We study the interaction between the precision of exogenous and market-generated information in a class of economies where firms display coordination motives in presence of dispersed information and where the outcome of the coordination is traded in a competitive asset market à-la Grossman and Stiglitz (1980). We show that when more private information is injected in the coordination economy the equilibrium asset price becomes less informative. To showcase the relevance of our result we present an application to a problem of endogenous information choice where the “Knowing What Others Know” property of information acquisition derived by Hellwig and Veldkamp (2009) breaks down in presence of market-generated information.

Keywords: dispersed information, information acquisition, complementarities, rational expectations, Grossman-Stiglitz asset market
1 INTRODUCTION

We study the interaction between private information and market-generated public information in the context of economies in which agents value, either positively or negatively, what other agents are doing in the aggregate. In our economy, firms play a coordination game of incomplete information (e.g. individual investment productivity function of aggregate investment) and use private and public signals to set optimally their actions. Simultaneously, an asset market is operating where risk averse traders exchange claims to dividends that are function of the outcome of the game for claims on a risk free asset. We let the equilibrium price be the main source of public information for the firms and we study how the information aggregating properties of the asset market are affected by the game played by firms.

Our main result reveals that when the private information held by firms in the incomplete information game is made more precise, the equilibrium asset price becomes less informative. Stating it differently: more precise private information leads to less precise public information.

The intuition behind this result goes as follows. In the coordination game the individual optimal strategies take the form of a linear combination of the private and public information. In the aggregate the noise in private information washes out and the game outcome is a function of the fundamentals and the public information. When private information is more precise, its relative weight in the individual optimal strategy is adjusted accordingly, which results in the outcome of the game being more sensitive to the unobserved fundamentals relative to the public information. Now consider the optimal strategies of risk averse traders in the asset market. Anything that makes the outcome of the game more predictable reduces the risk perceived by the traders and lead them to take more aggressive positions on the asset, i.e. positions that are more sensitive to the traders’ private information. With more aggressive individual positions the asset market aggregates traders’ private information more efficiently into the equilibrium price. When the private information of firms in the coordination game is increased the traders perceive that the dividend of the asset is riskier as it relies more on fundamentals, which are harder to predict. Traders react by taking less aggressive positions, thus using their private information less intensively, which finally results in the equilibrium price aggregating information less efficiently, and becoming less precise.

An additional result that we derive concerns the role of the exogenous precision of public information. In our setting this precision corresponds to volatility of the part of the net supply of the risky asset that
is unpredictable. We show that when the asset market is trading over claims on the outcome of the game, as opposed to trading directly on some underlying fundamental, any increase in the exogenous precision of public information results in a more than proportional increase in the precision of the market generated information. Once again, the mechanism that lies behind this magnification effect has to do with the way traders in the market react to the changes in the coordination game. Everything else equal, if public information become more precise firms rely more on it, which makes the game outcome more predictable for traders, which would now take more aggressive positions. The final result is a more efficient aggregation of information by the asset market, which magnifies the initial change in the exogenous precision.

To showcase the relevance of our results we present an information acquisition problem in presence of market-generated information. Information acquisition in our setting takes the form of a privately observed signal with a precision that is increasing in the costly effort spent on information; in this sense private information can be “produced” at the individual agent level by investing real resources. Hellwig and Veldkamp (2009) show that, in general, in economies with coordination motives, the coordination incentive is exactly transferred into the information acquisition incentives: the more agents invest in resources to produce precise private information in the aggregate, the more any individual agent has an incentive to increase (resp. decrease) her investment in private information when agents’ actions are complements (resp. substitutes). In the words of Hellwig and Veldkamp, agents want to “know what others know”.

We show that the exact transfer of incentives from the coordination game to the information acquisition breaks down when market generated information is introduced. More precisely, we show that in presence of complementarity the “knowing what others know” result is strengthen by market prices, while in presence of substitutability it is weakened, and for low enough substitutability it can be overturned.

We outline here the intuition for the result. As already mentioned, in a coordination game with incomplete information the optimal action of a firm takes the form of a linear combination of private and public signals with non-negative weights. Compared to the complete information case, in presence of strategic complementarity the agent optimally distorts the weights by reducing the relevance of private information in favor of the public signal which is a better predictor of coordination in actions, a valued characteristic of the game outcome independently of the true fundamentals. Under strategic
substitutability the opposite is true, the weight on private information is optimally distorted upward, and the weight on public information downward.

Consider now the effect of increasing the aggregate information acquisition effort of the firms on the incentive to acquire information of the individual firm in presence of either complementarity or substitutability in the underlying game. When aggregate private information is more precise all firms adjust their weighting towards private information and away from public information. Under complementarity in actions, the individual firm anticipates that public information has become a worse predictor of the coordinated outcome and thus assigns more value to its own private information, which means higher effort in private information acquisition. Under substitutability, on the other hand, as public information is a worst predictor of the coordinated outcome, the individual firm assigns a higher weight to the public signal, in the attempt to distance itself from the actions of others. The final result is less effort in private information acquisition.

Suppose now that public information is obtained from the equilibrium asset price. From our main result we know that an increase in the precision of firms’ private information reduces the information contained in the asset price. A less informative asset price has two effects on the optimal information choice of the individual firm. On the one hand, if all firms rely less on a less precise public signal, such signal becomes a worse predictor of the game outcome and firms will reduce its relevance when actions are complements, while increase its relevance if actions are substitute. We call this the “coordination” effect of price precision, as the direction of the effect depends on the type of coordination motives in the economy. Since the precision of the asset price declines when firms allocate more effort towards increasing private information precision, the coordination effect is increasing the value of private information under complementarity and decreasing it under substitutability. The coordination effect in the low precision equilibrium reinforces the “knowing what others know” mechanism. On the other hand, it is always true that a less precise public signal increases the value of a more precise private signal, which means that the incentive to invest in private information increases, no matter the type of coordination incentives. We call this effect the “individual” effect of price precision. If the individual effect dominates the coordination effects, the value of the effort in improving private information increases with the increase in the aggregate effort even when the coordination game displays strategic substitutability.

**Related Literature.** Several recent papers have studied, as we do here, the interaction between endogenous public signals and coordination games of incomplete information. Angeletos and Werning
(2006) study the effects of introducing an endogenous public signal in games of regime change in terms of the multiplicity of equilibria. Our modeling of the market that generates public information is similar to that of Angeletos and Werning (2006), but differently from them, we focus on economies with weak coordination motives and unique equilibria and we model players in the coordination game and traders in the market as different subjects. Vives (2012) studies the welfare effects of introducing an endogenous public signal in a game with coordination motives that is similar to ours. The endogeneity of the public signal is modeled by assuming that agents can observe a linear noisy statistics of the aggregate action. Differently from Vives we explicitly model an asset market that produces the public information as a market clearing price, and, most importantly, we consider traders that are risk averse, which makes the perceived uncertainty surrounding the coordination game outcome relevant for the equilibrium outcome. Our paper is also related to Amador and Weill (2012) who study the welfare implications of releasing exogenous public information in an environment in which agents learn from endogenous public information, such as market prices, and from endogenous private information, such as local interactions.

