Capital Taxation under Political Constraints*

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February 2014

Abstract
This paper studies optimal dynamic tax policy under the threat of political reform. A policy will be reformed ex post if a large enough political coalition supports reform; thus, credible policies are those that will continue to attract enough political support in the future. If the reform threat is to fully equalize consumption, we find that optimal marginal capital taxes are U-shaped, so that savings are subsidized for the middle class but are taxed for the poor and rich. If ex post the government may strategically propose a reform other than full equalization in order to secure additional political support, then optimal capital taxes are instead progressive throughout.

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1 Introduction

In a democratic society, the credibility of a course of public policy depends on the level of political support it can be expected to enjoy in the future. In a representative-agent economy, this level of support is degenerate: a policy is credible if and only if it is time-consistent from the representative agent’s perspective (Kydland and Prescott, 1977). If, on the other hand, society consists of diverse individuals, a democratic government needs to take the resulting heterogeneity of political preferences into account when formulating policy, and in particular has to realize that a policy is credible if it will continue to receive the support of a large enough coalition of citizens.

This paper adopts the above perspective on the credibility of public policy to reexamine the classical capital levy problem (Fischer, 1980) in a heterogeneous society. Several new insights emerge. In the simplest version of our model, marginal capital taxes are typically U-shaped in income: savings are taxed for the poor and the rich, but subsidized for the middle class. As discussed below, this prediction of U-shaped marginal capital taxes resonates well with policy in many advanced economies, once the savings incentives implied by means-tested government benefits are taken into account. In a second version of the model, the prediction of purely progressive capital taxes recently obtained in a related model due to Farhi et al. (2012) is recovered, although the mechanism is different from their paper.

We consider a two-period model where individuals have heterogeneous abilities as in Mirrlees (1971) and produce in period 0 only while consuming in both periods. In this model, the classical result of Atkinson and Stiglitz (1976) shows that zero capital taxes are optimal when the government can commit to any course of policy. Farhi et al. (2012) instead assume that the (utilitarian) government can always reform its policy (at a cost) in period 1. This leads the government to tie its hands by taxing capital at a progressive rate: the government is tempted to fully equalize period-1 consumption ex post, so it reduces this temptation by partially equalizing period-1 consumption under the status quo relative to the full-commitment optimum of zero capital taxes, which corresponds to taxing saving for the rich and subsidizing it for the poor.

In contrast, we assume that the government can reform its policy in period 1 if there is sufficient political support for a reform. In the simplest version of our model, the only possible (credible) reform is again full equalization of period-1 consumption: if voters empower the government to reform policy, the government always chooses its most-preferred reform, which remains full equalization. In this case, poor voters tend to support reform, while rich voters tend to oppose it, and middle-class voters tend to be close
to indifferent and thus pivotal. Thus, the government can make its policy credible by making the status quo as appealing as possible to the middle class, relative to full equalization. It does this both by subsidizing saving for middle class voters (so they have high period-1 consumption under the status quo) and by taxing capital for the poor and rich (so consumption under an equalizing reform is low). That is, the socially optimal manner for the government to forestall reform is to impose a U-shaped marginal tax on capital.

The assumption that full equalization is the only possible reform is important for this result. Indeed, this assumption is quite strong, as ex post the government may have a strong incentive to propose reforms other than full equalization in order to secure more political support; for example, the government may want to expropriate the richest 1% to sweeten the reform for the remaining 99%. To allow for this possibility, we next consider the polar case where the government is able to propose any reform in period 1, again assuming that the reform is implemented if it has sufficient political support relative to the status quo. Quite surprisingly, we find that in this case the progressive capital taxation result of Farhi et al. (2012) is restored, under some additional mild assumptions.

To establish this result, we first show that for any status quo period-1 consumption schedule, the most tempting reform consumption schedule is always more egalitarian than the status quo, even though it is no longer fully equalizing. Next, we show that the government’s problem may be written as a standard welfare maximization problem with an additional “no reform” constraint, where a marginal increase in a given individual’s period-1 consumption under the status quo relaxes the no reform constraint on net if and only if her period-1 consumption is higher under the reform than under the status quo. As the reform is more egalitarian than the status quo, it is the poor who have higher period-1 consumption under reform, and thus the poor face lower capital taxes under the government’s optimal policy.

Putting our results together, we find that when the government’s credibility is limited by the possibility of future reform, optimal capital taxes may be either U-shaped or progressive, depending on the government’s ability to credibly tailor reforms to popular demand. We formally consider the two polar versions of the model—where only fully equalizing reforms are credible, and where all reforms are—and reality is likely to lie somewhere in between. However, we can roughly map the two versions of our model to more representative and more direct versions of democratic government, respectively: in representative democracies, politicians retain the final say on fiscal policy, and are thus unlikely to be able to commit to reforms that they have a strong incentive to modify ex post; while in direct democracies, politicians may have the “agenda-setting” power to propose a specific policy, while being unable to modify it after it is approved by the voters.
With this interpretation, our model predicts that capital taxes are more likely to be U-shaped when fiscal policy is ultimately determined by representatives, and more likely to be progressive when fiscal policy is determined by direct referendum.\footnote{This prediction is hard to test directly for two reasons. First, effective marginal capital taxes are difficult to measure, as discussed below. Second, obvious ways of measuring whether a democracy is “direct” or “representative” are likely to pick up effects that are outside our model but that could also affect fiscal policy. For example, the recent empirical paper by Hinnerich and Pettersson-Lidbom (2013) argues that representative democracies are more redistributive than direct democracies because they are less easily captured by elites.}

Farhi et al. (2012) note that the level and progressivity of effective capital taxes in advanced economies are hard to measure, but that policies such as income, estate, and wealth taxes, as well as the tax treatment of retirement accounts and subsidies to savings and education by the poor, contribute to the progressivity of capital taxation. However, it seems quite plausible that overall capital taxes are U-shaped in many countries once means-tested government benefits are accounted for. For example, in the US, only individuals with sufficiently few assets qualify for Medicaid or Federal Student Aid, and only individuals with sufficiently low investment income qualify for the Earned Income Tax Credit. These asset tests can lead to very high effective savings distortions for the poor. More generally, the contribution to capital tax progressivity of subsidies to savings and education by the poor are at least partially offset by the phase-out of these subsidies, unless eligibility is solely determined by labor income.

In addition to the effect of the phase-out of these programs on marginal savings incentives, some of them are targeted more directly at middle-class voters than the very poor. Examples include subsidies to college education, many retirement savings programs (where the subsidy is increasing in the marginal income tax rate up to some caps) and the mortgage interest deduction, which subsidizes the accumulation of housing wealth. Finally, Doepke and Schneider (2006) show that the inflation tax effectively redistributes from rich, bond-holding households to middle class households with fixed-rate mortgage debt, consistent with the pattern predicted by our first model.

The paper proceeds as follows. Following a brief discussion of related literature, Section 2 introduces our basic framework, which is a standard two-period Mirrlees model as discussed above. Section 3 analyzes the version of the model with fully equalizing reforms only. Section 4 considers the version with arbitrary reforms. Section 5 concludes. Omitted proofs as well as several extensions of the model are presented in the appendix.

\footnote{An alternative interpretation is that the two versions of the model differ in the government’s degree of sophistication. In the second version of the model, the government is able to design (and commit to) sophisticated vote-buying strategies at the reform stage. In the first version, the government simply pursues its most preferred (fully equalizing) reform. This can capture naivety on the part of the government, as well as an inability to commit at the reform stage.}
Related Literature

This paper is related to several strands of the literature on political economy and public finance.

Most closely related is the large public finance literature on capital taxation with limited commitment. The most classical branch of this literature assumes a representative agent (Fischer, 1980, Klein et al., 2008), and thus clearly cannot address the coalition-formation concerns that are central to our results. Hassler et al. (2005) considers a two-type model, which again precludes non-trivial coalition-formation. A subset of this literature studies how “reputation” can mitigate the government’s time-inconsistency problem (Kotlikoff et al., 1988, Chari and Kehoe, 1990, Benhabib and Rustichini, 1997, and Phelan and Stacchetti, 2001); this line of work is less closely related, as we mostly consider a two-period model. More closely related are the few papers where the extent of commitment is explicitly determined by political economy factors. One such paper is that of Farhi et al. (2012), as discussed above. Another is due to Acemoglu et al. (2010), who analyze an infinite-horizon Mirrlees model with a self-interested politician and study whether the resulting distortions eventually vanish.3

There is also a classic positive political economy literature on capital taxation with heterogeneous voters and linear taxes (Bertola, 1993, Alesina and Rodrik, 1994, Persson and Tabellini, 1994a). One paper in this literature that emphasizes time-inconsistency is Persson and Tabellini (1994b), who consider a two-period model with linear taxes and show that voters may want to elect a government that is biased against taxing capital. Restricting to linear taxes again rules out the coalition-formation issues that underlie our model.

The prediction of our baseline model that optimal capital taxes are U-shaped recalls Director’s Law (Stigler, 1970), which observes that public redistribution tends to benefit the middle class rather than the poor. The literature on Director’s law does focus on the coalition-formation mechanisms underlying this pattern, but typically does so in a static setting (Lindbeck and Weibull, 1987; Dixit and Londregan 1996, 1998). To the best of our knowledge, our paper is the first to ask how such concerns about coalition formation affect optimal dynamic public policy. We also know of no antecedent to our result that progressivity is restored if strategic reforms are possible.

Our results about the shape of the non-linear capital tax schedule mirror an extensive

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3Similar commitment problems can arise in moral hazard models rather than the Mirrleesian adverse selection model considered here. See, for instance, Fudenberg and Tirole (1990) and Netzer and Scheuer (2010) for two-period models where, ex ante, the principal optimally offers incomplete insurance to a risk-averse agent in order to provide incentives for efficient effort, but ex post, once effort is sunk, prefers to provide full insurance.
literature on the shape of optimal income taxes in static Mirrleesian economies. In particular, many authors have found U-shaped marginal income taxes to be optimal (see e.g. Diamond, 1998, and Saez, 2001). However, whereas this property crucially depends on the shape of the underlying skill distribution (in particular an unbounded Pareto-tail at the top), our results about U-shaped or progressive marginal capital taxes are completely independent of the form of the skill distribution.

Finally, it should be emphasized that we study optimal taxes under political constraints, rather than taxes resulting from political competition. Competition among political parties offering non-linear tax schedules leads to a Colonel Blotto (or “divide-the-dollar”) game, which is notoriously difficult to analyze even in the simplest redistributive settings (Myerson, 1993, Lizzeri and Persico, 2001). Our approach is more tractable, in addition to being closer to the public finance literature on limited commitment.

2 Model

We consider a standard Mirrlees model with two periods, \( t = 0, 1 \). There is a continuum of individuals indexed by their ability \( \theta \in \Theta \). Assume that \( \Theta \) is an open subset of \( \mathbb{R} \) and that \( \theta \) has cumulative distribution function \( F \) with positive density \( f \) on \( \Theta \).

Individuals produce in period 0 only and consume in both periods. A type \( \theta \) individual has utility function

\[
u (c_0 (\theta)) + \beta u (c_1 (\theta)) - h (y (\theta), \theta) ,
\]

where \( c_0 (\theta) \) and \( c_1 (\theta) \) are her consumption in periods 0 and 1, \( u \) is a strictly increasing, concave, and twice-differentiable consumption utility function, \( \beta > 0 \) is the discount factor, \( y \) is her production in period 0, and \( h \) is a continuous function with strictly decreasing differences that measures the cost of production. To avoid corner solutions, we assume that either \( u \) is defined over all of \( \mathbb{R} \) or \( \lim_{c \to 0} u' (c) = \infty \).

Production is taken to be linear for simplicity, so the economy faces aggregate resource constraints in \( t = 0, 1 \) given by

\[
\int c_0 (\theta) \, dF + K \leq \int y (\theta) \, dF ,
\]

\[
\int c_1 (\theta) \, dF \leq RK ,
\]

where \( K \) is aggregate capital and \( R > 0 \) is its gross rate of return. These may be combined
to form a single intertemporal resource constraint
\[
\int \left( c_0 (\theta) + \frac{1}{R} c_1 (\theta) \right) dF \leq \int y (\theta) dF. \tag{1}
\]

In addition to the continuum of individuals, there is a government, assumed for now to be utilitarian (we relax this in Appendix C). As in Mirrlees (1971), the government may allocate resources among individuals arbitrarily but cannot observe ability. Therefore, the revelation principle implies that the government’s problem when it can fully commit to an intertemporal allocation is

\[
\max_{c_0, c_1, y} \int (u (c_0 (\theta)) + \beta u (c_1 (\theta)) - h (y (\theta), \theta)) dF
\]

subject to the intertemporal resource constaint (1) and a standard incentive compatibility constraint
\[
u (c_0 (\theta)) + \beta u (c_1 (\theta)) - h (y (\theta), \theta) \geq u (c_0 (\theta')) + \beta u (c_1 (\theta')) - h (y (\theta'), \theta) \text{ for all } \theta, \theta',
\tag{2}
\]
where \(c_0, c_1,\) and \(y\) are arbitrary measurable functions from \(\Theta\) to \(\mathbb{R}\).

Most of our results will concern the implicit marginal capital tax, defined by
\[
\tau_k (\theta) \equiv 1 - \frac{u' (c_0 (\theta))}{\beta Ru' (c_1 (\theta))} < 1. \tag{3}
\]
This “wedge” is well-defined in any allocation, and in addition can be interpreted as the actual marginal capital tax rate faced by agents of type \(\theta\) in a non-linear tax implementation of the optimal allocation, as we will discuss in Section 5. At a solution to the above full-commitment problem, Atkinson and Stiglitz’s (1976) uniform taxation result implies that \(\tau_k (\theta) = 0\) for all \(\theta\).\(^4\) As we will see, this result does not continue to hold when the government’s credibility is limited by the possibility of reform in period 1.

For some of our arguments, we will need to allow for randomized consumption schedules, even though ultimately deterministic solutions will exist for all versions of the model we consider. Formally, we allow the government to choose, for each \(\theta \in \Theta\), a distribution \(P (\theta)\) over consumption levels \((c_0 (\theta), c_1 (\theta))\) and assume that agents can choose any consumption level in the support of \(P\), so that (2) must now hold for all \((c_0 (\theta), c_1 (\theta)) \in \text{supp } P (\theta)\) and \((c'_0 (\theta), c'_1 (\theta)) \in \text{supp } P (\theta)\). In particular, since we do not consider ran-

\(^4\)Throughout, we omit caveats regarding measure-0 sets when stating results. We address this issue in various proofs where it may cause confusion.
domination over $y(\theta)$, this implies that total consumption utility

$$U(\theta) = u(c_0(\theta)) + \beta u(c_1(\theta))$$

is constant for all $(c_0(\theta), c_1(\theta)) \in \text{supp} \, P(\theta)$. We also require that $P(\theta)$ is right-continuous in $\theta$ (in the weak topology), and hence that $U(\theta)$ is right-continuous. The various restrictions on the form of allowable randomizations made in this paragraph can all be relaxed at a slight cost in terms of additional notation and caveats regarding measure-0 sets.

