Price markups are highly correlated with stock market values, whereas other financial measures of profitability exhibit much less variability and are weakly correlated with stock values. We propose a variant of the neoclassical growth model to study the role of innovation, price markups, and leverage as main determinants of stock market volatility. The model confers a rather limited role to other macroeconomic forces such as TFP shocks, adjustment costs, interest rate policies, input costs, taxes, and labor and financial frictions. We develop some numerical methods to provide a variance decomposition of stock market values.

Keywords: Technological innovations, stock market volatility, price markups, leverage, dividends, taxes, labor and financial frictions.

JEL Classification Numbers: E44, E32, G12.

1 Introduction

In this paper we are concerned with the main macroeconomic determinants of asset price volatility. Although in recent times a burgeoning literature has emerged in the frontier...
between macroeconomics and finance, there is no general consensus on the main driving forces of stock market prices – which remain a puzzle to economists. Furthermore, there seems to be a shortage of numerical methods to study these sources of volatility. There is an extensive empirical literature [e.g., Campbell and Shiller (1988, 1989), and Cochrane (1992)] using isolated sample paths or VAR models to estimate pricing kernels and to decompose the variance of stock values into changes in fundamentals and expectations of future prices. A numerical approximation can be evaluated over all future equilibrium paths and so it can exploit all the information provided by a full-fledged, non-linear model. Numerical non-linear approximations can be instrumental to bound errors from linearized solutions as well as sampling errors for testing an infinite-horizon asset pricing equation at a single equilibrium realization.

We focus on long-term asset price volatility. That is, the volatility corresponding to low-frequency fluctuations or secular trends of stock market values and real macroeconomic variables. Long-term volatility should be the easiest to predict since it can be isolated from daily noisy information and sudden changes in expectations of fundamental values. Still, the economics literature has struggled to come to terms with what it is usually dubbed as financial markets anomalies. First, fundamental variables (i.e., dividends and broader accounting measures of profitability) exhibit relatively low variability,\(^1\) and are weakly correlated with changes in stock market prices over the various time frequencies. While these fundamental variables lack predictive power, we provide new evidence on some measures of product price markups that comove in a parsimonious way with stock market values. Price markups may originate from product innovations, patents, and monopoly power.

Second, asset market volatility is also disconnected from real macroeconomic uncertainty. Again, real aggregates exhibit relatively low variability, and are weakly correlated with stock market values. Dynamic equilibrium models have been fairly successful in accounting for comovements of macroeconomic aggregates but have failed to offer plausible explanations.

\(^1\)See the earlier negative results of LeRoy and Porter (1981) and Shiller (1981). Some authors are somewhat more assertive on the correlation of stock prices with fundamental values; see Barsky and De Long (1993), Boldrin and Peralta-Alva (2009), Donaldson and Kamstra (1996), Hall (2001), and Larrain and Yogo (2008).
for the volatility of stock values based on macroeconomic uncertainty of the real economy. In the neoclassical growth model, changes in total factor productivity (TFP), the relative price of capital, taxes, and frictions in capital and labor markets hardly generate any volatility of stock values [see Rouwenhorst (1995) for some numerical exercises]. Indeed, all these macroeconomic variables do not affect significantly the volatility and persistence of dividends and earnings under observable variations in consumption. Viewed in another way, capital is the only asset in the economy, and investment must fluctuate enormously to get desirable levels of volatility in stock values. The model’s performance can be improved with adjustment costs [Christiano and Fisher (2003), and Jermann (1998)], but these costs need to be implausibly high [Hall (2001)] to attain reasonable levels of volatility in stock values. Gomme, Ravikumar and Rupert (2011) document that the volatility of physical capital returns of standard real business-cycle models is about one order of magnitude smaller than the volatility financial asset returns.

And third, asset price volatility is additionally disconnected from other financial variables. Risk-free or uncontingent interest rates do not fluctuate as much as stock market returns. Most of the volatility of stock market returns stems from fluctuations in the equity premium rather than in the risk-free rate. Hence, we can observe pronounced positive or negative equity premia over long-time episodes. A great deal of research has focused on the equity premium [e.g., Bansal and Yaron (2004), Boldrin, Christiano and Fisher (2001), Danthine and Donaldson (2002), and Guvenen (2009)]. A better understanding of risk premia should shed light on the determinants of asset price volatility.

Our empirical analysis considers publicly and privately held corporations in an effort to capture potential impacts of recently founded corporations. Thus, we include companies since their foundation dates; e.g., Google was founded in 1998 and went public in August 2004. Hence, Google would be included in our data since 1998. Furthermore, we focus on the market value of corporations (MVC): the sum of the market value of corporate equity and the book value of net debt. The joint consideration of equity and debt is convenient for our analysis as it avoids the introduction of arbitrary corporate debt policies and dividends.
As documented in Hall (2001), pay-outs to debt holders have been fairly erratic in recent decades.

In our general equilibrium model, asset prices may be driven by technological innovations, product price markups, and $TFP$ shocks. To explore the robustness of our results and the ability of the model to replicate some empirical regularities, we will later perturb the elasticity of risk aversion of the representative consumer and introduce supply shocks. As our asset pricing equation does not admit a closed-form solution, we propose some numerical methods to perform a variance decomposition of the stock market value.

Our model is a simplified variant of those in Romer (1990) and Comin and Gertler (2006), but our objectives are quite different. Romer (1990) is concerned with innovations and economic growth, and Comin and Gertler (2006) with a quantitative analysis of real economic fluctuations.\footnote{In a later paper, Comin, Gertler and Santacreu (2009) build their analysis on our asset pricing equation of Proposition 3.1 below. Their empirical implementation, however, is fairly independent from ours. They do not include price markups and leverage.} Technological innovations arrive exogenously to the economy. These innovations, however, cannot be readily put into use and undergo a process of adoption embedded in the production of new varieties of intermediate goods. Asset prices incorporate the option value of technological innovations that remain to be adopted. We decompose the value of the stock market into the value of installed capital, the value of technology goods, and the option value of adopting present and future innovations. Then, episodes of technology innovation, expected shocks to price markups and to real and financial variables may generate sudden fluctuations in the aggregate value of stocks. This propagation mechanism is somewhat present in the partial equilibrium setting of Abel and Eberly (2005), in the tree economy of Gärleanu, Panageas and Yu (2012), and in the learning model of Pástor and Veronesi (2009). In all these papers the value of the firm may differ from the replacement value of the stock of capital. In contrast to these authors, we carry out a quantitative general equilibrium analysis of the volatility of asset prices along with other macroeconomic fluctuations. Hence, the challenge for our model is to generate observed levels of volatility in stock markets while preserving the less pronounced volatility of real aggregates.
Our model can account for a sizable part of the volatility of stock market values. Moreover, asset price volatility stems from expectations of future stock prices rather than dividend growth. As in Greenwood and Jovanovic (1999) not all technological innovations will increase stock market prices, since the arrival of new technologies will depreciate the value of existing ones. To affect positively the stock market, technologies must incorporate high price markups. Apart from price markups, we also perform a quantitative study of the effects of various macroeconomic variables. A notable feature of these numerical exercises is that adjustment costs for capital investment, changes in input prices, taxes, leverage and interest rate policies, and labor and financial frictions may only have a significative impact on the volatility of asset values at the expense of implausible fluctuations in some other variables. The introduction of some of these frictions, however, may help explain the evolution of price-earning ratios and the observed weak correlation of stock values with real aggregates. The paper will proceed as follows. Section 2 gathers some empirical evidence on the volatility of stock market prices and various financial accounting measures of profitability as well as new evidence on the volatility of markups. Section 3 lays out a simple model of technology adoption and derives some qualitative properties of the solution with emphasis on a fundamental asset pricing equation in which the asset price is decomposed into the value of physical capital and the value of adopted and unadopted technologies for the production of intermediate products. Section 4 is concerned with the calibration of the model. Then, we report a battery of statistics on the performance of the model regarding the volatility and comovements of stock market values with macroeconomic variables. Various extensions of this basic numerical experiment are discussed in Section 5. Numerical methods to analyze asset price volatility are presented in Section 6. We conclude in Section 7 with a further evaluation of our findings. The Appendix lists definitions, data sources, and proofs of our analytical results.
2 Stock Market Volatility and Markups

Figure 1 plots the evolution of the S&P index and MVC. Both series have been adjusted for inflation,\(^3\) and have been filtered by taking out our best fit for a deterministic exponential trend. It can be observed that they both display similar long-term cyclical behavior. Peak values occur in 1900, 1929, 1965 and 2000. Therefore, the amplitude of these long-term cycles can be up to 35 years. Jovanovic and Rousseau (2001) associate these long fluctuations in the stock market with three technological revolutions: Electricity, World War II, and IT. These authors document long lags in the operation and diffusion of new technologies. Nicholas (2008) claims that innovation was a main driver of the stock market run-up of the late 1920s. Geanakoplos, Magill and Quinzii (2004) contend that changes in stock values could be driven by demographic trends, whilst other authors stress the importance of time-varying risk. For instance, Lustig and Nieuwerburgh (2006) cite credit access from home equity collateral that may affect attitudes toward risk.

Note that S&P and MVC growth rates are likely to differ at times of very high or very low returns on equity, significant changes in leverage ratios, and waves of potential entrants. This same figure also presents the evolution of the risk-free rate adjusted for inflation (R), and the leverage ratio (LR): value of net debt over MVC. (Recall that MVC includes both equity and net debt.) As shown below, in our sample period the volatility of the annual real interest rate is about ten times smaller than the volatility of MVC returns. Also, the volatility of MVC is about 20 percent lower than the volatility of equity. Hence, 20 percent of the volatility of equity could be ascribed to interest and debt policies. We will later discuss conditions under which a general equilibrium model may replicate the observed volatility of equity as a function of the leverage ratio.

In the remaining part of this section, we will present data from US companies in Compustat North America over the 1950-2012 period; see the Appendix for further details. Figure 2 decomposes the market value of equity for different company cohorts in the recent IT

\(^3\)The Appendix contains a comprehensive list of definitions for our macroeconomic aggregates and data sources.
revolution. Market capitalization relative to aggregate corporate value added is broken down into the values of four different groups of companies: (i) The incumbents, (ii) Companies listed in 1970-1980, (iii) Companies listed in 1980-1990, and (iv) Companies listed after 1990. As one can see, most added stock value belongs to new corporations. In our model these newcomers will reflect the value added of local technology adopters, but our story is not only about technology adoption since markups may be determined by other economic forces affecting supply and demand.

We next compare the financial performance of these cohorts over various measures of profitability. First, we consider the distribution of markups over time. To obtain markup estimates, companies are ranked by R&D intensity. Within the subsample of companies with positive R&D expenditures, we generate three subsamples comprising the top 50%, 75%, and 100% companies with the highest ratio of R&D expenditure over total revenue. The price markup of a company is defined as the ratio between total revenue and variable total cost; i.e., entries REV\textit{T} and COGS in Compustat. The aggregate markup is then obtained as a weighted average of company markups over the share of company revenues.

Figure 3 illustrates that the evolution of the relative size of each vintage within the aggregate stock market value shown in Figure 2 appears to be driven by the company markups of each vintage. Thus, this evidence reveals that the markup for the top 50% and 75% companies with the highest ratio of R&D expenditure over total revenue, which is closely associated with the evolution of the aggregate stock market value, is considerably higher for the new vintages. Therefore, some selected groups of newcomers command higher and steeper markups, and add up more incremental value to the stock market.

Table 1 offers a broader evaluation of firms’ financial performance by conditioning on various cash-flow measures. This table is intended to compare the relative value of each company cohort against commonly used measures of profitability for the aforementioned four groups of firms; i.e., firms listed before 1970, and within the 1970-1980, 1980-1990, and 1990-2000 periods. The table reports averages over three different time intervals: 1990-1994, 1995-1999, and 2000-2004. Again, all these cash-flow measures are detailed in the Appendix. Markups
are reported as fractions of the average markup in the corresponding sample. Hence, a value above 1 means that this vintage can secure a higher markup than the average markup in the economy for the given time period. $MU$ refers to the average company markup in our sample at a given date, $MU_{100}$ refers to the average markup of all the companies reporting $R&D$ activity, $MU_{75}$ refers to the average markup of the top 75% companies with the highest ratio of $R&D$ expenditure over total revenue, and $MU_{50}$ refers to the average markup of the top 50% companies with the highest ratio of $R&D$ expenditure over total revenue. For every other measure, the table reports the fraction or relative value belonging to this cohort over the total value. For instance, on the first column under the dividends ($D1$) entry of the table one can read that the value is 69.88. This means that the companies originating before 1970 were able to secure 69.88 percent of the total sum of dividends in the sample of companies over the 1990-1994 period. Note that regarding $MVC$ this cohort represents 66.70 percent of the total value. As we can see from Table 1 the youngest cohorts command the highest markups, but would be indistinguishable in terms of every other profitability measure.

