Reputation and Liquidity Traps

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Abstract

This paper studies credible policies in a New Keynesian economy in which the nominal interest rate is subject to the ZLB constraint and contractionary shocks hit the economy occasionally. The Ramsey policy involves keeping the policy rate low even after the shock disappears, but the central bank would be tempted to raise the rate to close consumption and inflation gaps if it could re-optimize. I find that the Ramsey policy is credible if the contractionary shock occurs sufficiently frequently. In the best credible plan, if the central bank reneges on the promise of low policy rates, it will lose reputation and the private sector will not believe such promises in future recessions. When the shock hits the economy sufficiently frequently, the incentive to maintain reputation outweighs the short-run incentive to close consumption and inflation gaps, and keeps the central bank on the originally announced path of low nominal interest rates.

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1 Introduction

A promise to keep the short-term nominal interest rate near zero for an extended period has been
an important feature of the recent monetary policy in the United States. FOMC has repeatedly
stated that it will keep the federal funds rate near zero for a considerable time after the economic
recovery strengthens.\(^1\) However, as the economic recovery strengthens, the inflationary pressures
arising from the reduced economic slack can potentially give the future Federal Reserve an incentive
to renege on the promise and start raising the federal funds rate.

The theory of optimal commitment policy in the New Keynesian economy nicely captures
this dilemma faced by the central bank. In the New Keynesian economy, in response to a large
contractionary shock, the central bank equipped with commitment technology promises to keep the
nominal interest rate low even after the contractionary shock disappears, as such promise reduces
the long-term real interest rate and stimulates the household’s spending.\(^2\) However, if the central
bank were to re-optimize again after the shock disappears, it would renege on the promise and raise
the rate in order to close consumption and inflation gaps. In other words, the policy of an extended
period of low nominal interest rates is time-inconsistent. In reality, despite this time-inconsistency,
financial and survey data suggests that the private sector expects the Federal Reserve to keep the
federal funds rate low for long. Although the theory of optimal commitment policy can explain why
the central bank should promise an extended period of low nominal interest rates, it can neither
explain why the central bank should fulfill such a promise, nor why the private sector should believe
it.

This paper provides a theory that explains why the central bank may want to fulfill the promise
of keeping the nominal interest rate low even after the economic recovery strengthens. The theory is
based on credible plans in a stochastic New Keynesian economy in which the nominal interest rate is
subject to the zero lower bound constraint and contractionary shocks hit the economy occasionally.
Credible plans can capture rich interactions between the government action and the private sector’s
belief, and I use this equilibrium concept to ask under what conditions, if any, the policy of keeping
the nominal interest rate low even after the economic recovery strengthens is time-consistent.\(^3\)

I find that the policy of keeping the nominal interest rate low for long is time-consistent if
the frequency of contractionary shocks is sufficiently high. The force that keeps the central bank
from raising the nominal interest rate is reputation. In the best credible equilibrium, if the central
bank reneges on its promise to keep the nominal interest rate low, it will lose reputation and the
private sector will never believe such promises in the face of future contractionary shocks. If the
private sector does not believe the promise of an extended period of low nominal interest rates, the
contractionary shock will cause large declines in consumption and inflation. Large consumption
collapse and deflation in the future liquidity traps reduce welfare even during normal times since

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\(^1\)See the FOMC statements from past few years.

\(^2\)See, for example, Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), Adam and Billi
(2006), and Werning (2012).

\(^3\)The equilibrium concept has been referred to as many different names, including sustainable plans, reputational
equilibria, sequential equilibria, and subgame Markov-Perfect equilibria.
the central bank cares about the discounted sum of future utility flows. Thus, the potential loss of reputation gives the central bank an incentive to fulfill the promise. When the frequency of shocks is sufficiently high, this incentive to maintain reputation outweighs the short-run incentive to raise the rate to close consumption and inflation gaps, and keeps the central bank on the originally announced path of low nominal interest rates.

I demonstrate this result in the following way. First, I construct a plan—a pair of government and private sector strategies—that induces the outcome that would prevail under the discretionary government. I will refer to this plan as the discretionary plan and show that this plan is time-consistent. Second, I propose a plan that guides the government to adhere to the Ramsey policy and the private sector to act accordingly, but instructs the private sector to believe that the government is following the discretionary outcome if the government ever deviates from the Ramsey policy. I will refer to this plan as the revert-to-discretion plan. By construction, this plan induces the Ramsey outcome. I then show by numerical simulations that the revert-to-discretion plan is time-consistent if the contractionary shock hits the economy sufficiently frequently.

This paper is related to the literature examining whether reputation can make the Ramsey equilibrium time-consistent in various macroeconomic models. In early contributions, Barro and Gordon (1983) and Rogoff (1987) asked whether reputation force can overcome inflation bias in models with a short-run trade-off between consumption and inflation. Chari and Kehoe (1990), Phelan and Stacchetti (2001), and Stokey (1991) have studied the time-consistency of the Ramsey policy in models of fiscal policy, while Chang (1998), Ireland (1997), and Orlik and Presno (2010) have studied the time-consistency of the Friedman rule in monetary models. More recently, Kurozumi (2008), Loisel (2008) and Sunakawa (2012) studied the time-consistency of the Ramsey policy in New Keynesian models with cost-push shocks but without the zero lower bound constraint.

The paper is closely related to a set of papers examining how the government can improve allocations at the zero lower bound in the absence of commitment technology. Eggertsson (2006) showed that, if the government has access to nominal debt, it chooses to issue nominal debt during the period of contractionary shocks so as to give the future government an incentive to lower the nominal interest rate and create inflation, and that this goes a long way toward achieving the Ramsey allocations. Using an open economy model, Jeanne and Svensson (2007) demonstrated that, if the government has concerns for the balance sheet, it can attain the Ramsey allocations by managing the balance sheet so as to give the future government an incentive to depreciate its currency and thus create inflation. More recently, Correia, Nicolini, and Teles (2013) have shown that labor income and consumption taxes, together with lump-sum taxes, can be manipulated in a way to achieve the first-best allocations even at the zero lower bound. This paper contributes to this body of work by proposing a new mechanism by which the central bank can attain the Ramsey allocations without a commitment technology. The proposed mechanism in this paper is novel in that it does not involve any additional policy instruments.

The paper is also related to Bodenstein, Hebden, and Nunes (2012) which study the consequence of imperfect credibility in the context of a New Keynesian economy with occasionally binding zero
lower bound constraints. While my paper and their paper are both motivated by the same idea that credibility of the central bank may be a key factor in understanding the effectiveness of the forward guidance policy, our approaches as well as the questions we ask are different. Their analysis is positive. They model imperfect credibility in a specific way—randomizing the timing of central bank’s optimization in a way reminiscent of the Calvo-pricing model—and ask what the effects of imperfect credibility on output and inflation are at the zero lower bound. On the other hand, my analysis is normative. I ask why the central bank may want to fulfill the promise and under what conditions the Ramsey outcome can prevail.