We will consider two versions of limited commitment on the part of the government, which differ in the government’s ability in period 1 to commit to the details of a proposed reform. In both versions, the timing of the model is as follows.

1. The government proposes consumption and production schedules $(c_0, c_1, y)$.
2. Production and period-0 consumption occurs.
3. The period-1 consumption schedule $c_1$ may be reformed to an alternative schedule $\hat{c}_1$.

The two versions of the model differ in how the “reform” consumption schedule $\hat{c}_1$ is specified; the details of this are described below. In both versions, the reform is defeated if and only if

$$\int H(c_1(\theta), \hat{c}_1(\theta)) \, dF \geq \alpha,$$  \hspace{1cm} (4)

for some function $H$ of status quo consumption $c_1$ and reform consumption $\hat{c}_1$ and some constant $\alpha \in [0,1]$. The interpretation is that the status quo is supported by fraction $H(c_1(\theta), \hat{c}_1(\theta)) \in [0,1]$ of those individuals with status quo consumption $c_1(\theta)$ and reform consumption $\hat{c}_1(\theta)$, and that the status quo prevails if it is supported by at least fraction $\alpha$ of the total population. In particular, one can interpret $H$ as the cumulative distribution function of taste shocks in a probabilistic voting model (e.g., Cox and McCubbins, 1986, Lindbeck and Weibull, 1987). An interesting extreme case obtains when these taste shocks are all zero, so that $H$ is a step function with

$$H(a,b) = \begin{cases} 1 \text{ if } a \geq b \\ 0 \text{ if } a < b. \end{cases}$$

\text{That is, we require that if } \theta' \downarrow \theta \text{ then } P(\theta') \text{ converges in distribution to } P(\theta).

\text{The assumption that the range of } H \text{ is } [0,1] \text{ is natural and is required for this interpretation, but is not technically required for the analysis.}
In this case, the “no-reform” constraint (4) simply requires that the fraction of the population who receive higher consumption (or utility) under the status quo than under the reform exceeds the critical threshold $\alpha$. For most of the analysis, we make the technically convenient assumption that $H$ is continuously differentiable in each argument, and thus admit step functions only as a limiting case. However, step functions themselves are also easy to work with—and indeed sometimes yields particularly sharp results—so we sometimes consider them separately. Another natural specification is where $H (c_1 (\theta), \hat{c}_1 (\theta))$ depends only on the difference $c_1 (\theta) - \hat{c}_1 (\theta)$ or the difference $u (c_1 (\theta)) - u (\hat{c}_1 (\theta))$ and its corresponding density is single-peaked at zero (so $H$ is S-shaped in the difference), which is a common assumption in the probabilistic voting literature. This captures a situation where taste-shocks can be non-zero but are concentrated around zero, for instance following a normal distribution. In any case, most of our results require only weak assumptions on $H$.

3 Equalizing Reforms

In this section, we assume that the only credible reform consumption schedule $\hat{c}_1$ involves full equalization in period 1, that is

$$\hat{c}_1 (\theta) = RK - \kappa$$

for all $\theta$, where $\kappa \geq 0$ is an exogenous cost of implementing a reform, which can equal 0. This assumption is appropriate if the government (rather than the voters) always has the final say on fiscal policy, as the (utilitarian) government’s most-preferred reform is always full equalization. In other words, the implicit assumption in this section is that the timing of a possible reform is as follows.

1. Individuals vote on whether to allow a reform to $c_1$.
2. If a reform is allowed, the government chooses the reform consumption schedule $\hat{c}_1$.

Before formulating the government’s problem, note that a reform never actually occurs in an equilibrium of this model. Otherwise, the government could have originally

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The text refers to $\kappa$ as the exogenous cost of implementing a reform, which can equal 0. It also mentions that in this section, the government always has the final say on fiscal policy, as the (utilitarian) government’s most-preferred reform is always full equalization. The assumption that individuals first vote on whether to allow a reform and then the government chooses the reform consumption schedule is made. The government’s problem is not formulated, but it is noted that a reform never actually occurs in an equilibrium of this model.
proposed the constant period-1 consumption schedule \( \hat{c}_1 \) ex ante—which will never be reformed—and saved the cost of reform \( \kappa \).\(^9\) The government’s problem is therefore

\[
\max_{c_0, c_1, y} \int (u(c_0(\theta)) + \beta u(c_1(\theta)) - h(y(\theta), \theta)) \, dF
\]

subject to (1), (2), and (4), where in (4) \( \hat{c}_1 \) is given by (5) with

\[
\int c_1(\theta) \, dF \leq RK. \tag{6}
\]

We note at the outset that the government’s problem is typically not concave, because \( H(c_1(\theta), \hat{c}_1(\theta)) \) is typically not concave in \( c_1(\theta) \); for example, step functions are not concave. This is not a “technical” problem (although it will lead to some mathematical complications), but rather a key economic aspect of the model. In particular, to design a credible policy, the government must in effect select a coalition of voters that will support this policy against a potential future reform. This coalition-formation problem is non-concave under natural assumptions, as it is natural to assume that the voters who are closest to indifferent between the status quo and the reform are the ones who are most sensitive to slight changes in these policies. For example, if \( H(c_1(\theta), \hat{c}_1(\theta)) \) is a cumulative distribution function that depends only on the difference \( c_1(\theta) - \hat{c}_1(\theta) \), then it is natural to assume that \( H \) is \( S \)-shaped in the difference, while it is not possible for \( H \) to be concave in the difference over the entire real line.

While the government’s problem is not concave, it is still true that any solution to it must also solve the dual problem

\[
\min_{c_0, c_1, y} \int \left( c_0(\theta) + \frac{1}{R} c_1(\theta) - y(\theta) \right) \, dF
\]

subject to

\[
\int (u(c_0(\theta)) + \beta u(c_1(\theta)) - h(y(\theta), \theta)) \, dF \geq V, \tag{7}
\]

(2), and (4), where \( V \) is the value of the primal.\(^10\) Note that in the constraints (2) and (7), \( c_0(\theta) \) and \( c_1(\theta) \) only enter through total consumption utility \( U(\theta) \). Hence, any solution

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\(^9\)If \( \kappa = 0 \) then reform may occur in equilibrium, but the government’s problem is unaffected by this possibility.

\(^{10}\)If not, then one could take a solution to the dual and vary \( c_0 \) so as to increase \( u(c_0(\theta)) \) to \( u(c_0(\theta)) + \varepsilon \) for all \( \theta \). This variation would increase the objective while leaving (2) and (4) unaffected, and would not violate (1) for small enough \( \varepsilon \).
must solve the subproblem

$$\min_{c_0,c_1,K} \int \left( c_0(\theta) + \frac{1}{R} c_1(\theta) \right) dF$$

subject to

$$u(c_0(\theta)) + \beta u(c_1(\theta)) = U(\theta),$$

$$\int H(c_1(\theta), RK - \kappa) dF \geq \alpha,$$

and (6). The first-order (necessary) conditions of this program deliver the following characterization.

**Lemma 1.** In any solution to the government’s problem, the intertemporal wedge $\tau_k(\theta)$ satisfies

$$\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = -R \eta \left[ H_2(c_1, RK - \kappa) + H_1(c_1(\theta), RK - \kappa) \right],$$

where $\eta \geq 0$ is the multiplier on (10), subscripts denote partial derivatives, and

$$\hat{H}_2(\tau_1, RK - \kappa) \equiv \int H_2(c_1(\theta), RK - \kappa) dF.$$

**Proof.** Substituting out for $c_0(\theta)$ using (9), and letting $\eta \geq 0$ and $\mu \geq 0$ be the multipliers on (10) and (6), respectively, we form the Lagrangian

$$\int \left( u^{-1}(U(\theta) - \beta u(c_1(\theta))) + \frac{1}{R} c_1(\theta) \right) dF - \eta \int H(c_1(\theta), RK - \kappa) dF - \mu \left( RK - \int c_1(\theta) dF \right).$$

The first-order (necessary) condition with respect to $K$ yields

$$\frac{\mu}{\eta} = -\hat{H}_2(c_1, RK - \kappa).$$

Next, rewrite the Lagrangian as

$$\int \left( u^{-1}(U(\theta) - \beta u(c_1(\theta))) + \frac{1}{R} c_1(\theta) - \eta \left( H(c_1(\theta), RK - \kappa) - \frac{\mu}{\eta} c_1(\theta) \right) \right) dF - \mu RK,$$

and differentiate under the integral with respect to $c_1(\theta)$ to obtain the necessary condition

$$u'(c_0(\theta)) = \frac{\beta R u'(c_1(\theta))}{1 - R \eta \left[ H_2(c_1, RK - \kappa) + H_1(c_1(\theta), RK - \kappa) \right]}.$$  

Finally, use (3) to rewrite this condition as in (11).
it is quite intuitive that $c_1(\theta)$ should be non-decreasing, this is not immediate in the current context because the problem is not concave. The following mathematical lemma will let us conclude that $c_1(\theta)$ is non-decreasing in some solution to the government’s problem.\footnote{The lemma also shows that deterministic allocations are optimal.} \footnote{That is, constraint C is a more general version of constraint $C'$ that allows the functions $y_x$ to depend on the aggregate allocation $A(P)$. Note also that constraint C may equivalently be written as}

We will make use of this lemma repeatedly in different contexts throughout the paper, so we state it in general language.

**Lemma 2.** Let $P$ be drawn from the set of right-continuous functions from an open set $\Theta \subseteq \mathbb{R}$ to $\Delta(\mathbb{R})$, the set of Borel distributions over real-valued allocations $a$. Let $X$ be an arbitrary index set, and consider the program

$$W = \sup_P \int \int w(a, t(\theta)) \, dP \, dF$$

subject to one of the following constraints

$$\int \int y_x(a, A(P)) \, dP \, dF \leq 0 \text{ for all } x \in X, \text{ where } A(P) = \int \int a \, dP \, dF, \quad (C)$$

or

$$\int \int y_x(a) \, dP \, dF \leq 0 \text{ for all } x \in X. \quad (C')$$

Assume that $w$ is continuous and has strictly increasing differences in $a$ and $t$, $y_x$ is continuous for all $x \in X$, and $t$ is right-continuous. Then

(i) In any solution $P$, if $t(\theta') < t(\theta'')$ then $a(\theta') \leq a(\theta'')$ for all $a(\theta') \in \text{supp } P(\theta')$ and $a(\theta'') \in \text{supp } P(\theta'')$.

(i') If the constraint takes the more restrictive form of constraint $C'$, then for any solutions $P'$ and $P''$, if $t(\theta') < t(\theta'')$ then $a(\theta') \leq a(\theta'')$ for all $a(\theta') \in \text{supp } P'(\theta')$ and $a(\theta'') \in \text{supp } P''(\theta'')$.

(ii) If $W < \infty$ and $t$ is non-decreasing, there exists a deterministic solution in which $a$ is non-decreasing.

**Proof.** See Appendix A.1. \hfill \blacksquare

To see the intuition for Lemma 2, restrict attention to deterministic allocations and assume that $t$ is the identity, $X$ is a singleton, and the constraint takes the form of constraint
C’. The program in the lemma is then

\[ \sup_{a: \Theta \rightarrow \mathbb{R}} \int w(a(\theta), \theta) \, dF \]

subject to

\[ \int y(a(\theta)) \, dF \leq 0. \]

The corresponding Lagrangian is

\[ \int (w(a(\theta), \theta) - \lambda y(a(\theta))) \, dF. \]

Standard monotone comparative statistics results (e.g., Theorem 4’ of Milgrom and Shannon, 1994) imply that any (right-continuous) function \( a^* \) that maximizes the Lagrangian is monotone in the sense of (i), even if \( y \) is not convex. However, if \( y \) is not convex, then a saddle point of the Lagrangian may not exist, so it does not immediately follow that any solution to the constrained optimization problem is monotone. The proof of Lemma 2 uses an explicit variational argument to show that this must nonetheless be the case. Lemma 2 is the key monotone comparative statics tool of this paper and seems like it could also be useful in other contexts, but it does require somewhat carefully chosen assumptions; in Appendix C, we point out both possible generalizations and limitations of the lemma.

We use Lemma 2 to prove the following result about the shape of \( c_1(\theta) \).

**Lemma 3.** There exists a deterministic solution to the planning problem in which \( c_1(\theta) \) is non-decreasing.

**Proof.** Allowing randomized consumption schedules, rewrite the dual problem as

\[
\min_{U, P, y} \int \int \left( u^{-1}(U(\theta) - \beta u(c_1)) + \frac{1}{R} c_1 - y(\theta) \right) dPdF
\]

subject to

\[
U(\theta) - h(y(\theta), \theta) \geq U(\theta') - h(y(\theta'), \theta) \quad \text{for all } \theta, \theta',
\]

\[
\int (U(\theta) - h(y(\theta), \theta)) \, dF \geq V,
\]

\[
\int \int H(c_1, \int c_1 dPdF - \kappa) \, dPdF \geq \alpha.
\]

Observe that at any solution, \( P \) must solve the subproblem

\[
\min_P \int \int \left( u^{-1}(U(\theta) - \beta u(c_1)) + \frac{1}{R} c_1 \right) dPdF.
\]
subject to
\[
\int \int H(c_1, \int c_1 dP dF - \kappa) dP dF \geq \alpha.
\]
Note that \(u^{-1}(U(\theta) - \beta u(c_1(\theta)))\) has strictly decreasing differences in \(U(\theta)\) and \(c_1(\theta)\) by strict concavity of \(u\), and that \(U(\theta)\) is non-decreasing and right-continuous in \(\theta\) by the incentive-compatibility constraint (2). The result then follows from Lemma 2 (ii) and the fact that the value of the objective is bounded below by the value of the first best solution from solving the relaxed program
\[
\min_{U, P, y} \int \int \left( u^{-1}(U(\theta) - \beta u(c_1)) + \frac{1}{R} c_1 - y(\theta) \right) dP dF
\]
subject to \(\int (U(\theta) - h(y(\theta), \theta)) dF \geq V\).  

