k - c\} - v_j(m_j)\right] + m_k\lambda\left[\bar{v}_j - v_j(m_j)\right]. \tag{8}$$

Using the assumption that $c \le z_1\lambda/(r+M\lambda) = \bar{v}_k$, simplifying equation (8), rearranging it, and using $m_1 + m_2 = M$, we obtain:

$$v_j(m_j) = \lambda\frac{z_j - cm_j + m_1\bar{v}_2 + m_2\bar{v}_1}{r+\lambda M}. \tag{9}$$

Focusing again on interior equilibria as in section 3, we suppose that $0 < \{m_1, m_2\} < M$. Then, again, a no-arbitrage condition holds in equilibrium: it must be that $v_1(m_1) = v_2(m_2)$. Simplifying, we obtain the simple condition:

$$c(m_2 - m_1) = z_2 - z_1. \tag{10}$$

The equilibrium shares m_1 and m_2 are set so that the net expected individual gain $z_2 - z_1$ for engaging in the more valuable line 2 is exactly offset by the additional individual expected switching cost $c(m_2 - m_1)$; note that, here, the probability of losing the R&D race 1 or 2, and paying the redeployment c exactly equals m_1 and m_2.

Let us now turn to determining the optimal allocation $\tilde{\mathbf{m}} = (\tilde{m}_1, m_2)$. When neither innovation $j = 1, 2$ has been discovered, the Belman equation that characterizes the optimal aggregate welfare $W(\tilde{\mathbf{m}})$ is:

$$rW(\tilde{\mathbf{m}}) = \tilde{m}_1\lambda\left[z_1 + \max\{0, \bar{w}_2 - c\tilde{m}_1\} - W(\tilde{\mathbf{m}})\right] + \tilde{m}_2\lambda\left[z_2 + \max\{0, \bar{w}_1 - c\tilde{m}_2\} - W(\tilde{\mathbf{m}})\right] \quad (11)$$

where $\bar{w}_k$ is the social optimal value of redeploying all researchers to line $k \neq j$ after innovation $j = 1, 2$ has been discovered (note that the social value $\bar{w}_k$ equals the private value $\bar{v}_j$ times the mass of employed researchers M).[39]

Highlighting the differences with respect to the equilibrium Belman equation (8), we note the following. First, here, the innovation values z_1 and z_2 are not divided by $\tilde{m}_1$ and $\tilde{m}_2$ respectively, unlike in equation (8). The value of the discovery of innovation j is z_j for the society, regardless of the individual value which depends on the identity of the innovator. Second, the cost borne in the aggregate by switching $\tilde{m}_j$ researchers from R&D line j to line k when innovation j is discovered is $c\tilde{m}_j$, while the analogous individual cost was c in the equilibrium Belman equation (8).

Rearranging the Belman equation (11), and noting that for both $k = 1, 2$, it is the case that $c \leq \bar{w}_k/\tilde{m}_k = \bar{v}_k M/\tilde{m}_k$, as the latter is larger than $\bar{v}_k$, we obtain the following expression for the optimal welfare:

$$W(\tilde{\mathbf{m}}) = \lambda \frac{\tilde{m}_1\left[z_1 + \bar{w}_2 - c\tilde{m}_2\right] + \tilde{m}_2\left[z_2 + \bar{w}_1 - c\tilde{m}_1\right]}{r + \lambda M}.$$

By equating the first-order conditions associated with maximization of $W(\tilde{\mathbf{m}})$ under the constraint $\tilde{m}_1 + \tilde{m}_2 = M$, we obtain the condition:

$$2c(\tilde{m}_2 - \tilde{m}_1) = (z_2 - z_1)\left(1 - \frac{M\lambda}{r + M\lambda}\right), \quad (12)$$

which characterizes the interior solutions $\tilde{\mathbf{m}}$, for which $0 < \{\tilde{m}_1, \tilde{m}_2\} < M$.

By comparing the optimal condition (12) with the no-arbitrage equilibrium condition (10), we see that the social net expected gain for engaging an additional researcher in the hot R&D line 2, $(z_2 - z_1)\left(1 - \frac{M\lambda}{r+M\lambda}\right)$, is smaller than the net individual expected gain $z_2 - z_1$. Eventually, the society achieves both innovations, but engaging an extra researcher in the hot patent race 2 makes it marginally more likely that 2 arrives before than 1. Hence,

[39] In fact, the the social value $\bar{w}_k$ is expressed as $\bar{w}_k = \int_0^\infty e^{-rt} z_k \left(M\lambda e^{-M\lambda t}\right) dt = \frac{\lambda M}{r+M\lambda} z_k = M\bar{v}_j$.

the net benefit $z_2 - z_1$ is multiplied by $1 - \frac{M\lambda}{r+M\lambda}$, which is the expected discount factor associated with the time that lapses between the discovery of the first innovation and that of the second one. This difference induces a 'postponement effect' that pushes towards overinvestment in the hot line 2, in equilibrium.

At the same time, the additional social marginal cost for engaging an additional researcher in the hot line, $2c\left(\tilde{m}_2 - \tilde{m}_1\right)$, is twice the private additional expected cost $c(\tilde{m}_2 - \tilde{m}_1)$ paid by the individual researcher. On top of this private cost, in fact, the society incurs also the extra redeployment cost paid in expectation by all researchers already engaged in the hot line, because the extra researcher increases the probability that each of them loses the hot R&D race 2. This 'redeployment cost externality' also pushes towards equilibrium overinvestment in the hot line 2.

Because the two effects we identified reinforce each others, we obtain again that $m_1 < \tilde{m}_1 < \tilde{m}_2 < m_2$, so that R&D firms overinvest in the more valuable line 2.

However, it is important to notice that, whenever c is small enough, it will be the case that neither the equilibrium and optimal solutions are interior, as can be seen by inspection of no-arbitrage condition (10) and the optimality condition (12). In this case, all researchers will be first engaged in the most valuable R&D line 2, in equilibrium. When innovation 2 is discovered, they will all be redeployed to the less valuable R&D line 1, until also innovation 1 is discovered. Most importantly, this unique equilibrium outcome is also socially optimal.

The following Proposition summarizes the analysis obtained so far in this subsection.

Proposition 5 *In equilibrium of the dynamic model with* 2 *R&D lines and cost c of redeploying researchers across R&D lines, firms overinvest in the hot R&D line* 2 *whenever the cost c is larger than a given threshold $\bar{c} > 0$.*[40] *If $0 < c < \bar{c}$, then all researchers are initially engaged in the hot R&D line* 2, *and when innovation* 2 *is discovered, they are all redeployed to R&D line* 1, *until also that innovation is discovered; and this outcome is socially optimal.*

This result identifies in the switching cost c an important substantive reason for the

[40]For consistency, it must be that $\bar{c} < z_1\lambda / \left(r + M\lambda\right)$. In the appendix, we prove that this is the case as long as $M > \frac{r}{2\lambda}\frac{z_2 - z_1}{z_1}$, which is a not a demanding condition.

market failure we identify in this paper (the finding that competing firms overinvest in hot R&D lines), beyond the mathematical condition on the generalized reverse hazard rate Γ presented in Proposition 3. If moving R&D resources across R&D lines were fully costless, then we would observe all R&D firms always compete on the hottest R&D line, but this would be socially optimal.

We now return to our canonical dynamic model with a continuum of R&D lines. For every innovation of value z and time t, we denote the mass of engaged researchers as $m(t,z)$, and let $z_0(t)$ to be the smallest active R&D line innovation value at time t — precisely, $z_0(t) = \inf_z\{m(t,z) > 0\}$. Let $v(z,t)$ be the equilibrium value for a researcher to engage in a R&D line of innovation value z at t. Here, $v(z,t)$ obeys a Belman equation analogous to equation (8):

$$rv(z,t) = \lambda m(z,t)\left[\frac{z}{m(z,t)} - c\right] + v_t(z,t), \tag{13}$$

where $v_t(z,t)$ denotes the time-derivative of $v(z,t)$.

For any time t, both the equilibrium value $v(z,t)$ and its derivative $v_t(z,t)$ are constant across all active R&D lines.[41] So, the analogue of the no-arbitrage equilibrium condition (10) is here that

$$\lambda[z - cm(z,t)] \tag{14}$$

be constant across all active R&D lines of innovation value $z \geq z_0(t)$.

Differentiating expression (14) with respect to z and equating the derivative to zero, we obtain that[42]

$$m_z(z,t) = 1/c. \tag{15}$$

Integrating this simple differential equation, we obtain the general solution $m(z,t) = [z - z_0(t)]/c$ for $z \geq z_0(t)$. This formula pins down the equilibrium allocation, together with the resource constraint $\int_{z_0(t)}^{\infty} m(z,t)\,dG(t,z) = M$, where $G(t,z)$ denote the

[41]These conditions are akin to value matching and smooth pasting conditions in stopping problems, see, for example, Dixit and Pindyck (1994).

