Does Trade Liberalization Increase Average Plant Productivity?

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ABSTRACT
In the standard heterogeneous firm model of Hopenhayn (1992) and Melitz (2003), more efficient firms export and trade liberalization leads to higher average efficiency among firms. In data for Columbian plants 1981–1991, we see that the average level of productivity among exporters is higher than that among non exporters and that trade liberalization led to an increase in average firm productivity. Are there facts in the data explained by the Hopenhayn-Meltiz model? The measure of product in the data differs from the measure of efficiency in the model. In fact, in a calibrated version of the Hopenhayn-Meltiz model, the average level of productivity among exporters is lower than that among non exporters and trade liberalization leads to an increase in average firm productivity. The impact of trade liberalization on productivity is a puzzle for the standard model.

* Kehoe thanks the National Science Foundation for support through SES-09-62865. The data presented is the figures are available at http://www.econ.umn.edu/~tkehoe/. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.
1. **Introduction**

In the standard heterogeneous firm model of Hopenhayn (1992) and Melitz (2003), more efficient firms export and trade liberalization leads to higher average efficiency among firms. In data for Columbian plants 1981–1991, we see that the average level of productivity among exporters is higher than that among non-exporters and that trade liberalization led to an increase in average firm productivity. Are there facts in the data explained by the Hopenhayn-Meltiz model? The measure of product in the data differs from the measure of efficiency in the model. In fact, in a calibrated version of the Hopenhayn-Meltiz model, the average level of productivity among exporters is lower than that among non-exporters and trade liberalization leads to an increase in average firm productivity. The impact of trade liberalization on productivity is a puzzle for the standard model.

2. **Model**

Our model is a static, two-country version of the canonical model presented in Melitz (2003). Each country has a representative household that inelastically supplies a unit of labor, and consumes a final good in their own country. Their decisions are the solution to the following problem:

\[
\max_{\mathcal{C}} u(\mathcal{C})
\]

subject to

\[
\mathcal{C} \geq wL + T
\]

where \(u\) is any increasing function and \(T\) is a lump sum rebate that the government gives to households from collected tariff revenues. The final consumption good in the domestic country is numeraire, and, by symmetry, the final good in the foreign country is as well.

In each country, there is a competitive producer that aggregates varieties from the domestic country and the foreign country into the final good. They solve the following problem:

\[
\max_{\mathcal{C}} \mathcal{C} - \int_0^t p_d(0)y_d(0)dt - \int_0^t (1 + \tau)p_x(0)y_x(0)dt
\]

2
subject to

$$C = \left( \int_{l}^{l^*} y_d(l)^{\frac{\sigma - 1}{\sigma}} \, dl + \int_{l^*}^{l_{max}} y_d(l)^{\frac{\sigma - 1}{\sigma}} \, dl \right)^{\frac{1}{\sigma - 1}}$$

where $\sigma > 1$, $l$ is the set of varieties available to the domestic country that are produced in the domestic country, and $l^*$ is the set of varieties available to the domestic country that are produced in the foreign country. The ad valorem tariff $\tau$ is collected on imported varieties and is lump sum rebated back to the household in the same country.\(^1\) The two countries are identical and the problem of the foreign households and competitive final producers are the same.

Both sets of varieties available in the domestic country and their prices are determined by the problems of monopolistically competitive firms. We assume that there is an exogenous set of potential firms $\Omega$ in the domestic country and $\Omega^*$ in the foreign country. They have efficiency levels $z(i)$, which are drawn from some distribution $F$.\(^2\) In order to operate, firms must pay a fixed cost $f$, which allows them to sell in their home market. Additionally, firms may pay an export fixed cost $f^*$ to sell to the other country.

The timing of the firms’ problem is as follows: first they observe their efficiency level $z$, then they choose which fixed costs to pay, then they choose the labor to employ and the quantities to sell in both places. Given the fixed costs chosen, the production functions are linear in labor.