The remaining results in this section concern such deterministic monotone solutions to the government’s problem; by Lemma 2, the only possible loss of generality involved in this restriction is that other solutions may involve randomization or non-monotonicity of \(c_1(\theta)\) in \(\theta\) over intervals where both \(U(\theta)\) and \(y(\theta)\) are constant (i.e. for types that are pooled).

Our first main result is the following. Note that \(H_1(c_1(\theta), R - \kappa)\) being single-peaked in \(c_1(\theta)\) corresponds to \(H(c_1(\theta), R - \kappa)\) being S-shaped in \(c_1(\theta)\).

**Proposition 1.** If \(H_1(c_1(\theta), R - \kappa)\) is single-peaked in \(c_1(\theta)\), then optimal marginal capital taxes are U-shaped in \(\theta\).

**Proof.** By (11), \(\tau_k(\theta)\) is non-increasing in \(H_1(c_1(\theta), R - \kappa)\). As \(c_1(\theta)\) is non-decreasing in \(\theta\), if \(H_1(c_1(\theta), R - \kappa)\) is single-peaked in \(c_1(\theta)\) then \(H_1(c_1(\theta), R - \kappa)\) is single-peaked in \(\theta\), and therefore \(\tau_k(\theta)\) is U-shaped in \(\theta\).  

Proposition 1 says that when a proposed policy is credible only if the period-1 allocation is preferred to full redistribution by a large enough share of the population, optimal marginal capital taxes are U-shaped. Although the derivation of this result was complicated by the non-concavity on the government’s problem, the intuition is quite simple. Ex post, most poor agents will support a fully equalizing reform, most rich agents will oppose it, and middle-class agents will tend to be pivotal; this feature that those in the middle are most sensitive to the details of their allocation under the status quo relative to the reform is captured by the assumption that \(H_1\) is single-peaked. Thus, in order to make the status quo credible, the government should ensure that middle-class agents’ period-1 consumption is high under the status quo and low under the reform. This is achieved by subsidizing capital for the middle-class (which increases middle-class period-1 consumption under the status quo) and taxing capital for the poor and rich (which decreases middle-class period-1 consumption under the reform). Of course, which agents end up
being “poor,” “rich,” and “middle-class” in terms of period-1 consumption is endoge-
nous to government policy, and one of the key lemmas behind Proposition 1 is that it is
indeed optimal for agents’ rankings in terms of their period-1 consumption to match their
rankings in terms of ability.

We now consider the implications of Proposition 1 for some leading specifications of
$H$. These stronger functional form assumptions will also let us sign optimal marginal
capital taxes.

**Definition 1.** $H$ depends on consumption differences if $H(c_1(\theta), \hat{c}_1(\theta)) = \tilde{H}(c_1(\theta) - \hat{c}_1(\theta))$ for some $\tilde{H}$. $H$ depends on utility differences if $H(c_1(\theta), \hat{c}_1(\theta)) = \tilde{H}(u(c_1(\theta)) - u(\hat{c}_1(\theta)))$ for some $\tilde{H}$.

$H$ will depend on consumption differences if it is the probability of supporting the
status quo in a probabilistic voting model with additive consumption shocks (i.e., individu-
als first learn the realization of a stochastic component of income under the status
quo and reform and then vote). Similarly, $H$ will depend on utility differences if it is the
probability of supporting status quo in a probabilistic voting model with additive taste
shocks.

Whether $H$ depends on consumption or utility differences, it is natural to assume
that $\tilde{H}$ is S-shaped, so that $\tilde{H}'$ is single-peaked. Under this assumption, we obtain the
following corollary of Proposition 1.$^{13}$

**Corollary 1.** If $H$ depends on consumption differences and $\tilde{H}'$ is single-peaked, then optimal marginal capital taxes are U-shaped in $\theta$. In addition, if the allocation $(c_0(\theta), c_1(\theta), y(\theta))$ is non-constant and $\tilde{H}'$ is strictly single-peaked, then optimal marginal capital taxes are negative for individuals with intermediate $\theta$, and are positive for individuals with low and/or high $\theta$.

If $H$ depends on utility differences and $\tilde{H}'u'$ is single-peaked, then optimal marginal
capital taxes are U-shaped in $\theta$. In addition, if the allocation $(c_0(\theta), c_1(\theta), y(\theta))$ is non-constant and $\tilde{H}'$ is strictly single-peaked, then optimal marginal capital taxes are negative for individuals with intermediate $\theta$, and are positive for individuals with low and/or high $\theta$.

**Proof.** If $H$ depends on consumption differences, then

$$H_1(c_1(\theta), RK - \kappa) = \tilde{H}'(c_1(\theta) - RK + \kappa)$$

and

$$\tilde{H}_2(c_1, RK - \kappa) = -\int \tilde{H}'(c_1(\theta') - RK + \kappa) dF,$$
so (11) becomes

\[
\frac{\tau_k (\theta)}{1 - \tau_k (\theta)} = R \eta \left[ \int \tilde{H}' (c_1 (\theta')) - RK + \kappa) \, dF - \tilde{H}' (c_1 (\theta)) - RK + \kappa) \right].
\]

It follows immediately that optimal marginal capital taxes are U-shaped in \( \theta \) in any deterministic monotone solution. In addition, if the allocation is non-constant and \( \tilde{H} \) is strictly single-peaked then \( \tilde{H}' (c_1 (\theta)) - RK + \kappa) \) must be greater than \( \int \tilde{H}' (c_1 (\theta')) - RK + \kappa) \, dF \) for some values of \( \theta \) and less than \( \int \tilde{H}' (c_1 (\theta')) - RK + \kappa) \, dF \) for others,\(^{14} \) so \( \tau_k (\theta) \) is negative for some individuals (who must be those with intermediate \( \theta \)) and positive for others.

The argument when \( H \) depends on utility differences is identical, except that now

\[
H_1 (c_1 (\theta), RK - \kappa) = \tilde{H}' (u (c_1 (\theta)) - u (RK - \kappa)) u' (c_1 (\theta))
\]

and

\[
\tilde{H}_2 (c_1, RK - \kappa) = - \int \tilde{H}' (u (c_1 (\theta')) - u (RK - \kappa)) u' (RK - \kappa) \, dF,
\]

so the relevant derivative is \( \tilde{H}' u' \) rather than \( \tilde{H}' \).

Since \( u \) is concave, the assumption required for U-shaped marginal capital taxes in Corollary 1 is stronger when \( H \) depends on utility differences than when it depends on consumption differences. In the former case, we have

\[
\frac{\tau_k (\theta)}{1 - \tau_k (\theta)} = R \eta \left[ \int \tilde{H}' (c_1 (\theta') - RK + \kappa) u' (RK - \kappa) \, dF - \tilde{H}' (c_1 (\theta)) - RK + \kappa) u' (c_1 (\theta)) \right].
\]

When \( \tilde{H} \) is uniform—so that \( \tilde{H}' \) is constant—we recover precisely the optimal capital tax formula of Farhi et al. (2012), which prescribes increasing marginal capital taxes. If \( \tilde{H}' \) is single-peaked, we then obtain a U-shaped adjustment to their progressive tax schedule, with taxes being U-shaped overall if and only if \( \tilde{H}' u' \) is single-peaked.

Indeed, the version of the model studied in this section is a strict generalization of Farhi et al.’s (2012) model whenever consumption utility can be bounded. To see this, let \( \underline{u} = \inf_{c \in C} u (c) \) and let \( \bar{u} = \sup_{c \in C} u (c) \), where \( C \) is some (large) set of relevant consumption levels. Suppose that

\[
H (c_1 (\theta), \tilde{c}_1 (\theta)) = \tilde{H} (u (c_1 (\theta)) - u (\tilde{c}_1 (\theta))) = \frac{1}{2} + \frac{1}{2} \frac{u (c_1 (\theta)) - u (\tilde{c}_1 (\theta))}{\bar{u} - \underline{u}},
\]

so that \( \tilde{H} \) is a uniform cumulative distribution function that is symmetric around zero in \( u(c_1(\theta)) - u(\tilde{c}_1(\theta)) \). Moreover, suppose \( \alpha = 1/2 \). Then the no-reform constraint in the

\(^{14} \)This follows because if the allocation is non-constant then \( U (\theta) \) must be non-constant by (2), and if \( U (\theta) \) is non-constant then \( c_1 (\theta) \) must be non-constant by (11).
current model, (4), becomes
\[
\int \left( \frac{1}{2} + \frac{1}{2} \frac{u(c_1(\theta)) - u(RK - \kappa)}{\bar{u} - u} \right) dF \geq \frac{1}{2},
\]
or equivalently
\[
\int u(c_1(\theta)) dF \geq u(RK - \kappa),
\]
which is precisely the no-reform constraint in Farhi et al. (2012). To understand this coincidence, recall that the no-reform constraint in Farhi et al. (2012) requires that a utilitarian government does not wish to equalize consumption at resource cost \(\kappa\), and observe that this is the case if and only if a simple majority of voters does not wish to equalize consumption at resource cost \(\kappa\) in a probabilistic voting model with uniform taste shocks. From this perspective, the results of this section may be viewed as a generalization of Farhi et al.’s (2012) analysis to the case where voters’ probabilities of supporting reform are not all equally sensitive to marginal policy changes. This in turn is exactly the case where political coalition formation matters.

We close this section by noting that the finding that optimal capital taxes are U-shaped holds in a particularly sharp way when \(H\) is a step function.

**Proposition 2.** If \(H\) is a step function, then in every monotone deterministic solution to the government’s problem there is an interval of types \([\theta_l, \theta_h]\) such that

1. Marginal capital taxes are positive and constant for \(\theta < \theta_l\) and \(\theta > \theta_h\), and in particular are given by
   \[
   \frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = R\eta \text{ for all } \theta < \theta_l, \theta > \theta_h
   \]
   where \(\eta \geq 0\) is the multiplier on (6).

2. Period-1 consumption \(c_1(\theta)\) equals \(RK - \kappa\) for all \(\theta \in [\theta_l, \theta_h]\). In addition, marginal capital taxes are non-decreasing on the interval \([\theta_l, \theta_h]\), and if \(c_1\) is non-constant then
   \[
   \frac{\tau_k(\theta)}{1 - \tau_k(\theta)} \leq R\eta \text{ for all } \theta \in [\theta_l, \theta_h].
   \]

**Proof.** See Appendix A.2. \(\blacksquare\)

Figure 1 illustrates the pattern derived in Proposition 2. To sustain support from a large enough fraction \(\alpha\) of the population, the government raises the consumption of the “middle class” (types between \(\theta_l\) and \(\theta_h\)) to \(RK - \kappa\), making them just indifferent to a
Figure 1: Equalizing reforms when $H$ is a step function

To achieve this, the government imposes a flat savings tax on the “poor” and “rich” (to depress $K$) and an increasing, lower tax (or a subsidy) on the middle class (to raise their $c_1(\theta)$ under the status quo). As in the case where $H$ is smooth, this leads to a U-shaped marginal capital tax schedule.

### 4 Strategic Reforms

In this section, we allow the government to (credibly) propose an arbitrary reform in period 1. While fully equalizing consumption remains the government’s most-preferred reform, it may wish to propose a reform other than full equalization in order to secure additional political support. Thus, in period 1 the government can commit to a “vote-buying” strategy that involves equalizing consumption as much as possible while still achieving sufficient support in the population relative to the status quo. As we assume that such proposals are credible, the implicit timing of the reform stage in this section is

1. The government proposes a reform consumption schedule $\hat{c}_1$.
2. Individuals vote on whether to implement the status quo $c_1$ or the reform $\hat{c}_1$. 
We will establish the unexpected result that in this version of the model, the purely progressive capital taxation result of Farhi et al. (2012) is restored under some mild additional assumptions, even if $H$ is strictly single-peaked. Indeed, a weak form of progressivity holds completely independently of the shape of $H$.

Before stating our results formally, we impose the assumption, maintained throughout this section, that the government would rather implement the optimal policy that forestalls reform than pay the resource cost $\kappa$ to implement the full-commitment solution. This assumption ensures that a reform does not actually occur in the best equilibrium for the government, as the government’s payoff in an equilibrium in which reform occurs is at most its full-commitment payoff minus $\kappa$. The assumption always holds if, for example, $\kappa > 0$ and $\alpha$ is sufficiently small, as in that case the full-commitment solution itself forestalls reform.\(^{15}\)

**Assumption 1** The value of the government’s problem described below is greater than the value of the government’s full-commitment solution minus $\kappa$.

We also maintain the following assumption on $H$ throughout this section.

**Assumption 2** $H$ either depends on consumption differences or utility differences, and $\tilde{H}'$ is single-peaked at 0.

Given that $\tilde{H}'$ is single-peaked, the assumption that the peak lies at 0 is essentially a normalization, as shifting $\tilde{H}$ and $\alpha$ by the same constant leaves the no-reform constraint (4) unchanged.

The following proposition summarizes our findings in this section. In what follows, a type $\theta$ is *non-pooled* if $c_1(\theta) \neq c_1(\theta')$ for all $\theta' \neq \theta$.

**Proposition 3.** Suppose Assumptions 1 and 2 are satisfied. Then in any monotone deterministic solution there is a threshold type $\theta^*$ such that capital is subsidized for all types $\theta < \theta^*$ and taxed for non-pooled types $\theta > \theta^*$. In addition, $\tau_k(\theta)$ is non-decreasing for all $\theta < \theta^*$ and all non-pooled $\theta > \theta^*$ if

(i) $H$ depends on consumption differences and $u''''(c) \geq 0$, or

(ii) $H$ depends on utility differences and $-u''(c) / u'(c)^2$ is non-increasing.

The most delicate issue in this result concerns the qualifications regarding non-pooled types. As we will see, these qualifications can be completely dispensed with in several leading cases, including when $H$ is a step function. Moreover, note that the progressivity\(^{15}\)It can also be easily shown that for any $\kappa > 0$, there exists an $\alpha > 0$ for which the assumption holds but the full-commitment solution does not forestall reform, so that the no-reform constraint is binding.
result in (i) goes through under the (weak) additional assumption that \( u''' \geq 0 \), which is a necessary condition for non-increasing absolute or relative risk aversion. The assumption in (ii) that \(-u''(c)/u'(c)^2\) is non-increasing is somewhat stronger than the assumption of non-increasing absolute risk-aversion, which says that \(-u''(c)/u'(c)\) is non-increasing. It is satisfied by CRRA utility with coefficient less than or equal to one, for example.

We analyze the case where \( H \) depends on consumption differences in the text, and defer the case where \( H \) depends on utility differences to Appendix B.