The rest of the paper is organized as follows. Section 2 describes the model and defines the competitive equilibria. Section 3 defines the discretionary and the Ramsey outcomes, and discusses their key features. Section 4 defines a plan and credibility and section 5 constructs the revert-to-discretionary plan that induces the Ramsey outcome. Section 6 presents the main results on the credibility of the Ramsey outcome. After a brief discussion on the scope of the paper in section 7, a final section concludes.

2 Model and competitive outcomes

The model is given by a standard New Keynesian economy. The environment features a representative household, monopolistic competition among a continuum of intermediate-goods producers, and sticky prices. The model abstracts from capital. Since this is a workhorse model, I will start with the well-known equilibrium conditions of the economy. Following the majority of the previous literature on the zero lower bound, I will conduct the analysis in a partially log-linearized version of the model, in which the equilibrium conditions are log-linearized except for the zero lower bound constraint on the nominal interest rate.

The only exogenous variable of the model is \( s_t \), interpreted as “the natural rate of interest.” I will also refer to \( s_t \) as the contractionary shock, the shock, or the state. \( s_t \) takes two values, H and L. H will be set to the steady-state real interest rate, and L will be assigned to a negative value so that the nominal interest rate that would keep inflation and consumption at the steady-state level is negative. When \( s_t = H \), the economy is said to be in the high state or the normal state. When \( s_t = L \), I will say that the economy is in the low state, or the economy is hit by the contractionary shock. A lower L will be interpreted as the shock being more severe. I will use \( s^t \) to denote a history of states up to period \( t \) (i.e. \( s^t := \{s_k\}_{k=1}^t \)) and \( S \) to denote the set of values \( s_t \) can take, i.e., \( S := \{L, H\} \).

The natural rate of interest rate evolves according to a two-state Markov process. Transition probabilities are given by

\[ \]
\[ Prob(s_{t+1} = L | s_t = H) = p_H \]  
\[ Prob(s_{t+1} = L | s_t = L) = p_L \]

\( p_H \) is the probability of moving to the low state next period when the economy is in the normal state today, and will be referred to as the *frequency* of the contractionary shocks. \( p_L \) is the probability of staying in the low state when the economy is in the low state today, and will be referred to as the *persistence* of the contractionary shocks. One key exercise of this paper will be to examine the credibility of the Ramsey policy in various economies with different values of \( p_H \) and \( p_L \).

I refer to the state-contingent sequence of consumption, inflation, and the nominal interest rate, \( \{c_t(s^t), \pi_t(s^t), r_t(s^t)\}_{t=1}^{\infty} \), as an *outcome*. Given a process of \( s_t \), a *competitive outcome*, or a *competitive equilibrium*, of the model is characterized by a sequence \( \{c_t(s^t), \pi_t(s^t), r_t(s^t)\}_{t=1}^{\infty} \) such that, for all \( t \geq 1 \) and \( s^t \in \mathcal{S} \), \( c_t(s^t) \in \mathbb{C} := [c_{min}, c_{max}] \), \( \pi_t(s^t) \in \mathcal{I} := [\pi_{min}, \pi_{max}] \) and \( r_t(s^t) \in \mathbb{R} := [r_{min}, r_{max}] \) and

\[ \chi_c c_t(s^t) = \chi_c E_t c_{t+1}(s^{t+1}) - \left[ r_t - E_t \pi_{t+1}(s^{t+1}) \right] + s_t \]  
\[ \pi_t = \kappa c_t + \beta E_t \pi_{t+1}(s^{t+1}) \]  

where \( c_t(s^t) \) and \( \pi_t(s^t) \) are consumption and inflation expressed as the log deviation from the deterministic steady-state and \( r_t(s^t) \) is the nominal interest rate.\(^6\) \( \chi_c \) is the inverse intertemporal elasticity of substitution of the representative household, and \( \kappa \) is the slope of the Phillips curve. The zero lower bound constraint on the nominal interest rate is imposed by setting

\[ r_{min} = 0 \]  

\( r_{max} \), \( c_{max} \), and \( \pi_{max} \) are set to arbitrarily large constants and \( c_{min} \) and \( \pi_{min} \) are set to arbitrarily small constants.

The government’s objective function at period \( t \) is given by

\[ w_t(s^t) := E_t \sum_{j=0}^{\infty} \beta^j u(\pi_{t+j}(s^{t+j}), c_{t+j}(s^{t+j})) \]  

where the utility flow at each period is given by the following function.

\[ u(\pi, c) := -\frac{1}{2} \left[ \pi^2 + \lambda e^2 \right] \]  

\(^6\)In the model without investment and government spending, consumption equals output. It is common in the New Keynesian literature to replace consumption with output in the Euler equation. However, I will depart from the common practice in presenting the model. Formulating credible plans requires us to specify who chooses what and when, and it is more natural to think of the household as choosing consumption, instead of output.
For any outcome, there is an associated state-contingent sequence of values, \( \{ w_t(s^t) \}_{t=1}^{\infty} \), which will be referred to as a value sequence.

**Notations**

Throughout the paper, I will use the following notations. For any variable \( x \), its state-contingent sequence is denoted by \( \mathbf{x} \). In other words,

\[
\mathbf{x} := \{ x_k(s^k) \}_{k=1}^{\infty}
\]

A state-contingent sequence up to time \( t \) and a continuation state-contingent sequence starting at time \( t \) are respectively denoted by \( \mathbf{x}^t \) and \( \mathbf{x}_t \). In other words,

\[
\mathbf{x}^t := \{ x_k(s^k) \}_{k=1}^{t}, \quad \mathbf{x}_t := \{ x_k(s^k) \}_{k=t}^{\infty}
\]

\( x^t \) (non-bold font) is used to denote a particular realization of \( \mathbf{x}^t \), and should not be confused with \( \mathbf{x}^t \) (bold font).

For any variable \( x \) with a range \( X \), \( x(s) \) denotes a state-contingent sequence with \( s_1 = s \), which is defined by a sequence of functions mapping a history of states with \( s_1 = s \) to \( X \). In other words,

\[
x_1 : s \to X \\
x_t : s \times S^{t-1} \to X
\]

\( x^t(s) \) denotes a state-contingent sequence with \( s_1 = s \) up to time \( t \). \( x_t(s) \) denotes a continuation state-contingent sequence starting at time \( t \) with \( s_t = s \), which is formally given by the following sequence of functions.

\[
x_t : s \to X \\
x_{t+k} : s \times S^{k-1} \to X
\]

\( CE(s) \) denotes the set of all competitive outcomes with \( s_1 = s \). That is, for each \( s \in S \),

\[
CE(s) := \{ (c(s), \pi(s), r(s)) \in C^\infty \times \Pi^\infty \times \mathbb{R}^\infty \\
| \text{Equations (3) and (4) hold for all } t \geq 1 \text{ and for all } s^t \in S^t \text{ with } s_1 = s \}
\]

\( CE_k(s) \) to denotes the set of continuation competitive outcomes starting at period \( k \) with \( s_k = s \). That is, for each \( s \in S \),
\[ CE_k(s) := \{(c_k(s), \pi_k(s), r_k(s)) \in C^\infty \times \Pi^\infty \times R^\infty \mid \text{Equations (3) and (4) hold for all } t \geq k \text{ and for all } s^t \in S^t \text{ with } s_k = s\} \]

3 The discretionary outcome and the Ramsey outcome

This section defines the discretionary and Ramsey outcomes, and discusses their key features. These outcomes will play a major role in the analysis of credible policies in later sections.

