Income Inequality, Political Polarization and Fiscal Policy Gridlock *

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Abstract

Over the past few decades, high income inequality and low output fluctuations have coincided with the rising domestic political polarization and policy gridlock in the United States. Motivated by the above fact, this paper analyzes the rigidity of tax policy in an economy with dynamic legislative bargaining and exogenous output fluctuations. First, by adopting the setup of bargaining with endogenous status quo, I account for the co-movement of economic inequality and policy gridlock. Second, by incorporating an economy-wide productivity shock, this model generates the feature that legislative stalemate occurs more frequently in times of reduced output volatility. Third, perhaps surprisingly, the model uncovers the property that equilibrium policy can be either ‘present-oriented’ or ‘far-sighted’. In the ‘far-sighted’ equilibrium, the policy maker is willing to sacrifice present well-being for the sake of achieving increased bargaining power in the future. Finally, I also find policy gridlock could be alleviated by introducing additional flexibility into the tax system.

Keywords: policy gridlock, legislative bargaining, endogenous status quo, income inequality, output volatility

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1 Introduction

Income inequality in the U.S. has been rising for decades. At the same time, domestic politics has become increasingly polarized, which has led to significant slowdown of the legislative process. This is particularly evident in the aftermath of the Great Recession. Households at opposing ends of the income distribution favor distinct policy responses to the negative shock. Gridlock, caused by inability of the legislature to find acceptable common ground, constantly stands in the way of effective fiscal policy responses.

Despite the broad consensus that policy gridlock prevents effective policy responses to economic recessions and threatens economic recovery, the phenomenon of legislative stalemate is largely ignored in the macroeconomic policy literature. Little work has addressed the interaction of policy behavior, legislative bargaining and economic fluctuations, leaving many important and interesting questions unanswered. For instance, how is fiscal policy set in an economy with polarized politics and legislative bargaining? What are the implications of rising income inequality on political polarization and policy gridlock? How do we explain the lack of change in tax legislation during the Great Moderation? How does policy stalemate affect the responsiveness of policy to exogenous shocks? What are the welfare implications of policy gridlock?

This paper achieves three objectives. First, by adopting a model of legislative bargaining with endogenous status quo, I account for the co-movement of economic inequality and policy gridlock. Second, by incorporating an economy-wide productivity shock, the model generates the feature that legislative stalemate occurs more frequently in times of reduced output fluctuation. Third, perhaps more surprisingly, the model uncovers two different types of equilibrium policies, one of which involves implementing an unpleasant policy in the present for the sake of increasing bargaining power in the future.

The model consists of two types of households, differentiated by their innate income generating ability. Households consume through labor income and value public goods. Each type of household is represented by one party. The policy gridlock is modeled through legislative bargaining with endogenous status quo. Parties with different welfare objectives are randomly selected to be the policy proposer. In each period, given the status quo tax rate, the incumbent makes a tax policy proposal. If the newly proposed policy is accepted by the opposing party, it is implemented immediately. Otherwise the economy remains at the status quo.

The relevance of modeling tax legislation through bargaining with endogenous status quo can be justified by two widely accepted facts. On the one hand, the majority of recent tax legislation, especially those passed after the 1970s, are designed to be permanent. On the other hand, since the 1970s, the phenomenon of divided government – a situation in which one party controls the White House and another party controls one or both houses of the U.S. Congress – has become increasingly common.

My analysis begins with analytical examples without economic uncertainty. I first characterize the one-period optimal tax policy and show its equivalence with infinite-horizon policy with fixed productivity shock realization. In line with Krehbiel (1998), I find gridlock occurs with moderate status quo policies and heterogeneous preferences. Next, I examine how the policy maker trades off between the present and the future when it is fully informed of potentially different future productivity shock realizations. I find the policy maker can be either ‘present-oriented’ or ‘far-sighted’ – in the first scenario, the policy maker simply implements policies closest to its ideal policy; in the second scenario, however, the policy maker implements policies he perceive to be unfavorable at present in return for increased bargaining power in the future.

1Two important exceptions are work by Krehbiel (1998) and Dziuda and Loeper (2010).
To gain deeper insight into how equilibrium policy behaves in an economic environment with both power fluctuation and a stochastic productivity shock, I calibrate the model to the U.S. economy and solve the full-fledged model quantitatively. I find that both rising income inequality and reduced output fluctuations raise the duration of policy gridlock and lower the probability of occurrence of new legislation. This is consistent with the stylized facts discussed in next section. Also, I numerically demonstrate the existence of both ‘present-oriented’ and ‘far-sighted’ equilibrium policies. Which type of policy will occur in equilibrium is determined jointly by the patience of the policy maker, the persistence of the shock and the current state of the economy. In terms of welfare, I find policy gridlock may improve the welfare of certain type of household by counteracting the high policy uncertainty created by political power fluctuation. I also show that once households are introduced into a system with more flexible tax policy, policy gridlock can be alleviated.

1.1 Stylized Facts

This section describes the salient facts motivating the theoretical model of this paper. I start by presenting the relationship between inequality and political polarization, as well as polarization and legislative stalemate following McCarty et al. (2006). Then I provide evidence of the coincidence of rising inequality, reduced output fluctuation and deepening inertia of U.S. tax legislation.

Fact 1: Over the past 100 years, U.S. political polarization is positively correlated with economic inequality. Figure 1 shows the simultaneous rise and drop in inequality and political polarization, which has been documented extensively in the political science literature (See McCarty et al. (2006)). In this figure, political polarization is measured by the distance between the first-dimension DW-NOMINATE score of the average Republican and Democrat in both chambers of congress. Income inequality is measured by income share of the top 1% households following Piketty and Saez (2003). In addition to the observed co-movement, also note that starting from the early 1980s substantial surge in both polarization and inequality persists to this day.

![Figure 1: Income Inequality and Political Polarization](image)

**Fact 2: The propensity of policy gridlock moves together with political polarization.**

One measure of the propensity of policy gridlock is in line with pivot theory (Krehbiel 1998, for example). In pivot theory models, policy making is driven by politicians whose support is pivotal for overcoming vetoes and
filibusters. Currently in the U.S., votes of three-fifths of the full Senate are required to overcome a filibuster. In order for Congress to pass new legislation, either the president needs to support the new legislation, or a coalition of two thirds of each chamber must vote to override a presidential veto. Following McCarty et al. (2006), propensity of gridlock is measured by the range of status quo lying between filibuster and veto override pivots according to Common Space DW-NOMINATE scores: if the president is on the left, propensity of gridlock is the preference distance between the 34th and 60th senator; if the president is on the right, it is the distance between the 41st and 67th senator. The larger the preference distance (i.e. the gridlock interval), the more difficult it is to achieve new legislation.

Figure 2: Political Polarization and Policy Gridlock

![Figure 2](image)

Figure 2 plots political polarization against the measure of propensity for policy gridlock as discussed above. As polarization rises, the propensity of policy gridlock is generally also moving upward. This implies that as political polarization increases, it becomes more difficult to pass new legislation. Figure 3 plots the number of cloture motions filed in the U.S. Senate as an alternative measure of legislative stalemate. Since the 1975 reform, the cloture motions filed has been expanding along with political polarization.

Figure 3: Political Polarization and Policy Gridlock (Cont’d)

![Figure 3](image)
Fact 3: Major tax legislation occurs less frequently in times of high economic inequality and low output fluctuations. In Figure 4 I plot U.S. postwar income inequality against occurrence of tax legislation changes as documented in Romer and Romer (2010). The shaded years correspond to periods in which at least one tax legislation change took place and the unshaded years correspond to periods when there is no legislative change. The figure shows that the period when tax legislation is produced frequently coincides with the periods when income inequality is relatively low. As income inequality grows in the 1980s, the frequency of major tax changes declines significantly.

Figure 4: Inequality and Tax Legislation Frequency

Another fact is the frequency of tax legislation changes correlates with the magnitude of output fluctuations. There is a sizable literature documenting the great moderation of U.S. business cycles and more generally, the reduction of volatility in the real GDP growth rate (See Stock and Watson (2003)). In particular, there was a sharp decline in the volatility of the U.S. GDP growth rate since the mid 1980s. In Figure 5 I plot the quarterly U.S. real GDP growth rate against the occurrence of tax legislation changes. Interestingly, parallel with the Great Moderation (as indicated by the light green region), the frequency of tax legislation falls.

Figure 5: Real GDP Growth and Tax Legislation Frequency

The facts stated above are summarized in Table 1. Before 1984 the standard deviation of the GDP growth was
approximately 4.9% while the top income share was 8.51%. During this period, the average duration of a period with no tax legislation change is less than a year. After 1984, however, the standard deviation of quarterly GDP growth declines to only around 2.5% whereas the top income share is 14.41%. The average duration of no change in tax legislation is 10 quarters, which is more than twice the duration of the period before the Great Moderation.

Table 1: Summary Statistics for Real GDP Growth, Inequality and Tax Legislation

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<thead>
<tr>
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<tbody>
<tr>
<td>Mean GDP Growth (%)</td>
<td>3.38</td>
<td>3.86</td>
<td>2.82</td>
</tr>
<tr>
<td>Standard Deviation of GDP Growth (%)</td>
<td>3.98</td>
<td>4.86</td>
<td>2.52</td>
</tr>
<tr>
<td>Mean Top 1% Income Share (%)</td>
<td>11.18</td>
<td>8.51</td>
<td>14.41</td>
</tr>
<tr>
<td>Mean Duration of No Change in Tax Legislation (quarters)</td>
<td>5.26</td>
<td>3.70</td>
<td>10.05</td>
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</table>

1.2 Related Literature

This paper brings together literature on dynamic legislative bargaining with endogenous status quo, dynamic efficiency of political power fluctuation, and optimal taxation with redistribution.

This paper is closely related to the literature adopting a dynamic legislative bargaining approach in analyzing optimal policy decisions. Battaglini and Coate (2008) and Barseghyan et al. (2013) introduce a legislature of representatives making decisions on income tax, debt and public spending by majority(or super-majority) rule. However, in their paper the dynamic linkage across policy making periods is the level of public debt and the default option in case of disagreement is exogenous.

A recent growing literature this paper relates more closely to is legislative bargaining with endogenous status quo. Baron (1996) first shows in a one-dimensional problem that policy making with endogenous status quo leads to the policy convergence towards that preferred by the median legislator. Further extensions include Kalandrakis (2004), Diermeier and Fong (2011), Dziuda and Loeper (2010), Bowen et al. (2012) and Piguillem and Riboni (2012), etc. Two papers that are particularly related to this paper are Bowen et al. (2012) and Piguillem and Riboni (2012). Bowen et al. (2012) analyze a model where two parties bargain over allocation of public good and transfer where mandatory spending is the endogenous status quo. Piguillem and Riboni (2012) looks at legislators bargaining over linear capital tax and found the distribution of wealth can lead to large variations in capital taxes, and power fluctuations generate political growth cycles.

Despite the similarity of focusing on legislative bargaining over fiscal policy and introducing power fluctuation between political parties, this paper differs from their settings in some important aspects. Note that the only form of uncertainty in the above papers is political power fluctuations. However, the actual legislative bargaining process is also affected by exogenous shocks such as business cycles. In addition to political turnover, this paper introduces an economy-wide productivity shock and examines how policy decisions are made under the changing environment. Moreover, this paper also sheds light on how introducing flexible tax system alleviates the political disagreement.

Another strand of literature that this paper builds upon focuses on the efficiency of political power fluctuations and how an incumbent government affects the policy carried out by subsequent government through certain endogenous state variable. The main argument in this literature is the political turnover could lead to policy inefficiency. For instance, Alesina and Tabellini (1990) show the incumbent overspends on public goods and this leads to over
accumulation of public debt. Azzimonti (2011) finds that disagreement and polarization between political parties leads to underinvestment in productive public capital, which slows down growth of the economy. Acemoglu et al. (2011) shows in a non-Markov setting that a smaller persistence of a party in power encourages political compromise and generates larger set of sustainable first-best allocations. In my paper, I find through numerical simulation that legislative bargaining and the incurred policy gridlock can counteract the volatile policy created by the political turnover. Therefore, under specific circumstances, certain households may benefit from policy gridlock thanks to the reduced policy uncertainty.

More generally, this paper relates to the macro and political economy literature on income inequality and redistribution. In both political economy (e.g. Meltzer and Richard (1981), Krusell and Rios-Rull (1999), Persson and Tabellini (1994), Hassler et al. (2007) ) and macroeconomic literature(e.g. Werning (2007), Bhandari et al. (2013)) there are papers addressing optimal redistributive policy. While the former focuses on politico-economic aspect of income inequality and typically assumes away aggregate uncertainty, the latter allows for aggregate uncertainty yet typically derives policy based on the benevolent social planner assumption. This paper seeks to bring together the above two aspects and looks at how fiscal policy behaves when there is both dynamic legislative bargaining and a changing aggregate environment. I find that not only does equilibrium policy and occurrence of policy gridlock depend on the aggregate state of the economy, but also that the magnitude of the shock determines whether or not temporary discretionary fiscal policy is implemented.

The rest of the paper is organized as follows. Section 2 outlines the setup of the model and defines the equilibrium. Section 3 provides analytical examples to convey the basic intuition. Section 4 calibrates the model and presents the main results. Section 5 discusses the further extension of the model. Section 6 concludes.

2 The Model

This section begins by describing the model’s economic environment and characterizing the competitive equilibrium. Then I specify the political system and define the Markov perfect equilibrium.

2.1 The Economy

2.1.1 Households

Time is discrete and infinite horizon, indexed by $t = 0, 1, 2, \cdots$. The economy is populated by two types of infinitely lived households, indexed by $i = \{1, 2\}$. The size of each type of household is normalized to 1. Households consume consumption good $c^i$, supply labor $l^i$ and benefit from public good $g$. Each household’s periodic utility is

$$u(c^i_t, l^i_t, g_t) = \frac{1}{1-\gamma} \left( c^i_t - \frac{l^i_t}{1+\nu} \right)^{1-\gamma} + A g_t^{1-\xi}$$

(2.1)

where $\gamma > 0, \nu > 0, A > 0$ and $0 \leq \xi < 1$. By assumption, $u(c^i, l^i, g)$ is increasing in $c^i$ and $g$ and decreasing in $l^i$.

Household of type $i$ supplies $l^i_t$ units of labor and produces $\epsilon_i i^i_t l^i_t$ units of output, where $\theta_i > 0$ is a non-negative scalar capturing the innate income generating ability of each household. Without loss of generality, I assume
\( \bar{\theta}_1 > \bar{\theta}_2 \). Random variable \( \epsilon_t > 0 \) is an economy-wide productivity shock. It follows log-normal distribution where

\[
\begin{align*}
\log \epsilon_t &= \rho \log \epsilon_{t-1} + u_t, \quad u_t \sim N(0, \sigma^2) \\
\end{align*}
\] (2.2)

Assume there is no borrowing or lending in this economy. Given an arbitrary linear tax \( \tau_t \) imposed on labor income, household \( i \)'s budget constraint is

\[
c_i^t = (1 - \tau_t) \epsilon_t \bar{\theta}_i l_i^t \quad \forall t \\
\] (2.3)

2.1.2 Government

The government holds balanced budget each period. It finances public good \( g_t \) through linear taxation \( \tau_t \) on households’ labor income

\[
g_t = \tau_t (\epsilon_t \bar{\theta}_1 l_1^t + \epsilon_t \bar{\theta}_2 l_2^t) \\
\] (2.4)

2.1.3 Competitive Equilibrium

**Definition 1.** A competitive equilibrium is an allocation \( \{c_i^t, l_i^t\}_{t=0}^{\infty} \) for \( i = \{1, 2\} \), a price system \( \{w_i^t = \epsilon_t \bar{\theta}_i\}_{t=0}^{\infty} \) and a sequence of fiscal policies \( \{\tau_t, g_t\}_{t=0}^{\infty} \) such that: 1. Given the price system and the government’s fiscal policy, the allocation for each household maximizes its expected discounted life-time utility subject to its budget constraint; 2. Given the allocation and the price system, the government holds balanced budget each period; 3. Goods market clears.

Since households are assumed not to have access to any borrowing or lending facilities, their optimal consumption and labor supply are determined statically. Maximizing periodic utility of household \( i \) subject to its budget constraint as in Equation 2.3, we obtain the optimal consumption \( C_i^*(\tau_t, \epsilon_t) \) and labor supply \( L_i^*(\tau_t, \epsilon_t) \):

\[
\begin{align*}
C_i^*(\tau_t, \epsilon_t) &= \left[ (1 - \tau_t) \bar{\theta}_i \epsilon_t \right]^{\frac{1+\nu}{\nu}} \\
L_i^*(\tau_t, \epsilon_t) &= \left[ (1 - \tau_t) \bar{\theta}_i \epsilon_t \right]^{\frac{1}{\nu}} \\
\end{align*}
\] (2.5, 2.6)

Using government budget constraint Equation 2.4 and households’ optimal labor supply Equation 2.6, the equilibrium level of public good can also be expressed as a function of tax rate and the aggregate productivity shock

\[
\begin{align*}
G_i^*(\tau_t, \epsilon_t) &= \tau_t \epsilon_t \bar{\theta}_1 \bar{\theta}_2 \left[ (1 - \tau_t) \bar{\theta}_1 \epsilon_t \right]^{\frac{1}{\nu}} + \tau_t \epsilon_t \bar{\theta}_2 \left[ (1 - \tau_t) \bar{\theta}_2 \epsilon_t \right]^{\frac{1}{\nu}} \\
&= \tau_t \epsilon_t \bar{\theta}_1 \epsilon_t \left[ (1 - \tau_t) \bar{\theta}_1 \epsilon_t \right]^{\frac{1}{\nu}} + \tau_t \epsilon_t \bar{\theta}_2 \epsilon_t \left[ (1 - \tau_t) \bar{\theta}_2 \epsilon_t \right]^{\frac{1}{\nu}} \\
\end{align*}
\] (2.7)

Plugging \( C_i^*(\tau_t, \epsilon_t), L_i^*(\tau_t, \epsilon_t) \) and \( G_i^*(\tau_t, \epsilon_t) \) into household \( i \)'s periodic utility function Equation 2.1, we can write
household’s indirect utility as a function of $\tau_t$ and $\epsilon_t$

$$u_i^s(\tau_t, \epsilon_t) = \frac{1}{1 - \gamma} \left[ C_i^s(\tau_t, \epsilon_t) - \frac{L_i^s(\tau_t, \epsilon_t)}{1 + \nu} \right]^{1 - \gamma} + A \frac{G_i^s(\tau_t, \epsilon_t)}{1 - \xi}$$

$$= \frac{\nu^{1 - \gamma}}{(1 - \gamma)(1 + \nu)^{1 - \gamma}} \left[ (1 - \tau_t)\theta_1 \epsilon_t \right]^{1 + \nu(1 - \gamma)} + \frac{A}{1 - \xi} \left\{ \tau_t \theta_1 \epsilon_t \left[ (1 - \tau_t)\theta_1 \epsilon_t \right]^\frac{\nu}{\gamma} + \tau_t \theta_2 \epsilon_t \left[ (1 - \tau_t)\theta_2 \epsilon_t \right]^\frac{\nu}{\gamma} \right\}^{1 - \xi} \tag{2.8}$$

$$\frac{\nu^{1 - \gamma}}{(1 - \gamma)(1 + \nu)^{1 - \gamma}} \left[ (1 - \tau_t)\theta_1 \epsilon_t \right]^{1 + \nu(1 - \gamma)} + \frac{A}{1 - \xi} \left\{ \tau_t \theta_1 \epsilon_t \left[ (1 - \tau_t)\theta_1 \epsilon_t \right]^\frac{\nu}{\gamma} + \tau_t \theta_2 \epsilon_t \left[ (1 - \tau_t)\theta_2 \epsilon_t \right]^\frac{\nu}{\gamma} \right\}^{1 - \xi} \tag{2.9}$$

### 2.2 Political System

The political system of the economy consists of two parties labeled by $i = \{1, 2\}$, each seeking to maximize a weighted sum of expected discounted life-time utility of the households

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \alpha_i u(c_t^i, l_t^i, g_t) + (1 - \alpha_i) u(c_t^{-i}, l_t^{-i}, g_t) \right] \tag{2.10}$$

where $\alpha_i$ is the weight party $i$ places on its own constituents. $(c_t^i, l_t^i)$ denotes the consumption and labor supply of the constituents of party $i$, $(c_t^{-i}, l_t^{-i})$ denotes the consumption and labor supply of the constituents of the opponent of party $i$.

