Abstract

DSGE models are used for analyzing policy and the sources of business cycles. A competing approach uses VARs that are partially identified using, for example, narrative shock measures and are often viewed as imposing fewer restrictions on the data. Narrative shocks are identified non-structurally through information external to particular models. This uses non-structural narrative shock measures to inform the structural estimation of DSGE models. Since fiscal policy has received much recent attention but the foundations of the fiscal side of DSGE models are less well studied than their monetary building block, fiscal DSGE models are a particularly promising application. Preliminary results from a standard medium-scale DSGE model support this argument: Structurally identified monetary shocks line up well with narrative measures, whereas government spending shocks do not. Extending the model to include distortionary taxes and more general fiscal policy processes, I find that model implied labor tax shocks line up well with narrative tax shocks. Including different narrative shock measures affects parameter identification and implied measures such as fiscal multipliers.

Keywords: Fiscal policy; DSGE model; Bayesian estimation; narrative shocks; Bayesian VAR

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1 Introduction

With monetary policy constrained by the Zero Lower Bound, “stimulating” fiscal policy has gained a lot of attention. To analyze its efficiency, DSGE models commonly used for monetary policy analysis have also been applied to the analysis of fiscal policy, see Christiano et al. (2009). While some papers have extended the fiscal sector of DSGE models for such a policy analysis, it is not clear whether their implications for fiscal policy are as plausible as the well-studied implications for monetary policy rules. Here, I ask whether the fiscal policy shocks are well identified and what features of DSGE models are crucial to replicate identified non-structural impulse-responses. A corollary result is an updated answer to the question of which shocks historically drive business cycles.¹

To give these answers, I make use of recent advances in VAR-based shock identification using narrative measures as instruments (Stock and Watson, 2012; Mertens and Ravn, 2013)). Narrative shocks are defined here to include literally narrative measures based on the reading of FOMC-meeting briefs in Romer and Romer (2004) or congressional records (Romer and Romer, 2010) as well as other time-series measures of surprise variation associated with shocks, such as government spending forecast errors in Ramey (2011). I embed the narrative shock identification scheme in a Bayesian VAR, which also allows me to naturally quantify the posterior uncertainty involved in the narrative identification scheme. When applying the new identification scheme based on the narrative fiscal measures in Ramey (2011), I show that the estimated DSGE model implies shocks which line up poorly with the narrative shock measures with a correlation of 0.15.² This is not by construction: The partially identified VAR implies correlations between narrative shock measures and the identified shocks between 0.46 and 0.58 (see the leftmost and rightmost graphs in Figure 1). Also, narrative measures of monetary policy and productivity line up well with estimated shocks. I extend the existing methodology for narrative shock identification in VARs by applying it to DSGE models: Narrative measures are interpreted as noisy measurements of the actual structural shocks (think: “instruments”).

¹Del Negro and Schorfheide (2009) find when comparing VAR evidence to DSGE model evidence in a framework comparable to Smets and Wouters (2007) that “most of the misspecification is concentrated in the response to government spending/demand shocks (g_t) and is large relative to the posterior uncertainty.” (p. 1433)
²Unless otherwise stated, estimates are reported at the posterior mean.
Structural DSGE models serve as a tool to assess the quantitative plausibility of “stories” about the causes of economic fluctuations and consequences of different policies. Sims (2005) cautions that economists may be set back “[i]f Bayesian DSGE’s displace methods that try to get by with weak identification and in the process reinforce the excess weight we give to story-spinning” (p. 2). The idea here is to discipline these stories further by learning from methods which have been developed to identify shocks with weak assumptions – without giving up on using structural models per se. It complements more heavily non-structural research such as Mian and Sufi (2012), who only use micro data to identify demand shocks, but then feed the micro-level shocks into a partially specified macro model.3

Formally, this paper is related to the literature on how different sets of observable variables influence the estimation of DSGE models. Boivin and Giannoni (2006) propose to augment the observation equations with additional measures of economic variables to aid in the identification of latent variables and parameters.

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3 Two recent job market papers also use DSGE model in their analysis, but then use non-traditional identification methods: Ferreira (2013) in his analysis of risk shocks also shies away of using an estimated DSGE model to back out shocks and instead uses only model-derived sign restrictions for fear of the DSGE model misspecification. Drautzburg (2013) identifies shocks from cross-sectional data relying only on part of the structural model and then uses them as an input into a dynamic structural model.
Guerron-Quintana (2010) and Canova et al. (2013) examine how different measures of endogenous variables affect the model estimation. Unlike Boivin and Giannoni (2006), I do not propose to use a wide range of noisy measures of general economic indicators, but focus instead on incorporating indicators of structural shocks directly. Insofar as I examine how these noisy observables affect inference, this mirrors Guerron-Quintana (2010) and Canova et al. (2013).\(^4\) The key difference is, however, that I aim to incorporate insights from non-structural identification schemes into the structural model estimation.

Information on narrative shocks affects structural parameters in economically meaningful ways, when applied to the Smets and Wouters (2007) model. Filtered shocks, for example those plotted on the vertical axes in Figure 1, enter the likelihood function and inference about structural shocks typically interacts with inference about parameters. I show that, for example, when including the Romer and Romer (2004) monetary policy shock measures as noisy shock measurements, the estimated monetary policy function changes significantly. Similarly, when using the Fernald (2012)-measure of capacity-adjusted TFP growth, the estimates of the aggregate production function change significantly (see Table 1 on page 22). Similar results hold for fiscal policy rules in an extended model which combines labor taxes and capital taxes as in Drautzburg and Uhlig (2011) with fiscal feedback rules for taxes and government consumption as in Leeper et al. (2010). Using narrative shocks from Mertens and Ravn (2013) for labor and capital taxes with and without additional government spending forecast errors from Ramey (2011) produces significant differences in the inferred fiscal and monetary policy rules as well as for the capital share and adjustment cost.\(^5\)

In preliminary results, matching narrative government spending shocks and BVAR evidence on the effect of government spending shocks has been particularly challenging. In ongoing work I consider both generalizing shock processes as in Del Negro and Schorfheide (2009) and introducing productive or complementary government spending as in Baxter and King (1993), Drautzburg and Uhlig

\(^4\)Also Negro et al. (2013) find that including extra data on bond spreads significantly changes the estimated degree of nominal rigidities in their model. Guerron-Quintana (2013) uses information on a cross-section of countries in addition to the time dimension to aid in the estimation.

\(^5\)Popular summary measures of the fiscal policy implications of my model are also affected by the choice of narrative instrument. Preliminary results indicate that using an external instrument to identify the government spending shock yields a median impact multiplier of 1.04, whereas using an external instrument to identify TFP shocks changes parameter estimates to yield a lower median impact multiplier of 0.91.
(2011), and Coenen et al. (2012). Generalizing the shock processes would leave the conclusions in standard DSGE models for counterfactual fiscal policy experiments unchanged, but preliminary results find little support in the data for an exogenous government spending process resembling the hump-shaped VAR response.

Anticipation (“news”) effects of fiscal policy can be important and future extensions will incorporate measures on fiscal news as well. A narrative measure on future defense spending shocks are available from Ramey (2011). Tax news can be extracted from municipal bond spreads as in Kueng (2013).

Extensions to other fiscal instruments are natural. Using the tax instruments in Mertens and Ravn (2013) provides an alternative way to identify fiscal shocks compared to the parametric approach in Leeper et al. (2010). As argued by Drautzburg and Uhlig (2011), the financing matters for evaluating fiscal policy even in the presence of the Zero Lower Bound. Additionally, the survey forecast errors used to identify federal spending shocks in Ramey (2011) are also available from the Survey of Professional forecasters for state and local government spending on consumption and investment, which makes up more than half of overall government spending since 1972 and has risen to over 60% in 2013.

Fully structural and partial identification of fiscal shocks can lead to widely different conclusions about the drivers of business cycles: Rossi and Zubairy (2011) document that when applying a Blanchard and Perotti (2002)-type identification of government spending shocks in a VAR, the fraction of the variance of GDP driven by those shocks rises with the forecast horizon from below 5% at four quarters to 35% at 40 quarters (their Table 2), while the DSGE-model based variance decomposition in (Smets and Wouters, 2007, Figure 1) implies that the output variance explained by GDP falls from roughly 30% at one quarter to about 15% at four quarters and less than 5% at 40 quarters. Future extensions are going to assess how narrative shocks affect inference on the estimated causes of business cycles. This can be expected to become more important when measurement error is present, as discussed in Section 2.

This paper is structured as follows: Section 2 discusses shock identification in DSGE models in a simple example. Section 3 describes the methodology for the

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6 Assuming policy-invariance of the remaining estimated parameters.
7 The posterior density falls by 18 to 20 log points when an AR(2) process is imposed whose roots are chosen to resemble the government spending response in a VAR.
8 See, among others, Ramey (2011) for government spending shocks in VARs and Christiano et al. (2014) for an analysis in an estimated DSGE model.
narrative DSGE and BVAR estimation. Section 4 presents results from estimating the basic Smets and Wouters (2007) model and an extension with distortionary taxes and government debt with and without using narrative shocks. A comparison to the results from a narrative VAR is provided. Section 5 concludes.

2 Shock and parameter identification in the workhorse New Keynesian model

This section discusses in the context of a simple New Keynesian model how narrative shocks affect parameter estimation and shock identification. To fix ideas, consider the canonical 3-equation New Keynesian model, e.g. (Galí, 2009, ch. 3). Since I am interested in fiscal policy, I consider a version with government spending shocks $e^g_t$ and monetary policy shocks $e^m_t$. Shocks are, to simplify, iid standard normally distributed. The log-deviations of output $y_t$ and inflation $\pi_t$ are determined by the following pair of equations

$$
\begin{bmatrix}
y_t \\
\pi_t
\end{bmatrix} = A E_t \begin{bmatrix} y_{t+1} \\
\pi_{t+1}
\end{bmatrix} + B \begin{bmatrix} a_t \\
v_t
\end{bmatrix},
\begin{bmatrix}
E_t[e^g_t - e^g_{t+1}] \\
e^m_t
\end{bmatrix} \sim N(0, I_2),
$$

(2.1)

where $A$ is a stable matrix, and $B$ is of full rank. Given that shocks are $iid$, the locally stable New Keynesian model solution is simply given by:

$$
\begin{bmatrix}
y_t \\
\pi_t
\end{bmatrix} = B \begin{bmatrix} e^g_t \\
e^m_t
\end{bmatrix}.
$$

(2.2)

Clearly, since $B$ is invertible, observing output and inflation perfectly reveals the underlying shocks exactly as it does in a fundamental, identified VAR. Identification depends, however, on the structural parameters which determine

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9To simplify, I set the labor share in Galí (2009) to one.