3.1 The discretionary outcome

At each time \( t \geq 1 \), the discretionary government chooses today’s consumption, inflation, and nominal interest rate in order to maximize its objective function, taking as given the value function and policy functions for consumption, inflation, and the nominal interest rate in the next period.\(^7\) The Bellman equation of the government’s problem is given by

\[ w_t(s_t) = \max_{\{c_t \in C, \pi_t \in \Pi, r_t \in R\}} u(c_t, \pi_t) + \beta E_t w_{t+1}(s_{t+1}) \] (8)

where the optimization is subject to the equations characterizing the competitive equilibria (i.e., equations (3) and (4)). Let \( \{w_d(\cdot), c_d(\cdot), \pi_d(\cdot), r_d(\cdot)\} \) be the set of time-invariant value function and policy functions for consumption, inflation, and the nominal interest rate that solves this problem in which the ZLB only binds in the low state.\(^8\) The discretionary outcome is defined as, and denoted by, the state-contingent sequence of consumption, inflation, and the nominal interest rate, \( \{c_{d,t}(s_t) := c_d(s_t), \pi_{d,t}(s_t) := \pi_d(s_t), r_{d,t}(s_t) := r_d(s_t)\} \) such that \( c_{d,t}(s^t) := c_d(s^t), \pi_{d,t}(s^t) := \pi_d(s^t), \) and \( r_{d,t}(s^t) := r_d(s^t) \) and the discretionary value sequence is defined as, and denoted by, \( \{w_{d,t}(s^t)\} \) such that \( w_{d,t}(s^t) := w_d(s^t) \).

Figure 1 shows the discretionary outcome and value sequence for a particular realization of \( s^{10} \in S^{10} \) in which \( s_1 = L \), and \( s_t = H \) for \( 2 \leq t \leq 10 \). It also plots the sequence of contemporaneous utility, \( \{u(c_{d,t}, \pi_{d,t})\} \) associated with the consumption and inflation sequence. In each panel, solid black and dashed red lines are for the economy with \( p_H = 0.01 \) and \( p_H = 0 \). Values for other parameters are the same in both black and red lines, and are listed in Table 1.

In the model without commitment, as soon as the contractionary shock disappears, the government raises the nominal interest rate in order to stabilize consumption and inflation. In the model where the high state is an absorbing state (i.e., \( p_H = 0 \)), the government raises the nominal interest rate to \( H \), and consumption and inflation are fully stabilized at zero at time 2. Accordingly, the

\(^7\)Following the literature, I assume that the discretionary government acts as a planner and chooses the policy instrument and allocations without being explicit about the within-period timing assumption of the government and the private sector. While it is not important here, the within-period timing will be crucial in analyzing credible plans in later sections.

\(^8\)In the Appendix, I demonstrate the existence of a time-invariant solution to this discretionary government’s problem in which the ZLB binds in both states.
contemporaneous utility is zero as well. In the model with a positive $p_H$, the household and firms will have an incentive to lower consumption and prices in the normal period, as they expect that consumption and inflation will decline in some states tomorrow. The government tries to prevent those declines by reducing the nominal interest rates from the deterministic steady-state level, and in equilibrium, consumption and inflation are respectively slightly positive and negative. As a result, the contemporaneous utility flows are slightly negative.

One key feature of the model with recurring shocks is that the discretionary value remain negative even after the shock disappears, as captured in the the dashed red line bottom-right panel. The discretionary value stays negative even during the normal times for two reasons. First, consumption and inflation are slightly positive and negative due to the anticipation effects described above, pushing down contemporaneous utility flow below zero in the high state. Second, and more quantitatively importantly, the possibility that consumption and inflation will decline in response to the future contractionary shock tomorrow lowers the discretionary value by reducing the continuation value of the government. This is in a sharp contrast to the economy in which the contractionary shock never hits after the initial shock. In such a model, the discretionary value becomes zero after the shock disappears, as shown in the red line in the bottom-right panel. This feature of the economy with recurring shocks—that the discretionary values remain negative even after the shock disappears—will be important in understanding the reputational force present in the credible plans.

### 3.2 The Ramsey outcome

The Ramsey planner chooses a state-contingent sequence of consumption, inflation, and the nominal interest rate in order to maximize the expected discounted sum of future utility flows at time one. For each $s_1 \in S$, the Ramsey planner’s problem is given by

$$
\max_{(c(s_1), \pi(s_1), r(s_1)) \in CE(s_1)} w_1(s_1)
$$

where the optimization is subject to the equations characterizing the competitive equilibria (i.e., equations (3) and (4)). The **Ramsey outcome** is defined as the state-contingent sequence of consumption, inflation, the nominal interest rate that solves the problem above. In other words, the Ramsey outcome is a competitive outcome with the highest time-one value. I will denote the Ramsey outcome by $\{c_{ram,t}(s^t), \pi_{ram,t}(s^t), r_{ram,t}(s^t)\}_{t=1}^{\infty}$. At each period $t$ and for each $s^t \in S^t$, the value associated with the Ramsey outcome is given by

$$
w_{ram,t}(s^t) := E_t \sum_{j=0}^{\infty} \beta^j u(\pi_{ram,t+j}(s^{t+j}), c_{ram,t+j}(s^{t+j}))
$$

I will refer to $\{w_{ram,t}(s^t)\}_{t=1}^{\infty}$ as the **Ramsey value sequence**.

Solid black lines in Figure 2 shows the Ramsey outcome and value sequence in the economy with $p_H = 0.01$ for a particular realization of $s^{10} \in S^{10}$, together with the sequence of contemporaneous
utility, \( \{u(c_{ram,t}(s^t), \pi_{ram,t}(s^t))\}_{t=1}^{\infty} \), associated with the outcome sequence. The figure shows that the Ramsey planner keeps the nominal interest rate at zero even after the contractionary shock disappears. An extended period of low nominal interest rates, together with consumption boom and above-trend inflation at time 2, mitigates the declines in consumption and inflation during the period of the contractionary shock.

Since the contemporaneous utility flow is maximized when consumption and inflation are stabilized at zero, the consumption boom and above-trend inflation are undesirable ex post. Thus, if the Ramsey planner was hypothetically given an opportunity to re-optimize again after the shock disappears, the planner would choose to stabilize consumption and inflation. This is captured in the dashed red lines which show the sequence of consumption, inflation and the nominal interest rate the Ramsey planner would choose in the hypothetical reoptimization at time 2. The planner would renege on the promise of the low nominal interest rate and raise the rate in order to stabilize consumption and inflation. This discrepancy between the pre-announced policy path (solid black lines) and the policy path the government would like to choose in the future (dashed red lines) captures the time-inconsistency of the Ramsey policy.

In the rest of the paper, we will study credible plans, which allow the private sector’s belief to shift if the government reneges on the promise it has made in the past. By allowing the private sector’s belief to depend on the history of policy actions, credible plans can give the government an incentive to fulfill the promise of the low nominal interest rate, and can make the Ramsey policy time-consistent.