As discussed above, no reform occurs in an optimal equilibrium for the government under Assumption 1. Thus, the government’s problem is exactly same as in Section 3, except that the credibility constraint is no longer

\[
\int H (c_1 (\theta), RK - \kappa) dF \geq \alpha,
\]

but rather the condition that there does not exist any consumption schedule \( \hat{c}_1 : \Theta \rightarrow \mathbb{R} \) such that

\[
\int u (\hat{c}_1 (\theta)) dF > \int u (c_1 (\theta)) dF,
\]

\[
\int \hat{c}_1 (\theta) dF \leq \int c_1 (\theta) dF - \kappa,
\]

\[
\int \tilde{H} (c_1 (\theta) - \hat{c}_1 (\theta)) dF \leq \alpha.
\]

This constraint is clearly equivalent to the value of the following deviation program (DP), which we denote by \( V_D (c_1) \), being less than \( \int u (c_1 (\theta)) dF \).

\[
\max_{\hat{c}_1} \int u (\hat{c}_1 (\theta)) dF
\]

subject to (12) and (13).\(^{16}\)

As with the government’s period-0 problem, non-convexity is an unavoidable feature of the deviation program under natural specifications of \( H \), and hence its solution \( \hat{c}_1 (\theta) \) may not be unique or deterministic. Letting \( x (\theta) \equiv \hat{c}_1 (\theta) - c_1 (\theta) \), it is therefore useful to

\[^{16}\text{Given the convention from the previous section that the status quo is supported when (13) is satisfied with equality, the correct constraint here would involve a strict inequality in (13), so that the reform is supported. However, whenever there exists some } \hat{c}_1 \text{ that satisfies (12) and}
\]

\[
\int H (c_1 (\theta), \hat{c}_1 (\theta)) dF < \alpha,
\]

then \( \sup_{\hat{c}_1} \int u (\hat{c}_1 (\theta)) dF \) s.t. (12) and (14) equals \( V_D (c_1) \) by continuity of \( F \) and \( \tilde{H} \), so using the weak inequality in (13) does not make a difference. If on the other hand there does not exist any \( \hat{c}_1 \) that satisfies (12) and (14), then the “no-reform” constraint is not binding and the Atkinson-Stiglitz solution applies.
rewrite (DP), allowing for randomization over $x$-schedules, as

$$\max_P \int \int u( c_1(\theta) + x ) \, dP \, dF$$

subject to

$$\int \int x \, dP \, dF \leq -\kappa,$$

$$\int \int \tilde{H}( -x ) \, dP \, dF \leq \alpha.$$ (17)

Observe that (DP) does not depend on heterogeneity in $\theta$ directly but only on the distribution of $c_1$ in the population.\(^{17}\) Let

$$X(\theta) \equiv \{ x : x \in \text{supp } P(\theta) \text{ for some solution } P \text{ to (DP)} \}.$$  

Using our general Lemma 2, we can collect the following results about solutions to (DP).

**Lemma 4.** (i) A solution exists.
(ii) If $c_1(\theta) < c_1(\theta')$ then $\inf X(\theta) \geq \sup X(\theta').$
(iii) For any $c_1$-schedule and any $\bar{c}_1 \in \mathbb{R}$, let $\Theta_{\bar{c}_1} \equiv \{ \theta : c_1(\theta) = \bar{c}_1 \}$. Then for almost all $\bar{c}_1$, $\bigcup_{\theta \in \Theta_{\bar{c}_1}} X(\theta)$ is a singleton.
(iv) For every $\bar{c}_1 \in \mathbb{R}$, there exists at most one $s > 0$ such that $s \in \bigcup_{\theta \in \Theta_{\bar{c}_1}} X(\theta)$.

**Proof.** See Appendix A.3. \(\blacksquare\)

With these preliminary observations about the deviation program in hand, we consider the following dual formulation of the government’s problem.\(^{18}\)

$$\min_{c_0, c_1, y} \int \left( c_0(\theta) + \frac{1}{R} c_1(\theta) - y(\theta) \right) dF$$

subject to

$$\int u( c_1(\theta) ) \, dF \geq V_D( c_1 ),$$ (2) and (7). As in the equalizing reforms case, Lemma 2 can be applied to show the existence of a deterministic monotone solution.

**Lemma 5.** There exists a deterministic solution to the government’s problem in which $c_1(\theta)$ is non-decreasing.

\(^{17}\)Consequently, ratchet effects again do not arise here, as $c_1$ is observable and the government cannot gain from information about $\theta$ revealed in period 0.

\(^{18}\)The same variation as in footnote 9 shows that a solution to the primal must also solve the dual.
Lemma 6. (i) In any deterministic solution $c_1$, for almost all $\theta$ where $X(\theta)$ is single-valued, we have

$$\frac{\tau_k (\theta)}{1 - \tau_k (\theta)} = R \eta \left[ u' (c_1 (\theta) + x (\theta)) - u' (c_1 (\theta)) \right], \quad \text{where } X(\theta) = \{ x(\theta) \}. \quad (19)$$

(ii) For almost all non-pooled types $\theta$, $X(\theta)$ is single-valued, so (19) holds.

Proof. (i) Suppose toward a contradiction that (19) fails on a positive measure set of types with single-valued $X(\theta)$ for some optimal consumption schedule $c_1$. Then there exists either a positive measure set $\Theta'$ on which $\tau_k (\theta) / (1 - \tau_k (\theta))$ is less than the right-hand side of (19) (and $X(\theta)$ is single-valued for all $\theta \in \Theta'$), or a positive measure set $\Theta''$ on which $\tau_k (\theta) / (1 - \tau_k (\theta))$ exceeds it (and $X(\theta)$ is single-valued for all $\theta \in \Theta''$). Assume the first case applies; the argument for the second case is symmetric.

Consider the variant consumption schedule $\tilde{c}_1$ given by

$$\tilde{c}_1 (\theta) = \begin{cases} c_1 (\theta) & \text{for } \theta \notin \Theta', \\ c_1 (\theta) + t & \text{for } \theta \in \Theta', \end{cases} \quad (20)$$

for $t \in \mathbb{R}$. Slightly abusing notation, for variants $\tilde{c}_1$ of this form, let $V_D (t)$ be the corresponding value function in (DP). Letting $V_D' (0-)$ and $V_D' (0+)$ denote the left- and right-derivative of $V_D (t)$ at $t = 0$, respectively, Corollary 4 of Milgrom and Segal (2002) implies that $V_D' (0-)$ and $V_D' (0+)$ exist and satisfy

$$V_D' (0-) = \inf_{p \text{ that solve (DP)}} \int_{\Theta'} \int u' (c_1 (\theta) + x) \, dPdF,$$

$$V_D' (0+) = \sup_{p \text{ that solve (DP)}} \int_{\Theta'} \int u' (c_1 (\theta) + x) \, dPdF.$$

Since $X(\theta)$ is single-valued for all $\theta \in \Theta'$ by hypothesis, the supremum and infimum must fall together, so
\( V_D(t) \) is differentiable at \( t = 0 \) with

\[
V_D'(0) = \int_{\Theta'} u'(c_1(\theta) + x(\theta))dF.
\]

Therefore, a necessary condition for optimality of \( c_1 \) is that \( t = 0 \) is a stationary point (over \( t \in \mathbb{R} \)) of the Lagrangian

\[
\int_{\Theta'} \left( u^{-1} (U(\theta) - \beta u(c_1(\theta) + t)) + \frac{1}{R} (c_1(\theta) + t) - \eta (u(c_1(\theta) + t) - u'(c_1(\theta) + t + x(\theta))) \right) dF.
\]

The corresponding first-order condition is

\[
\int_{\Theta'} \left( -\frac{\beta u'(c_1(\theta))}{u'(c_0(\theta))} + \frac{1}{R} - \eta [u'(c_1(\theta)) - u'(c_1(\theta) + x(\theta))] \right) dF = 0.
\]

Multiplying through by \( R \) and using the definition (3), this implies

\[
\int_{\Theta'} \left( -\frac{1}{1 - \tau_k(\theta)} + 1 - R\eta [u'(c_1(\theta)) - u'(c_1(\theta) + x(\theta))] \right) dF = 0.
\]

Together with the fact that \( \Theta' \) has positive measure, this contradicts the initial hypothesis that

\[
\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} < R\eta [u'(c_1(\theta) + x(\theta)) - u'(c_1(\theta))]
\]

for all \( \theta \in \Theta' \). Hence, no such set \( \Theta' \) can exist.

(ii) If (19) fails on a positive measure set of non-pooled agents for some optimal consumption schedule \( c_1 \), there exists either a positive measure set \( \Theta' \) on which \( \tau_k(\theta)/(1 - \tau_k(\theta)) \) is less than the right-hand side of (19) (and \( c_1(\theta) \) is strictly increasing in \( \theta \in \Theta' \)), or a positive measure set \( \Theta'' \) on which \( \tau_k(\theta)/(1 - \tau_k(\theta)) \) exceeds it (and \( c_1(\theta) \) is strictly increasing in \( \theta \in \Theta'' \)). Consider again the first case and recall that \( \cup_{\theta,c_1(\theta)=c_1} X(\theta) \) is single-valued for almost all \( \bar{c}_1 \), by Lemma 4 (iii). Since \( c_1(\theta) \) is strictly increasing on \( \Theta' \), this implies that \( X(\theta) \) is single-valued for almost all \( \theta \in \Theta' \), so the argument from (i) can be applied.

In what follows, we restrict attention to monotone deterministic solutions. We will show that such solutions feature progressive marginal capital taxation in the following sense.

**Proposition 4.** In a monotone deterministic solution, there exists a threshold type \( \theta^* \) such that capital is subsidized (\( \tau_k(\theta) < 0 \)) for agents with \( \theta < \theta^* \) and capital is taxed (\( \tau_k(\theta) \geq 0 \)) for all non-pooled agents with \( \theta > \theta^* \).

**Proof.** Let \( c_1^* = \inf \{ c_1 : \exists \theta \text{ such that } c_1(\theta) = c_1 \text{ and } x(\theta) \leq 0 \text{ for some } x(\theta) \in X(\theta) \} \) and \( \theta^* = \inf \{ \theta : c_1(\theta) \geq c_1^* \} \). Note that if \( \theta \) satisfies equation (19) then

\[
\text{sign} (\tau_k(\theta)) = \text{sign} (-x(\theta)).
\]

We will show that \( \theta \) satisfies (19) for (almost) all \( \theta < \theta^* \) and non-pooled \( \theta > \theta^* \), and that \( x(\theta) > 0 \) for \( \theta < \theta^* \) while \( x(\theta) \leq 0 \) for non-pooled \( \theta > \theta^* \). This will complete the proof.
If $\theta < \theta^*$ then $c_1(\theta) < c_1^*$ and hence, by definition of $c_1^*$, $x(\theta) > 0$ for all $x(\theta) \in X(\theta)$. By Lemma 4 (iv), for such types $X(\theta)$ is single-valued if $\tilde{H}'$ is single-peaked at 0. Hence by Lemma 6, such types satisfy (19). In sum, types with $\theta < \theta^*$ satisfy (19) and $x(\theta) > 0$.

If $\theta > \theta^*$ is non-pooled then $c_1(\theta) > c_1^*$ and therefore $x(\theta) \leq 0$ for some $x(\theta) \in X(\theta)$, by definition of $c_1^*$ and Lemma 4 (ii). By Lemma 6 (ii), for almost all non-pooled types $X(\theta)$ is singleton and satisfies (19). In sum, non-pooled types with $\theta > \theta^*$ satisfy (19) and $x(\theta) \leq 0$.

The intuition for Proposition 4 relies on the fact that the capital tax, which follows formula (19), is designed to make individuals of each type $\theta$ internalize the effect of an additional unit of their saving on the no-reform constraint. This involves comparing the effect on period-1 welfare under the best deviation, given by $u'(c_1(\theta) + x(\theta))$, with the effect on welfare under the status quo, $u'(c_1(\theta))$. Since by Lemma 4 any reform will equalize period-1 consumption relative to the status quo (i.e., $x(\theta)$ is decreasing), low $\theta$ types face $x(\theta) > 0$, so their saving relaxes the no-reform constraint (as $u'(c_1(\theta) + x(\theta)) < u'(c_1(\theta))$ when $x(\theta) > 0$), motivating the capital subsidy. In contrast, high $\theta$ types face $x(\theta) < 0$, so their saving tightens the no-reform constraint, which makes it optimal for them to face a capital tax.

Note that this logic is independent from the shape of the function $\tilde{H}$; notably, it does not depend on whether $\tilde{H}'$ is single-peaked or not. This is in contrast to our results in Section 3, where the shape of $\tilde{H}$ was crucial for the U-shaped pattern of the intertemporal wedge. The reason for this difference is that, in Section 3, the key comparison for determining the capital tax is between $c_1(\theta)$ and $RK - \kappa$, which implies that agents with intermediate $c_1(\theta)$ are “pivotal” when $\tilde{H}'$ is single-peaked, and are therefore subsidized. In contrast, in the current section the key comparison is between $u'(c_1(\theta))$ and $u'(c_1(\theta) + x(\theta))$, so that agents with $x(\theta) > 0$ are more sensitive to their period-1 consumption under the status quo than under the reform—regardless of the shape of $\tilde{H}$—which leads them to be subsidized (and $x(\theta)$ is decreasing regardless of the shape of $\tilde{H}$, as for any status quo schedule $c_1$, the most tempting reform schedule is more egalitarian than $c_1$).

In Appendix B, we show that a similar tradeoff determines the shape of $\tau_k$ when $\tilde{H}$ depends on utility rather than consumption differences. In this case, both the government’s planning problem and deviation program can most conveniently be written in dual utility space, where period 1 consumption utility $u_1(\theta) \equiv u(c_1(\theta))$ is chosen for each individual. The no-reform constraint can then be framed as requiring that the optimal deviation must

---

$^{19}$The only place where single-peakedness of $\tilde{H}$ matters in this section so far is part (iv) of Lemma 4, which says that $X(\theta)$ have at most one positive element for almost all $\theta$. This in turn implies that formula (19) applies for almost all $\theta < \theta^*$ without restricting to non-pooled types. Without this assumption, Proposition 4 would still hold for non-pooled types.
be more costly in terms of resources than the status quo when achieving at least the same level of welfare. Hence, the key tradeoff becomes $\Phi'(u_1(\theta) + x(\theta))$ versus $\Phi'(u_1(\theta))$, where $\Phi$ is the inverse of $u(\cdot)$ and $x(\theta) = \hat{u}_1(\theta) - u_1(\theta)$ is reform minus status quo consumption utility. Since $x(\theta)$ will again be decreasing, the same qualitative results apply in that model. Table 1 summarizes these key tradeoffs and results.