Our application relates to a recent strand of literature on endogenous information acquisition and coordination motives. Several papers have reviewed, as we do, the generality of Hellwig and Veldkamp (2009)’s “Knowing What Others Know” result by considering different settings. Myatt and Wallace (2012) consider the setting of Hellwig and Veldkamp (2009) but endogenize the choice of information along two dimensions: signal choice and attention to be paid to a signal. Their analysis focuses on the effects of information choice to the emergence of a unique equilibrium in situations where Hellwig and Veldkamp (2009) obtain multiple equilibria with respect to information choice. Colombo, Femminis, and Pavan (2012) consider the endogenous information choice for economies with a flexible quadratic-Gaussian structure with the objective of studying the welfare implications of different public information policies. The information coming from public signals in their setting is set by a public authority rather than being the outcome of a competitive market, and therefore it does not react to changes in the private information of agents through market interactions as it is the case in our setting. Szkup and Trevino (2012) is closest to the results that we obtain as they also show conditions where the “knowing what others know” property can break down. In particular, in their setting information acquisition can exhibit substitutability even when the underlying game exhibits strategic complementarity. Two features distinguish our model from their model. First, they consider coordination games with ‘strong’
complementarity, sometimes known as “regime-attack” games, while the model we consider exhibits ‘weak’ coordination motive as Hellwig and Veldkamp (2009) do. Second, the key mechanisms driving the results are different. In our model, the key effect is the decline of the informativeness of the public signal due to an increase in the precision of private signal of the players in the coordination game. In Szkup and Trevino (2012), instead, other players’ better information sometimes decreases the cost of incurring in a prediction error for the individual players. As a consequence, a player, everything else equal, has an incentive to choose less precise information.

The rest of the paper is organized as follows. Section 2 models and studies the coordination economy. Section 3 characterizes the information properties of the equilibrium asset price and contains our main result. Section 4 presents an application of our result to a problem of endogenous information acquisition. Section 5 concludes. Proofs can be found in Appendix A if not reported in the main text.

2 Coordination Economies

We focus on a class of coordination economies as modeled by Angeletos and Pavan (2007). In the economy there are a continuum of agents $i \in [0,1]$, the individual action of agent $i$ is denoted by $a_i$. Let $\Psi(a)$ denote the cumulative distribution function for individual actions across the population; the average action is $A \equiv \int a d\Psi(a)$. Let $\theta \in \mathbb{R}$ represent an exogenous payoff relevant state of the world, which we will refer to as “the fundamentals”. We assume that player’s $i$ payoff function is

$$U(a, A, \theta) = -\frac{1}{2} \left( a - (1 - r)\theta - rA \right)^2. \quad (2.1)$$

where $r \in (-1,1)$. Under (2.1) the payoff of the agent is higher the smaller is the distance between her action $a$ and a linear combination of the aggregate action and the fundamentals. The parameter $r$ measures the degree of desired coordination between the individual and the aggregate action: when $r > 0$ agents actions are complements, when $r < 0$ they are substitutes. Note that $U$ is concave at the individual level, which ensures that the optimal best response is bounded, and the slope of the best response with respect to the aggregate action $A$ is smaller than 1, implying a unique symmetric equilibrium.\(^1\)

\(^1\)The specific form of the payoff function is assumed for analytical convenience and it is without loss of generality for the results derived below. In the Appendix we discuss a general form of the payoff function whose key features are all captured by (2.1)
In what follows we will refer to the agents choosing action \( a \) under (2.1) as firms, in order to distinguish them from the traders that participate in the asset market (see Section 3). Firms do not observe the fundamentals \( \theta \), nor the average action \( A \). It is assumed that each firm forms expectations using two signals, a private signal \( x_i = \theta + (\alpha_{x,i})^{-\frac{1}{2}} \varepsilon_i \) with \( \varepsilon_i \sim \mathcal{N}(0,1) \), and a public signal \( p = \theta + (\alpha_p)^{-\frac{1}{2}} \varepsilon \) with \( \varepsilon \sim \mathcal{N}(0,1) \). For expositional purposes it is useful to allow the signal of an arbitrary firm \( i \) to have a precision that can in principle be different from the precision of the private signals received by the other firms. In line with the existing literature we will restrict our focus on symmetric linear equilibria. As a consequence, here we proceed by solving for firm \( i \) equilibrium action assuming that all the remaining players have a private signal with a common precision \( \alpha_x \). Since the noise in private information is assumed to be i.i.d. across agents, in any symmetric equilibrium, the aggregate action \( A \) will be a function of \((\theta, p)\) only. Following Angeletos and Pavan (2007) we define the equilibrium of the economy as follows.

**Definition.** A linear equilibrium is a strategy \( a : \mathbb{R}^2 \to \mathbb{R} \) linear in \( x \) and \( p \) such that, for all \((x, p)\)

\[
a(x, p) = \arg \max_{a'} \mathbb{E} \left[ U(a', A(\theta, p), \theta) | x, p \right],
\]

where \( A(\theta, p) = \mathbb{E} \left[ a(x, p) | \theta, p \right] \) and \( \Psi(x|\theta, p) \) denotes the conditional distribution of \( x \) given \((\theta, p)\).

Under complete information the equilibrium action is given by \( a^* = \theta \). Angeletos and Pavan (2007) show that under incomplete information an individual strategy \( a : \mathbb{R}^2 \to \mathbb{R} \) is an equilibrium if, and only if, for all \((x, p)\) it satisfies

\[
a(x, p) = \mathbb{E} \left[ (1 - r) \cdot \theta + r \cdot A(\theta, p) | x, p \right],
\]

where \( A(\theta, p) = \mathbb{E} \left[ a(x, p) | \theta, p \right] \) for all \((\theta, p)\). The following proposition then immediately follows.

**Proposition 1.** Suppose that the precision of the private signal of firm \( i \) is given by \( \alpha_{x,i} \), while the precision of the private signal for all the other firms is \( \alpha_x \). For any given value of \((\theta, p)\) a linear equilibrium exists and is unique. The equilibrium action of firm \( i \) is given by

\[
a_i(x_i, p) = \psi_i \Lambda x_i + (1 - \psi_i \Lambda) p
\]
with
\[ \psi_i = \frac{\alpha_{x,i}}{\alpha_{x,i} + \alpha_p} \quad \text{and} \quad \Lambda = \frac{\alpha_x + \alpha_p}{\alpha_x + \frac{\alpha_p}{1-r}}. \] (2.5)

In the linear equilibrium the aggregate action across firms is

\[ A(\theta, p) = \lambda \theta + (1 - \lambda)p \] (2.6)

where

\[ \lambda = \frac{\alpha_x}{\alpha_x + \frac{\alpha_p}{1-r}}. \] (2.7)

Compared to the complete information case, the equilibrium action (2.4) substitutes \( \theta \) with a linear combination of the information available \((x, p)\). The weights that a standard signal extraction exercise would suggest, \( \psi_i \), are distorted by the term \( \Lambda \). In presence of strategic substitutability, i.e. for \( r < 0 \), \( \Lambda > 1 \) and private information \( x_i \) receives a disproportionately higher weight than public information \( p \). In presence of strategic complementarity, i.e. for \( r > 0 \), \( \Lambda < 1 \) and the opposite is true, public information \( p \) receives a disproportionately higher weight than private information \( x_i \). The distortion is common across firms and it carries over to the aggregate so that the average action places more importance on the public signal rather than on the true fundamentals \( \theta \) as shown by equations (2.6) and (2.7).