Firms are monopolistically competitive and take the inverse demand functions of the households in both countries as given. Solving the competitive intermediary’s problem in each country yields inverse demand functions:

$$p_d(y_d(\ell)) = \left( \frac{C}{y_d(\ell)} \right)^{1/\sigma}$$

$$p_n(y_n(\ell)) = \frac{1}{1 + \tau} \left( \frac{C^*}{y_n(\ell)} \right)^{1/\sigma}$$

where $C^*$ is the consumption level in the foreign country. By symmetry, it is equal to $C$.

Given these inverse demand functions we can write the problem of a firm $i$ as:

\(^1\) For the purposes this paper, we could instead have iceberg transportation costs with no changes to our results.

\(^2\) Throughout, we will refer to efficiency as the parameter that enters the firm’s production function and productivity as what one would measure using value added and input data.
subject to

\[
\begin{align*}
\gamma_d(0) + \gamma_w(0) & \leq \sigma(0) t(0) \\
\xi(0, \gamma_d(0), \gamma_w(0)) & \geq 0; \xi(0, t(0)) \in (0, 1)
\end{align*}
\]

The problem is analogous for firms in the other country (solutions denoted with stars).

As is well-known in this model, potential firms fall into three categories. The lowest efficiency firms choose not to operate because the fixed cost required to operate exceeds any profits they could make. A second category of firms are sufficiently efficient to operate, but their efficiency is not high enough for their profit from exporting to justify the export fixed cost. Lastly, all firms that have a sufficiently high efficiency pay both fixed costs and operate in both markets. Furthermore, the amount (both quantity and value) that any given firm sells to each market in which they operate is increasing in that firm’s efficiency.

Therefore, given the solutions to the problem above, we can define the sets of operating firms as:

\[
I = \{i \in \Omega | \xi(i) = 1\}
\]

\[
I^* = \{i \in \Omega^* | \xi^*(i) = 1\}
\]

Lastly, wages are determined by a market clearing condition in the labor market. Each country has an equal endowment of labor \( L \), and labor market clearing in the domestic country is given by:

\[
L = \int_{\Omega} \xi(0) x_d(i) \, di
\]

3. Productivity Measurement

We measure productivity at the firm level in both the model and data as value added per worker. Value added is defined as revenue less production cost, plus labor cost and depreciation. In the model, the only production cost is labor and there is no depreciation. The total labor used by the
firm is all labor applied to the production function, plus all labor used to pay fixed costs.\textsuperscript{3} Therefore, the productivity of operating firm $i$ is defined as:

$$m(i) = \frac{w(i) y(i)}{l(i) + f + x(i) f_r}$$

Importantly, this is distinct from the efficiency of firm $i$, which is the parameter $z(i)$. Hsieh-Klenow (2009) refers to $z(i)$ as $QTFP(i)$ and $m(i)$ as $RTFP(i)$. As the model does not include other factors, such as intermediate goods, capital or energy, we do not need to parameterize factor shares, which is the focus of the production function estimation literature (see Pavcnik, 2002).

We can easily show the relationship between the efficiency and the measured productivity of a firm by using the well-known fact that the profit-maximizing price of each monopolistically competitive firm is a constant markup over marginal cost, and that the gross markup is equal to $\sigma/(\sigma - 1)$. Then we can write the productivity of firm $i$, denoted $m(i)$, as:

$$m(i) = \frac{\sigma w}{\sigma - 1} z(i) \frac{y(i)}{l(i) + f + x(i) f_r} = \frac{\sigma w}{\sigma - 1} \frac{l(i)}{l(i) + f + x(i) f_r}$$

For any operating firm, $l(i)$ is:

$$l(i) = z(i) \sigma^{-1} C \left( \frac{\sigma - 1}{W} \right)^{\sigma} \left[ 1 + x(i)(1 + \tau)^{-1} \right]$$

Hence, the relationship between measured productivity and underlying efficiency is given by:

$$m(i) = \frac{\sigma w}{\sigma - 1} \left( 1 + \frac{z(i) 1 - \sigma}{C} \left( \frac{W}{\sigma - 1} \right)^{\sigma} \frac{f + x(i) f_r}{1 + x(i)(1 + \tau)^{-1}} \right)^{-1}$$

Measured productivity only depends on the fraction of all employed labor that is used in production, rather than to pay fixed costs.\textsuperscript{4} Therefore, as $\sigma > 1$, for all exporters (that is, for all firms

\textsuperscript{3} In the appendix we consider an alternative case in which the fixed costs are an expense to firms, so that the cost is subtracted from value added and does not appear in the firm's labor. Our results are robust to this specification.