We also have the following stronger result about tax progressivity, which completes the proof of Proposition 3 for the case where $\hat{H}$ depends on consumption differences.

**Proposition 5.** In a monotone deterministic solution, the following hold.

(i) If $u'''' > 0$, then $\tau_k(\theta)$ is non-decreasing on $\{\theta : \theta < \theta^*\}$.

(ii) $\tau_k(\theta)$ is non-decreasing on $\{\theta : \theta$ is non-pooled and $\theta > \theta^*\}$.

**Proof.** Note that if (almost) all $\theta$ in a set $\Theta' \subseteq \Theta$ satisfy (19), then $\tau_k(\theta)$ is (almost everywhere) non-decreasing on $\Theta'$ if and only if $u'(\hat{c}_1(\theta)) - u'(c_1(\theta))$ is (almost everywhere) non-decreasing.

For the first part, recall that types $\theta < \theta^*$ satisfy (19) as well as $x(\theta) > 0$. If $u'(c_1(\theta) + x(\theta)) - u'(c_1(\theta))$ is differentiable at $\theta$, then (omitting the $\theta$-arguments) its derivative equals
\[
(u''(c_1 + x) - u''(c_1))c_1' + u''(c_1 + x)x'.
\]

Note that $c_1'(\theta) \geq 0$ by Lemma 5 and $x'(\theta) \leq 0$ by Lemma 4 (i), so if $x(\theta) > 0$ and $u'''' > 0$ then this derivative is non-negative. In addition, if $u'(c_1(\theta) + x(\theta)) - u'(c_1(\theta))$ is discontinuous at $\theta$, then either $c_1(\theta)$ jumps up or $x(\theta)$ jumps down, and when $x(\theta) > 0$ and $u'''' \geq 0$ either of these jumps increases $u'(c_1(\theta) + x(\theta)) - u'(c_1(\theta))$. Hence, $u'(c_1(\theta) + x(\theta)) - u'(c_1(\theta))$ is non-decreasing on $\{\theta : \theta < \theta^*\}$.

For the second part, we first claim that $\hat{c}_1(\theta)$ is non-increasing on $\Theta' = \{\theta : c_1(\theta) > c_1^* \text{ and } \theta > \theta^*\}$ (which is a superset of $\{\theta : \theta$ is non-pooled and $\theta > \theta^*\}$). To see this, write (DP) as
\[
\min_{\hat{c}_1} \int \hat{H}(c_1(\theta) - \hat{c}_1(\theta))\, dF
\]
subject to
\[
\begin{align*}
\int \hat{c}_1(\theta)\, dF & \leq RK - \kappa, \\
\int u(\hat{c}_1(\theta))\, dF & \geq \int u(c_1(\theta))\, dF. 
\end{align*}
\]

Table 1: Comparison of key tradeoffs and tax results

<table>
<thead>
<tr>
<th>Model</th>
<th>Tradeoff</th>
<th>Shape of $\tau_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equalizing reforms</td>
<td>$H_1$ vs. $\bar{H}_2$</td>
<td>U-shaped</td>
</tr>
<tr>
<td>Strategic reforms – consumption differences</td>
<td>$u'(c_1 + x)$ vs. $u'(c_1)$</td>
<td>progressive</td>
</tr>
<tr>
<td>Strategic reforms – utility differences</td>
<td>$\Phi'(u_1 + x)$ vs. $\Phi'(u_1)$</td>
<td>progressive</td>
</tr>
</tbody>
</table>
Fix a deterministic solution $\hat{c}_1^* : \Theta \rightarrow \mathbb{R}$. Let

$$
\begin{align*}
\hat{K} &= \int_{\theta \in \Theta^*} \hat{c}_1^* (\theta) \, dF, \\
\hat{V} &= \int_{\theta \in \Theta^*} u (\hat{c}_1^* (\theta)) \, dF.
\end{align*}
$$

Then a necessary condition for optimality is that the restriction of $\hat{c}_1^*$ to $\Theta^*$ solves the subproblem

$$
\min_{\hat{c}_1 : \Theta^* \rightarrow \mathbb{R}} \int_{\Theta^*} H (c_1 (\theta) - \hat{c}_1 (\theta)) \, dF
$$

subject to

$$
\begin{align*}
\hat{K} + \int_{\Theta^*} \hat{c}_1 (\theta) \, dF &\leq RK - \kappa, \\
\hat{V} + \int_{\Theta^*} u (\hat{c}_1 (\theta)) \, dF &\geq \int u (c_1 (\theta)) \, dF.
\end{align*}
$$

If $c_1 (\theta) > c_1^*$ then $c_1 (\theta) \geq \hat{c}_1^* (\theta)$ for every solution to (DP), by the definition of $c_1^*$ and Lemma 4 (ii). Hence, a necessary condition for optimality is that $\hat{c}_1^*$ still solves the above subproblem when $\hat{c}_1$ is restricted to satisfy $c_1 (\theta) \geq \hat{c}_1 (\theta)$ for all $\theta \in \Theta^*$. Now, since $\hat{H}$ is single-peaked at 0, the objective in this subproblem has strictly increasing differences in $c_1 (\theta)$ and $\hat{c}_1 (\theta)$ over this range, while $c_1 (\theta)$ does not enter in the constraints except through the constant $\int u(c_1(\theta))dF$, so Lemma 2 (i) implies that at every solution $\hat{c}_1 (\theta)$ is non-increasing in $c_1 (\theta)$, and hence in $\theta$.

Finally, note that if $u' (\hat{c}_1 (\theta)) - u' (c_1 (\theta))$ is differentiable at $\theta$, then (omitting the $\theta$-arguments) its derivative equals

$$
u'' (\hat{c}_1) \hat{c}_1^* - u'' (c_1) c_1^*.$$

Thus, if $c_1$ is non-decreasing and $\hat{c}_1$ is non-increasing, this derivative is non-negative. If $u' (\hat{c}_1 (\theta)) - u' (c_1 (\theta))$ is discontinuous at $\theta$, then either $c_1 (\theta)$ jumps up or $\hat{c}_1 (\theta)$ jumps down, and either or these jumps increases $u' (\hat{c}_1 (\theta)) - u' (c_1 (\theta))$. Hence, $u' (\hat{c}_1 (\theta)) - u' (c_1 (\theta))$ is non-decreasing on $\{ \theta : c_1 (\theta) > c_1^* \}$ and $\theta > \theta^*$. Finally, non-pooled types satisfy (19), so $\tau_k (\theta)$ is non-decreasing on $\{ \theta : \theta$ is non-pooled, $c_1 (\theta) > c_1^*$, and $\theta > \theta^* \} = \{ \theta : \theta$ is non-pooled and $\theta > \theta^* \}$. \[\square\]

The proof of Proposition 5 shows that, if $\hat{H}$ is single-peaked, the optimal deviation for the government (which is not implemented in equilibrium) is a reform with a consumption schedule $\hat{c}_1$ that is weakly decreasing in $\theta$ for the taxed agents (i.e. those with $\theta > \theta^*$ and hence $x(\theta) \leq 0$ and $\tau_k (\theta) \geq 0$). An analogous argument can be used to show that $\hat{c}_1$ is increasing in $\theta$ for the subsidized agents with $\theta < \theta^*$. Hence, the (off-path) post-reform consumption $\hat{c}_1 (\theta)$ is always single-peaked in $\theta$ with the peak occurring at $\theta^*$, the type for which reform and status quo consumption coincide. Of course, this single-peaked reform consumption pattern $\hat{c}_1$ does not translate into a U-shaped pattern of marginal capital taxes: the intertemporal wedge is progressive throughout because what matters is not $\hat{c}_1$ per se but the difference $\hat{c}_1 - c_1$, and this is monotonically non-increasing as discussed above.
We close this section with a discussion of the caveats regarding non-pooled types in Proposition 3. It is first worth recalling that \( \tau_k(\theta) \) is non-decreasing at all types \( \theta \) for which formula (19) applies, which by Lemma 6 occurs whenever there is a unique solution to the deviation program for type \( \theta \), or, even if this not the case, type \( \theta \) is not pooled with other types in terms of \( c_1 \). The reason why we need one of these two assumptions is that we use an envelope theorem in the proof of Lemma 6, and the deviation value function \( V_D(c_1) \) may not be differentiable if \( X(\theta) \) is not single-valued (left- and right-derivatives are always well-defined, but they may not coincide).

However, by Lemma 4 (iii), we know that the solution to (DP) is unique for almost all \( c_1 \)-levels (recall that (DP) does not directly depend on \( \theta \) but only through the implied \( c_1 \)-schedule). Hence, potential non-differentiability of \( V_D \) can occur almost nowhere in terms of \( c_1 \), and problems could only arise when a non-unique solution to (DP) happens to occur at a \( c_1 \)-level at which there is a strictly positive mass of agents, i.e. where there is pooling. This is illustrated in Figure 2, where types in \( [\theta_1, \theta_2] \) are pooled at \( \bar{c}_1 \), and \( X \) happens to be non-singleton at \( \bar{c}_1 \) (note that, by Lemma 4 (iv), such non-uniqueness can only occur for negative \( x \)-values, i.e. for types \( \theta > \theta^* \)). In this case, the marginal capital tax could be non-monotone overall, although it is still increasing among the non-pooled types. There is no obvious reason, however, to expect that non-unique solutions to (DP) are more or less likely to emerge at \( c_1 \)-levels at which there is pooling, so we view this as a technicality that needs to be accounted for but not as a particularly important economic feature of our model.

![Figure 2: Potential non-monotonicity of \( \tau_k \) with pooled individuals](image)
In fact, these caveats can be completely dispensed with if for every period-1 consumption schedule \( c_1 \), one of the following two conditions holds.

1. The set of solutions to the deviation program \( X(\theta) \) is single-valued for almost all \( \theta \).

2. \( \int u(c_1(\theta))\,dF - V_D(c_1) \) is convex in \( c_1 \) (which implies the government’s problem is convex).

Both of these points require some explanation. For the first one, note that if \( X(\theta) \) is (almost) always single-valued, then the argument of Lemma 6 (i) applies—and therefore (19) holds—for (almost) all types \( \theta \). Simpler versions of the proofs of Propositions 4 and 5 then imply that both of these results will hold without the qualifications concerning non-pooled types.

Furthermore, \( X(\theta) \) is indeed single-valued almost everywhere in some natural cases. For example, if \( \hat{H} \) is a step function, then it may be shown that \( X(\theta) \) is single-valued almost everywhere, and in particular that it is flat at \( x(\theta) = 0 \) for all \( \theta \) in an interval \((\theta_l, \theta_h)\) (corresponding to those types that are indifferent between the status quo and the reform), and is decreasing on the intervals \((-\infty, \theta_l)\) and \((\theta_h, \infty)\) (in order to fully equalize final consumption under the reform for types in those intervals).\(^{21}\) Figure 3 depicts the resulting shape of \( \hat{c}_1 \) compared to \( c_1 \) as well as the intertemporal wedge \( \tau_k \).

For the second point, observe that the government’s problem is convex if \( \int u(c_1(\theta))\,dF - V_D(c_1) \) is convex. The intuition is that, unlike the government’s problem in the model with equalizing reforms or the deviation problem in the current model, the government’s problem in the current model does not directly depend on the non-concave political support function \( \hat{H} \) (though of course it depends indirectly on \( \hat{H} \) through \( V_D \)), so there is some hope that this problem might actually be convex. If it is convex, then an explicit variational approach can be taken to minimizing the Lagrangian (21) in the proof of Lemma 6 (as now an extremum of the government’s problem must also be an extremum of the Lagrangian), and it may be shown that the following version of (19) holds for almost all types \( \theta \) (whether or not \( \theta \) is pooled), where \( \bar{x}(\theta) = \sup X(\theta) \) and \( \underline{x}(\theta) = \inf X(\theta) \):

\[
\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} \in \left[ R\eta \left[ u'(c_1(\theta) + \bar{x}(\theta)) - u'(c_1(\theta)) \right], R\eta \left[ u'(c_1(\theta) + \underline{x}(\theta)) - u'(c_1(\theta)) \right] \right].
\]

Lemma 4 may then be used to show that Propositions 4 and 5 hold without the qualifications about non-pooled types.

\(^{21}\)Unlike in the equalizing reforms section, we do not need a separate result to cover the step function case here, as the optimal tax formulas derived in this section do not assume that \( \hat{H} \) is differentiable.
A last remark is that the condition that \( \int u(c_1(\theta)) \, dF - V_D(c_1) \) is convex is not overly strong. For example, it may be checked that it holds if \( u \) is quadratic.\(^{22}\)

5 Discussion

This section discusses three possible extensions of our model: implementing the optimal allocation with taxes, generalizing the government’s objective, and moving from two periods to overlapping generations.

5.1 Tax Implementation

Our analysis so far has implicitly considered direct mechanisms, where the government allocates \( c_0(\theta), c_1(\theta) \) and \( y(\theta) \) conditional on individual reports about \( \theta \), taking into account technological, incentive compatibility and political credibility constraints. As in

\(^{22}\)One might think that if \( u \) is quadratic then the comparative statics in Proposition 5 (i) would never hold strictly, as that result assumes \( u''' \geq 0 \). However, inspecting the proof reveals that there is slack in this sufficient condition, and that in fact the comparative static holds strictly at any type \( \theta \) where \( x(\theta) \) is differentiable and non-constant, even if \( u''' = 0 \).
Farhi et al. (2012), it is straightforward to show that these allocations can alternatively, and more realistically, be implemented through a tax system where each individual is confronted with the same budget set and picks her preferred allocation within this set. In particular, with a non-linear labor income tax \( T_y \) and a non-linear capital income tax \( T_k \), individuals are faced with the budget constraint \( c_0 + k \leq y - T_y(y) \) in period 0 and \( c_1 \leq Rk - T_k(Rk) \) in period 1 and choose \( c_0, c_1, y, k \) to maximize \( u(c_0) + \beta u(c_1) - h(y, \theta) \) subject to these two constraints.