4 Definition of a plan and credibility

This section defines a plan, credibility, and related concepts. The definitions closely follow Chang (1998).

4.1 Plan

A government strategy, denoted by \( \sigma_g := \{\sigma_{g,t}\}_{t=1}^{\infty} \), is a sequence of functions that maps a history of the nominal interest rates up to the previous period and a history of states up to today into today’s nominal interest rate. Formally, \( \sigma_{g,t} \) is given by

\[
\sigma_{g,1} : S \rightarrow \mathbb{R} \\
\sigma_{g,t} : \mathbb{R}^{t-1} \times S^t \rightarrow \mathbb{R}
\]

The first period is a special case since there is no previous policy action. Given a particular realization of \( \{s_t\}_{t=1}^{\infty} \), a sequence of nominal interest rates will be determined recursively by \( r_1 = \sigma_{g,1} \) and \( r_t = \sigma_{g,t}(r^{t-1}, s^t) \) for all \( t > 1 \) and for all \( s^t \in S^t \). A government strategy is said to induce a sequence of the nominal interest rates.
A private sector strategy, denoted by $\sigma_p := \{\sigma_{p,t}\}_{t=1}^\infty$, is a sequence of functions mapping a history of nominal interest rates up to today and a history of states up to today into today’s consumption and inflation. Formally, $\sigma_{p,t}$ is given by

$$\sigma_{p,t} : \mathbb{R}^t \times S^t \rightarrow (\mathbb{C}, \Pi)$$

Given a government and private-sector strategy, a sequence of consumption and inflation will be determined recursively by $(c_t, \pi_t) = \sigma_{p,t}(r^t, s^t)$ for all $t \geq 1$ and for all $s^t \in S^t$. A private sector strategy, together with a government strategy, is said to induce a sequence of consumption and inflation.

Notice that, while the nominal interest rate today depends on the history of nominal interest rates up to the previous period, consumption and inflation today depend on the history of nominal interest rates up to today. The implicit within-period-timing protocol behind this setup is that the government moves before the private sector does.

**Definition of a plan:** A plan is defined as a pair of government and private sector strategies, $(\sigma_g, \sigma_p)$.

Notice that a plan induces an outcome—a state-contingent sequence of consumption, inflation, and the nominal interest rate. As discussed earlier, there is a value sequence $\{w_t(s^t)\}_{t=1}^\infty$, associated with any outcome. A plan is said to imply a value sequence.

### 4.2 Credibility

A few more concepts and notations need to be introduced before defining credibility. Let us use $CE^R_t(s)$ to denote a set of state-contingent sequences of the nominal interest rate consistent with the existence of a competitive equilibrium when $s_t = s$. In other words, for each $s \in S$

$$CE^R_t(s) \equiv \{r_t(s) \in \mathbb{R}^\infty : \exists (c_t(s), \pi_t(s)) \text{ such that } (c_t(s), \pi_t(s), r_t(s)) \in CE_t(s)\}$$

**Definition of admissibility:** $\sigma_g$ is said to be admissible if, after any history of policy actions, $r^{t-1}$, and any history of states, $s^t$, $r_t(s)$ induced by the continuation of $\sigma_g$ belongs to $CE^R_t(s_t)$.

**Definition of credibility:** A plan, $(\sigma_g, \sigma_p)$, is credible if

- $\sigma_g$ is admissible.
- After any history of policy actions, $r^t$, and any history of states, $s^t$, the continuation of $\sigma_p$ and $\sigma_g$ induce a $(c_t(s_t), \pi_t(s_t), r_t(s_t)) \in CE_t(s_t)$.
- After any history $r^{t-1}$ and $s^t$, $r_t(s_t)$ induced by $\sigma_g$ maximizes the government’s objective over $CE^R_t(s_t)$ given $\sigma_p$. 

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An outcome is said to be *credible* if there is a credible plan that induces it. When a certain plan $A$ is credible and the plan $A$ induces a certain outcome $\alpha$, we say that the outcome $\alpha$ can be made time-consistent by the plan $A$.

5 The discretionary plan and the revert-to-discretion plan

In the first subsection, I will define the discretionary plan and demonstrate that it is credible. In the second subsection, I will define the revert-to-discretion plan and discusses a general condition under which it is credible.

5.1 The discretionary plan

The discretionary plan, $(\sigma^d_g, \sigma^d_p)$, consists of the following government strategy

- $\sigma^d_{g,1} = r_d(s_1)$ for any $s_1 \in S$
- $\sigma^d_{g,t}(r^{t-1}, s^t) = r_d(s_t)$ for any $s^t \in S^t$ and any $r^{t-1} \in \mathbb{R}^{t-1}$

and the following private-sector strategy

- $\sigma^d_{p,t}(r^t, s^t) = (c_d(s_t), \pi_d(s_t))$ if $r_t = r_d(s_t)$
- $\sigma^d_{p,t}(r^t, s^t) = (c_{br}(s_t, r_t), \pi_{br}(s_t, r_t))$ otherwise

where $c_{br}(s_t, r_t) = E_{t+1}c_{d,t+1}(s^{t+1}) - \frac{1}{\chi_c}\left[ r_t(s^t) - E_{t+1}\pi_{d,t+1}(s^{t+1}) \right] - s_t$

$$\pi_{br}(s_t, r_t) = \kappa c_{br}(s_t, r_t) + \beta E_{t+1}\pi_{d,t+1}(s^{t+1})$$

The government strategy instructs the government to choose the nominal interest rate consistent with the discretionary outcome, regardless of the history of past nominal interest rates. The private sector strategy instructs the household and firms to choose consumption and inflation consistent with the discretionary outcome, as long as today’s nominal interest rate chosen by the government is consistent with the discretionary outcome. If the government chooses an interest rate that is not consistent with the discretionary outcome, then the private sector strategy instructs the household and firms to optimally choose today’s consumption under the belief that the government in the future will not deviate again.

By construction, the discretionary plan induces the discretionary outcome, and the value sequence implied by the discretionary plan is identical to the discretionary value sequence.

**Proposition 1**: The discretionary plan is credible.

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Subscript *br* stands for best response.
See the Appendix for the proof. The discretionary plan will be a key ingredient in constructing the revert-to-discretion plan, which we will discuss now, and this proposition will be essential in analyzing the credibility of the revert-to-discretion plan.

5.2 The revert-to-discretion plan

The revert-to-discretion plan, \((\sigma^{rtd}_g, \sigma^{rtd}_p)\), consists of the following government strategy

- \(\sigma^{rtd}_{g,1} = r_{ram,1}(s_1)\) for any \(s_1 \in S\)
- \(\sigma^{rtd}_{g,t}(r^{t-1}, s^t) = r_{ram,t}(s^t)\) if \(r_j = r_{ram,j}(s^j)\) for all \(j \leq t - 1\)
- \(\sigma^{rtd}_{g,t}(r^{t-1}, s^t) = \sigma^{d}_{g,t}(r^{t-1}, s^t)\) otherwise.

and the following private-sector strategy

- \(\sigma^{rtd}_{p,t}(r^t, s^t) = c_{ram,t}(s^t)\) if \(r_j = r_{ram,j}(s^j)\) for all \(j \leq t\)
- \(\sigma^{rtd}_{p,t}(r^t, s^t) = \sigma^{d}_{p,t}(r^t, s^t)\) otherwise.