10In terms of primitives, the matrices $A$ and $B$ are given by:

$$
A = \begin{bmatrix}
\tilde{\sigma} & 1 - \beta \phi_x
\\
\kappa \tilde{\sigma}(\tilde{\sigma} + \psi) & \beta(\tilde{\sigma} + \phi_y) + \kappa(\sigma + \psi)
\end{bmatrix} \Omega, 
B = \begin{bmatrix}
\tilde{\sigma} + \kappa \tilde{\sigma} \psi \phi_x \\
\kappa \tilde{\sigma}(\psi - \phi_y) - \kappa(\sigma + \psi)
\end{bmatrix} \Omega, 
\Omega = \frac{1}{\sigma + \phi_y + \kappa \phi_x (\sigma + \psi)}.
$$

where $\kappa$ is a measure of price flexibility, $\tilde{\sigma} = \frac{1}{\bar{c} / \bar{y}}$ is the inverse of the IES of substitution divided by the consumption share in GDP, $\psi$ is (for $\sigma = 1$) the inverse Frisch elasticity of labor supply, and $\phi_x, \phi_y$ are the reaction coefficients in the Taylor rule with respect to inflation and output. $\beta$ is the discount factor.

11See Fernandez-Villaverde et al. (2007) for general conditions.
the entries of the $B$ matrix. To see how having access to narrative information in this context helps in inference, assume that we know the (scaled) inverse intertemporal elasticity of substitution $\tilde{\sigma}$ and the elasticity of labor supply $\psi^{-1}$, but were uncertain about the degree of price stickiness indexed by $\kappa^{-1}$.

Observing narrative shocks translates into additional information about the structural parameters. Assume that the narrative shock measure for government spending is $z^g_t$ and satisfies the following equation:

$$z^g_t = c_g z^g_t + \omega_g u^g_t, \quad u^g_t \sim N(0, 1), \quad (2.3)$$

where the loading on the structural shock $c_g$ and the size of the measurement error $\omega_g$ are considered known.\(^\text{12}\) Following the example above with known preference parameters $\sigma, \psi$, the narrative shock (2.3) together with the New Keynesian Philips curve can be inverted to yield the price stickiness $\kappa^{-1}$ in terms of (noisy) observables and otherwise given parameters:\(^\text{13}\)

$$\kappa^{-1} = \frac{y_t(\bar{\sigma} + \psi) - \tilde{\sigma} g_t}{\pi_t} = \frac{y_t(\bar{\sigma} + \psi) - \tilde{\sigma} (c^{-1}_g z^g_t - \omega_g u^g_t)}{\pi_t} \sim N \left( \frac{y_t(\bar{\sigma} + \psi) - \tilde{\sigma} c^{-1}_g z^g_t}{\pi_t c_g}, \left( \frac{\tilde{\sigma}_g}{\pi_t c_g} \right)^2 \right). \quad (2.4)$$

With a flat prior on $\kappa^{-1}$ and known preference parameters, (2.4) gives the posterior over $\kappa^{-1}$ after one observation of the narrative shock.

In practice, other structural parameters are typically not known exactly a priori, and there is feedback between the inference on the different structural parameters as there are competing estimates of the government spending shock in the data:

$$\epsilon^g_t = \frac{1}{\sigma} ((\bar{\sigma} + \psi) \hat{y}_t - \kappa^{-1} \pi_t) = \frac{1}{c_g} (z^g_t - \omega_g u^g_t). \quad (2.5)$$

Averaging out over the measurement error, the posterior over the structural parameters adjusts to ensure consistency between the model-based and the narrative

\(^\text{12}\) Whether the loading $c_c$ is known depends on the narrative instrument at hand. Measures based on survey or market expectational errors of policy suggests a loading of $c_c = 1$. Knowing one of the parameters reveals the other, with enough data. Fix the standard deviation of the measurement error and impose a sign restriction common in the VAR literature, to get $c_g = \sqrt{\frac{\text{var}[z^g_t] - \omega^2_g}{\text{var}[\epsilon^g_t]}}$.\(^\text{13}\) The NKPC is given by $\pi_t = \beta E_t[\pi_{t+1}] + \kappa(\bar{\sigma} + \psi) \hat{y}_t - \kappa \tilde{\sigma} g_t$. The result can also be derived from (2.2).
shock measure. In the case of the standard Smets and Wouters (2007) model, Table 1 shows that several key parameters are sensitive to the inclusion of noisy narrative “instruments” for the structural shocks.

In addition to indirect effects on shock identification via structural parameters, narrative shocks can affect shock identification directly when endogenous variables are observed with error. If, as in one of the two cases considered in Sargent (1989), the econometrician does not observe output and inflation perfectly, identification of shocks (given structural parameters) is no longer exact: With measurement error, the true economic shocks become latent factors. Boivin and Giannoni (2006) and Del Negro and Schorfheide (2009) make a case for allowing this type of measurement error in DSGE models – and therefore, effectively, to relax the strict identifying assumptions and to prevent exact identification exemplified in (2.2).

Formally, assume that $Y_t^{obs} = [y_t, \pi_t] = B[e_t^q, e_t^m] + CU_t, U_t \sim N(0, I_2)$. Then knowledge of $Y_t^{obs}$ induces the following belief:

$$
\epsilon_t \equiv \begin{bmatrix} e_t^q \\ e_t^m \end{bmatrix} | Y_t^{obs} \sim N(B^{-1}Y_t, \text{Var}), \quad \text{Var} \equiv B^{-1}CC'(B^{-1})'.
$$

(2.6)

Narrative shocks generally lower the posterior uncertainty in this case with given parameters and measurement error on all observables. Generalize equation (2.3) to

$$
Z_t = c\epsilon_t + \omega\tilde{U}_t, \quad \tilde{U}_t \sim N(0, I).
$$

(2.7)

Given the prior (2.6) and the signal $Z_t$, the posterior over the structural shocks is given by:

$$
\epsilon_t | Y_t^{obs}, Z_t \sim N(B^{-1}Y_t^{obs} + K(Z_t - cB^{-1}Y_t^{obs}), \text{Var}')
$$

(2.8)

$$
\text{Var}' \equiv \text{Var} - \text{Var} c'(c \text{Var} c' + \omega\omega')^{-1} c \text{Var} \leq \text{Var},
$$

$$
K \equiv \text{Var} c'(c \text{Var} c' + \omega\omega')^{-1},
$$

where the matrix inequality implies that the matrix difference is positive semi-

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14I am assuming that there is a finite sample so that even if parameters are identified, the narrative measures help to increase the posterior precision. Beyond this case, an interesting question remains for future research whether narrative shocks can alleviate some of the weak or non-identification problems documented, for example, in Canova and Sala (2009).

15Note that allowing for measurement error generally requires restricting the dynamics of the measurement error when parameters are unknown. Consider $u_t = \epsilon_t + f_t$ with $u_t$ the sole observable. Identifying the parameters governing the evolution of $\epsilon_t$ requires restricting the dynamics of $f_t$. 

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definite: There is less uncertainty about the structural shocks when narrative instruments are available. This is useful when considering historical decompositions, one of the applications of structural models.

Besides measurement error, also variable transformations prior to estimation can preempt exact shock identification by introducing uncertain initial conditions. For example, in Smets and Wouters (2007), only first difference of (log) output and its components are observed. However, this seems quantitatively less important: Moving beyond the above toy model, Figure 12 in the appendix investigates the posterior precision for the smoothed shocks in the full-fledged Smets and Wouters (2007) model. It shows that the posterior (Kalman-smoothed) standard deviation of structural innovations in the Smets and Wouters (2007) model varies between 2% and 7% of the estimated prior standard deviation.\footnote{Applying the “ABCD” test in Fernandez-Villaverde et al. (2007) verifies that changes the observables from levels to growth rates implies that the model has an invertible VAR-representation. Figure ?? shows that when only levels, rather than growth rates, are used in the estimation, all shocks which are not MA-processes themselves are perfectly revealed by the data, as in the simple example above.}

3 DSGE and BVAR estimation with narrative instruments

3.1 Narrative DSGE model

Consider the canonical state-space representation of linearized DSGE models (e.g. Fernandez-Villaverde et al., 2007) with observed endogenous variables $Y_t$ and state variables $X_t$:

$$
X_t = AX_{t-1} + B\epsilon_t \quad \quad \epsilon_t \overset{iid}{\sim} N(0, I_m), \quad (3.1a)
$$

$$
Y_t = CX_{t-1} + D\epsilon_t + Eu_t \quad \quad u_t \overset{iid}{\sim} N(0, I_{m'}) \quad (3.1b)
$$

where measurement error $u_t$ is treated separately from structural shocks $\epsilon_t$.

In the baseline specification and following Smets and Wouters (2007), the observation equations do not contain any measurement error, i.e. $E = 0$. However, as discussed in Section 2, since the observation equation consists of several growth rates rather than level equations, uncertain initial conditions enter the inference about shocks.
To implement the idea of a narrative DSGE-model estimation, a subset $Z_t$ of the observables $Y_t$ contains signals about the underlying structural shocks. Ordering these narrative shocks first in the vector $Y_t$, the associated observation equations take the following form:

$$Z_t^{(i)} = C^{i,0} X_{t-1} + \begin{bmatrix} D^{(i,j)} & 0 & \ldots & 0 \end{bmatrix} \epsilon_t + \begin{bmatrix} E^{(i,i)} & 0 & \ldots & 0 \end{bmatrix} u_t$$

Here, the narrative measure $i$ is a signal of shock $j$. As becomes clear when looking at particular examples in (3.2), typically the row vector $C^{i,0}$ equals the zero vector, but when there is no unit root in the model for TFP or government spending does not follow a simple AR(1) process, $C^{i,0}$ is still sparse, but has some non-zero entries. For $(E^{(i,i)})^2 \rightarrow 0$, the observation equation (3.1b) reveals the structural shock arbitrarily well.\(^{17}\)

The additional measurement equations I consider in practice are all or some of the following:

\[
\begin{align*}
    z_t^m &= c_m \epsilon_t^m + \omega_m u_t^m, \\
    z_t^{\tau,n} &= c_{\tau,n} \epsilon_t^{\tau,n} + \omega_{\tau,n} u_t^{\tau,n}, \\
    z_t^{\tau,k} &= c_{\tau,k} \epsilon_t^{\tau,k} + \omega_{\tau,k} u_t^{\tau,k}, \\
    z_t^a &= c_a \Delta a_t + \omega_a u_t^a, \\
    z_t^g &= c_g (g_t - g_{t-1} - E_{t-1}[g_t - g_{t-1}]) + \omega_g u_t^g,
\end{align*}
\]

where $u_t^i \text{ iid } \sim N(0, 1)$. The constants $c_o > 0$ reflect the fact that the scale of the narrative shocks is unknown, but the sign of its correlation with the underlying disturbance is known.\(^{18}\)

Note that (3.1b) in general and (3.2) specifically do not require $Y_t^{\text{obs}}$ to be fully observed in every time period. There is also no requirement that missing observations be missing at random. (3.2) only states that conditional on observing a narrative shock, it is conditionally normally distributed. It is therefore consistent

\[^{17}\text{In that case, however, if the number of shocks matches the number of observables, the system comes close to being singular.}\]

\[^{18}\text{Given that } \epsilon_t^m \text{ follows some normal prior distribution without narrative information, i.e. } \epsilon_t^m \sim N(\mu_t, \sigma_t^2), \text{ given a signal } z_t^m \sim N(\mu_m \epsilon_t^m, \sigma_m^2 + \sigma_t^2), \text{ it follows that the posterior after observing the signal is given by: } \epsilon_t^m \sim N(\mu_t + \frac{c_m \sigma_t^2}{\sigma_m^2 + \sigma_t^2} (z_t^m - \epsilon_m \mu_t), \frac{\sigma_m^2}{\sigma_m^2 + \sigma_t^2} / \sigma_m^2)).\]

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with a scenario in which a narrative measure is more likely to be observed when structural shocks are “large”. This could be the case if, for example, the researcher compiling the narrative series searches more carefully for narrative measures for times with a known large variation in, say, nominal interest rates.

