Optimal Dynamic Contracts in Financial Intermediation: With an Application to Venture Capital Financing

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Monitoring and financial intermediation

- Monitoring is done by specialized agents (e.g., venture capitalists, bankers) in presence of other investors:

  Investors → Intermediary → Entrepreneur
  (pension funds) → (venture capitalists) → (startups)
  (depositors) → (bankers) → (borrowers)
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- Entrepreneurs form long-term relationships with intermediaries (i.e., use dynamic long-term contracts)
Environment

- Entrepreneur has project that needs financing
- Intermediary and entrepreneur are constrained in wealth
- Project has multiple investment/payoff periods
- Entrepreneur can divert resources or shirk
- Intermediary can monitor at cost (as in Costly State Verification)
- Investor does not observe whether intermediary enforces no diversion or shares benefits
Contracts

Contracts specify:

▶ state-contingent compensations, monitoring, and liquidation
▶ dependence on history

Characterize optimal dynamic contracts that:

▶ Motivate entrepreneur to work efficiently
▶ Motivate intermediary to monitor diligently
▶ Give sufficient return to investor
▶ Minimize monitoring and liquidation costs
Results: seniority structure

Seniority structure:
- Low payoff $\implies$ project is liquidated and only investor is paid
- Medium payoff $\implies$ intermediary monitors and shares additional payoff
- High payoff $\implies$ entrepreneur shares additional payoff

Implementation:
- Entrepreneur holds equity
- Intermediary holds junior debt (triggers monitoring if not paid)
- Investors hold senior debt (triggers liquidation if not paid) + share of junior debt and equity
Results: dynamics

Bad *intermediate* performance gives more control (monitoring) and cash flow rights to intermediaries

- Covenants and clauses: (VC) Kaplan and Stromberg [2003], (banking) Nini, Smith and Sufi [2009]
- Maturity structure: Implementable with debt of multiple maturities and trading in intermediate periods (mature firms)
- Sale of securities: Implementable with sale of securities in intermediate periods when additional financing required (startups)
Results: comparative statics

- Higher payoff variance $\implies$ compensating for performance is more costly $\implies$ more monitoring (short-term debt) (eg, Barclay and Smith [1995])

- Higher payoff variance $\implies$ intermediaries contribute higher share of capital in smaller scale projects

- Higher costs of monitoring $\implies$ use liquidation instead $\implies$ less contingent control (eg, fewer covenants in public debt contracts than bank loans (Kahan and Tuckman [1993]))
Related literature

Outline of talk

- Three-period model
- Characterization of optimal contracts
  - optimal actions
  - optimal second-period contract
  - optimal first-period contract
- Implementations and links to stylized facts
  - implementation with debt of different seniorities and maturities
  - implementation with issuance of debt
- Overview of results: infinite-horizon model, continuous-time model
Three-period model: agents

- Dates $t = 0, 1, 2$
- Agents: Entrepreneur (E), Monitor (M), Investor (I)
- Risk neutral, zero interest rate
- Endowments: $E^E = 0$, $E^M = \text{const}$, $E^I = \infty$
- Project requires entrepreneur and initial investment $I$
Generates (random, independent) outputs $a_1$ and $a_2$

$a_t$ are observed by entrepreneur and monitor, but not verifiable

After observing $a_t$, entrepreneur diverts $x_t \geq 0$ and destroys $z_t \geq 0$

$x_t$ gives entrepreneur $\varphi x_t$ of utility, $0 \leq \varphi \leq 1$, $z_t$ gives no utility

Verifiable payoff is $y_t = a_t - x_t - z_t$

If project is terminated, $y_t = 0$, $x_t = 0$ and no continuation
Monitoring

- Monitoring at $t$ is ability to enforce $x_t = 0$ (but not $z_t = 0$)
- Monitoring is part of contract, contingent on performance $M_t(y^t) \in \{0, 1\}$
- Monitoring costs $c_t$
- Examples: participating in board meetings, being on site with entrepreneur
- Investor does not observe enforcement
Side agreements between entrepreneur and monitor

- Bargain to choose $x_t$, $z_t$ and compensating transfer $Tr$
- $Tr$ can include current and future $x_t$ or compensations
- There are no costs of such transfers
Side agreements between entrepreneur and monitor