\textsuperscript{4} Notice that if there are no fixed costs, then measured productivity is constant across all firms.
that choose \(x(i)=1\), \(m(i)\) is increasing in \(z(i)\), as more productive firms use more labor in production and pay the same fixed costs. Therefore, a greater fraction of labor is employed in production for firms with greater \(z(i)\). The same is true within the set of non-exporting firms.

The observation made in this paper is that the model predicts that this relationship between \(m(i)\) and \(z(i)\) breaks down when one considers the union of the set of exporting firms with the set of non-exporting firms. In particular, consider exporting firm \(i\) and non-exporting firm \(j\):

\[
m(i) > m(j) \iff \frac{\frac{f}{l(i)} + \frac{fx}{l(i)}}{\frac{l(i)}{l(j)}} > 1 + \frac{\frac{fx}{f}}{\frac{l(i)}{l(j)}} = \left[1 + (1 + \tau)^{-\frac{z(i)}{l(i)}}\right]^{\frac{z(i)}{l(i)} - 1}
\]

As this demonstrates, \(l(i) > l(j)\) for two reasons. First, the exporting firm operates in two markets, so employs more labor to produce for both. Second, in this model the fact that firm \(i\) is an exporter and firm \(j\) is not implies that \(z(i) > z(j)\). Therefore firm \(i\) employs more labor in every market in which they operate. However, in this model the only way that the higher efficiency, exporting firm has higher measured productivity is if these two factors swamp the effect of having to pay more in fixed costs. Clearly, if firm \(i\) is the least efficient exporter, and firm \(j\) is just below the cutoff to be an exporter, then firm \(j\) has higher measured productivity than firm \(i\). Therefore, while there is a clean cutoff in efficiency that separates exporters and non-exporters, there is not a corresponding cutoff in measured productivity between the two groups.

While this observation is clear for firms around the export cutoff, we show a quantitative result that is much stronger than that. In the calibrated model, for a value of \(f_x\) large enough to match the export participation rate of firms, the average exporter has lower measured productivity than the average non-exporter. Therefore, the disconnect between efficiency and productivity of exporters and non-exporters is not only true at the margin, but is also true for the average.\(^5\)

4. **Calibrated Results**

As we demonstrated before, the measured productivity of a firm depends on the ratio of labor used in production to labor used to pay fixed costs. Exporters use more labor because they are more efficient and supply two markets, but they also pay more in fixed costs than do non-exporters. Therefore, whether exporters or non-exporters have higher measured productivity on

\(^5\) Notice that there are clear special cases in which this result is obvious. For instance, if \(f = 0\) and \(f_x > 0\), then an even stronger result holds: every non-exporter is more productive than every exporter.
average is ambiguous. In this section, we measure the average productivity of both groups in a calibrated model.

We calibrate the model using data from the Colombian Survey of Manufacturers covering the years 1981-1991. This survey provides information on all manufacturing plants in Colombia with at least ten employees. We observe value added, number of employees and export sales, among other variables at the plant level. Colombia underwent a major policy reform in 1985, which resulted in a substantial increase in firm export activity (see Fernandez (2007), Roberts (1996)). For that reason, we calibrate the model to the 1981-1984 data, and use the data from the end of the sample to calibrate the trade liberalization exercise of the next section.