By Proposition 3 in Farhi et al. (2012), any incentive compatible allocation \((c_0(\theta), c_1(\theta), y(\theta))\) that is non-decreasing in \( \theta \) can be implemented using such a tax system. Since we show that \( c_1(\theta) \) is always non-decreasing in a (deterministic and monotone) optimal allocation and \( y(\theta) \) is non-decreasing by incentive compatibility, their result can be applied to our framework.²³ More importantly, the first-order conditions from the above utility-maximization problem imply

\[
u'(c_0(\theta)) = \beta R(1 - T'_k(Rk(\theta))) \nu'(c_1(\theta))\]

for all \( \theta \) whenever \( T_k \) is differentiable, so the wedge \( \tau_k(\theta) \) defined in (3) and characterized throughout this paper coincides with the actual marginal capital income tax rate \( T'_k(Rk(\theta)) \) faced by individuals of type \( \theta \) in this implementation.²⁴

### 5.2 General Objective Function

Our maintained assumption that the government is utilitarian can be relaxed substantially. This is shown in Appendix C, where we relax the utilitarian assumption in two ways. First, we let the government use general Pareto weights \( G \) to evaluate welfare, which can differ from the population distribution \( F \). Second, we let individuals differ along a second dimension \( \rho \) in addition to \( \theta \), where \( \rho \) is observable to the government and enters into the government’s Pareto weights, but is otherwise payoff-irrelevant. This second dimension of heterogeneity allows for a government that is “non-benevolent,” in that it favors certain groups in society over others: for example, \( \rho \) could capture an individual’s race, ethnicity, or other minority status, her age, her geographic region of origin, or any other observable marker of membership in some group that the government may favor or disfavor. We show that our main results go through within each group \( \rho \) if the

²³The statement of Proposition 3 in Farhi et al. (2012) also requires \( c_0(\theta) \) to be non-decreasing, which is true at the optimum in their framework but may or may not be true in our model. However, inspecting the proof reveals that this condition is in fact not needed for the result.

²⁴If \( T_k(Rk(\theta)) \) is not differentiable because there is pooling at consumption level \( c_1(\theta) \) (so \( T_k \) has a convex kink), \( \tau_k(\theta) \) is still bounded between the (well-defined) left- and right-derivatives of \( T_k \).
government’s redistributive preferences are at least as inequity averse as the utilitarian criterion would imply. Formally, if $g(\theta, \rho)$ is the density corresponding to the Pareto weights $G$ over the two dimensions of heterogeneity, then our main results go through whenever $g(\theta, \rho)/f(\theta, \rho)$ is decreasing in $\theta$, holding $\rho$ fixed, where $f$ is the joint density corresponding to $F$.

For example, this implies that, perhaps somewhat surprisingly, the U-shaped pattern of marginal capital taxes from the first version of our model emerges even if the government puts very high weight on low-$\theta$ types (as with a Rawlsian objective).

5.3 Overlapping Generations

It is straightforward to extend the two-period model here to an infinite-horizon overlapping generations (OLG) setting, as in Farhi et al. (2012). This would allow us to endogenize the cost of reform $\kappa$ as a “reputational” cost borne by the government when it deviates by reforming its proposed policy. In particular, consider an OLG-version of our model where individuals of each generation live for two periods. Suppose that whenever a reform occurs, play reverts to the worst continuation equilibrium for the government. This worst continuation equilibrium is the one where no further production takes place, and the government fully equalizes future consumption, spending down the remaining capital stock optimally. In such “grim trigger” equilibria, the results from our two-period model, both from Sections 3 and 4, would continue to hold.

One technicality here is that, unlike in Farhi et al. (2012), grim trigger equilibria would not necessarily be the best equilibria overall from the perspective of the government in an OLG version of our model. To see this, note that an optimal equilibrium for the government is one in which it is punished as harshly as possible for any deviation. In grim trigger equilibria, the government gets its lowest possible continuation payoff (corresponding to full equalization) starting in period $t+1$ after a deviation in period $t$, but in period $t$ voters approve the proposed reform if it is myopically optimal for them to do so. In general, it might be possible to punish the government more harshly by specifying that continuation play after a deviant reform proposal that is myopically appealing to voters is such that the reform is not approved, while continuation play is something other than full equalization (for example, continuation play might specify that the government rewards “pivotal” voters in the future if they do not support a deviant reform today).

Note that this issue would go away if, for example, we assumed that only the old gener-

\footnote{Such a punishment might be harsher than grim trigger because it gives a lower instantaneous payoff for the government, even though it also gives a higher continuation payoff (note that there is no contradiction with the results of Abreu (1988) and others here, as the current model is not a repeated game).}
ation votes, which is a relatively common in OLG models of political economy (Glomm and Ravikumar, 1992, Saint-Paul and Verdier, 1997, Benabou, 2000).

6 Conclusion

This paper has studied dynamic non-linear taxation under the assumption that a policy is credible if it maintains the support of a large enough political coalition. Optimal taxes in this setting differ starkly from those in settings where the government can fully commit to policy, or where it can always change policy at a fixed cost. Rather than predicting zero capital taxes (as in Atkinson and Stiglitz, 1976) or purely progressive capital taxes (as in Farhi et al., 2012), the simplest version of our model (which is still a generalization of both Atkinson and Stiglitz, 1976, and Farhi et al., 2012) predicts U-shaped capital taxes, so that saving is subsidized for the middle class but taxed for the poor and rich, recalling Director’s law of redistribution (Stigler, 1970). In a more complicated version of the model where the government can engage in sophisticated vote-buying schemes, we find that purely progressive capital taxation re-emerges. These versions of the model can be interpreted as capturing varying degrees of government commitment at the reform stage, as for instance resulting from more direct versus indirect forms of democracy. More generally, our analysis suggests that the nature of potential political reforms is an important determinant of the progressivity and middle-class bias of capital taxes, and of redistribution more broadly.

References


A Appendix: Omitted Proofs

A.1 Proof of Lemma 2

Part (i). To obtain a contradiction, suppose that there exist \( \theta', \theta'' \in \Theta \) such that \( t(\theta') < t(\theta'') \) and \( a(\theta') > a(\theta'') \) for some \( a(\theta') \in \text{supp} \ P(\theta'), a(\theta'') \in \text{supp} \ P(\theta''). \) Since \( t \) and \( P \) are right-continuous and \( \Theta \) is open, there exist disjoint closed intervals of positive length \( \Theta' \subseteq \Theta \) and \( \Theta'' \subseteq \Theta \) such that \( t(\theta') < t(\theta'') \) and \( a(\theta') > a(\theta'') \) for some \( a(\theta') \in \text{supp} \ P(\theta'), a(\theta'') \in \text{supp} \ P(\theta'') \) for all \( \theta' \in \Theta', \theta'' \in \Theta''. \) Let \( \bar{a}(\theta) = \sup \{ \text{supp} \ P(\theta') \} \), \( a(\theta) = \inf \{ \text{supp} \ P(\theta') \} \), and

\[
\nu \equiv \inf_{\theta' \in \Theta', \theta'' \in \Theta''} \bar{a}(\theta') - a(\theta'') > 0.
\]

Without loss of generality, let the lengths of \( \Theta' \) and \( \Theta'' \) be equal. Define \( \phi : \Theta' \to \Theta'' \) by \( \phi(\theta) = \theta + \theta' - \theta'' \), so that in particular \( \phi \) is an invertible bijection. Given a distribution \( P(\theta) \), let \( \bar{P}(\theta) \) and \( \underline{P}(\theta) \) denote the truncation of \( P(\theta) \) on \([\bar{a}(\theta) - \nu/4, \bar{a}(\theta)]\) and \([\underline{a}(\theta), \underline{a}(\theta) + \nu/4]\), respectively. Define a new randomized schedule \( \hat{P} \) by

\[
\hat{P}(\theta) \equiv \begin{cases} 
    P(\theta) + \int_{\Theta'} \left( \overline{\gamma}(\theta) \bar{P}(\phi(\theta)) - \underline{\gamma}(\phi(\theta)) \underline{P}(\theta) \right) & \text{if } \theta \in \Theta' \\
    P(\theta) + \int_{\Theta''} \left( \overline{\gamma}(\theta) \bar{P}(\phi^{-1}(\theta)) - \underline{\gamma}(\phi^{-1}(\theta)) \underline{P}(\theta) \right) & \text{if } \theta \in \Theta'' \\
    P(\theta) & \text{if } \theta \notin \Theta' \cup \Theta''
\end{cases}
\]

where the factors

\[
\overline{\gamma}(\theta) \equiv \int d\bar{P}(\theta) \quad \text{and} \quad \underline{\gamma}(\theta) \equiv \int d\underline{P}(\theta)
\]

ensure that \( \hat{P}(\theta) \) integrates to one for each \( \theta \), and we fix some \( \epsilon > 0 \) such that \( \epsilon < \inf_{\theta \in \Theta' \cup \Theta''} f(\theta) \), which is positive and (together with \( \overline{\gamma}(\theta), \underline{\gamma}(\theta) \leq 1 \) for all \( \theta \)) makes sure that \( \hat{P}(\theta) \geq 0 \) for all \( a \) and \( \theta \).

This variation is constructed such that, for any \( x \in X \), we have

\[
\int \int y_x(a) d\hat{P}dF = \int \int y_x(a) dPdF,
\]

because

\[
\int \int y_x(a) d\hat{P}dF - \int \int y_x(a) dPdF = \epsilon \int_{\Theta'} \int y_x(a) \left( \overline{\gamma}(\theta) d\bar{P}(\phi(\theta)) - \underline{\gamma}(\phi(\theta)) d\bar{P}(\theta) \right) d\theta + \epsilon \int_{\Theta''} \int y_x(a) \left( \overline{\gamma}(\theta) d\bar{P}(\phi^{-1}(\theta)) - \underline{\gamma}(\phi^{-1}(\theta)) d\bar{P}(\theta) \right) d\theta = 0.
\]
In particular, this implies that $A(\hat{P}) = A(P)$ and therefore
\[
\int \int y_x(a, A(\hat{P})) \, d\hat{P} dF = \int \int y_x(a, A(P)) \, dP dF
\]
for all $x \in X$, i.e. if $P$ satisfies constraint C or $C'$, then so does $\hat{P}$. In addition,
\[
\int \int w(a, t(\theta)) \, d\hat{P} dF - \int \int w(a, t(\theta)) \, dP dF
\]
\[
= \epsilon \int_{\Theta'} \left[ \int w(a, t(\theta)) \gamma(\theta) \, d\hat{P}(\phi(\theta)) - \int w(a, t(\theta)) \gamma(\phi(\theta)) \, d\hat{P}(\theta) \right] \, d\theta
\]
\[
+ \epsilon \int_{\Theta'} \left[ \int w(a, t(\theta)) \gamma(\theta) \, d\hat{P}(\phi(\theta)) - \int w(a, t(\theta)) \gamma(\phi(\theta)) \, d\hat{P}(\theta) \right] \, d\theta
\]
\[
= \epsilon \int_{\Theta'} \left[ \int [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \gamma(\theta) \, d\hat{P}(\phi(\theta)) - \int [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \gamma(\phi(\theta)) \, d\hat{P}(\theta) \right] \, d\theta.
\]
For each $\theta \in \Theta'$, $t(\theta) < t(\phi(\theta))$, and because $w(a, t)$ has increasing differences, $w(a, t(\theta)) - w(a, t(\phi(\theta)))$ is decreasing in $a$. Therefore,
\[
\int [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \gamma(\theta) \, d\hat{P}(\phi(\theta)) > [w(a, \phi(\theta)) + v/4, t(\theta)) - w(a, \phi(\theta)) + v/4, t(\phi(\theta)))] \gamma(\theta) \gamma(\phi(\theta))
\]
where we used $\int d\hat{P}(\phi(\theta)) = \gamma(\theta(\phi))$. Similarly, for each $\theta \in \Theta'$,
\[
\int [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \gamma(\phi(\theta)) \, d\hat{P}(\theta) < [w(a, \phi(\theta)) - v/4, t(\theta)) - w(a, \phi(\theta)) - v/4, t(\phi(\theta)))] \gamma(\phi(\theta)) \gamma(\phi(\theta)).
\]
Hence,
\[
\epsilon \int_{\Theta'} \left[ \int [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \gamma(\theta) \, d\hat{P}(\phi(\theta)) - \int [w(a, t(\theta)) - w(a, t(\phi(\theta)))] \gamma(\phi(\theta)) \, d\hat{P}(\theta) \right] \, d\theta
\]
\[
> \epsilon \int_{\Theta'} \left[ w(a, \phi(\theta)) + v/4, t(\theta)) - w(a, \phi(\theta)) + v/4, t(\phi(\theta)))] \gamma(\phi(\theta)) \gamma(\theta) \, d\theta
\]
\[
> 0,
\]
where the last inequality follows because $a(\phi(\theta)) + v/4 < a(\theta) - v/4$ for all $\theta \in \Theta'$ and $w(a, t(\theta)) - w(a, t(\phi(\theta)))$ is decreasing in $a$ (as $w(a, t)$ has increasing differences). Therefore, $\hat{P}$ achieves a strictly higher value of the objective than $P$, so $P$ cannot be a solution.

Part (i'). Under Constraint C', if $P'$ and $P''$ are both solutions then so is the function $\frac{1}{2}P' + \frac{1}{2}P''$ given by $\left(\frac{1}{2}P' + \frac{1}{2}P''\right)(\theta) = \frac{1}{2}P'(\theta) + \frac{1}{2}P''(\theta)$ for all $\theta$. Noting that $\text{supp} \, P'(\theta') \subseteq \text{supp} \, \left(\frac{1}{2}P' + \frac{1}{2}P''\right)(\theta')$ and $\text{supp} \, P''(\theta'') \subseteq \text{supp} \, \left(\frac{1}{2}P' + \frac{1}{2}P''\right)(\theta'')$, the result follows from applying (i) to $\frac{1}{2}P' + \frac{1}{2}P''$.

Part (ii). If $W < \infty$, the supremum in the objective is attained by some $P$ because the objective is continuous (as $w$ is continuous) and the constraint set is closed (as $y_x$ is continuous for all $x \in X$).

To see that a monotone deterministic solution exists, let $P$ be any solution. Taking $\theta'' \downarrow \theta'$ and recalling that $P(\theta)$ is right-continuous, (i) implies that $P$ is already deterministic and monotone over every interval $\Theta' \subseteq \Theta$ on which $t$ is strictly increasing. It remains only to show that $P$ may be replace by a deterministic and monotone allocation on those intervals $\Theta'$ on which $t$ is constant. To see that this is possible, fix such
an interval $\Theta' = [\underline{\theta}', \bar{\theta}']$, and let
\[
a = \inf \{ a_0 : a_0 \in \text{supp } P(\theta), \theta \in \Theta' \}.
\]

Now define the deterministic and monotone allocation $a : \Theta' \to \mathbb{R}$ by
\[
a(\theta) = \inf \left\{ a_0 : \int_{\Theta'} I\{a \in [a_0]\} \, dPdF \geq F(\theta) - F(\bar{\theta}') \right\},
\]
where $I\{\cdot\}$ denotes the indicator function. It follows that for every interval of allocations $A = [\underline{a}, a_0]$
\[
\int_{\Theta'} I\{a(\theta) \in A\} \, dF = \int_{\Theta'} I\{a \in A\} \, dPdF,
\]
and therefore that the same holds for every measurable set of allocations $A \subseteq \mathbb{R}$. Since $t$ is constant on $\Theta'$, this implies that replacing $P$ with $a$ on $\Theta'$ does not affect the objective or the constraints of our program. Therefore, performing this replacement on all intervals on which $t$ is constant yields a deterministic and monotone solution.