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- $Tr$ can include current and future $x_t$ or compensations
- There are no costs of such transfers
- Bargaining is efficient: monitor and entrepreneur choose side agreement that maximizes their combined expected utilities:

$$x_t, z_t \in \arg \max [U^E + U^M]$$
Side agreements between entrepreneur and monitor

- Bargain to choose $x_t, z_t$ and compensating transfer $Tr$
- $Tr$ can include current and future $x_t$ or compensations
- There are no costs of such transfers
- Bargaining is efficient: monitor and entrepreneur choose side agreement that maximizes their combined expected utilities:

$$x_t, z_t \in \arg \max [U^E + U^M]$$

- Entrepreneur is not worse off than with $x_t = 0$ or best non-monitored outcome:

$$x_t', z_t' \in \arg \max [U^E]_{M(y^t)=0}$$
Timing of the model

\[ t=0 \]
- M and E discuss actions
- residual output \( y_1 \) is observed by investors

- contract signed, \( I \) paid
- output \( a_1 \) is realized
- E decides on diversion \( x_1 \) and destruction \( z_1 \)
- M is in control?

\[ t=2 \]
- M and E discuss actions
- residual output \( y_2 \) is observed by investors
- consumption by E and M

- output \( a_2 \) is realized
- E decides on diversion \( x_2 \) and destruction \( z_2 \)
- M is in control?

...
Contracts

Contracts specify monitoring $M_t \in \{0, 1\}$, termination $D_t \in \{0, 1\}$, compensations at $t = 2$, $C^E$ and $C^M$, all conditional on performance history $y^t$
Contracts

Contracts specify monitoring $M_t \in \{0, 1\}$, termination $D_t \in \{0, 1\}$, compensations at $t = 2$, $C^E$ and $C^M$, all conditional on performance history $y^t$

Constraints:

- Feasibility: $I^I + E^M = I$, $C^E + C^M + C^I = \sum_{t=1}^{T} y_t$

- Limited liability: $C^E \geq 0$ and $C^M \geq 0$

- Individual rationality: $U^E = \mathbb{E}[C^E + \sum_{t=0}^{T} \varphi x_t - Tr] \geq 0$
  
  $U^M = \mathbb{E}[C^M + Tr] \geq E^M$

  $U^I = \mathbb{E}[C^I - \sum_{\tau=t}^{T} c_t M_t] \geq I^I$

- Incentive compatibility: $x_t$, $z_t$ and $Tr$ are chosen optimally
Optimal contracts

- All Pareto-optimal contracts are solutions to:

\[
\max_{\gamma \in \Gamma} \mathbb{E}\left[ \sum_{t=1}^{T} (a_t - (1 - \varphi)x_t - z_t - c_t M_t) \right],
\]

s.t. \[ \mathbb{E}\left[ \sum_{t=1}^{T} \varphi x_t - Tr + C^E \right] \geq U^E, \mathbb{E}[C^M + Tr] \geq U^M. \]

- For all \( U^E \geq 0 \) and \( U^M \geq 0 \)

- \( \Gamma \) is set of all feasible, incentive compatible, individually rational contracts with limited liability

- Surplus

\[
\mathbb{E}\left[ \sum_{t=1}^{T} (a_t - (1 - \varphi)x_t - z_t - c_t M_t) \right] = U^E + U^M + U^I
\]
Outline of talk

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- Overview of results: infinite-horizon model, continuous-time model
Optimal actions

*Result. (Efficient actions, no side agreements)* Optimal contracts implement $x_t = 0, z_t = 0$ and $Tr = 0$. 
Optimal actions

Result. (Efficient actions, no side agreements) Optimal contracts implement $x_t = 0$, $z_t = 0$ and $Tr = 0$.