We choose values of parameters to hit important targets in the micro data, and to be consistent with standard values from the literature. We set \( \sigma \) (the elasticity of substitution across goods) to 2 following Ruhl (2004). The fixed cost \( f \) is set to 1, and \( L \) is 1,814,556, which is the total number of workers employed by all firms in our sample period (the measure of firms is the same in the model as in the data). The distribution of efficiency draws is log-normal\(^6\) where log-efficiency has mean 0 and standard deviation \( s \). We calibrate the values of \( f_x \) (the fixed cost to export), \( s \) (the standard deviation of log-efficiency), and \( \tau \) (the tariff rate) to match the following moments from the micro data: the fraction of plants that export (12.2 percent), the standard deviation of the log of labor (1.08), and exports as a fraction of total sales (5.5 percent) in the 1981-1984 subset of the data. All parameter values are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of Substitution</td>
<td>( \sigma )</td>
<td>2</td>
</tr>
<tr>
<td>Fixed Cost to Export</td>
<td>( f_x )</td>
<td>16.56</td>
</tr>
<tr>
<td>Fixed Cost to Operate</td>
<td>( f )</td>
<td>1</td>
</tr>
<tr>
<td>St. Dev. of Log-Efficiency</td>
<td>( s )</td>
<td>1.09</td>
</tr>
<tr>
<td>Tariff, pre-reform</td>
<td>( \tau )</td>
<td>1.84</td>
</tr>
<tr>
<td>Tariff, post-reform</td>
<td>( \tau' )</td>
<td>1.29</td>
</tr>
</tbody>
</table>

Our main result is that the average measured productivity of non-exporting firms is higher than that of exporting firms. This is the case, even though the underlying efficiency of every

\(^6\) We choose log-normal (rather than Pareto, for instance) to match the firm-size distribution in both the left and right tails of the data. Also, the analytical tractability of the Pareto distribution is not important because average measured productivity does not have a closed form, so it must be computed in any case.
exporter is higher than that of every non-exporter. The reason for this is that the calibrated export fixed cost $f_x$ must be so high to match the export participation rate that the ratio of labor used in production to labor used to pay fixed costs is higher for non-exporters than it is for exporters.

Figure 1 shows the relationship between true, underlying efficiency $z$ and measured productivity $m$. In this graph, the efficiency threshold above which all firms are exporters is obvious. As described above, productivity falls at that cutoff because firms pay the fixed cost to export at that point. The supremum of measured productivity is $w\sigma/(\sigma-1)$, and productivity approaches this value from below as efficiency goes to infinity.

Figure 1: Productivity and Efficiency

The fact that average productivity is lower for non-exporters is not obvious from Figure 1 because it does not take the density of efficiency into account. Instead, Figure 2 divides the distribution of log-efficiency into 50,000 bins such that the total density in each bin is equal, then computes average log-productivity in each.
Here it is obvious that the average productivity of non-exporters (those to the left of the productivity break) is higher than that of exporters. For all but the least efficient non-exporter the fixed cost to operate is small relative to their operating size. However, the export fixed cost is large relative to the scale of operation for exporting firms, except for the largest of them. But there are relatively few of those large exporters. Hence, on average exporters have lower measured productivity than do non-exporters.

Additionally, the model is unsuccessful at matching the cross-sectional distribution of productivity. The standard deviation of log-productivity in the data (measured as the ratio of value added to labor) is 0.81. In the model, it is 0.06. This is true even though the model is calibrated to exactly match the standard deviation of log-labor usage in the data. Figure 3 shows the distribution of log-productivity in the model and data.
Figure 3: Histogram of Log-Productivity in the Model and Data

![Histogram of Log-Productivity](image)

Though the distribution of log-productivity in the model looks degenerate, in fact, as Figures 1 and 2 show, it does have positive variance. However, the variation in productivity is nowhere near what it is in the data.

These two facts that the model fails to capture are statements about the cross-section. This model is also commonly used to understand the effects of trade liberalization. In the next section, we point out an important limitation of the model in matching how the measured productivity of firms changes in response to a reduction in tariffs.