3 Asset Market

3.1 Demand, Supply and Equilibrium We model the asset market using the CARA-Gaussian framework of Grossman and Stiglitz (1980).\(^2\) The market is made of a large number of traders that have initial wealth \( w_0 \) and decide how to allocate it into a risky asset with price \( p \) and dividend \( f(\theta) \) and a riskless asset with return equal to 1. The utility of the trader \( i \) is \( V(w_i) = -e^{-\gamma w_i} \) with \( w_i = w_0 + fk - pk \), where \( k \) is the demand for the risky asset. Traders base their prediction of the dividend on the information conveyed by the equilibrium price \( p \) and by a private signal \( y = \theta + (\alpha_y)^{-\frac{1}{2}} \varepsilon_y \), where \( \alpha_y \) is the precision of the signal and \( \varepsilon_y \sim N(0, 1) \). Traders individual demands take the standard

\(^2\)In modeling the asset market we follow closely Angeletos and Werning (2006).
“mean-variance” form

\[ k^d(y, p) = \frac{\mathbb{E}(f|y, p) - p}{\gamma \sqrt{\mathbb{V}(f|y, p)}} \]  

(3.1)

The supply in the asset market is assumed to be stochastic and equal to \( K^s(\varepsilon) = (\alpha_\varepsilon)^{-\frac{1}{2}}\varepsilon \). Traders do not observe \( \varepsilon \) directly, but they try to infer it from the information that they have available. The presence of a stochastic supply in the asset market prevents the equilibrium price to reveal perfectly the fundamental \( \theta \): the lower is \( \alpha_\varepsilon \) the lower the information about \( \theta \) in \( p \), everything else equal. In what follows we refer to \( \alpha_\varepsilon \) as the “exogenous” precision of the public information \( p \). The equilibrium market price ensures that the following market clearing condition holds,

\[
\int_{-\infty}^{\infty} k^d(y, p)\phi(y)dy \equiv K^d = K^s(\varepsilon),
\]  

(3.2)

where \( \phi(y) \) represents the pdf of a Normal distribution.

3.2 Exogenous Dividend

We consider first the case of an exogenous dividend, which corresponds to the function \( f \) being exogenously specified rather than being the outcome of some endogenous process. To maintain normality of the distributions we set \( f = \theta \). We conjecture a price that is linear in \( \theta \) and \( \varepsilon \) of the form \( p = \theta - (\alpha_p)^{-\frac{1}{2}}\varepsilon \). The posterior of \( \theta \) conditional on \( y \) and \( p \) is normally distributed with mean \( \delta y + (1 - \delta)p \) and variance \( (\alpha_y + \alpha_p)^{-1} \). The individual demand for the asset is therefore given by

\[ k^d(y, p) = \frac{\alpha_y}{\gamma} (y - p). \]  

(3.3)

The aggregate asset demand is obtained by aggregating over individual traders’ demands, which gives

\[ K^d(\theta, p) = \frac{\alpha_p}{\gamma} (\theta - p). \]  

Imposing market clearing one obtains \( p = \theta - \frac{1}{\alpha_y} (\alpha_\varepsilon)^{-\frac{1}{2}}\varepsilon \) which results in the equilibrium precision of the asset price being

\[ \alpha_p = \alpha_\varepsilon \left( \frac{\alpha_y}{\gamma} \right)^2 \]  

(3.4)

In equilibrium \( \alpha_p \) depends on the precision of the private information of traders and on their degree of risk aversion. In particular, the higher the precision of the private signal of the traders \( \alpha_y \), the higher the precision of the price. This follows from the form of the individual demand (3.3): a more precise private signal makes the demand more sensitive to the difference between \( y \) and \( p \), which means that the
aggregate demand is more sensitive to $\theta$ and thus more information is carried by the equilibrium price. A similar intuition applies for the risk aversion parameter $\gamma$: as traders become more risk averse, their demand is less sensitive to private information and so aggregate demand is less sensitive to $\theta$, and a less informative price results. Finally, we note that the equilibrium precision of the market generated public information depends linearly on the exogenous precision of public information $\alpha_\varepsilon$. As the exogenous precision is increased, the equilibrium precision is proportionally increased as well. Interestingly, for a given realization of $p$, $\alpha_\varepsilon$ does not affect the asset demand, which suggests that the exogenous precision factors into the price precision only through the supply side of the market clearing condition.

3.3 Endogenous Dividend Consider now the case of a dividend that is equal to the aggregate action in the coordination game, namely $f = A(\theta, p)$. The endogeneity of the dividend to the outcome of the coordination game is a simplified way to capture a potentially deeper interconnection between a competitive speculative market and an economic environment with coordination incentives. Using (2.6) the conditional average and variance of the aggregate action can be written as

$$\mathbb{E}[A(\theta, p)|y, p] = \lambda \frac{\alpha_y}{\alpha_y + \alpha_p} (y - p) + p \quad \text{and} \quad \mathbb{V}[A(\theta, p)|y, p] = \frac{\lambda^2}{\alpha_y + \alpha_p}$$

(3.5)

The individual asset demand is then given by

$$k^d(y, p) = \frac{\alpha_y}{\gamma \lambda} (y - p)$$

(3.6)

The expression differs from the exogenous dividend case for the presence of $\lambda$. Recall that $\lambda$ measures how reactive is the individual (and average) action in the coordination game to changes in $\theta$. When $\lambda$ is high the dividend is very reactive to the underlying fundamentals. As a consequence, risk averse traders perceive the asset as riskier, and assume less aggressive positions. A less sensitive individual asset demand to the difference between the private signal and the price results in an aggregate demand that is less sensitive to $\theta$. Aggregating demands one obtains $K^d(\theta, p) = \frac{\alpha_y}{\gamma \lambda} (\theta - p)$. Imposing market

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3One possible micro-foundation of this feature can be obtained by considering a “market monitoring” contractual agreement as in Tirole (2006), where a principal-agent (or more appropriately “agents” in our case) problem is addressed by allowing speculative traders to trade on claims on the outcome of the project. We abstain from modeling the equilibrium resolution of the incentives problem explicitly here, but we focus on the informational consequences of allowing a market populated by agents that are different from those involved in the game whose outcome the market is pricing.
clearing for the asset the equilibrium price is \( p = \theta - \lambda \frac{\alpha_y}{\alpha_p} \alpha_x \) and the precision of the asset price becomes

\[
\alpha_p = \alpha_x \left( \frac{\alpha_y}{\lambda \gamma} \right)^2
\]

Note that \( \alpha_p \) is decreasing in \( \lambda \). Everything else equal, the more firms aggregate action \( A \) responds to fundamentals, the smaller the traders positions on the asset market, which results in less information being encoded into the equilibrium asset price.

Substituting the expression for \( \lambda \) from Proposition 1 the fixed point condition for \( \alpha_p \) takes the quadratic form

\[
\alpha_p = \alpha_x \left( \frac{\alpha_y}{\gamma} \right)^2 \left( \frac{\alpha_p}{(1 - r)\alpha_x} + 1 \right)^2
\]

It can be showed that equation (3.8) has always two positive real solutions, a high precision one and a low precision one, provided that \( 2 \alpha_y \sqrt{\alpha_x} < \sqrt{(1 - r)\alpha_x} \). When the latter condition is not satisfied, an equilibrium for the asset market does not exist. Multiplicity in the asset market is generated by traders trying to predict a dividend that is itself the outcome of a signal extraction problem which depends on the asset price, and thus on traders dividend predictions.\(^4\) To outline an intuition for multiplicity, let us abstract from strategic incentive for a moment and consider the case of \( r = 0 \), so that \( \lambda = \frac{\alpha_y}{\alpha_x + \alpha_p} \). When the precision of the asset price is low, firms allocate a low weight to the public signal and a high weight to their own private signal. This corresponds to a high \( \lambda \), which makes the dividend very reactive to the fundamental \( \theta \). Risk averse traders, everything else equal, will respond by taking more cautious positions on the asset and this, in turn, will lead to a less reactive asset demand and a less informative price, which reinforces the initial low precision. When the precision of the asset is high, exactly the opposite argument holds: firms will rely more on the price, which will make the dividend more predictable, which will make traders take larger positions and thus make the price more informative.\(^5\)

While both levels of price precisions are legitimate equilibria, they turn out to have different properties around the equilibrium point. To be clear, if one considers the stability of the equilibria with

\(^4\)The multiplicity in the precision of the asset price is similar to the multiplicity obtained in Ganguli and Yang (2009).