[42]The expression $m_z(z,t)$ denotes the derivative of the allocation $m(z,t)$ with respect to z.

cumulative distribution function of the innovation not discovered yet at time t; so that the initial condition $z_0(t)$ is pinned down by the equation:[43]

$$\int_{z_0(t)}^{\infty} z dG(t,z) - z_0(t)[1 - G(t, z_0(t))] = cM. \tag{16}$$

Further, as we prove in appendix B, the equilibrium comparative statics is such that $z_0'(t) < 0$ and $m_t(z,t) > 0$: over time, there is increasing congestion in every active R&D line, and less valuable R&D lines become active; and that $v_t(z,t) < 0$: the value of each R&D line decreases over time. These results are intuitive. Over time, active R&D lines with innovation value $z \geq z_0(t)$ get exhausted, and this leads to more researchers engaging in the remaining lines, i.e., in $m_t(z,t) > 0$, for $z \geq z_0(t)$, in less valuable lines becoming active, i.e., in $z_0'(t) < 0$, and in each active research line becoming less valuable, i.e. in $v_t(z,t) < 0$.

The value $v(z,t)$ decreases over time until matching and smoothly pasting the stationary value $\bar{v}(z, m(z,t))$ at the time $T(z)$ such that $\bar{v}(z, m(z,T)) = c$. At that time, the cost of c moving an additional researcher to a R&D line of innovation value z equals the stationary expected discounted value $\bar{v}(z, m(z,t))$ of the innovation per researcher, when $m(z,t)$ researchers are engaged from time t onwards. From the time $T(z)$ onwards, R&D firms do not add more researchers to any R&D line of innovation value z any longer. These R&D lines have become so congested, that the cost c of switching individual researchers into any of the lines cannot be recovered any longer.

The next Proposition summarizes the above equilibrium analysis.

Proposition 6 *The equilibrium allocation function m of the canonic dynamic model with cost c of researcher redeployment is*

$$m(z,t) = \frac{z - z_0(t)}{c}, \text{ for all } z \geq z_0(t), \tag{17}$$

where the boundary $z_0(t)$ solves equation (16). Researchers are moved to innovations of value z until time $T(z)$ such that $z_0(T) = rc/\lambda$.

[43] This last equation is obtained by simplifying the expression

$$cM = c\int_{z_0(t)}^{\infty} m(z,t)\, dG(t,z) = c\int_{z_0(t)}^{\infty} [z(t) - z_0(t)] dG(t,z).$$

We now turn to assess the welfare properties of the market equilibrium allocation m. To do so, we determine the first best allocation function $\tilde{m}$ by solving the problem of a social planner who remunerates individual researchers for allocating across R&D lines optimally over time. In absence of moral hazard, and with complete information, this program achieves the first best (see, for example, Hurwicz, 1960).

Further, as we discussed earlier, this approach to solve for the first best allocation clarifies that the source of market inefficiency, here, are missing property rights and missing markets. Suppose that markets rewarded research upfront, i.e., the *attempt* to achieve marketable innovations, instead of only the *achievement and marketing* of these innovations, through IP rights. Then, by the Coase Theorem (Coase, 1960), the negative externality exerted on the R&D firms by the individual researchers who engage in hot R&D lines would be internalized. So, researchers in less profitable R&D lines would be remunerated above and beyond the expected market prospects of their R&D lines. That extra remuneration would rebalance remunerations across R&D lines and achieve the first best allocation. But of course, this is impossible: it is well understood that markets cannot efficiently reward research upfront, because of impediments based on incomplete information, moral hazard, and R&D secrecy.

Suppose that a social planner provides the flow remuneration $u(t)$ to each individual researcher, regardless of the R&D line in which she is engaged.[44] The social planner problem can be expressed by the following program:

$$r\tilde{v}(t,z) = \max_{\hat{m}\in\mathbb{R}} \lambda\hat{m}\left[z - \tilde{v}(t,z) - \hat{m}c\right] - u(t)\,\hat{m} + \tilde{v}_t(t,z)\,, \tag{18}$$

for all active R&D lines of innovation value $z \geq \tilde{z}_0(t)$, under the constraint that $u(t)$ satisfies the clearing condition in the R&D researchers labor market. The expression can be related to the market equilibrium Belman equation (13), and the solution of the constrained maximization problem is the optimal allocation function $\tilde{m}(z,t)$. Here however, $\tilde{v}(t,z)$ denotes the aggregate value of the R&D lines of innovation value z. Further, the arrival

[44] The remuneration $u(t)$ need to be decreasing over time, because of the measure of undiscovered R&D lines decreases over time, here.

of an innovation of value z accrues the value z to the R&D firms aggregate, but leads to the aggregate cost $\tilde{m}(z,t)c$ of redeplying all $\tilde{m}(z,t)$ researchers previously engaged in the R&D of the discovered innovation. Finally, the aggregate remuneration of researchers, $u(t)\,\tilde{m}(z,t)$, is made explicit in the planner's problem.

The solution of problem (18) leads to the first order conditions

$$\lambda\left[z-\tilde{v}\left(t,z\right)-2\tilde{m}(z,t)c\right]=u\left(t\right),\text{ for every } z\geq\tilde{z}_0(t). \tag{19}$$

Equating these first order condictions leads to the differential equation

$$\tilde{m}_z\left(z,t\right)=\left(\frac{1-\tilde{v}_z\left(t,z\right)}{2c}\right). \tag{20}$$

Comparing this differential equation with equation (15), we recover the sources of market inefficiency we identified by comparing the optimal condition (12) with the no-arbitrage equilibrium condition (10) in the simpler model with only two R&D lines, presented earlier. Namely, we recover the 'postponement effect' and the 'redeployment cost externality' we discussed earlier.

In a later part of this section, we modify the canonical dynamic model with redeployment of researchers we developed here. We stipulate that novel profitable R&D lines exogeneously arise over time, and replace exhausted R&D lines with discovered innovations, so as to keep the economy in steady state. Before doing, we devote the next subsection to consider duplicative efforts. We show that duplicative efforts constitute a further source for the market inefficiency we identified in this paper (over-investment in hot R&D lines), beyond and independently of the postponement effect and the redeployment cost externality we singled out above.

Duplicative Efforts As discussed in the introduction, duplicative efforts have long been recognized as a possible source of market inefficiency in R&D competition, leading to the possibility of excessive private investment in R&D. We now show that they reinforce our finding that firms overinvest in hot R&D lines. To model duplicative efforts, we modify our canonical dynamic model so that the arrival rate of an innovation increases less than linearly in the number of engaged researchers. In the process of achieving a patentable innovation, competing firms often need to go through the same intermediate steps, and this occurs independently of every other firms intermediate results, which are jealously kept secret by the competing firms. Hence, the arrival rate of an innovation often does not double if twice as many firms compete in the same R&D race.

Specifically, maintaining the assumption that innovation arrivals are independent across R&D firms, we make the following assumptions. For each researcher engaged in a R&D line of innovation value z at time t, the innovation discovery arrival rate $\lambda\left(m(z,t)\right)$ is a twice differentiable and decreasing function of the mass of engaged researchers $m(z,t)$. Further, the aggregate innovation discovery arrival rate $\lambda\left(m(z,t)\right)m(z,t)$ is a concave function of $m(z,t)$. To isolate the effect of duplicative efforts from the effect of redeployment costs, we here focus on $c=0$.

Obvious modifications in the formulation and analysis of expression (13) imply that, here, the expected innovation value $\lambda(m(z,t))z$ is constant across all active R&D lines with innovation values $z \geq z_0(t)$, in equilibrium. So, differentiating $\lambda(m(z,t))z$ with respect to z, and equating the derivative to zero, we obtain that the equilibrium allocation function m is pinned down by the differential equation

$$m_z(z,t) = -\frac{\lambda(m(z,t))/m(z,t)}{\lambda'(m(z,t))}\frac{1}{z/m(z,t)}. \tag{21}$$

In words, the increase of the allocation function $m(z,t)$ in z equals the inverse of the average individual innovation value $z/m(z,t)$ at t, times the inverse of the elasticity $\epsilon\left(\hat{m}\right) = -\hat{m}\lambda'\left(\hat{m}\right)/\lambda\left(\hat{m}\right)$ of the individual arrival rate $\lambda(\hat{m})$ with respect to any researcher mass value $\hat{m}$, calculated for $\hat{m} = m(z,t)$.