5. Trade Liberalization

Now we consider the effect of a bilateral tariff reduction on measured productivity in the model and in the data. We calibrate the change in tariffs to match the change in exports in Colombia following their reforms in the mid-1980s. Because the reforms took time to be fully implemented, we look at the data from the period 1990-1991 for the post-reform data.

Our exercise is to hold all parameters constant, then reduce tariffs to a level sufficient to match the increase in exports between the pre-reform period (1981-1984) and the post-reform period (1990-1991), which is an increase in exports as a fraction of total sales from 5.5 percent to 10.9 percent. In the data, we then measure the change in productivity separately for firms that exported both before and after the liberalization, for firms that started exporting after the reform, for firms that stopped exporting after, and for firms that exported neither before nor after. The results of this are reported in Table 2. Continuing exporters and new exporters had the largest increases in productivity, while export stoppers and continuing non-exporters had lower increases. Using one-tailed t tests for the inequality of means, we find that growth for new exporters and continuing
exporters is higher than for non-exporters, but we fail to reject the hypothesis that the growth for continuing exporters is the same or lower than for new exporters. We interpret this to mean that growth is higher for both new exporters and continuing exporters than for non-exporters in the data. The fact that productivity rises for every group is likely due to technological progress and general economic growth during that time period.

Next, we compare these results to what happens after the liberalization in the model. Inspection of the equation for measured productivity gives some insights into how productivity changes:

\[
m(i) = \frac{\sigma^W}{\sigma - 1} \left( 1 + \frac{1 - \frac{1}{\sigma}}{C} \left( \frac{w^C}{\sigma - 1} \right) \left( 1 + \frac{\sigma (1 - x(i))}{f + x(i) A} \right) \right)^{-1}
\]

Clearly, this term is increasing in \( w \) and \( C \) for all firms, and is decreasing in \( x(i) \) if \( x(i) = 1 \). Therefore, since the reform is a decrease in \( x(i) \), productivity rises for all continuing exporters. Also, the reform causes both \( C \) and \( w \) to rise. This pushes the productivity of even non-exporters higher. For firms switching from being non-exporters to being exporters (switching \( x(i) \) from 0 to 1), however, the effect is ambiguous because now those firms must pay the fixed cost to export.

Figure 4 shows the effect that the reform has across the set of firm efficiencies:

Table 2: Increase in Productivity after Reform by Group, Data

<table>
<thead>
<tr>
<th>Pre-Liberalization</th>
<th>Post-Liberalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exporter</td>
<td>Exporter</td>
</tr>
<tr>
<td></td>
<td>145%</td>
</tr>
<tr>
<td></td>
<td>(6.162)</td>
</tr>
<tr>
<td></td>
<td>850</td>
</tr>
<tr>
<td>Non-Exporter</td>
<td>Exporter</td>
</tr>
<tr>
<td></td>
<td>96%</td>
</tr>
<tr>
<td></td>
<td>(6.047)</td>
</tr>
<tr>
<td></td>
<td>173</td>
</tr>
<tr>
<td>Exporter</td>
<td>128%</td>
</tr>
<tr>
<td></td>
<td>(5.807)</td>
</tr>
<tr>
<td></td>
<td>742</td>
</tr>
<tr>
<td>Non-Exporter</td>
<td>62%</td>
</tr>
<tr>
<td></td>
<td>(4.233)</td>
</tr>
<tr>
<td></td>
<td>4941</td>
</tr>
</tbody>
</table>

Sample standard deviation in parentheses, count of each group in italics.
Here we see that the productivity of every exporter and every non-exporter increases, for reasons already explained. We also see that the productivity of every new exporter is lower than before the reform. As illustrated in Table 3, this contrasts sharply with the data. While the model and data agree that continuing exporters have a larger increase in productivity than do continuing non-exporters (though the magnitudes are very different), they disagree qualitatively in the gains in productivity for new exporters. In the data, exporters have the largest gains, and there is no statistical difference between their gains and those of new exporters. Both are larger than those of continuing non-exporters. This is not true in the model, where new exporters actually experience productivity losses.