\(^5\)It is possible to show that as the precision of private information for the firms is made arbitrarily large, the high precision equilibrium would disappear and the low precision equilibrium would coincide with the case of the exogenous dividend.
respect to variations in the asset price, both equilibria are stable in the sense that the asset demand is always downward sloping at the equilibrium point in both cases (see figure 3.3b). On these grounds, both prices could be considered as acceptable from a theoretical point of view. This is for instance the notion of stability that is advocated by Ganguli and Yang (2009). We instead follow Manzano and Vives (2011) and argue that a different notion of stability needs to be applied for fixed point conditions such as (3.8). Manzano and Vives (2011) propose that, if the equilibrium fixed point condition can be written as $\alpha \equiv f(\alpha)$, stability should be defined as

**Definition.** A fixed point $\alpha$ is stable if $|f'(\alpha)| < 1$.

In our case $f(\alpha) \equiv \alpha^2 \left( \frac{\alpha_0}{\gamma} \right)^2 \left( \frac{\alpha}{(1-\gamma)x_1} + 1 \right)^2$, which leads to the following result.

**Proposition 2.** The high precision solution to (3.8) is unstable; the low precision solution is stable.

Figure 3.3a shows the graphical representation of the fixed point in (3.8). At the low precision equilibrium the mapping $f$ has slope less than 1, while at the high precision equilibrium the slope is greater than 1. According to our definition of stability, $\alpha_L$ is stable, while $\alpha_H$ is unstable.

Following the result of our stability analysis, from now on we restrict our attention to the low precision equilibrium. It should be noted, however, that our results would be reversed if we were to consider the case of the unstable high precision equilibria.
3.4 Exogenous Private Precision and Price Precision  We are now in a position to analyze the relationship between the precision of private information at the coordination game stage and the precision of the asset market generated information. The following proposition states our main result.

Proposition 3. The precision of the equilibrium asset price is decreasing in the precision of the private information of firms. Formally

\[
\frac{\partial \alpha_p}{\partial \alpha_x} < 0.
\] (3.9)

The intuition for this result can be gained by recalling our discussion about the effect of \( \lambda \) on the perceived riskiness of the asset dividend. When more precise private information is available to firms, i.e. when \( d \alpha_x > 0 \), they rely more on their private signals, which makes the aggregate action in the coordination economy more responsive to the actual economic fundamentals as opposed to the signal \( p \); this, in turn, makes the dividend riskier from the perspective of the traders and therefore reduces the sensitivity of aggregate asset demand to the fundamental \( \theta \), which eventually results in a noisier price. The negative relationship between the information available to firms and the information contained in the asset price emerges in the context of a coordination game of incomplete information and it is, to the best of our knowledge, novel to the literature.\(^6\)

3.5 Exogenous Public Precision and Price Precision  The interaction between the coordination economy and the asset market also affects the ways in which the exogenous precision of public information \( \alpha_\varepsilon \) translates into the market generated precision of the asset price. To see this recall that in the case of an exogenous dividend the precision of the asset price is a linear function of \( \alpha_\varepsilon \), so that the derivative of the asset price precision with respect to the exogenous public precision is constant and equal to \( \left( \frac{\alpha_\varepsilon}{\gamma} \right)^2 \). The following proposition shows that the presence of the coordination economy enhances the increase in public information precision.

Proposition 4. The precision of the equilibrium asset price is increasing in the exogenous precision of

\(^6\)In Rondina and Shim (2013) we show that a similar result holds when the dividend of the asset is a function of the outcome of a regime change game, such as a currency attack game or a bank run game. We use the result to show that when information precision of players in the game is made arbitrarily precise, the introduction of an endogenous public signal does not necessarily imply multiplicity of equilibria, a result that qualifies the criticism of Angeletos and Werning (2006) to the uniqueness result of Morris and Shin (1998).
public information according to the expression

\[
\frac{\partial \alpha_p}{\partial \alpha_x} = v \left( \frac{\alpha_p}{\gamma} \right)^2 > 0
\]  \hspace{1cm} (3.10)

where

\[
v \equiv \left( \frac{\alpha_p}{(1-r)\alpha_x} + 1 \right)^2 \frac{1}{{1 - f'(\alpha_p)}} > 1.
\]  \hspace{1cm} (3.11)

Once again, the intuition for this result lies in the role of \( \lambda \) in determining the perceived riskiness of the asset. When more precise public information is available firms reduce the weight they assign to their own private information, which makes their actions less sensitive to the true fundamentals. This makes traders’ positions more sensitive on their private information and thus enhances the precision of the equilibrium price. Note that while in the exogenous dividend case \( \alpha_x \) operates only through the supply side of the market, in the endogenous dividend case the behavior of firms in response to a change in \( \alpha_x \) is inducing traders to use their own private information more intensively, thereby changing the demand side of the market as well. The more-than-proportional enhancement of public information precision is thus demand-driven in the endogenous dividend case.

4 APPLICATION: ENDGENOUS INFORMATION ACQUISITION

To showcase the relevance of the results just presented we study a problem of endogenous information acquisition. In particular we allow firms to choose the precision of the private signal they receive, \( \alpha_x \), in exchange for exerting a costly effort. We are interested in understanding whether in presence of market-generated information the information acquisition effort choice of an individual firm is complement or substitute with respect to the information choice of other firms and with respect to the exogenous precision of public information. We think of time as made of three stages. In stage 1 firms choose how much precision to acquire and nature draws the vector of stochastic shocks. In stage 2 traders observe their private signal \( y \) and set their demand strategies and the asset market clears at the competitive equilibrium price. In stage 3, firms observe their private signal \( x \), the equilibrium price and, conditional on both, set their optimal strategies in the coordination economy. We focus on symmetric perfect Bayesian equilibria of the three-stage extensive form game\(^7\) and use backward induction to characterize

\(^7\)Because each individual agent is infinitesimal beliefs off-equilibrium path need not be specified as any deviation from a single player cannot possibly be detected.
the equilibrium strategies. We begin by solving the coordination game at stage 3, given a pre-specified symmetric informational choice across agents and an arbitrary asset price. Then, given the outcome of the coordination game we solve for the equilibrium asset price of stage 2, and finally, we study the information choice at stage 1 under the equilibrium restrictions that we obtained in stages 2 and 3.

4.1 Private Information Acquisition and Market Generated Information

We assume that the information precision of a firm $i$ depends on the amount of effort $n_i \geq 0$ that is exerted in stage 1. We capture this relationship by denoting the private precision with $\alpha_x(n_i)$ where the function $\alpha_x(\ )$ is assumed to be non-negative, non-decreasing, weakly concave and differentiable. Introducing the relationship between precision and effort is not strictly necessary for our analysis, but it allows us to distinguish more easily what signal precisions are controlled by the individual firms and what are taken as given but still endogenous in equilibrium. The private signal received by a firm $i$ can be then written as

$$x(n) = \theta + \alpha_x(n)^{-\frac{1}{2}} \epsilon_x$$  \hspace{1cm} (4.1)

Firms choose $n$ taking into account the effect that the private information precision has on the utility they expect from playing the coordination game once the signals are received. The expected utility at the information choice stage is $E_0[U^*(a, A, \theta)]$. Here $U^*$ denotes the fact that, given any information choice $n$, the payoff function is specified using the function describing the optimal strategy at stage 3, while $E_0$ denotes that expectations are taken with respect to the common prior before signals are realized.