The finance literature has introduced various cash-flow measures to account for the volatility of the stock market. The most common ones are dividends $D1$ and accounting earnings – frequently referred as net income ($NI$). As shown in Barsky and DeLong (1993), both measures are reasonably correlated with stock values at low frequencies. $D1$ and $NI$ are narrow measures of profitability, which may be subject to optimal payout policies [cf. Marsh and Merton (1987)]. Although Fama and French (2006) find that $NI$ has some predictive power for cross-section company returns, Novy-Marx (2013) argues that gross profitability ($GP$) would be a more suitable concept for economic purposes. Hence, following Novy-Marx (2013), we include $GP$ and earnings before extraordinary items (IB). These are cleaner accounting measures. They represent the total firm’s cash-flow to enhance growth by investing in physical capital and R&D activities under various tax treatments. Additionally, we include some popular cash-flow measures like $EBITDA$, $EBIT$, and operating income (OI) before and after depreciation.

As seen in Table 1 the youngest cohorts display high markups, but behave quite similarly
regarding every other profitability measure. Roughly, for every sample period the relative value of the cohort determines the relative value of every other financial accounting measure besides markups. For instance, \( D_1 \) and \( GP \) follow no clear patterns and do not vary substantially across vintages as we condition by vintage value. In conclusion, apart from markups all these other financial accounting measures would seem unable to sort out those company cohorts with the highest present and future returns.

The lack of correlation at the cross-section level is also validated in a further time series analysis for our sample of companies. Here, we will provide formal quantitative estimates over the available 1950-2002 period. Table 2 records changes in \( MVC \) against changes in the financial accounting variables and markup measures over 10-year time intervals. More specifically, the table records log differences – and hence approximate growth rates – of the financial accounting variables and four markup measures over 10-year time periods.\(^4\) We include an additional cash-flow measure, \( D_2 \), which is computed by taking out investment and wages from aggregate corporate value added. A similar measure of dividends has been reported by various authors and includes interest payments to debt holders [cf. Boldrin and Peralta-Alva (2009), Larrain and Yogo (2008), and McGrattan and Prescott (2005)]. Observe that all measures do very poorly in the 1950s in which the change in \( MVC \) was 103 percent. This is termed the World War II revolution by Jovanovic and Rousseau (2001), but the run-up of the 1950s may be related to time-varying risk premiums and some problems with our limited sample of companies in the Compustat data base. For all the other time intervals, financial accounting variables appear to be poorly correlated with \( MVC \) and display very low variability, but our markup measures show high correlation coefficients and more variability as reported in the last two columns of the table. Therefore, while our markup measures are closely associated with the evolution of \( MVC \), the considered cash-flow measures (e.g., \( D_1, D_2, NI, IB \)) exhibit very low correlation with stock market values and very low variability.

Table 3 reports correlation coefficients for the above log differences of \( MVC \) and the log differences of every variable considered above over various time periods. Roughly, these are

\(^4\)\( MVC \) and financial accounting measures are not stationary, and so their growth rates are adjusted by aggregate corporate value added.
correlations between the average growth rate of $MVC$ and the average growth rate of every other variable over various time intervals. While markups may display correlation coefficients of about 0.80 over the range of 5- to 30-year frequencies, financial accounting measures such as $D1$ display correlation coefficients of about 0.10. Hence, changes in $MVC$ are mildly correlated with changes in every cash-flow measure. Newey-West corrected standard errors appear in the parentheses below the correlation coefficients. (Hansen-Hodrick tests were also performed with no noticeable changes in these estimates.) Most correlation coefficients of the cash-flow measures are not statistically significant at conventional confidence levels.

Finally, to check for the robustness of these findings we include some sectoral data. From the Fama-French 5-sector industrial decomposition, we break down our initial set of companies into three sectors. Fama and French 5-industry classification specifies **Cnsmr**: Consumer Durables, Non Durables, Wholesale, Retail, and Some Services (Laundries, Repair Shops); **Manuf**: Manufacturing, Energy, and Utilities; **HiTec**: Business Equipment, and Telephone and Television Transmission; **Hlth**: Healthcare, Medical Equipment, and Drugs; **Other**: Mines, Construction, Building Materials, Transportation, Hotels, Business Services, and Entertainment. Based on this classification we define our sectors as **Sector 1**: Cnsmr and Manuf, **Sector 2**: HiTec, and **Sector 3**: Hlth and Other. We further identify two categories for high-tech companies: (1) Companies with SIC codes 281, 283, 284, 289, 357, 367, 381, 384, and (2) Companies whose stocks are currently traded in the NASDAQ stock exchange. Table 4 reports correlation coefficients for the log differences of $MVC$ and the log differences of three markup measures over five and ten-year periods. Almost all correlations are statistically significant, and the correlations over the ten-year period are slightly higher with coefficients ranging from 0.60 to 0.80. We therefore obtain that the strong correlation between changes in $MVC$ and the markup measures extends to sectoral data.

We have included four markup measures ($MU$, $MU100$, $MU75$, $MU50$) because long-term fluctuations in stock market values may be driven by some groups of companies rather than by a representative or average firm (Figure 2). Hence, it may be possible to construct some other related profitability measures over some company groups that are highly correlated
with stock market values. As opposed to other endogenous profitability measures, in our modeling strategy markups can be microfounded: optimal markups may arise from pricing rules and price rigidities, and can be related to changes in elasticities because of income and substitution effects, shocks to monetary and fiscal policies, and technology adoption. Several methods have been proposed to measure markups [cf. Nekarda and Ramey (2013)]. Our estimation procedure for markups is closest to Domowitz et al. (1986) and Boulholn (2008). We shall later report on statistical properties of our markup measures. Price markups are positively correlated with corporate value added and are fairly persistent with autocorrelation coefficients ranging from 0.92 to 0.98 [cf., Kim (2010)].

3 A Simple Model of Technology Adoption

We start with a very simple model of technology adoption to analyze some basic sources of volatility of the stock market value. At a later stage, we contemplate various extensions as we introduce adjustment costs, supply shocks, taxes, and debt and interest rate policies. The economy is populated by a continuum of identical households. At every time $t = 0, 1, \ldots$, a representative agent demands quantities of the aggregate consumption good, supplies labor inelastically, and trades in the equity and bond markets. The aggregate consumption good is produced by a single firm with a constant returns to scale technology. Three inputs are involved in the production of this final commodity: capital accumulated by the firm, labor, and a composite intermediate good. Both the firm and the consumer act competitively in all markets, but the sector of intermediate goods is composed of a continuum of monopolistic competitors. The range of available intermediate goods can be expanded by a fixed set of local adopters upon the arrival of new technologies. As in Romer (1990), an increase in the varieties of intermediate goods allows for a more efficient use of resources and enhances capital and labor productivity. The remaining source of change in productivity is an exogenous $TFP$ shock of the final good production function. Proposition 3.1 below puts forward an asset pricing equation which will be a main building block in our empirical investigation. This asset pricing equation includes the value of physical capital as well as the value of the
existing technologies. This is a main departure from the neoclassical growth model: raw capital generates rather low stock market volatility.

3.1 The household

The representative household supplies one unit of labor inelastically, and has preferences over infinite streams of consumption. Preferences are represented by the expected discounted objective:

$$
E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{c_{t+1}^{1-\sigma}}{1-\sigma} \right]
$$

(1)

where \( c_t \geq 0 \) denotes the quantity of consumption at \( t \), with \( 0 < \beta < 1 \), and \( \sigma \geq 0 \).

The agent may participate in financial markets by trading shares of an aggregate stock \( a_t \). The aggregate stock yields a stochastic dividend \( d_t \). For initial asset holdings \( a_0 \), the optimization problem of the agent is to choose a stochastic sequence of consumption and shares of the aggregate stock \( \{c_t, a_{t+1}\}_{t \geq 0} \) to attain the maximum utility in (1) subject to the sequence of budget constraints

$$
c_t + q_t a_{t+1} = \omega_t + (q_t + d_t) a_t
$$

(2)

$$
q_t a_{t+1} \geq 0, \ t = 0, 1, 2, ..., \tag{3}
$$

for given stock prices \( q_t \), and exogenous wages \( \omega_t \). Note that (3) is a simple borrowing limit which in this representative agent economy entails no loss of generality.

3.2 The production sector

The firm producing the final good accumulates capital and buys labor and intermediate goods. Gross production \( Y_t \) occurs under a CES production function subject to a TFP shock represented by a random variable \( \theta_t \). At every date \( t \) there is a mass \( A_t \) of intermediate goods that enter into the production of the final good. These intermediate goods are bundled together in a composite good \( M_t \) defined by a CES technology \( M_t = [\int_0^{A_t} m_{s,t}^{\frac{1}{\gamma_s}} ds]^{\theta_t} \) where
$m_{s,t}$ denotes the amount of intermediate good $s$ bought by the firm at time $t$ and $\vartheta_t > 1$ follows an exogenous stochastic process to be specified below.

Given initial condition $k_0$, the firm chooses stochastic sequences of investment, labor, and intermediate goods $\{i_t, l_t, (m_{s,t})_{s \in [0, A_t]}\}_{t \geq 0}$ so as to maximize the present value of dividends:

$$E_0 \left[ \sum_{t=0}^{\infty} \eta_t d^f_t \right]$$

subject to

$$d^f_t \equiv Y_t - \left( i_t + \omega_t + \int_0^{A_t} p_{s,t} m_{s,t} ds \right)$$

$$Y_t \equiv \theta_t \left[ \gamma \left( k_t^{1-\alpha} \right)^{\rho} + (1 - \gamma) M_t^\rho \right]^{\frac{1}{\rho}}, \; 0 < \gamma < 1, \; -\infty < \rho < 1$$

$$k_{t+1} = (1 - \delta) k_t + i_t.$$  

Here, $\eta_t$ is a state price converting income of period $t$ to period 0, and $p_{s,t}$ denotes the price of intermediate good $s$ at time $t$. Our definition of dividends in (5) will later include financial leverage as well as the possibility of a tax on dividends $0 \leq \tau < 1$ and a supply shock or friction $\Omega_t$ which may reflect trends in factor prices or financial costs. Note that physical capital stock $k_t$ depreciates at a constant rate $0 \leq \delta < 1$. We do not impose adjustment costs for physical capital accumulation in this simple version of the model.

Besides state prices and wages $\{\eta_t, \omega_t\}_{t \geq 0}$, the firm considers that TFP shocks and price markups evolve exogenously. Stochastic variables $\theta_t$ and $\vartheta_t$ are governed by the following stationary first-order autoregressive processes

$$\ln (\theta_t) = \psi^\theta \ln (\theta_{t-1}) + \sigma^\theta \varepsilon^\theta_t$$

$$\ln (\vartheta_t) = \psi_1^\theta \ln (\vartheta_{t-1}) + \psi_2^\theta \varepsilon^\theta_t$$

where $\psi^\theta, \psi_1^\theta \in (0, 1)$, $\psi_2^\theta > 0$, $\varepsilon^\theta_t \sim N(0, 1)$, and $\ln (\varepsilon^\theta_t) \sim N(0, \sigma^\theta)$.

Monopolistic competition prevails in the market for intermediate goods. Each variety $s$ is supplied by an independent producer. For simplicity, the production process adopts the following form: one unit of good $s$ requires only one unit of the final good. Then, producer of variety $s$ picks an optimal pricing strategy $p_{s,t}$ and quantity $m_{s,t}$ from inspection of the
downward-sloping demand for the product by the firm producing the aggregate commodity – after assuming a fixed set of prices and quantities for all other varieties. More precisely, for each time period \( t \) producer of variety \( s \) maximizes the amount of profits:

\[
\pi_{s,t} \equiv \max_{m_{s,t} \geq 0} \{p_{s,t}m_{s,t} - m_{s,t}\}
\]

(10)

where \( p_{s,t} \) should be viewed as a function of \( m_{s,t} \) from the inverse demand

\[
p_{s,t} = \left( \frac{m_{s,t}}{M_t} \right)^{1-\vartheta_t} p_t
\]

(11)

with \( p_t = \left( \int_0^{A_t} \frac{1}{\vartheta_s} ds \right)^{1-\vartheta_t} \).