Turning to the calculation of the optimal allocation function $\tilde{m}$, obvious modifications in the formulation and analysis of expression (18) yield the following first-order conditions:

$$\left[\lambda(\tilde{m}(z,t)) + \lambda'(\tilde{m}(z,t))\tilde{m}(z,t)\right]\left[z - \tilde{v}\left(t,z\right)\right] = u(t) \tag{22}$$

for all active R&D lines $z \geq \tilde{z}_0(t)$, at the optimal solution $\tilde{m}$.

In order to assess the welfare properties of equilibrium, we pick any pair of active R&D lines 1 and 2, with values $z_1 < z_2$, and directly compare the no arbitrage market equilibrium condition

$$\lambda(m(z_1,t))z_1 = \lambda(m(z_2,t))z_2 \tag{23}$$

with the condition obtained by equating first-order conditions in the optimal planning solution

$$\begin{aligned} & [\lambda(\tilde{m}(z_1,t)) + \lambda'(\tilde{m}(z_1,t))\tilde{m}(z_1,t)]\,[z_1 - \tilde{v}\,(t,z_1)] \\ = \;& [\lambda(\tilde{m}(z_2,t)) + \lambda'(\tilde{m}(z_2,t))\tilde{m}(z,t)]\,[z_2 - \tilde{v}\,(t,z_2)]\,. \end{aligned} \tag{24}$$

The comparison of equations (23) and (24) is simple and leads to the identification of two effects, both leading to the result that R&D firms overinvest in the more valuable hot line 2, i.e., that $m_1 < \tilde{m}_1 < \tilde{m}_2 < m_2$. The first effect is due to the fact that $\tilde{v}\,(t,z_1) > \tilde{v}\,(t,z_2)\,.$ In fact, $\tilde{v}\,(t,z_j)$ is the optimal social value of the continuation after innovation $j = 1,2$ has been discovered. This optimal continuation value is larger when the least valuable innovation 1 has been discovered, so that the more valuable innovation 2 is yet to be discovered. Because $\tilde{v}\,(t,z_1) > \tilde{v}\,(t,z_2)\,,$ the social relative benefit $\frac{z_2-\tilde{v}(t,z_2)}{z_1-\tilde{v}(t,z_1)}$ for engaging in the most valuable R&D line 2 is smaller than the individual relative benefit z_2/z_1. Hence, the private incentive of joining the more valuable R&D line 2 is larger than the social incentive.

This 'continuation-value effect' is reinforced by a 'technological congestion' effect, when the elasticity $\epsilon\,(\hat{m})$ of the arrival rate $\lambda\,(\hat{m})$ weakly increases in $\hat{m}$. As we show in appendix B, this is because of the following argument. For both equations (23) and (24) to hold simultaneously when the elasticity $\epsilon\,(\hat{m})$ does not decrease in $\hat{m}$, it must be that $m(z_2,t)$ is $\tilde{m}(z_2,t)$, so as to reduce $[\lambda m(z_2,t)]/[\lambda m(z_1,t)]$ relative to $[\tilde{m}(z_2,t)\lambda'\,(\tilde{m}(z_2,t)) + \lambda\,(\tilde{m}(z_2,t))]/[m(z_1,t)\lambda'\,(m(z_1,t)) + \lambda\,(m(z_1,t))]$.

The following propositions sums up our findings on the case of duplicative effort.

Proposition 7 *Consider a canonical dynamic model with costless redeployment of researchers across R&D lines, but with duplicative effort within each R&D line: the per-researcher discovery arrival rate* $\lambda\,(m(z,t))$ *of each innovation of value* z *at any time* t

decreases in the mass $m(z,t)$ of engaged researchers. Suppose that $\lambda(\hat{m})\hat{m}$ is a concave function of $\hat{m}$, and that the elasticity $\epsilon(\hat{m}) = -\hat{m}\lambda'(\hat{m})/\lambda(\hat{m})$ does not decrease in $\hat{m}$. Then, in equilibrium, R&D firms overinvest in the hot R&D lines: there exists a twice differentiable threshold function $\bar{z}$ such that $m(z,t) < \tilde{m}(z,t)$ for $z < \bar{z}(t)$ and $m(z,t) > \tilde{m}(z,t)$ for $z > \bar{z}(t)$.

To summarize our findings so far, within the canonical dynamic framework formulated in this section, we have singled out two separate substantive sources for the kind of market inefficiency identified in this paper. Research firms overinvest in the hot R&D lines in equilibrium, because the redeployment of researchers is not perfectly costless and/or because of duplicative effort in the research of innovations. The latter source of market inefficiency is exacerbated by R&D firms' secrecy policies about their intermediate findings.

Steady State Economy We now return to the canonical dynamic model with costly redeployment of researchers across R&D lines, and without duplication efforts within R&D lines. We introduce the possibility of successive innovation vintages. We suppose that novel profitable R&D lines arise over time and replace exhausted R&D lines that already led to the discovery of profitable innovations. We focus on R&D line replacement that keeps the economy in steady state. We denote the stationary market equilibrium and optimal allocation functions $m(z)$ and $\tilde{m}(z)$, respectively. We let the flow arrival rate of R&D lines be denoted by α, and the cumulative distribution function of novel R&D line innovation values be denoted by F, with associated density f. Thus, we impose the stationality condition

$$\lambda m(z)\, g(z) = \alpha f(z), \text{ for all } z \geq z_0, \tag{25}$$

on the stationary cumulative distribution G of undiscovered innovation values, and we denote by g the density of G. Likewise, the stationary distribution $\tilde{G}$ of R&D lines available for discovery satisfies condition

$$\lambda\tilde{m}(z)\,\tilde{g}(z) = \alpha f(z), \text{ for all } z \geq \tilde{z}_0. \tag{26}$$

Thus, for any innovation value $z \geq z_0$, the total mass of researchers allocated to R&D lines of innovation value z in the steady state equilibrium and optimal allocations, respectively $m(z)\, g(z)$ and $\tilde{m}(z)\, \tilde{g}(z)$, are both equal to $(\alpha/\lambda)\, f(z)$, the net inflow of R&D lines of innovation value z. Of course, this does not mean that also the welfare is the same in the equilibrium and in the optimal allocations, because the mass of researchers engaged in each R&D line may differ: it need not be that $m(z) = \tilde{m}(z)$ for any active R&D line of innovation value z. But it needs to be the case that, in steady state, the support of $m(z)\, g(z)$ and $\tilde{m}(z)\, \tilde{g}(z)$ coincide, so that the smallest active R&D line innovation values z_0 and $\tilde{z}_0$ also coincide.

Turning to equilibrium analysis, obvious modifications of the analysis in the previous part of the section imply that, again, the expression $\lambda\, [z - m(z)\, c]$ is constant across all active R&D lines of innovation value $z \geq z_0$, that the equilibrium satisfies the differential equation $m_z(z) = 1/c$, and that the equilibrium expression is $m(z) = (z - z_0)/c$, for all $z \geq z_0$.

When the resource feasibility constraint $\int_{z_0}^{+\infty} m(z)\, g(z)\, dz \leq M$ binds, all R&D firms make a non-negative profit by participating to the economy, and the threshold z_0 is pinned down by plugging the stationarity condition (25) into the binding resource feasibility constraint, so as to obtain the equation

$$\alpha\, (1 - F(z_0)) = \lambda M.$$

As a result, we see that, here, z_0 is determined independently of the allocation function m.[45,46] Finally, by returning to the stationarity condition (25), we calculate the density of

[45] Also the stationary equilibrium value $\bar{v}$ of engaging researchers in every R&D line of innovation value $z \geq z_0$ can be easily calculated, here. From the expression,

$$\bar{v} = \int_0^{\infty} e^{-rt} \left[\frac{z}{m(z)} + \bar{v} - c \right] m(z)\, \lambda e^{-m(z)\lambda t} dt = \frac{\lambda m(z)}{r + \lambda m(z)} \left(\frac{z}{m(z)} + \bar{v} - c \right),$$

where $\bar{v} - c$ is the value for redeploying researchers once the innovation is discovered, we obtain: $\bar{v} = (\lambda/r)\, [z - m(z)\, c]$, for all $z \geq z_0$.

[46] If the resource constraint is satisfied with a strict inequality, $\int_{z_0}^{\infty} m(z)\, dG(z) < M$, then the economy cannot support entry by all firms, in the steady state equilibrium. So the participation constraint $\bar{v} \geq c$ binds, and pins down z_0 through the equality $c = \bar{v} = (\lambda/r) z_0$.