At the beginning of each period, one party is randomly assigned to be the incumbent. The process of political turnover is governed by a 2-state Markov process: with probability $p$, the incumbent stays in power in the next period. With probability $1 - p$, the incumbent loses power and the opponent takes power.

Each period, tax policy is determined through bargaining between the incumbent and opponent. Let $s_t$ denote the status quo tax rate in period $t$, i.e., the tax rate implemented in period $t - 1$. After realization of the identity of the incumbent $i_t$, given the status quo $s_t$ and economy-wide productivity shock $\epsilon_t$, the incumbent makes a take-it-or-leave-it offer $\tau_t$. If the opponent agrees with the proposed policy $\tau_t$, it is implemented immediately as the tax policy for period $t$; if the opponent fails to agree, period $t$ tax policy stays at the status quo, i.e., $\tau_t = s_t$.

The timing of events is as follows. At the beginning of each period, upon realization of productivity shock and identity of incumbent party, the tax policy is determined through the legislative bargaining process as described above. Once tax policy is implemented, households consume, work and benefit from public good.

### 2.3 Markov Perfect Equilibrium

This paper focuses on stationary Markov perfect equilibria. Strategies of each agent are assumed to be dependent on current state only and each agent is assumed to be able to correctly forecast the future incumbent’s tax policy. At time $t$, the state variables are the status quo tax rate $s_t$, the economy-wide shock $\epsilon_t$, and the identity of the incumbent $i_t$. Given $(s_t, \epsilon_t, i_t)$, the strategy profile of party $i$ can be summarized by $\sigma_i^t = \{T_i(s_t, \epsilon_t, i); A_i(T_{-i}(s_t, \epsilon_t, -i); s_t, \epsilon_t, -i)\}$. If party $i$ is the incumbent, i.e. $i_t = i$, then party $i$ proposes tax policy $T_i(s_t, \epsilon_t, i)$. If party $i$ is the opponent, i.e. $i_t = -i$, then party $i$ determines to accept $(A_i(T_{-i}(s_t, \epsilon_t, -i); s_t, \epsilon_t, -i) = 1)$ or reject $(A_i(T_{-i}(s_t, \epsilon_t, -i); s_t, \epsilon_t, -i) = 0)$ the incumbent’s proposal.

For each $(\sigma_i^t, \sigma_i^{-i})$, there are two payoff functions associated with each party. $V_i(s_t, \epsilon_t, i; \sigma_i^t, \sigma_i^{-i})$ is the payoff for party $i$ when it is the incumbent, and $W_i(s_t, \epsilon_t, -i; \sigma_i^t, \sigma_i^{-i})$ is the payoff for party $i$ when it is the opponent.
Suppose party \( i \) is the incumbent. For an arbitrary tax policy \( \tau_t \) proposed by party \( i \), opponent \( -i \) accepts the proposal if and only if the welfare obtained from the proposed policy \( \tau_t \) is at least as high as staying with the status quo \( s_t \)

\[
W_{-i}(s_t, \epsilon_t, i; \tau_t, 1) \geq W_{-i}(s_t, \epsilon_t, i; \tau_t, 0)
\]  

(2.11)

Given the best response of the opponent, now consider the incumbent’s decision. The incumbent chooses the policy \( \tau_t \) such that it maximizes incumbent’s welfare while taking into consideration the opposing party’s strategy:

\[
V_i(s_t, \epsilon_t, i; \sigma_t^i, \sigma_t^{-i}) = \max_{\tau_t} \alpha_i u(C_i^i((\tau_t, \epsilon_t), L_i^i((\tau_t, \epsilon_t))) + (1 - \alpha_i)u(C_{-i}^i((\tau_t, \epsilon_t), L_{-i}^i((\tau_t, \epsilon_t))) + AG_i^i(\tau_t, \epsilon_t)
\]

\[
+ \beta [pE[V_i(s_{t+1}, \epsilon_{t+1}, i; \sigma_{t+1}^i, \sigma_{t+1}^{-i})] + (1 - p)E[V_i(s_{t+1}, \epsilon_{t+1}, -i; \sigma_{t+1}^i, \sigma_{t+1}^{-i})]]
\]

s.t.

\[
A_{-i}(T_i(s_t, \epsilon_t, i); s_t, \epsilon_t, i) = \begin{cases} 
1 & \text{if } W_{-i}(s_t, \epsilon_t, i; \tau_t, 1) \geq W_{-i}(s_t, \epsilon_t, i; \tau_t, 0) \\
0 & \text{if } W_{-i}(s_t, \epsilon_t, i; \tau_t, 1) < W_{-i}(s_t, \epsilon_t, i; \tau_t, 0) 
\end{cases}
\]

\[
s_{t+1} = \tau_t
\]

\[
\tau_t \in [0, 1]
\]

where

\[
W_{-i}(s_t, \epsilon_t, i; \tau_t, 1) = \alpha_{-i} u(C_{-i}^i(\tau_t, \epsilon_t), L_{-i}^i(\tau_t, \epsilon_t)) + (1 - \alpha_{-i})u(C_i^i(\tau_t, \epsilon_t), L_i^i(\tau_t, \epsilon_t)) + AG_{-i}^i(\tau_t, \epsilon_t) + \\
\beta [pE[W_{-i}(s_{t+1}, \epsilon_{t+1}, i; \sigma_{t+1}^i, \sigma_{t+1}^{-i})] + (1 - p)E[V_{-i}(s_{t+1}, \epsilon_{t+1}, -i; \sigma_{t+1}^i, \sigma_{t+1}^{-i})]]
\]

Following Bowen, Chen and Eraslan (2013), this paper assumes the opponent accepts any proposal if it is indifferent as a function of the state variables only and suppressing \( V(\cdot) \) as a function of the state variables and incumbent’s policy, I define the Markov perfect equilibrium as follows:

**Definition 2.** A Markov Perfect Equilibrium with Legislative Bargaining is an allocation \( \{C_i^j(s, \epsilon, i)\}_{j=\{1,2\}} \), \( \{L_i^j(s, \epsilon, i)\}_{j=\{1,2\}} \), an incumbent strategy \( \{T_i(s, \epsilon, i), G_i(s, \epsilon, i)\}_{i=\{1,2\}} \), an opponent strategy \( \{A_{-i}(T_i(s, \epsilon, i); s, \epsilon, i)\}_{i=\{1,2\}} \), and value functions \( \{V_i(s, \epsilon, i), W_{-i}(s, \epsilon, i; \cdot)\}_{i=\{1,2\}} \) such that:

1. Given incumbent’s policy \( \{T_i(s, \epsilon, i), G_i(s, \epsilon, i)\} \), the allocation for each consumer maximizes their utility;
2. Given incumbent’s policy, the opponent accepts the proposal if and only if

\[
W_{-i}(s, \epsilon, i; \tau) \geq W_{-i}(s, \epsilon, i; s)
\]  

(2.12)
Given opponent’s best response, the incumbent i’s policy solves the following problem

\[ V_i(s, \epsilon, i) = \max_{\tau} \alpha_i u(C^{i*}(\tau, \epsilon), L^{i*}(\tau, \epsilon)) + (1 - \alpha_i)u(C^{-i*}(\tau, \epsilon), L^{-i*}(\tau, \epsilon)) + AG^*(\tau, \epsilon) + \beta [pEV_i(s', \epsilon', i) + (1 - p)EW_i(s', \epsilon', -i)] \]

s.t.

\[ W_{-i}(s, \epsilon, i; \tau) \geq W_{-i}(s, \epsilon, i; s) \]
\[ s' = \tau \]
\[ \tau \in [0, 1] \]

where for arbitrary \( \tau \),

\[ W_{-i}(s, \epsilon, i; \tau) = \alpha_{-i} u(C^{-i*}(\tau, \epsilon), L^{-i*}(\tau, \epsilon)) + (1 - \alpha_{-i})u(C^{i*}(\tau, \epsilon), L^{i*}(\tau, \epsilon)) + AG^*(\tau, \epsilon) + \beta [pEW_{-i}(s', \epsilon', i; T_i(s', \epsilon', i)) + (1 - p)EW_{-i}(s', \epsilon', -i; T_{-i}(s', \epsilon', -i))] \]

and in equilibrium \( C^{j*}(T_i(s, \epsilon, i), \epsilon) = C^j(s, \epsilon, i), \ L^{i*}(T_i(s, \epsilon, i), \epsilon) = L^i(s, \epsilon, i), \ G^*(T_i(s, \epsilon, i), \epsilon) = G(s, \epsilon, i) \) for \( i, j = \{1, 2\} \).

Now let’s define the policy gridlock following above notation. First, for arbitrary policy \( \tau \in [0, 1] \), the payoff of the incumbent is summarized by \( V_i(s, \epsilon, i; \tau) \) where

\[ V_i(s, \epsilon, i; \tau) = \alpha_i u(C^{i*}(\tau, \epsilon), L^{i*}(\tau, \epsilon)) + (1 - \alpha_i)u(C^{-i*}(\tau, \epsilon), L^{-i*}(\tau, \epsilon)) + AG^*(\tau, \epsilon) + \beta [pEV_i(\tau, \epsilon', i; T_i(\tau, \epsilon', i)) + (1 - p)EW_i(\tau, \epsilon', -i; T_{-i}(\tau, \epsilon', -i))] \]

**Definition 3.** Given status quo tax rate \( s \), aggregate state of the economy \( \epsilon \) and identity of the incumbent party \( i \), the tax policy is gridlocked if there is no \( \tau \) other than \( s \) such that

1) both parties under policy \( \tau \) are at least as well-off as under status quo policy \( s \), i.e.

\[ V_i(s, \epsilon, i; \tau) \geq V_i(s, \epsilon, i; s) \]

and

\[ W_{-i}(s, \epsilon, i; \tau) \geq W_{-i}(s, \epsilon, i; s) \]

2) at least one party is strictly better-off under policy \( \tau \) than under status quo policy \( s \), i.e.

\[ V_i(s, \epsilon, i; \tau) > V_i(s, \epsilon, i; s) \]

or

\[ W_{-i}(s, \epsilon, i; \tau) > W_{-i}(s, \epsilon, i; s) \]
3 Mechanism in Simple Analytic Examples

In this section, I present the main idea of this paper through simplified analytical examples. I start with legislative bargaining problem in one-period economy and illustrate the mechanism of policy gridlock. Comparative static analysis is performed to examine the role of income inequality and output volatility. Then I extend the one-period economy to infinite horizon under two different productivity shock realization assumptions and show the existence of equilibrium policy that sacrifices current well being for purpose of large welfare gain in the future.

3.1 One-Period Benchmark

Consider a one-period economy with household’s optimization problem and government’s budget same as previous section. Let the aggregate productivity shock \(\epsilon\) follow log normal distribution with \(E(\log \epsilon) = 0\) and \(\text{var}(\log \epsilon) = \sigma^2\). Suppose there is no political turnover. The rich(with productivity \(\bar{\theta}_1\)) is the incumbent and the poor(with productivity \(\bar{\theta}_2 < \bar{\theta}_1\)) is the opponent. Given status quo tax rate \(\tau_0\), the rich makes a take-it-or-leave-it offer \(\tau\). If the poor accepts the proposed policy, \(\tau\) is implemented immediately. If the poor rejects the proposed policy, \(\tau_0\) is implemented instead. Assume \(\alpha_1 = \alpha_2 = 1\) so each party only cares welfare of its own constituents. To simplify analytical derivation, I assume \(\gamma = 0.5, \nu = 1\) and \(\xi = 0\) in this section.

Following households’ indirect utility as in Equation 2.8 household \(i\)’s indirect utility as a function of tax policy \(\tau\) and productivity shock \(\epsilon\) is given by

\[
u_i(\tau, \epsilon) = \sqrt{2}(1 - \tau)\theta_i \epsilon + A\tau(1 - \tau)(\theta_1^2 + \theta_2^2)\epsilon^2
\]  

(3.1)

For arbitrary tax policy \(\tau\) proposed by the rich, the poor accepts the policy proposal if and only if the indirect utility of the poor obtained from \(\tau\) is at least as high as that obtained from the status quo tax policy \(\tau_0\), i.e.

\[
\sqrt{2}(1 - \tau_0)\theta_2 \epsilon + A\tau_0(1 - \tau_0)(\theta_1^2 + \theta_2^2)\epsilon^2 \geq \sqrt{2}(1 - \tau)\theta_2 \epsilon + A\tau(1 - \tau)(\theta_1^2 + \theta_2^2)\epsilon^2
\]  

(3.2)

The rich chooses the tax policy \(\tau\) to maximize the welfare of its constituents taking into consideration the best response of the poor. Assuming the optimal tax rate is non-negative and no greater than 1, the rich’s optimization problem is summarized by

\[
\max_{\tau} \sqrt{2}(1 - \tau)\theta_1 \epsilon + A\tau(1 - \tau)(\theta_1^2 + \theta_2^2)\epsilon^2
\]  

s.t.

\[
\sqrt{2}(1 - \tau)\theta_2 \epsilon + A\tau(1 - \tau)(\theta_1^2 + \theta_2^2)\epsilon^2 \geq \sqrt{2}(1 - \tau_0)\theta_2 \epsilon + A\tau_0(1 - \tau_0)(\theta_1^2 + \theta_2^2)\epsilon^2
\]

\[
0 \leq \tau \leq 1
\]

**Tax Policy without Bargaining** First consider what happens when there is no legislative bargaining between the rich and the poor. In this case, the rich simply maximizes its indirect utility subject to the constraint \(\tau \in [0, 1]\).
The ideal tax policy for the rich is
\[
\tau_1^* = \frac{1}{2} \left[ 1 - \frac{\sqrt{2}\tilde{\theta}_1}{A(\theta_1^2 + \theta_2^2)\epsilon} \right]
\]
(3.4)

Similarly, if the poor is in power and the rich is the opponent, the ideal tax policy for the poor is
\[
\tau_2^* = \frac{1}{2} \left[ 1 - \frac{\sqrt{2}\tilde{\theta}_2}{A(\theta_1^2 + \theta_2^2)\epsilon} \right]
\]
(3.5)

Using assumption \( \tilde{\theta}_1 > \tilde{\theta}_2, A > 0 \) and \( \epsilon > 0 \), we can obtain from Equation 3.4 and Equation 3.5 that for arbitrary shock \( \epsilon \) and status quo \( \tau_0 \), the ideal tax level of the rich is lower than the poor:
\[
\tau_1^* < \tau_2^* < \frac{1}{2}
\]

In particular, the ideal tax rates are independent of the status quo \( \tau_0 \).

**Tax Policy with Bargaining** Next consider the tax policy proposed by the rich when it bargains with the poor for given status quo \( \tau_0 \) and aggregate productivity shock \( \epsilon \). Solving optimization problem as defined in Equation 3.3, we can obtain the rich’s tax policy \( T(\tau_0, \epsilon) \) as follows:

**Lemma 1.** Assume there is no political turnover and the rich is the incumbent. For given status quo \( \tau_0 \) and productivity realization \( \epsilon \), the equilibrium tax policy for the one period economy with legislative bargaining is
\[
T(\tau_0, \epsilon) = \begin{cases} 
\tau_1^* & \text{if } \tau_0 \leq \tau_1^* \\
\tau_0 & \text{if } \tau_1^* < \tau_0 \leq \tau_2^* \\
2\tau_2^* - \tau_0 & \text{if } \tau_2^* < \tau_0 < 2\tau_2^* - \tau_1^* \\
\tau_1^* & \text{if } \tau_0 \geq 2\tau_2^* - \tau_1^* 
\end{cases}
\]
(3.6)

**Proof.** See Appendix.

Note the tax policy proposed by the rich with bargaining is dependent on the status quo \( \tau_0 \). When the status quo tax rate \( \tau_0 \) is below the ideal tax rate of both the rich and the poor, the two parties agree to raise the tax rate. Since the rich is making a take-it-or-leave-it offer and has the full bargaining power, the rich simply sets the tax policy at its ideal level, i.e. \( T(\tau_0, \epsilon) = \tau_1^* \). As the status quo \( \tau_0 \) moves between the ideal of the rich and the poor, the rich prefers lower tax rate while the poor prefers higher tax rate. Two parties fail to reach any agreement on policy adjustment so the equilibrium tax policy simply stays at the status quo, i.e. \( T(\tau_0, \epsilon) = \tau_0 \). When status quo \( \tau_0 \) goes above the ideal tax rate of the poor, both parties agree to lower the tax rate. As the status quo gradually moves further above the ideal tax level of the poor, the rich is regaining bargaining power and the equilibrium tax rate falls. When the status quo is sufficiently high such that the rich regains full bargaining power, it sets the tax policy at its ideal level, i.e. \( T(\tau_0, \epsilon) = \tau_1^* \).

**Corollary 1.** *(identical policy gridlock zone)* Let \( i \) denote the identity of the incumbent and \( T(\tau_0, \epsilon, i) \) denote equilibrium tax policy by incumbent \( i \) in the one period economy. For a given \( \tau_0 \), if \( T(\tau_0, \epsilon, 1) = \tau_0 \), then \( T(\tau_0, \epsilon, 2) = \tau_0 \).
Intuitively, policy gridlock occurs if two parties want to shift tax policy in opposite directions. Despite the identity of the incumbent, disagreement occurs only if the status quo tax rate is between the ideal tax rate of the rich and the poor. This is independent of the identity of the incumbent. Thus the gridlock zones of the rich and the poor coincide.

3.2 Role of Inequality and Output Volatility

In this section, I perform comparative statics to investigate the role of income inequality and output volatility on behavior of policy gridlock. Later in this paper, parallel results for the full-fledged model will be shown through quantitative exercise.

3.2.1 Role of Income Inequality

**Corollary 2. (role of income inequality)** For given $\epsilon$, both gridlock region and compromise region expand as income inequality (as measured by income share of the rich) rises.

**Proof.** See Appendix.

This result is quite intuitive. As income inequality rises, the ideal tax policies of the rich and the poor shift away from each other. These increasingly polarized ideal policies naturally generate a larger region of gridlock and compromise.

More specifically, in order to examine the role of income inequality, I vary the income share of the two types of households through introducing a mean preserving spread $\delta (0 < \delta < min\{\bar{\theta}_1, \bar{\theta}_2\})$ to individual productivity levels, i.e. $\bar{\theta}_1 = \bar{\theta} + \delta$ and $\bar{\theta}_2 = \bar{\theta} - \delta$. This transformation keeps average productivity of the economy constant.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig6.png}
\caption{Equilibrium Tax Policy: Rich}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig7.png}
\caption{Rich Vs Poor}
\end{figure}
while increases the income share of the rich. Also note that under current households’ preference specification, the income share as a measure of inequality is \textit{independent} of the aggregate productivity shock \( \epsilon \).