Production of intermediate goods may be discontinued because of exogenous factors. Let \( \phi \) be the probability of survival of a technology at every date \( t \). Let \( V_{s,t} \) be the present value of operating technology \( s \) from the beginning of time \( t \):

\[
V_{s,t} = \mathbb{E}_t \left[ \sum_{r=t}^{\infty} \frac{\eta_r}{\eta_t} \phi^{r-t} \pi_{s,r} \right]
\]

(12)

By the symmetry embedded in our model, \( \pi_{s,t} \) and \( V_{s,t} \) are the same for all \( s \).

### 3.3 Technology adoption

Technological innovations arrive exogenously to the economy. The average stock of technological innovations \( Z_t \) evolves according to the law of motion

\[
Z_t = \phi Z_{t-1} + \mu x_{t-1}
\]

(13)

with normalizing constant \( \mu > 0 \) and

\[
\ln x_t = \psi^x \ln x_{t-1} + \sigma_x \varepsilon_t^x
\]

(14)

where \( \psi_x \in (0, 1), \sigma_x > 0, \) and \( \varepsilon_t^x \overset{iid}{\sim} N(0, 1) \).

Technologies are put into use by local adopters. The adoption sector is composed of a continuum of agents \( i \in [0,1] \) that behave competitively. Each adopted technology sells at
price $V_t$ to a monopolistic producer of intermediate goods. Let $A^i_t$ be the stock of already adopted technologies by agent $i$, and $\lambda(H^i_t)$ the probability of adopting a new technology after investing the amount of resources $H^i_t$. An adopter can undertake a diversified menu of projects, and hence we assume that her aggregate productivity is not subject to uncertainty. The stock $A^i_{t+1}$ follows the law of motion

$$A^i_{t+1} = \lambda(H^i_t)\phi [Z^i_t - A^i_t] + \phi A^i_t. \tag{15}$$

The optimal amount of expenditure $H^i_t$ is derived from the following Bellman equation in which the value function is the option value $J^i_t$ of a new technology:

$$J^i_t = \max_{H^i_t} \left\{ -H^i_t + \phi \mathbb{E}_t \left[ \frac{\eta_{t+1}}{\eta_t} \left( \lambda(H^i_t)V_{t+1} + (1 - \lambda(H^i_t)) J^i_{t+1} \right) \right] \right\}. \tag{16}$$

As is well known, this equation can be computed recursively by the method of successive approximations. It follows that the optimal amount of expenditure $H^i_t$ is the same for all $i$. Hence, without loss of generality, we assume $Z^i_t = Z_t$ for all $i$. We then let the aggregate stock of adopted technologies $A_{t+1} = \int A^i_{t+1} di$.  

### 3.4 Equilibrium and asset prices

In the present model, the exogenous state variables are the stock of available technologies $Z_t$, the addition of new technologies $x_t$, the TFP index $\theta_t$, and the price markup $\vartheta_t$. The endogenous state variables are the capital stock $k_t$, and the stock of adopted technologies $A_t$. The remaining variables are determined as solutions of the model from the above optimization problems, market clearing and feasibility conditions. 

As suggested above, we adopt the convention that the stock market value includes all the aforementioned production sectors. That is, $q_t a_{t+1}$ comprises the value of the objective (4) for the firm producing the final good, plus the discounted net value of profits over the set of intermediate goods, and technology adoption. Hence, the aggregate dividend $d_t \equiv d^f_t + \pi_t A_t - H_t(Z_t - A_t)$. In what follows we assume that the aggregate net supply of the asset equals one (i.e. $a_{t+1} = 1$) so that $q_t$ corresponds to the value of the stock market.
For the aggregate commodity, market clearing holds if

\[ YN_t \equiv Y_t - A_t m_t = c_t + i_t + H_t (Z_t - A_t) \] (17)

where \( Y_t \) denotes gross production of the final good and \( m_t \) is the quantity of each variety of intermediate good produced. Hence, \( A_t m_t \) is the cost of producing the composite intermediate good, and \( YN_t \) denotes aggregate value added. Therefore, output can just be broken down into consumption, gross investment in physical capital, and investment in adopting new technologies.

The first-order conditions for the representative household correspond to the usual no-arbitrage conditions for the aggregate stock:

\[ 1 = E_t \left[ \frac{\eta_{t+1}}{\eta_t} \left( \frac{d_{t+1} + q_{t+1}}{q_t} \right) \right] . \] (18)

For positive consumption, \( \eta_t \) can be identified with the marginal utility. The firm producing the final good will always demand positive amounts of each factor. Hence, for positive physical investment the first-order conditions for the maximization of the objective in (4) will always hold with equality. In the adoption sector, optimal positive expenditure in new varieties requires:

\[ 1 = \lambda'(H_t) \phi E_t \left[ \frac{\eta_{t+1}}{\eta_t} (V_{t+1} - J_{t+1}) \right] . \] (19)

It follows that for a concave function \( \lambda(H) \) the optimal expenditure \( H \) is positively correlated with the expected difference between the value of adopted and non-adopted varieties.

The next proposition is central to our study. Iterating forwards over our above asset pricing equations we show that the value of the stock market comprises the value of adopted technologies and the option value to adopt new technologies.\(^5\)

**Proposition 3.1 The stock market value**

\[ q_t = k_{t+1} + V^+_t A_t + J^+_t (Z_t - A_t) + \xi_t \] (20)

where \( V^+_t \equiv V_t - \pi_t, J^+_t \equiv J_t + H_t, \) and \( \xi_t \equiv E_t \left[ \sum_{r=t+1}^{\infty} \eta_{t+1} J_r (Z_r - \phi Z_{r-1}) \right] . \)

\(^5\)Following our approach, this asset pricing equation has been used in the empirical work of Comin, Gertler and Santacreu (2009), and Kung and Schmid (2011).
Therefore, the value of the stock market incorporates four components: The value of installed capital, the value of adopted technologies, the option value of inventions currently available but not yet adopted, and the present value of future inventions expected to happen. These latter components are further sources of volatility in the stock market over the value of capital and adopted technologies. We will analyze the dynamic evolution of these components – as well as their correlation with real macro aggregates – under perturbations of the exogenous state variables. For the purposes of our quantitative analysis, we consider corporate equity and debt, which is referred as the market value of corporations MVC. This avoids modeling of corporate debt policies and pay-outs to debt holders. As already discussed, pay-outs to debt holders have been quite erratic in recent decades [Hall (2001)].

3.5 Equilibrium dynamics

The dynamics of deterministic endogenous growth models has been widely studied in the literature. For instance, for the Lucas-Uzawa model Garcia-Belenguer (2007) shows that without externalities and state-dependent taxes there is a unique balanced growth path which is globally convergent. These results can readily be extended to the Romer model, as this latter model becomes mathematically isomorphic to the Lucas-Uzawa model. Much less is known regarding the stochastic dynamics of this family of endogenous growth models. But under suitable assumptions, it seems plausible to establish a global convergence result. For economies with real and financial frictions, Santos and Peralta-Alva (2013) provide some regularity conditions to ensure existence of an ergodic invariant distribution as well as the convergence of the simulated moments from numerical approximations to the population moments of the original model.

From numerical experimentation, in all calibrations below the model appears to have a unique stable ergodic invariant distribution. The existence of a globally convergent invariant distribution implies that all exogenous and endogenous variables will eventually reach the ergodic set with probability one. It seems then adequate to simulate the model using a high-order perturbation method [Schmitt-Grohé and Uribe (2004)] that takes into account
the high volatility of stock market prices. To check for accuracy of the computed solution, we have combined this approximation method with a numerical dynamic programming algorithm [Santos (1999)] for the computation of Bellman’s equation (16).

4 A Numerical Experiment

In this section we present a straightforward, easy-to-follow calibration of the above simple model of technology adoption to highlight some basic forces driving stock market volatility. The baseline calibration assumes a moderate degree of risk aversion, a Cobb-Douglas aggregate production function, and all shocks to be independently distributed. The exogenous markup process is calibrated from various estimates from the Compustat data set of Section 2. As before, definitions and data sources are gathered together in the Appendix.

4.1 Baseline calibration

Parameter $\sigma$ in the utility function is set to 5, which is within the range of empirical estimates for many studies. Parameter $\beta$ is fixed at 0.95, leading to an annual interest rate of 5.26%. We assume that the production function of the final good producer (6) is Cobb-Douglas (i.e., $\rho \approx 0$). We make $\alpha = 0.26$ based on evidence of the average share of labor costs over corporate value added. The share of materials in gross output is usually assumed to be around 50% [e.g., Comin and Gertler (2006) and Jaimovich and Floetotto (2008)] from data on the manufacturing sector. We let $\gamma = 0.7$ so that the share of materials in gross output is here lowered to 0.30. This seems a fairly conservative estimate, which would be in accordance with our interpretation of a specialized set of producers supplying technology goods to all sectors of the economy. We reproduce the average investment to capital ratio in the data by assuming an annual depreciation rate $\delta$ of 0.09 in (7). The persistence parameter $\psi^\theta$ in the $TFP$ stochastic process (8) is assumed to be 0.95, and $\sigma_\theta$ is set to 0.0128 to match the volatility of the Solow residual under an inelastic labor supply.

For the process of technology adoption, we focus on the volatility and persistence of the
expenditure process $H(Z - A)$ rather than on the increment on the stock of adopted technologies (i.e., $A_{t+1} - \phi A_t$) which is non-observable. Barlevy (2007) shows a high degree of correlation between R&D activities as reported by the NSF and Compustat data. We use the NSF data. The literature usually considers the number of patent applications as a proxy for the degree of technological innovation. This is controversial since patent volumes are just a quantitative measure, and patent applications are noisy measures of technological progress. Nicholas (2008) and Alexopoulos (2011) are recent examples of new measures intended to capture the intensity of technological innovation.

Following empirical estimates in Hall (2007), the survival rate of each intermediate product $\phi$ is set to 0.98. The probability of adoption is determined by an exponential function

$$\lambda(H_t) = \Lambda H_t^\kappa$$

with $\Lambda > 0$ and $\kappa \in (0, 1)$. We assign parameter values in conjunction with the law of motion for $x$ to replicate the volatility and persistence of patents and the ratio of R&D expenditures over corporate output. The steady-state value for probability $\lambda(H)$ is 0.166, which yields an average adoption time of six years. Hence, we impose much less persistence in our calibration exercise than other studies such a Comin and Gertler (2006) who consider a steady-state value for probability $\lambda(H)$ equal to 0.10. Parameter $\kappa$ then determines the volatility of expenditures in technology adoption. We come close to this volatility for $\kappa = 0.80$. This calibration implies a mean value for the ratio of adoption expenditures over net output of about 0.91 percent. This figure is roughly the ratio found in the data. Indeed, the expenditure share of applied research over corporate output for the period 1960-2007 is 0.47 percent, and the expenditure share of development is 1.56 percent. The remaining parameters in the stochastic process (14) will be discussed below (Table 7), and are chosen to match the autocorrelation and volatility of these expenditures in the data.

For the stochastic process governing the evolution of markups (9), we create six subsamples of our Compustat data set comprising the top 50%, 60%, 70%, 80%, 90%, and 100% companies with the highest ratio of R&D expenditure over total revenue. For each subsample, the price markup of a company is defined as the ratio of total revenue over variable cost. The
aggregate markup is then obtained as a weighted average of company markups, using the share of company revenues. After taking logs and detrending, Table 5 presents estimates of the AR(1) processes. Observe from this table that the persistence parameter $\psi_1^\vartheta$ lies in the range of estimated values $\hat{\psi}_1^\vartheta \in [0.9434, 0.9827]$, and the volatility parameter $\sigma_\vartheta$ lies in the range of estimated values $\hat{\sigma}_\vartheta \in [0.1413, 0.3862]$. These estimates remain the same for certain groups of high-tech companies. For instance, the aggregate markup could be defined over all companies with SIC codes: 281, 283, 284, 289, 357, 367, 381, and 384. In this case, we obtain the following point estimates for the AR(1) process: $\hat{\psi}_1^\vartheta = 0.9490$, and $\hat{\sigma}_\vartheta = 0.1805$. In light of all this evidence we let $\psi_1^\vartheta = 0.968$ and $\sigma_\vartheta = 0.2135$. In the model with no uncertainty, we get that the steady-state value for the intermediate producers’ gross markup $\vartheta$ is equal to 1.18. But as the markup shock stems from a log-normal distribution, by Jensen’s inequality the simulated mean for parameter $\vartheta$ is 1.33 which again is in line with previous estimations for these models. These values are consistent with parameter estimates of New-Keynesian models with price rigidities. Thus, Smets and Wouters (2007, Table 4, p. 597) report a mean autocorrelation parameter equal to 0.90 in a model with price stickiness. However, this parameter jumps to 0.97 when the degree of price stickiness is moved to a minimal value.