For simplicity we consider symmetric equilibria in information choice at the aggregate level and so we let $n$ represent the information acquisition effort across firms, or aggregate information acquisition effort. The individual firm $i$ chooses effort taking the aggregate effort $n$ as given. The infinitesimal dimension of each individual firm is such that the individual deviation from the aggregate information choice has no detectable effect. On the other hand, the aggregate information effort $n$ affects the precision of the asset price and thus the asset price itself. We embed this into our notation by denoting the asset price by $p(n)$ and its precision by $\alpha_p(n)$. Using Proposition 1 the optimal strategy for an individual firm $i$ is given by

$$a(x_i(n), p(n)) = \lambda(n, n)x_i(n) + (1 - \lambda(n, n))p(n)$$  \hspace{1cm} (4.2)
where
\[ \lambda(n, n) = \Lambda(n) \psi(n, n); \quad \psi(n, n) = \frac{\alpha_x(n)}{\alpha_x(n) + \alpha_p(n)}; \quad \Lambda(n) = \frac{\alpha_x(n) + \alpha_p(n)}{\alpha_x(n) + \frac{\alpha_p(n)}{1 - r}}. \quad (4.3) \]

Defining \( \lambda(n, n) \equiv \lambda(n) \) from the above relationships it follows that the aggregate action of firms is
\[ A(\theta, p(n)) = \lambda(n) \theta + (1 - \lambda(n)) p(n) \quad \text{where} \quad \lambda(n) = \frac{\alpha_x(n)}{\alpha_x(n) + \frac{\alpha_p(n)}{1 - r}}. \quad (4.4) \]

At stage 1 firms anticipate that the equilibrium of the coordination satisfies (4.4) together with market clearing in the asset market. Information sets at stage 1 are the same across all agents and consist of the structure of the game and the common degenerate prior over \( \theta \). Because the utility of firms is quadratic, the expected utility at stage 1 depends only on unconditional second moments. Using the equilibrium relationships above it is possible to show the following Lemma.

**Lemma 1.** Suppose that at stage 1 in the economy the aggregate information acquisition effort is \( n \).

The expected utility for an individual firm with information acquisition effort \( n \) is given by
\[ E_0[U^*(a, A, \theta)] \equiv u(n, n) = \frac{1}{2} \lambda(n, n) \left[ 1 + r \left( 1 - \lambda(n) \right) \right] + r \left( 1 - \lambda(n, n) \right) \left[ \frac{1 - \lambda(n)}{\alpha_p(n)} \right] + \bar{u}(n) \quad (4.5) \]

where \( \bar{u}(n) \) is a constant term that does not depend on the choice variable \( n \).

To understand the terms on the right hand side of (4.5) note that the effort choice \( n \) at the individual level affects only the individual action \( a \) by redistributing the weight \( \lambda(n, n) \) between the private signal \( x \) and the public signal \( p \). Since \( U \) is quadratic, it follows that the choice of \( n \) can affect only the terms in the payoff function that depend on \( a \); these are the variance of the individual action, \( E_0[a^2] \), and the covariance of the individual action with the aggregate action, \( E_0[aA] \), and with the state, \( E_0[a\theta] \). Using (4.2) and \( p = \theta - \alpha_x^{-1/2} \varepsilon \) one can obtain expressions for these three terms. First, the covariance between the individual action and the state \( E_0[a\theta] \) is just a function of the common prior over the variance of \( \theta \), which is unaffected by the information choice and it is therefore included in \( \bar{u}(n) \). Second, the variance of the individual action is given by the common prior over the variance of the state \( \theta \), plus a linear combination of the variance of the noise in the public and private signal, which can be expressed as the first term on the right hand side of (4.5). Finally, the covariance between the individual action and the aggregate action equals the unconditional variance of the state \( \theta \) plus a covariance term that emerges because both the individual and the aggregate action rely on the noisy public signal. This is the second
term on the right hand side.

The form of (4.5) is very useful in isolating the effects of investing in private information precision on the payoff of the players. There are two opposite effects at play when private information is changed. On the one hand, as \( n \) is increased the effect on \( \lambda(n, \mathbf{n}) \) is always positive: as private information is more precise the individual player will rely more on it and thus incur in a smaller prediction error, which always reduces the ex-ante variance of the individual action choice \( a \). This effect is captured in the first term on the right hand side of (4.5). Note that for \( r \in (-1, 1) \), the term is always positive, which means that, no matter the strategic interactions in the coordination economy, higher effort in private information precision acquisition always increases expected utility by reducing the variance of the individual action. On the other hand, as \( n \) is increased the player will rely less on public information and thus reduce the covariance of her individual action with the aggregate action. This is captured in the second term at the right hand side of (4.5). Whether the reduction in covariance is desirable or not depends on the type of strategic interactions in the coordination economy. If actions are complement \((r > 0)\) a reduction in covariance will reduce the ex-ante payoff; while the opposite is true if actions are substitutes \((r < 0)\).

Complementarity or substitutability in information acquisition for firms can be evaluated by studying the sign of the cross-partial derivative

\[
\frac{\partial^2 u}{\partial n \partial \mathbf{n}} \equiv u_{nn} \quad (4.6)
\]

When the sign of the cross-partial is positive, the value of additional information at the individual firm level is increasing in the precision of firms’ private information at the aggregate level. In such case we say that the coordination economy displays complementarity in information acquisition with respect to other firms. When the sign is negative, the value of additional information at the individual level is decreasing with the precision of information at the aggregate level. In this case we say that the economy displays substitutability in information acquisition.

We develop our analysis in two steps. First we consider the case of an exogenous dividend for the asset price. For the first term in (4.10) this corresponds the case considered by Hellwig and Veldkamp (2009); we show that their result holds our setting. Next, we allow the market price to have an endogenous precision through the equilibrium interaction. We show that the result of Hellwig and Veldkamp (2009) is modified in a very specific direction.
Knowing what Others Know. The following proposition corresponds to a version of Hellwig and Veldkamp (2009) main proposition for environments where information acquisition is continuous.

Proposition 5. Assume that the precision of the asset market price does not depend on the precision of the private information of firms as in Section 3.2.

1. If actions are complementary in the coordination economy \((r > 0)\), the value of additional effort in private information acquisition at the individual firm level \(n\) is increasing in the aggregate effort across firms \(n\).

2. If actions are substitutes in the coordination economy \((r < 0)\), the value of additional effort in private information acquisition at the individual firm level \(n\) is decreasing in the aggregate effort across firms \(n\).

3. If actions are independent of each other in the coordination economy \((r = 0)\), the value of additional effort in private information acquisition at the individual firm level \(n\) is independent of the aggregate effort across firms \(n\).

Proof. The proof of 1-3 consists in showing that \(\text{sign}(u_{nn}) = \text{sign}(r)\). Holding \(\alpha_p\) constant the cross partial derivative of (4.5) is \(u_{nn} \propto 2\Lambda(n)\frac{\partial\Lambda(n)}{\partial n} \frac{\alpha_z'(n)}{(\alpha_x(n)+\alpha_p)}\), where \(\alpha_z'(n) = \frac{\partial\alpha_z(n)}{\partial n}\). The sign of this expression depends entirely on the sign of \(\frac{\partial\Lambda(n)}{\partial n}\) as both \(\alpha_z'(n)\) and \(\Lambda(n)\) are always positive. Since \(\alpha_p\) is constant it can be showed that

\[
\frac{\partial\Lambda(n)}{\partial n} = r \frac{(1-r)\alpha_z'(n)\alpha_p}{(\alpha_p + (1-r)\alpha_x(n))^2}.
\]

The sign of the expression on the right hand side depends only the sign on \(r\), which completes the proof.

Recall that the term \(\Lambda(n)\) represents the adjustment that individual firms apply to their actions to take into account that other firms under-react or over-react to the common noise that is present in the public signal. In presence of complementarity the adjustment is downward, so that \(\Lambda(n) < 1\), while in presence of substitutability the adjustment is upward, \(\Lambda(n) > 1\). When aggregate private information acquisition across firms \(n\) is increased, all firms devote more importance to their private signals and less to their public signal. In presence of strategic complementarity individual firms recognize this and
adjust $\Lambda(n) < 1$ upwards in order to take into account the reduced correlation across actions from the common noise of the public signal. In presence of strategic substitutability individual firms act exactly in the opposite way and adjust $\Lambda(n) > 1$ downward. As a consequence, when $n$ is increased, the value of private information is enhanced for the individual firm in the economy with complementarities, and this creates an incentive in increasing the investment in private information. On the contrary, the value of private information is reduced for the individual player in the economy with substitutability, and so the incentive is now to reduce the effort in private information. The strategic motives in the coordination economy carry over to the information acquisition stage.