Recall from Equation 3.6 the equilibrium tax policy with bargaining is essentially characterized by the ideal tax policy of both parties (\( \tau^*_1 \) and \( \tau^*_2 \)) and the status quo \( \tau_0 \). After raising the rich income share through introduction of \( \delta \), the optimal policy as well as the gridlock and compromise zone are all a function of \( \delta \), and hence a function of rich income share.

\[
T(\tau_0, \epsilon; \delta) = \begin{cases} 
\tau_1^*(\delta) & \text{if } \tau_0 \leq \tau_1^*(\delta) \\
\tau_0 & \text{if } \tau_1^*(\delta) < \tau_0 \leq \tau_2^*(\delta) - \tau_1^*(\delta) \\
2\tau_2^*(\delta) - \tau_0 & \text{if } \tau_2^*(\delta) < \tau_0 < 2\tau_2^*(\delta) - \tau_1^*(\delta) \\
\tau_1^*(\delta) & \text{if } \tau_0 \geq 2\tau_2^*(\delta) - \tau_1^*(\delta)
\end{cases}
\]  
(3.7)

where \( \tau_1^*(\delta) = \frac{1}{2} \left[ 1 - \frac{\sqrt{\tau_1(\theta_1 + \delta)}}{A((\theta_1 + \delta)^2 + (\theta_2 - \delta)^2)\epsilon} \right] \) and \( \tau_2^*(\delta) = \frac{1}{2} \left[ 1 - \frac{\sqrt{\tau_2(\theta_2 - \delta)}}{A((\theta_1 + \delta)^2 + (\theta_2 - \delta)^2)\epsilon} \right] \). Differentiating \( \tau_1^*(\delta) \) and \( \tau_2^*(\delta) \) with respect to \( \delta \), we see when \( \delta \) is relatively small, as \( \delta \) rises \( \tau_1^*(\delta) \) is decreasing while \( \tau_2^*(\delta) \) is increasing. This implies as inequality rises, both gridlock zone and compromise zone are expanding. As \( \delta \) grows sufficiently large, \( \tau_1^*(\delta) \) starts to increase with \( \delta \). This is because when the productivity of the poor is too low, the rich is willing to sacrifice part of its income for purpose of benefiting from government public goods provision.

\footnotetext{Using the optimal consumption and labor decision from the household, we can obtain the income share of the rich after introduction of mean preserving spread \( \delta \): \[ \frac{g_1}{g_1 + g_2} = \frac{\left(1 - \tau_1^*\theta_1\right)^2}{\left(1 - \tau_2^*\theta_2\right)^2 + \left(1 - \tau_2^*\theta_2\right)^2} = \frac{\theta_1^2}{\theta_1^2 + \theta_2^2} = \frac{(\theta_1 + \delta)^2}{(\theta_1 + \delta)^2 + (\theta_2 - \delta)^2}. \] It is not difficult to show \( \frac{g_1}{g_1 + g_2} \) is independent of \( \epsilon \) and it is an increasing function of \( \delta \) when \( \delta \in (0, \min(\theta_1, \theta_2)) \).}
3.2.2 Role of Output Volatility

To examine the role of output volatility $\sigma$, I first characterize the probability of policy gridlock for arbitrary status quo $\tau_0 \in [0,1]$ as follows:

$$Pr(T(\tau_0, \epsilon) = \tau_0) = \begin{cases} 
\Phi(\epsilon_1^*) - \Phi(\epsilon_2^*) & \text{if } \tau_0 < \frac{1}{2} \\
0 & \text{if } \tau_0 \geq \frac{1}{2}
\end{cases}$$

where $\epsilon_1^* = \frac{1}{\sigma} \log \left[ \sqrt{2A(\bar{\theta}_1(\bar{\theta}_1^2 + \bar{\theta}_2^2)(\frac{1}{2} - \tau_0))} \right]$, $\epsilon_2^* = \frac{1}{\sigma} \log \left[ \sqrt{2A(\bar{\theta}_2(\bar{\theta}_1^2 + \bar{\theta}_2^2)(\frac{1}{2} - \tau_0))} \right]$ and $\Phi(\cdot)$ denotes the cumulative density function of standard normal distribution. From above expression, we see that when $\tau_0$ is sufficiently large, policy gridlock will not occur irrespective of the shock realization. When $\tau_0$ is relatively small, however, the gridlock is likely to occur. In particular, we can investigate the relationship between the probability of gridlock and the output volatility by looking at the partial derivative of the gridlock probability with respect to $\sigma$.

**Corollary 3 (role of output volatility).** Assume $\phi(\epsilon_1^*) > \phi(\epsilon_2^*)$ with $\phi(\cdot)$ denoting the probability density function of standard normal distribution. For arbitrary status quo tax rate $\tau_0$, the probability of policy gridlock is a non-increasing function of output volatility $\sigma$. In particular, if $\tau_0 < \frac{1}{2}$, the probability of policy gridlock is strictly decreasing.

**Proof.** See Appendix.

All else equal, the rise in output volatility increases the chance the economy experiences large changes in aggregate productivity. When sufficiently large changes in aggregate productivity occur, the ideal level of both the rich and the poor shift significantly. This is likely to drive the economy away from the status quo tax rate.

3.3 Infinite-Horizon Economy and ‘Far-Sighted’ Equilibrium Policy

Now let’s consider what happens when the one-period economy is extended to infinite-horizon. Two scenarios will be considered: one with constant productivity shock realization throughout time and the other with productivity shock realization differ only between the first and rest periods.

3.3.1 Infinite-Horizon Problem with Constant Productivity Shock Realization

Consider an infinite-horizon economy with the same setup as the one-period economy. Assume there is no political turnover and the rich (with productivity $\bar{\theta}_1$) is the incumbent throughout time. Furthermore, assume the productivity shock realization stays constant forever (i.e. $\epsilon_t \equiv \epsilon_0, \forall t$). Each period given status quo tax rate $\tau_{t-1}$, the rich makes a take-it-or-leave-it offer $\tau_t$. If the poor accepts the proposed policy, $\tau_t$ is implemented immediately. If the poor rejects the proposed policy, $\tau_{t-1}$ is implemented instead.

**Proposition 1.** Assume there is no political turnover and the productivity shock stays constant (i.e. $\epsilon_t \equiv \epsilon, \forall t$). The equilibrium tax policy for the infinite horizon problem coincides with the equilibrium tax policy in the one-period...
economy. If the rich is in power, then

\[ T(\tau_{t-1}, \epsilon) = \begin{cases} 
\tau_1^* & \text{if } \tau_{t-1} \leq \tau_1^* \\
\tau_{t-1} & \text{if } \tau_1^* < \tau_{t-1} \leq \tau_2^* \\
2\tau_2^* - \tau_{t-1} & \text{if } \tau_2^* < \tau_{t-1} < 2\tau_2^* - \tau_1^* \\
\tau_2^* & \text{if } \tau_{t-1} \geq 2\tau_2^* - \tau_1^* 
\end{cases} \]

where \( \tau_1^* = \frac{1}{2} \left[ 1 - \frac{\sqrt{2\theta_1}}{A(\theta_1^2 + \theta_2^2)\epsilon} \right] \) and \( \tau_2^* = \frac{1}{2} \left[ 1 - \frac{\sqrt{2\theta_2}}{A(\theta_1^2 + \theta_2^2)\epsilon} \right] \)

Furthermore, the steady state of the economy is \([\tau_1^*, \tau_2^*]\).

**Proof.** See Appendix.

According to above proposition, the rich will implement the same tax policy as above forever. To understand this results, first notice that despite the status quo, the equilibrium tax policy always falls into the gridlock region \([\tau_1^*, \tau_2^*]\). Once the economy reaches gridlock region, the equilibrium policy is to stay at the status quo. This implies once the policy enters the gridlock region, it will stay there from then on. i.e \( T(\tau_{t-1}, \epsilon) \in [\tau_1^*, \tau_2^*] \) implies \( T(T(\tau_{t-1}, \epsilon), \epsilon) = T(\tau_{t-1}, \epsilon) \). Essentially, in this scenario the infinite horizon problem boils down to repetition of one period problem.

**Corollary 4 (closest-to-ideal-policy rule).** Assume there is no political turnover and the productivity shock stays constant (i.e. \( \epsilon_t \equiv \epsilon \forall t \)). The equilibrium tax policy is the policy that is closest to proposer’s ideal tax level within the constraint set.

More specifically, suppose the rich is the incumbent. Denote the constraint set for arbitrary status quo \( \tau_{t-1} \in [0, 1] \) as \( C(\tau_{t-1}, \epsilon) \). Let the ideal tax policy of the rich be \( \tau_1^* \) and the equilibrium tax policy by the incumbent be \( T(\tau_{t-1}, \epsilon) \).

Then for all \( \tau_{t-1} \in [0, 1] \), there does not exist any \( \tau \in C(\tau_{t-1}, \epsilon) \) such that \( \| \tau - \tau_1^* \| < \| T(\tau_{t-1}, \epsilon) - \tau_1^* \| \). In addition, when the incentive compatibility constraint for the opponent binds, \( T(\tau_{t-1}, \epsilon) = \min C(\tau_{t-1}, \epsilon) \).

**Proof.** See Appendix.

This result is quite intuitive. Suppose the rich is the incumbent. When the incentive compatibility constraint of the poor is not binding, the rich always implements its ideal tax policy, i.e. \( \tau_1^* \). When the constraint binds, for example, \( \tau_1^* < \tau_{t-1} < \tau_2^* \), the rich stays with status quo according to equilibrium tax policy function \( T(\tau_{t-1}, \epsilon) \). Note in this scenario, the constraint set is \([\tau_{t-1}, 2\tau_2^* - \tau_{t-1}]\). Since the ideal tax level \( \tau_1^* \) is not within the constraint set, \( \tau_{t-1} \) as the tax level closest to the ideal \( \tau_1^* \) now becomes the policy that maximizes the objective of the rich.

### 3.3.2 ‘Present-Oriented’ Vs ‘Far-Sighted’ Equilibrium Policy

To make the problem more interesting, let’s consider now an economy same as above but with aggregate productivity shock \( \epsilon_t \equiv 1 \) at \( t = 1 \) and \( \epsilon_t \equiv \epsilon < 1 \) for \( t > 1 \).

Following results in above section, we know from period \( t=2 \) onwards, the economy will be following same equilibrium policy since \( \epsilon_t \) stays constant. So let’s focus on the initial period \( t = 1 \). For arbitrary tax policy \( \tau_1 \)
Suppose there is no political turnover. Assume productivity realization follows Proposition 2.

\( \text{above is quite burdensome. However, we can still characterize the property of the equilibrium tax policy.} \)

\[ \sqrt{2} (1 - \tau_1) \bar{\theta}_2 + A \tau_1 (1 - \tau_1) (\bar{\theta}_1^2 + \bar{\theta}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2} (1 - T(\tau_1, \epsilon)) \bar{\theta}_2 \epsilon + AT(\tau_1, \epsilon)(1 - T(\tau_1, \epsilon)) (\bar{\theta}_1^2 + \bar{\theta}_2^2) \epsilon^2 \right] \geq \]

\[ \sqrt{2} (1 - \tau_0) \bar{\theta}_2 + A \tau_0 (1 - \tau_0) (\bar{\theta}_1^2 + \bar{\theta}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2} (1 - T(\tau_0, \epsilon)) \bar{\theta}_2 \epsilon + AT(\tau_0, \epsilon)(1 - T(\tau_0, \epsilon)) (\bar{\theta}_1^2 + \bar{\theta}_2^2) \epsilon^2 \right] \]

The rich chooses the tax policy \( \tau_1 \) that maximizes the welfare of its constituents taking into consideration the best response of the poor. Assuming the optimal tax rate is non-negative and no greater than 1, the rich’s optimization problem can be summarized by

\[
\max_{\tau_1} \sqrt{2} (1 - \tau_1) \bar{\theta}_1 + A \tau_1 (1 - \tau_1) (\bar{\theta}_1^2 + \bar{\theta}_2^2) + \frac{\bar{\theta}_1}{1 - \beta} \left[ \sqrt{2} (1 - T(\tau_1, \epsilon)) \bar{\theta}_1 \epsilon + AT(\tau_1, \epsilon)(1 - T(\tau_1, \epsilon)) (\bar{\theta}_1^2 + \bar{\theta}_2^2) \epsilon^2 \right]
\]

\text{s.t.}

\[
\sqrt{2} (1 - \tau_1) \bar{\theta}_2 + A \tau_1 (1 - \tau_1) (\bar{\theta}_1^2 + \bar{\theta}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2} (1 - T(\tau_1, \epsilon)) \bar{\theta}_2 \epsilon + AT(\tau_1, \epsilon)(1 - T(\tau_1, \epsilon)) (\bar{\theta}_1^2 + \bar{\theta}_2^2) \epsilon^2 \right] \geq \]

\[ \sqrt{2} (1 - \tau_0) \bar{\theta}_2 + A \tau_0 (1 - \tau_0) (\bar{\theta}_1^2 + \bar{\theta}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2} (1 - T(\tau_0, \epsilon)) \bar{\theta}_2 \epsilon + AT(\tau_0, \epsilon)(1 - T(\tau_0, \epsilon)) (\bar{\theta}_1^2 + \bar{\theta}_2^2) \epsilon^2 \right] \]

\[ 0 \leq \tau \leq 1 \]

where

\[ T(\tau, \epsilon) = \begin{cases} 
\tau_1^* & \text{if } \tau \leq \tau_1^* \\
\tau & \text{if } \tau_1^* < \tau \leq \tau_2^* \\
2\tau_2^* - \tau & \text{if } \tau_2^* < \tau < 2\tau_2^* - \tau_1^* \\
\tau_1^* & \text{if } \tau \geq 2\tau_2^* - \tau_1^* 
\end{cases} \]

with \( \tau_1^* = \frac{1}{2} \left[ 1 - \frac{\sqrt{2} \bar{\theta}_1}{A(\bar{\theta}_1^2 + \bar{\theta}_2^2) \epsilon} \right] \) and \( \tau_2^* = \frac{1}{2} \left[ 1 - \frac{\sqrt{2} \bar{\theta}_2}{A(\bar{\theta}_1^2 + \bar{\theta}_2^2) \epsilon} \right] \)

Since \( T(\tau_0, \epsilon) \) and \( T(\tau_1, \epsilon) \) may take potentially different functional forms, solving the entire problem as specified above is quite burdensome. However, we can still characterize the property of the equilibrium tax policy.

**Proposition 2.** Suppose there is no political turnover. Assume productivity realization follows \( \epsilon_1 \equiv 1 \) and \( \epsilon_t \equiv \epsilon < 1, \forall t > 1 \). The equilibrium tax policy at \( t = 1 \) may involve policy maker sacrificing well being at \( t = 1 \) for purpose of higher welfare in the future.

In particular, suppose the rich is the incumbent. Denote the constraint set at \( t = 1 \) for arbitrary status quo tax rate \( \tau_0 \in [0, 1] \) as \( \tilde{C}(\tau_0, \epsilon) \) and the equilibrium tax policy \( T(\tau_0, \epsilon) \). When the incentive compatibility constraint for the opponent binds, there may exist certain \( \tau_0 \in [0, 1] \) such that \( T(\tau_0, \epsilon) = \max \tilde{C}(\tau_0, \epsilon) \).

**Proof.** See Appendix.

To put it simply, after introduction of different shock realizations over time, the incumbent has to take into
account the trade off between utility at $t = 1$ and the bargaining power in the future. Figure 10 illustrates three possible scenarios of the equilibrium tax policy at $t = 1$. Comparing with the one period equilibrium policy, we see now depending on the model parameter, there can be two different types of equilibria. One type of equilibria basically follows from equilibrium policy in the one period problem, i.e. when the status quo is between the ideal of the rich and ideal of the poor, the policy maker simply stays at the status quo. An alternative equilibrium policy, however, involves implementing tax policy higher than the status quo when the status quo tax rate is inbetween the ideal of two political parties.

Figure 10: ‘Present-Oriented’ Vs ‘Far-Sighted’ Equilibrium

Intuitively, if the discounted expected utility gain of implementing high tax rate is less than the current utility loss, the incumbent will choose to implement the tax policy that follows status quo at gridlock zone. If, on the other hand, the discounted expected utility gain of implementing high tax rate turns out to outweigh the current utility loss of staying with the status quo, the policy maker will sacrifice the current utility for purpose of higher bargaining power in the future. Since the policy maker in the second type of equilibrium is sacrificing the current for the future, we label it ‘far-sighted equilibrium’. The other type where the incumbent behaves in the opposite way is thus labeled as ‘present-oriented equilibrium’.

There are several factors affecting which type of equilibrium the economy can achieve. According to the conditions governing different types of equilibrium tax policy in appendix, we see the higher the discount factor $\beta$, the more the party cares about the future, hence the more likely ‘far-sighted equilibrium’ will occur. Similarly, the worse the negative shock $\epsilon$ in the second period, the lower the future tax rate will be if high tax rate is implemented in current period, hence the more incentive for the proposer to implement the ‘far-sighted equilibrium tax policy’.

Another important issue worth emphasizing is the ‘far-sighted equilibrium policy’ only occurs when the future shock realization is worse than the current realization. Suppose on the contrary the ‘far-sighted equilibrium tax policy’ is implemented in the first period when $\epsilon_t > 1$ for $t > 1$ and the rich is in power, the rich will find the future tax policy ends up stuck at a higher status quo between the ideal of the rich and ideal of the poor. This only generates utility loss for both current and future periods. In other words, there is no utility gain of implementing ‘far-sighted equilibrium tax policy’ when the future turns out to be better than current.
4 Quantitative Results

In this section I parameterize the model and solve it numerically through value function iteration. The purpose is to both illustrate the theoretical results in previous sections and provide further insights into the model. In addition to examining the implications of income inequality and output volatility for policy gridlock behavior in the full-fledged model, I also provide a quantitative example of the ‘far-sighted equilibrium’. Finally, I also explore the role of political power fluctuation.

4.1 Parameterization

Now we turn to the calibration of the model. The length of model period is one calendar year.

Preference I set discount factor \( \beta \) at 0.9, risk aversion parameter \( \gamma \) equal to 0.5, labor elasticity parameter \( \nu \) equal to 1, and government spending elasticity parameter \( \xi \) equal to 0.8. The income generating ability parameters \( \bar{\theta}_1 \) and \( \bar{\theta}_2 \) are chosen so that the average income share of the rich and poor from the model matches the average income share of the second (as a proxy for the median income of bottom 50%) and fourth quintile (as a proxy for the median income of top 50%) of U.S. before tax income distribution. I choose valuation of public good parameter \( A \) so that the average government spending as a share of GDP is around 21%. \( \alpha \) is determined through matching the average spell of policy gridlock generated by the model with the time length of no individual income tax legislation change in United States. Using tax legislation changes summarized by Romer and Romer (2010) I obtain the average spell for inaction in U.S. individual income tax legislation is 2.689 years.