The government strategy instructs the government to choose the nominal interest rate consistent with the Ramsey outcome, but chooses the interest rate consistent with the discretionary outcome if it has deviated from the Ramsey outcome at some point in the past. The private sector strategy instructs the household and firms to choose consumption and inflation consistent with the Ramsey outcome as long as the government has never deviated from the Ramsey outcome. If the government has ever deviated from the nominal interest rate consistent with the Ramsey outcome, the private sector strategy instructs the household and firms to choose consumption and inflation today based on the belief that the government in the future will choose the nominal interest rate consistent with the discretionary outcome.

By construction, the revert-to-discretion plan induces the Ramsey outcome, and the implied value sequence is identical to the Ramsey value sequence. The main exercise of the paper is to characterize the conditions under which the revert-to-discretion plan is credible. The following proposition will be useful in answering this question.

**Proposition 2**: The revert-to-discretion plan is credible if and only if \(w_{ram,t}(s^t) \geq w_{d,t}(s^t)\) for all \(t \geq 1\) and all \(s^t \in S^t\).

See the Appendix for proof. The condition that \(w_{ram,t}(s^t) \geq w_{d,t}(s^t)\) for all \(t \geq 1\) and all \(s^t \in S^t\) makes sure that the government does not have an incentive to deviate from the instruction given by the government strategy after any history \(r^{t-1}\) and \(s^t\) in which the Ramsey policy has been followed.
It is useful to decompose $w_{\text{ram},t}(s^t)$ and $w_{\text{d},t}(s^t)$ into two components in order to gain insights on this proposition. Notice that, after the history in which the Ramsey policy has been followed, $w_{\text{ram},t}(s^t)$ is the value of following the instruction given by the revert-to-discretionary plan and $w_{\text{d},t}(s^t)$ is the best value the government can attain if the government deviates from the instruction. $w_{\text{ram},t}(s^t)$ and $w_{\text{d},t}(s^t)$ can both decomposed into today’s utility flows ($u(c_{\text{ram},t}(s^t), \pi_{\text{ram},t}(s^t))$, $u(c_{\text{d},t}(s^t), \pi_{\text{d},t}(s^t))$) and the discounted continuation values ($\beta E_t w_{\text{ram},t+1}(s^{t+1})$, $\beta E_t w_{\text{d},t+1}(s^{t+1})$) as follows.

$$w_{\text{ram},t}(s^t) = u(c_{\text{ram},t}(s^t), \pi_{\text{ram},t}(s^t)) + \beta E_t w_{\text{ram},t+1}(s^{t+1})$$

$$w_{\text{d},t}(s^t) = u(c_{\text{d},t}(s^t), \pi_{\text{d},t}(s^t)) + \beta E_t w_{\text{d},t+1}(s^{t+1})$$

Thus, the restriction $w_{\text{ram},t}(s^t) \geq w_{\text{d},t}(s^t)$ can be written as

$$u(c_{\text{ram},t}(s^t)) + \beta E_t w_{\text{ram},t+1}(s^{t+1}) \geq u(c_{\text{d},t}(s^t), \pi_{\text{d},t}(s^t)) + \beta E_t w_{\text{d},t+1}(s^{t+1})$$

$$\iff \beta E_t [w_{\text{ram},t+1}(s^{t+1}) - w_{\text{d},t+1}(s^{t+1})] \geq [u(c_{\text{d},t}(s^t), \pi_{\text{d},t}(s^t)) - u(c_{\text{ram},t}(s^t), \pi_{\text{ram},t}(s^t))]

The left-hand and right-hand sides of this last inequality constraint respectively capture the loss in the continuation value and the gain in today’s utility flow if the government deviates from the Ramsey policy. Thus, the aforementioned proposition can be restated as “If the loss in the continuation value caused by the deviation from the Ramsey prescription is larger than the gain in today’s utility flow, the revert-to-discretion plan is credible.”

According to this proposition, in order to check whether or not the revert-to-discretion plan is credible given a particular set of parameter values, it suffices to solve for the discretionary and Ramsey value sequences and check whether or not $w_{\text{ram},t}(s^t) \geq w_{\text{d},t}(s^t)$ for all $t \geq 1$ and all $s^t \in S^t$. While the partial log-linearization framework allows us to derive the discretionary value sequence in closed-form, the Ramsey value sequence cannot be characterized analytically.\(^{10}\) Thus, it is not feasible to analytically characterize the conditions under which $w_{\text{ram},t}(s^t) \geq w_{\text{d},t}(s^t)$ for all $t \geq 1$ and all $s^t \in S^t$. In the next section, we will use numerical simulations to characterize the set of parameter values—particularly ones governing the natural rate process—for which the revert-to-discretion plan is credible.

6 Results

In this section, I solve the discretionary and Ramsey value sequences for various combinations of parameter values and characterize the circumstances under which $w_{\text{ram},t}(s^t) \geq w_{\text{d},t}(s^t)$ for all

\(^{10}\)See Eggertsson and Woodford (2003).
\[ t \geq 1 \text{ and all } s^t \in \mathbb{S}^t, \] and thus the revert-to-discretionary plan is credible. I organize the results in the following way. I first describe the set of \((p_H, p_L)\) under which the revert-to-discretionary plan is credible, given the baseline values for other parameters of the model (i.e. \(L, \beta, \chi, \kappa, \lambda\)) as listed in Table 1. I then describe how this set varies when other parameters take alternative values.

First-order necessary conditions of the discretionary government’s problem is given by a system of linear equations, and thus the discretionary outcome and values can be computed by linear algebra. The Ramsey outcomes and value sequence are solved globally by a time-iteration method of Coleman (1991). For each set of parameter values considered, I simulate the model until I observe one million episodes of contractionary shocks, and decides that \(w_{\text{ram},t}(s^t) \geq w_{\text{d},t}(s^t) \) for all \(t \geq 1\) and all \(s^t \in \mathbb{S}^t\) if the simulated Ramsey values are always above the simulated discretionary values.

Figure 3 show whether the revert-to-discretion plan is credible or not for the set of \((p_H, p_L) \in \mathbb{P}_H \times \mathbb{P}_L\) where \(\mathbb{P}_H\) is 101 equally spaced grid points between \([0, 0.01]\) and \(\mathbb{P}_L\) is 51 equally spaced grid points between \([0, 1]\). Blank areas indicate combinations of \((p_H, p_L)\) for which the revert-to-discretion plan is credible. Blue dots indicate the combinations of \((p_H, p_L)\) for which the revert-to-discretion plan is not credible. Black dots indicate the combinations of \((p_H, p_L)\) for which the revert-to-discretion plan is not defined because the discretionary outcome does not exist.\(^{11}\)

### 6.1 Frequency

**Result 1:** For any given \(p_L \in \mathbb{P}_L\), there exists \(p_H^*\) such that the revert-to-discretion plan is credible if \(p_H \geq p_H^*\) and is not credible otherwise.