4.2 Impulse-response functions

To learn about the influence of the exogenous shocks in the model dynamics, Figure 5 displays impulse-response functions for the TFP index $\theta$, the stock of available technologies $Z$, and the price markup $\vartheta$, respectively.

The impact of an increase of one standard deviation in $\theta$ on MVC is a bit stronger and more persistent than in the neoclassical growth model because of the extra propagation mechanism in the market of intermediate goods. An increase in TFP stimulates consumption and the value of installed capital $k$. Thus, the interest rate $R$ goes down to accommodate convergence back to the steady state. Also, positive changes in $\theta$ and the quantity of physical capital increase the demand for intermediate goods and raise profits per variety $\pi$. This increment in the demand of intermediate products along with the lower interest rates leads to increases in
the different components of MVC characterized in Proposition 3.1 (i.e., $V, J$, and $\xi$). Hence, there are further incentives to invest in technology adoption: $H$ will go up, and the amount of adopted varieties $A$ will increase over time. This last additional channel generates a more persistent response of physical capital $k$.

However, an increase of one standard deviation in $Z$ reduces MVC as the arrival of new technologies depresses the amount of profits $\pi$ per variety. Actually, as a result of the arrival of new technologies, investment in technology adoption $H(Z - A)$ increases at the expense of consumption and capital investment. Output peaks later on because adoption of new technologies stirs up the productivity of capital and labor, so both consumption and investment must rebound.

Finally, an increase in one standard deviation in $\vartheta$ boosts MVC considerably. Capital investment and consumption go down, and investment in technology adoption increases. Even in spite of the wealth effect from the rise in MVC, consumption goes down slightly because the aggregate firm purchases less intermediate goods, and so the productivity of capital and labor go down. This diminishes the flow of income to the consumer.

Therefore, for our baseline calibration of the model, widening the range of available technologies $Z$ decrease MVC, as the arrival of new technologies depresses the price of existing ones. Positive changes in $\theta$ and $\vartheta$ lead to extended increases in MVC and its components, but the quantitative impact of $\vartheta$ on MVC is about four times bigger than that of the change in $\theta$. From Figure 5, note that a positive change in $\vartheta$ has a relatively small impact in output and consumption. As in Gârleanu, Panageas and Yu (2012), a large increase in stock market values is compatible with small changes in consumption, whereas in many general equilibrium models such an increase in wealth produces a big jump in consumption.

4.3 Second-order moments

The foregoing impulse-response functions will be helpful to gain some intuition for the simulated moments that we now pass to present. Simulated moments are obtained from equilibrium paths over 3000 observations, where the first 1000 observations have been dropped.
to avoid influence of initial conditions. These moments are computed from log values, and both data and model’s simulations have been filtered within a frequency band of 2-50 years. The following variables are considered: Output (YN), consumption (C), Investment (I), the Solow residual (SR), R&D expenditures (RDE), market value of corporations (MVC), annual return of the market value of corporations (RC), dividends (D), price-dividend ratio (PD), and the annual risk-free interest rate (R). For all these variables, Table 6 reports standard deviations, first-order autocorrelation coefficients, correlation coefficients with YN, and correlation coefficients with MVC.

From standard real business-cycle models [Cooley and Prescott (1995)] the volatility of real macroeconomic variables reported in this table is somewhat expected. For all the financial variables (MVC, RC, PD, and R) the standard deviation generated by the model is over one third of that in the data. For instance, the model generates a volatility for MVC of about 9.51 percent as opposed to 23.96 percent in the data – without compromising the volatility of the real economy. Standard calibrations of the neoclassical model, however, yield standard deviations for these financial variables of one-order of magnitude smaller than the data. Therefore, we have isolated a new channel (variation in product price markups) that leads to about a four-fold increase in the volatility of the financial variables.

Second, all autocorrelation coefficients are within the 95-percent confidence intervals given by the data. Hence, the volatility of these financial variables generated by the model does not depend on near unit-root autocorrelation or excessive persistence of the endogenous variables. Third, as compared to the data, financial variables are highly correlated with YN, while MVC is unduly correlated with our main macroeconomic variables: YN, C, I, and SR. And fourth, the risk-free rate displays very low volatility, and appears to be very negatively correlated with MVC. We next explore some other channels that may bring these correlations closer to their data counterparts.
5 Extensions of the Simple Model of Technology Adoption

The business cycles literature has explored various market frictions and additional propagation mechanisms amplifying output fluctuations. In this section we extend our basic model in the following directions: capital adjustment costs, a general CES specification of the final good production function, a supply shock affecting input costs, an increase in the risk aversion coefficient, taxes, and interest and debt policies. We also allow external shocks to be correlated.

5.1 Adjustment costs, a CES production function, and a more general specification of the shock process

We first consider adjustment costs in physical capital accumulation, a general CES specification of the final good production function, and a supply shock affecting input costs. We also allow external shocks to be correlated.

Let us assume the following law of motion for physical capital accumulation [cf., Jermann (1998)]:

\[ k_{t+1} = (1 - \delta) k_t + g(i_t/k_t) k_t, \]  \hspace{1cm} (22)

where

\[ g(i/k) = \frac{\delta}{1-\frac{1}{\varsigma}} \left( \frac{i}{k} \right)^{1-\varsigma} + \frac{\delta}{1-\varsigma}, \]  \hspace{1cm} (23)

and the positive parameter \( \varsigma \) is the elasticity of the investment to capital ratio with respect to Tobin’s \( q \).

Additionally, let us suppose that the cost of factors of production is no longer deterministic. We introduce a stochastic variable \( \Omega_t \) which may capture trends in factor prices or financial costs so that dividends in the final good sector are expressed as:

\[ d^f_t \equiv Y_t - \Omega_t \left( i_t + \omega_t l_t + \int_0^{A_t} p_{s,t} m_{s,t} ds \right). \]  \hspace{1cm} (24)

This input markup can actually be restricted to some factors of production so that the quantitative impact of this distortion may depend on the functional form of the aggregate
production function (6). Stochastic variable $\Omega_t$ follows a first-order autoregression:

$$\ln(\Omega_t) = \varphi \ln(\Omega_{t-1}) + \varepsilon_t^\Omega$$
with $\varepsilon_t^\Omega \sim iid N(0, \sigma^\Omega)$. \hfill (25)

For simplicity, the proceeds of this distortion are rebated back to the consumer as a lump-sum transfer.

The Appendix below provides an extension of Proposition 3.1 encompassing all these extensions. The fundamental asset pricing equation takes on the form:

$$q_t = p_t^k k_{t+1} + V_t^+ A_t + J_t^+ (Z_t - A_t) + \xi_t$$ \hfill (26)

where $p_t^k = \Omega g_t^\prime$ is the market value of installed capital at the end of period $t$, and $g_t^\prime$ denotes the first-order derivative of the adjustment cost function $g$.

Our calibration of parameter values is displayed in the second column of Table 7. There are two salient features of this calibration exercise. First, parameters $\beta, \sigma, \alpha, \gamma, \delta, \kappa, \lambda, \phi$, are just taken from the previous section. Second, for the estimation of the elasticity of substitution parameter $\rho$ in the production function (6), the elasticity parameter $\varsigma$ in the adjustment cost function (23), and the laws of motion for TFP (8), markups (9), technology innovation (14), and the supply shock (25), we use a simulation-based estimation procedure along the lines of Santos (2010). In this simulation-based estimation exercise parameters are selected from a loss function defined over weighted second-order moments of output $YN$, investment $I$, and market value of corporations $MVC$. This exercise yields an optimal estimation of the covariance matrix for the shocks which may be of independent interest as it suggests how unobservable shocks may be correlated in the data. We would like to remark that the evolution of the markup process is consistent with our previous econometric estimations from Compustat data (see Table 7). Also, the estimated value of $\rho$ in the production function (6) is $-0.6$. Krusell et al. (2000) provide some estimates for the elasticity of substitution between capital and skilled labor which are consistent with our estimation of $\rho$.

As shown in Table 8, these extensions increase the volatility of the financial variables $MVC$, $RC$, $PD$, and more substantially the volatility of the risk-free interest rate $R$. Specifically, the volatility of $MVC$ jumps from 9.51 percent to 13.07 percent, and hence this extended model
accounts for about half of the variation of MVC observed in the data while preserving first-order autocorrelation coefficients and the volatility of real macroeconomic variables within data confidence intervals. A simulation without the supply shock Ω reveals that this shock does not increase the volatility of MVC but increases the volatility of PD and R, and brings various correlations of MVC and PD with real macroeconomic variables much closer to the data. Likewise, adjustment costs have a minor effect on the volatility of MVC. These costs reduce the volatility of investment and crowds out expenditure in technology adoption.

Figure 6 decomposes MVC into the various components of our fundamental asset pricing equation (26). Roughly, the value of installed physical capital ranges between 15 and 30 percent of total corporate value, the value of the sector of intermediate goods ranges between 30 and 65 percent of total corporate value, whereas the option value of adopting future technologies lies between 10 and 30 percent of total corporate value. These figures seem quite plausible. For instance, Hall (2001) argues that in periods of high technological activity the weight of capital in total stock value may get down to one fourth of its peak value. Of course, these fluctuations in the relative value of capital reflect changes in the valuation of technological goods since the replacement value of capital is fairly smooth.

Figure 7 plots the evolution of the income shares of labor, capital, and intermediate goods. We can see that the income share of labor in this calibration of the model hovers around 60 percent and it is also reasonably volatile [cf. Krusell et al. (2000)]. The model with variable labor does not significantly improve upon the volatility of the stock market. Various labor market frictions have been considered in the literature to increase the risk of equity holdings [see Rouwenhorst (1995), and Danthine and Donaldson (2002)]. In our model we find that sticky wages, labor market rigidities and additional shocks to labor markets seem to have a minor influence on the long-term volatility of the stock market. As a proxy for labor distortions, we have experimented with a persistent shock in the shares of labor and capital income. This distortion generated too much volatility in capital investment.
5.2 Risk aversion

Table 9 presents second-order moments for the extended model under a relative risk-aversion coefficient $\sigma$ that has been increased from 5 to 10. It can be seen that a higher degree of risk aversion for the representative consumer brings about more volatility for the financial variables (except dividends $D$) and leaves unchanged the volatility of the real economy. In other words, since a risk-averse consumer dislikes uncertainty, for the same degree of fluctuations in the real economy we observe more volatility in financial prices. Specifically, the volatility of $MVC$ jumps to 16.14 percent, which is about 65 percent of the volatility observed in the data. In contrast, a logarithmic utility function yields a volatility for $MVC$ of 8.27 percent, which is about 35 percent of the volatility observed in the data. Therefore, increasing the degree of risk aversion generates considerable volatility of financial variables while preserving the uncertainty of the real economy.

5.3 Taxes, leverage, and interest rate policies

Taxes on corporate profits and dividends could greatly affect the stock market value as well as endogenous investment and dividends [cf. McGrattan and Prescott (2005) and Poterba (2004)]. We have considered an exogenous process for taxes that is meant to fit the evolution of taxes on dividends in the US as reported in McGrattan and Prescott (2003). This tax policy had a very small effect on the stock market. We should also remark that some activist fiscal policies on taxes and allowances for depreciation [Auerbach (2009)] have a damping effect on stock market values. After analyzing various arbitrary tax policies, we have concluded that taxes may strongly affect the volatility of asset values as they can change optimal dividend policies, but these desirable changes in the volatility of stock values are only obtained at the expense of excessive volatility in some real variables such as capital investment and consumption.

Leverage – short-term and long-term debt – can be an important source of stock market volatility. As is well known, the biggest impact of leverage on the volatility of equity occurs
when debt and the interest rate are negatively correlated with equity. We have considered several debt policies negatively correlated with stock market values. It turns out that a sizable impact of leverage on the volatility of equity is only observed in our model for leverage ratios of about 40 percent, which is well above our mean estimate of 13.91 in our data set.\textsuperscript{6} It appears that the relative low effect of the leverage ratio on the volatility of equity stems from our representative agent framework. In this restricted setting a high level of debt univocally produces a large effect on the volatility of dividends [Danthine and Donaldson (2002)]. The distribution of the leverage ratio over consumers and firms could also matter. Our measure of net debt (see the Appendix) is quite close to Hall (2001), McGrattan and Prescott (2005), and Peralta-Alva (2007) and may seem appropriate in our aggregate model of economic activity.