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<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta = 0.9 )</td>
<td>discount factor</td>
<td>-</td>
</tr>
<tr>
<td>( \gamma = 0.5 )</td>
<td>preference parameter</td>
<td>-</td>
</tr>
<tr>
<td>( \nu = 1 )</td>
<td>labor elasticity parameter</td>
<td>-</td>
</tr>
<tr>
<td>( \xi = 0.8 )</td>
<td>spending elasticity parameter</td>
<td>-</td>
</tr>
<tr>
<td>( A = 0.4158 )</td>
<td>valuation of public goods</td>
<td>spending as a share of GDP is on average 21%</td>
</tr>
<tr>
<td>( \bar{\theta}_1 = 0.9657 )</td>
<td>productivity of rich</td>
<td>average before tax income share of top 50% is around 68.4%</td>
</tr>
<tr>
<td>( \bar{\theta}_2 = 0.6886 )</td>
<td>productivity of poor</td>
<td>average before tax income share of bottom 50% is around 31.6%</td>
</tr>
<tr>
<td>( \rho = 0.95 )</td>
<td>persistence of productivity shock</td>
<td>-</td>
</tr>
<tr>
<td>( \sigma = 0.0791 )</td>
<td>std. deviation of productivity shock</td>
<td>estimate from Heathcote et al. (2010)</td>
</tr>
<tr>
<td>( \alpha = 0.62 )</td>
<td>welfare weight on own constituents</td>
<td>average spell of income tax legislation inaction is 2.689 years</td>
</tr>
<tr>
<td>( p = 0.76 )</td>
<td>persistence of power</td>
<td>average probability of staying in office is 0.76</td>
</tr>
</tbody>
</table>

Productivity Shock The process of log normal economy-wide productivity shock \( \epsilon_t \) is approximated with a 51 state Markov chain and the transition probabilities are chosen following Tauchen (1987). The persistence parameter \( \rho \) is set at 0.95. The standard deviation parameter \( \sigma \) is set at 0.0791 to match the empirical estimates of the variance of wage increase according to Heathcote et al. (2010).

---

3 For empirical counterparts calculation, unless otherwise mentioned, the sample period is 1970 - 2012. As shown in the motivation, starting 1970s, the U.S. economy has witnessed rising inequality, rising political polarization, and increasing divided government.

4 For purpose of completeness, sensitivity analysis of \( \xi \) is provided in the appendix.

5 Source: Congressional Budget Office Website, supplemental data spreadsheet for The Distribution of Household Income and Federal Taxes, 2008 and 2009

6 Source: Congressional Budget Office website, the U.S. federal budget: 1971 – 2011

7 Modifications have been made to include most recent income tax legislation changes and drop tax changes irrelevant to individual income tax
**Power Persistence** To figure out power persistence parameter \( p \), I first compute separately the probability of staying in office for both Republican and Democrats by looking at historical party control of White house and both houses of United States Congress. In appendix, I summarize the the computed probability of Republican and Democrats staying in control of White house, Senate and House using party control information of U.S. government from 1969 to 2012. Since the difference between the two parties is trivial, arithmetic mean of above probabilities \( p = 0.76 \) is used as the power persistence parameter in my model. Table 2 summarizes the calibration targets and parameter values.

### 4.2 Properties of Stationary Distribution

To obtain the stationary distribution of tax policy, I simulate the economy as calibrated above for 1000 times and each realization involves 1000 periods. Table 3 summarizes the properties of tax policy stationary distribution and compare it with two benchmarks: one is an economy where a social planner with utilitarian welfare objective determines tax policy each period, and the other is an economy with power fluctuation as in our model yet without bargaining.

<table>
<thead>
<tr>
<th>Economy</th>
<th>( \bar{\tau} )</th>
<th>( \sigma_\tau )</th>
<th>corr(( \tau_t, \tau_{t-1} ))</th>
<th>gridlock duration</th>
<th>( P(\tau_t \neq \tau_{t-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Planner; No Turnover</td>
<td>0.1989</td>
<td>0.0237</td>
<td>0.9484</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>No Bargain; Turnover</td>
<td>0.1990</td>
<td>0.0246</td>
<td>0.9202</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Bargain; Turnover</td>
<td>0.1987</td>
<td>0.0228</td>
<td>0.9639</td>
<td>2.7847</td>
<td>0.3592</td>
</tr>
</tbody>
</table>

Comparing the stationary distribution statistics of above three economies, we can see even though the mean tax rates are almost the same, power fluctuation and legislative bargaining can generate different volatility and persistence of the tax policy. In terms of volatility, we can see while power fluctuation alone can generate volatile tax policy (0.0246 > 0.0237), legislative bargaining with endogenous status quo, however, reduces the variation of tax policy (0.0228 < 0.0246). In particular, the model is parameterized such that the role of bargaining dominates that of political turnover (0.0228 < 0.0237). Therefore, the economy with both bargaining and political turnover faces least variation of tax rates.

In terms of policy persistence, we see since the social planner is determining tax policy each period upon realization of the economy-wide shock, the persistence of tax rate closely follows that of the productivity shock (recall \( \rho = 0.95 \)). After introduction of political turnover, assuming no legislative bargaining, each political party simply implements its ideal tax policy. Since the rich and the poor have different ideal policies in mind, the political turnover reduces the persistence of the tax policy (0.9202 < 0.9484). Legislative bargaining with endogenous status quo, however, increases the policy persistence. This is because whenever two parties fail to achieve any agreement, the economy simply experiences periods of policy inaction.

Table 4 presents and compares the welfare of above three economies. Not surprisingly, political turnover reduces both the welfare of the rich and the poor. After introducing the additional friction of legislative bargaining with endogenous status quo, we can see both groups of households are still worse-off than the social planner economy. One interesting observation, however, is while the rich is getting even worse-off as compared to the second economy, the poor is experiencing a slight welfare gain. This is potentially due to the reduced volatility of tax policy, which
in turn reduces the volatility of consumption. In this sense, as compared to the economy with turnover only, the economy with both turnover and legislative bargaining may deliver a slight welfare improvement.

Table 4: Summary Statistics of Welfare

<table>
<thead>
<tr>
<th>Economy</th>
<th>rich</th>
<th>poor</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Planner; No Turnover</td>
<td>2680.800</td>
<td>2354.536</td>
<td>5035.336</td>
</tr>
<tr>
<td>No Bargain; Turnover</td>
<td>2680.630</td>
<td>2354.402</td>
<td>5035.032</td>
</tr>
<tr>
<td>Bargain; Turnover</td>
<td>2680.621</td>
<td>2354.465</td>
<td>5035.086</td>
</tr>
</tbody>
</table>

4.3 Role of Income Inequality

To investigate the role of income inequality, I first plot the simulated tax series of economies with different rich income shares. Comparing Panel A with Panel B in Figure 11, we can see despite the tax policy moves with the aggregate productivity shock over time in general, policy gridlock creates periods of policy inaction. Comparing Panel B with Panel C, we can see that higher income inequality as measured by higher rich income share leads to less frequent tax policy changes and longer spell of policy gridlock.

Figure 11: Simulated Tax Series

To further characterize the relation between income inequality and properties of policy gridlock, I plot simulated duration of policy inaction and probability of policy change against income inequality in Figure 12. Note that as income share of the rich increases, the duration of policy gridlock is rising at an increasing speed. Meanwhile, the probability of policy change is gradually decreasing.
4.4 Role of Shock Magnitude and Output Volatility

In this section, I examine how shock magnitude affects the responsiveness of the equilibrium tax policy. I first examine given the process of the shock, how different sizes of shock realization affect the responsiveness of the equilibrium tax policy. Then I look at the role of output volatility by varying \( \sigma \) parameter and compute simulated statistics as in previous section.

One way of understanding whether or not the equilibrium tax policy responds to aggregate productivity shock is through looking at the policy function. Figure 14 plots the policy function of both rich and poor for three different shock realizations. For both types of incumbents, when the difference of shock sizes is relatively small (e.g. \( \epsilon_t = 1 \)}
and $\epsilon_{t+1} = 1.1293$), the inaction region of two policy functions overlap. This implies for $\epsilon_t = 1$, starting with any status quo within the overlapped region, the shock $\epsilon_{t+1} = 1.1293$ cannot generate any policy response. When the difference of shock sizes is relatively large (e.g. $\epsilon_t = 0.8855$ and $\epsilon_{t+1} = 1.1293$), the inaction region of two policy functions do not overlap. This implies for $\epsilon_t = 0.8855$, no matter what the status quo tax rate is, two parties will always agree on changing the tax policy on occurrence of $\epsilon_{t+1} = 1.1293$.

An alternative way of illustrating above idea is by looking at the impulse response of the economy towards shocks of different magnitude. Let the economy start with the no shock state at $T = 0$. Starting from period $T = 2$, there is a temporary unexpected negative aggregate shock. The shock lasts for 3 periods and the economy returns back to the no shock state afterwards. In Figure 14, the left column illustrates policy response when the size of negative shock is small and the right column illustrates policy response when the size of negative shock is large. When the shock is small, depending on the status quo, the two parties may or may not reach an agreement. For instance, starting with status quo $\tau_0 = 0.2049$, the incumbent fails to respond to the exogenous shock and the tax policy stays constant throughout the experiment periods. When shock is large, however, despite the status quo tax level, the two parties manage to achieve an agreement on occurrence of the shock.

Another interesting observation from this exercise is that after the shock is over and the economy goes back to no shock state, the tax policy may fail to reverse back to the pre-shock level. For instance, in the small shock scenario, for status quo $\tau_0 = 0.1901$, the tax policy fails to drop after the shock is over and simply stays constant afterwards. This is because given the status quo, two parties fail to reach an agreement and the policy gets stuck at the gridlock region.

Figure 14: Impulse Responses: Rich In Power, Small Vs Large Shock
To further characterize the relation between properties of policy gridlock and role of shock magnitude, I plot simulated duration of policy inaction and probability of policy change against volatility of the shock measured by $\sigma$ in Figure 15. We can see that as volatility of the shock rises, the duration of policy gridlock is declining. Meanwhile, the probability of policy change is increasing. This is consistent with the findings in the analytical example. As $\sigma$ rises, the economy is more likely to experience shocks sufficiently large to drive the economy away from the status quo.

Figure 15: Role of Shock Magnitude

4.5 ‘Far-Sighted’ Equilibrium Tax Policy

In this section, I show numerically that in certain scenarios the equilibrium tax policy proposed by the incumbent may exhibit the ‘far-sighted’ feature as illustrated in previous analytical example.

Recall the equilibrium tax policy plotted earlier for the parameterized economy. As illustrated by the left panel in Figure 16, when the incentive compatibility constraint of the opponent binds, the rich always chooses to prescribe the minimum tax policy within its constraint set. This is not surprising since if the policy maker fails to implement its ideal policy, a natural way is to implement the tax level that is closest to its ideal level.

Implementing the policy closest to the incumbent’s ideal level, however, may not always be the equilibrium behavior. As shown in previous analytical example, under certain parameter assumptions, it is likely for the rich to trade off the current and future in a different way. The right panel of Figure 16 illustrates the equilibrium policy under the parameterization $\bar{\theta}_1 = 1.1725$, $\bar{\theta}_2 = 0.4818$ and $\rho = 0$. Note over certain range of status quo the rich chooses the maximum tax rate within its constraint set when incentive compatibility constraint binds. Intuitively, the rich is implementing this ‘seemingly unreasonable’ tax policy so that it can obtain higher expected utility gain in the future.

To further explain the mechanism behind the ‘far-sighted equilibrium’, in Figure 17 and Figure 18 I compare the utility gains and losses obtained through equilibrium tax policy with ‘closest-to-ideal-policy’ rule. In both figures, the solid horizontal red line indicates the equilibrium tax level. The dashed horizontal red line indicates the tax level prescribed by ‘closest-to-ideal-policy’ rule. Assume the productivity shock is i.i.d (i.e. $\rho = 0$), the economy is
at a bad state in period $t$ and the rich is in power. Given status quo $\tau_{t-1}$, by following equilibrium tax policy the proposer suffers from current utility loss due to proposing a higher tax rate than the status quo. Since we assume there is no persistence for the productivity shock, in period $t + 1$ the shock $\epsilon$ will on average revert to its expected level which is slightly above 1. Using the policy function for state $\epsilon = 1$, we see higher tax policy in period $t$ implies a much lower tax policy in period $t + 1$, and hence discounted expected welfare gain in the future. As long as the discounted expected welfare gain outweighs the current utility loss, the rich will choose the maximum tax policy now for purpose of lower tax policy tomorrow.
There are several factors affecting the occurrence of the ‘far-sighted equilibrium’ tax policy. First, note this type of policy only occurs when the economy is in relatively bad economic state within the state space. This is because if the economy is in a relatively good state, the rich will expect in general it will be gridlocked at a higher status quo in the future by proposing a high tax rate, so there is little expected future welfare gain. Second, as shown in the example, the equilibrium is found when $\rho = 0$. As compared to $\rho = 0.9$, conditional on being at a bad state of the economy, the policy proposer expects a higher chance of getting to a good state in the future. This in fact raises its expected future welfare gain. Another factor that also affects the occurrence of the ‘far-sighted equilibrium’ is the discount factor $\beta$. All else equal, the higher the discount factor $\beta$, the larger the expected welfare gain in the future, and hence the more likely the occurrence of the ‘far-sighted equilibrium’.

4.6 Role of Power Fluctuation

Finally, I examine the role of political turnover on incumbent’s optimal tax policy through varying power persistence parameter $p$ and see how equilibrium tax policy behaves as $p$ changes. As $p$ increases, the likelihood of current incumbent stays in power increases. In particular, when $p = 1$, we are essentially facing the problem with rich (poor) being the incumbent forever. To isolate the role of power fluctuation, economy-wide productivity shock is shut off and $\epsilon$ is assumed to be 1 for all time and states.

In Figure 19, I plot the policy functions for both parties while varying the power persistence. Note that as $p$ drops from 1 to 0.5, the bargaining power of the incumbent declines while the bargaining power of the opponent rises. This can be observed from the expansion of the compromise region and the shift of equilibrium policy towards the ideal level of the opponent within the compromise region. Compromise can also be observed in a small region on the other side of the gridlock region. Another observation worthy mentioning is despite the difference in compromise regions, the inaction region stays the same irrespective of the incumbent.
5 Further Extension: Progressive Tax-Transfer System

In this section, I examine the policy gridlock behavior under the progressive tax-transfer system. Following Heathcote et al. (2010), I assume the government operates a two-parameter tax function to finance the provision of public goods

\[
T(y_i) = y_i - \lambda_t y_i^{(1 - \tau_t)}
\]  

(5.1)

This implies an individual with pre-tax income \(y_i\) receives post-tax income \(\lambda_t y_i^{(1 - \tau_t)}\). Here \(\lambda_t\) captures the average level of tax, while \(\tau_t\) determines the progressivity of the tax system.

For simplicity, in rest of this section I focus on the scenario where there is no stochastic shock or political turnover and the rich party is in power. Under above tax system, assuming households preferences take the same functional form as previous sections

\[
u(c_t^i, l_t^i, g_t) = \frac{1}{1 - \gamma} \left( c_t^i - \frac{l_t^i}{1 + \nu} \right)^{1-\gamma} + A \frac{g_t^{1 - \xi}}{1 - \xi}
\]  

(5.2)

the optimal consumption and labor allocations for household with productivity \(\bar{\theta}_t\) are given by

\[
C_t^i(\tau_t, \lambda_t) = \lambda_t (\bar{\theta}_t L_t^i)^{(1 - \tau_t)}
\]  

(5.3)

\[
L_t^i(\tau_t, \lambda_t) = (\lambda_t (1 - \tau_t) \bar{\theta}_t^{1 - \tau_t})^{1/\tau_t}
\]  

(5.4)
Let \((s_\tau, s_\lambda)\) denote the status quo value of \((\tau, \lambda)\). The incumbent’s problem is as follows:

\[
V_1(s_\tau, s_\lambda, 1) = \max_{\tau, \lambda} \frac{1}{1-\gamma} \left( C^{1*}(\tau, \lambda) - \frac{L^{1*}(\tau, \lambda)^{1+\nu}}{1+\nu} \right)^{1-\gamma} + A \frac{G^*(\tau, \lambda)^{1-\xi}}{1-\xi} + \beta V_1(s'_\tau, s'_\lambda, 1)
\]

s.t.

\[
W_2(s_\tau, s_\lambda, 1; \tau, \lambda) \geq W_2(s_\tau, s_\lambda, 1; s_\tau, s_\lambda)
\]

\[
s'_\tau = \tau
\]

\[
s'_\lambda = \lambda
\]

where for arbitrary \((\tau, \lambda)\)

\[
W_2(s_\tau, s_\lambda, 1; \tau, \lambda) = \frac{1}{1-\gamma} \left( C^{2*}(\tau, \lambda) - \frac{L^{2*}(\tau, \lambda)^{1+\nu}}{1+\nu} \right)^{1-\gamma} + A \frac{G^*(\tau, \lambda)^{1-\xi}}{1-\xi} + \beta W_2 (s'_\tau, s'_\lambda, 1; T(s'_\tau, s'_\lambda, 1), \Lambda(s'_\tau, s'_\lambda, 1))
\]

In Figure 20 I plot the equilibrium average and marginal tax rates for the rich against status quo average and marginal tax rates. Since now the incumbent have access to two tax policy parameters, the mapping from status quo tax rate to equilibrium tax rate is not necessarily one to one. This is because different \((s_\tau, s_\lambda)\) combinations may generate same status quo average tax rate yet different equilibrium \((\tau, \lambda)\) policies. While some \((s_\tau, s_\lambda)\) combination can lead to full agreement, others may incur policy gridlock.

**Figure 20: Equilibrium Average and Marginal Tax Rate\((\xi = 0)\)**

For both average tax rate and marginal tax rate, we can see similar to the linear taxation benchmark, when the status quo is far beyond the ideal level of the incumbent, two parties manage to reach policy agreement. When the status quo tax rates are between the ideal of the rich and the poor, however, agreement on implementing \((\tau, \lambda)\) policy to achieve the ideal average or marginal tax rate of the rich is still possible for certain \((s_\tau, s_\lambda)\) combinations.
Therefore, instead of being plotted as a single line in the linear taxation benchmark, now the equilibrium policy is indicated by the entire red region. In fact, we can see all values between the ideal level of the incumbent and the level where gridlock of average or marginal tax rates occurs are possible to be achieved. To some extent, the flexibility of the tax system alleviates the occurrence of policy gridlock.

Moreover, for equilibrium average tax rate, when the status quo average tax rate is around the ideal of the opponent, the equilibrium policy may turn out to be higher than the status quo. This could be explained by the fact that in face of multi-dimensional policy choices, the decision maker may choose to sacrifice one dimension for the other. In this case, the rich may be willing to take a higher average tax rate in return for lower marginal tax rate.

6 Conclusion

This paper examines tax policy behavior in an economy with dynamic legislative bargaining and exogenous economy-wide productivity shock. By adopting a model of bargaining with endogenous status quo, I account for the co-movement of economic inequality and policy gridlock. By incorporating economy-wide productivity shock, not only does this model generate the feature that legislative stalemate occurs more frequently in times of lower output fluctuation, but it also uncovers two different types of equilibrium policy, one of which involves the policy maker implementing a policy unpleasant in the present for the sake of high bargaining power in the future. As an extension, I also find that introducing additional flexibility to the tax system can alleviate policy gridlock. In this model, the inequality is determined exogenously. One potentially interesting extension would be to consider an economy where inequality is endogenous and verify whether and how the policy maker fights inequality in face of legislative bargaining.
References


T. Renee Bowen, Ying Chen, and Hulya Eraslan. Mandatory versus discretionary spending: The status quo effect. Research Papers 2121, Stanford University, Graduate School of Business, October 2012.