G:

$$g(z) = (\alpha/\lambda)\frac{f(z)\,c}{z - z_0}.$$

Turning to the calculation of the optimal allocation $\tilde{m}$, with obvious modifications of the Bellman equations (18), we see that the social planner problem takes the following form, here:

$$r\tilde{v}(z) = \max_{\hat{m}\in\mathbb{R}} \lambda\hat{m}\left[z - \tilde{v}(z) - \hat{m}c\right] - u\hat{m}, \tag{27}$$

for all $z \geq z_0$, under the constraint that u satisfies the clearing condition in the R&D researchers labor market. The associated first-order conditions are:

$$\lambda\left[z - \tilde{v}(z) - 2\tilde{m}(z)c\right] = u, \quad \text{for every } z \geq z_0. \tag{28}$$

Plugging the solution $\tilde{m}(z)$ of the program (27) in the program, we solve for the optimal value $\tilde{v}(z)$ and obtain:

$$\tilde{v}(z) = \frac{\lambda\tilde{m}(z)\left[z - \tilde{m}(z)c\right] - \tilde{m}(z)u}{r + \lambda\tilde{m}(z)}, \tag{29}$$

substituting the optimal value $\tilde{v}(z)$ into the first-order conditions (28),

$$\lambda\left[z - \frac{\lambda\tilde{m}(z)\left[z - \tilde{m}(z)c\right] - \tilde{m}(z)u}{r + \lambda\tilde{m}(z)} - 2\tilde{m}(z)c\right] = u,$$

so that, solving for u and simplifying, we have:

$$\lambda z - \lambda\tilde{m}(z)(\lambda\tilde{m}(z)/r)c - 2\lambda\tilde{m}(z)c = u, \quad \text{for every } z \geq z_0. \tag{30}$$

These equations are analogous to the first-order conditions (19) in the canonical dynamic model without R&D line replacement we solved earlier. The only difference is that the term $\tilde{v}(t,z)$ takes the constant form $\tilde{m}(z)(\lambda\tilde{m}(z)/r)c$, here. The latter is the discounted cost of all future switches for the mass $\tilde{m}(z)$ of researchers engaged in every R&D line of innovation value z —the term $(\lambda\tilde{m}(z)/r)c$ is the individual discounted cost. So, we can identify as $\lambda\tilde{m}(c)(1 + \lambda\tilde{m}(z)/r)c$, the externality that an additional researcher imposes on the $\tilde{m}(z)$ researchers engaged on the R&D line.

Specifically, the first term in expression (30), λz, represents the expected value of engaging this researcher on a R&D line of innovation value z. It equals the innovation value

z times the hazard rate of the innovation discovery arrival to the researcher. The term $\lambda\tilde{m}(c)(1+\lambda\tilde{m}(z)/r)c$ represents the current and future discounted redeployment costs borne by the other $\tilde{m}(z)$ researchers, if the innovation is discovered by the marginal researcher. The remaining term, $\lambda\tilde{m}(z)c$, is the cost of redeploying the marginal researcher; grouped with the term λz, it gives the expression $\lambda[z-\tilde{m}(z)]c$ which is exactly what is equated across research lines in the market equilibium, as we reported above.

Applying the implicit function theorem to expression (30), we obtain:

$$\begin{aligned}\tilde{m}'(z) &= -\frac{\lambda}{-2\lambda\tilde{m}(z)(\lambda/r)c-2\lambda c} \\ &= \frac{1}{2c}\left(\frac{1}{\tilde{m}(z)(\lambda/r)+1}\right). \end{aligned} \tag{31}$$

Further, the expression for the optimal allocation $\tilde{m}(z)$ can be solved explicitly, here, and takes the closed form:[47]

$$\tilde{m}(z)=\frac{r}{\lambda}\left(\sqrt{\lambda\frac{z-\tilde{z}_0}{rc}+1}-1\right), \text{ for all } z\geq z_0. \tag{32}$$

Evidently, the derivative of $\tilde{m}$, reported in expression (31), is strictly smaller than $1/2c$, which is smaller than $1/c$, the derivative $m(z)$ reported above. Further, the equilibrium allocation function m and the optimal allocation function $\tilde{m}$ satisfy the same resource feasibility constraints

$$\int_{z_0} m(z)\,g(z)\,dz = M = \int_{z_0}\tilde{m}(z)\,\tilde{g}(z)\,dz.$$

These two facts imply that, once again, the allocation functions m and $\tilde{m}$ cross only once, so that R&D firms overinvest in the hot R&D lines in equilibrium.

We single out this result in the following proposition.

Proposition 8 *Consider the canonic dynamic model with costly redeployment of researchers, and replacement of exhausted R&D lines that leads to a stationary distribution of R&D lines with undiscovered innovation. In equilibrium, R&D firms overinvest in the hot R&D lines: there exists a threshold $\bar{z}$ such that $m(z)<\tilde{m}(z)$ for $z<\bar{z}$ and $m(z)>\tilde{m}(z)$ for $z>\bar{z}$.*

[47] This expression follows by first solving equation (30) for $\tilde{m}(z)$ to obtain, for all $z\geq z_0$: $\tilde{m}(z)=(r/\lambda)\left(\sqrt{(\lambda z-u)(rc)^{-1}+1}-1\right)$, and then by noting that researchers are a perfectly divisible factor in our model, so that $\tilde{m}(z)=0$ at $z=z_0$, and, hence, $u=\lambda z_0$.

Interestingly, here, it is also the case that the thresholds z_0 and $\bar{z}$ coincide. Because the support of the allocation functions m and $\tilde{m}$ coincide, in this model, it must be that the functions m and $\tilde{m}$ cross at z_0, the smallest innovation value for which the R&D lines are active. This fact, together with the fact that $m' > \tilde{m}'$ also imply that $m > \tilde{m}$, $g < \tilde{g}$ and that $g' < \tilde{g}'$, here. And to make up for the fact that $m(z_0) = \tilde{m}(z_0) = 0$, it also needs to be the case that $\lim_{z \to z_0^+} g(z) = \lim_{z \to z_0^+} \tilde{g}(z) = +\infty$.

In other words, the density of R&D lines with undiscovered innovations is very large for very small innovation values. Innovation discoveries arrive with a very low rate, but this is compensated by a very high density of R&D lines. Researchers allocated to these R&D lines of small innovation value rarely make a discovery and are redeployed, because they are very few. As the innovation value grows larger, the density of R&D lines with undiscovered innovations decreases. The rate of decrease is larger for the competitive equilibrium than for the optimal regime. So, the market suboptimally exhausts too many high value R&D lines too early, and leaves too few for future discovery.

We now turn to assessing the welfare properties of the equilibrium of the canonical dynamic model with stochastic replacement of exhausted R&D lines that leads to a stationary distribution of R&D lines with undiscovered innovation. We prove in Appendix B the following result.

Proposition 9 *Consider the canonic dynamic model with costly redeployment of researchers and stationary distribution of R&D lines with undiscovered innovations. The aggregate equilibrium welfare is*[48]

$$W(m) = (\lambda/r)\underline{z}M \tag{33}$$

The aggregate welfare associated with the optimal allocation $\tilde{m}$ *is:*

$$W(\tilde{m}) = \frac{\alpha}{r}\int_{z_0}^{\infty} zf(z)\,dz + cM - \frac{\alpha}{\lambda}\int_{z_0}^{\infty}\sqrt{\frac{c\lambda}{r}(z - z_0) + c^2} \cdot f(z)\,dz. \tag{34}$$

[48]It may appear surprising that the steady state equilibrium welfare $W(m)$ is independent of c. This follows from the fact that the equilibrium redeployment cost $m(z)\,c$ equals $z - z_0$ independently of c for any innovation value $z \geq z_0$, as it can seen by inspecting the equilibrium expression $m(z) = (z - z_0)/c$.

These closed-form expressions make welfare assessments simple and precise in 'mature' R&D industries for which it is appropriate to assume a stationary distribution of undiscovered innovations, through the steady emergence of innovation opportunities in successive vintages.

In the limit as the redeployment cost c vanishes, we further have:

$$\begin{aligned}\lim_{c\to 0^+} W(\tilde{m}) &= (\alpha/r)\int_{z_0}^{\infty} zf(z)\,dz = (\alpha/r)E(z|z\geq z_0)\left[1-F(z_0)\right] \\ &= (\lambda/r)E(z|z\geq z_0)\,M,\end{aligned}$$

where the expectation is taken according to the innovation value cumulative distribution function F. Thus, for small redeployment cost c, the welfare ratio $W(m)/W(\tilde{m})$ takes the simple form

$$\lim_{c\to 0^+}\frac{W(m)}{W(\tilde{m})} = \frac{z_0}{E(z|z\geq z_0)}.$$

In words, the welfare ratio converges to the smallest active R&D line innovation value z_0, divided by the expected active R&D line innovation value. It is intuitive that this quantity can be significantly small, for standard cumulative distributions F.[49]

5 Conclusion

Research on the efficiency of innovation markets is usually concerned on whether the level of R&D firm investment is socially optimal. This paper has asked a novel, important question: Does R&D go in the right direction? In a simple, general model, we have demonstrated that R&D competition pushes firms to disproportionately engage in the hot research lines, characterized by higher expected rates of return. The identification of this form of market failure is a novel result of this paper, as we explained in details in the introduction and literature review.