To implement it (denote $\bar{C}(y_t) = \mathbb{E}_t[C|y^t]$):

1. Incentive compatible for entrepreneur:

$$0 \in \arg \max_{x_t \geq 0} [\varphi x_t (1 - D_t(a_t - x_t))(1 - M_t(a_t - x_t)) + \bar{C}^E(a_t - x_t)]$$

2. Collusion proof:

$$0 \in \arg \max_{x_t \geq 0} [\varphi x_t (1 - D(a_t - x_t)) + \bar{C}^E(a_t - x_t) + \bar{C}^M(a_t - x_t)]$$

3. Monotonic: $C^E$ is non-decreasing in $y_t$
Optimal second-period contract

Note: payoffs below $y_2^D(y_1)$ are terminated and between $y_2^D(y_1)$ and $y_2^M(y_1)$ are monitored; positive slopes are $\varphi$. 
Optimal second-period contract and securities

Consider only distributions with second-period payoffs always $\geq c_2$

<table>
<thead>
<tr>
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**Result (Optimal second-period contract):**

- Entrepreneur holds share $\varphi$ of equity
- Monitor holds share $\varphi$ of second-period junior debt
- Investor holds second-period senior debt and shares $1 - \varphi$ of second-period junior debt and equity
- Face values are contingent on $y_1$
Optimal first-period contract

Note: $\bar{C}$ is expected compensation (paid at second period); payoffs below $y_1^D$ are terminated and between $y_1^D$ and $y_1^M$ are monitored; positive slopes are $\varphi$. 
Optimal first-period contract

Note: \( \bar{C} \) is expected compensation (paid at second period); payoffs below \( y_1^D \) are terminated and between \( y_1^D \) and \( y_1^M \) are monitored; positive slopes are \( \varphi \).
Results: (Optimal first-period contract)

1. Termination and monitoring rules are threshold rules, i.e.,
   \( D_1(y_1) = 1[y_1 < y_1^D], \quad M_1(y_1) = 1[y_1^D \leq y_1 < y_1^M] \).

2. Entrepreneur’s equity value is non-decreasing in \( y_1 \)

3. \( y_2^D(y_1) \) and \( y_2^M(y_1) \) are non-increasing, i.e., good first-period performance leads to less termination and control by intermediary in future

4. Monitoring in intermediate stage does not result in complete loss of cash flow rights by entrepreneur
Outline of proof

**Assumption.** Hazard rate of second-period payoff distribution, $f_2(a_2)/(1 - F_2(a_2))$, is non-decreasing

- Continuation value of project is concave in entrepreneur’s expected compensation
- Implies only sufficient cash incentives to entrepreneur and monitor
- “No holes” in monitoring and liquidation regions
- In monitored region all incentives given to monitor
Summary of optimal contracts

- Optimal contracts implement efficient actions
- Investors hold senior debt, monitors hold junior debt and entrepreneurs hold equity
- Low payoffs $\implies$ liquidation, medium payoffs $\implies$ monitoring
- Bad intermediate performance $\implies$ more debt and more monitoring in future
Outline of talk

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▶ Overview of results: infinite-horizon model, continuous-time model
Implementation with debt of different maturities

Consider only distributions with first-period payoffs always \( \geq 0 \) and when \( y_1^H = y_1^D \).

**Result.** Optimal contracts can be replicated with initial allocation:

- Entrepreneur holds share \( \varphi \) of equity
- Monitor holds share \( \varphi \) of some amounts of first- and second-period junior debt
- Investor holds some amounts of first- and second-period senior debt + shares \( 1 - \varphi \) of monitor’s and entrepreneur’s securities

And trade after period 1 realization:

- Agents with lower cash priority use period one cash distributions to buy securities from agents with higher priority
- $1 of payment increases value of buyer’s claim by $1
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And trade after period 1 realization:

- Agents with lower cash priority use period one cash distributions to buy securities from agents with higher priority
- $\$1$ of payment increases value of buyer’s claim by $\$1$
Example: $a_t$ uniformly distributed on $[1, 3]$, $\varphi = 1$

Transactions at $t = 1$ when $a_1 = 2$:

- Investor is paid 1, monitor is paid 0.22, entrepreneur is paid 0.78
- Entrepreneur buys junior debt with value 0.78 from monitor
- Monitor buys senior debt with face value 1 from investor
Example: $a_t$ uniformly distributed on $[1, 3]$, $\varphi = 1$
## Comparative statics of maturity

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Implementation with issuance of debt (startups)

- Consider only distributions with first period payoffs always $\leq 0$
- Same initial allocation of second period securities as before

Trade after period 1 realization:
- In region without monitoring entrepreneur sells junior debt to monitor and monitor sells senior debt to investor
- In region with monitoring monitor sells senior debt to investor
- $1$ of investment decreases value of claim of seller by $1$
### Comparative statics of VC contracts