**MARKET-GENERATED INFORMATION.** We now allow for the precision of the public information to vary with the aggregate informational choice $n$ by introducing the asset market of Section 3.3. Differently from the exogenous precision case, as $n$ is changed, the precision $\alpha_p(n)$ is changed as well. We know from Proposition 3 that the change in precision of the asset price is inversely related to the precision of private information for firms. Introducing the endogeneity of information precision, a corollary immediately follows.

**Corollary 1.** The precision of the equilibrium asset price is decreasing in the aggregate information acquisition effort of firms, formally

$$\alpha_p^n \equiv \frac{\partial \alpha_p(n)}{\partial n} < 0. \quad (4.8)$$

**Proof.** Follows immediately from Proposition 3 and $\alpha'_x(n) > 0$. \hfill \Box

Using (4.5) the cross-partial $u_{nn}$ can be written as

$$u_{nn} \propto \Lambda_r \alpha'_x(n) + (\Lambda_r - \Lambda) \alpha_p^n \quad (4.9)$$

where

$$\Lambda_r \equiv r \frac{(1-r)\alpha_p(n)}{(\alpha_p(n) + (1-r)\alpha_x(n))^2} \quad \text{and} \quad \lambda_\alpha \equiv -\frac{\Lambda(n)}{\alpha_x(n) + \alpha_p(n)}$$

We refer to $\Lambda_r$ as the “coordination effect” term, as its sign and magnitude depend on $r$, and to $\lambda_\alpha$ as the “individual effect” of the market-generated information.
The expression for $u_{nn}$ shows how the coordination effect operates across public versus private signals. If all firms increase the precision of their private signals, the value of allocating effort to private information is higher for the individual firm under complementarity. If the increase in precision concerns the public signal, then there is less value in allocating effort to private information, hence the minus sign in front of $\Lambda_r$. In addition to these two contrasting effects, there is a third effect measured by $\lambda_\alpha$ capturing the substitutability across different signals typical of any signal extraction problem: the relative value of a signal is reduced when another signal becomes more precise. Note that the magnitude of $\lambda_\alpha$ depends on the individual precision of the private signal, $\alpha_x(n)$, and its sign is independent of the strategic interaction motives. Since the sign of $\lambda_\alpha$ is always negative, its contribution to the information acquisition incentive depends on the direction of the change in market price precision. When $\alpha_p(n)$ increases, the value of private information is always reduced as better public information is available, while the opposite is true when $\alpha_p(n)$ decreases. In addition, the relevance of the individual substitution effect $\lambda_\alpha$ depends on the distortionary term $\Lambda(n)$. More precisely, the individual effect matters more in presence of substitutability, when $\Lambda(n) > 1$, than in presence of complementarity, when $\Lambda(n) < 1$. The reason being that the change in precision concerns a public signal, which, while desirable in presence of positive coordination motives, it remains undesirable under substitutability.

The following proposition summarizes how the interaction of the coordination and individual effect with the asset price precision shapes the value of firms’ effort in private information acquisition.

**Proposition 6.** Assume that the precision of the public signal $p$ is market-generated as in Section 3.3.

1. If actions are complementary in the coordination economy ($r > 0$), the value of additional effort in private information acquisition at the individual firm level $n$ is always increasing in the aggregate effort across firms $n$.

2. If actions are weakly substitutes in the coordination economy ($r \leq 0$), the value of additional effort in private information acquisition at the individual firm level $n$ is increasing in the aggregate effort across firms $n$ if the individual effect is stronger than the coordination effect, i.e. $\lambda_\alpha \alpha_p^n > -\Lambda_r \alpha_x(n) + \Lambda_r \alpha_p^n > 0$

**Proof.** Follows immediately from (4.9), the definitions for $\Lambda_r$ and $\lambda_\alpha$ and Corollary 1.

Under strategic complementarity $\Lambda_r > 0$ and $\lambda_\alpha < 0$, but the precision of the market price is
decreasing in the aggregate effort on information acquisition of firms, which implies that both the coordination and individual effect reinforce the “knowing what others know” motive. Compared to the exogenous information case of Proposition 5, the complementarity in information acquisition effort is magnified through the decrease in precision of the public signal, which turns the coordination effect on public information in the direction of “knowing what others know”. Under strategic substitutability the coordination effects reverse their signs. However, the individual effect remains overall positive and it is larger since $\Lambda(n) > 1$. It follows that information acquisition effort can display complementarity even in presence of substitutability in the coordination economy. Essentially, if the deterioration in the precision of the public signal is large enough (which is a consequence of more private aggregate information precision), the individual firm finds it valuable to increase its private information acquisition effort in order to contrast the deterioration in its overall information precision due to a less precise market price. If this happens, the “knowing what others know” property does not hold anymore.

4.2 Private Information Acquisition and Exogenous Precision of Public Information

In the study of information acquisition it is important to understand how exogenous changes in the precision of public information, possibly obtained as the result of a public information policy, affect the incentive to acquire information at the individual agent level. In the present application one can think of $\alpha_\varepsilon$ as the information parameter on which a public policy can intervene. The consequences on the information acquisition incentives can thus be studied by looking at the behavior of the cross-partial

$$u_{n\alpha_\varepsilon} \equiv \frac{\partial^2 u}{\partial n \partial \alpha_\varepsilon}.$$ (4.10)

When the sign $u_{n\alpha_\varepsilon}$ is positive, the value of additional information at the individual firm level is increasing in the exogenous precision of public information. In such case we say that the coordination economy displays complementarity in information acquisition with respect to public information. When the sign is negative, the value of additional information at the individual level is decreasing with the exogenous precision of public information. In this case we say that the economy displays substitutability in information acquisition with respect to public information. Following our previous analysis it is immediate to see that the cross partial can be written as

$$u_{n\alpha_\varepsilon} \propto (-\Lambda_r + \lambda_\alpha \frac{\partial \alpha_p}{\partial \alpha_\varepsilon})$$ (4.11)
The following result immediately follows.

**Proposition 7.** Assume that the precision of the public signal $p$ is market-generated as in Section 3.3.

1. If actions are complementary in the coordination economy ($r > 0$), the value of additional effort in private information acquisition at the individual firm level $n$ is always decreasing in the exogenous precision of public information $\alpha_\varepsilon$.

2. If actions are weakly substitutes in the coordination economy ($r \leq 0$), the value of additional effort in private information acquisition at the individual firm level $n$ is decreasing in the exogenous precision of public information $\alpha_\varepsilon$ if the individual effect is weaker than the coordination effect, i.e. $|\lambda_\alpha| < |\Lambda_r|$.  

3. Everything else equal, the change in the value of additional effort in private information acquisition at the firm level is always larger when the dividend of the asset price is endogenous to the outcome of the coordination economy.

**Proof.** Follows immediately from (4.11), the definitions for $\Lambda_r$ and $\lambda_\alpha$, Corollary 1 and Proposition 4.

To gain intuition for points 1-2 of the result, recall that $\lambda_\alpha$ is always negative, while the sign of $\Lambda_r$ depends on $r$. When $r > 0$ both the individual and coordination effects agree and point towards a lower value for private information. For the case of $r < 0$, it is possible that the individual effect is so small that the coordination effect dominates. In a symmetric equilibrium in the information choice one has $n = n$ which makes the latter condition not possible, so that the individual effect always dominates the coordination effects. In other words, the incentive to choose information so to receive signals that would ensure the maximum distance from the aggregate action is always weaker than the effect of having better information available, independently of the source. It follows that, even in the presence of substitutability, an increase in the precision of public information reduces the value of allocating effort to the acquisition of public information.