Models with heterogeneous agents, borrowing limits and arbitrary interest rate policies offer new channels for leverage to impact the volatility of equity. These latter models may actually bring the correlation of MVC with some other financial variables closer to the data. As compared with our data set (see Table 8) in our model the correlation between MVC and \( C \) is too high, the correlation between MVC and \( PD \) is too low, and the correlation between MVC and the risk-free rate \( R \) is too high. The correlation between MVC and \( C \) may be improved with consumer heterogeneity and tight borrowing limits. Similar considerations should apply to improve the correlation between \( PD \) and MVC. In the recent economic crisis we have seen low \( PD \) ratios associated with tighter borrowing limits, a lower quality of collateral, as well as higher credit risk. Hence, from our original computations it seems that the introduction of some monetary factors may lower the correlation between \( PD \) and \( MVC \) and between \( MVC \) and \( R \).

\textsuperscript{6}In our data set the correlation coefficient of equity and debt is -0.37. The maximum level of leverage was 32.97 percent in 1982 with imploding stock market values. Considering 5-year debt, the high risk aversion model of Table 9 is able to reproduce the volatility of the market value of equity with an average leverage ratio of about 43 percent. These debt policies can be specified as functions of capital, output, and a fraction of MVC. Roughly, they all provide similar results. Jermann (1998) analyzes leverage and the equity premium.
A lot of research has been devoted to explore sources of volatility of stock prices, e.g., see Gilles and LeRoy (1991) and references therein. Some authors [Campbell and Shiller (1988) and Cochrane (1992, 2008)] argue that \textit{PD} can mainly be explained by expected future returns, whereas expected future dividend growth lacks explanatory power. We should nevertheless remark that these results are not generally accepted. Some authors have questioned their econometric significance [see Ang (2002) and Ang and Bekaert (2007)] and others have found strong inconsistencies over data samples [see Boudoukh \textit{et al.} (2007) and Larrain and Yogo (2008)].

This ongoing debate provides further motivation for our numerical work. We focus on the computation of the population moments of the model’s invariant distribution for asset prices and returns. Hence, the computed population moments are not subject to the sampling error found in data analysis. Making use of the information provided by the numerical simulation of a general equilibrium model we shall address the following issues: \textit{(i) Accuracy of the linear approximation}: The variance decomposition of \textit{PD} of Campbell and Shiller (1988) rests upon a linear approximation, but it is known from numerical work [e.g., Christiano and Fisher (2001) and Aruoba \textit{et al.} (2007)] that a linear approximation may not be sufficiently accurate for the computation of some financial variables such as the equity premium. \textit{(ii) Sampling error}: The variance decompositions of \textit{PD} is usually calculated over a single realization of the data generating process. In model simulation, however, we can generate all future equilibrium paths and construct confidence intervals for a single realization. And \textit{(iii) Behavior of the above model}: We shall provide a variance decomposition of \textit{PD} for our model and for some versions of the neoclassical growth model. We will use empirical data to test these model predictions.

Following Campbell and Shiller (1988), we now derive an approximate expression for \textit{PD} by a log linearization over an observed sample path. Let $pd_t \equiv \ln \left( \frac{MVC_t}{D_t} \right)$, $\Delta d_t \equiv \ln \left( \frac{D_t}{D_{t-1}} \right)$, $r_t \equiv \ln \left( \frac{MVC_t + D_t}{MVC_{t-1}} \right)$, where $D_t$ refers to our definition of dividends from Section 3, and $MVC_t$.
is the market value of corporations at the end of \( t \). Then, from these definitions, we get the following equation

\[ pd_t = -r_{t+1} + \Delta d_{t+1} + \ln \left[ 1 + \exp \left( pd_{t+1} \right) \right]. \]  

(27)

By a first-order Taylor approximation for \( \ln \left[ 1 + \exp \left( pd_{t+1} \right) \right] \) at the expected value \( \mathbb{E} \left[ pd_t \right] \), it must hold that

\[ pd_t \approx \nu - r_{t+1} + \Delta d_{t+1} + \rho pd_{t+1}, \]

(28)

where \( \nu = \ln \left[ 1 + \exp \left( \mathbb{E} \left[ pd_t \right] \right) \right] - \rho \exp \left( \mathbb{E} \left[ pd_t \right] \right) \) and \( \rho \equiv \exp \left( \mathbb{E} \left[ pd_t \right] \right) / \left( 1 + \exp \left( \mathbb{E} \left[ pd_t \right] \right) \right) \). Iterating forward over this difference equation over \( pd_{t+1} \) for \( N \) periods, we must have

\[ pd_t \approx \nu \frac{1 - \rho^N}{1 - \rho} - \sum_{s=1}^{N} \rho^{s-1} r_{t+s} + \sum_{s=1}^{N} \rho^{s-1} \Delta d_{t+s} + \rho^N pd_{t+N}. \]

(29)

Applying the conditional expectations operator \( \mathbb{E}_t \), multiplying both sides of (29) by \( pd_t - \mathbb{E} \left[ pd_t \right] \), and dividing by \( \text{Var}(pd_t) \), we obtain the variance decomposition [e.g., see Cochrane (1992)]:

\[ CVAR_{N,r} \equiv \frac{-\text{Cov} \left\{ \mathbb{E}_t \left[ \sum_{s=1}^{N} \rho^{s-1} r_{t+s} \right], pd_t \right\}}{\text{Var}(pd_t)} \times 100, \]

(30)

\[ CVAR_{N,d} \equiv \frac{\text{Cov} \left\{ \mathbb{E}_t \left[ \sum_{s=1}^{N} \rho^{s-1} \Delta d_{t+s} \right], pd_t \right\}}{\text{Var}(pd_t)} \times 100, \]

(31)

\[ CVAR_{N,pd} \equiv \frac{\text{Cov} \left\{ \mathbb{E}_t \left[ \rho^N pd_{t+N} \right], pd_t \right\}}{\text{Var}(pd_t)} \times 100. \]

(32)

Observe that these ratios represent the fraction of the variance of \( pd \) that can be attributed to fluctuations of expected future returns, dividend growth, and the volatility of the terminal component \( pd_{t+N} \), respectively. It is important to realize that the above variance decomposition is based upon the following assumptions: (i) Equation (27) considers the realized return \( r_{t+1} \) whereas our pricing equation (18) holds under the discounting operator \( \mathbb{E}_t \left[ \frac{n_t}{n_s} \left( \cdot \right) \right] \) for \( s \geq t \), and (ii) Equation (28) comes from a first-order Taylor approximation. In order to circumvent these shortcomings we now propose new techniques of analysis based upon numerical simulation of our asset pricing equation (18). We provide variance decompositions of \( pd \) under the following two methods:
Method 1: This procedure builds upon a linear approximation of the summation terms (30)-(31); see the Appendix for further technical details. The evolution of our state variables and the pricing kernel \( \mathbb{E}_t \left[ \frac{n_t}{\eta_t} (\cdot) \right] \) are computed through a high-order approximation. Hence, the linear approximation only applies for the computation of the summation terms in the variance decomposition of (30)-(31) evaluated by the expectations operator over the non-linear law of motion of the state variables. The presumption is that the effect of higher order terms will be small for the variance decomposition. Then, \( \mathbb{E}_t \left[ \sum_{s=1}^{\infty} \rho^{s-1} \hat{r}_{t+s} \right] \) and \( \mathbb{E}_t \left[ \sum_{s=1}^{\infty} \rho^{s-1} \Delta d_{t+s} \right] \) are expressed as linear functions of the state variables, where \( \hat{r}_{t+1} \equiv -\ln \left( \frac{n_{t+1}}{n_t} \right) \). Using long simulations we compute the following ratios:

\[
NCVAR_{1,r} \equiv \frac{-\text{Cov} \left\{ \mathbb{E}_t \left[ \sum_{s=1}^{\infty} \rho^{s-1} \hat{r}_{t+s} \right], pd_t \right\}}{\text{Var}(pd_t)} \times 100 \tag{33}
\]

\[
NCVAR_{1,d} \equiv \frac{\text{Cov} \left\{ \mathbb{E}_t \left[ \sum_{s=1}^{\infty} \rho^{s-1} \Delta d_{t+s} \right], pd_t \right\}}{\text{Var}(pd_t)} \times 100 \tag{34}
\]

Method 2: This is a simple procedure to assess the size of the non-linear approximation errors neglected under Method 1. Again, the stock price is defined as the expected discounted value of dividends under operator \( \mathbb{E}_t \left[ \frac{n_t}{\eta_t} (\cdot) \right] \) rather than using the realized return \( r_{t+1} \). Indeed, both \( \frac{n_t}{\eta_t} \) and \( d_t \) are functions of the state variables, and both terms interact in a nonlinear way in the computation of \( pd \). Hence, we propose the following numerical approximation of the variance decomposition based on the computation of two objects: A constant-dividend ratio \( pd^r \) and a constant-discounting ratio \( pd^d \). More precisely, \( pd^r \) is computed from the exact \( pd \) ratio of the model by letting \( \Delta d_t = 0 \), for all \( t \geq 0 \), and \( pd^d \) is computed from the exact \( pd \) ratio by letting \( \frac{n_{t+1}}{n_t} = \beta \), so that expected future dividends are discounted by \( \beta \) from the model. We can then define the following ratios:

\[
NCVAR_{2,r} \equiv \frac{-\text{Cov} \left\{ pd_t^r, pd_t \right\}}{\text{Var}(pd_t)} \times 100 \tag{35}
\]

\[
NCVAR_{2,d} \equiv \frac{\text{Cov} \left\{ pd_t^d, pd_t \right\}}{\text{Var}(pd_t)} \times 100 \tag{36}
\]

As before, these second-order population moments can be calculated by model simulation. As already stressed, these statistics do not depend on the Campbell-Shiller approximation (29) and are computed using operator \( \mathbb{E}_t \left[ \frac{n_t}{\eta_t} (\cdot) \right] \).
For the high-risk-aversion model (HRA), under (33)-(34) we get $NCVAR_{1,r} = 88.22$ and $NCVAR_{1,d} = 14.26$. Hence, for Method 1 about 88 percent of the variance decomposition corresponds to changes in the expected value of future state prices, whilst only 14 percent of the variance decomposition corresponds to changes in expected dividend growth. For Method 2, under (35)-(36) we get $NCVAR_{2,r} = 78.31$ and $NCVAR_{2,d} = 21.20$. Hence, almost 78 percent of the variance decomposition corresponds to changes in the expected value of future state prices, whilst only 21 percent of the variance decomposition corresponds to changes in expected dividend growth. Interestingly, for both Methods 1 and 2 the sum of the components $NCVAR_r$ and $NCVAR_d$ is very close to 100. Furthermore, under both methods the variance of $pd$ is mainly explained by news associated with the discounting factor. Method 1 attributes more variability to asset returns, which may stem from computational errors of the linear approximation, but these errors are relatively small.

These numerical methods allow us to compute the sources of volatility of $PD$ in our model. Now, let us compare our numerical procedures with commonly used econometric methods. Under (30)-(32) we calculate $CVAR_{N,r}$, $CVAR_{N,d}$ and $CVAR_{N,pd}$ for both the data and arbitrarily long simulated paths under the HRA model. Data statistics are calculated over our annual set of observations for the time period 1960-2007. Model statistics are calculated using an equilibrium path. The estimated values are presented in Table 10 for several terminal periods $N$. According to these estimates, for the HRA model the fraction of the variance of $pd$ that can be associated with expected dividend growth is never greater than 20 percent. Observe from this table that actual data attaches a negative weight to expected dividend growth, which suggests an over-reaction of stock prices to changes in expected future returns with values around 120 percent. Although these estimates may change over data samples, they underscore the role of fluctuations of expected asset returns in the volatility of the price-dividend ratio.

In conclusion, we get similar variance decompositions for the above methods and for the computation of $CVAR_{N,r}$ and $CVAR_{N,d}$ in Table 10. In both cases the variability attributed to expected changes in state prices in the HRA model is around 80 percent. As in Campbell
and Shiller (1988), this confirms that the approximation errors for these empirical tests seem to be small. From a methodological point of view, it is therefore reassuring that we get similar variance decomposition results for both computational and commonly used econometric methods. As a matter of fact, our numerical procedures provide a further validation of empirical tests because our computations of the variance decomposition for the HRA model are not subject to sampling error.

We carried out the same analysis for the baseline and the extended models and found similar quantitative results. Hence, these variance decompositions do not seem to crucially depend on high risk aversion levels or input market frictions, and should be ascribed to the evolution of the price markup (9) and other variables affecting asset prices and dividends. This is most clearly seen when we perform a similar analysis of the neoclassical growth model. In the real business cycle model of Rouwenhorst (1995) with a log utility function, we find that 90 percent of the pd volatility is explained by expected dividend growth. In this latter model, it takes a coefficient of risk aversion $\sigma = 10$ for fluctuations of expected asset returns to account for half of the volatility of pd. Therefore, traditional business cycle models cannot generate desired levels of volatility of asset prices, and such volatility is basically driven by expected dividend growth.