While a host of narrative, or external, instruments is available (cf. Stock and Watson, 2012), the validity of external instruments in a structural model cannot be taken for granted. For example, the identification of government spending shocks in Ramey (2011) using Survey of Professional Forecasters data fails to identify exogenous government spending components in a world in which government spending has an endogenous component which is not predetermined. One example violating this assumption is if government spending reacts contemporaneously to endogenous drops in GDP, as in Leeper et al. (2010). To address this, (3.2e) lines up the observed mean forecast error in the data with the model analogue.\footnote{If genuine government spending shocks are particularly important for the forecast errors, (3.2e) can still be expected to be very informative about \( \epsilon^g_t \). For other external instruments, such as technology shocks identified with the Gali (1999) long-run restrictions, one might be concerned whether they are reliable in finite samples in the presence of shocks to distortionary taxes (Uhlig, 2004), even though they are asymptotically valid with stationary tax rates.}

Once priors over the parameters of (3.2) are specified\footnote{I normalize the sample standard deviation of \( z^\circ_t \) prior to the estimation to unity, and have the prior that each \( c_\circ \) follows an inverse Gamma distribution with mean 0.5 and a relatively large standard deviation of 2. I either fix \( \omega_\circ = 0.25 \) (or 0.125 for tax shocks) or impose an inverse Gamma prior with mean of 0.1 and standard deviation of 0.2.\label{footnote1}}, standard Bayesian procedures for linear DSGE model (DelNegro and Schorfheide, 2011) can be applied. Here, I implement the estimation with Adjemian et al. (2011).

### 3.2 Narrative BVAR

In this section, I follow the exposition in Mertens and Ravn (2013) to derive identifying restrictions based on narrative shock measures.\footnote{Section ?? in the appendix follows the derivation in Stock and Watson (2012).} I first discuss population results without parameter uncertainty to fix ideas. Then I proceed to discuss the sampling scheme and compare it to the sampling scheme in Mertens and Ravn (2013).

I use the following notation for the data generating process:

\[
Y_t = BY_{t-1} + v_t \quad \text{(3.3a)}
\]

\[
v_t = A\epsilon^{str}_t, \epsilon^{str}_t \sim \text{iid} N(0, I_m) \quad \text{(3.3b)}
\]
\[ z_t = F v_t + \Omega^{-1/2} u_t, u_t \overset{iid}{\sim} \mathcal{N}(0, I_k) \]  

(3.3c)

Here, \( Y_t \) is the observed data, \( B \) is a matrix containing the (possible stacked) lag coefficient matrices of the equivalent VAR(p) model as well as constants and trend terms, \( v_t \) is the \( m \)-dimensional vector of forecast errors, and \( z_t \) contains \( k \) narrative shock measures.

Note that \( v_t \) is data conditional on \( D \). We can thus also observe \( \text{Var}[v_t] = AA' \equiv \Sigma \). Note that \( A \) is identified only up to an orthonormal rotation: \( \tilde{A}\Phi(\tilde{A}\Phi)' = AA' \) for \( \tilde{A} = \text{cho}(\Sigma) \) and \( \Phi\Phi' = I \).

The observation equation for the narrative shocks (3.3c) can alternatively be written as:

\[ z_t = G \epsilon_t^{str} + \Omega^{-1/2} u_t = \underbrace{G \tilde{A}^{-1}\Phi^{-1} v_t}_{F} + \Omega^{-1/2} u_t \]  

(3.4)

\( F \) can be estimated. \( G \), or equivalently \( A \), is of interest.

### 3.2.1 Population results

Following Mertens and Ravn (2013), consider the case of as many instruments as shocks to be identified, with \( k \leq m \).

Following them, partition \( A = [\alpha^{[1]}, \alpha^{[2]}] = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \), \( \alpha^{[1]} = [\alpha_{11}', \alpha_{21}']' \) with both \( \alpha_{11}(k \times k) \) being invertible and \( \alpha_{21}((m - k) \times k) \).

As before:

\[ \text{Cov}[z_t, v_t] = F\Sigma \Rightarrow F = \text{Cov}[z_t, v_t]\Sigma^{-1} = \text{Cov}[z_t, v_t](A')^{-1}A^{-1}. \]

Also:

\[ \text{Cov}[z_t, v_t] = \text{Cov}[z_t, \epsilon_t^{str}] = \begin{bmatrix} G & 0 \end{bmatrix} A' = G\alpha_1' = [G\alpha_{11}', G\alpha_{21}'] \]  

(3.5)

Consequently, since \( \alpha_{11} \) is invertible:

\[ G\alpha_{11}' = \text{Cov}[z_t, v_t]_1, \]  

(3.6)
and

$$\alpha'_{21} = G^{-1} \text{Cov}[z_t, v_t]_2 = \alpha'_{11} (\text{Cov}[z_t, v_t]_1^{-1} \text{Cov}[z_t, v_t]_2), \equiv \alpha'_{11} \kappa'$$  \hspace{1cm} (3.7)

where \( \text{Cov}[z_t, v_t] = F \Sigma \) is in terms of observables.

Hence the (structural) impulse-vector to shocks 1, \ldots, \( k \) satisfies:

$$\alpha^{[1]} = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \end{bmatrix} = \begin{bmatrix} I_k \\ (\text{Cov}[z_t, v_t]_1^{-1} \text{Cov}[z_t, v_t]_2)' \end{bmatrix} \alpha_{11},$$  \hspace{1cm} (3.8)

where \( \text{Cov}[z_t, v_t]_2 \) is \( k \times \( m - k \) \) and \( \text{Cov}[z_t, v_t]_1 \) is \( k \times k \). This \( m \times k \) dimensional vector is a known function of the \( k^2 \) parameters in \( \alpha_{11} \). It therefore restricts \( (m-k)k \) elements of \( A \).

When each of the \( k \) instruments identifies only one of the \( m \) shocks, then each instrument delivers \( (m - 1) \) identifying restrictions: Each instrument then identifies one column of \( A \) up to scale. To see this, consider (3.5) for the case of \( G = \text{diag}([g_1, \ldots, g_k, 0, \ldots, 0]) \) and let \( \alpha^{[i]} \) now denote the \( i \)'th column of \( A \):

$$\text{Cov}[z^{[i]}_t, v_t] = e_i G A' = g_i e_i e_i' A' = g_i (\alpha^{[i]})'.$$  \hspace{1cm} (3.9)

The following proposition summarizes the above results:

**Proposition 1** *Stock and Watson, 2012; Mertens and Ravn, 2013* The impact of shocks with narrative instruments is generally identified up to a \( k \times k \) scale matrix \( \alpha_{11} \) whose outer product \( \alpha_{11} \alpha'_1 \) is known, requiring an extra \( \frac{(k-1)k}{2} \) identifying restrictions and the impulse vector is given by (3.8). If instruments are known to predict only one shock, the scale matrix is diagonal and response vectors are identified up to sign and scale. The identified impact vectors then each satisfy (3.9).

*Proof:* See Appendix A.1.

The next section considers how inference is affected when the covariances underlying the identification scheme has to be estimated.

### 3.2.2 Posterior uncertainty

Note that conditional on \( F \) and \( \Sigma \), the above identification is well defined. Here, I consider the case when the posterior over \( F \) is non-degenerate. For now, I abstract
from potential weak instruments, which is discussed among others in Kleibergen and Zivot (2003) and surveyed in Lopes and Polson (2014).

Recall equation 3.3c:

\[ z_t = Fv_t + \Omega^{-1/2}u_t, u_t \overset{iid}{\sim} \mathcal{N}(0, I_k). \] (3.3c)

The object of interest in the above algorithm is \( \text{Cov}[: ] = F\Sigma \). I now describe the hierarchical procedure to draw from the posterior of \( \Sigma \) (and \( B \)) and \( F \).

Following, for example, Uhlig (1994), the posterior for \( \Sigma^{-1} \) follows from the stacked form of the VAR and is given by this hierarchical model:

\[
\begin{align*}
\Sigma^{-1} &\sim \mathcal{W}_n((\nu_T S_T)^{-1}, \nu_T), \quad \text{(3.10a)} \\
\nu_T &= \nu_0 + T, \quad \text{(3.10b)} \\
N_T &= N_0 + X'X, \quad \text{(3.10c)} \\
S_T &= \frac{1}{\nu_T} (\hat{B} - B_0)' N_0^{-1} X' X (\hat{B} - B_0) + \frac{\nu_0}{\nu_T} S_0 + \frac{\nu_T - \nu_0}{\nu_T} \hat{\Sigma} \quad \text{(3.10d)}
\end{align*}
\]

Here, \( \hat{B} = (X'X)^{-1}X'Y, B_T = N_T^{-1}(N_0B_0 + X'X\hat{B}), \hat{\Sigma} = T^{-1}Y'(I - X(X'X)^{-1}X')Y \) and

\[ B|\Sigma \sim \mathcal{N}(B_T, \Sigma \otimes N_T^{-1}) \] (3.11)

In the computations I am using the flat prior suggested by Uhlig (1994) with \( \nu_0 = 0, N_0 = 0 \).

Conditional on \( B, \Sigma \), I know \( V = Y_t - XB \):

\[
\begin{align*}
\Omega^{-1} &\sim \mathcal{W}_k((\nu_{TF} S_{TF})^{-1}, \nu_{TF}^F), \quad \text{(3.12a)} \\
\nu_{TF}^F &= \nu_0^F + T_F, \quad \text{(3.12b)} \\
N_{TF}^F &= N_0^F + V'V, \quad \text{(3.12c)} \\
S_{TF}^F &= \frac{1}{\nu_{TF}^F} (\hat{B} - B_0)' N_0^F (N_{TF}^F)^{-1} V' V (\hat{B} - B_0) + \frac{\nu_0^F}{\nu_{TF}^F} S_0^F + \frac{\nu_{TF}^F - \nu_0^F}{\nu_{TF}^F} \hat{\Omega} \quad \text{(3.12d)}
\end{align*}
\]

Here, \( \text{vec}(F) = (V'V)^{-1}V' \mathbf{n}, \text{vec}(F)_{TF} = (N_{TF}^F)^{-1}(N_0^F \text{vec}(F)_0 + V'V \text{vec}(F)), \hat{\Omega} = T_F^{-1}(\mathbf{n})(I - V(V'V)^{-1}V')\mathbf{n} \) and

\[ \text{vec}(F)|\Omega \sim \mathcal{N}(\text{vec}(F_{TF}), \Omega \otimes (N_{TF}^F)^{-1}) \] (3.13)
Consequently, conditional on $\Sigma$, $\text{vec}(F)$, the covariance is known:

$$
\text{Cov}[n, V] = F \Sigma^{-1} \Rightarrow \text{vec}(\text{Cov}[n, V]) = (\Sigma^{-1} \otimes I)\text{vec}(F)
$$

$$
\Rightarrow \text{vec}(\text{Cov}[n, V])|_{\Omega, \Sigma} \sim \mathcal{N}((\Sigma^{-1} \otimes I)\text{vec}(F_{T_T}), (\Sigma^{-1} \otimes I)(\Omega \otimes (N_{T_T}^F)^{-1})(I \otimes \Sigma^{-1}))
$$

(3.14)

Note that a weak instrument problem may materialize if $F \approx 0$.