After demonstrating the result that competing R&D firms ovrinvest in the R&D areas within the framework of a simple model, we have embedded our analysis in a canonical

[49] We performed a back-of-the-envelope calculation of the welfare ratio $W(m)/W(\tilde{m})$, under the assumption that the distribution F is lognormal with mean equal to 7 and standard deviation equal to 1.5, consistently with the estimates provided by Schankerman (1998). With cost $c = 1$ million, the welfare ratio $W(m)/W(\tilde{m}) \approx 0.28$. As the cost c vanishes, the ratio $W(m)/W(\tilde{m})$ converges to 0.17 approximately.

dynamic framework that can be directly compared with extant R&D race models, and that we modified and extended in several directions. Our framework is populated with a continuum of innovation R&D lines, whose discovery is an independent random event, equally likely across all engaged researchers, and with time constant hazard rate. We considered different cases in which researchers may be moved across R&D lines costly, and in which the innovation discovery hazard rate grows with the number of engaged researchers proportionally, or less than proportionally because of duplicative R&D effort. We have recovered our market inefficiency result that competing R&D firms overinvest in hot research areas in all these cases, with the exception of the knife-hedge case without duplicative R&D effort and with small researcher redeployment costs. Focusing on the case without without duplicative R&D effort for expositional simplicity, we have finally recovered our market inefficiency results in a model with successive innovation vintages that may represent 'mature' R&D industries.[50]

The implications of our study in terms of policy are transparent. Because of our prediction that competing R&D firms overinvest in hot research lines in equilibrium, this paper advocates for market intervention that rebalances remuneration across R&D lines, so as to subsidize R&D lines with less profitable or less feasible innovations. Importantly, our case for R&D subsidization is based on a framework without any market frictions. We argue that even frictionless markets and competition forces cannot solve the form of market failure identified here, because of an unavoidable missing market institutional constraint. Efficiency could be achieved without external intervention, here, only if markets were capable to fully remunerate R&D effort upfront, instead of just rewarding innovation discovery. But of course, this is not what profit motivated entrepreneurs would do, because of several well-understood impediments based on incomplete information, moral hazard, and R&D secrecy.

Details of the existing forms and mechanisms of non-market R&D subsidization have been discussed in the literature review (section 2). The main sources of R&D funding are research grants and fiscal incentives, in the form of subsidies or tax breaks. Prizes,

[50]Specifically, this model stipulates that novel profitable R&D lines arise over time, replacing discovered innovation research lines, so as to keep the economy in steady state.

procurements and the funding of academia also serve to subsidize R&D, but they do not seem plausibly effective in allevating the form of market inefficiency identifies here (over-investment in the hot R&D areas). It is even possible that prizes, procurement and career concerns in academia exacerbate the market inefficiency singled out in this paper. Plausibly, they may bias incentives of individual researchers so that they disproportionately compete on a small set of high-profile breakthroughs, instead of spreading their efforts more evenly across valuable innnovations.[51]

Returning to comparing grants and fiscal incentives, the relative merits of these two mechanisms are known in the literature. Fiscal incentives have the advantage that they leave the choice of the direction of R&D to the informed parties, the competing R&D firms. Unfortunately, this is exactly the source of market inefficiency identified in this model. It is therefore possible that grants would fare better than fiscal incentives, for the purposes advocated here. The verification of this concluding conjecture will require extensive work, both in terms of formal modelling and data-based quantitative assessments, that are beyond the boundaries of this paper.

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Appendix A: General Utilities, Externalities and Technological Spillovers

Here, we show that our findings hold for general specifications of the innovations' social values. Given that each innovation $j = 1, 2$ can be achieved or not, let $0 = u(0,0)$, $z_1 = u(1,0)$, $z_2 = u(0,1)$, and $\bar{z} = u(1,1)$ be the innovations' social values, where the index 1 stands for the discovery of innovation j and the index 0 stands for non discovery. Now, the social planner problem's is to set $\tilde{m}_1$ and $\tilde{m}_2$ so as to maximize:

$$W(\tilde{m}_1, \tilde{m}_2) = P(\tilde{m}_1)[1 - P(\tilde{m}_2)] z_1 + [1 - P(\tilde{m}_1)] P(\tilde{m}_2) z_2 + P(\tilde{m}_1) P(\tilde{m}_2) \bar{z} \text{ s.t. } \tilde{m}_2 + \tilde{m}_1 = 1$$

Equating the first-order conditions, we obtain:

$$\begin{aligned} \frac{\partial}{\partial \tilde{m}_1} W(\tilde{m}_2, \tilde{m}_1) &= P'(\tilde{m}_1)[1 - P(\tilde{m}_2)] z_1 - P'(\tilde{m}_1) P(\tilde{m}_2) z_2 + P'(\tilde{m}_1) P(\tilde{m}_2) \bar{z} \\ &= P'(\tilde{m}_1) \{[1 - P(\tilde{m}_2)] z_1 + P(\tilde{m}_2)[\bar{z} - z_2]\}. \\ &= P'(\tilde{m}_2) \{[1 - P(\tilde{m}_1)] z_2 + P(\tilde{m}_1)[\bar{z} - z_1]\} = \frac{\partial}{\partial \tilde{m}_2} W(\tilde{m}_2, \tilde{m}_1) \end{aligned} \quad (35)$$

The equilibrium condition, instead, is simply:

$$\frac{P(m_2)}{m_2} \{[1 - P(m_1)] z_2 + P(m_1) p_2\} = \frac{P(m_1)}{m_1} \{[1 - P(m_2)] z_1 + P(m_2) p_1\}, \quad (36)$$

where p_1 and p_2 are the prices of innovations 1 and 2 if both innovations are made, in which case, almost surely, the two innovations are found by different firms.

In determining the values of p_1 and p_2 we distinguish two cases. In the first one, the innovations are weak substitutes, so that $\bar{z} \leq z_1 + z_2$. Here, we set $p_1 = \bar{z} - z_2$ and $p_2 = \bar{z} - z_1$, as this is the unique outcome of competition in which each innovator fully appropriate of the marginal product of its innovation. Indeed, the core is made of all positive prices (p_1, p_2) such that $p_1 + p_2 \leq \bar{z}$ and $p_1 \leq \bar{z} - z_2$ and $p_2 \leq \bar{z} - z_1$. Hence, the prices $p_1 = \bar{z} - z_2$ and $p_2 = \bar{z} - z_1$ is the best outcome for the innovators in the core, because $p_2 + p_1 = \bar{z} - z_2 + \bar{z} - z_1 \leq \bar{z}$, as is implied by the condition $\bar{z} \leq z_1 + z_2$. When $p_1 = \bar{z} - z_2$ and $p_2 = \bar{z} - z_1$, dividing the equilibrium condition (36) by the social planner's solution condition (35), we obtain equality (3), again. Hence, the proof of Proposition 1 implies that, again, too many researchers join the hot industry 2, whenever $\Gamma(m) = mP'(m)/P(m)$ strictly decreases in m.

In the second case, innovations are strict complements, so that $\bar{z} > z_1 + z_2$. Here, the prices $p_1 = \bar{z} - z_2$ and $p_2 = \bar{z} - z_1$ are not in the core, because $p_1 + p_2 = \bar{z} - z_2 + \bar{z} - z_1 > \bar{z}$. So, we suppose that the two innovators split the joint innovation profits according to the Nash bargaining solution. The feasible bargaining set corresponds to the core: the set of all positive prices (p_1, p_2) such that $p_1 + p_2 \leq \bar{z}$ and $p_1 \leq \bar{z} - z_2$ and $p_2 \leq \bar{z} - z_1$. The two innovators threat points are z_1 and z_2, because each innovator can at least guarantee to sell its good at its value —recall that $\bar{z} > z_1 + z_2$, so that the consumer is willing to pay the total price $z_1 + z_2$ to buy both innovations. Hence the Nash bargaining solution (p_1, p_2) is such that $p_1 - z_1 = p_2 - z_2$ and $p_1 + p_2 = \bar{z}$. Solving out these two equations, we obtain: $p_1 = \frac{1}{2}[\bar{z} + z_1 - z_2]$, $p_2 = \frac{1}{2}[\bar{z} + z_2 - z_1]$.