The second part of the result is a direct consequence of Proposition 4. When the dividend of the asset is endogenous there is a demand-driven information precision enhancement that reduces (or increases) the value of acquiring private information even further.
We conclude this section with a comment on the relevance of the results of our application for information policy problems. Generally speaking, a policymaker is always faced with the choice of spending resources to provide incentives to private information acquisition at the individual agent level or using resources to increase the precision of public information. Propositions 4, 6 and 7 provide reasons for caution when designing an optimal policy in presence of market-generated information. On the one hand, Proposition 4 says that increasing private information acquisition deteriorates the precision of the asset price. To the extent that the asset price precision is of relevance to the policymaker in its own right, the proposition warns the policymaker that providing private information acquisition incentives might backfire through the endogenous public information. At the same time, Proposition 6 informs that for a given amount of resources spent in providing incentives for private information acquisition, the final outcome is magnified in presence of market-generated information, irrespectively of the sign of coordination incentives in the economy. On the other hand, Proposition 7 warns that if resources are spent on increasing the precision of public information, this may come at the cost of reducing the acquisition of private information at the firms level. In particular, when the precision of public information affects a market-generated public signal, it matters how the market forces take into account the outcome of the underlying coordination economy. If such an interaction is overlooked, the chosen release of public information might “overshoot” the optimal level by not taking into account the ensuing excessive reduction in private information acquisition.

5 Conclusion

There is little doubt that market prices are the primary source of publicly available information in the economy. A common presumption is that when more private information is injected in the economy, market prices reflect the increase in information and become a better source of public information. In this paper we have presented a setting in which this presumption is proven incorrect: if more private information is available to firms in the economy, the economic outcome is perceived as more uncertain to risk averse asset traders, which leads to a reduction in traders’ use of private information in the asset market. The final result is a less informative market price. Our application to a simple information choice problem highlights the importance of our result for a policymaker that needs to allocate resources to design the proper informational incentives to maximize welfare, as in Colombo, Femminis, and Pavan...
Disregarding the interaction between private and public information through market forces is likely to let the policymaker choose an allocation that is sub-optimal. We leave such welfare analysis to future work.

**REFERENCES**


A Appendix A: Proofs

A.1 Proof of Proposition 1  

The individual optimal action is \( a(x, p) = (1 - r)E[\theta | x, p] + rE[A(\theta, p) | x, p] \).

We guess the individual and aggregate solutions \( a(x, p) = \lambda^i_x x + \lambda^i_p p \) and \( A(\theta, p) = \lambda_x \theta + \lambda_p p \). Since \( E[\theta | x, p] = \psi_i x_i + (1 - \psi_i)p \), where \( \psi_i = \alpha_{x,i}/(\alpha_{x,i} + \alpha_p) \), it follows that

\[
E[A(\theta, p) | x, p] = \lambda_x (\psi_i x_i + (1 - \psi_i)p) + \lambda_p p. \tag{A.1}
\]

The individual action becomes

\[
a_i(x_i, p) = \left[(1 - r)\rho_1 \psi_i + r\lambda_x \psi_i\right] x_i + \left[(1 - r)\rho_1 (1 - \psi_i) + r\lambda_x (1 - \psi_i) + r\lambda_p\right] p. \tag{A.2}
\]

The fixed point conditions are

\[
\lambda^i_x = [(1 - r) + r\lambda_x] \psi_i \quad \text{and} \quad \lambda^i_p = (1 - r)(1 - \psi_i) + r\lambda_x (1 - \psi_i) + r\lambda_p \tag{A.2}
\]

In the symmetric equilibrium \( \lambda^i_x = \lambda_x, \psi_i = \psi \) and \( \lambda^i_p = \lambda_p \), so from (A.2) one obtains

\[
\lambda_x = \frac{(1 - r)\alpha_x}{(1 - r)\alpha_x + \alpha_p} \quad \text{and} \quad \lambda_p = \frac{\alpha_p}{(1 - r)\alpha_x + \alpha_p}. \tag{A.3}
\]

The solution for the aggregate action \( A(\theta, p) \) is

\[
A(\theta, p) = [\lambda \theta + (1 - \lambda)p] \tag{A.4}
\]
with \( \lambda = \frac{\alpha_x + \alpha_p}{\alpha_x + \alpha_y} \). It follows that \( \lambda' = \psi \Lambda \) where \( \Lambda = \frac{\alpha_x + \alpha_p}{\alpha_x + \alpha_y} \) and \( \lambda_p \) is given by

\[
\lambda_p = \frac{1}{(1-r)\alpha_x + \alpha_p} \left[(1-r)\alpha_x + \alpha_p - \psi(1-r)(\alpha_x + \alpha_p)\right]
\]

which results in \( \lambda_p = [1 - \psi \Lambda] \).

**A.2 Proof of Proposition 2** Let \( \delta = \sqrt{\alpha_x \alpha_y / \gamma} \). The derivative \( f'(\alpha_p) \) is

\[
f'(\alpha_p) = 2 \left( \frac{\delta}{(1-r)\alpha_x} \right)^2 \cdot \alpha_p + \frac{2\delta^2}{(1-r)\alpha_x} \tag{A.5}
\]

The solutions satisfying (\( ?? \)) are

\[
\alpha_p = \frac{1 - \frac{2\delta^2}{(1-r)\alpha_x}}{2 \left( \frac{\delta}{(1-r)\alpha_x} \right)^2} \pm \sqrt{1 - 4\delta^2 \frac{1}{(1-r)\alpha_x}} \tag{A.6}
\]

Substituting (A.6) into (A.5) yields

\[
f'(\alpha_p) = 1 \pm \sqrt{1 - 4\delta^2 \frac{1}{(1-r)\alpha_x}} \tag{A.7}
\]

Existence of the equilibria requires \( 1 - 4\delta^2 \frac{1}{(1-r)\alpha_x} > 0 \) from which the result immediately follows.

**A.3 Proof of Proposition 3** Given our definition of \( \delta \) the fixed point condition for \( \alpha_p \) is given by

\[
\alpha_p = \delta^2 \left( \frac{\alpha_p}{(1-r)\alpha_x} + 1 \right)^2 \tag{A.8}
\]

and rearranging equation (A.6) the solution for the fixed point condition can be written as

\[
2 \left( \frac{\delta}{(1-r)\alpha_x} \right)^2 \alpha_p - \left(1 - \frac{2\delta^2}{(1-r)\alpha_x}\right) = \pm \sqrt{1 - 4\delta^2 \frac{1}{(1-r)\alpha_x}}. \tag{A.9}
\]

The expression for \( \partial \alpha_p / \partial \alpha_x \) is obtained by differentiating equation (A.8) with respect to \( \alpha_x \) to get

\[
\frac{\partial \alpha_p}{\partial \alpha_x} = \frac{2 \delta^2 \left( \frac{\alpha_p}{(1-r)\alpha_x} + 1 \right) \frac{\alpha_p}{\alpha_x^2}}{\left(1 - \frac{2\delta^2}{(1-r)\alpha_x}\right) - 2 \left( \frac{\delta}{(1-r)\alpha_x} \right)^2 \alpha_p} \tag{A.10}
\]

Substituting equation (A.9) into (A.10) yields
\[
\frac{\partial \alpha_p}{\partial \alpha_x} = \begin{cases} 
\frac{\beta^2 (1 - r) \alpha p (1 - \alpha_x x + 1) \alpha_x}{\alpha p} & \text{for High Precision Equilibrium} \\
\frac{\sqrt{1 - 4 \beta^2 (1 - r) \alpha_x}}{1 - \beta^2 (1 - r) \alpha_x} & \text{for Low Precision Equilibrium}
\end{cases}
\] (A.11)

Therefore, \( \frac{\partial \alpha_p}{\partial \alpha_x} < 0 \) (resp. \( \frac{\partial \alpha_p}{\partial \alpha_x} > 0 \)) for the low (resp. high) precision equilibrium.