A Proofs

A.1 Lemma 1

Lemma 2. Assume there is no political turnover and the rich is the incumbent. For given status quo \( \tau_0 \) and productivity realization \( \epsilon \), the equilibrium tax policy for the one period economy with legislative bargaining is

\[
T(\tau_0, \epsilon) = \begin{cases} 
\tau_1^* & \text{if } \tau_0 \leq \tau_1^* \\
\tau_0 & \text{if } \tau_1^* < \tau_0 \leq \tau_2^* \\
2\tau_2^* - \tau_0 & \text{if } \tau_2^* < \tau_0 < 2\tau_2^* - \tau_1^* \\
\tau_1^* & \text{if } \tau_0 \geq 2\tau_2^* - \tau_1^* 
\end{cases}
\] (A.1)

where \( \tau_1^* = \frac{1}{2} \left[ 1 - \frac{\sqrt{2} \bar{\theta}_1}{A(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon} \right] \) and \( \tau_2^* = \frac{1}{2} \left[ 1 - \frac{\sqrt{2} \bar{\theta}_2}{A(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon} \right] \).

Proof. Assume the rich is in power. The one-period problem is as follows

\[
\max_{\tau} \frac{(1 - \tau) \bar{\theta}_1 \epsilon}{\sqrt{2}} + A\tau(1 - \tau)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \\
\text{s.t.} \\
\frac{(1 - \tau) \bar{\theta}_2 \epsilon}{\sqrt{2}} + A\tau(1 - \tau)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \geq \frac{(1 - \tau_0) \bar{\theta}_1 \epsilon}{\sqrt{2}} + A\tau_0(1 - \tau_0)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2
\]

Assume \( \bar{\theta}_1 > \bar{\theta}_2 \). Depending on whether or not the inequality constraint binds, we look at the following two cases.

**Case 1: Full Concensus** Assume the inequality constraint does not bind. Then the rich simply implement its ideal tax policy by solving the unconstrained maximization problem:

\[
\max_{\tau} \frac{(1 - \tau) \bar{\theta}_1 \epsilon}{\sqrt{2}} + A\tau(1 - \tau)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \\
\text{s.t.} \\
\frac{(1 - \tau) \bar{\theta}_2 \epsilon}{\sqrt{2}} + A\tau(1 - \tau)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \geq \frac{(1 - \tau_0) \bar{\theta}_1 \epsilon}{\sqrt{2}} + A\tau_0(1 - \tau_0)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2
\]

(A.2)

The optimal tax rate is given by

\[
\tau_1^* = \frac{1}{2} \left[ 1 - \frac{\bar{\theta}_1}{\sqrt{2}A(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon} \right]
\] (A.3)

Similarly, if the poor is in power, the ideal tax policy of the poor is given by

\[
\tau_2^* = \frac{1}{2} \left[ 1 - \frac{\bar{\theta}_2}{\sqrt{2}A(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon} \right]
\] (A.4)

In order for \( \tau^* \) to be the optimal tax policy with endogenous status quo, we also need

\[
\frac{(1 - \tau^*) \bar{\theta}_2 \epsilon}{\sqrt{2}} + A\tau^*(1 - \tau^*)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 > \frac{(1 - \tau_0) \bar{\theta}_1 \epsilon}{\sqrt{2}} + A\tau_0(1 - \tau_0)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2
\]
which implies

\[ \tau_0 < \tau_1^* \quad \text{or} \quad \tau_0 > \tau_2^* \]

with \( \tau_1^* = \min \{ \frac{1}{2} - \frac{\sqrt{2A \theta_1}}{4A(\theta_1^2 + \theta_2^2)c}, \frac{1}{2} + \frac{\sqrt{2A \theta_1}}{4A(\theta_1^2 + \theta_2^2)c}, \frac{1}{2} - \frac{\sqrt{2A \theta_2}}{4A(\theta_1^2 + \theta_2^2)c}, \frac{1}{2} + \frac{\sqrt{2A \theta_2}}{4A(\theta_1^2 + \theta_2^2)c} \} \) and \( \tau_2^* = \max \{ \frac{1}{2} - \frac{\sqrt{2A \theta_1}}{4A(\theta_1^2 + \theta_2^2)c}, \frac{1}{2} + \frac{\sqrt{2A \theta_1}}{4A(\theta_1^2 + \theta_2^2)c}, \frac{1}{2} - \frac{\sqrt{2A \theta_2}}{4A(\theta_1^2 + \theta_2^2)c}, \frac{1}{2} + \frac{\sqrt{2A \theta_2}}{4A(\theta_1^2 + \theta_2^2)c} \} \). Using the assumption \( \bar{\theta}_1 > \bar{\theta}_2 \), the inequality above can be simplified as

\[ \tau_0 < \frac{1}{2} - \frac{\sqrt{2A \theta_1}}{4A(\theta_1^2 + \theta_2^2)c} \quad \text{or} \quad \tau_0 \geq \frac{1}{2} + \frac{\sqrt{2A \theta_1}}{4A(\theta_1^2 + \theta_2^2)c} \]  

(A.5)

**Case 2: Policy Gridlock and Compromise**

Assume the inequality constraint binds. Then the optimal tax policy implemented is implied by the quadratic equation

\[ \frac{(1 - \tau)(\bar{\theta}_2 \epsilon)}{\sqrt{2}} + A\tau(1 - \tau)(\bar{\theta}_1^2 + \bar{\theta}_2^2)c^2 = \frac{(1 - \tau_0)(\bar{\theta}_2 \epsilon)}{\sqrt{2}} + A\tau_0(1 - \tau_0)(\bar{\theta}_1^2 + \bar{\theta}_2^2)c^2 \]

the solution of which is given by

\[ \tau_1 = \tau_0 \]

(A.6)

\[ \tau_2 = 1 - \tau_0 - \frac{\bar{\theta}_2}{\sqrt{2A(\theta_1^2 + \theta_2^2)c}} \]  

(A.7)

since the two roots are not equal in general, the optimal tax policy is then determined by the one that gives rich higher utility. Note that under the assumption \( \bar{\theta}_1 > \bar{\theta}_2 \),

\[ \frac{(1 - \tau_1)(\bar{\theta}_1 \epsilon)}{\sqrt{2}} + A\tau_1(1 - \tau_1)(\bar{\theta}_1^2 + \bar{\theta}_2^2)c^2 \geq \frac{(1 - \tau_2)(\bar{\theta}_1 \epsilon)}{\sqrt{2}} + A\tau_2(1 - \tau_2)(\bar{\theta}_1^2 + \bar{\theta}_2^2)c^2 \]

if and only if

\[ \tau_0 \leq \frac{1}{2} - \frac{\bar{\theta}_2}{2\sqrt{2A(\theta_1^2 + \theta_2^2)c}} \]

Hence we have

\[ \tau^* = \begin{cases} 
\tau_0 & \text{if } \tau_0 \leq \frac{1}{2} - \frac{\bar{\theta}_2}{2\sqrt{2A(\theta_1^2 + \theta_2^2)c}} \\
1 - \tau_0 - \frac{\bar{\theta}_2}{\sqrt{2A(\theta_1^2 + \theta_2^2)c}} & \text{if } \tau_0 > \frac{1}{2} - \frac{\bar{\theta}_2}{2\sqrt{2A(\theta_1^2 + \theta_2^2)c}} 
\end{cases} \]

First observe the cutoff point is the ideal tax policy of the poor. Also, note that within the interval \( \left[ \frac{1}{2} - \frac{\bar{\theta}_1}{2\sqrt{2A(\theta_1^2 + \theta_2^2)c}}, \frac{1}{2} - \frac{\bar{\theta}_2}{2\sqrt{2A(\theta_1^2 + \theta_2^2)c}} \right] \), the optimal tax policy is to implement previous period’s tax rate, i.e. the tax policy is completely gridlocked. Within the interval \( \left[ \frac{1}{2} - \frac{\bar{\theta}_2}{2\sqrt{2A(\theta_1^2 + \theta_2^2)c}}, \frac{1}{2} + \frac{\bar{\theta}_1 - 2\theta_2}{2\sqrt{2A(\theta_1^2 + \theta_2^2)c}} \right] \), the optimal tax policy is implemented at the level that is lower than the status quo, i.e. \( 1 - \tau_0 - \frac{\bar{\theta}_2}{\sqrt{2A(\theta_1^2 + \theta_2^2)c}} < \tau_0 \).
Summing up Case 1 and Case 2 above, the optimal tax policy is characterized as

\[
\tau^* = \begin{cases} 
\tau_0 & \text{if } \tau_0 < \frac{1}{2} - \frac{\sqrt{2} \delta_0}{2A(\theta^2_1 + \theta^2_2)\epsilon} \\
1 - \tau_0 - \frac{\delta_2}{\sqrt{2A(\theta^2_1 + \theta^2_2)\epsilon}} & \text{if } \frac{1}{2} - \frac{\delta_2}{2\sqrt{2A(\theta^2_1 + \theta^2_2)\epsilon}} \leq \tau_0 \leq \frac{1}{2} - \frac{\delta_0}{2\sqrt{2A(\theta^2_1 + \theta^2_2)\epsilon}} \\
\frac{1}{2} \left[1 - \frac{\delta_0}{\sqrt{2A(\theta^2_1 + \theta^2_2)\epsilon}}\right] & \text{if } \tau_0 > \frac{1}{2} + \frac{\delta_0}{2\sqrt{2A(\theta^2_1 + \theta^2_2)\epsilon}}
\end{cases}
\]

(A.8)

\[\square\]

A.2 Corollary 1

**Corollary 5. (identical policy gridlock zone)** Let \( i \) denote the identity of the incumbent and \( T(\tau_0, \epsilon, i) \) denote equilibrium tax policy by incumbent \( i \) in the one period economy. For a given \( \tau_0 \), if \( T(\tau_0, \epsilon, 1) = \tau_0 \), then \( T(\tau_0, \epsilon, 2) = \tau_0 \).

**Proof.** By symmetry, the equilibrium tax policy of the poor is given by

\[
T(\tau_0, \epsilon) = \begin{cases} 
\tau^*_2 - \tau_0 & \text{if } \tau_0 \leq 2\tau^*_1 - \tau^*_2 \\
2\tau^*_1 - \tau_0 & \text{if } 2\tau^*_1 - \tau^*_2 < \tau_0 < \tau^*_1 \\
\tau_0 & \text{if } \tau^*_1 < \tau_0 \leq \tau^*_2 \\
\tau^*_2 & \text{if } \tau_0 \geq \tau^*_2
\end{cases}
\]

(A.9)

where \( \tau^*_1 = \frac{1}{2} \left[1 - \frac{\sqrt{2} \delta_1}{A(\theta^2_1 + \theta^2_2)\epsilon}\right] \) and \( \tau^*_2 = \frac{1}{2} \left[1 - \frac{\sqrt{2} \delta_2}{A(\theta^2_1 + \theta^2_2)\epsilon}\right] \).

Comparing with Equation A.1, note the region where \( T(\tau_0, \epsilon) = \tau_0 \) coincides. i.e. As long as \( \tau^*_1 < \tau_0 < \tau^*_2 \), \( T(\tau_0, \epsilon) = \tau_0 \) no matter whether the rich or the poor is in power. \[\square\]

A.3 Corollary 2

**Corollary 6. (role of income inequality)** For given \( \epsilon \), both gridlock region and compromise region expands as income inequality (as measured by income share of the rich) rises.

**Proof.** Recall the equilibrium tax policy as a function of mean preserving spread \( \delta \) can be rewritten as

\[
T(\tau_0, \epsilon; \delta) = \begin{cases} 
\tau^*_1(\delta) & \text{if } \tau_0 \leq \tau^*_1(\delta) \\
\tau_0 & \text{if } \tau^*_1(\delta) < \tau_0 \leq \tau^*_2(\delta) \\
2\tau^*_2(\delta) - \tau_0 & \text{if } \tau^*_2(\delta) < \tau_0 < 2\tau^*_2(\delta) - \tau^*_1(\delta) \\
\tau^*_1(\delta) & \text{if } \tau_0 \geq 2\tau^*_2(\delta) - \tau^*_1(\delta)
\end{cases}
\]

(A.10)

where \( \tau^*_1(\delta) = \frac{1}{2} \left[1 - \frac{\sqrt{2} \delta_1}{A((\theta_1 + \delta)^2 + (\theta_2 - \delta)^2)\epsilon}\right] \) and \( \tau^*_2(\delta) = \frac{1}{2} \left[1 - \frac{\sqrt{2} \delta_2}{A((\theta_1 + \delta)^2 + (\theta_2 - \delta)^2)\epsilon}\right] \). Differentiating \( \tau^*_1(\delta) \) and \( \tau^*_2(\delta) \) with respect to \( \delta \), we see when \( \delta < \frac{\sqrt{2} (\theta_1 + \delta)}{4A(\theta_1 + \delta)^2 + (\theta_2 - \delta)^2)\epsilon} \), as \( \delta \) rises \( \tau^*_1(\delta) \) is decreasing while \( \tau^*_2(\delta) \) is increasing. Since
the gridlock zone is characterized by \([\tau_1^*(\delta), \tau_2^*(\delta)]\), the gridlock zone expands. Similarly, since compromise zone is characterized by \([\tau_2^*(\delta), 2\tau_2^*(\delta) - \tau_1^*(\delta)]\), the region is also expanding since the upper bound is increasing at a faster speed than the lower bound.

When \(\delta \geq \frac{2\sqrt{2(\theta_1^2 + \theta_2^2)} e^{-4\theta_1 \epsilon}}{4} \), \(\tau_1^*(\delta)\) starts to increase. However, both regions are still expanding since \(\tau_2^*(\delta)\) is increasing at a faster speed than \(\tau_1^*(\delta)\).

\[\text{A.4 Corollary 3}\]

\[\text{Corollary 7 (role of output volatility). Assume } \phi(\epsilon_1^*)\epsilon_1^* > \phi(\epsilon_2^*)\epsilon_2^* \text{ with } \phi(\cdot) \text{ denoting the probability density function of standard normal distribution. For arbitrary status quo tax rate } \tau_0, \text{ the probability of policy gridlock is a non-increasing function of output volatility } \sigma. \text{ In particular, if } \tau_0 < \frac{1}{2}, \text{ the probability of policy gridlock is strictly decreasing.}\]

\[\text{Proof.}\] To examine the role of output volatility \(\sigma\), I first characterize the probability of policy gridlock for arbitrary status quo \(\tau_0 \in [0, 1]\) as follows:

\[\Pr (T(\tau_0, \epsilon) = \tau_0) = \begin{cases} \Phi(\epsilon_1^*) - \Phi(\epsilon_2^*) & \text{if } \tau_0 < \frac{1}{2} \\ 0 & \text{if } \tau_0 \geq \frac{1}{2} \end{cases}\]

where \(\epsilon_1^* = \frac{1}{\sigma} \log \left[ \frac{\theta_1}{\sqrt{2A(\theta_1^2 + \theta_2^2)}} \right] \), \(\epsilon_2^* = \frac{1}{\sigma} \log \left[ \frac{\theta_2}{\sqrt{2A(\theta_1^2 + \theta_2^2)}} \right] \), and \(\Phi(\cdot)\) denotes the cumulative density function of standard normal distribution. From above expression, we see that when \(\tau_0\) is sufficiently large, policy gridlock will not occur irrespective of the shock realization. When \(\tau_0\) is relatively small, however, the gridlock is likely to occur.

Now let’s focus on the status quo with \(\tau_0 < \frac{1}{2}\). Taking derivative of the gridlock probability with respect to \(\sigma\), we get

\[\frac{d\Pr(T(\tau_0, \epsilon))}{d\sigma} = \frac{d[\Phi(\epsilon_1^*) - \Phi(\epsilon_2^*)]}{d\sigma} = -\frac{1}{\sigma^2} \left\{ \phi(\epsilon_1^*) \log \left[ \frac{\theta_1}{\sqrt{2A(\theta_1^2 + \theta_2^2)}} \right] - \phi(\epsilon_2^*) \log \left[ \frac{\theta_2}{\sqrt{2A(\theta_1^2 + \theta_2^2)}} \right] \right\}
\]

\[= -\frac{1}{\sigma} \{ \phi(\epsilon_1^*)\epsilon_1^* - \phi(\epsilon_2^*)\epsilon_2^* \}\]

\[\frac{d\Pr(T(\tau_0, \epsilon))}{d\sigma} < 0 \text{ if and only if } \phi(\epsilon_1^*)\epsilon_1^* > \phi(\epsilon_2^*)\epsilon_2^*.\]

\[\text{A.5 Proposition 1}\]

\[\text{Proposition 3. Assume there is no political turnover and the productivity shock stays constant (i.e. } \epsilon_t = \epsilon, \forall t). \text{ The equilibrium tax policy for the infinite horizon problem coincides with the equilibrium tax policy in the one-period}\]
economy. If the rich is in power, then

\[ T(\tau_{t-1}, \epsilon) = \begin{cases} 
\tau_{i}^* & \text{if } \tau_{t-1} \leq \tau_{1}^* \\
\tau_{t-1} & \text{if } \tau_{1}^* < \tau_{t-1} \leq \tau_{2}^* \\
2\tau_{2}^* - \tau_{t-1} & \text{if } \tau_{2}^* < \tau_{t-1} < 2\tau_{2}^* - \tau_{1}^* \\
\tau_{1}^* & \text{if } \tau_{t-1} \geq 2\tau_{2}^* - \tau_{1}^* 
\end{cases} \]

where \( \tau_{i}^* = \frac{1}{2} \left[ 1 - \frac{\sqrt{2\theta_1}}{A(\theta_1^2 + \theta_2^2)\epsilon} \right] \) and \( \tau_{2}^* = \frac{1}{2} \left[ 1 - \frac{\sqrt{2\theta_2}}{A(\theta_1^2 + \theta_2^2)\epsilon} \right] \)

Furthermore, the steady state of the economy is \([\tau_{1}^*, \tau_{2}^*]\).

**Proof.** To prove above claim, I proceed in three steps. First, I show \( T(\tau_{t-1}, \epsilon) \in [\tau_{1}^*, \tau_{2}^*] \) and once \( \tau_{t} \in [\tau_{1}^*, \tau_{2}^*] \), \( T(\tau_{t}, \epsilon) = \tau_{t} \). Second, I show \( V(T(\tau_{t}, \epsilon), \epsilon) = \frac{1}{1-\beta} \left\{ \frac{(1-T(\tau_{t}, \epsilon))\theta_1\epsilon}{\sqrt{2}} + AT(\tau_{t}, \epsilon)(1 - T(\tau_{t}, \epsilon)) (\theta_1^2 + \theta_2^2)\epsilon^2 \right\} \) and \( W(T(\tau_{t}, \epsilon), \epsilon) = \frac{1}{1-\beta} \left\{ \frac{(1-T(\tau_{t}, \epsilon))\theta_2\epsilon}{\sqrt{2}} + AT(\tau_{t}, \epsilon)(1 - T(\tau_{t}, \epsilon)) (\theta_1^2 + \theta_2^2)\epsilon^2 \right\} \). Last, I guess and verify \( T(\tau_{t-1}, \epsilon) \) as specified above is the optimal tax policy for incumbent’s problem.