Section A.1.1 in the appendix describes the corresponding simple hierarchical sampling algorithm.

3.2.3 Comparison with Mertens and Ravn (2013) (under construction)

Mertens and Ravn (2013) procedure is characterized as follows:

1. For $t = 1, \ldots, T$, draw $\{e^b_1, \ldots, e^b_T\}$, where $e^b_t \sim \text{iid}$ with $\Pr\{e^b_t = 1\} = \Pr\{e^b_t = -1\} = 0.5$.

2. Construct the artificial data for $Y^b_t$. In a VAR of lag length $p$, build $Y^b_t, t > p$ as:

   • For $t = 1, \ldots, p$ set $Y^b_t = Y_t$.
   • For $t = p + 1, \ldots, T$ construct recursively $Y^b_t = \sum_{j=1}^p \hat{B}_j Y^b_{t-j} + e^b_t \hat{u}_t$.

3. Construct the artificial data for the narrative instrument:

   • For $t = 1, \ldots, T$ construct recursively $z^b_t = e^b_t z^b_t$.

4 Empirical specification and preliminary results

In this section I investigate the effect of varying the econometrician’s information set on DSGE model estimates. The baseline model largely follows Smets and Wouters (2007). Two different model specifications are considered: The plain Smets and Wouters (2007) model, and a model extension distortionary labor and capital taxes,
as well as endogenous government spending, following Drautzburg and Uhlig (2011), and with fiscal policy rules as in Leeper et al. (2010).  

For each of the two specifications, I re-estimate the model with and without narrative shocks and examine the implications on structural parameters. In ongoing work, I also consider implications for fiscal and monetary policy as well as for the drivers of business cycles.

4.1 Data

I use an updated version of the Smets and Wouters (2007) dataset for the baseline DSGE model estimation. I follow their variable definitions, with the exception of consumer durables, which I include among the investment goods. I also extend the sample period to 1947:Q1 to 2007:Q4, stopping before the ZLB became binding. Sources for narrative shock measures are Romer and Romer (2004) for monetary policy shocks, Fernald (2012) for TFP growth, and the Survey of Professional Forecasters’ government spending forecast errors from Ramey (2011) for government spending. In ongoing work I update the Leeper et al. (2010) dataset for average tax rates and debt and extend it to cover debt and revenue by state and local governments uses NIPA and Flow of Fund data. The corresponding tax instruments are taken from Mertens and Ravn (2013) subset of the shocks in Romer and Romer (2010).

While it is arguably important to include a long sample covering WWII and the Korean War to have important instances of exogenous variation in government spending (cf. Ramey, 2011), I consider subsamples of the data before and after 1982 in robustness exercises.

I also consider an alternative measure of hours worked, taken from Francis and Ramey (2009). They show that government hours differ in the early post-war period

---

23 As discussed below, when extending the model to include fiscal policy rules, I assume that the monetary authority does not consider the output gap as in Smets and Wouters (2007), but looks instead at detrended output.

24 A significant downside of this strategy is that one period of significant fiscal changes during peacetime is omitted.

25 The estimated BVAR models also use data from Ramey (2011) and Mertens and Ravn (2013).

26 Bohn (1991) uses both annual data from 1792 to 1988 and quarterly data from 1954 to 1988. He argues that long samples are preferable because debt is often slow moving and because it contains more episodes such as wars which might otherwise appear special. However, he finds that the results for the shorter quarterly sample are qualitatively unchanged.
systematically from private hours worked. This can be particularly relevant when examining the effect of fiscal spending on the economy.

I follow Leeper et al. (2010) in constructing time series for taxes, except for including state and local tax revenue in the calculation of revenue, tax, and debt data, similar to Fernandez-Villaverde et al. (2011). The significant share of state and local governments in both government debt and spending motivates this broader definition: Using Flow of Fund (FoF) data to initialize the time series of government debt at par value, I calculate debt levels using the cumulative net borrowing of all three levels of government. Municipal debt as a share of total government debt has increased from little over 5% in 1945 to around 35% in 1980 before falling to about 20% in 2012 according to FoF data. The share of state and local governments in government consumption and investment has been above 50% since 1972. Details are given in Appendix A.2.

4.2 Model specification (under construction)

The empirical model builds on the well-known Smets and Wouters (2007) model with monopolistic competition in intermediate goods markets and the labor market and Calvo frictions to price and wage adjustment, partial price and wage indexation, and real frictions such as investment adjustment cost and habit formation. Ongoing work includes labor, capital, and consumption taxes as in Drautzburg and Uhlig (2011) and fiscal rules as in Leeper et al. (2010).

4.2.1 Fiscal and monetary policy

The monetary authority sets interest rates according to the following rule:

\[ \hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r) \left( \psi_{r,\pi} \hat{\pi}_t + \psi_{r,y} \hat{y}_t + \psi_{r,\Delta y} \Delta \hat{y}_t \right) + \varepsilon_t \]

(4.1)

where \( \rho_r \) controls the degree of interest rate smoothing and \( \psi_{r,x} \) denotes the reaction of the interest rate to deviations of variable \( x \) from its trend.\(^{28}\)


\(^{28}\)Money supply is assumed to adjust to implement the interest rate and fiscal transfers are adjusted to accommodate monetary policy. Unlike Smets and Wouters (2007), I assume that the
Fiscal rules:

\[
\begin{align*}
\hat{g}_t &= -\psi_{g,y} \hat{y}_{t-\tau} - \psi_{g,b} \hat{b}_{t-1} + \xi^g_t \tag{4.2a} \\
\hat{s}_t &= -\psi_{s,y} \hat{y}_{t-\tau} - \psi_{s,b} \hat{b}_{t-1} + \xi^s_t \tag{4.2b} \\
\frac{\bar{w}\bar{n}}{y} d\tau^n_t &= \psi_{\tau^n,y} \hat{y}_{t-\tau} + \psi_{\tau^n,b} \hat{b}_{t-1} + \xi^\tau^n_t \tag{4.2c} \\
\frac{(\hat{r}^k - \delta)\hat{k}}{y} d\tau^k_t &= \psi_{\tau^k,y} \hat{y}_{t-\tau} + \psi_{\tau^k,b} \hat{b}_{t-1} + \xi^\tau^k_t \tag{4.2d}
\end{align*}
\]

for \( \tau \in \{0, 1\} \). \(^{29}\) The \( \xi^i_t \) are exogenous AR(1) processes: \( \xi^i_t = \rho \xi^i_{t-1} + \epsilon^i_t \). Note that the sign of the coefficients in the expenditure components \( g_t, s_t \) are flipped so that positive estimates always imply consolidation in good times (\( \psi_{o,y} > 0 \)) or when debt is high (\( \psi_{o,b} > 0 \)).

The consolidated government budget constraint is: \(^{30}\)

\[
\begin{align*}
\bar{b} (\hat{b}_t - \hat{r}) + \frac{\bar{w}\bar{n}}{y} (d\tau^n + \bar{n}_t) + \frac{\bar{c}}{\gamma} \hat{c}_t + \frac{(\hat{r}^k - \delta)\hat{k}}{y} (d\tau^k + \bar{k}_t + \hat{\tau}^k_t) = \hat{g}_t + \hat{s}_t + \bar{b} (\hat{b}_{t-1} - \hat{\pi}_t) \tag{4.3}
\end{align*}
\]

### 4.2.2 Households

The law of motion for capital:

\[
\hat{k}^p_t = (1 - \bar{x}) \hat{k}^p_{t-1} + \bar{x} \hat{k}^p_{t+1}
\]

Household wage setting:

\[
\hat{w}_t = \frac{\bar{w}_t - \hat{r}}{1 + \beta \gamma} + \frac{\bar{v} \pi_t}{1 + \beta \gamma} \times \hat{\pi}_t + \frac{\bar{c}_t - (h/\gamma)\hat{c}_{t-1}}{1 - h/\gamma} + \nu \hat{n}_t - \hat{w}_t + \frac{d\tau^n_t}{1 + \tau^n_t} + \frac{d\tau^c_t}{1 + \tau^c_t} \tag{4.5}
\]

central bank compares output to detrended output rather than looking at the output gap relative to the flexible price economy. This simplifies the model without changing substantive results.

\(^{29}\) Leeper et al. (2010) use \( \tau = 0 \), Fernandez-Villaverde et al. (2011) use \( \tau = 1 \).

\(^{30}\) Seigniorage revenue for the government enters negatively in the lump-sum transfer to households \( \hat{s}_t \).
Household consumption Euler equation:

\[
E_t[\hat{\xi}_{t+1} - \hat{\xi}_t] + E_t[d\tau^c_{t+1} - d\tau^c_t] = \\
= \frac{1}{1 - h/\gamma}E_t((\sigma - 1)\frac{1}{1 + \lambda_w} 1 + \tau^c - \frac{\hat{\xi}_{t+1} - \hat{\xi}_t}{\hat{\xi}_t}) - \sigma \left[ \hat{c}_{t+1} - \left( 1 + \frac{h}{\gamma} \right) c_t + \frac{h}{\gamma} \hat{c}_{t+1} \right],
\]

(4.6)

Other FOC (before rescaling of \(q^b_t\)):

\[
E_t[\hat{\xi}_{t+1} - \hat{\xi}_t] = -q^b_t - \hat{R}_t + E_t[\hat{\pi}_{t+1}],
\]

(4.7)

\[
\hat{Q}_t = \frac{1}{\tau^k(1 - \tau^k) + \delta \tau^k + 1 - \delta} \times \\
\times \left[ (\tau^k(1 - \tau^k) + \delta \tau^k)\hat{q}^k_t - (\tau^k - \delta)\hat{c}^k_{t+1} + \\
\tau^k(1 - \tau^k)E_t(\hat{c}^k_{t+1}) + (1 - \delta)E_t(\hat{Q}_{t+1}) \right],
\]

(4.8)

\[
\hat{x}_t = \frac{1}{1 + \beta \gamma} \left[ \hat{x}_{t-1} + \hat{\beta} \gamma \hat{E}_t(\hat{x}_{t+1}) + \frac{1}{\gamma^2 S''(\gamma)}[\hat{Q}_t + \hat{q}^c_t] \right],
\]

(4.9)

\[
\hat{u}_t = \frac{a'(1)}{a''(1)} \frac{1}{\psi_w} \hat{r}^k_t.
\]

(4.10)

\[
4.2.3 \quad \text{Production side and price setting}
\]

The linearized aggregate production function is:

\[
\hat{y}_t = \frac{\bar{y} + \Phi}{\bar{y}} \left( \varepsilon^o_t + \zeta \hat{\xi}^o_{t-1} + \alpha(1 - \zeta)\hat{k}_t + (1 - \alpha)(1 - \zeta)\hat{n}_t \right),
\]

(4.12)

where \(\Phi\) are fixed costs. Fixed costs, in steady state, equal the profits made by intermediate producers.