A.2 Proof of Proposition 2

When $H$ is a step function, the government’s dual problem is
\[
\min_{c_0, c_1, y} \int \left( c_0(\theta) + \frac{1}{R} c_1(\theta) - y(\theta) \right) \, dF
\]
subject to (2), (6), (7) and the no-reform constraint
\[
\int I\{c_1(\theta) \geq RK - \kappa\} \, dF \geq a,
\]
where $I\{\cdot\}$ is the indicator function. As in the case where $H$ is differentiable, any solution must solve the subproblem
\[
\min_{c_0, c_1, K} \left( c_0(\theta) + \frac{1}{R} c_1(\theta) \right) \, dF
\]
subject to (6), (9) and (21). Substituting out for $c_0(\theta)$ using (9) and letting $\eta \geq 0$ and $\phi \geq 0$ be the multipliers on (6) and (21), respectively, the Lagrangian for this problem is
\[
\int \left( u^{-1} \left( U(\theta) - \beta u(c_1(\theta)) \right) + \left( \frac{1}{R} + \eta \right) c_1(\theta) - \phi I\{c_1(\theta) \geq RK - \kappa\} \right) \, dF.
\]
If $c_1(\theta) \neq RK - \kappa$, then differentiating under the integral with respect to $c_1(\theta)$ yields first-order condition
\[
\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = R\eta.
\]

Hence, in any solution either (23) holds or $c_1(\theta) = RK - \kappa$. Furthermore, in any monotone deterministic solution, the set of types $\theta$ with $c_1(\theta) = RK - \kappa$ forms an interval $[\theta_l, \theta_h]$, so it remains only to show that marginal capital taxes are non-decreasing on $[\theta_l, \theta_h]$ and (if $c_1$ is non-constant) satisfy $\tau_k(\theta) / (1 - \tau_k(\theta)) \leq \frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = R\eta$. 

36
\( R \eta \) on \([\theta_1, \theta_2]\). The former statement follows immediately from the fact that \( U(\theta) \) is non-decreasing on \([\theta_1, \theta_2]\) and (3). For the latter statement, note that the restriction of any solution \((c^*_0, c^*_1, y^*)\) to \(\Theta' = \{\theta : c^*_1(\theta) \geq RK - \kappa\}\) must solve the subproblem

\[
\min_{c_0 \in \mathbb{R}} \left( c_0(\theta) + \frac{1}{R} c_1(\theta) \right) dF
\]

subject to (9),

\[
\int_{\Theta'} c_1(\theta) dF \leq \int_{\Theta'} c_1^*(\theta) dF,
\]

and

\[
c_1(\theta) \geq RK - \kappa \text{ for all } \theta \in \Theta'.
\]

Letting \( \tilde{\eta} \) be the multiplier on (24) and letting \( \psi(\theta)(f(\theta) \geq 0 \) be the multiplier on (25), the Lagrangian for this subproblem is

\[
\int \left( u^{-1}(U(\theta) - \beta u(c_1(\theta))) + \left( \frac{1}{R} + \tilde{\eta} - \psi(\theta) \right) c_1(\theta) \right) dF.
\]

The first order condition for \( c_1(\theta) \) and the fact that \( \psi(\theta) \geq 0 \) immediately imply that \( \tau_k(\theta)/(1 - \tau_k(\theta)) \leq R\tilde{\eta} \) for all \( \theta \in \Theta' \), and hence for all \( \theta \) such that \( c_1(\theta) = RK - \kappa \). Finally, since \( c_1 = c^*_1 \) at a solution to the subproblem and \( \psi(\theta) = 0 \) for all \( \theta \) such that \( c_1(\theta) > RK - \kappa \), we have

\[
\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = R\tilde{\eta} = R\eta \text{ for all } \theta \text{ such that } c_1(\theta) > RK - \kappa.
\]

As such a type \( \theta \) exists whenever \( c_1 \) is non-constant (by (6) and \( \kappa \geq 0 \)), we may conclude that \( \tilde{\eta} = \eta \) and hence that \( \tau_k(\theta)/(1 - \tau_k(\theta)) \leq R\eta \) for all \( \theta \) such that \( c_1(\theta) = RK - \kappa \).

### A.3 Proof of Lemma 4

(i) follows from the fact that the (15) is continuous, the constraint set defined by (16) and (17) is closed (by continuity of \( \tilde{H} \) and \( F \)) and \( V_D(c_1) \) is bounded above by the value of solving the relaxed program

\[
\max_x \int u(c_1(\theta) + x(\theta)) dF \text{ s.t. } \int x(\theta) dF \leq -\kappa,
\]

which is clearly finite. (ii) follows from Lemma 2 (i') because \( u(c_1(\theta) + x) \) has strictly decreasing differences in \( c_1(\theta) \) and \( x \), due to the concavity of \( u \). For (iii), let \( \tilde{x}(\tilde{c}_1) = \sup_{\theta \in \Theta_1} X(\theta) \) and \( \underline{x}(\tilde{c}_1) = \inf_{\theta \in \Theta_1} X(\theta) \). It follows from (ii) that \( \tilde{x} \) and \( \underline{x} \) are monotone, and thus continuous for almost all \( \tilde{c}_1 \), and also that \( \tilde{x}(\tilde{c}_1) = \underline{x}(\tilde{c}_1) \) whenever \( \tilde{x} \) and \( \underline{x} \) are continuous at \( \tilde{c}_1 \). Hence, \( \tilde{x}(\tilde{c}_1) = \underline{x}(\tilde{c}_1) \) for almost all \( \tilde{c}_1 \), so \( \bigcup_{\theta \in \Theta_1} X(\theta) \) is single-valued for almost all \( \tilde{c}_1 \). Finally, to see (iv), note that a necessary condition for \( x(\theta) \in X(\theta) \) is that

\[
-u'(c_1(\theta) + x(\theta)) + \lambda = \mu \tilde{H}'(-x(\theta)),
\]

where \( \lambda \geq 0 \) and \( \mu \geq 0 \) are the multipliers on (16) and (17). Since the left-hand side of (26) is strictly increasing in \( x(\theta) \) whereas the right-hand side is weakly decreasing in \( x(\theta) \) for \( x(\theta) > 0 \) when \( \tilde{H}' \) is single-peaked at 0, there can be at most one solution with \( x(\theta) > 0 \).
A.4 Proof of Lemma 5

Note that the constraint $\int u(c_1(\theta))dF \geq V_D(c_1)$ can be written as $\int u(c_1(\theta))dF \geq \int u(c_1(\theta) + x(\theta))dF$ for all $x \in X$, where $X$ is the set of all $x$-schedules that satisfy (16) and (17). The constraint set therefore takes the same form as (C') in Lemma 2, so the result follows from Lemma 2 (ii) by exactly the same same argument as in Lemma 3.

B Appendix: Dependence on Utility Differences

This appendix shows that results very similar to Propositions 4 and 5 hold when $\tilde{H}$ depends on utility differences in the model of Section 4. The argument that no reform occurs in equilibrium is as in the text. The credibility constraint is now that there does not exist a scheme $\hat{c}_1$ such that

$$\int u(\hat{c}_1(\theta))dF > \int u(c_1(\theta))dF,$$

$$\int \hat{c}_1(\theta)dF \leq \int c_1(\theta)dF - \kappa, \quad (27)$$

$$\int \tilde{H}(u(\hat{c}_1(\theta)) - u(c_1(\theta)))dF \leq \alpha. \quad (28)$$

This constraint is equivalent to the value of the following deviation program being less than $\int u(c_1(\theta))dF$.

$$\max_{\hat{c}_1} \int u(\hat{c}_1(\theta))dF$$

subject to (27) and (28). Letting $x(\theta) = u(\hat{c}_1(\theta)) - u(c_1(\theta))$, $\Phi = u^{-1}$, $u_t(\theta) = u(c_1(\theta))$ and allowing for randomization, this is in turn equivalent to the value of the following dual program, which we denote by $R_D(u_1)$, being greater than $\int \Phi(u_1(\theta))dF - \kappa$.

$$\min_P \int \int \Phi(u_1 + x)dPdF$$

subject to

$$\int \int xdPdF \geq 0,$$

$$\int \int \tilde{H}(-x)dPdF \leq \alpha.$$ 

This deviation program—which we denote by (DP)—is framed in utility space, while the deviation program (DP) in the text is framed in consumption space. The key feature which makes (DP) tractable is that the status quo utility schedule $u_1$ does not enter the constraints, just as the status quo consumption schedule $c_1$ does not enter the constraints in (DP).

Denote the set of possible solutions to (DP) at each $\theta$ by

$$X(\theta) \equiv \{ x : x \in \text{supp} \ P(\theta) \text{ for some solution } P \text{ to (DP')}. \}$$

The next lemma again collects properties of these solutions:

Lemma 7. (i) A solution exists.
(ii) If \( u_1(\theta) < u_1(\theta') \) then \( \inf X(\theta) \geq \sup X(\theta') \).

(iii) For any \( u_1 \)-schedule and any \( \pi_1 \), let \( \Theta_{\pi_1} \equiv \{ \theta : u_1(\theta) = \pi_1 \} \). Then for almost all \( \pi_1 \), \( \bigcup_{\theta \in \Theta_{\pi_1}} X(\theta) \) is singleton.

(iv) For every \( \pi_1 \in \mathbb{R} \), there exists at most one \( s > 0 \) such that \( s \in \bigcup_{\theta \in \Theta_{\pi_1}} X(\theta) \).

Proof. Noting that the objective has strictly increasing differences in \( u_1 \) and \( x \) by convexity of \( \Phi \), the proof is analogous to the proof of Lemma 4.

We can now write the planner’s problem as follows.

\[
\min_{u_1,y} \int \left( \Phi(U(\theta) - \beta u_1(\theta)) + \frac{1}{R} \Phi(u_1(\theta)) - y(\theta) \right) dF \text{ s.t. } R_D(u_1) \geq \int \Phi(u_1(\theta)) dF - \kappa,
\]

subject to

\[
\int (U(\theta) - h(y(\theta),\theta)) dF \geq V, \tag{29}
\]

\[
U(\theta) - h(y(\theta),\theta) \geq U(\theta') - h(y(\theta'),\theta). \tag{30}
\]

Since (29) and (30) only depend on \( U \) and \( y, u_1 \) must solve

\[
\min_{u_1} \int (\Phi(U(\theta) - \beta u_1(\theta))) dF \text{ s.t. } R_D(u_1) \geq \int \Phi(u_1(\theta)) dF - \kappa. \tag{31}
\]

Next, note that Lemma 5 goes through because \( \Phi(U(\theta) - \beta u_1(\theta)) \) has strictly decreasing differences in \( U(\theta) \) and \( u_1(\theta) \), so there exists a deterministic solution in which \( u_1 \) is non-decreasing. We can therefore reproduce Lemma 6, denoting by \( \eta \geq 0 \) the multiplier on the constraint in (31).

Lemma 8. (i) In any deterministic solution \( u_1 \), for almost all \( \theta \) where \( X(\theta) \) is single-valued, we have

\[
\frac{\tau_k(\theta)}{1 - \tau_k(\theta)} = \eta \left( 1 - \Phi'(u_1(\theta) + x(\theta)) / \Phi'(u_1(\theta)) \right) \text{ where } X(\theta) = \{ x(\theta) \}. \tag{32}
\]

(ii) For almost all non-pooled \( \theta \) (i.e. \( \theta \) such that \( u_1(\theta) \neq u_1(\theta') \) for all \( \theta' \neq \theta \), \( X(\theta) \) is single-valued, so (32) holds.

The proof is analogous to the proof of Lemma 6. This immediately leads to the following result reproducing Proposition 4.

Proposition 6. In any monotone deterministic solution, there exists a threshold type \( \theta^* \) such that capital is subsidized for agents with \( \theta < \theta^* \) and capital is taxed for all non-pooled agents with \( \theta > \theta^* \).

Proof. Analogous to the proof of Proposition 4, using Lemmas 7 and 8 in place of Lemmas 4 and 6. The intuition is that, as in the case where \( \bar{H} \) depends on consumption differences, \( \text{sign} \left( \tau_k(\theta) \right) = \text{sign}(-x(\theta)) \) whenever \( \theta \) satisfies (32), and \( x(\theta) \) is non-increasing whenever \( X(\theta) \) is a singleton.

We can also reproduce Proposition 5 if the condition that \( u'' \geq 0 \) is strengthened to non-increasing \( -u''(c) / u'(c)^2 \).

Proposition 7. In a monotone deterministic solution, the following hold.

(i) if \( -u''(c) / u'(c)^2 \) is non-increasing, then \( \tau_k(\theta) \) is non-decreasing on \( \{ \theta : \theta < \theta^* \} \).

(ii) \( \tau_k(\theta) \) is non-decreasing on \( \{ \theta : \theta \) is non-pooled and \( \theta > \theta^* \} \).
Proof. For simplicity, we prove the proposition under the additional hypothesis that the functions $u_1$ and $x$ are differentiable. This hypothesis can be dispensed with as in the proof of Proposition 5.

Note that $\tau_k(\theta)$ is non-decreasing if (32) is satisfied and $\Phi'(u_1(\theta) + x(\theta)) / \Phi'(u_1(\theta))$ is non-increasing.

For the first part, this holds whenever (omitting $\theta$-arguments)

$$\Phi'(u_1) \Phi''(u_1 + x)(u_1' + x') - \Phi'(u_1 + x) \Phi''(u_1) u_1' \leq 0,$$

(33)

or

$$\frac{\Phi''(u_1 + x)}{\Phi'(u_1 + x)} \frac{(u_1' + x')}{u_1'} \leq \frac{\Phi''(u_1)}{\Phi'(u_1)}.$$

As $x' \leq 0$ by Lemma 7, a sufficient condition for this is

$$\frac{\Phi''(u_1 + x)}{\Phi'(u_1 + x)} \leq \frac{\Phi''(u_1)}{\Phi'(u_1)},$$

or, since $x \geq 0$ for $\theta < \theta^*$, $\Phi''(u_1) / \Phi'(u_1)$ non-increasing. This is easily seen to be equivalent to $-u''(c) / u'(c)^2$ being non-increasing.