So, the equilibrium conditions are:

$$\begin{aligned} & \frac{P(m_2)}{m_2}\left\{[1-P(m_1)]z_2+\frac{1}{2}P(m_1)[\bar{z}+z_2-z_1]\right\} \\ = & \frac{P(m_1)}{m_1}\left\{[1-P(m_2)]z_1+\frac{1}{2}P(m_2)[\bar{z}-z_2+z_1]\right\}. \end{aligned} \quad (37)$$

Comparing these conditions with the optimality conditions (35), we conclude in the Appendix that again, too many researchers join the hot industry 2, whenever $\Gamma(m) = mP'(m)/P(m)$ strictly decreases in m.[52]

We summarize the analysis of this section in the following proposition

Proposition 10 *Suppose that the success probability $P(m)$ is increasing and weakly concave, and that $P(m)/m$ weakly decreases in m. For any specification of the innovations' social values, if innovations'* generalized reverse hazard rate $\Gamma(m) = mP'(m)/P(m)$ *strictly decreases in m, R&D firms overinvest in the hot R&D line 2, in equilibrium, i.e., $m_2 > \tilde{m}_2 > \tilde{m}_1 > m_1$.*

Proof of Proposition 10. The proof for the case in which $\bar{z} \le z_1+z_2$ is entirely analogous to the proof of Proposition 1. Considering the case in which $\bar{z} > z_1 + z_2$, proceding by contradiction, suppose $m_2 \le \tilde{m}_2$, so that $P(\tilde{m}_1) \le P(m_1) < P(m_2) \le P(\tilde{m}_2)$. We first show that this implies:

$$\frac{[1-P(\tilde{m}_1)]z_2+P(\tilde{m}_1)[\bar{z}-z_1]}{[1-P(\tilde{m}_2)]z_1+P(\tilde{m}_2)[\bar{z}-z_2]} < \frac{2[1-P(m_1)]z_2+P(m_1)[\bar{z}-z_1+z_2]}{2[1-P(m_2)]z_1+P(m_2)[\bar{z}-z_2+z_1]},$$

because

$$\begin{aligned} & ([1-P(\tilde{m}_1)]z_2+P(\tilde{m}_1)[\bar{z}-z_1])(2[1-P(m_2)]z_1+P(m_2)[\bar{z}-z_2+z_1]) \\ & -(2[1-P(m_1)]z_2+P(m_1)[\bar{z}-z_1+z_2])([1-P(\tilde{m}_2)]z_1+P(\tilde{m}_2)[\bar{z}-z_2]) \\ = & (\bar{z}-z_1-z_2)\{[2P(\tilde{m}_1)-P(\tilde{m}_1)P(m_2)+P(\tilde{m}_2)P(m_1)-P(m_1)]z_1 \\ & -[2P(\tilde{m}_2)-P(\tilde{m}_2)P(m_1)+P(\tilde{m}_1)P(m_2)-P(m_2)]z_2+[P(\tilde{m}_1)p_2-P(\tilde{m}_2)P(m_1)]\bar{z}\} \\ \propto & P(\tilde{m}_1)z_1+(P(\tilde{m}_1)[1-P(m_2)]-P(m_1)[1-P(\tilde{m}_2)])z_1 \\ & -P(\tilde{m}_2)z_2-(P(\tilde{m}_2)[1-P(m_1)]-P(m_2)[1-P(\tilde{m}_1)])z_2 \\ & +[P(\tilde{m}_1)P(m_2)-P(\tilde{m}_2)P(m_1)]\bar{z} \\ < & P(\tilde{m}_1)z_1-P(\tilde{m}_2)z_2<0. \end{aligned}$$

This result, together with the chain of inequalities (4), contradicts the equality

$$\frac{P(m_2)/m_2}{P(m_1)/m_1}\frac{[1-P(m_1)]z_2+\frac{1}{2}P(m_1)[\bar{z}+z_2-z_1]}{[1-P(m_2)]z_1+\frac{1}{2}P(m_2)[\bar{z}-z_2+z_1]} = \frac{P'(\tilde{m}_2)}{P'(\tilde{m}_1)}\frac{[1-P(\tilde{m}_1)]z_2+P(\tilde{m}_1)[\bar{z}-z_1]}{[1-P(\tilde{m}_2)]z_1+P(\tilde{m}_2)[\bar{z}-z_2]},$$

which is implied by the equilibrium condition (**??**) and by the social planner's solution condition (35). ■

The significance of this result goes beyond showing that researchers overinvest in hot R&D lines also when innovations do not enter welfare additively.

[52]It is easy to appreciate that this 'overinvestment in hot areas result' holds beyond the case of Nash bargaining, here, as long as the share of profits of the holder of the hot innovation patent is not too small.

Appendix B: Omitted Proofs

Proof of Proposition 3. The no-arbitrage conditions are, for all z,

$$z\frac{P(m)}{m} = z\frac{\lambda}{r + m\lambda} = c,$$

for some constant c, so that, solving out, we obtain

$$m\left(z\right) = \max\left\{0, z/c - r/\lambda\right\} = \max\left\{0, z/z_0 - 1\right\} r/\lambda$$

where $z_0 = rc/\lambda$ is the smallest-value R&D line z such that $m\left(z\right) > 0$, and is pinned down by the resource contraint:

$$M = \int_1^\infty m(z)f(z)dz = \frac{r}{\lambda}\int_{z_0}^\infty \left(\frac{z}{z_0} - 1\right)\frac{1}{z^{\eta+1}}\eta dz = \frac{r}{\lambda}\frac{z_0}{\eta - 1}.$$

Hence, the equilibrium welfare is simply:

$$W\left(m\right) = Mc = \frac{z_0^{1-\eta}}{\eta - 1},$$

as each firm collects the expected profit c, and there is a continuum of mass M of firms.

A social planner chooses $\tilde{m}\left(\cdot\right)$ to maximize the social welfare:

$$W\left(\tilde{m}\left(\cdot\right)\right) = \int_1^\infty z\frac{\tilde{m}(z)\lambda}{r + \tilde{m}(z)\lambda} f\left(z\right) dz s.t. \int_1^\infty \tilde{m}\left(z\right) f(z)dz = M.$$

Hence, the Euler conditions are that, for all z,

$$z\frac{r\lambda}{\left(r + \tilde{m}\left(z\right)\right)\lambda)^2} = \mu,$$

where μ is the Lagrange multiplier of the resource constraint.

Solving out, we thus obtain:

$$\tilde{m}\left(z\right) = \max\left\{0, \sqrt{\left(z/\mu\right)\left(r/\lambda\right)} - r/\lambda\right\} = \max\left\{0, \sqrt{z/\tilde{z}_0} - 1\right\} r/\lambda,$$

where $\tilde{z}_0 = r\mu/\lambda$ is the smallest-value R&D line z such that $\tilde{m}\left(z\right) > 0$, again, pinned down by the resource constraint

$$M = \int_1^\infty \tilde{m}\left(z\right) dF(z) = \frac{r}{\lambda}\int_{\tilde{z}_0}^\infty \left(\sqrt{\frac{z}{\tilde{z}_0}} - 1\right)\frac{1}{z^{\eta+1}}\eta dz = \frac{r}{\lambda}\frac{\tilde{z}_0^{-\eta}}{2\eta - 1}.$$

Here, because the expected social value of employing $m\left(z\right)$ on R&D line is

$$\begin{aligned} zP(\tilde{m}(z)) &= z\frac{\tilde{m}\left(z\right)\lambda}{r + \tilde{m}\left(z\right)\lambda} = z\frac{\frac{r}{\lambda}\left(\sqrt{\frac{z}{\tilde{z}_0}} - 1\right)\lambda}{r + \frac{r}{\lambda}\left(\sqrt{\frac{z}{\tilde{z}_0}} - 1\right)\lambda} \\ &= z - \sqrt{z\tilde{z}_0}, \end{aligned}$$

integrating over z, we obtain that the optimal welfare is:

$$\begin{aligned}\tilde{W} &= W(\tilde{m}) = \int_{\tilde{z}_0}^{\infty} zP(\tilde{m}(z))dF(z) = \int_{\tilde{z}_0}^{\infty} \left(z - \sqrt{z\tilde{z}_0}\right) \frac{1}{z^{\eta+1}}\eta dz \\ &= \eta \left[\frac{\tilde{z}_0^{1-\eta}}{(\eta-1)(2\eta-1)}\right].\end{aligned}$$

Hence, the welfare gain over the equilibrium is:

$$\tilde{W}/W = \frac{\eta}{\eta-1}\left(\frac{2\eta-1}{\eta-1}\right)^{-1/\eta}.$$

■

Proof of Proposition 4. Here, the equilibrium arbitrage conditions require that the expression $\frac{P(z,m(z))}{m(z)} = \frac{z\lambda(z)}{r+m(z)\lambda(z)}$ is constant in z. Likewise, equating the first-oder conditions to find the optimal allocation $\tilde{m}$ implies that the expression $P_m(z, \tilde{m}(z))\, z = \frac{rz\lambda(z)}{[r+\tilde{m}(z)\lambda(z)]^2}$ is constant in z.