The capital-labor ratio:

\[
\hat{k}_t = \hat{n}_t + \hat{w}_t - \hat{r}^k_t.
\]

(4.13)

Price setting:

\[
\hat{\pi}_t = \frac{\xi_p}{\xi_p + \beta \gamma} \hat{\pi}_{t-1} + \frac{1 - \zeta_p \beta \gamma}{\xi_p + \zeta_p} A_p(\hat{m}c_t + \hat{\xi}^{\lambda p}_t) + \frac{\beta \gamma}{1 + \xi_p \beta \gamma} \hat{E}_t \hat{\pi}_{t+1}.
\]

(4.14)
4.2.4 Market clearing

Goods market clearing requires:

\[ \hat{y}_t = \frac{\hat{c}}{\hat{y}} x_t + \frac{\hat{x}}{\hat{y}} \hat{y}_t + \hat{\pi}_t + \frac{\hat{r}_k}{\hat{y}} \hat{u}_t. \]  \hspace{1cm} (4.15)

4.2.5 Observation equations

The observation equations are given by (3.2) as well as the following seven observation equations from Smets and Wouters (2007) and three additional equations (4.17) on fiscal variables:

\[ \Delta \ln y^{\text{obs}}_t = y_t - y_{t+1} + (\gamma - 1), \] \hspace{1cm} (4.16a)
\[ \Delta \ln x^{\text{obs}}_t = x_t - x_{t+1} + (\gamma - 1), \] \hspace{1cm} (4.16b)
\[ \Delta \ln c^{\text{obs}}_t = c_t - c_{t+1} + (\gamma - 1), \] \hspace{1cm} (4.16c)
\[ \hat{\pi}^{\text{obs}}_t = \hat{\pi}_t + \bar{\pi}, \] \hspace{1cm} (4.16d)
\[ \hat{w}^{\text{obs}}_t = \hat{w}_t + \bar{w}, \] \hspace{1cm} (4.16e)
\[ \hat{n}^{\text{obs}}_t = \hat{n}_t + \bar{n}, \] \hspace{1cm} (4.16f)
\[ \hat{R}_{t}^{\text{obs}} = \hat{R}_t + (\beta^{-1} - 1), \] \hspace{1cm} (4.16g)

I use the deviation of debt to GDP and revenue to GDP, detrended prior to the estimation, as observables:

\[ b^{\text{obs}}_t = \frac{\bar{b}}{\bar{y}} (\hat{b} - \hat{y}) + \bar{y}^{\text{obs}}; \] \hspace{1cm} (4.17a)
\[ rev^{n,\text{obs}}_t = \bar{r}^{n} \frac{\hat{w} \bar{n}}{\bar{c}} \frac{\hat{e}}{\hat{y}} \frac{dt^{n}}{\bar{y}} + \hat{n}_t - \bar{y}^{\text{obs}}; \] \hspace{1cm} (4.17b)
\[ rev^{k,\text{obs}}_t = \frac{\hat{k}}{\bar{y}} (\hat{p}^k - \delta) \left( \frac{dt^{k}}{\bar{r}^k} + \frac{\hat{r}^k}{\bar{r}^k - \delta} \hat{r}_t^k + \hat{k}_t^p - \hat{y}_t \right) + \bar{r}^{\text{obs}}; \] \hspace{1cm} (4.17c)

4.3 Narrative DSGE model results

As a first check to gauge shock identification, I estimate the baseline Smets and Wouters (2007) model and compare the correlation of the structurally identified
shock with the narrative shock measures for the monetary (3.2a), technology growth (3.2d), and government spending shocks (3.2e). I then consider the extended model with distortionary taxes and fiscal policy rules, using also the narrative tax measures in (3.2c) and (3.2b).

4.3.1 Smets and Wouters (2007) with narrative shocks

Throughout this section, I consider the baseline Smets and Wouters (2007) model without distortionary taxes: $\bar{\tau} = d_\tau = 0$.

Figure 2 compares structurally identified shocks (or their linear transforms) with their non-structural counterparts, given three different information sets for the econometrician. The top row does not use any of the narrative measures in the estimation. The middle row presents the results from three different estimates, using one narrative shock at a time. Last, the bottom row uses all three narrative measures at once.

Looking at the correlation of government spending shocks with the corresponding SPF forecast errors from Ramey (2011) suggests that the rudimentary fiscal sector in Smets and Wouters (2007) identifies shocks which do not line up well with the narrative measure: The correlation between the two measures is in the order of 0.5 for the monetary and technology shocks, but only 0.14 for government spending.

Expanding the econometrician’s one narrative shock measure at a time yields the results shown in the middle panel. It leads to small increases in the correlation of the monetary and technology shocks with their instruments, but brings a substantial increase in the case of the government spending shock from 0.14 to 0.41.

In the above exercise, the measurement error variance $\omega^2_\phi$ is fixed at a small number, i.e. $\omega_\phi = \frac{1}{4}$. The bottom panel in Figure 2 uses all narrative measures at once and estimates the measurement error of the narrative shocks. The fit of the monetary policy measure is virtually unchanged, the fit of the technology shock increases again moderately, while the fit of the government spending shock falls almost back to the level of government spending in the baseline estimates.

Changing the econometrician’s information set has substantive effects on parameter estimates. Table 1 examines the implications for the monetary policy rule, and the production technology when narrative instruments are used one at a time. The first column presents the baseline estimates for comparison. Introducing the government spending instrument leaves the estimates of the monetary policy rule virtually unchanged, but implies a significant higher fixed cost in production (sec-
Romer & Romer monetary shock  Fernald TFP  Ramey government spending
Baseline: narrative shocks not used in estimation

narrative shocks used in estimation: one shock at a time, small measurement error

narrative shocks used in estimation: all at once, estimated measurement error

Figure 2: Comparison of correlation of “narrative” shock measures with Smets-Wouters analogues: Varying the econometrician’s information set
When the monetary policy instrument is introduced (third column), only the investment adjustment cost changes significantly on the production side, while the estimated monetary policy rule differs substantially: The Fed is estimated to react less to inflation, the output gap, and output growth than in the baseline model. Lastly, when the technology growth instrument is used, the monetary policy rule is again largely unchanged, but the estimated fixed costs in production are now substantially lower, while capacity utilization is harder to adjust.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Smets and Wouters (2007)</th>
<th>g-signal</th>
<th>R&amp;R m-signal</th>
<th>a-signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adj. cost</td>
<td>6.82 (5.37, 8.31)</td>
<td>6.98 (5.42, 8.48)</td>
<td>8.75 (7.16, 9.97)</td>
<td>6.98 (5.40, 8.16)</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>1.32 (1.22, 1.40)</td>
<td>1.43 (1.31, 1.54)</td>
<td>1.33 (1.24, 1.45)</td>
<td>1.08 (1.04, 1.12)</td>
</tr>
<tr>
<td>Ela cap util.</td>
<td>0.36 (0.21, 0.52)</td>
<td>0.38 (0.22, 0.51)</td>
<td>0.29 (0.17, 0.43)</td>
<td>0.93 (0.86, 0.98)</td>
</tr>
<tr>
<td>Taylor rule: $\pi_t$</td>
<td>1.86 (1.58, 2.13)</td>
<td>1.82 (1.56, 2.07)</td>
<td>1.5 (1.25, 1.74)</td>
<td>1.9 (1.63, 2.16)</td>
</tr>
<tr>
<td>Taylor rule: $y_t$</td>
<td>0.10 (0.07, 0.14)</td>
<td>0.11 (0.07, 0.14)</td>
<td>0.06 (0.03, 0.08)</td>
<td>0.1 (0.07, 0.14)</td>
</tr>
<tr>
<td>Taylor rule: $\Delta y_t$</td>
<td>0.14 (0.11, 0.17)</td>
<td>0.14 (0.11, 0.17)</td>
<td>0.04 (0.03, 0.07)</td>
<td>0.17 (0.14, 0.21)</td>
</tr>
<tr>
<td>AR(1) mon. shock</td>
<td>0.30 (0.20, 0.40)</td>
<td>0.29 (0.20, 0.39)</td>
<td>0.42 (0.37, 0.48)</td>
<td>0.3 (0.22, 0.40)</td>
</tr>
</tbody>
</table>

Note: The Smets and Wouters (2007) is re-estimated with my updated dataset, resulting in small differences compared to the original paper.

Table 1: Comparing selected parameter estimates: Varying the econometrician’s information set

Figure 1 on page 2 contrasts the correlation of identified shocks and narrative instruments in the baseline DSGE model and the narrative BVAR. The BVAR is estimated using the data in Ramey (2011) and using the one quarter ahead forecast errors for government spending committed by the participants of the Survey of Professional Forecasters as the narrative instrument. The plotted shocks at the posterior mean of the estimated parameters readily reveal a high, positive correlation and sampling from the posterior yields a 90% coverage set of $(-0.59, 0.46)$.

How does the shock identification affect substantive implications of the model? To answer this question, Figure 3 compares the median response of labor productivity to a 1% (peak) increase in government spending in the BVAR to four estimates of the Smets and Wouters (2007) model. The different estimates use the four different information sets for the econometrician as in Figure 2. The visual comparison of the posterior medians shows that the different information sets change the responses to a government in intuitive ways. While using monetary policy instruments hardly changes the results compared to the baseline estimates (shown in green), including

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31 While the BVAR identification in the Mertens and Ravn (2013) case is largely uncertain when taking first stage uncertainty into account, the posterior uncertainty in my application is minor: The correlation is robust to sampling uncertainty. Table 3 presents the correlation between the sampled and the posterior mean of the coefficient matrix $F$ is consistently very high.
The baseline specification uses the postwar data in Ramey (2011), which is based on a measure of total hours worked (incl. the government) and the real wage for all private businesses. The long and short sample specification use total business hours and the manufacturing real wage starting in 1939 and 1948, respectively. The specifications use both a monetary and government spending instrument, except the “single instrument” specification, which just uses the government spending instrument.

Figure 3: Labor productivity response to a 1% peak government spending shock: BVAR vs. DSGE model
government spending shocks increases the response (red, dashed-dotted line) and brings it closer to the BVAR response (blue line with circles). In contrast, including TFP-growth signals lowers the response of labor productivity (purple, dashed).

Table 1 reveals why: With government spending shock observations a higher fixed cost is estimated, while it is lower with TFP-growth instruments. Higher fixed costs imply a larger response of labor productivity because returns to scale are larger.

Figure 4: Comparing individual IRFs: Pointwise medians in BVAR vs. DSGE model

Figure 6 suggests that particularly the responses of hours and the dynamics of the government spending shock are problematic. These issues are related: The DSGE model implies an immediate peak and then a very slow decline in government spending, whereas the VAR implies a year-long buildup in government spending and a subsequently quick decline in government spending. Consequently, the wealth effect of a 1% peak government spending shock is very different. Since the negative wealth effect is stronger in the DSGE model, it is not surprising that households work harder in the DSGE model. However, as Figure 7 shows, simply forcing a comparable exogenous AR(2) process on the model does not generate significant differences in the level of the hours reply, even though it generates a similarly hump-shaped response. This suggests (partly) endogenizing government spending to allow for richer dynamics in government spending, as in Leeper et al. (2010).