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Summary of optimal contracts

- Dynamic features of optimal contracts can be replicated with initial allocation of securities and trading in intermediate periods
- For mature firms this gives a theory of maturity structure of debt
- For startups this gives a theory of staged financing
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Infinite-horizon model, $E^M \to \infty$

- Project generates payoffs for many periods until sold (in IPO) to outsiders
- Payoffs are normal, project is sold with some probability each period
- Optimal contracts have liquidation and monitoring thresholds
- Same allocation of marginal cash flow rights
- History dependence is expressed through dependence on $C_t^E$
- Same implementation with debt (of multiple maturities)
- Liquidation and monitoring thresholds are decreasing in $C_t^E$

(slides)
Alternative contracts

- minimal current monitoring: objective is to minimize $a^M$
- static contract: $a^M$ does not depend on $C^E$, only on $U^E$
Continuous-time model

- Time interval between periods shrinks to zero, per-period parameter values shrink proportionally
- Investor’s contract for $\varphi = 1$ becomes: investor gives cash at $t = 0$ and receives fixed coupon each period until project stops
- Project’s value and monitoring intensity solve a second-order linear differential equation

(slides)
Conclusion

- Extend model of costly state verification to dynamic setting with hierarchy of investor, intermediary and entrepreneur
- Explains seniority structure: entrepreneur holds junior claim; intermediary holds middle claim and investor holds senior claim
- Dynamics: bad past performance leads to allocation of more cash flow and control rights to monitor
- Replicate optimal contracts with debt of different maturities and seniorities
- Generates testable predictions on capital structure and control
Role of monitor’s capital

**Result. (monitor’s capital)** If $E^M = 0$ monitor does not increase project’s expected value. Moreover, this value is non-decreasing in $E^M$ and for $E^M = \infty$ project is financed only by monitor.

Simulate project’s net value for different $E^M$.
Parameters are:
\[ a_t \sim N(1.5, 1), \]
\[ c = 0.6, I = 1.5. \]
Optimal leverage

Comparative statics: fix available monitor’s capital $E^M$ and find scale $I$ that maximizes total surplus (parameters are scaled with $I$)

<table>
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<td>$\mu$</td>
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![Graph showing the relationship between net project value and monitor's share of investment.](image)
Result. *(Optimal cash flow rights)*

1. Compensations are zero on \( y_t < y_t^D \) and continuous on \( y_t \geq y_t^D \).

2. In monitoring region \( (y_t^D \leq y_t < y_t^M) \) monitor holds marginal cash flow rights. *(Denoting derivatives \( \overline{C}_1 \)) If \( \overline{C}^M (y_1^D) > 0 \), \( \overline{C}_1 = \varphi \) and \( \overline{C}_1^E = 0 \).

Exception: When \( \overline{C}^M (y_1^D) = 0 \), there exist \( y_t^H \in [y_t^D, y_t^M) \) such that on \( y_t^D \leq y_t < y_t^H \), \( \overline{C}^M = 0 \), \( \overline{C}_1^M = 0 \) and \( \overline{C}_1^E = \varphi \).

3. In non-monitoring region \( (y_t \geq y_t^M) \) entrepreneur holds marginal cash flow rights, \( \overline{C}_1^M = 0 \) and \( \overline{C}_1^E = \varphi \).
Infinite horizon model

Additional elements:

- Dates \( t = 0, 1..\infty \), interest rate \( r \)
- With probability \( 1 - P_{IPO} \) project generates iid \( a_t \sim N(\mu, \sigma) \)
- With probability \( P_{IPO} > 0 \) project generates verifiable \( V_{IPO} \geq \mu/r \) and stops
- Terminal period is uncertain \( \tau \), \( C_T^E((a - x)^\tau) \), \( C_T^M((a - x)^\tau) \) and \( C_T^I((a - x)^\tau) \)
- Constraint (IPO is preferable): \( C_t^E \geq \mathbb{E}_t[C_T^E] \) and \( C_t^M \geq \mathbb{E}_t[C_T^M] \)

Result (no time dependence) Constraints above are binding, i.e. \( C_t^E = \mathbb{E}_t[C_T^E] \) and \( C_t^M = \mathbb{E}_t[C_T^M] \). (extensions)
Dynamic program