7 Concluding Remarks

This paper explores macroeconomic determinants of long-term asset price volatility in a general equilibrium model with lags in technology adoption, product price markups, and leverage. The challenge for our model is to generate observed levels of volatility for asset prices while preserving low degrees of volatility for some financial and real economic aggregates as well as their rather weak correlations with stock market values.

As shown in Section 2, product price markups are highly correlated with stock market values. In the recent IT revolution we find that most of the stock price gains were propelled by new company cohorts. These newcomers command the highest and steepest price markups but are otherwise indistinguishable in terms of popular cash-flow measures such as dividends,
EBITDA, and gross profits. Hence, these financial accounting measures of profitability appear to be rather limited to forecast current and future returns. This evidence is further supported in a time-series analysis of our Compustat data over the 1950-2012 period: both narrow and broad financial accounting measures of profitability are weakly correlated with stock market values at high and low frequencies while our product price markup measures are always statistically significant. For low frequencies these markups display correlation coefficients with stock market values in the range of 0.60 to 0.80. The predictive power of our markup measures is enhanced when companies are grouped by R&D intensity, and so we recognize that not all new corporations are equally affecting stock market values.

In our basic model, unexpected shocks to the markup process have a bigger quantitative impact on stock market values than technology shocks to innovation and aggregate production. Therefore, the estimation of the price markup process becomes critical in our quantitative exercises. We calibrate the variability and persistence of product price markups from various estimates of markup measures over our Compustat sample of companies with positive R&D investment. Overall, our basic model generates a volatility for the market value of corporations of about 9.51 percent as opposed to 23.96 percent in the data – without compromising the volatility of the real economy. Standard calibrations of the neoclassical model, however, yield a volatility of physical capital of one-order of magnitude smaller than the data. Therefore, this new channel (variation in product price markups) leads to about a four-fold increase in the long-term volatility of asset values.

As we allow for correlated shocks, a CES specification for the aggregate production sector, and higher degrees of risk aversion, the model can account for a volatility of the market value of corporations of about 70 percent of the volatility observed in the data. It should be stressed, however, that our general equilibrium setting imposes severe discipline in our numerical experiments. Various market distortions commonly studied in models of real business cycles [cf. Smets and Wouters (2007)] appear to have minor impact on the volatility of the market value of corporations. Indeed, desired levels of volatility of asset prices usually come with pronounced changes in macroeconomic fluctuations. Thus, TFP shocks and the
arrival of new technologies, taxes, interest rate policies, and real and financial frictions have minor effects on the long-term volatility of asset values under various CES formulations of the aggregate production function for capital and labor. Technological innovations can have significant effects if they come along with high markups and TFP changes. The introduction of supply shocks brings the correlation between real and financial variables closer to the data. It seems that these cross-correlations can be further improved by the consideration of monetary factors, borrowing constrains, and further real and financial frictions.

We find that about 80 percent of the volatility of the price-dividend ratio can be explained by fluctuations of expected asset returns – leaving the remaining 20 percent to changes in expected dividend growth. These values are in line with our data estimates. In contrast, many variations of the neoclassical growth model predict low volatility for the price-dividend ratio, which is driven by dividend growth. In these models, to attain reasonable levels of volatility for the price-dividend ratio we may need close to unit-root behavior in the state variables and rather high coefficients of risk aversion.

Leverage is also an important factor to account for the volatility of stock market prices. As documented by several authors, in recent times dividends have lost their allure and payouts to debt holders gained importance over the last two decades. In our representative agent economy it seems more adequate to consider the market value of corporations: the sum of the market value of corporate equity and the book value of net debt. Further, corporate equity includes both publicly and privately held companies so as to capture the effects on stock values of newly founded companies.

Since the early analysis of Chen, Roll and Ross (1986), economists have been interested in quantifying the macroeconomic sources of investment risk. From a modeling perspective, there is a widely-rooted belief that to explain various asset pricing anomalies one should depart from the representative-agent paradigm, and explore the quantitative implications of economies with heterogeneous agents and market frictions (e.g., incomplete markets, endogenous and exogenous borrowing constraints, and collateral requirements). It should be stressed that the numerical simulation of these latter economies is usually quite difficult. In
the available computational experiments [cf., Heaton and Lucas (1996)], it is usually found that agents cumulate enough liquidity so as to minimize the adverse welfare effects of these financial frictions. Hence, economies with market frictions seem unable to generate observed levels of volatility of stock market values. There is a more successful line of research that uses non-additive preferences to explain asset price anomalies [cf., Boldrin, Christiano and Fisher (2001), Campbell and Cochrane (1999) and Miao (2012)]. But the empirical evidence reported in Section 2 suggests that for long-term asset price volatility a more fruitful research avenue would be to focus on models of technology adoption and vintage capital.

8 Appendix

8.1 Data Appendix for Section 2

**SP:** S&P 500 price index – deflated by the Consumer Price Index (CPI). Taken from Robert Shiller’s web page.

**MVC:** Market value of corporations. This measure captures the market value of quoted and unquoted shares of corporate equities in the US. It is the sum of corporate’s market value of equity and book value of net debt. Following Peralta-Alva (2007), this measure has been computed from data of the Flow of Funds Accounts of the United States (FOF) issued by the Board of Governors of the Federal Reserve System (FRB). This measure is transformed in real terms by applying the GDP implicit price deflator taken from NIPA, Table 1.1.4. Similar measures of market value of corporations were computed by Hall (2001), and McGrattan and Prescott (2005).

**LR:** Leverage Ratio: Net debt over MVC.

**R:** Six-month real commercial paper rate from Robert Shiller’s web page.

**D2:** Corporate value added less investment and wages. Corporate value added and wages are taken from the NIPA, Table 1.1.4, and investment is the sum of Investment in Private Nonresidential and Residential Fixed Assets of US Corporations (Standard Fixed Asset Ta-
bles 4.7 and 5.7). A similar measure has been considered by Peralta-Alva and Boldrin (2007), and McGrattan and Prescott (2005). In correspondence with our model, we do not include taxes in this cash-flow definition.

The following variables have been computed from the US set of companies in Compustat (FIC=USA and non-ADR).s.

**MVC:** Sum of the stock market value and book value of net debt. Net debt is defined as the total liabilities (LT) minus total current assets (ACT) minus investments and advances with the equity method (IVAEQ) minus other investments and advances (IVAO) minus other assets (AO). This measure of market value of corporations is in the spirit of our aggregate measure of market value (MVC defined before) and substracts the value of financial assets. Note that it differs from the measure in Larrain and Yogo (2008) which does not consider the net financial position.

**MU:** Aggregate markup. It is a weighted average of company markups. Individual markups are computed as the revenues (REVT) over production cost (COGS) ratios. The aggregate markup is then a weighted average of company markups, using the share of company revenues.

**MU50, MU75, and MU100:** These markup measures have been computed over the set of US corporations in Compustat reporting positive R&D (positive values on XRD) and non-financial (SIC codes out of the interval 6000–6999). We create three subsamples comprising the top 50%, 75%, and 100% companies with the highest ratio of R&D expenditure over revenue. Individual markups are defined as in MU as the ratio of revenue over cost of goods sold. For each subsample, the aggregate markup is then obtained as a weighted average of company markups, using the share of company revenues.

**D1:** Dividends distributed by the companies in US corporations listed in Compustat.

**NI:** Net income.

**GP:** Gross profits defined as total revenue (REVT) minus cost of goods sold (COGS).

**IB:** Earnings before extraordinary items.

**EBITDA:** Earnings before interest, taxes and depreciation.
**EBIT:** Earnings before interest and taxes.

**OIBDP:** Operating income before depreciation.

**OIADP:** Operating income after depreciation.

These cash-flow measures satisfy the following identities:

- Operating Income Before Depreciation (OIBDP) = GP – Other Operating Costs.
- Operating Income After Depreciation (OIADP) = OIBDP – Depreciation.
- EBITDA = OIABDP – Nonoperating, Nonfinancial and Nonextraordinary Costs.
- EBIT = EBITDA – Depreciation.
- Earnings Before Extraordinary Items (IB) = NI + Extraordinary Items (after tax).

### 8.2 Data Appendix for Sections 4-6

We use annual data from 1960 to 2007. Each nominal variable is transformed in real terms under the GDP implicit price deflator taken from NIPA, Table 1.1.4, and then expressed in per-capita terms. Population is taken from NIPA, Table 7.1. When appropriate, financial and economic aggregates are expressed in logs. Then, we use the band pass filter of Christiano and Fitzgerald (2003) over various frequencies. Output (YN) is the corporate value added from NIPA, Table 1.14. Investment (I) is the sum of Investment in Private Nonresidential and Residential Fixed Assets of US Corporations (Standard Fixed Asset Tables 4.7 and 5.7). The replacement value of corporate capital (K) is the sum of nonresidential and residential tangible corporate fixed assets (Standard Fixed Asset Tables 4.1 and 5.1). Consumption (C) is measured as the sum of non-durables and services (NIPA, Table 1.1.5). The Solow residual (SR) is taken from the Bureau of Labor Statistics (BLS) private business sector. Wages are measured as compensation of employees from the NIPA, Table 1.14. (R&D) in non-federally funded R&D taken from National Science Foundation (NSF). The interest rate (R) is the short-term commercial paper rate from Robert Shiller’s web page. Finally, (MVC), and (D)
are defined as the corporate sector (MVC) and (D2) in Appendix 8.1.

8.3 Proof of Proposition 3.1

We here provide an extended version of Proposition 3.1 that includes adjustment costs as in (23), a cost friction $\Omega_t$ as in (24), a tax $\tau$ on dividends distributed by the final good firm, and a predetermined corporate debt $B_{t+1}$ issued by the final good firm. For this extended version of Proposition 3.1, the market value of corporations satisfies

$$q_t = (1 - \tau) \left( p_t^k k_{t+1} - B_{t+1} \right) + V^+_t A_t + J^+_t (Z_t - A_t) + \xi_t.$$  

To prove this result, recall that $d_t \equiv d_t^f + d_t^i$, where $d_t^f$ and $d_t^i \equiv \pi_t A_t - H_t (Z_t - A_t)$ are dividends generated by the final and intermediate input sectors, respectively. Then, first order condition (18) for the household’s problem jointly with the equilibrium condition $a_{t+1} = 1$ and the transversality condition $\lim _{T \to \infty} E_t \left[ \frac{\nu_{T+1}}{\eta_t} a_{T+1} \right] = 0$ imply

$$q_t = E_t \left[ \sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} d_r \right].$$  

(37)

Similarly, using the Euler equation for capital accumulation and the transversality condition for the aggregate firm

$$\lim _{T \to \infty} E_t \left[ \frac{\eta_T}{\eta_t} (p_T^k k_{T+1} - B_{T+1}) \right] = 0,$$  

(38)

we obtain

$$(1 - \tau) \left( p_t^k k_{t+1} - B_t \right) = E_t \left[ \sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} d_r^f \right].$$  

(39)

Finally, we must show

$$V^+_t A_t + J^+_t (Z_t - A_t) + \xi_t = E_t \left[ \sum_{r=t+1}^{\infty} \frac{\eta_r}{\eta_t} d_r^i \right].$$  

(40)
To this end, note that

$$V_t^+ A_t + J_t^+ (Z_t - A_t) + \xi_t$$

$$= E_t \left[ \frac{\eta_{t+1}}{\eta_t} \phi V_{t+1} \right] A_t + E_t \left[ \frac{\eta_{t+1}}{\eta_t} \phi [\lambda(H_t)V_{t+1} + (1 - \lambda(H_t))J_{t+1}] \right] (Z_t - A_t) +$$

$$+ E_t \left[ \frac{\eta_{t+1}}{\eta_t} [J_{t+1}(Z_{t+1} - \phi Z_t) + \xi_{t+1}] \right]$$

$$= E_t \left[ \frac{\eta_{t+1}}{\eta_t} [V_{t+1}A_{t+1} + J_{t+1}(Z_{t+1} - A_{t+1}) + \xi_{t+1}] \right],$$

where the last equality comes after rearranging terms and letting $A_{t+1} = \phi \lambda(H_t)[Z_t - A_t] + \phi A_t$, and $Z_{t+1} - A_{t+1} = Z_{t+1} - \phi \lambda(H_t)[Z_t - A_t] - \phi A_t$. Hence,

$$V_t^+ A_t + J_t^+ (Z_t - A_t) + \xi_t$$

$$= E_t \left[ \frac{\eta_{t+1}}{\eta_t} [d_{t+1}^i + V_{t+1}^+ A_{t+1} + J_{t+1}^+(Z_{t+1} - A_{t+1}) + \xi_{t+1}] \right].$$

Then, iterating forward this equation and ruling out bubbles in equilibrium [see Santos and Woodford (1997)] we get that (40) is satisfied.