### A.4 Proof of Proposition 4

Totally differentiating the fixed point condition (3.8) on gets

\[
d \alpha_p = \left( \frac{\alpha_y}{\gamma} \right)^2 d \alpha_x \left( \frac{\alpha_p}{1 - r \alpha_x} + 1 \right)^2 + 2 \left( \frac{\alpha_y}{\gamma} \right) \alpha_x \left( \frac{\alpha_p}{1 - r \alpha_x} + 1 \right) \frac{1}{1 - r \alpha_x} d \alpha_p.
\] (A.12)

Rearranging terms and using (A.5) one obtains the expression in the proposition. The inequality follows by noticing that along a stable solution it is always the case that \( 0 < f'(\alpha_p) < 1 \).

### A.5 Proof of Lemma 1

Let \( U^* = U(a^*, A^*, \theta) \), be the utility function of individual firms when \( a_i \) and \( A \) are specified in the equilibrium functional forms of the second stage game. Let the generic form of the quadratic utility function \( U \) be specified as

\[
U(a, A, \sigma_a, \theta) = \begin{pmatrix} a & A \end{pmatrix} \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} \begin{pmatrix} a \\ A \end{pmatrix}.
\] (A.13)

The equilibrium strategy for the second stage game is

\[
a_i(x(n), p(n)) = \lambda(n, n) x_i + (1 - \lambda(n, n)) p(n)
\] (A.14)

\[
A(\theta, p(n)) = \lambda(n) \theta + (1 - \lambda(n)) p(n)
\] (A.15)

Now

\[
a(x(n), p(n)) = \lambda(n, n) (\theta + \alpha_x(n)^{-1/2} \varepsilon_i) + (1 - \lambda(n, n))(\theta - \alpha_p(n)^{-1/2} \varepsilon)
\]

thus

\[
a(x(n), p(n)) = \theta + \lambda(n, n) \alpha_x(n)^{-1/2} \varepsilon_i - (1 - \lambda(n, n)) \alpha_p(n)^{-1/2} \varepsilon
\] (A.16)

Similarly,

\[
A(\theta, p(n)) = \theta - (1 - \lambda(n)) \alpha_p(n)^{-1/2} \varepsilon
\] (A.17)

Then

\[
a^2 = \theta^2 + \lambda(n, n)^2 \alpha_x(n)^{-1/2} \varepsilon_i^2 + (1 - \lambda(n, n))^2 \alpha_p(n)^{-1/2} \varepsilon^2 + C(\theta, \varepsilon_i, \varepsilon)
\] (A.18)
where \( C(\theta, \varepsilon_i, \varepsilon) \) summarizes all the relevant covariance terms which become zero once we take expectations, 

\[
A^2 = \theta^2 + (1 - \lambda(n))^2 \alpha_p(n)^{-1} \varepsilon^2 + \text{cov}(\theta, \varepsilon), \quad (A.19)
\]

\[
aA = \theta^2 + (1 - \lambda(n)) \alpha_p(n)^{-1} \varepsilon^2 + C(\theta, \varepsilon_i, \varepsilon), \quad (A.20)
\]

\[
a\theta = \theta^2 + C(\theta, \varepsilon_i, \varepsilon) \quad \text{and} \quad A\theta = \theta^2 - \text{cov}(\theta, \varepsilon) \quad (A.21)
\]

Substituting into expected utility and rearranging

\[
EU^* = \left( \frac{\lambda(n, n)^2}{\alpha_x(n)} + \frac{(1 - \lambda(n, n))^2}{\alpha_p(n)} \right) M_{11} + (1 - \lambda(n, n)) \frac{1 - \lambda(n)}{\alpha_p(n)} (M_{21} + M_{12}) \nonumber \\
+ E[\theta^2 (M_{11} + M_{22} + M_{33} + (M_{21} + M_{12}) + (M_{31} + M_{13}) + (M_{32} + M_{23})) + \frac{(1 - \lambda(n))^2}{\alpha_p(n)} M_{22}] 
onumber
\]

From the above function, one can see that all the terms in the expectations are not relevant to the optimization since all the terms are out of control from the perspective of each individual. For the specific utility function we have chosen \( M_{11} = -\frac{1}{2}, M_{21} + M_{12} = r, \) therefore

\[
EU^* = -\frac{1}{2} \left[ \frac{\lambda(n, n)^2}{\alpha_x(n)} + \frac{(1 - \lambda(n, n))^2}{\alpha_p(n)} \right] - 2r(1 - \lambda(n, n)) \frac{1 - \lambda(n)}{\alpha_p(n)} + \bar{u}(n) \quad (A.22)
\]

where \( \bar{u}(n) \) is a constant term that does not depend on the choice variable \( n. \) Recall that \( \lambda(n, n) = \Lambda(n) \psi(n, n), \) then

\[
\frac{\lambda(n, n)^2}{\alpha_x(n)} + \frac{(1 - \lambda(n, n))^2}{\alpha_p(n)} = \frac{1}{\alpha_p(n)} \left[ \Lambda(n) \psi(n, n)(\lambda(n) - 2) + 1 \right] \nonumber
\]

Using the definition of \( \Lambda(n), \)

\[
\frac{1}{\alpha_p(n)} \left[ \Lambda(n) \psi(n, n)(\lambda(n) - 2) + 1 \right] = -\frac{\lambda(n, n)}{\alpha_p(n)} (1 + r(1 - \lambda(n))) + \frac{1}{\alpha_p(n)} \nonumber
\]

Hence,

\[
EU^* = \frac{1}{2} \frac{\lambda(n, n)}{\alpha_p(n)} \left[ 1 + r(1 - \lambda(n)) \right] + r(1 - \lambda(n, n)) \frac{1 - \lambda(n)}{\alpha_p(n)} + \bar{u}(n) \quad (A.23)
\]

### A.6 General Payoff Function

Consider the general payoff function \( U(a, A, \sigma_a, \theta) \) where \( U \) is quadratic with cross-partial derivatives \( U_{a\sigma_a} = U_{A\sigma_a} = U_{\theta\sigma_a} = 0 \) and \( U_{a}(a, A, 0, \theta) = 0 \) for any \((a, A, \theta),\) so that dispersion has only second order, non-strategic effects. The quadratic form is made for analytical convenience as it allows closed form solutions once incomplete information is considered. The payoff function \( U \) can be usefully considered as a second order approximation to more general concave economies. To ensure that an equilibrium under complete
information is unique and bounded it is sufficient to assumed that $U_{aa} < 0$, so that the payoff function is concave at the individual level and thus the optimal response bounded, and $-U_{aA}/U_{aa} < 1$, so that the slope of the best response with respect to the aggregate action is less than one. The two assumptions together also imply concavity at the aggregate level, which makes the efficient aggregate action well defined. In addition, for the fundamentals to matter for the individual action it must be that $U_{a\theta} \neq 0$. These restrictions on the payoff function are very mild. In fact, the payoff function $U$ can exhibit either strategic complementarity, when $U_{aA} > 0$, or strategic substitutability, when $U_{Aa} < 0$. To map the general payoff function with the properties just described into (2.1) it is sufficient to set $U_{aa} = -1$, $U_{aA} = r$, $U_{a\theta} = (1 - r) > 0$ and $U_{a\sigma} = 0$. 