- State variables: \( C_{t-1}^E \) (equity value), \( C_{t-1}^M \)
- Monitoring, termination and compensations at \( t \) are functions of \( a_t \) and state variables
- Solve dynamic program:

\[
V^{tot}(C_{t-1}^E, C_{t-1}^M) = \max_{\gamma \in \Gamma} \mathbb{E}[(1 - D(a_t))(a_t - cM(a_t) + \frac{1}{1+r} P_{IPO} V_{IPO} + \frac{1}{1+r} (1 - P_{IPO}) V^{tot}(C_t^E(a_t), C_t^M(a_t))]
\]

s.t. \( C_{t-1}^E = \mathbb{E}[C_t^E(a_t)] \) and \( C_{t-1}^M = \mathbb{E}[C_t^M(a_t)] \)
- Use the same steps as in three-period model
Optimal contract

Result. (Optimal dynamic contract) If $V^{\text{tot}}(C^E, C^M)$ is concave and $V_{22}^{\text{tot}} \leq V_{12}^{\text{tot}} \leq 0$,

- Termination and control are threshold rules, i.e.
  
  $D(C^E_{t-1}, C^M_{t-1}) = 1[a_t < a^D(C^E_{t-1}, C^M_{t-1})]$ and
  $M(C^E_{t-1}, C^M_{t-1}) = 1[a^D(C^E_{t-1}, C^M_{t-1}) \leq a_t < a^M(C^E_{t-1}, C^M_{t-1})]$

- Compensations are zero on $a_t < a^D$, continuous on $a_t \geq a^D$ and satisfy:
  
  - on $a^D \leq a_t < a^M$, $C^E_1 = \xi \in [0, \varphi]$ and $C^M_1 = \varphi - \xi$
  - on $a_t \geq a^M$, $C^E_1 = \varphi$ and $C^M_1 = 0$
Monitor-only financing \( (E^M = \infty) \)

**Result. (Optimal dynamic contract)** If \( V^{\text{tot}}(C^E) \) is concave

- Termination and control are threshold rules, i.e.
  \[
  D(C^E_{t-1}) = 1[a_t < a^D(C^E_{t-1})] \quad \text{and} \\
  M(C^E_{t-1}) = 1[a^D(C^E_{t-1}) \leq a_t < a^M(C^E_{t-1})]
  \]

- Compensations are zero on \( a_t < a^D \), continuous on \( a_t \geq a^D \) and satisfy:
  - on \( a^D \leq a_t < a^M \), \( C_1^E = 0 \)
  - on \( a_t \geq a^M \), \( C_1^E = \varphi \)
Numerical simulation: monitoring probability

Parameters are:
\( \mu = -2, \sigma = 1, \)
\( r = 0.05, \)
\( \varphi = 0.5, \)
\( P_{IPO} = 0.068 \)
and \( V_{IPO} = 40 \)
Monitoring costs and monitoring probability

\[ P(q_t^M < a_t^c < a_t^D) \]

- \( c = \frac{1}{3} \)
- \( c = \frac{2}{3} \)
- \( c = 3 \)
Uncertainty and monitoring probability

Monitoring probability $P(a_t^M < a_t^D)$

- $\sigma = 1$
- $\sigma = 0.5$
- $\sigma = 1.5$

Equity value $C_{t-1}$
Alternative contracts

- **minimal current monitoring**: objective is to minimize $a^M$
- **static contract**: $a^M$ does not depend on $C^E$, only on $U^E$
Frequent reporting

- Period length $\Delta \rightarrow 0$
- Payoffs are $N(\mu\Delta, \sigma\sqrt{\Delta})$
- IPO probability $P_{IPO} = (1 - e^{-\lambda\Delta}) \approx \lambda\Delta$
- $\lambda V_{IPO} + \mu > c$, i.e., per period value is higher than per period monitoring cost

Results. (Frequent reporting)

- Project is never terminated as long as $C_t^E + C_t^M > 0$
- For $\varphi = 1$ and $\lambda \rightarrow 0$ investor’s compensation can be replicated by him paying $I + U^E + U^M$ at $t = 0$ and receiving $\mu\Delta$ each period
- For $E^M \rightarrow \infty$, $V^{tot}$ solves second order differential equation and is concave

(extensions)