### 8.4 Linearization Procedure

By a first-order Taylor expansion of the model’s policy function, it follows that

$$\tilde{d}_t \approx g_d^T \tilde{s}_t,$$

$$\tilde{q}_t \approx g_q^T \tilde{s}_t,$$

$$\tilde{\varrho}_t \approx g_{\varrho}^T \tilde{s}_{t-1} + \epsilon_t,$$

where for each variable $x_t$ we define $\tilde{x}_t \equiv ln \left( \frac{x_t}{x_{ss}} \right)$, and $x_{ss}$ is a deterministic steady-state value, $s_t$ is a $(n_s \times 1)$ vector of state variables, $\varrho_t \equiv \frac{\eta_t}{\eta_{t-1}}$ is the model’s discount factor between $t - 1$ and $t$, $g_q, g_d, g_{\varrho}$ are $(n_s \times 1)$ vectors, and $\epsilon_t \overset{iid}{\sim} (\mu_\epsilon, \Sigma_\epsilon)$. The vector of state variables $\tilde{s}_t \equiv [\tilde{k}_t, \tilde{A}_t, \tilde{Z}_t, \tilde{\theta}_t, \tilde{x}_t, \tilde{\varrho}_t, \tilde{\Omega}_t, \tilde{\epsilon}_{t-1}]^T$ follows the law of motion

$$\tilde{s}_t = h \tilde{s}_{t-1} + \epsilon_t,$$
where $h$ is a $(n_s \times n_s)$ constant matrix. For these linear approximations, we calculate the ratios $NCVAR_{1,r}$ and $NCVAR_{1,d}$ in (33)-(34). These ratios are evaluated over the non-linear motion of the state variables.

REFERENCES


Figure 1: Evolution of $S&P$, $MVC$, $LR$, and $R$

Notes: Detrended $S&P$ 500 price index, market value of corporations (MVC), leverage ratio (LR), and $R$. The market value of corporations is defined as the sum of corporate’s market value of equity and book value of net debt. Definitions and data sources are contained in the Appendix.
Figure 2: Market Value of Equity for Various Cohorts

Sources: Compustat and NIPA.
Notes: Market value of corporate equity over corporate value added for different vintages. Definitions and data sources are contained in the Appendix.
Figure 3: Markup of different Vintages

Sources: Compustat.
Notes: Average markup for the top 50% and 75% companies with the highest ratio of R&D expenditure over total revenue. Average markups are computed as weighted averages using the share of company revenues. The price markup of a company is defined from the revenue over cost ratio. Definitions and data sources are contained in the Appendix.
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Notes: Share of each cohort over the aggregate value. Reported values are averages for different time intervals. MVC: market value of corporations. MU: aggregate markup. MU 50, MU 75, MU 100: average markup for the top 50%, 75%, and 100% companies with the highest ratio of R&D expenditure over total revenue. D1: dividends. NI: net income. GP: gross profit. IB: earnings before extraordinary items. EBITDA: earnings before interest taxes and depreciation. EBIT: earnings before interest and taxes. OIBDP: operating income before depreciation. OIADP: operating income after depreciation. Definitions and data sources are contained in the Appendix.
Table 2: Ten-year Changes in MVC and Financial Measures

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Notes: All variables except markups have been scaled by corporate value added. Growth rates are ten-year log differences. MVC: market value of corporations. MU: aggregate markup. MU50, MU75, MU100: average markup for the top 50%, 75%, and 100% companies with the highest ratio of R&D expenditure over total revenue. D1: dividends. D2: corporate value added less investment and wages. NI: net income. GP: gross profit. IB: earnings before extraordinary items. EBITDA: earnings before interest taxes and depreciation. EBIT: earnings before interest and taxes. OIBDP: operating income before depreciation. OIADP: operating income after depreciation. Definitions and data sources are contained in the Appendix.
Table 3: Correlation Coefficient of MVC with Financial Measures

<table>
<thead>
<tr>
<th>Time Interval:</th>
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<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>20</th>
<th>30</th>
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</thead>
<tbody>
<tr>
<td><strong>MU</strong></td>
<td>0.14</td>
<td>0.18</td>
<td>0.08</td>
<td>0.07</td>
<td>0.06</td>
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</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.11)</td>
<td>(0.14)</td>
<td>(0.21)</td>
<td>(0.22)</td>
<td>(0.10)</td>
<td>(0.07)</td>
</tr>
<tr>
<td><strong>MU50</strong></td>
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<td>0.35</td>
<td>0.50</td>
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<td>0.70</td>
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<td>0.74</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.15)</td>
<td>(0.14)</td>
<td>(0.14)</td>
<td>(0.11)</td>
<td>(0.07)</td>
<td>(0.05)</td>
</tr>
<tr>
<td><strong>MU75</strong></td>
<td>0.14</td>
<td>0.36</td>
<td>0.39</td>
<td>0.49</td>
<td>0.51</td>
<td>0.72</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.15)</td>
<td>(0.19)</td>
<td>(0.19)</td>
<td>(0.20)</td>
<td>(0.14)</td>
<td>(0.08)</td>
</tr>
<tr>
<td><strong>MU100</strong></td>
<td>0.46</td>
<td>0.46</td>
<td>0.53</td>
<td>0.62</td>
<td>0.68</td>
<td>0.82</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
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<td>(0.17)</td>
<td>(0.17)</td>
<td>(0.15)</td>
<td>(0.07)</td>
<td>(0.10)</td>
</tr>
<tr>
<td><strong>D1</strong></td>
<td>0.15</td>
<td>0.05</td>
<td>-0.25</td>
<td>-0.50</td>
<td>-0.51</td>
<td>-0.56</td>
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</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.12)</td>
<td>(0.13)</td>
<td>(0.16)</td>
<td>(0.19)</td>
<td>(0.21)</td>
<td>(0.14)</td>
</tr>
<tr>
<td><strong>D2</strong></td>
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<td>0.03</td>
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<td>-0.09</td>
<td>0.03</td>
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</tr>
<tr>
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<td>(0.17)</td>
<td>(0.24)</td>
<td>(0.29)</td>
<td>(0.21)</td>
<td>(0.17)</td>
</tr>
<tr>
<td><strong>NI</strong></td>
<td>0.35</td>
<td>0.29</td>
<td>0.13</td>
<td>-0.05</td>
<td>-0.07</td>
<td>-0.08</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.10)</td>
<td>(0.13)</td>
<td>(0.12)</td>
<td>(0.13)</td>
<td>(0.07)</td>
</tr>
<tr>
<td><strong>GP</strong></td>
<td>0.47</td>
<td>0.35</td>
<td>0.21</td>
<td>0.02</td>
<td>-0.11</td>
<td>-0.42</td>
<td>-0.55</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(0.21)</td>
<td>(0.20)</td>
<td>(0.20)</td>
<td>(0.17)</td>
<td>(0.08)</td>
<td>(0.04)</td>
</tr>
<tr>
<td><strong>EBITDA</strong></td>
<td>0.46</td>
<td>0.38</td>
<td>0.23</td>
<td>0.01</td>
<td>-0.08</td>
<td>-0.33</td>
<td>-0.38</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.22)</td>
<td>(0.26)</td>
<td>(0.28)</td>
<td>(0.26)</td>
<td>(0.11)</td>
<td>(0.13)</td>
</tr>
<tr>
<td><strong>EBIT</strong></td>
<td>0.36</td>
<td>0.36</td>
<td>0.27</td>
<td>0.12</td>
<td>0.10</td>
<td>0.17</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.20)</td>
<td>(0.22)</td>
<td>(0.25)</td>
<td>(0.24)</td>
<td>(0.14)</td>
<td>(0.12)</td>
</tr>
<tr>
<td><strong>OIBDP</strong></td>
<td>0.46</td>
<td>0.38</td>
<td>0.23</td>
<td>0.01</td>
<td>-0.08</td>
<td>-0.33</td>
<td>-0.38</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.22)</td>
<td>(0.26)</td>
<td>(0.28)</td>
<td>(0.26)</td>
<td>(0.11)</td>
<td>(0.13)</td>
</tr>
<tr>
<td><strong>OIADP</strong></td>
<td>0.40</td>
<td>0.36</td>
<td>0.19</td>
<td>-0.05</td>
<td>-0.11</td>
<td>-0.23</td>
<td>-0.25</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.20)</td>
<td>(0.24)</td>
<td>(0.25)</td>
<td>(0.25)</td>
<td>(0.17)</td>
<td>(0.15)</td>
</tr>
</tbody>
</table>

*Notes:* All variables except markups have been scaled by corporate value added. Reported values are contemporaneous correlations with MVC of growth rates for different time intervals. MVC: market value of corporations. MU: aggregate markup. MU50, MU75, MU100: average markup for the top 50%, 75%, and 100% companies with the highest ratio of R&D expenditure over total revenue. D1: dividends. D2: corporate value added less investment and wages. NI: net income. GP: gross profit. IB: earnings before extraordinary items. EBITDA: earnings before interest taxes and depreciation. EBIT: earnings before interest and taxes. OIBDP: operating income before depreciation. OIADP: operating income after depreciation. Figures in parentheses are Newey-West standard errors. Definitions and data sources are contained in the Appendix.
### Table 4: Correlation Coefficient of Sectoral MVC with Markups

<table>
<thead>
<tr>
<th>Time Interval:</th>
<th>Sector 1</th>
<th>Sector 2</th>
<th>Sector 3</th>
<th>Nasdaq</th>
<th>SIC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MU</td>
<td>-0.18</td>
<td>-0.19</td>
<td>0.07</td>
<td>0.10</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.17)</td>
<td>(0.15)</td>
<td>(0.25)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>MU50</td>
<td>0.71</td>
<td>0.89</td>
<td>0.27</td>
<td>0.43</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.04)</td>
<td>(0.21)</td>
<td>(0.22)</td>
<td>(0.27)</td>
</tr>
<tr>
<td>MU75</td>
<td>0.72</td>
<td>0.90</td>
<td>0.21</td>
<td>0.30</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.03)</td>
<td>(0.24)</td>
<td>(0.24)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>MU100</td>
<td>0.76</td>
<td>0.89</td>
<td>0.16</td>
<td>0.29</td>
<td>0.41</td>
</tr>
<tr>
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<td>(0.15)</td>
<td>(0.11)</td>
<td>(0.17)</td>
<td>(0.16)</td>
<td>(0.22)</td>
</tr>
</tbody>
</table>

**Notes:** Reported values are contemporaneous correlations with MVC of log differences for different time intervals. MVC is adjusted by aggregate corporate value. MU: aggregate markup. MU50, MU75, MU100: average markup for the top 50%, 75%, and 100% companies with the highest ratio of R&D expenditure over total revenue. Figures in parentheses are Newey-West standard errors. Definitions and data sources are contained in the Appendix.
Table 5: Point-Estimates of the Markup Process

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MU50</th>
<th>MU60</th>
<th>MU70</th>
<th>MU80</th>
<th>MU90</th>
<th>MU100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{\psi}_1 )</td>
<td>0.9491</td>
<td>0.9648</td>
<td>0.9718</td>
<td>0.9827</td>
<td>0.9434</td>
<td>0.9643</td>
</tr>
<tr>
<td>( \bar{\sigma}_\theta )</td>
<td>0.3862</td>
<td>0.3485</td>
<td>0.2334</td>
<td>0.1713</td>
<td>0.1831</td>
<td>0.1413</td>
</tr>
</tbody>
</table>

Notes: \( MU_{50}, MU_{60}, MU_{70}, MU_{80}, MU_{90}, MU_{100} \) : average markup for the top 50%, 60%, 70%, 80%, 90%, and 100% companies with the highest ratio of R&D expenditure over total revenue. Figures in parentheses are Newey-West standard errors. Definitions and data sources are contained in the Appendix.
Figure 4: Market Value over Revenue and Markup Measures: NASDAQ Companies

Sources: Compustat.
Notes: All variables have been detrended. MU50, MU75, MU100: average markup for the top 50%, 75%, and 100% companies with the highest ratio of R&D expenditure over total revenue. PR denotes the market value over revenue ratio for the considered sample of companies. Definitions and data sources are contained in the Appendix.
Figure 5: Impulse–Response Functions