In other words, for any given \(p_L \in \mathbb{P}_L\), the revert-to-discretionary plan is credible if and only if the contractionary shock hits the economy sufficiently frequently.

To gain insights on this result, Figure 4 compares the discretionary and Ramsey outcomes/value sequences for two economies—one with frequent shocks (i.e., a small \(p_H\)) and the other with infrequent shock (i.e., a large \(p_H\)). The left column shows the Ramsey and discretionary outcomes/value sequences in the model with infrequent shocks, while the right column shows those in the model with frequent shocks.

Top three rows show that the discretionary and Ramsey outcomes are very similar across two models with infrequent and frequent shocks. However, according to the bottom row, the discretionary and Ramsey value sequences behave differently when the shock frequencies are different. In particular, in the model with frequent shocks, the discretionary value stays below the Ramsey value at time 2 and remains so afterwards. In contrast, in the model with infrequent shocks, the discretionary value exceeds the Ramsey value at time 2. Thus, the revert-to-discretionary plan is not credible when the contractionary shock occurs infrequently.

To understand why the discretionary value stays below the Ramsey value in the model with frequent shocks, it is useful to examine how the loss in the continuation value and the gain in today’s utility flow at time 2 vary with the frequency of the shock. The black and red lines in

\(^{11}\)The Appendix explains in detail why the solution does not exist for certain combinations of \((p_H, p_L)\).
Figure 5 respectively depict these two objects for various values of $p_H$. Since the frequency of the shock does not substantially affects the discretionary and Ramsey outcomes at time 2, the gain in today’s utility flow of deviating from the Ramsey policy are essentially unaltered by the shock frequency, as seen in the constant red line. However, the frequency of the shock does alter the loss in the continuation value associated with the deviation from the Ramsey prescription. In particular, the loss in the continuation value increases with frequency shocks. For sufficiently frequent shocks (i.e., sufficiently large $p_H$), the losses in the continuation value becomes larger than the short-run gain, making the revert-to-discretionary plan credible.

To understand why the loss in the continuation value increases as the shock becomes more frequent, Figure 6 shows how the Ramsey continuation value and the continuation value in the case of deviation vary with the frequency at period 2. The panel shows that the discretionary continuation value declines more rapidly as $p_H$ gets larger than the Ramsey continuation value does. As seen in Figure 4, the contractionary shock leads to a larger decline in consumption and inflation in the discretionary outcome than in the Ramsey outcome in the face of contractionary shock. Thus, a higher probability of contractionary shocks reduce the expected discounted sum of future utility flows associated with the discretionary outcome by more than that associated with the Ramsey outcome, making the loss in the continuation value an increasing function of the frequency.

6.2 Persistence

**Result 2**: For a sufficiently high $p_H \in \mathbb{P}_H$, the revert-to-discretion plan is credible regardless of the value of $p_L$. For a sufficiently small $p_H \in \mathbb{P}_H$, there exists $p^*_L$ such that the revert-to-discretion plan is credible if $p_L \geq p^*_L$ and is not credible otherwise.

This result says that, even when the frequency of shock is small, the revert-to-discretion plan is credible if the contractionary shock is sufficiently persistent. For example, when $p_H = 0.05$, the revert-to-discretionary plan is credible regardless of the values of $p_L$. When $p_H = 0.005$, the revert-to-discretionary plan is credible if $p_L > 0.5$, but is not credible otherwise. Another way of phrasing this result is that the threshold value of $p_H$ above which the revert-to-discretionary plan is credible is decreasing in $p_L$.\(^{12}\)

To understand the mechanism behind this result, Figure 7 compares the discretionary and Ramsey outcomes/value sequences for two economies—one with transient shocks (i.e., a small $p_L$) and the other with persistent shock (i.e., a large $p_L$). The left column shows the Ramsey and discretionary outcomes/value sequences in the model with transient shocks, while the right column shows those in the model with persistent shocks.

When the persistence is high, the household and firms expect to stay in the low state for long. Since marginal costs and inflation are low in the low state, such expectation implies lower expected

\(^{12}\)There is a discontinuity at $p_L = 0$. When the $p_L$ is low, the marginal changes in $p_L$ affects whether or not the discretionary value exceeds the Ramsey values only after a long-lasting spell of low states. When the probability of staying at the low state is zero, you never observe the low state lasting longer than one period.
marginal costs and higher expected real interest rates. Accordingly, the household and firms in
the low state choose lower consumption and inflation. However, the Ramsey planner can mitigate
this effect by promising a higher inflation, a larger consumption boom, and a longer period of zero
nominal interest rates after the shock disappears. Thus, the declines in consumption and inflation
from marginal increases in persistence is larger in the discretionary outcome than in the Ramsey
outcome, as captured in the second and third rows in Figure 7. As a result, the continuation value
of reverting back to the discretionary plan declines more rapidly with $p_L$ than that of staying with
the Ramsey outcome, as depicted in Figure 9. This implies that the long-run loss of reverting back
to a discretionary plan is higher with more persistent shocks, as depicted by the solid black line
in Figure 8. In the meantime, the promise of higher inflation and consumption increases in the
economy with persistent shocks means that the short-run incentive to deviate from the promise is
larger, as illustrated by Figure 8. Quantitatively, the long-run loss increases more rapidly than the
short-run gain for large values of $p_L$, the former exceeds the latter for sufficiently large values of
$p_L$.

6.3 Sensitivity Analysis

Figure 10 shows how alternative values of other parameters alter the set of $(p_H, p_L)$ under which
the revert-to-discretionary plan is credible. For the sake of brevity, I will discuss the results only
casually in this section, and will delegate to the Appendix detailed analyses on how each parameter
affects the outcomes and value sequences as well as the short-run gain and the long-run loss of
deviating from the Ramsey policy.

Severity of the shock ($L$)

A larger shock (a larger $|L|$) means larger declines in consumption and inflation under both
discretionary and Ramsey outcomes. However, the Ramsey planner can promise a higher inflation
and a larger consumption boom to mitigate the declines in consumption and inflation during the
period of contractionary shocks. Thus, a marginal increase in the shock severity leads to larger
marginal declines in low-state consumption and inflation under the discretionary outcome than
under the Ramsey outcome, leading to larger marginal declines in the both high-state and low-
state values. Accordingly, the long-run loss from reneging on the Ramsey promise and reverting
back to the discretionary outcome is larger in the economy with more severe shocks.

On the other hand, as the Ramsey promise entails a higher inflation and larger consumption
boom, the short-run gain from reneging on the promise is also larger with a larger shock. As such,
the overall effects are mixed. According to the figure, while the threshold frequency is higher when
the shock is larger in the economy with highly persistent shocks, the threshold frequency is lower
when the shock is larger in the economy in which the shock persistence is low.
Discount rate ($\beta$)

With a higher $\beta$, the same difference between the discretionary and Ramsey continuation values translates into a larger difference between discounted continuation values. As a result, a high discount factor implies a larger long-run loss of reneging on the promise. The discount rate also affects the short-run gain from reneging on the promise as it alters inflation booms the Ramsey planner would promise, but this effect is quantitatively negligible. As a result, credible region expands with larger $\beta$. Figure 10 shows that the threshold $p_H$ above which the revert-to-discretion plan is credible is lower in the economy with a larger $\beta$. This result is consistent with the previous literature on credible plans which has shown that a sufficiently large $\beta$ can make the Ramsey policy credible in various contexts (see, for example, Chari and Kehoe (1990), Phelan and Stacchetti (2001), and Kurozumi (2008)).