Figure 5 shows that there are significant differences in the total hours measure.

---

32Interestingly, Del Negro and Schorfheide (2009) document the same pattern for the demand shock in their model. The same qualitative hump-shaped AR(2) pattern matches well the spending plan for the 2009 stimulus plan (cf. Uhlig (2010)). Here, however, allowing for AR(2) dynamics as in Del Negro and Schorfheide (2009) does not address the problem when estimating the model with the generalized shock structure when the prior for the second root is centered at zero.
from Francis and Ramey (2009) and used in Ramey (2011), arising both from the hours time series itself (blue solid line vs. red dotted line) and from the difference between population measures (dashed vs. solid lines). These different measures of the same concept in the model imply different estimates of the fixed cost and persistence of government spending in the model (not shown) and consequently also in the implied IRFs. Figure 6 illustrates this. Using the Francis and Ramey (2009) hours measure with the Smets and Wouters (2007) population measure lowers the prediction of the DSGE model for the response of hours to a government spending shock.

Figure 6: Comparing individual IRFs: Pointwise medians in DSGE model with different hours definitions
Figure 7 addresses whether a different government spending process with a shorter half-life and build-up changes the mismatch of the hours response. A shorter half-life and better matched time profile relative to the VAR results would imply a smaller shock to the PDV of government spending and therefore a smaller increase in labor supply via the wealth effect. For the AR(2) processes in the figure, I chose the parameters to minimize the distance between the VAR and DSGE responses of government spending to an exogenous government spending shock. While, by construction, the model now matches the hump-shaped responses, the level of hours worked is still off. Introducing a noisy signal of government spending shocks continues to narrow the difference to the VAR response, but falls short of making a qualitative difference. Note also that, allowing for separate time trends in the observables by itself implies less persistent AR(1) processes. The lower PDV of government spending shocks does not affect the qualitative responses of hours.

4.3.2 Extended model with distortionary taxes and fiscal rules

When estimating the model with tax shocks, I fix the elasticity of labor supply at \( \nu^{-1} = 1. \) In this section I consider the following three information sets for the

\[ \frac{\nu (1 - \sigma^{-1})}{(1 - h)(1 - \sigma^{-1})} \]
Figure 8: Comparing individual IRFs: Pointwise medians in DSGE model after 1982 with different shock processes

econometrician: (1) No narrative shocks, (2) narrative tax shocks only, (3) narrative tax and government spending shocks. I fix the measurement error of the narrative shocks throughout: at $\omega_{r,o} = 0.125$ for the infrequently observed tax shocks and $\omega_g = 0.25$ for the government spending shocks.

Figure 9 examines how the filtered structural shocks line up with the narrative shocks when the econometrician’s information set is varied. The upper panel shows that the correlation of labor tax shocks with the narrative measure for personal income taxes is reasonably high at 0.40, given that personal income taxes are just one of the components of labor taxes. However, the correlation of the corporate income tax narrative measure with capital tax rate shocks is only 0.20, and that of government spending shocks with forecast errors even lower (0.09).\textsuperscript{36} Introducing the narrative tax rate (middle panel) measures increases the filtered correlation significantly: It rises to 0.92 for labor tax rates and 0.67 for capital tax rates. However, the correlation of narrative and structural government spending shocks declines further, to 0.05. Using also narrative government spending shocks in addition to narrative tax shocks (bottom panel) leaves the correlations of tax shocks virtually unchanged, but brings the structural government spending shocks closer to the data.

The different information sets induce significantly different posterior beliefs. Table 2 displays the posterior means and standard deviations for the structural parameters. Examining the first group of parameters examining the production technology

\textsuperscript{36}Figure 20 on page 51 in the Appendix compares subsamples. It shows that after 1982, both narrative measures have a correlation with the implied structural shocks above 0.8, even though neither is used in the estimation. However, the results for government spending shocks deteriorate even further.
Figure 9: Comparison of correlation of “narrative” shock measures with extended fiscal model analogues: Varying the econometrician’s information set
shows that with and without narrative shocks, the inferred capital share and investment adjustment costs imply a very limited role for capital in the dynamics: The capital share is very low (below 0.20) and the posterior mean investment adjustment costs between 9.5 and 11.0, compared with prior mean of 0.3 and 4, respectively. The lack of a role for capital is particularly pronounced when using all three narrative fiscal shocks. The second group of parameters concerns preferences over consumption. A very high degree of habit formation is estimated, whereas $\sigma$ is close to log utility. The third group of parameters relates to nominal rigidities. The model implies very little price indexation, but reasonable estimates of the Calvo parameters. The fourth and fifth group of parameters characterize the monetary and fiscal policy rules. Both sets of rules are heavily affected by the use of fiscal instruments in the estimation.

With fiscal instruments, the monetary policy rule is estimated to respond less to inflation and output. This is likely the case because tax rates and government spending are estimated to react significantly to output and debt when fiscal instruments are included. Labor taxes and capital taxes are rising when output rises ($\psi_{g,y} > 0$ implies procyclical tax rates), thereby stabilizing the economy. The coefficients for government consumption $g_t$ switch signs when estimated with and without the narrative government spending signal. When estimated without the narrative government spending signal, the posterior implies that government spending is adjusted downward when debt is high or output is high, as in Leeper et al. (2010). However, when the narrative shock is used in the estimation, the systematic part of government spending is actually estimated to be procyclical and increase when debt is high. Instead, lump-sum transfers adjust to debt.

Figure 10 plots the response of output and government consumption to a one percentage point tax cut corresponding to the different posterior means associated with different information sets. While all three estimates imply that output rises significantly, the estimate with only narrative tax shocks implies a slow rises, peaking at about 15 quarters, while the estimate with all fiscal shocks implies a steep increase in output, peaking after about five quarters. The right pane shows that these different responses can be explained, to a large extent, by the estimated behaviour of government consumption, which is estimated to fall after a tax cut when narrative...
Table 2: Posterior mean (standard deviation) of extended model: Varying the econometrician’s information set

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No narrative shocks</th>
<th>Narrative tax shocks</th>
<th>Narrative tax &amp; g shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share $\alpha$</td>
<td>0.172 (0.020)</td>
<td>0.204 (0.024)</td>
<td>0.150 (0.017)</td>
</tr>
<tr>
<td>Fixed cost $\theta$</td>
<td>1.428 (0.069)</td>
<td>1.401 (0.071)</td>
<td>1.432 (0.068)</td>
</tr>
<tr>
<td>Capacity utilization</td>
<td>0.400 (0.110)</td>
<td>0.409 (0.102)</td>
<td>0.549 (0.120)</td>
</tr>
<tr>
<td>Investment adj. cost $S''(\gamma)$</td>
<td>9.552 (0.975)</td>
<td>11.009 (1.018)</td>
<td>10.539 (1.025)</td>
</tr>
<tr>
<td>Inverse IES $\sigma$</td>
<td>1.044 (0.031)</td>
<td>1.035 (0.029)</td>
<td>0.959 (0.028)</td>
</tr>
<tr>
<td>Habit</td>
<td>0.916 (0.012)</td>
<td>0.914 (0.011)</td>
<td>0.915 (0.011)</td>
</tr>
<tr>
<td>Price indexation $\iota_p$</td>
<td>0.133 (0.053)</td>
<td>0.141 (0.051)</td>
<td>0.130 (0.053)</td>
</tr>
<tr>
<td>Wage indexation $\iota_w$</td>
<td>0.540 (0.112)</td>
<td>0.617 (0.118)</td>
<td>0.622 (0.118)</td>
</tr>
<tr>
<td>Calvo, prices $\zeta_o$</td>
<td>0.742 (0.042)</td>
<td>0.705 (0.055)</td>
<td>0.615 (0.063)</td>
</tr>
<tr>
<td>Calvo, wages $\zeta_w$</td>
<td>0.740 (0.035)</td>
<td>0.744 (0.046)</td>
<td>0.720 (0.046)</td>
</tr>
<tr>
<td>Taylor rule, inflation: $\psi_{R,\pi}$</td>
<td>1.547 (0.162)</td>
<td>1.415 (0.136)</td>
<td>1.303 (0.114)</td>
</tr>
<tr>
<td>Taylor rule, output: $\psi_{R,y}$</td>
<td>0.077 (0.026)</td>
<td>0.011 (0.007)</td>
<td>-0.003 (0.004)</td>
</tr>
<tr>
<td>Taylor rule, output change: $\psi_{R,\Delta y}$</td>
<td>0.097 (0.015)</td>
<td>0.061 (0.014)</td>
<td>0.074 (0.015)</td>
</tr>
<tr>
<td>Taylor rule, smoothing: $\rho_R$</td>
<td>0.877 (0.024)</td>
<td>0.815 (0.020)</td>
<td>0.809 (0.020)</td>
</tr>
<tr>
<td>Covariance($g$,TFP)</td>
<td>0.257 (0.074)</td>
<td>0.260 (0.089)</td>
<td>0.128 (0.066)</td>
</tr>
<tr>
<td>Gov. spending, debt: $\psi_{g,b}$</td>
<td>-0.014 (0.023)</td>
<td>0.042 (0.024)</td>
<td>-0.066 (0.024)</td>
</tr>
<tr>
<td>Gov. spending, output: $\psi_{g,y}$</td>
<td>-0.085 (0.070)</td>
<td>0.074 (0.104)</td>
<td>-0.159 (0.064)</td>
</tr>
<tr>
<td>Capital taxes, debt: $\psi_{r,k,b}$</td>
<td>-0.017 (0.012)</td>
<td>-0.064 (0.011)</td>
<td>-0.069 (0.010)</td>
</tr>
<tr>
<td>Capital taxes, output: $\psi_{r,k,y}$</td>
<td>0.163 (0.019)</td>
<td>0.204 (0.020)</td>
<td>0.199 (0.018)</td>
</tr>
<tr>
<td>Labor taxes, debt: $\psi_{r,n,b}$</td>
<td>0.022 (0.011)</td>
<td>-0.068 (0.016)</td>
<td>-0.061 (0.007)</td>
</tr>
<tr>
<td>Labor taxes, output: $\psi_{r,n,y}$</td>
<td>-0.019 (0.020)</td>
<td>0.310 (0.020)</td>
<td>0.298 (0.020)</td>
</tr>
<tr>
<td>Transfers, debt: $\psi_{s,b}$</td>
<td>0.402 (0.075)</td>
<td>0.083 (0.026)</td>
<td>0.285 (0.066)</td>
</tr>
</tbody>
</table>

Note: Only the posterior distributions of structural parameters are shown.

Figure 10: IRF of output and government consumption to a 1 p.p. labor tax cut at posterior mean: Varying the econometrician’s information set
government spending shocks are not taken into account.

5 Conclusion

A key problem in macroeconomics is to translate forecast errors into structural economic shocks. This paper examines whether the inference on shocks from a standard fully structural DSGE model lines up with the conclusions based on methods with much weaker identifying assumptions, such as narrative methods. I find that for monetary policy shocks, productivity shocks, and, to a lesser extent, tax shocks, the shocks line up reasonably well with narrative measures. However, since there is feedback between shock identification and parameter identification in structural DSGE models, extending the econometrician’s information set by narrative shock measures changes important structural and policy parameters in the production function and policy rules.
References


DelNegro, Marco and Frank Schorfheide, Bayesian Macroeconometrics Oxford Handbooks in Economics, Oxford University Press, 2011.