**Lemma 1.** \( T(\tau_{t-1}, \epsilon) \in [\tau_{1}^*, \tau_{2}^*] \) and \( T(T(\tau_{t-1}, \epsilon), \epsilon) = T(\tau_{t-1}, \epsilon) \forall \tau_{t-1} \). Proof:

When \( \tau_{t-1} \leq \tau_{1}^* \) or \( \tau_{t-1} \geq 2\tau_{2}^* - \tau_{1}^* \), \( T(\tau_{t-1}, \epsilon) = \tau_{t}^* \in [\tau_{1}^*, \tau_{2}^*] \).

When \( \tau_{t-1} \in (\tau_{1}^*, \tau_{2}^*], T(\tau_{t-1}, \epsilon) = \tau_{t-1} \in [\tau_{1}^*, \tau_{2}^*] \).

When \( \tau_{t-1} \in (\tau_{2}^*, 2\tau_{2}^* - \tau_{1}^*], T(\tau_{t-1}, \epsilon) = 2\tau_{2}^* - \tau_{t-1} \in [\tau_{1}^*, \tau_{2}^*] \).

Therefore, \( T(\tau_{t-1}, \epsilon) \in [\tau_{1}^*, \tau_{2}^*] \).

Since \( T(\tau_{t-1}, \epsilon) = \tau_{t-1} \) for \( \tau_{t-1} \in [\tau_{1}^*, \tau_{2}^*] \) and \( T(T(\tau_{t-1}, \epsilon), \epsilon) \in [\tau_{1}^*, \tau_{2}^*] \), we have \( T(T(\tau_{t-1}, \epsilon), \epsilon) = T(\tau_{t-1}, \epsilon) \).

**Lemma 2.** Assume \( T(\tau_{t-1}, \epsilon) \) is the optimal tax policy for incumbent’s problem. Then for any \( \tau_{t} \), we have

\[
V(T(\tau_{t}, \epsilon), \epsilon) = \frac{1}{1-\beta} \left\{ \frac{(1-T(\tau_{t}, \epsilon))\theta_1\epsilon}{\sqrt{2}} + AT(\tau_{t}, \epsilon)(1 - T(\tau_{t}, \epsilon)) (\theta_1^2 + \theta_2^2)\epsilon^2 \right\} \\
W(T(\tau_{t}, \epsilon), \epsilon) = \frac{1}{1-\beta} \left\{ \frac{(1-T(\tau_{t}, \epsilon))\theta_2\epsilon}{\sqrt{2}} + AT(\tau_{t}, \epsilon)(1 - T(\tau_{t}, \epsilon)) (\theta_1^2 + \theta_2^2)\epsilon^2 \right\}
\]

Proof:
Recall that \( V(T(\tau_{t}, \epsilon), \epsilon) = \frac{(1-T(T(\tau_{t}, \epsilon), \epsilon))\theta_1\epsilon}{\sqrt{2}} + AT(T(\tau_{t}, \epsilon), \epsilon)(1 - T(T(\tau_{t}, \epsilon), \epsilon)) (\theta_1^2 + \theta_2^2) + \beta V(T(T(\tau_{t}, \epsilon), \epsilon), \epsilon) \).

Using Lemma 1, we know \( T(T(\tau_{t}, \epsilon), \epsilon) = T(\tau_{t}, \epsilon) \forall \tau_{t} \). Hence

\[
V(T(\tau_{t}, \epsilon), \epsilon) = \frac{(1-T(\tau_{t}, \epsilon))\theta_1\epsilon}{\sqrt{2}} + AT(\tau_{t}, \epsilon)(1 - T(\tau_{t}, \epsilon)) (\theta_1^2 + \theta_2^2)\epsilon^2 + \beta V(T(\tau_{t}, \epsilon), \epsilon) \] (A.11)

Rearrange and solve for \( V(T(\tau_{t}, \epsilon), \epsilon) \), we get for any \( \tau_{t} \),

\[
V(T(\tau_{t}, \epsilon), \epsilon) = \frac{1}{1-\beta} \left\{ \frac{(1-T(\tau_{t}, \epsilon))\theta_1\epsilon}{\sqrt{2}} + AT(\tau_{t}, \epsilon)(1 - T(\tau_{t}, \epsilon)) (\theta_1^2 + \theta_2^2)\epsilon^2 \right\} \] (A.12)

Following similar logic and using \( W(T(\tau_{t}, \epsilon), \epsilon) = \frac{(1-T(T(\tau_{t}, \epsilon), \epsilon))\theta_2\epsilon}{\sqrt{2}} + AT(T(\tau_{t}, \epsilon), \epsilon)(1 - T(T(\tau_{t}, \epsilon), \epsilon)) (\theta_1^2 + \theta_2^2)\epsilon^2 + \beta W(T(T(\tau_{t}, \epsilon), \epsilon), \epsilon) \),
we can obtain for any $\tau_t$

$$W(T(\tau_t, \epsilon), \epsilon) = \frac{1}{1 - \beta} \left\{ \frac{(1 - T(\tau_t, \epsilon))\bar{\theta}_2\epsilon}{\sqrt{2}} + AT(\tau_t, \epsilon)(1 - T(\tau_t, \epsilon))(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right\} \quad (A.13)$$

Now I will proceed to show $T(\tau_{t-1}, \epsilon)$ is the optimal tax policy for incumbent’s problem given status quo tax rate $\tau_{t-1}$. The idea is first guess $T(\tau_t, \epsilon)$ to be the optimal tax policy for status quo $\tau_t$ and obtain value functions $V(T(\tau_t, \epsilon), \epsilon)$ and $W(T(\tau_t, \epsilon), \epsilon)$, then show given $V(T(\tau_t, \epsilon), \epsilon)$ and $W(T(\tau_t, \epsilon), \epsilon)$, the optimal tax rate for status quo $\tau_{t-1}$ takes same form as $T(\cdot, \epsilon)$, implying $V(\cdot, \epsilon)$ and $W(\cdot, \epsilon)$ are the fixed points of this problem.

First note that

$$V(\tau_t, \epsilon) = \frac{(1 - T(\tau_t, \epsilon))\bar{\theta}_1\epsilon}{\sqrt{2}} + AT(\tau_t, \epsilon)(1 - T(\tau_t, \epsilon))(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 + \beta V(T(\tau_t, \epsilon), \epsilon) \quad (A.15)$$

Replacing $V(T(\tau_t, \epsilon), \epsilon)$ using Lemma 2, we get

$$V(\tau_t, \epsilon) = \frac{1}{1 - \beta} \left\{ \frac{(1 - T(\tau_t, \epsilon))\bar{\theta}_1\epsilon}{\sqrt{2}} + AT(\tau_t, \epsilon)(1 - T(\tau_t, \epsilon))(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right\} \quad (A.16)$$

Similarly,

$$W(\tau_t, \epsilon) = \frac{1}{1 - \beta} \left\{ \frac{(1 - T(\tau_t, \epsilon))\bar{\theta}_2\epsilon}{\sqrt{2}} + AT(\tau_t, \epsilon)(1 - T(\tau_t, \epsilon))(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right\} \quad (A.17)$$

Now the incumbent’s problem can be rewritten as

$$V(\tau_{t-1}, \epsilon) = \max \frac{(1 - \tau_t)\bar{\theta}_1\epsilon}{\sqrt{2}} + AT_t(1 - \tau_t)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 + \frac{\beta}{1 - \beta} \left\{ \frac{(1 - T(\tau_t, \epsilon))\bar{\theta}_1\epsilon}{\sqrt{2}} + AT(\tau_t, \epsilon)(1 - T(\tau_t, \epsilon))(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right\} \quad (A.18)$$

s.t.

$$W(\tau_t, \epsilon) \geq W(\tau_{t-1}, \epsilon)$$

where

$$W(\tau_t, \epsilon) = \frac{(1 - \tau_t)\bar{\theta}_2\epsilon}{\sqrt{2}} + AT_t(1 - \tau_t)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 + \frac{\beta}{1 - \beta} \left\{ \frac{(1 - T(\tau_t, \epsilon))\bar{\theta}_2\epsilon}{\sqrt{2}} + AT(\tau_t, \epsilon)(1 - T(\tau_t, \epsilon))(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right\} \quad (A.19)$$

$$W(\tau_{t-1}, \epsilon) = \frac{(1 - \tau_{t-1})\bar{\theta}_2\epsilon}{\sqrt{2}} + AT_{t-1}(1 - \tau_{t-1})(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 + \frac{\beta}{1 - \beta} \left\{ \frac{(1 - T(\tau_{t-1}, \epsilon))\bar{\theta}_2\epsilon}{\sqrt{2}} + AT(\tau_{t-1}, \epsilon)(1 - T(\tau_{t-1}, \epsilon))(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right\} \quad (A.20)$$

Case 1: $\tau_{t-1} \leq \tau_t^*$
Since \( \tau_{t-1} \leq \tau^*_t \), \( T(\tau_{t-1}, \epsilon) = \tau^*_t \). Hence

\[
W(\tau_{t-1}, \epsilon) = \frac{(1 - \tau_{t-1})\beta \epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\beta_1 + \beta_2)^2 + \beta W(T(\tau_{t-1}, \epsilon), \epsilon)
\]

\[
= \frac{(1 - \tau_{t-1})\beta \epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\beta_1^2 + \beta_2^2)^2 + \beta W(\tau^*_t, \epsilon)
\]

\[
= \frac{(1 - \tau_{t-1})\beta \epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\beta_1^2 + \beta_2^2)^2 + \beta \frac{1}{1 - \beta} \left\{ \frac{(1 - \tau^*_t)\beta \epsilon}{\sqrt{2}} + A\tau^*_t(1 - \tau^*_t)(\beta_1^2 + \beta_2^2)^2 \right\}
\]

Since the functional form of \( T(\tau_t, \epsilon) \) depends on the value of \( \tau_t \), we will consider different scenarios separately.

1) suppose \( \tau_t \leq \tau^*_t \), then \( T(\tau_t, \epsilon) = \tau^*_t \). The optimization problem becomes

\[
V(\tau_{t-1}, \epsilon) = \max_{\tau_t} \frac{(1 - \tau_t)\beta \epsilon}{\sqrt{2}} + A\tau_t(1 - \tau_t)(\beta_1^2 + \beta_2^2)^2 + \beta \frac{1}{1 - \beta} \left\{ \frac{(1 - \tau^*_t)\beta \epsilon}{\sqrt{2}} + A\tau^*_t(1 - \tau^*_t)(\beta_1^2 + \beta_2^2)^2 \right\}
\]

\text{s.t.}

\[
W(\tau_t, \epsilon) \geq W(\tau_{t-1}, \epsilon)
\]

where

\[
W(\tau_{t-1}, 2) = \frac{(1 - \tau_{t-1})\beta \epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\beta_1^2 + \beta_2^2)^2 + \beta \frac{1}{1 - \beta} \left\{ \frac{(1 - \tau^*_t)\beta \epsilon}{\sqrt{2}} + A\tau^*_t(1 - \tau^*_t)(\beta_1^2 + \beta_2^2)^2 \right\}
\]

which could be simplified into following static problem

\[
V(\tau_{t-1}, \epsilon) = \max_{\tau_t} \frac{(1 - \tau_t)\beta \epsilon}{\sqrt{2}} + A\tau_t(1 - \tau_t)(\beta_1^2 + \beta_2^2)^2
\]

\text{s.t.}

\[
\frac{(1 - \tau_t)\beta \epsilon}{\sqrt{2}} + A\tau_t(1 - \tau_t)(\beta_1^2 + \beta_2^2)^2 \geq \frac{(1 - \tau_{t-1})\beta \epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\beta_1^2 + \beta_2^2)^2
\]

From the result for one period problem, we know that when \( \tau_{t-1} \leq \tau^*_t \), the inequality constraint does not bind and \( \tau_t^* = \tau^*_t \). Since \( \tau_t^* \) corresponds to the unconstrained maximum, we know that there does not exist alternative tax policy that deliver higher life-time utility.

\text{Case 2: } \tau_{t-1} \geq 2\tau^*_t - \tau^*_t

When \( \tau_{t-1} \geq 2\tau^*_t - \tau^*_t \), \( T(\tau_{t-1}, \epsilon) = \tau^*_t \). Again,

\[
W(\tau_{t-1}, \epsilon) = \frac{(1 - \tau_{t-1})\beta \epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\beta_1^2 + \beta_2^2)^2 + \beta W(T(\tau_{t-1}, \epsilon), \epsilon)
\]

\[
= \frac{(1 - \tau_{t-1})\beta \epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\beta_1^2 + \beta_2^2)^2 + \beta \frac{1}{1 - \beta} \left\{ \frac{(1 - \tau^*_t)\beta \epsilon}{\sqrt{2}} + A\tau^*_t(1 - \tau^*_t)(\beta_1^2 + \beta_2^2)^2 \right\}
\]
Suppose $\tau_t \leq \tau_1^*$, the optimization problem again boils down to static problem:

$$V(\tau_{t-1}, \epsilon) = \max_{\tau_t} \frac{(1 - \tau_t)\tilde{\theta}_1\epsilon}{\sqrt{2}} + A\tau_t(1 - \tau_t)(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2$$

s.t.

$$\frac{(1 - \tau_t)\tilde{\theta}_2\epsilon}{\sqrt{2}} + A\tau_t(1 - \tau_t)(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 \geq \frac{(1 - \tau_{t-1})\tilde{\theta}_2\epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2$$

Using similar argument as in Case 1, we can show when $\tau_{t-1} \geq 2\tau_2^* - \tau_1^*$, $\tau_t^* = \tau_1^*$.

**Case 3:** $\tau_1^* \leq \tau_{t-1} \leq \tau_2^*$

When $\tau_1^* \leq \tau_{t-1} \leq \tau_2^*$, $T(\tau_{t-1}, \epsilon) = \tau_{t-1}$ and

$$W(\tau_{t-1}, \epsilon) = \frac{(1 - \tau_{t-1})\tilde{\theta}_2\epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 + \beta W(\tau_{t-1}, \epsilon), \epsilon$$

$$= \frac{(1 - \tau_{t-1})\tilde{\theta}_2\epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 + \beta W(\tau_{t-1}, \epsilon)$$

$$= \frac{1}{1 - \beta} \left\{ \frac{(1 - \tau_{t-1})\tilde{\theta}_2\epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 \right\}$$

1) Suppose $\tau_1^* < \tau_t \leq \tau_2^*$. Then $T(\tau_t, \epsilon) = \tau_t$. Then maximization problem can be rewritten as

$$V(\tau_{t-1}, \epsilon) = \max_{\tau_t} \frac{1}{1 - \beta} \left\{ \frac{(1 - \tau_t)\tilde{\theta}_1\epsilon}{\sqrt{2}} + A\tau_t(1 - \tau_t)(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 \right\}$$

s.t.

$$\frac{1}{1 - \beta} \left\{ \frac{(1 - \tau_t)\tilde{\theta}_2\epsilon}{\sqrt{2}} + A\tau_t(1 - \tau_t)(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 \right\} \geq \frac{1}{1 - \beta} \left\{ \frac{(1 - \tau_{t-1})\tilde{\theta}_2\epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 \right\}$$

Again, the problem simplifies into a static problem. Using the fact $\tau_1^* < \tau_{t-1} \leq \tau_2^*$ and the result for one period problem, we get $\tau_t^* = \tau_{t-1}$.

2) Suppose $\tau_t \leq \tau_1^*$. Then $T(\tau_t, \epsilon) = \tau_1^*$. The optimization problem can be simplified into

$$V(\tau_{t-1}, \epsilon) = \max_{\tau_t} \frac{(1 - \tau_t)\tilde{\theta}_1\epsilon}{\sqrt{2}} + A\tau_t(1 - \tau_t)(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 + \frac{\beta}{1 - \beta} \left\{ \frac{(1 - \tau_1^*)\tilde{\theta}_1\epsilon}{\sqrt{2}} + A\tau_1^*(1 - \tau_1^*)(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 \right\}$$

s.t.

$$\frac{(1 - \tau_t)\tilde{\theta}_2\epsilon}{\sqrt{2}} + A\tau_t(1 - \tau_t)(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 + \frac{\beta}{1 - \beta} \left\{ \frac{(1 - \tau_1^*)\tilde{\theta}_2\epsilon}{\sqrt{2}} + A\tau_1^*(1 - \tau_1^*)(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 \right\} \geq$$

$$\frac{1}{1 - \beta} \left\{ \frac{(1 - \tau_{t-1})\tilde{\theta}_2\epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 \right\}$$

If the inequality constraint does not bind, $\tau_t^* = \tau_1^*$. This, however, could not be true since for $\tau_1^* < \tau_{t-1} \leq \tau_2^*$,

$$\frac{1}{1 - \beta} \left\{ \frac{(1 - \tau_{t-1})\tilde{\theta}_2\epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 \right\} > \frac{1}{1 - \beta} \left\{ \frac{(1 - \tau_1^*)\tilde{\theta}_2\epsilon}{\sqrt{2}} + A\tau_1^*(1 - \tau_1^*)(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 \right\}$$
(In fact, $\tau_2^*$ maximizes $u(\tau_l, 2) = \frac{(1-\tau_l)\theta_2}{\sqrt{2}} + A\tau_l(1-\tau_l)(\theta_1^2 + \theta_2^2)c^2$. ) Now consider the case when inequality constraint does bind, i.e.

$$
\frac{(1 - \tau_l)\theta_2}{\sqrt{2}} + A\tau_l(1 - \tau_l)(\theta_1^2 + \theta_2^2)c^2 + \frac{\beta}{1 - \beta} \left\{ \frac{(1 - \tau_l^*)\theta_2}{\sqrt{2}} + A\tau_l^*(1 - \tau_l^*)(\theta_1^2 + \theta_2^2)c^2 \right\} = 
$$

$$
\frac{1}{1 - \beta} \left\{ \frac{(1 - \tau_{l-1})\theta_2}{\sqrt{2}} + A\tau_{l-1}(1 - \tau_{l-1})(\theta_1^2 + \theta_2^2)c^2 \right\}
$$
or

$$
\frac{(1 - \tau_l)\theta_2}{\sqrt{2}} + A\tau_l(1 - \tau_l)(\theta_1^2 + \theta_2^2)c^2 =
$$

$$
\frac{1}{1 - \beta} \left\{ \frac{(1 - \tau_{l-1})\theta_2}{\sqrt{2}} + A\tau_{l-1}(1 - \tau_{l-1})(\theta_1^2 + \theta_2^2)c^2 \right\} - \frac{\beta}{1 - \beta} \left\{ \frac{(1 - \tau_l^*)\theta_2}{\sqrt{2}} + A\tau_l^*(1 - \tau_l^*)(\theta_1^2 + \theta_2^2)c^2 \right\}
$$

Using the fact $\tau_1^* \leq \tau_{l-1}$, we know

$$
\frac{1}{1 - \beta} \left\{ \frac{(1 - \tau_{l-1})\theta_2}{\sqrt{2}} + A\tau_{l-1}(1 - \tau_{l-1})(\theta_1^2 + \theta_2^2)c^2 \right\} - \frac{\beta}{1 - \beta} \left\{ \frac{(1 - \tau_l^*)\theta_2}{\sqrt{2}} + A\tau_l^*(1 - \tau_l^*)(\theta_1^2 + \theta_2^2)c^2 \right\} \geq
$$

$$
\left\{ \frac{(1 - \tau_{l-1})\theta_2}{\sqrt{2}} + A\tau_{l-1}(1 - \tau_{l-1})(\theta_1^2 + \theta_2^2)c^2 \right\}
$$

which implies $\tau_l^* > \tau_{l-1} > \tau_1^*$. This contradicts the assumption $\tau_l \leq \tau_1^*$.