Slope of the Phillips Curve ($\kappa$)

When the slope of the Phillips curve is high (i.e. prices are flexible), declines in low-state consumption and inflation are exacerbated under both discretionary and Ramsey outcomes. While the Ramsey planner mitigates those declines by promising a higher inflation and consumption boom, the discretionary government cannot. Thus, a marginal increase in the slope parameter leads to larger marginal declines in consumption and inflation, and thus values, in the discretionary outcome than in the Ramsey outcome. Accordingly, the long-run loss from reverting back to the discretionary plan is larger in the economy with more flexible prices. On the other hand, the Ramsey promise of higher inflation and larger consumption booms means that short-run gain from reneging on the promise once the shock disappears is higher under a more flexible price environment. Quantitatively, for the calibration considered in this paper, the second effects dominates the first effect. The threshold value of $p_H$ above which the revert-to-discretionary plan is credible is lower for any given $p_L$ as shown in Figure 10.

Inverse IES ($\chi_c$)

When the inverse IES is high, the household’s consumption decision is more sensitive to the fluctuations in $s_t$. Since firms’ pricing today depends on consumption today, inflation today is more sensitive to the fluctuations in $s_t$ with a higher $\chi_c$. Thus, a higher $\chi_c$ implies larger declines in consumption and inflation in the low state under both discretionary and Ramsey outcomes. While the Ramsey planner can mitigate those additional declines by future promises, the discretionary government has no tool to mitigate them. As a result, a marginal increase in the inverse IES leads to larger marginal declines in low-state consumption and inflation under the discretionary outcome than in the Ramsey outcome. Since these lower low-state consumption and inflation reduce values in both states. On the other hand, higher promised consumption and inflation with a larger $\chi_c$ mean a larger short-run gain from reneging on the promise. Thus, the effects are mixed. Similarly to the severity of shocks, while the threshold frequency is higher when the inverse IES is larger in
the economy with highly persistent shocks, the threshold frequency is lower when the inverse IES is larger in the economy in which the shock persistence is low.

**Weight on consumption volatility (λ)**

A larger $\lambda$ means that the government cares more about consumption volatility relative to inflation volatility. Under the discretionary government, a greater concern for consumption volatility exacerbates the deflation bias in the high state, in turn magnifying deflation and consumption decline in the low state.\(^{13}\) The Ramsey planner can mitigate this effect by promising a higher inflation and consumption boom in the future, and marginal increases in the weight on consumption volatility reduces the low-state consumption and inflation, and thus values in both states, by more under the discretionary outcome than under the Ramsey outcome. Accordingly, the long-run loss of reneging on the promise, and therefore accepting the continuation value associated with the discretionary outcome, is higher. On the other hand, promises of higher inflation and consumption hikes means that the short-run gain of deviating from the promise is larger. Quantitatively, the second effect dominates the first effect unless the persistence of the shock is very high, as shown in Figure 10. For most values of $p_L$, the threshold frequency above which the revert-to-discretionary plan is credible is higher when the central bank places a greater weight on consumption volatility in its objective function.

7 Additional results and discussion

7.1 The revert-to-discretion(N) plan

One may feel that the private sector’s punishment strategy of reverting to the discretionary outcome forever after the government’s deviation may be too harsh and unrealistic. In reality, the household and firms do not live forever. The head of the central bank changes at some frequencies. Even if the same central banker is in charge for an extended period of time, central bank doctrines can change over the course of his/her tenure.\(^{14}\) Based on these considerations, I define and analyze the revert-to-discretion($N$) plan in which the punishment regime lasts for a finite period of time ($N$) and the economy reverts back to the Ramsey outcome afterwards. Since a formal definition of this plan is involved, I relegate it to the Appendix for the sake of brevity. Here, I report the main results from the analysis.

Figure 11 shows how the credible regions vary with the number of punishment periods. Black and red lines are respectively the threshold frequencies above which the revert-to-discretion($N$) plans are credible with $N=40$ and 200, while the blue line depicts the threshold frequency for the standard revert-to-discretion plan. Not surprisingly, given $p_L$, the threshold frequency increases with the number of punishment periods. A smaller punishment period is associated with a larger

\(^{13}\)See Nakata and Schmidt (2014) for more detailed analyses

\(^{14}\)For example, consider the gradual move toward transparency during the tenure of Alan Greenspan at the Federal Reserve.
value for the government in the case of defection, and therefore with a smaller long-run loss from reneging on the Ramsey promise. Thus, with a less severe punishment, the contractionary shock needs to be more frequent in order to make the Ramsey policy credible.

### 7.2 The revert-to-deflation plan

Throughout the paper, I focus on the question of whether or not the Ramsey outcome can be made time-consistent by the revert-to-discretion plan. However, there is an alternative plan that induces the Ramsey outcome and that is credible under a different set of conditions than those for credibility of the revert-to-discretion plan. In the Appendix, I construct a plan called the *revert-to-deflation* plan in which the government’s deviation from the Ramsey prescription is punished by reverting to the Markov-Perfect equilibrium in which the zero lower bound binds in both states, and show that it induces the Ramsey outcome and it is credible regardless of the parameter values. Since an outcome is defined to be credible if there is a credible plan that induces it, the result that the revert-to-deflation plan is credible regardless of the parameter values means that the Ramsey outcome is credible regardless of the parameter values.

I focus on the revert-to-discretion plan, instead of the revert-to-deflation plan, for two reasons. First, in the revert-to-deflation plan, there is no short-run gain from reneging on the promise after the contractionary shock disappears. As the plan instructs the private sector to expect deflationary outcome to persistent in the future after the government’s deviation, the forward-looking household and firms would respond to the government’s deviation by lowering consumption and inflation immediately in the period of deviation. When one thinks about time-consistency of the Ramsey promise in this model, the premise is that the government can stabilize consumption and inflation in the period of defection by reneging on the Ramsey promise. If this short-run gain does not exist in the specified plan, then such a plan is not economically interesting.