Notes: Response to a positive perturbation to each shock (θ, Z, and ϑ) by one standard deviation. MVC: market value of corporations. K: stock of capital. C: consumption. The y-axis measures percentage deviation from the deterministic steady-state value and the x-axis measures time in years.
<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>First-order Autocorrelation</th>
<th>Correlation with YN</th>
<th>Correlation with MVC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>YN</td>
<td>3.41 (2.77, 4.06)</td>
<td>2.71 (0.54, 0.87)</td>
<td>0.71</td>
<td>0.78</td>
</tr>
<tr>
<td>C</td>
<td>1.88 (1.44, 2.31)</td>
<td>2.37 (0.65, 0.95)</td>
<td>0.80</td>
<td>0.83</td>
</tr>
<tr>
<td>I</td>
<td>9.71 (7.76, 11.66)</td>
<td>12.92 (0.63, 0.92)</td>
<td>0.77</td>
<td>0.65</td>
</tr>
<tr>
<td>SR</td>
<td>2.48 (2.04, 2.93)</td>
<td>2.48 (0.68, 0.91)</td>
<td>0.80</td>
<td>0.71</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>6.49 (4.99, 7.99)</td>
<td>8.56 (0.75, 0.90)</td>
<td>0.83</td>
<td>0.84</td>
</tr>
<tr>
<td>MVC</td>
<td>23.96 (19.34, 28.58)</td>
<td>9.51 (0.63, 0.95)</td>
<td>0.79</td>
<td>0.89</td>
</tr>
<tr>
<td>RC</td>
<td>14.80 (10.19, 19.41)</td>
<td>4.37 (-0.24, 0.25)</td>
<td>0.01</td>
<td>0.08</td>
</tr>
<tr>
<td>D</td>
<td>8.08 (5.96, 10.21)</td>
<td>5.74 (0.48, 0.85)</td>
<td>0.66</td>
<td>0.88</td>
</tr>
<tr>
<td>PD</td>
<td>23.17 (17.85, 28.48)</td>
<td>7.73 (0.60, 0.95)</td>
<td>0.78</td>
<td>0.72</td>
</tr>
<tr>
<td>R</td>
<td>2.48 (1.69, 3.26)</td>
<td>0.64 (0.54, 0.80)</td>
<td>0.67</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Notes: YN: corporate value added; C: consumption; I: investment; SR: Solow residual; R&D: expenditures on R&D. MVC: market value of corporations; RC: return of MVC; D: dividends; PD: price-dividend ratio; and R: risk-free interest rate. Statistics are log-values for a frequency band of 2-50 years. Conventional confidence intervals (95-percent) appear in parentheses. Definitions and data sources are contained in the Appendix.
Table 7: Calibration of Parameter Values

<table>
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<th>Parameter</th>
<th>Baseline Calibration</th>
<th>Extended Model</th>
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<td>$\beta$</td>
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<td>0.95</td>
</tr>
<tr>
<td>$\sigma$</td>
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<td>5</td>
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<tr>
<td>Technology</td>
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<tr>
<td>$\alpha$</td>
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<tr>
<td>$\gamma$</td>
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</tr>
<tr>
<td>$\delta$</td>
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<td>0.09</td>
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<tr>
<td>$\varsigma$</td>
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<tr>
<td>$\rho$</td>
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<tr>
<td>$\kappa$</td>
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<td>0.80</td>
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<tr>
<td>$\lambda$</td>
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<td>0.166</td>
</tr>
<tr>
<td>$\phi$</td>
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<td>Exogenous shocks</td>
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<td>0.91</td>
</tr>
<tr>
<td>$\psi^\theta_0'$</td>
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<td>-0.145</td>
</tr>
<tr>
<td>$\psi^\theta_1$</td>
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<td>0.968</td>
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<td>$\psi^\theta_2$</td>
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<td>$\psi^x$</td>
<td>0.72</td>
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<tr>
<td>$\psi^\Omega$</td>
<td>–</td>
<td>0.79</td>
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<tr>
<td>$\sigma_\theta$</td>
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<td>0.0123</td>
</tr>
<tr>
<td>$\sigma_\vartheta$</td>
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</tr>
<tr>
<td>$\sigma_x$</td>
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<td>0.20</td>
</tr>
<tr>
<td>$\sigma_\Omega$</td>
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<td>$\text{Corr } {\varepsilon_\theta, \ln(\varepsilon_\theta)}$</td>
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<td>0.81</td>
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<tr>
<td>$\text{Corr } {\varepsilon_\theta, \varepsilon^x}$</td>
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<td>0.75</td>
</tr>
<tr>
<td>$\text{Corr } {\varepsilon^x, \ln(\varepsilon_\theta)}$</td>
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<td>0.70</td>
</tr>
<tr>
<td>$\text{Corr } {\varepsilon_\theta, \varepsilon_\Omega}$</td>
<td>–</td>
<td>0.55</td>
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Table 8: Second-Order Moments: An Extended Model

<table>
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<th>Standard Deviation</th>
<th>First-order Autocorrelation</th>
<th>Correlation with YN</th>
<th>Correlation with MVC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data Model</td>
<td>Data Model</td>
<td>Data Model</td>
<td>Data Model</td>
</tr>
<tr>
<td>YN</td>
<td>3.41 (2.77, 4.06)</td>
<td>0.71 (0.54, 0.87)</td>
<td>1.00</td>
<td>0.17 (-0.25, 0.60)</td>
</tr>
<tr>
<td>C</td>
<td>1.88 (1.44, 2.31)</td>
<td>0.8 (0.65, 0.95)</td>
<td>0.72 (0.46, 0.98)</td>
<td>-0.02 (-0.42, 0.36)</td>
</tr>
<tr>
<td>I</td>
<td>9.71 (7.76, 11.66)</td>
<td>0.77 (0.63, 0.92)</td>
<td>0.75 (0.55, 0.95)</td>
<td>-0.08 (-0.56, 0.38)</td>
</tr>
<tr>
<td>SR</td>
<td>2.48 (2.04, 2.93)</td>
<td>0.8 (0.68, 0.91)</td>
<td>0.52 (0.20, 0.85)</td>
<td>0.15 (-0.31, 0.62)</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>6.49 (4.99, 7.99)</td>
<td>0.83 (0.75, 0.90)</td>
<td>0.42 (0.02, 0.81)</td>
<td>0.49 (0.23, 0.74)</td>
</tr>
<tr>
<td>MVC</td>
<td>23.96 (19.34, 28.58)</td>
<td>0.79 (0.63, 0.95)</td>
<td>0.17 (-0.25, 0.60)</td>
<td>1.00 1.00</td>
</tr>
<tr>
<td>RC</td>
<td>14.8 (10.19, 19.41)</td>
<td>0.01 (-0.24, 0.25)</td>
<td>-0.26 (-0.50, -0.03)</td>
<td>0.17 (-0.09, 0.42)</td>
</tr>
<tr>
<td>D</td>
<td>8.08 (5.96, 10.21)</td>
<td>0.66 (0.48, 0.85)</td>
<td>0.02 (0.29, 0.34)</td>
<td>0.26 (-0.15, 0.68)</td>
</tr>
<tr>
<td>PD</td>
<td>23.17 (17.85, 28.48)</td>
<td>0.78 (0.60, 0.95)</td>
<td>0.17 (-0.30, 0.64)</td>
<td>0.94 (0.79, 1.00)</td>
</tr>
<tr>
<td>R</td>
<td>2.48 (1.69, 3.26)</td>
<td>0.67 (0.54, 0.80)</td>
<td>-0.09 (-0.35, 0.16)</td>
<td>-0.02 (-0.49, 0.43)</td>
</tr>
</tbody>
</table>

Notes: YN: corporate value added; C: consumption; I: investment; SR: Solow residual; R&D: expenditures on R&D. MVC: market value of corporations; RC: return of MVC; D: dividends; PD: price-dividend ratio; and R: risk-free interest rate. Statistics are log-values for a frequency band of 2-50 years. Conventional confidence intervals (95-percent) appear in parentheses. Definitions and data sources are contained in the Appendix.
Figure 6: Components of the Stock Market Value

Notes: The bottom area is the relative value of physical capital (i.e., $p_t^k k_{t+1}$), the middle area is the relative value of adopted technologies (i.e., $V_t^A A_t$), and the top area is the relative value of unadopted technologies (i.e., $J_t^i (Z_t - A_t) + \xi_t$).
Notes: The bottom area is the labor income share, the middle area is the capital income share, and the top area is the share of intermediate sector’s profits.
<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>First-order Autocorrelation</th>
<th>Correlation with YN</th>
<th>Correlation with MVC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>YN</td>
<td>3.41</td>
<td>2.65</td>
<td>0.71</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>(2.77, 4.06)</td>
<td></td>
<td>(0.54, 0.87)</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1.88</td>
<td>2.12</td>
<td>0.8</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>(1.44, 2.31)</td>
<td></td>
<td>(0.65, 0.95)</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>9.71</td>
<td>10.72</td>
<td>0.77</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>(7.76, 11.66)</td>
<td></td>
<td>(0.63, 0.92)</td>
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</tr>
<tr>
<td>SR</td>
<td>2.48</td>
<td>2.51</td>
<td>0.8</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>(2.04, 2.93)</td>
<td></td>
<td>(0.68, 0.91)</td>
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</tr>
<tr>
<td>R&amp;D</td>
<td>6.49</td>
<td>9.09</td>
<td>0.83</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>(4.99, 7.99)</td>
<td></td>
<td>(0.75, 0.90)</td>
<td></td>
</tr>
<tr>
<td>MVC</td>
<td>23.96</td>
<td>16.14</td>
<td>0.79</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>(19.34, 28.58)</td>
<td></td>
<td>(0.63, 0.95)</td>
<td></td>
</tr>
<tr>
<td>RC</td>
<td>14.8</td>
<td>10.32</td>
<td>0.01</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>(10.19, 19.41)</td>
<td></td>
<td>(-0.24, 0.25)</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>8.08</td>
<td>10.83</td>
<td>0.66</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>(5.96, 10.21)</td>
<td></td>
<td>(0.48, 0.85)</td>
<td></td>
</tr>
<tr>
<td>PD</td>
<td>23.17</td>
<td>17.24</td>
<td>0.78</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>(17.85, 28.48)</td>
<td></td>
<td>(0.60, 0.95)</td>
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</tr>
<tr>
<td>R</td>
<td>2.48</td>
<td>1.93</td>
<td>0.67</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>(1.69, 3.26)</td>
<td></td>
<td>(0.54, 0.80)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: YN: corporate value added; C: consumption; I: investment; SR: Solow residual; R&D: expenditures on R&D. MVC: market value of corporations; RC: return of MVC; D: dividends; PD: price-dividend ratio; and R: risk-free interest rate. Statistics are log-values for a frequency band of 2-50 years. Conventional confidence intervals (95-percent) appear in parentheses. Definitions and data sources are contained in the Appendix.
Table 10: Variance Decomposition of $PD$: High-Risk-Aversion Extended Model

<table>
<thead>
<tr>
<th>Time Horizon</th>
<th>$CVAR_{N,r}$</th>
<th>$CVAR_{N,d}$</th>
<th>$CVAR_{N,pd}$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>N = 1</td>
<td>19.37</td>
<td>10.64</td>
<td>-1.37</td>
<td>6.18</td>
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<tr>
<td></td>
<td>(7.78, 30.95)</td>
<td>(-8.42, 5.68)</td>
<td>(67.5, 94.94)</td>
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</tr>
<tr>
<td>N = 5</td>
<td>58.7</td>
<td>39.99</td>
<td>6.81</td>
<td>16.61</td>
</tr>
<tr>
<td></td>
<td>(43.70, 73.69)</td>
<td>(-8.18, 21.80)</td>
<td>(18.90, 44.97)</td>
<td></td>
</tr>
<tr>
<td>N = 10</td>
<td>95.13</td>
<td>59.66</td>
<td>-10.17</td>
<td>18.24</td>
</tr>
<tr>
<td></td>
<td>(62.28, 127.97)</td>
<td>(-22.71, 2.37)</td>
<td>(-11.25, 31.55)</td>
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<tr>
<td>N = 20</td>
<td>128.58</td>
<td>76.43</td>
<td>-16.41</td>
<td>16.57</td>
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<tr>
<td></td>
<td>(108.94, 148.21)</td>
<td>(-34.52, 1.70)</td>
<td>(-16.97, 5.40)</td>
<td></td>
</tr>
<tr>
<td>N = 30</td>
<td>118.27</td>
<td>81.85</td>
<td>-16.86</td>
<td>16.06</td>
</tr>
<tr>
<td></td>
<td>(98.18, 138.36)</td>
<td>(-35.83, 2.11)</td>
<td>(-1.37, 2.55)</td>
<td></td>
</tr>
<tr>
<td>N = 100</td>
<td>85.21</td>
<td>16.27</td>
<td></td>
<td>0.12</td>
</tr>
</tbody>
</table>

Notes: Variance decomposition for different horizons $N$. Conventional confidence intervals (95-percent) appear in parentheses.