Pick two values of z, z_1 and z_2 and say without loss of generality that $z_2\lambda(z_2) > z_1\lambda(z_1)$. Because the function $\frac{r}{(r+\tilde{m}\lambda)^2}$ decreases in $\tilde{m}\lambda$, it follows that $\tilde{m}_2\lambda_2 > \tilde{m}_1\lambda_1$, where we write $\tilde{m}_j$ instead of $\tilde{m}(z_j)$ and λ_j instead of $\lambda(z_j)$ for brevity. Dividing the no arbitrage condition

$$\frac{P_1(m_1)}{m_1}z_1 = z_1\lambda_1\frac{1}{r+m_1\lambda_1} = z_2\lambda_2\frac{1}{r+m_2\lambda_2} = \frac{P_2(m_2)}{m_2}z_2,$$

where, again, we write m_j instead of $m(z_j)$, by the social planner's solution condition

$$P_1'(\tilde{m}_1)\, z_1 = z_1\lambda_1\frac{r}{(r+\tilde{m}_1\lambda_1)^2} = z_2\lambda_2\frac{r}{(r+\tilde{m}_2\lambda_2)^2} = P_2'(\tilde{m}_2)\, z_2, \tag{39}$$

we obtain:

$$\frac{(r+\tilde{m}_1\lambda_1)^2}{r(r+m_1\lambda_1)} = \frac{(r+\tilde{m}_2\lambda_2)^2}{r(r+m_2\lambda_2)}.$$

Now suppose, by contradiction, that $m_2 \leq \tilde{m}_2$, so that $m_1 \geq \tilde{m}_1$. Then, as in the proof of Proposition 1, we obtain the contradiction:

$$\frac{(r+\tilde{m}_1\lambda_1)^2}{r(r+m_1\lambda_1)} \leq \frac{(r+\tilde{m}_1\lambda_1)^2}{r(r+\tilde{m}_1\lambda_1)} < \frac{(r+\tilde{m}_2\lambda_2)^2}{r(r+\tilde{m}_2\lambda_2)} \leq \frac{(r+\tilde{m}_2\lambda_2)^2}{r(r+m_2\lambda_2)},$$

using the fact that the function $\frac{(r+\tilde{m}\lambda)^2}{r(r+\tilde{m}\lambda)}$ increases in $\tilde{m}\lambda$. ■

Proof of Proposition 5. Given the analysis presented in the main body before the statement of this result, we only need to show that there exists a threshold $\bar{c} > 0$ such that, for all $c > \bar{c}$, there exists at least a set of innovations J and a pair $j, k \in J$, with $j < k$, such that $0 < \tilde{m}_j < \tilde{m}_k < M$ and that condition (12) holds as an equality. Indeed, if this were not the case, because as $z_1 < z_2 < ... < z_N$, there would then be a single innovation

$k\in J$ such that $\tilde{m}_k=M$ and for all other innovations $j\in J$, it would be the case that $\tilde{m}_j=0$. But this would contradict the Kuhn-Tucker condition:

$$2c\left(\tilde{m}_k-\tilde{m}_j\right)\leq\left(z_k-z_j\right)\left(1-\frac{M\lambda}{r+M\lambda}\right),$$

for $c>\frac{(z_k-z_j)\left(1-\frac{M\lambda}{r+M\lambda}\right)}{2M}$. Hence, the result is proved by setting $\bar{c}=\min_{j,k}\left\{\frac{(z_k-z_j)\left(1-\frac{M\lambda}{r+M\lambda}\right)}{2M}\right\}=\frac{(z_2-z_1)\left(1-\frac{M\lambda}{r+M\lambda}\right)}{2M}$. Finally, we note that $z_1\lambda/\left(r+M\lambda\right)>\frac{(z_2-z_1)\left(1-\frac{M\lambda}{r+M\lambda}\right)}{2M}$ as long as $M>\frac{r}{2\lambda}\frac{(z_2-z_1)}{z_1}$. ■

Proof of Proposition 6. The equilibrium analysis is presented in the main body of the paper. We now turn to prove the comparative statics results. By the implicit function theorem, totally differentiating equation (16), we obtain:

$$\begin{aligned}z_0'(t) &= -\frac{D_t\left(\int_{z_0}^{\infty}zdG\left(t,z\right)-z_0[1-G\left(t,z_0\right)]\right)}{D_{z_0}\left(\int_{z_0}^{\infty}zdG\left(t,z\right)-z_0[1-G\left(t,z_0\right)]\right)}\\ &= \frac{\int_{z_0}^{\infty}zdG_t\left(t,z\right)+z_0G_t\left(t,z_0\right)}{1-G\left(t,z_0\right)}. \qquad (40)\end{aligned}$$

Now, we express $G\left(t,z\right)$, the cumulative distribution function of the innovation not discovered yet at time t, as a function of the un-normalized cumulative distribution function of the innovation not discovered yet at time t, which we denote by $H\left(t,z\right)$, so that $G\left(t,z\right)=\int_{-\infty}^{z}dH\left(t,x\right)/\int_{-\infty}^{\infty}dH\left(t,x\right)$. For clarity, we smooth H and consider the (Radon–Nikodym) derivative h, rewriting $G\left(t,z\right)$ as $G\left(t,z\right)=\int_{-\infty}^{z}h\left(t,x\right)dx/\int_{-\infty}^{\infty}h\left(t,x\right)dx$. Because the arrival rate of each innovation x is $\lambda m\left(z,x\right)$, we have that, for small $\Delta>0$,

$$\frac{h\left(t+\Delta,x\right)-h\left(t,x\right)}{\Delta}\approx-\lambda m\left(x,t\right)h\left(t,x\right),$$

or,

$$h\left(t+\Delta,x\right)\approx h\left(t,x\right)[1-\lambda m\left(x,t\right)\Delta].$$

Now, we can express $G_t\left(t,z\right)$ as:

$$G_t\left(t,z\right)=\lim_{\Delta\to0}\left(\frac{1}{\Delta}\left(\frac{\int_{-\infty}^{z}[1-\lambda m\left(x,t\right)\Delta]h\left(t,x\right)dx}{\int_{-\infty}^{\infty}[1-\lambda m\left(x,t\right)\Delta]h(t,x)dx}-\frac{\int_{-\infty}^{z}h\left(t,x\right)dx}{\int_{-\infty}^{\infty}h\left(t,x\right)dx}\right)\right),$$

by De L'Hopital rule,

$$
\begin{aligned}
G_t(t,z) &= \lim_{\Delta\to 0}\frac{d}{d\Delta}\left(\frac{\int_{-\infty}^{z}[1-\lambda m(x,t)\Delta]h(t,x)\,dx}{\int_{-\infty}^{\infty}[1-\lambda m(x,t)\Delta]h(t,x)dx}-\frac{\int_{-\infty}^{z}h(t,x)\,dx}{\int_{-\infty}^{\infty}h(t,x)\,dx}\right)\\
&= \lim_{\Delta\to 0}\left(\frac{\left[\begin{array}{c}\int_{-\infty}^{z}[-\lambda m(x,t)]h(t,x)\,dx\int_{-\infty}^{\infty}[1-\lambda m(x,t)\Delta]h(t,x)dx\\ -\int_{-\infty}^{\infty}[1-\lambda m(x,t)]h(t,x)dx\int_{-\infty}^{z}[1-\lambda m(x,t)\Delta]h(t,x)\,dx\end{array}\right]}{\left(\int_{-\infty}^{\infty}[1-\lambda m(x,t)\Delta]h(t,x)dx\right)^2}\right)\\
&= \lambda\frac{\int_{-\infty}^{\infty}m(x,t)\,h(t,x)dx\int_{-\infty}^{z}h(t,x)\,dx-\int_{-\infty}^{z}m(x,t)\,h(t,x)\,dx\int_{-\infty}^{\infty}h(t,x)dx}{\left(\int_{-\infty}^{\infty}h(t,x)dx\right)^2}.
\end{aligned}
$$