The second reason, somewhat related to the first one, is that the private sector’s punishment strategy of the revert-to-deflation plan in which the nominal interest rate is zero and consumption and inflation are below steady-state even in the high state can be seen as too harsh and unrealistic. Of course, inside the model, there is nothing unrealistic about this punishment regime. A proposition in the Appendix indeed shows that no one would have incentives to deviate from their strategies in this regime with permanently binding ZLB. Nevertheless, outside the model, a natural question emerges as to why the private sector and the government do not want renegotiate to move to a better outcome. Within the theoretical literature on repeated games, the same concern regarding the plausibility of punishment strategy led to the development of appropriate concepts of *renegotiation-proof* equilibrium in which players are allowed to renegotiate after the defection is detected.\(^{15}\) Introducing this concept may render the revert-to-deflation plan incredible and could formally justify my focus on the revert-to-discretion plan. However, such analyses are beyond the scope of this paper.

\[^{15}\text{See, for example, Abreu and Pearce (1991) and Farrell and Maskin (1989)}\]
7.3 A Broader Interpretation

In the model, the loss of reputation gives the central bank an incentive to fulfill the promise because it would make the forward guidance on the nominal interest rates ineffective in future recessions. However, readers can also interpret the model more broadly as suggesting that, if the loss of reputation makes the conduct of any policy involving the manipulation of future expectations ineffective, then the central bank would have an incentive to fulfill the promise of low nominal interest rates. For example, to the extent that the private sector’s trust in the Fed’s announcement on the likely course of the asset purchase programs makes the Fed’s financial policy more effective, and to the extent that reneging on the promise of keeping the nominal interest rate low will lead the private sector to distrust the Fed’s announcement on the financial policy as well, then the loss of reputation may be more damaging to the central bank than the analysis of this paper indicates. One could formally incorporate this possibility into the model, and may be able to obtain new insights.

8 Conclusion

Why should the Federal Reserve fulfill the promise of keeping the nominal interest rate low even after the economic recovery strengthens? What is the force that will prevent the future central bank from reneging on this promise? To shed light on these questions, this paper has analyzed credible plans in a stochastic New Keynesian economy in which the nominal interest rate is subject to the zero lower bound constraint and contractionary shocks hit the economy occasionally.

I have demonstrated that the policy of keeping the nominal interest rate low for long is credible if the contractionary shocks hit the economy sufficiently frequently. In the best credible plan, if the central bank reneges on its promise to keep the nominal interest rate low, it will lose reputation and the private sector will never believe such promises in the face of future contractionary shocks. If the private sector does not believe the promise of an extended period of low nominal interest rates, the contractionary shock will cause large declines in consumption and inflation. Large declines in consumption and inflation in the future recessions reduce welfare even during normal times since the agents care about the discounted sum of future utility flows. Thus, the potential loss of reputation gives the central bank an incentive to fulfill the promise. When the frequency or severity of shocks is sufficiently large, this incentive to maintain reputation outweighs the short-run incentive to raise the rate to close consumption and inflation gaps, and keeps the central bank on the originally announced path of low nominal interest rates.
References


Table 1: Baseline Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>$\frac{1}{1+0.0075} \approx 0.9925$</td>
</tr>
<tr>
<td>$\chi_c$</td>
<td>Inverse intertemporal elasticity of substitution</td>
<td>1</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>The slope of the Phillips curve</td>
<td>0.024</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>the relative weight on output volatility</td>
<td>0.003</td>
</tr>
<tr>
<td>$H$</td>
<td>the natural rate of interest in the high (normal) state</td>
<td>$\frac{1}{\beta} - 1 (=0.0075)$</td>
</tr>
<tr>
<td>$L$</td>
<td>the natural rate of interest in the low (contractionary) state</td>
<td>$-0.0125$</td>
</tr>
</tbody>
</table>


Figure 1: The discretionary outcome and value sequence

$r_t$: Nominal Interest Rate (Ann. %)

$\pi_t$: Inflation

$c_t$: Consumption

$u(\pi_t, c_t)$: Contemporaneous utility flow

$w_t$: the value sequence

$(p_H, p_L) = (0.01, 0.5)$

$(p_H, p_L) = (0, 0.5)$

*Solid blue vertical lines show the period with contractionary shocks.*
Figure 2: The Ramsey outcome and value sequence

- $r_t$: Nominal Interest Rate (Ann. %)
- $c_t$: Consumption
- $\pi_t$: Inflation
- $u(\pi_t, c_t)$: Contemporaneous utility flow
- $w_t$: the value sequence

*Solid blue vertical lines show the period with contractionary shocks.*
Figure 3: Credibility of the revert-to-discretion plan

- The revert-to-discretion plan is not credible
- The revert-to-discretion plan is credible
- The discretionary outcome does not exist
Figure 4: The discretionary and Ramsey outcomes/value sequences:
Frequent vs. infrequent shocks

Solid black line: The discretionary outcome and value sequence
Dashed red line: The Ramsey outcome and value sequence
**The long-run loss** shows the loss in the continuation value if the government deviates from the Ramsey policy at $t=2$, given by $\beta E_2[w_{ram,3}(s^3) - w_{d,3}(s^3)]$, and the short-run gain shows the gain in today’s utility flow if the government deviates from the Ramsey policy at $t=2$, given by $[u(c_{d,2}(s^2), \pi_{d,2}(s^2)) - u(c_{ram,2}(s^2), \pi_{ram,2}(s^2))]$, where $s_1 = L$ and $s_2 = H$.

**The continuation values of following and deviating from the Ramsey plan at $t=2$ are respectively given by $\beta E_2 w_{ram,3}(s^3)$ and $\beta E_2 w_{d,3}(s^3)$ where $s_1 = L$ and $s_2 = H$.**
Figure 7: The discretionary and Ramsey outcomes/value sequences:
Transient vs. persistent shocks

Solid black line: The discretionary outcome and value sequence
Dashed red line: The Ramsey outcome and value sequence
Figure 8: The short-run gain and the long-run loss of deviating from the Ramsey policy (with alternative shock persistence)

**The long-run loss** shows the loss in the continuation value if the government deviates from the Ramsey policy at \( t=5 \), given by \( \beta E_5[w_{\text{ram},6}(s^6) - w_{d,6}(s^6)] \), and the **short-run gain** shows the gain in today’s utility flow if the government deviates from the Ramsey policy at \( t=5 \), given by \( [u(c_{d,5}(s^5), \pi_{d,5}(s^5)) - u(c_{\text{ram},5}(s^5), \pi_{\text{ram},5}(s^5))] \), where \( s_t = L \) for \( 1 \leq t \leq 4 \) and \( s_5 = H \).

Figure 9: The continuation values of following versus deviating from the Ramsey policy (with alternative shock persistence)

**The continuation values of following and deviating from the Ramsey plan at \( t=5 \) are respectively given by \( \beta E_5 w_{\text{ram},6}(s^6) \) and \( \beta E_5 w_{d,6}(s^6) \) where \( s_t = L \) for \( 1 \leq t \leq 4 \) and \( s_5 = H \).
Figure 10: Credibility of the revert-to-discretion plan: Sensitivity Analysis

*L: Severity of the shock

\[ L = -0.0025 \]

\[ L = -0.025 \]

\[ \beta = 0.98 \]

\[ \beta = 0.999 \]

\[ \kappa = 0.012 \]

\[ \kappa = 0.036 \]

\[ \chi = 0.25 \]

\[ \chi = 1.5 \]

\[ \lambda = 0.0003 \]

\[ \lambda = 0.03 \]

*In all charts, colored lines (and dots for the case with \( p_L = 0 \)) show the threshold frequency above which the revert-to-discretion plan is credible.
Figure 11: Credibility of the revert-to-discretion(N) plans (i.e., plans with finite-periods punishment)

*Colored lines (and dots for the case with \( p_L = 0 \)) show the threshold frequency above which the revert-to-discretion plan is credible. N is the punishment periods. Grey areas represent combinations of \( p_H \) and \( p_L \) for which the discretionary outcome does not exist.