Francis, Neville and Valerie A. Ramey, “Measures of per Capita Hours and Their Implications for the Technology-Hours Debate,” Journal of Money, Credit and Banking, 09 2009, 41 (6), 1071–1097.


A Appendix

A.1 Narrative Shock identification

Note that (3.6) does not restrict $\alpha_{11}$ for any value of $G$, assuming that $\text{Cov}[z_t, v_t]$ is of maximal column rank $k \leq m$. The LHS depends on $k^2$ different parameters of $\alpha_{11}$, and $\text{Cov}[z_t, v_t]_1$ has also $k^2$ elements.

$\Sigma$ provides an extra $m \frac{m+1}{2}$ equations, which can be used to solve for the remaining $k \leq m$ elements in $\alpha_{11}$ plus the $m \times (m-k)$ elements in $\alpha_2$. Thus, in general, an extra $m(m-1) - k(m-k) \geq 0$ restrictions are needed.\footnote{For $m = 2$ shocks and $k = 1$ instruments, the model is exactly identified.} For $m = 2$ shocks and $k = 1$ instruments, the model is exactly identified.

Define

$$\kappa = (\text{Cov}[z_t, v_t]_1^{-1} \text{Cov}[z_t, v_t]_2)', \tag{A.1}$$

so that $\alpha_{21} = \kappa \alpha_{11}$. Then:

$$\Sigma = \begin{bmatrix} \alpha_{11} \alpha_{11}' + \alpha_{12} \alpha_{12}' & \alpha_{11} \alpha_{11}' \kappa' + \alpha_{12} \alpha_{22}' \\ \kappa \alpha_{11} \alpha_{11}' + \alpha_{22} \alpha_{12}' & \kappa \alpha_{11} \alpha_{11}' \kappa' + \alpha_{22} \alpha_{22}' \end{bmatrix}$$

The covariance restriction identifies the impulse response (or component of the forecast error) up to a $k \times k$ square scale matrix $\alpha_{11}$:

$$u_t = A \epsilon_t = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{11} & \alpha_{12} \end{bmatrix} \epsilon_t = \alpha_{11} \epsilon_t^{[1]} + \alpha_{12} \epsilon_t^{[2]} = \begin{bmatrix} I_k & 0 \\ \kappa & \alpha_{11} \end{bmatrix} \alpha_{11} \epsilon_t^{[1]} + \begin{bmatrix} I_k & 0 \\ \kappa & \alpha_{12} \end{bmatrix} \alpha_{12} \epsilon_t^{[2]}$$

Given that $\epsilon_t^{[1]} \perp \epsilon_t^{[2]}$ it follows that:

$$\text{Var}[u_t \mid \epsilon_t^{[1]}] = \alpha_{11}^2 (\alpha_{12}^2)' = \begin{bmatrix} \alpha_{12} \alpha_{12}' & \alpha_{12} \alpha_{22}' \\ \alpha_{22} \alpha_{12}' & \alpha_{22} \alpha_{22}' \end{bmatrix}$$

$$\text{Var}[u_t \mid \epsilon_t^{[2]}] = \alpha_{11}^2 (\alpha_{11})' = \begin{bmatrix} \alpha_{11} \alpha_{11}' \\ \kappa \alpha_{11} \alpha_{11}' \alpha_{11} \alpha_{11}' \kappa' + \alpha_{11} \alpha_{11}' \kappa' + \alpha_{22} \alpha_{22}' \end{bmatrix}$$

$$\Sigma = \text{Var}[u_t] = \text{Var}[u_t \mid \epsilon_t^{[1]}] + \text{Var}[u_t \mid \epsilon_t^{[2]}] = \begin{bmatrix} \Sigma_{12} \Sigma_{12}' & \Sigma_{12} \Sigma_{22}' \\ \Sigma_{22} \Sigma_{12}' & \Sigma_{22} \Sigma_{22}' \end{bmatrix}$$

Note that:

$$u_{t|_{true}} = u_t - E[u_t \mid \epsilon_t^{[1]}] \perp E[u_t \mid \epsilon_t^{[1]}] = \begin{bmatrix} I_k \\ \kappa \end{bmatrix} \alpha_{11} \epsilon_t^{[1]}$$

Any vector in the nullspace of $[I_k \ \kappa']$ satisfies the orthogonality condition.

Note that $\begin{bmatrix} I_k \\ \kappa' \end{bmatrix}$ is an orthogonal basis for $\mathbb{R}^m$.

\footnote{For $m \geq 1, m \geq k \geq 0$.}
Define

\[
Z \equiv [Z]_{1} [Z]_{2} = \left[ I_{k} \kappa^{'} I_{m-k} \right] \left[ I_{k} \kappa^{'} I_{m-k} \right]^{-1} \equiv \tilde{Z} \parallel \tilde{Z}^{-1}
\]

is an orthogonal matrix, where \(\parallel X \parallel = \sqrt{\text{tr}(X^{T} X)}\).

Note that \(Z^{[2]}\) is in the Nullspace of \(\alpha^{[1]}\). Hence, \((Z^{[2]})^{T} v_{t}\) projects \(v_{t}\) into the Nullspace of the instrument-identified shocks \(\epsilon^{[1]}_{t}\).

\[
(Z^{[2]})^{T} v_{t} = (Z^{[2]})^{T} A \epsilon_{t} = (Z^{[2]})^{T} \left( \alpha^{[1]} \alpha^{[2]} \right) \epsilon_{t}
\]

\[
= (Z^{[2]})^{T} \left[ Z^{[1]} \| \tilde{Z} \|_{\alpha_{11}} \alpha^{[2]} \right] \epsilon_{t} = 0 \left( Z^{[2]} \right)^{T} \alpha^{[2]} \epsilon_{t}
\]

\[
= 0 \times \epsilon^{[1]}_{t} + (Z^{[2]})^{T} \alpha^{[2]} \epsilon^{[2]}_{t} \perp \epsilon^{[1]}_{t}
\]

Note that \((Z^{[2]})^{T} \alpha^{[2]}\) is of full rank and I can therefore equivalently consider \(\epsilon^{[2]}_{t}\) or \((Z^{[2]})^{T} v_{t}\). Thus, the expectation of \(v_{t}\) given \(\epsilon^{[2]}_{t}\) is given by:

\[
E[v_{t} | \epsilon^{[2]}_{t}] = \text{Cov}[v_{t}, (Z^{[2]})^{T} v_{t}] \text{Var}[(Z^{[2]})^{T} v_{t}]^{-1} (Z^{[2]})^{T} v_{t},
\]

\[
v_{t} - E[v_{t} | \epsilon^{[2]}_{t}] = (I - \text{Cov}[v_{t}, (Z^{[2]})^{T} v_{t}] \text{Var}[(Z^{[2]})^{T} v_{t}]^{-1} (Z^{[2]})^{T} v_{t},
\]

\[
\text{Cov}[v_{t}, (Z^{[2]})^{T} v_{t}] = \Sigma(Z^{[2]}) = \Sigma \left[ \kappa^{-1} I_{m-k} \right] \parallel \tilde{Z} \parallel
\]

\[
\text{Var}[v_{t} | \epsilon^{[2]}_{t}] = E[(I - \text{Cov}[v_{t}, (Z^{[2]})^{T} v_{t}] \text{Var}[(Z^{[2]})^{T} v_{t}]^{-1} (Z^{[2]})^{T} v_{t} v_{t}^{T}]
\]

\[
= E[v_{t} v_{t}^{T}] - \text{Cov}[v_{t}, (Z^{[2]})^{T} v_{t}] \text{Var}[(Z^{[2]})^{T} v_{t}]^{-1} E[(Z^{[2]})^{T} v_{t} v_{t}^{T}]
\]

\[
= \Sigma - \text{Cov}[v_{t}, (Z^{[2]})^{T} v_{t}] \text{Var}[(Z^{[2]})^{T} v_{t}]^{-1} \text{Cov}[v_{t}, (Z^{[2]})^{T} v_{t}]
\]

\[
= \Sigma - \Sigma \left[ \kappa^{'} I_{m-k} \right] \left[ \kappa^{-1} I_{m-k} \Sigma \left[ \kappa^{'} I_{m-k} \right] \right]^{-1} \left[ \kappa^{-1} I_{m-k} \Sigma \right]
\]

\[
= \left[ \alpha_{11} \alpha_{11}^{'} \alpha_{11} \alpha_{11}^{'} \right]
\]

This gives a solution for \(\alpha_{11} \alpha_{11}^{'}\) in terms of observables: \(\Sigma\) and \(\kappa = \text{Cov}[z_{t}, v_{t}]^{-1} \times \text{Cov}[z_{t}, v_{t}]_{2}\).

In general, \(\alpha_{11}\) itself is unidentified: An additional \(\frac{(k-1)k}{2}\) restrictions are needed to pin down its \(k^2\) elements from the \(\frac{(k+1)k}{2}\) independent elements in \(\alpha_{11} \alpha_{11}^{'}\). Given \(\alpha_{11}\), the impact-response to a unit shock is given by:

\[
\left[ I_{k} \kappa \right] \alpha_{11}
\]

Note that this leaves \(m(m-k)\) elements in \(\alpha^{[2]}\) unrestricted, for which there exist \(\frac{m(m+1)}{2}\) equations in \(\Sigma - \text{Var}[v_{t} | \epsilon^{[1]}_{t}]\), requiring an extra \(\frac{m}{2}(m - (1 + 2k))\) restrictions.
(Note: if \( k \geq \frac{1}{2}(m - 1) \), this implies over-identification.)

Intuitively: \( \alpha^{[1]} \) has \( km \) parameters. There are \( (m - k) \times k \) covariances with other structural shocks (\( \Sigma_{12} \)) which help to identify the model, leaving \( k^2 \) parameters undetermined. An extra \( \frac{k(k+1)}{2} \) restrictions comes from the Riccatti equation via the residual variance, leaving \( \frac{k(k-1)}{2} \) parameters to be determined. (This should also work when identifying one shock at a time with instruments.) The problem with multiple instrumented shocks is that even if I know that each instruments is only relevant for one specific structural shock, they are silent on the covariance between shocks – the covariance between shocks could be contained in the first stage residual.

When each instruments exactly identifies one shock, then each instrument delivers \( (n - 1) \) identifying restrictions: Each instrument then identifies one column of \( A \) up to scale. To see this, consider (3.5) for the case of \( G = \text{diag}([g_1, \ldots, g_k, 0, \ldots, 0]) \) and let \( \alpha^{[i]} \) now denote the \( i \)’th column of \( A \):

\[
\text{Cov}[z_t^{[i]}, v_t] = e_i G A' = g_i e_i e_i' A' = g_i (\alpha^{[i]})'.
\]  
(A.4)

Since the LHS is observable, (3.9) identifies \( \alpha^{[i]} \) up to scale, imposing \( n - 1 \) restrictions on \( A \) for each instrument. In general, this leads to overidentifying restrictions on \( \Sigma \).