Substituting in the above expression the formula $m(x,t)=\max\{[x-z_0(t)]/c,0\}$, we obtain that for $z\geq z_0(t)$,

$$
\begin{aligned}
G_t(t,z) &= \frac{\lambda}{c}\frac{\int_{-\infty}^{z}h(t,x)\,dx\int_{-\infty}^{\infty}[x-z_0(t)]\,h(t,x)dx-\int_{-\infty}^{z}[x-z_0(t)]\,h(t,x)\,dx\int_{-\infty}^{\infty}h(t,x)dx}{\left(\int_{-\infty}^{\infty}h(t,x)dx\right)^2}\\
&= \frac{\lambda}{c}\frac{\int_{-\infty}^{z}h(t,x)\,dx\int_{-\infty}^{\infty}xh(t,x)dx-\int_{-\infty}^{z}xh(t,x)\,dx\int_{-\infty}^{\infty}h(t,x)dx}{\left(\int_{-\infty}^{\infty}h(t,x)dx\right)^2},
\end{aligned}
$$

and

$$
dG_t(t,z)=\frac{\lambda}{c}\frac{h(t,z)\int_{-\infty}^{\infty}xh(t,x)dx-zh(t,z)\int_{-\infty}^{\infty}h(t,x)dx}{\left(\int_{-\infty}^{\infty}h(t,x)dx\right)^2}
$$

Now, we can substitute in expression (40) the above expressions for $G_t(t,z)$ and $dG_t(t,z)$,

to obtain:

$$
\begin{aligned}
z_0'(t) &\propto \int_{z_0(t)}^{\infty}zh(t,z)\left[\int_{-\infty}^{\infty}xh(t,x)dx-zx\int_{-\infty}^{\infty}h(t,x)dx\right]dz\\
&\quad+\int_{-\infty}^{z_0(t)}z_0(t)\left[h(t,z)\int_{-\infty}^{\infty}xh(t,x)dx-zh(t,z)\int_{-\infty}^{\infty}h(t,x)dx\right]dz
\end{aligned}
$$

$$
\propto\int_{z_0(t)}^{\infty}h(t,z)\,z\left[\frac{\int_{-\infty}^{\infty}xh(t,x)dx}{\int_{-\infty}^{\infty}h(t,x)dx}-z\right]dz+z_0(t)\int_{-\infty}^{z_0(t)}h(t,z)\left[\frac{\int_{-\infty}^{\infty}xh(t,x)dx}{\int_{-\infty}^{\infty}h(t,x)dx}-z\right]dz.
$$

Letting $\bar{z}(t)=\int_{-\infty}^{\infty}xh(t,x)dx\ /\int_{-\infty}^{\infty}h(t,x)dx$, we rewrite

$$
\begin{aligned}
z_0'(t) &\propto \frac{\int_{z_0(t)}^{\infty}zh(t,z)\,[\bar{z}(t)-z]\,dz}{\int_{-\infty}^{\infty}h(t,z)dz}+z_0(t)\frac{\int_{-\infty}^{z_0(t)}h(t,z)\,[\bar{z}(t)-z]dz}{\int_{-\infty}^{\infty}h(t,z)dz}\\
&< z_0(t)\frac{\int_{z_0(t)}^{\infty}h(t,z)\,[\bar{z}(t)-z]\,dz}{\int_{-\infty}^{\infty}h(t,z)dz}+z_0(t)\frac{\int_{-\infty}^{z_0(t)}h(t,z)\,[\bar{z}(t)-z]dz}{\int_{-\infty}^{\infty}h(t,z)dz}=0,
\end{aligned}
$$

where the strict inequality folllows because $\int_{z_0(t)}^{\infty} zh(t,z) \; / \; \int_{-\infty}^{\infty} h(t,z)dz > z_0(t)$ and the final equality follows by definition of $\bar{z}(t)$.

Once concluded that $z_0'(t) < 0$, it immediately follows that $m_t(z,t) > 0$ for all $z \geq z_0(t)$. Then, it must be the case that $v_t(z,t) < 0$. Further, because $m_t(z,t) > 0$, it follows that $\bar{v}(z, m(z,t))$, which equals $\frac{\lambda}{r+\lambda m(z,t)} z$, decreases in t, and this leads to the existence of a time $T(z)$, after which R&D firms do not engage researchers in line z any further. Solving the equation $c = \frac{\lambda}{r+\lambda m(z,t)} z$, with $m(z,t) = \frac{z-z_0(t)}{c}$, we obtain the expression reported in the statement of the Proposition. ■

Proof of Proposition 7. Consider any R&D pair of R&D lines $1, 2$, that are active at time t, i.e., $\{z_0(t), \tilde{z}_0(t)\} < \{z_1, z_2\}$, and such that $z_1 < z_2$, without loss of generality.

Suppose by contradiction that $m(z_2,t) \leq \tilde{m}(z_2,t)$ and that $\tilde{m}(z_1,t) \leq m(z_1,t)$. Then, because $\lambda(\hat{m})\hat{m}$ is concave in $\hat{m}$, so that $\lambda(\hat{m}) + \hat{m}\lambda'(\hat{m})$ decreases in $\hat{m}$, we obtain the chain of inequality:

$$
\begin{aligned}
& \frac{z_2/z_1}{[z_2 - v(z_2,t)] / [z_1 - v(z_1,t)]} \frac{\lambda(m(z_2,t))}{\lambda(m(z_2,t)) + m(z_2,t)\lambda'(m(z_2,t))} \\
> \; & \frac{\lambda(m(z_2,t))}{\lambda(m(z_2,t)) + m(z_2,t)\lambda'(m(z_2,t))} = \frac{1}{1-\epsilon(m(z_2,t))} \\
\geq \; & \frac{1}{1-\epsilon(m(z_1,t))} = \frac{\lambda(m(z_1,t))}{\lambda(m(z_1,t)) + m(z_1,t)\lambda'(m(z_1,t))} \\
\geq \; & \frac{\lambda(m(z_1,t))}{\lambda(\tilde{m}(z_1,t)) + \tilde{m}(z_1,t)\lambda'(\tilde{m}(z_1,t))},
\end{aligned}
$$

which contradicts the equality

$$
\begin{aligned}
& \frac{\lambda(m(z_2,t))\, z_2}{[\lambda(\tilde{m}(z_2,t)) + \tilde{m}(z_2,t)\lambda'(\tilde{m}(z_2,t))]\,[z_2 - v(z_2,t)]} \\
= \; & \frac{\lambda(m(z_1,t))\, z_1}{[\lambda(\tilde{m}(z_1,t)) + \tilde{m}(z_1,t)\lambda'(\tilde{m}(z_1,t))]\,[z_1 - v(z_1,t)]},
\end{aligned}
$$

obtained dividing the equilibrium condition (23) by the optimality condition (24). ■

Proof of Proposition 9. We begin by calculating the aggregated welfare $W(m)$ associated with any allocation function m and associated density g.

The flow of aggregate welfare is expressed as:

$$
rW(m) = \int_{z_0}^{\infty} \lambda m(z) [z - m(z)c] g(z) \, dz.
$$

In this expression, innovations of value z are discovered at arrival rate $\lambda m(z)$, upon discovery they accrue value z to the aggregate welfare but the induce the aggregate cost $m(z)c$ as $m(z)$ researchers need to be allocated to different R&D lines (possibly of the same innovation value).

Substituting in the expression $m(z) g(z) = (\alpha/\lambda) f(z)$, and rearranging, we obtain:

$$
rW(m) = \alpha \int_{z_0}^{\infty} z f(z) \, dz - \alpha c \int_{z_0}^{\infty} m(z) f(z) \, dz. \tag{41}
$$

Now, we consider the equilibrium allocation function $m(z) = (z - z_0)/c$, so the second term in the expression (41) takes the form:

$$\alpha c \int_{z_0}^{\infty} m(z) f(z) dz = \alpha \int_{z_0}^{\infty} (z - z_0) f(z) dz,$$

which, substituted back into the expression (41), gives

$$rW(m) = \alpha z_0 [1 - F(z_0)] . \tag{42}$$

To gain some intuition for this expression, note that in equilibrium the researchers' expected payoffs for engaging in any R&D line are equal to the payoffs associate with engaging in a line with innovation value z_0. Continuing the analysis, we integrate condition (25) across z to obtain the expression $\lambda M = \alpha[1 - F(z_0)]$, and substitute this last expression into (42), so as to obtain expression (33) for the aggregate equilibrium welfare.

We now consider the aggregate welfare $W(\tilde{m})$ associated with the optimal allocation $\tilde{m}$. Taking the second term expression (41), simple algebraic manipulations show that:

$$\begin{aligned} \alpha c \int_{z_0}^{\infty} m(z) f(z) dz &= \alpha c \int_{z_0}^{\infty} \left(\frac{\sqrt{c\lambda r (z - z_0) + c^2 r^2} - cr}{c\lambda} \right) f(z) dz \\ &= (\alpha/\lambda) \int_{z_0}^{\infty} \sqrt{c\lambda r (z - z_0) + c^2 r^2} \cdot f(z) dz - rcM, \end{aligned}$$

using, again, $\lambda M = \alpha[1 - F(z_0)]$. By simplifying, we obtain the aggregate optimal welfare expression (34). ■