3) Suppose $\tau_2^* < \tau_l \leq 2\tau_2^* - \tau_1^*$. Then $T(\tau_l, \epsilon) = 2\tau_2^* - \tau_l$. The optimization problem can be rewritten as

$$
V(\tau_{l-1}, \epsilon) = \max_{\tau_l} \frac{(1 - \tau_l)\theta_1}{\sqrt{2}} + A\tau_l(1 - \tau_l)(\theta_1^2 + \theta_2^2)c^2 + \frac{\beta}{1 - \beta} \left\{ \frac{[1 - (2\tau_2^* - \tau_l)]\theta_1}{\sqrt{2}} + A(2\tau_2^* - \tau_l)[1 - (2\tau_2^* - \tau_l)](\theta_1^2 + \theta_2^2)c^2 \right\}
$$

s.t.

$$
\frac{(1 - \tau_l)\theta_2}{\sqrt{2}} + A\tau_l(1 - \tau_l)(\theta_1^2 + \theta_2^2)c^2 + \frac{\beta}{1 - \beta} \left\{ \frac{[1 - (2\tau_2^* - \tau_l)]\theta_2}{\sqrt{2}} + A(2\tau_2^* - \tau_l)[1 - (2\tau_2^* - \tau_l)](\theta_1^2 + \theta_2^2)c^2 \right\} \geq
$$

$$
\frac{1}{1 - \beta} \left\{ \frac{(1 - \tau_{l-1})\theta_2}{\sqrt{2}} + A\tau_{l-1}(1 - \tau_{l-1})(\theta_1^2 + \theta_2^2)c^2 \right\}
$$

If inequality constraint does not bind, $\tau_1^* = (1 - 2\beta)\tau_1^* + 2\beta\tau_2^* \in (\tau_2^*, 2\tau_2^* - \tau_1^*)$. It is not difficult to show if $\tau_1^* \in (\tau_2^*, 2\tau_2^* - \tau_{l-1})$, the value function delivered by $\tau_1^*$ is less than that of $\tau_{l-1}$ [draw graph].

If constraint binds, note the constraint can simplify to

$$
\frac{(1 - \tau_l)\theta_2}{\sqrt{2}} + A\tau_l(1 - \tau_l)(\theta_1^2 + \theta_2^2)c^2 = \frac{(1 - \tau_{l-1})\theta_2}{\sqrt{2}} + A\tau_{l-1}(1 - \tau_{l-1})(\theta_1^2 + \theta_2^2)c^2
$$

which implies $\tau_l^* = 2\tau_2^* - \tau_{l-1}$ and

$$
V(\tau_{l-1}, \epsilon) = \frac{[1 - (2\tau_2^* - \tau_{l-1})]\theta_1}{\sqrt{2}} + A(2\tau_2^* - \tau_{l-1})[1 - (2\tau_2^* - \tau_{l-1})](\theta_1^2 + \theta_2^2)c^2 + \frac{\beta}{1 - \beta} \left\{ \frac{(1 - \tau_{l-1})\theta_1}{\sqrt{2}} + A\tau_{l-1}(1 - \tau_{l-1})(\theta_1^2 + \theta_2^2)c^2 \right\}
$$
We can show that as long as \( \tau_1^* < \tau_{t-1} \leq \tau_2^* \), above value function is less than the value function derived with \( \tau_t^* = \tau_{t-1} \). We can view above \((1 - \beta)V(\tau_{t-1}, \epsilon)\) as a linear combination of \( u(\tau_{t-1}) \) and \( u(2\tau_2^* - \tau_{t-1}) \), which is less than \( u(\tau_{t-1}) \).

4) Suppose \( \tau_t \geq 2\tau_2^* - \tau_1^* \). Then \( T(\tau_t, \epsilon) = \tau_1^* \). The optimization problem simplifies into same as 2).

If the inequality constraint does not bind, \( \tau_t^* = \tau_1^* \), which contradicts with the assumption.

If the inequality constraint does bind, we can show by argument similar to 2) that \( \tau_t^* < 2\tau_2^* - \tau_{t-1} < 2\tau_2^* - \tau_1^* \), which contradicts our assumption.

Case 4: \( \tau_2^* < \tau_{t-1} < 2\tau_2^* - \tau_1^* \). Since \( \tau_2^* < \tau_{t-1} < 2\tau_2^* - \tau_1^* \), \( T(\tau_{t-1}, \epsilon) = 2\tau_2^* - \tau_{t-1} \). Hence

\[
W(\tau_{t-1}, \epsilon) = \frac{(1 - \tau_{t-1})\bar{\theta}_2\epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 + \beta W(T(\tau_{t-1}, \epsilon), 2)
\]

\[
= \frac{(1 - \tau_{t-1})\bar{\theta}_2\epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 + \beta W(2\tau_2^* - \tau_{t-1}, 2)
\]

\[
= \frac{(1 - \tau_{t-1})\bar{\theta}_2\epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 + \frac{\beta}{1 - \beta} \left\{ \frac{1 - (2\tau_2^* - \tau_{t-1})}{\sqrt{2}} \bar{\theta}_2\epsilon + A(2\tau_2^* - \tau_{t-1})[1 - (2\tau_2^* - \tau_{t-1})](\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right\}
\]

1) Suppose \( \tau_t^* < \tau_t \leq \tau_2^* \). Then \( T(\tau_t, \epsilon) = \tau_t \). The optimization problem can be rewritten as

\[
V(\tau_t, \epsilon) = \max_{\tau_t} \frac{1}{1 - \beta} \left\{ \frac{(1 - \tau_{t-1})\bar{\theta}_1\epsilon}{\sqrt{2}} + A\tau_t(1 - \tau_t)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right\}
\]

s.t.

\[
\frac{1}{1 - \beta} \left\{ \frac{(1 - \tau_t)\bar{\theta}_2\epsilon}{\sqrt{2}} + A\tau_t(1 - \tau_t)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right\} \geq \frac{(1 - \tau_{t-1})\bar{\theta}_2\epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 + \frac{\beta}{1 - \beta} \left\{ \frac{1 - (2\tau_2^* - \tau_{t-1})}{\sqrt{2}} \bar{\theta}_2\epsilon + A(2\tau_2^* - \tau_{t-1})[1 - (2\tau_2^* - \tau_{t-1})](\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right\}
\]

Note the inequality constraint can be rewritten as

\[
\frac{(1 - \tau_t)\bar{\theta}_2\epsilon}{\sqrt{2}} + A\tau_t(1 - \tau_t)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \geq \frac{(1 - \tau_{t-1})\bar{\theta}_2\epsilon}{\sqrt{2}} + A\tau_{t-1}(1 - \tau_{t-1})(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 + \frac{\beta}{1 - \beta} \left\{ \frac{1 - (2\tau_2^* - \tau_{t-1})}{\sqrt{2}} \bar{\theta}_2\epsilon + A(2\tau_2^* - \tau_{t-1})[1 - (2\tau_2^* - \tau_{t-1})](\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right\}
\]

If inequality constraint does not bind, then \( \tau_t^* = \tau_1^* \), which contradicts the assumption \( \tau_1^* < \tau_t \leq \tau_2^* \). If inequality constraint does bind, then \( \tau_t^* = 2\tau_2^* - \tau_{t-1} \in (\tau_1^*, \tau_2^*) \).
2) Suppose $\tau_t \leq \tau_1^*$. Then $T(\tau_t, \epsilon) = \tau_1^*$. The optimization problem can be rewritten as

$$
V(\tau_{t-1}, \epsilon) = \max_{\tau_t} \frac{(1-\tau_t)\theta_t \epsilon}{\sqrt{2}} + A\tau_t(1-\tau_t)(\theta_1^2 + \theta_2^2)\epsilon^2 + \frac{\beta}{1-\beta} \left\{ \frac{(1-\tau_1^*)\theta_1 \epsilon}{\sqrt{2}} + A\tau_1^*(1-\tau_1^*)(\theta_1^2 + \theta_2^2)\epsilon^2 \right\}
$$

s.t.

$$
\frac{(1-\tau_t)\theta_2 \epsilon}{\sqrt{2}} + A\tau_t(1-\tau_t)(\theta_1^2 + \theta_2^2)\epsilon^2 + \frac{\beta}{1-\beta} \left\{ \frac{(1-\tau_1^*)\theta_2 \epsilon}{\sqrt{2}} + A\tau_1^*(1-\tau_1^*)(\theta_1^2 + \theta_2^2)\epsilon^2 \right\} \geq \frac{1}{1-\beta} \left\{ \frac{(1-\tau_{t-1})\theta_2 \epsilon}{\sqrt{2}} + A\tau_{t-1}(1-\tau_{t-1})(\theta_1^2 + \theta_2^2)\epsilon^2 \right\}
$$

If the inequality constraint does not bind, $\tau_t^* = \tau_1^*$. This, however, violates the inequality constraint.

If the inequality constraint binds, $\tau_t^* \in (2\tau_2^* - \tau_{t-1}, \tau_{t-1})$. This contradicts $\tau_t \leq \tau_1^*$ since $\tau_{t-1} \in (\tau_1^*, 2\tau_2^* - \tau_1^*)$. To show $\tau_t^* \in (2\tau_2^* - \tau_{t-1}, \tau_{t-1})$, note that

$$
\frac{(1-\tau_t)\theta_2 \epsilon}{\sqrt{2}} + A\tau_t(1-\tau_t)(\theta_1^2 + \theta_2^2)\epsilon^2 = \frac{1}{1-\beta} \left\{ \frac{(1-\tau_{t-1})\theta_2 \epsilon}{\sqrt{2}} + A\tau_{t-1}(1-\tau_{t-1})(\theta_1^2 + \theta_2^2)\epsilon^2 \right\} - \frac{\beta}{1-\beta} \left\{ \frac{(1-\tau_1^*)\theta_2 \epsilon}{\sqrt{2}} + A\tau_1^*(1-\tau_1^*)(\theta_1^2 + \theta_2^2)\epsilon^2 \right\}
$$

which implies $\tau_t^* \in (2\tau_2^* - \tau_{t-1}, \tau_{t-1})$.

3) Suppose $\tau_t \geq 2\tau_2^* - \tau_1^*$. Then $T(\tau_t, \epsilon) = \tau_1^*$. The optimization problem can be rewritten as same in 2).

If the inequality constraint does not bind, $\tau_t^* = \tau_1^*$. Again, this violates the inequality constraint.

If the inequality constraint binds, similar to 2), $\tau_t^* \in (2\tau_2^* - \tau_{t-1}, \tau_{t-1})$. Since $\tau_{t-1} < 2\tau_2^* - \tau_1^*$, this contradicts with our assumption.

4) Suppose $\tau_2^* < \tau_t < 2\tau_2^* - \tau_1^*$. Then $T(\tau_t, \epsilon) = 2\tau_2^* - \tau_t$. The optimization problem becomes

$$
V(\tau_{t-1}, \epsilon) = \max_{\tau_t} \frac{(1-\tau_t)\theta_t \epsilon}{\sqrt{2}} + A\tau_t(1-\tau_t)(\theta_1^2 + \theta_2^2)\epsilon^2 + \frac{\beta}{1-\beta} \left\{ \frac{1}{A}(2\tau_2^* - \tau_t)(1-(2\tau_2^* - \tau_t))(\theta_1^2 + \theta_2^2)\epsilon^2 \right\}
$$

s.t.

$$
\frac{(1-\tau_t)\theta_2 \epsilon}{\sqrt{2}} + A\tau_t(1-\tau_t)(\theta_1^2 + \theta_2^2)\epsilon^2 \geq \frac{(1-\tau_{t-1})\theta_2 \epsilon}{\sqrt{2}} + A\tau_{t-1}(1-\tau_{t-1})(\theta_1^2 + \theta_2^2)\epsilon^2
$$

If the inequality constraint does not bind, then $\tau_t^* = (1-2\beta)\tau_1^* + 2\beta\tau_2^*$. We can show that $\tau_t^* > \tau_2^*$ and $\tau_t^* < 2\tau_2^* - \tau_1^*$. If $\tau_t^* > \tau_{t-1}$, inequality constraint is violated. If $\tau_t^* < \tau_{t-1}$, we can show that the life-time utility delivered by this policy is less than that delivered by $\tau_t^* = 2\tau_2^* - \tau_{t-1}$. [To show above argument, note $(1-\beta)V(t_{t-1}, \epsilon) = (1-\beta)u[(1-2\beta)\tau_1^* + 2\beta\tau_2^*] + \beta u[2\tau_2^* - ((1-2\beta)\tau_1^* + 2\beta\tau_2^*)] < u(2\tau_2^* - \tau_{t-1})].$

If the inequality constraint binds, $\tau_t^* = \tau_{t-1}$. Again, we can show the life-time utility delivered by this policy is less
than that delivered by \( \tau^*_t = 2\tau^*_2 - \tau_{t-1} \). [To show above argument, note \((1-\beta)u(\tau_{t-1})+\beta u[2\tau^*_2 - \tau_{t-1}]] < u(2\tau^*_2 - \tau_{t-1}) \].

\[\] \[\]

**A.6 Corollary 4**

**Corollary 8 (closest-to-ideal-policy rule).** Assume there is no political turnover and the productivity shock stays constant (i.e. \( \epsilon_t \equiv \epsilon \forall t \)). The equilibrium tax policy is the policy that is closest to proposer’s ideal tax level within the constraint set.

More specifically, suppose the rich is the incumbent. Denote the constraint set for arbitrary status quo \( \tau_{t-1} \in [0,1] \) as \( C(\tau_{t-1}, \epsilon) \). Let the ideal tax policy of the rich be \( \tau^*_1 \) and the equilibrium tax policy by the incumbent be \( T(\tau_{t-1}, \epsilon) \). Then for all \( \tau_{t-1} \in [0,1] \), there does not exist any \( \tau \in C(\tau_{t-1}, \epsilon) \) such that \( \|\tau - \tau^*_1\| < \|T(\tau_{t-1}, \epsilon) - \tau^*_1\| \). In addition, when the incentive compatibility constraint for the opponent binds, \( T(\tau_{t-1}, \epsilon) = \min C(\tau_{t-1}, \epsilon) \).

**Proof.** Recall the equilibrium tax policy of the rich is given by

\[
T(\tau_{t-1}, \epsilon) = \begin{cases} 
\tau^*_1 & \text{if } \tau_{t-1} \leq \tau^*_1 \\
\tau_{t-1} & \text{if } \tau^*_1 < \tau_{t-1} \leq \tau^*_2 \\
2\tau^*_2 - \tau_{t-1} & \text{if } \tau^*_2 < \tau_{t-1} < 2\tau^*_2 - \tau^*_1 \\
\tau^*_1 & \text{if } \tau_{t-1} \geq 2\tau^*_2 - \tau^*_1 
\end{cases}
\]

where \( \tau^*_1 = \frac{1}{2} \left[1 - \frac{\sqrt{2\theta_1}}{A(\theta_1^2 + \theta_2^2)\epsilon}\right] \) and \( \tau^*_2 = \frac{1}{2} \left[1 - \frac{\sqrt{2\theta_2}}{A(\theta_1^2 + \theta_2^2)\epsilon}\right] \).

(1) For \( \tau_{t-1} \leq \tau^*_1 \) and \( \tau_{t-1} \geq 2\tau^*_2 - \tau^*_1 \), the above result is obvious. This is because \( T(\tau_{t-1}) = \tau^*_1 \) hence \( \|T(\tau_{t-1}, \epsilon) - \tau^*_1\| = 0 \).

(2) For \( \tau^*_1 < \tau_{t-1} \leq \tau^*_2 \), the incentive compatibility constraint binds. Therefore, \( C(\tau_{t-1}, \epsilon) = \{ \tau : \tau \in [\tau^*_1, 2\tau^*_2 - \tau_{t-1}] \} \). Since \( T(\tau_{t-1}, \epsilon) = \tau_{t-1} \), and \( \tau_{t-1} = \min C(\tau_{t-1}, \epsilon) \), we know \( \|\tau - \tau^*_1\| < \|\tau_{t-1} - \tau^*_1\| \) for all \( \tau \in C(\tau_{t-1}, \epsilon) \).

(3) For \( \tau^*_2 < \tau_{t-1} < 2\tau^*_2 - \tau^*_1 \), the incentive compatibility constraint also binds. \( C(\tau_{t-1}, \epsilon) = \{ \tau : \tau \in [2\tau^*_2 - \tau_{t-1}, \tau_{t-1}] \} \). Since \( T(\tau_{t-1}, \epsilon) = 2\tau^*_2 - \tau_{t-1} \), and \( 2\tau^*_2 - \tau_{t-1} = \min C(\tau_{t-1}, \epsilon) \), we know \( \|\tau - \tau^*_1\| < \|2\tau^*_2 - \tau_{t-1} - \tau^*_1\| \) for all \( \tau \in C(\tau_{t-1}, \epsilon) \). Summarizing up above three scenarios, we have for all \( \tau_{t-1} \in [0,1] \), there does not exist any \( \tau \in C(\tau_{t-1}, \epsilon) \) such that \( \|\tau - \tau^*_1\| < \|T(\tau_{t-1}, \epsilon) - \tau^*_1\| \).

\[\]

**A.7 Proposition 2**

**Proposition 4.** Suppose there is no political turnover. Assume productivity realization follows \( \epsilon_t \equiv 1 \) and \( \epsilon_t \equiv \epsilon < 1 \), \( \forall t > 1 \). The equilibrium tax policy at \( t = 1 \) may involve policy maker sacrificing well being at \( t = 1 \) for purpose of higher welfare in the future.

In particular, suppose the rich is the incumbent. Denote the constraint set at \( t = 1 \) for arbitrary status quo tax rate \( \tau_0 \in [0,1] \) as \( \tilde{C}(\tau_0, \epsilon) \) and the equilibrium tax policy \( T(\tau_0, \epsilon) \). When the incentive compatibility constraint for the opponent binds, there may exist certain \( \tau_0 \in [0,1] \) such that \( T(\tau_0, \epsilon) = \max \tilde{C}(\tau_0, \epsilon) \).

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Proof. For purpose of showing the existence of ‘far-sighted’ equilibrium tax policy, I will first solve for the equilibrium tax policy over \([\tau_1^*, \tau_2^*]\). Then I will show for certain \(\tau_0 \in [\tau_1^*, \tau_2^*]\), the equilibrium tax policy fails to be the policy closest to the ideal of the incumbent.

Recall the problem of infinite horizon economy with distinct productivity realizations for first and rest periods. Assume the rich is the incumbent. The optimization problem is given by

\[
\max_{\tau_1} \sqrt{2}(1 - \tau_1)\theta_1 + A\tau_1(1 - \tau_1)(\theta_1^2 + \theta_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2}(1 - T(\tau_1, \epsilon))\bar{\theta}_1\epsilon + AT(\tau_1, \epsilon)(1 - T(\tau_1, \epsilon))(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right]
\]

s.t.

\[
\sqrt{2}(1 - \tau_1)\bar{\theta}_2 + A\tau_1(1 - \tau_1)(\bar{\theta}_1^2 + \bar{\theta}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2}(1 - T(\tau_1, \epsilon))\bar{\theta}_2\epsilon + AT(\tau_1, \epsilon)(1 - T(\tau_1, \epsilon))(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right] \geq 0
\]

0 \leq \tau \leq 1

where

\[
T(\tau, \epsilon) = \begin{cases} 
\tau_1^* & \text{if } \tau \leq \tau_1^* \\
\tau & \text{if } \tau_1^* < \tau \leq \tau_2^* \\
2\tau_2^* - \tau & \text{if } \tau_2^* < \tau < 2\tau_2^* - \tau_1^* \\
\tau_1^* & \text{if } \tau \geq 2\tau_2^* - \tau_1^* 
\end{cases}
\]

with \(\tau_1^* = \frac{1}{2} \left[ 1 - \frac{\sqrt{2}\bar{\theta}_1}{A(\theta_1^2 + \theta_2^2)\epsilon} \right]\) and \(\tau_2^* = \frac{1}{2} \left[ 1 - \frac{\sqrt{2}\bar{\theta}_2}{A(\theta_1^2 + \theta_2^2)\epsilon} \right]\).