The (normalized) identified shocks are given by \( Fu \), where \( F = \text{Cov}[z_t, v_t] \Sigma^{-1} \).

### A.1.1 Sampling with partial- or just-identification

For \( i = 1, \ldots, M \), repeat (1) and (2):

1. Sample from the posterior over \( (B, \Sigma) \):
   - (a) Draw \( (\Sigma^{(i)})^{-1} \) from the marginal Wishart distribution (3.10).
   - (b) Conditional on \( (\Sigma^{(i)})^{-1} \), draw \( B^{(i)} \) from the Normal distribution (3.11).

2. Sample from the posterior over \( \text{Cov}[n, V] \):\(^{39}\)
   - (a) Draw \( (\Omega^{(i)})^{-1} \) from (3.12), computing the different quantities conditional on \( (B^{(i)}, \Sigma^{(i)}) \).
   - (b) Conditional on \( (\Omega^{(i)})^{-1}, (\Sigma^{(i)})^{-1} \), draw \( \text{Cov}[n, V]^{(i)} \) from (3.14).

Given partial identification or just identification, the impulse-vector can be computed using (A.3).

Note that this sampling mechanism is simpler than procedures for Bayesian instrumental variables estimation as described in, for example, Rossi et al. (2005, ch. 7). The reason is that the procedure above samples only from the reduced form involving \( F \), rather than the structural form involving \( G \) and does not restrict \( \Sigma \), the

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\(^{39}\)Note that it is immaterial whether one or multiple samples are drawn in this second stage. For a numerical illustration, see Figure ?? in the appendix.
covariance matrix drawn in the first stage. In contrast, over-identifying restrictions
add a feedback between the implied covariance \( F \Sigma \) and \( \Sigma \) itself via the implied
structural matrix \( A \) satisfying \( \Sigma = AA' \).

Compared to the usual IV setup, I am specifying just the “first stage”.\(^{40}\)

A.1.2 Pointwise IRFs

The procedure outlined above readily gives coverage probabilities for pointwise IRFs.

A.1.3 Implementing Inoue and Kilian (2013) (under construction)

A.2 Data construction

I follow Smets and Wouters (2007) in constructing the variables of the baseline
model, except for allocating durable consumption goods to investment rather than
consumption expenditure. Specifically:

\[
\begin{align*}
y_t &= \frac{(\text{nominal GDP}: \text{NIPA Table 1.1.5Q, Line 1})_t}{(\text{Population above 16: FRED CNP16OV})_t \times (\text{GDP deflator: NIPA Table 1.1.9Q, Line 1})_t} \\
c_t &= \frac{(\text{nominal PCE on nondurables and services: NIPA Table 1.1.5Q, Lines 5+6})_t}{(\text{Population above 16: FRED CNP16OV})_t \times (\text{GDP deflator: NIPA Table 1.1.9Q, Line 1})_t} \\
i_t &= \frac{(\text{Durables PCE and fixed investment: NIPA Table 1.1.5Q, Lines 4 + 8})_t}{(\text{Population above 16: FRED CNP16OV})_t \times (\text{GDP deflator: NIPA Table 1.1.9Q, Line 1})_t} \\
\pi_t &= \Delta \ln(\text{GDP deflator: NIPA Table 1.1.9Q, Line 1})_t \\
r_t &= \begin{cases} 
\frac{1}{4} (\text{Effective Federal Funds Rate: FRED FEDFUNDS})_t & t \geq (1954:Q3) \\
\frac{1}{4} (\text{3-Month Treasury Bill: FRED TB3MS})_t & \text{(else.)}
\end{cases} \\
n_t &= \frac{(\text{Nonfarm business hours worked: BLS PRS85006033})_t}{(\text{Population above 16: FRED CNP16OV})_t} \\
w_t &= \frac{(\text{Nonfarm business hourly compensation: BLS PRS85006103})_t}{(\text{GDP deflator: NIPA Table 1.1.9Q, Line 1})_t}
\end{align*}
\]

When using an alternative definition of hours worked from Francis and Ramey
(2009), I compute:

\[
n_{t FR} = \frac{(\text{Total hours worked: Francis and Ramey (2009)})_t}{(\text{Population above 16: FRED CNP16OV})_t}
\]

Fiscal data is computed following Leeper et al. (2010), except for adding state and local
governments (superscript “s&l”) to the federal government account (superscript “f”), similar to Fernandez-Villaverde et al. (2011). Since in the real world

\(^{40}\)Note that there is no “2SLS” here as Bayesian inference is based on a joint posterior when the
covariance matrix is unknown.
\[
\tau_t^c = \frac{(\text{production} \& \text{imports taxes: Table 3.2, Line 4})_t^f + (\text{Sales taxes})_t^{s\&l}}{((\text{Durables PCE})_t + c_t) \times (\text{GDP deflator})_t - (\text{production} \& \text{imports taxes})_t^f - (\text{Sales taxes})_t^{s\&l}}
\]

\[
\tau_t^p = \frac{1}{2}(\text{Proprietors' income})_t + (\text{wage income})_t + (\text{wage supplements})_t + (\text{capital income})_t
\]

\[
\tau_t^n = \frac{\tau_t^p \left( \frac{1}{2}(\text{Proprietors' income})_t + (\text{wage income})_t + (\text{wage supplements})_t \right) + (\text{wage taxes})_t^f}{(\text{wage income})_t + (\text{wage supplements})_t + (\text{wage taxes})_t^f + \frac{1}{2}(\text{Proprietors' income})_t}
\]

\[
\tau_t^k = \frac{\tau_t^p (\text{capital income})_t + (\text{corporate taxes})_t^f + (\text{corporate taxes})_t^{s\&l}}{(\text{Capital income})_t + (\text{Property taxes})_t^{s\&l}}
\]

where the following NIPA sources were used:

- (Federal) production & imports taxes: Table 3.2Q, Line 4
- (State and local) sales taxes: Table 3.3Q, Line 7
- (Federal) personal current taxes: Table 3.2Q, Line 3
- (State and local) personal current taxes: Table 3.3Q, Line 3
- (Federal) taxes on corporate income minus profits of Federal Reserve banks: Table 3.2Q, Line 7 – Line 8.
- (State and local) taxes on corporate income: Table 3.3Q, Line 10.
- (Federal) wage tax (employer contributions for government social insurance): Table 1.12Q, Line 8.
- Proprietors' income: Table 1.12Q, Line 9
- Wage income (wages and salaries): Table 1.12Q, Line 3.
- Wage supplements (employer contributions for employee pension and insurance): Table 1.12Q, Line 7.
- Capital income = sum of rental income of persons with CCAdj (Line 12), corporate profits (Line 13), net interest and miscellaneous payments (Line 18, all Table 1.12Q)
Note that the tax base for consumption taxes includes consumer durables, but to be consistent with the tax base in the model, the tax revenue is computed with the narrower tax base excluding consumer durables.

\[
(\text{rev})^c_t = \tau^c_t \times (c_t - (\text{Taxes on production and imports})^f_t - (\text{Sales taxes})^{s&l}_t) / ((\text{Population above 16})_t \times ((\text{GDP deflator})_t
\]

\[
(\text{rev})^n_t = \tau^n_t \times ((\text{wage income})_t + (\text{wage supplements})_t + (\text{wage taxes})^f_t + \frac{1}{2}(\text{Proprietors’ income})_t)
\]

\[
(\text{rev})^k_t = \tau^k_t \times ((\text{Capital income})_t + (\text{Property taxes})^{s&l}_t)
\]

I construct government debt as the cumulative net borrowing of the consolidated NIPA government sector and adjust the level of debt to match the value of consolidated government FoF debt at par value in 1950:Q1. A minor complication arises as federal net purchases of nonproduced assets (NIPA Table 3.2Q, Line 43) is missing prior to 1959Q3. Since these purchases typically amount to less than 1% of federal government expenditures with a minimum of -1.1%, a maximum of 0.76%, and a median of 0.4% from 1959:Q3 to 1969:Q3, two alternative treatments of the missing data leads to virtually unchanged implications for government debt. First, I impute the data by imposing that the ratio of net purchases of nonproduced assets to the remaining federal expenditure is the same for all quarters from 1959:Q3 to 1969:Q4. Second, I treat the missing data as zero.

In 2012 the FoF data on long term municipal debt was revised up. The revision covers all quarters since 2004, but not before, implying a jump in the debt time series.\(^{41}\) I splice together a new smooth series from the data before and after 2004 by imposing that the growth of municipal debt from 2003:Q4 to 2004:Q1 was the same before and after the revision. This shifts up the municipal and consolidated debt levels prior to 2004. The revision in 2004 amounts to $840bn, or 6.8% of GDP.

The above data is combined with data from the web appendices of Romer and Romer (2004), Fernald (2012), Ramey (2011), and Mertens and Ravn (2013) on narrative shock measures. I standardize the different narrative shock measures to have unit standard deviation.

A.3 Model equations (under construction)
A.4 Additional Figures

\(^{41}\)http://www.bondbuyer.com/issues/121_84/holders-municipal-debt-1039214-1.html
“Data Show Changes in Muni Buying Patterns” by Robert Slavin, 05/01/2012 (retrieved 01/24/2014).
Romer and Romer (2004) FFR

Filtering shocks

Fernald TFP

Ramey gov. spending

Figure 11: Correlation of “narrative” shock measures with Smets-Wouters analogues
Figure 12: Kalman-smoothed posterior standard deviations relative to prior standard deviations (in parentheses: end values)

Note: The Fed began publishing a press release after FOMC meetings in 1994. The Gürkaynak et al. (2005)-measure is aggregated over a given quarter by summing individual shocks.

Figure 13: Monetary policy shock measures in comparison
Romer & Romer monetary shock  
Fernald TFP  
Ramey government spending  
Post 1982 sample: no narrative shocks used  
Post 1982 sample: narrative shocks used one by one  

Figure 14: Comparison of correlation of “narrative” shock measures for full sample and post 1982 sample
The shock is normalized such that the maximal median government spending response is unity.

Figure 15: Comparison: Ramey (2011)-data BVAR with Choleski “standard identification” (red, solid) vs narrative SPF shock identification (blue, dashed)
The shock is normalized such that the maximal median government spending response is unity.

Figure 16: Comparison: Ramey (2011)-data BVAR with different “1st stage” samples
The shock is normalized such that the maximal median government spending response is unity.

Figure 17: Comparison: Ramey (2011)-data BVAR with narrative SPF shock identification. Certain (red, solid) and uncertain covariance (blue, dashed)
Correlation Median 90% credible set

| 1st stage sampling: $\text{corr}[\hat{F}, \hat{F}^{(i)}]$ | 0.950 | (0.828, 0.987) |
| IRF identification: $\text{corr}[A_{\pi,1}^{IV}, A_{\pi,1}^{\text{Cholesky}}]$ | 0.989 | (0.946, 0.998) |

Table 3: Diagnostic correlations for instrumented fiscal shock

Figure 18: Comparing individual IRFs when only detrended output enters the Taylor Rule: Pointwise medians in DSGE model with different hours definitions
Figure 19: Comparison of correlation of “narrative” shock measures with fiscal model analogues: subsamples