**Table 3: Increase in Productivity after Reform by Group, Model**

<table>
<thead>
<tr>
<th>Pre-Liberalization</th>
<th>Post-Liberalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exporter</td>
<td>Non-Exporter</td>
</tr>
<tr>
<td>9.9%</td>
<td>-3.2%</td>
</tr>
<tr>
<td>(0.0003)</td>
<td>(0.0130)</td>
</tr>
<tr>
<td>823</td>
<td>604</td>
</tr>
<tr>
<td>9.5%</td>
<td>5279</td>
</tr>
<tr>
<td>(0.0044)</td>
<td>(0.0044)</td>
</tr>
</tbody>
</table>

All moments reported above are computed from a discrete grid (50,000 points) over productivity levels. The counts are over grid points, then multiplied to give the same number of observations in the model and data.
References


Appendix: Alternative Measure of Productivity

The measure developed in the body of the paper assumes that fixed costs, which are in terms of labor, are measured as employment within the firm. This is only one way to map the data into the model. Alternatively, one could measure fixed costs as an expense to firms. For example, imagine that the fixed cost to export captures the legal and accounting overhead necessary to operate in a new market. Then the firm may hire lawyers and accountants “in house” as workers of the firms, or they may hire a third party law or accounting firm. In both cases, labor is used to pay those costs, but in the second case, these overhead costs are deducted from value added, rather than being added to the labor usage of the firm.

In the model, suppose there is an industry separate from the firm that uses labor to produce the commodity used to pay the fixed cost, so that the labor employed to pay fixed costs is not measured as firm labor. Clearly, this is equivalent to the environment described above when that outside industry is competitive. In this case, the expense charged by the fixed cost industry is subtracted from revenue when computing value added. As such, this alternative measure is given by:

\[ \hat{\mu}(i) = \frac{p(i)y(i) - w_f - x(i)w_{fe}}{l(i)} \]

As above, this can be rewritten as

\[ \hat{\mu}(i) = \frac{\sigma w}{\sigma - 1} \frac{l(i)}{l(i)} - \frac{w_f - x(i)w_{fe}}{l(i)} = \frac{\sigma w}{\sigma - 1} \left( 1 - \frac{1 - \frac{x(i)w_{fe}}{l(i)}}{\sigma} \right) \]

Though this measure is not equivalent to the one given above, they are similar enough that our results are robust to either specification. In both cases, all variation in measured productivity comes from variation in the ratio of labor used to pay fixed costs to labor used in production. Also, both measures are decreasing in that ratio.

Formally, here we show that the log-difference between the measured productivity of any firm \( i \) from any other firm \( j \) is approximately the same under both measures up to a constant proportion:
Therefore, the relationship between the two is:

\[
\ln \left( \frac{m(t)}{m(0)} \right) = \ln \left( 1 - \frac{\sigma - 1}{\sigma} \frac{\mu + \chi \gamma \rho}{\lambda} \right) - \ln \left( 1 - \frac{\sigma - 1}{\sigma} \frac{\mu + \chi \gamma \rho}{\lambda} \right) \approx \frac{\sigma - 1}{\sigma} \left[ \frac{\mu + \chi \gamma \rho}{\lambda} - \frac{\mu + \chi \gamma \rho}{\lambda} \right]
\]

\[
\ln \left( \frac{m(t)}{m(0)} \right) = -\ln \left( 1 + \frac{\mu + \chi \gamma \rho}{\lambda} \right) + \ln \left( 1 + \frac{\mu + \chi \gamma \rho}{\lambda} \right) \approx \frac{\mu + \chi \gamma \rho}{\lambda} - \frac{\mu + \chi \gamma \rho}{\lambda}
\]

Therefore, the relationship between the two is:

\[
\ln \left( \frac{m(t)}{m(0)} \right) \approx \frac{\sigma - 1}{\sigma} \ln \left( \frac{m(0)}{m(0)} \right)
\]

Hence, the difference in these two measures only shows up in the dispersion of measured productivity. This difference affects our results quantitatively, but not qualitatively.