Assuming \(\tau_0 \in [\tau_1^*, \tau_2^*]\), we know \(T(\tau_0, \epsilon) = \tau_0\). Therefore, the incentive compatibility constraint is simplified into

\[
\sqrt{2}(1 - \tau_1)\bar{\theta}_2 + A\tau_1(1 - \tau_1)(\bar{\theta}_1^2 + \bar{\theta}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2}(1 - T(\tau_1, \epsilon))\bar{\theta}_2\epsilon + AT(\tau_1, \epsilon)(1 - T(\tau_1, \epsilon))(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right] \geq 0
\]

Since \(T(\tau_1, \epsilon)\) may take different functional forms depending on the value of \(\tau_1\), I’ll proceed by solving the equilibrium \(\tau_1\) under alternative assumptions. Then I will check whether the \(\tau_1\) obtained is consistent with the assumption. If more than one valid \(\tau_1\) is found through different \(T(\tau_1, \epsilon)\) assumption, the equilibrium \(\tau_1\) will be determined by the one which maximizes the rich’s objective.

**Case 1:** \(\tau_1 \leq \tau_1^*\)
Under above assumption, we know $T(\tau_1, \epsilon) = \tau_1^*$. This implies the rich’s optimization problem can be rewritten as

$$\max_{\tau_1} \sqrt{2}(1 - \tau_1)\bar{\theta}_1 + A\tau_1(1 - \tau_1)(\bar{\theta}_1^2 + \bar{\theta}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2}(1 - \tau_1^*)\bar{\theta}_1\epsilon + A\tau_1^*(1 - \tau_1^*)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right]$$

s.t.

$$\sqrt{2}(1 - \tau_1)\bar{\theta}_2 + A\tau_1(1 - \tau_1)(\bar{\theta}_1^2 + \bar{\theta}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2}(1 - \tau_1^*)\bar{\theta}_2\epsilon + A\tau_1^*(1 - \tau_1^*)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right] \geq 0 \leq \tau \leq 1$$

(1) Suppose the incentive compatibility constraint is not binding, the ideal tax policy for the rich is

$$\tau_1^{o1} = \frac{1}{2} \left[ 1 - \frac{\sqrt{2}\bar{\theta}_1}{A(\bar{\theta}_1^2 + \bar{\theta}_2^2)} \right] > \tau_1^*$$

(2) Suppose the incentive compatibility constraint does bind, using the fact $\tau_1^* < \tau_0 < \tau_2^*$, it is not difficult to show the equilibrium tax policy is greater than $\tau_0$. Note, however, both above results are invalid since they violate the assumption that $\tau_1 < \tau_1^*$.

Case 2: $\tau_1^* < \tau_1 < \tau_2^*$

Under above assumption, we know $T(\tau_1, \epsilon) = \tau_1$. This implies the rich’s optimization problem can be rewritten as

$$\max_{\tau_1} \sqrt{2}(1 - \tau_1)\bar{\theta}_1 + A\tau_1(1 - \tau_1)(\bar{\theta}_1^2 + \bar{\theta}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2}(1 - \tau_1)\bar{\theta}_1\epsilon + A\tau_1(1 - \tau_1)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right]$$

s.t.

$$\sqrt{2}(1 - \tau_1)\bar{\theta}_2 + A\tau_1(1 - \tau_1)(\bar{\theta}_1^2 + \bar{\theta}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2}(1 - \tau_1)\bar{\theta}_2\epsilon + A\tau_1(1 - \tau_1)(\bar{\theta}_1^2 + \bar{\theta}_2^2)\epsilon^2 \right] \geq 0 \leq \tau \leq 1$$

(1) Suppose the incentive compatibility constraint does not bind, the ideal tax policy for the rich would be

$$\tau_1^{o2} = \frac{1}{2} \left[ 1 - \frac{(1 + \frac{\beta}{1 - \epsilon^2})\sqrt{2}\bar{\theta}_1}{(1 + \frac{\beta}{1 - \epsilon^2})A(\bar{\theta}_1^2 + \bar{\theta}_2^2)} \right]$$

Similarly, suppose the poor is the incumbent, its ideal tax policy is

$$\tau_2^{o2} = \frac{1}{2} \left[ 1 - \frac{(1 + \frac{\beta}{1 - \epsilon^2})\sqrt{2}\bar{\theta}_2}{(1 + \frac{\beta}{1 - \epsilon^2})A(\bar{\theta}_1^2 + \bar{\theta}_2^2)} \right]$$

It is not difficult to show $\tau_1^{o2} > \tau_1^*$, $\tau_2^{o2} > \tau_2^*$ and $\tau_2^{o2} > \tau_1^{o2}$.

(2) Now consider the scenario when incentive compatibility constraint binds. When constraint binds, there are two
roots to the equality: \( \tau_0 \) and \( 2\tau_2^* - \tau_0 \). Combining results from (1) and (2), we obtain the equilibrium tax policy \( \tilde{T}^2(\tau_0, \epsilon) \) for period \( t = 1 \) is as follows

\[
\tilde{T}^2(\tau_0, \epsilon) = \begin{cases} 
\tau_1^{o2} & \text{if } \tau_0 \leq \tau_1^{o2} \\
\tau_0 & \text{if } \tau_1^{o2} < \tau_0 \leq \tau_2^{o2} \\
2\tau_2^* - \tau_0 & \text{if } \tau_2^{o2} < \tau_0 < 2\tau_2^* - \tau_1^{o2} \\
\tau_2^{o2} & \text{if } \tau_0 \geq 2\tau_2^* - \tau_1^{o2}
\end{cases}
\]

Now check the validity of this solution. Since we assumed \( \tau_1^* < \tau_0 < \tau_2^* \), there are following two possibilities:

1) if \( \tau_1^{o2} < \tau_2^* \)

\[
\tilde{T}^2(\tau_0, \epsilon) = \begin{cases} 
\tau_1^{o2} & \text{if } \tau_1^* \leq \tau_0 \leq \tau_1^{o2} \\
\tau_0 & \text{if } \tau_1^{o2} < \tau_0 \leq \tau_2^* 
\end{cases}
\]

2) if \( \tau_1^{o2} \geq \tau_2^* \)

\[
\tilde{T}^2(\tau_0, \epsilon) = \tau_1^{o2} \text{ if } \tau_1^* \leq \tau_0 \leq \tau_2^*
\]

However, the second scenario is invalid. This is because in this case, \( \tilde{T}^2(\tau_0, \epsilon) > \tau_1^* \), violating the assumption \( \tau_1^* < \tau_1 < \tau_2^* \). Therefore, the equilibrium policy under the assumption \( \tau_1^* < \tau_1 < \tau_2^* \) is

\[
\tilde{T}^2(\tau_0, \epsilon) = \begin{cases} 
\tau_1^{o2} & \text{if } \tau_1^* \leq \tau_0 \leq \tau_1^{o2} \\
\tau_0 & \text{if } \tau_1^{o2} < \tau_0 \leq \tau_2^* 
\end{cases}
\]

and this requires \( \tau_1^{o2} < \tau_2^* \).

**Case 3:** \( \tau_2^* < \tau_1 < 2\tau_2^* - \tau_1^* \) Under this assumption, we know \( T(\tau_1, \epsilon) = 2\tau_2^* - \tau_1 \). This implies the rich’s optimization problem is as follows:

\[
\max_{\tilde{\tau}_1} \sqrt{2}(1 - \tilde{\tau}_1)\tilde{\theta}_1 + A\tau_1(1 - \tilde{\tau}_1)(\tilde{\theta}_1^2 + \tilde{\theta}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2}(1 - (2\tau_2^* - \tau_1))\tilde{\theta}_1\epsilon + A(2\tau_2^* - \tau_1)[1 - (2\tau_2^* - \tau_1)](\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 \right]
\]

s.t.

\[
\sqrt{2}(1 - \tilde{\tau}_1)\tilde{\theta}_2 + A\tau_1(1 - \tilde{\tau}_1)(\tilde{\theta}_1^2 + \tilde{\theta}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2}(1 - \tau_0)\tilde{\theta}_2\epsilon + A\tau_0(1 - \tau_0)(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)\epsilon^2 \right] \geq 0
\]

Note for the incentive compatibility constraint, I invoked symmetry between \( \tau_1 \) and \( 2\tau_2^* - \tau_1 \) for poor’s welfare in the future.

1) Suppose the incentive compatibility constraint does not bind, the ideal tax policy for the rich would be

\[
\tau_1^{o3} = \frac{1}{2} - \frac{2\sqrt{2}\tilde{\theta}_2 - \beta\epsilon + \sqrt{2}\tilde{\theta}_1(1 - \frac{\beta}{1 - \beta}\epsilon)}{2A(\tilde{\theta}_1^2 + \tilde{\theta}_2^2)(1 + \frac{\beta}{1 - \beta}\epsilon^2)}
\]
Maximizing the objective of the poor, we get

\[ \tau_2^{o3} = \tau_2^{o2} = \frac{1}{2} \left[ 1 - \frac{(1 + \frac{\beta}{1-\beta} \epsilon) \sqrt{2\theta_2}}{(1 + \frac{\beta}{1-\beta} \epsilon^2) A(\theta_1 + \theta_2)} \right] \]

Taking difference between \( \tau_1^{o3} \) and \( \tau_2^{o3} \), we obtain

\[
\begin{cases} 
\tau_1^{o3} \geq \tau_2^{o3} & \text{if } \epsilon \geq \frac{1-\beta}{\beta} \\
\tau_1^{o3} < \tau_2^{o3} & \text{if } \epsilon < \frac{1-\beta}{\beta}
\end{cases}
\]

Suppose the incentive compatibility constraint binds, there are two roots for the equality \( \tau_0 \) and \( 2\tau_2^{o3} - \tau_0 \). Combining results from (1) and (2), we obtain depending on parameter assumptions, the equilibrium tax policy can be as follows

1) if \( \epsilon < \frac{1-\beta}{\beta} \)

\[
\tilde{T}_3(\tau_0, \epsilon) = \begin{cases} 
\tau_1^{o3} & \text{if } \tau_0 \leq \tau_1^{o3} \\
\tau_0 & \text{if } \tau_1^{o3} < \tau_0 \leq \tau_2^{o3} \\
2\tau_2^{o3} - \tau_0 & \text{if } \tau_2^{o3} < \tau_0 \leq 2\tau_2^{o3} - \tau_1^{o3} \\
\tau_1^{o3} & \text{if } \tau_0 > 2\tau_2^{o3} - \tau_1^{o3}
\end{cases}
\]

Now check the validity of this solution. Using \( \epsilon < \frac{1-\beta}{\beta} \), we can show that \( \tau_1^* < \tau_1^{o3} < \tau_2^* \). Under the assumption \( \tau_1^* < \tau_0 < \tau_2^* \), we have

\[
\tilde{T}_3(\tau_0, \epsilon) = \begin{cases} 
\tau_1^{o3} & \text{if } \tau_1^* < \tau_0 \leq \tau_1^{o3} \\
\tau_0 & \text{if } \tau_1^{o3} < \tau_0 \leq \tau_2^*
\end{cases}
\]

However, this violate the assumption that \( \tau_1 \in [\tau_2^*, 2\tau_2^* - \tau_1^*] \), hence the solution above is not a valid solution.

2) if \( \epsilon > \frac{1-\beta}{\beta} \)

\[
\tilde{T}_3(\tau_0, \epsilon) = \begin{cases} 
\tau_1^{o3} & \text{if } \tau_0 \leq 2\tau_2^{o3} - \tau_1^{o3} \\
2\tau_2^{o3} - \tau_0 & \text{if } 2\tau_2^{o3} - \tau_1^{o3} < \tau_0 \leq \tau_2^{o3} \\
\tau_0 & \text{if } \tau_2^{o3} < \tau_0 \leq \tau_1^{o3} \\
\tau_1^{o3} & \text{if } \tau_0 > \tau_1^{o3}
\end{cases}
\]

Depending on parameter assumptions, for \( \tau_1^* < \tau_0 < \tau_2^* \), the valid equilibrium tax policy may take two different forms:

A) if \( \frac{2\epsilon}{1-\epsilon} \leq \frac{\eta_1}{\eta_2} < \frac{1 + \frac{\beta}{1-\beta} \epsilon^2 - 2\epsilon}{\epsilon(\frac{1}{1-\epsilon} - 1)} \)

\[
\tilde{T}_3(\tau_0, \epsilon) = \tau_1^{o3} \text{ if } \tau_1^* < \tau_0 < \tau_2^*
\]
B) if \( \frac{\bar{b}_2}{\bar{b}_1} \geq \frac{1 + \sqrt{2]}(1 + \bar{b}^2 - 2\bar{b})}{\sqrt{1 + \bar{b}^2 - 1}} \)

\[
\tilde{T}^3(\tau_0, \epsilon) = \begin{cases} 
\tau_1^0 & \text{if } \tau_1^0 < \tau_0 \leq 2\tau_2^0 - \tau_1^0 \\
2\tau_2^0 - \tau_0 & \text{if } 2\tau_2^0 - \tau_1^0 < \tau_0 < \tau_2^0
\end{cases}
\]

**Case 4:** \( \tau_1 \geq 2\tau_2^0 - \tau_1^* \)

Under above assumption, we know \( T(\tau_1, \epsilon) = \tau_1^* \). This implies the rich’s optimization problem can be rewritten as

\[
\max_{\tau_1} \sqrt{2}(1 - \tau_1)\bar{b}_1 + A\tau_1(1 - \tau_1)(\bar{b}_1^2 + \bar{b}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2}(1 - \tau_1^*)\bar{b}_1 \epsilon + A\tau_1^*(1 - \tau_1^*)(\bar{b}_1^2 + \bar{b}_2^2)\epsilon^2 \right] \\
\text{s.t.} \\
\sqrt{2}(1 - \tau_1)\bar{b}_2 + A\tau_1(1 - \tau_1)(\bar{b}_1^2 + \bar{b}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2}(1 - \tau_1^*)\bar{b}_2 \epsilon + A\tau_1^*(1 - \tau_1^*)(\bar{b}_1^2 + \bar{b}_2^2)\epsilon^2 \right] \geq 0 \\
0 \leq \tau \leq 1
\]

Following similar argument as in Case 1, we can also rule out the results from Case 4 as possible solution of the problem.

Now I will show it is possible to have equilibrium solution that is not the minimum policy available in the constraint set. Consider the scenario when \( 2\tau_2^0 - \tau_1^0 < \tau_0 < \tau_2^* \) and \( \epsilon > \frac{1 - \beta}{\bar{b}^2} \). There are two potential solutions to the problem

\[
\tilde{T}^2(\tau_0, \epsilon) = \tau_0 \quad \tilde{T}^3(\tau_0, \epsilon) = 2\tau_2^0 - \tau_0
\]

To determine which one is the equilibrium solution, we simply plug the above two solutions into the objective of the rich and compare which delivers the highest welfare. Comparing above two policies, we obtain there exist a cutoff point \( \bar{\tau} \) such that

\[
\tilde{T}(\tau_0, \epsilon) = \begin{cases} 
\tilde{T}^2(\tau_0, \epsilon) & \text{if } \tau_0 \leq \bar{\tau} \\
\tilde{T}^3(\tau_0, \epsilon) & \text{if } \tau_0 > \bar{\tau}
\end{cases}
\]

where

\[
\bar{\tau} = \frac{2\sqrt{2}\tau_2^0 \bar{b}_1 - 2A\tau_2^0(1 - \tau_2^0)(\bar{b}_1^2 + \bar{b}_2^2) + \frac{\beta}{1 - \beta} \left[ \sqrt{2}(2\tau_2^* - 2\tau_2^0)\bar{b}_1 \epsilon - A(2\tau_2^* - 2\tau_2^0)(1 - 2\tau_2^2 + 2\tau_2^0)^2(\bar{b}_1^2 + \bar{b}_2^2)\epsilon^2 \right]}{2\sqrt{2}\bar{b}_1 - 2A\bar{b}_1^2 + 2\tau_2^0 - 1 + \frac{\beta}{1 - \beta} \epsilon^2(2\tau_2^* - 2\tau_2^0)^2} - A(\bar{b}_1^2 + \bar{b}_2^2)\frac{\beta}{1 - \beta} \epsilon^2
\]

Recall in Case 3, when incentive compatibility constraint binds, the constraint set \( \tilde{C}(\tau_0, \epsilon) \) is characterized by \( \tilde{C}(\tau_0, \epsilon) = [\tau_0, 2\tau_2^0 - \tau_0] \). Therefore, when \( \bar{\tau} < \tau_0 < \tau_2^* \), \( \tilde{T}(\tau_0, \epsilon) = 2\tau_2^0 - \tau_0 = \max \tilde{C}(\tau_0, \epsilon) \).
B Sensitivity Analysis

In this section, I perform robustness checks on preference parameter $\xi$. Recall the policy function of the rich and the poor in Figure 13. Since the government spending over GDP (i.e., the tax rate) goes up in recessions and goes down in booms, the tax policy by both potential incumbents is countercyclical. This, however, depends on the parameterization of $\xi$. Since households obtain utility from both private and public good consumption, they are trading off potential utility gain from private good consumption with that from public good consumption. When $\xi$ is relatively low, households place less emphasis on public good consumption. Hence when economy is bad, households simply give up public good consumption for sake of higher private good consumption. When $\xi$ is relatively high, households place more emphasis on public good consumption. Hence when economy is bad, households simply give up some of their private good consumption for purpose of utility gain from public good provision. In Figure 21 I plot the policy function of the rich with $\xi = 0.8$, $\xi = 0.5$ and $\xi = 0.2$. As explained above, when $\xi$ is high, the policy is countercyclical and when $\xi$ is low, the policy is procyclical. In particular, when $\xi = 0.5$, the two effect completely cancel out each other and the policy is independent of the state of the economy.

Figure 21: Role of $\xi$
C Power Persistence

To figure out power persistence parameter $p$, I compute separately the probability of staying in office for both Republican and Democrats by looking at historical party control of White house and both houses of United States Congress. In the table below, I summarize the the computed probability of Republican and Democrat staying in control of White house, Senate and House using party control information of U.S. government from 1969 to 2012.

Table 5: Summary of Power Persistence

<table>
<thead>
<tr>
<th>Power Persistence</th>
<th>White House</th>
<th>Senate</th>
<th>House</th>
</tr>
</thead>
<tbody>
<tr>
<td>Republican</td>
<td>0.79</td>
<td>0.63</td>
<td>0.75</td>
</tr>
<tr>
<td>Democrat</td>
<td>0.75</td>
<td>0.79</td>
<td>0.87</td>
</tr>
</tbody>
</table>