For the second part, we first argue that $\hat{u}_1(\theta) = u_1(\theta) + x(\theta)$ is non-increasing for $\theta > \theta^*$. To see this, write the deviation program (DP') as

$$\min_{\hat{u}_1} \int \hat{H}(u_1(\theta) - \hat{u}_1(\theta)) \ dF$$

subject to

$$\int \Phi(\hat{u}_1(\theta)) \ dF \leq RK - \kappa,$$

$$\int \hat{u}_1(\theta) \ dF \geq \int u_1(\theta) \ dF,$$

and apply the same argument (using Lemma 2) as in the proof of Proposition 5, which implies that every solution $\hat{u}_1(\theta)$ is non-increasing in $u_1(\theta)$, and hence in $\theta$, for $\theta > \theta^*$. Finally, rewrite (33) as

$$\Phi'(u_1) \Phi''(\hat{u}_1) \hat{u}_1' - \Phi'(\hat{u}_1) \Phi''(u_1) u_1' \leq 0.$$

Since $\Phi' \geq 0$, $\Phi'' \geq 0$, $\hat{u}_1' \leq 0$ and $u_1' \geq 0$, this is satisfied. 

C Appendix: General Welfare Weights

In this appendix, we discuss situations where the government uses general Pareto weights $G$ to evaluate welfare, which could be different from the population distribution $F$. Moreover, we allow the government’s Pareto weights to depend not only on $\theta$, but also on a second dimension of heterogeneity $\rho$, which is assumed to be payoff-irrelevant and observable to the government. As discussed in the text, this second dimension of heterogeneity allows for a government that is “non-benevolent.” We let $F(\theta, \rho)$ be the joint cdf over $\theta$ and $\rho$ and let $G(\theta, \rho)$ be the corresponding cdf of the government’s welfare weights. We assume that $F$ and $G$ are absolutely continuous with respect to each other, so that the government puts positive weight
on the welfare of all groups in society (although these weights may be arbitrarily close to 0).\textsuperscript{26} We will show that all of our main results continue to hold \textit{within each group} $\rho$, with little or no modification, under this more general specification of the government’s objective, so long as $g(\theta, \rho) / f(\theta, \rho)$ is non-increasing in $\theta$ for each $\rho$: that is, we require that, within each group, the government is at least as redistributive as a utilitarian would be. This shows that our results are not overly sensitive to either the assumption that the government has utilitarian preferences for redistribution, or that the government cares equally about all groups in society.

\textbf{C.1 Non-Strategic Reforms}

Let us begin with the case where the government cannot commit to the details of a reform. This corresponds to the “equalizing reforms” model of Section 3, except that the government’s most-preferred reform is no longer full equalization of consumption due to the Pareto weights $G(\theta, \rho)$. Instead, the dual problem for $c_0, c_1$, and $K$ becomes

$$\min_{c_0,c_1,K} \int \left( c_0(\theta, \rho) + \frac{1}{K} c_1(\theta, \rho) \right) dF$$
subject to

$$u(c_0(\theta, \rho)) + \beta u(c_1(\theta, \rho)) = U(\theta, \rho),$$

$$\int c_1(\theta, \rho) dF \geq RK,$$

and

$$\int H(c_1(\theta, \rho), \hat{c}_1(\theta, \rho)) dF \geq \alpha,$$

where reform consumption $\hat{c}_1$ will solve

$$\max_{\hat{c}_1} \int u(\hat{c}_1(\theta, \rho)) dG \text{ s.t. } \int \hat{c}_1(\theta, \rho) dF \leq RK - \kappa.$$

It should be pointed out that in this model abstracting from ratcheting is no longer without loss of generality. This is because the government in period 1 allocates consumption using the information about $\theta$ revealed in period 0 (as a consequence, the deviation program does not involve incentive constraints). In principle, the government may therefore do better by offering an allocation in period 0 that reveals less information about types. It would thereby tie its hands and prevent itself from offering perfectly $\theta$-targeted consumption at the reform stage, which may relax the no-reform constraint. Nonetheless, we feel that the most direct way to study how our main results change with a non-utilitarian government is to continue to assume that the government employs a fully revealing direct mechanism, so we maintain this assumption throughout.

This leads to the following result about the implicit marginal capital tax:

\textbf{Lemma 9.} \textit{With Pareto weights $G(\theta, \rho)$, the intertemporal wedge at any solution to the government’s problem satis-}

\textsuperscript{26}The absolute continuity also rules out the possibility that the government may put non-zero weight on the welfare of a measure-zero set of agents, such as for example the leader of the government herself. This differentiates the model from one with a self-interested politician, such as Acemoglu, Golosov, and Tsyvinski (2010).
Implicitly differentiating this equation and (39) yields
\[
\frac{\tau_k (\theta, \rho)}{1 - \tau_k (\theta, \rho)} = -R \eta [\hat{H}_2 (c_1, \hat{c}_1) - H_1 (c_1 (\theta, \rho), \hat{c}_1 (\theta, \rho))], \quad (38)
\]
where \( \eta \geq 0 \) is the multiplier on (37) and
\[
\mathcal{H}_2(c_1, \hat{c}_1) \equiv \int H_2(c_1(\theta, \rho), \hat{c}_1(\theta, \rho))m(\theta, \rho)dF
\]
with
\[
m(\theta, \rho) = \frac{(u''(\hat{c}_1(\theta, \rho))g(\theta, \rho)/f(\theta, \rho))^{-1}}{\int (u''(\hat{c}_1(\theta', \rho''), \rho'))/f(\theta'))^{-1} dF}.
\]

**Proof.** Note that the reform consumption \( \hat{c}_1 \) depends on the choice variables \( c_0, c_1, K \) only through \( K \), so we may write \( \hat{c}_1(\theta, \rho, K) \) in the following. \( \hat{c}_1(\theta, \rho, K) \) must satisfy the first order condition
\[
u' (\hat{c}_1(\theta, \rho, K)) = \lambda(K)f(\theta, \rho)/g(\theta, \rho),
\]
where \( \lambda(K) \) is the multiplier on the post-reform resource constraint as a function of \( K \). We can use (39) to solve for \( \hat{c}_1(\theta, \rho, K) = u^{-1}(\lambda(K)f(\theta, \rho)/g(\theta, \rho)) \), so \( \lambda(K) \) is implicitly determined by
\[
\int u^{-1}(\lambda(K)f(\theta, \rho)/g(\theta, \rho))dF = RK - \kappa.
\]
Implicitly differentiating this equation and (39) yields
\[
\frac{\partial \hat{c}_1(\theta, \rho, K)}{\partial K} = \frac{\partial \hat{c}_1(\theta, \rho)}{\partial \lambda} \frac{\partial \lambda}{\partial K} = R \left( \frac{u''(\hat{c}_1(\theta, \rho))g(\theta, \rho)/f(\theta, \rho))^{-1}}{\int (u''(\hat{c}_1(\theta', \rho''), \rho'))/f(\theta'))^{-1} dF} \right) = Rm(\theta, \rho).
\]
Using this in the first order condition for \( K \) corresponding to the planning problem (34) to (37) and combining it with the first order conditions for \( c_0(\theta, \rho) \) and \( c_1(\theta, \rho) \) delivers the result. \( \blacksquare \)

Observe that, with utilitarian welfare weights \( G = F \), (39) implies full equalization of post-reform consumption \( \hat{c}_1 \), so \( m(\theta, \rho) = 1 \) for all \( \rho, \theta \), and (38) collapses back to the standard formula from Section 3. However, even if \( G \neq F \), we will see that the results from Section 3 will continue to hold within each group \( \rho \). For instance, if \( H \) depends on consumption differences with \( H(c_1, \hat{c}_1) = \hat{H}(c_1 - \hat{c}_1) \) and \( \hat{H}' \) is single-peaked, marginal capital taxes are U-shaped in \( \theta \) for any given \( \rho \) whenever \( c_1(\theta, \rho) - \hat{c}_1(\theta, \rho) \) is monotone in \( \theta \) (a similar result holds when \( H \) depends on utility differences and \( \hat{H}'u' \) is single-peaked). This will be established in the following proposition:

**Proposition 8.** Suppose \( H(c_1, \hat{c}_1) = \hat{H}(c_1 - \hat{c}_1) \) depends on consumption differences and \( \hat{H}' \) is single-peaked. Then for any given \( \rho \), marginal capital taxes are U-shaped in \( \theta \) if \( g(\theta, \rho)/f(\theta, \rho) \) is non-increasing in \( \theta \) given \( \rho \).

**Proof.** Fix a deterministic solution \( c_0, c_1, K \) to (34)-(37), with reform consumption schedule \( \hat{c}_1 \). Consider any given \( \rho \) and let \( c_1^\rho \equiv c(., \rho), F^\rho \equiv F(., \rho) \) and analogously for \( G^\rho \), and let
\[
\mathcal{R}^\rho \equiv \int c_1(\theta, \rho)dF^\rho/R, \quad \mathcal{F}^\rho \equiv \int H(c_1(\theta, \rho), \hat{c}_1(\theta, \rho, K))dF^\rho
\]
(i.e., we fix $\rho$ and only integrate in the $\theta$-dimension). Using (35) to substitute $c_0(\theta, \rho) = u^{-1}(U(\theta, \rho) - \beta u(c_1(\theta, \rho)))$, we see that $c^p_1$ has to solve the subproblem

$$\min_{c_1} \int \left( u^{-1}(U(\theta, \rho) - \beta u(c_1(\theta, \rho))) + \frac{1}{R} c_1(\theta, \rho) \right) dF^p$$

s.t.

$$\int c_1(\theta, \rho) dF^p \leq R \bar{K}^p,$$

$$\int H(c_1(\theta, \rho), \hat{c}_1(\theta, \rho, K)) dF^p \geq \bar{\alpha}^p.$$  

Since $c^p_1$ does not depend on $c^p_1$ (holding $K$ constant), we can apply Lemma 2, which implies that $c_1(\theta, \rho)$ is non-decreasing in $U(\theta, \rho)$ and hence $\theta$. A sufficient condition for $c_1(\theta, \rho) - \hat{c}_1(\theta, \rho, K)$ to be non-decreasing in $\theta$ is therefore that $\hat{c}_1(\theta, \rho, K)$ is non-increasing in $\theta$. By (39), $\hat{c}_1(\theta, \rho, K)$ is non-increasing in $\theta$ if and only if $g(\theta, \rho) / f(\theta, \rho)$ is non-increasing in $\theta$, for any given $\rho$ and $K$. 

Proposition 8 shows that, unsurprisingly, we can think of the problem for each group $\rho$ separately, and thus, for any given $\rho$, the marginal tax rate is U-shaped in $\theta$ whenever the relative social welfare weights $g(\theta, \rho) / f(\theta, \rho)$ are non-increasing in $\theta$. This is because these relative welfare weights determine the shape of post-reform consumption. With utilitarian weights $G = F$, they are constant and equal to one, leading to fully equalized $\hat{c}_1$. Whenever $g(\theta, \rho) / f(\theta, \rho)$ is decreasing, $\hat{c}_1(\theta, \rho)$ will be decreasing in $\theta$ for any given $\rho$. This is because the government values low $\theta$-types relatively more, and therefore allocates them higher consumption. Since $c_1(\theta, \rho)$ is still weakly increasing in $\theta$ by Lemma 2, this makes sure that $c_1(\theta, \rho) - \hat{c}_1(\theta, \rho)$ is also weakly increasing. The case where $\bar{H}$ depends on utility differences may be treated analogously.

Hence, our main result about the shape of the marginal capital tax from Section 3 continue to hold within each group $\rho$ whenever the government is utilitarian or more redistributive than utilitarian within each group, in the sense of putting higher Pareto weights on low-skill types. What does change is the average level of the distortion because of the additional weighting factor $m(\theta, \rho)$ in $\bar{H}_2(c_1, \hat{c}_1)$ in equation (38). Intuitively, $m(\theta, \rho)$ measures the effect of a marginal increase in period 1 aggregate resources $RK$ on post-reform consumption $\hat{c}_1(\theta, \rho)$ of an individual of type $(\theta, \rho)$. The overall effect of an additional unit of $RK$ on political support for the status quo is then given by the weighted average $\int H_2(c_1(\theta, \rho), \hat{c}_1(\theta, \rho)) m(\theta, \rho) dF$, which includes both the effect of $RK$ on post-reform consumption (through $m(\theta, \rho)$) and the effect of a change in post-reform consumption on the probability of supporting the status quo (through $H_2$). For instance, if $g(\theta, \rho) / f(\theta, \rho)$ is decreasing in $\theta$ and $u'' \geq 0$, then $m(\theta, \rho)$ is increasing in $\theta$, so more weight is put on the political responsiveness of high-$\theta$ individuals (those whose post-reform consumption moves more in response to aggregate savings).

### C.2 Strategic Reforms

We now turn to the case where the government can commit to the details of a reform, corresponding to Section 4 of the text. We will see that most of our results in that section can also be extended to allow for general Pareto weights, although the analysis is somewhat more complicated. In this appendix, we restrict attention to the case where $H$ depends on consumption differences; the case where $H$ depends on utility differences can again be treated similarly.
With general Pareto weights, the planning subproblem (18) becomes
\[
\min_{c_1} \int \left( u^{-1}(U(\theta, \rho) - \beta u(c_1(\theta, \rho))) + \frac{1}{R} c_1(\theta, \rho) \right) dF \quad \text{s.t.} \quad \int u(c_1(\theta, \rho))dG \geq V_D(c_1)
\]
where
\[
V_D(c_1) \equiv \max_x \int u(c_1(\theta, \rho) + x(\theta, \rho))dG
\]
\[
\text{s.t. } \int x(\theta, \rho)dF \leq -\kappa
\]
\[
\int \tilde{H}(-x(\theta, \rho))dF \leq \alpha.
\]
The following result shows that Lemma 4 extends to this more general framework for any given \( \rho \) if \( g(\theta, \rho) / f(\theta, \rho) \) is non-increasing in \( \theta \):
\[\text{Lemma 10. If } g(\theta, \rho) / f(\theta, \rho) \text{ is non-increasing in } \theta \text{ for a given } \rho, \text{ then } x(\theta, \rho) \equiv \hat{c}_1(\theta, \rho) - c_1(\theta, \rho) \text{ is non-increasing in } c_1(\theta, \rho), \text{ for any selection } x(\theta, \rho) \in X(\theta, \rho), \text{ where } X(\theta, \rho) \text{ is the set of solutions to the deviation program.}\]
\[\text{Proof. We first show that, as in the non-strategic reforms case, the subproblem for each } \rho \text{ can be considered separately. To see this, fix a deterministic solution } c_1 \text{ with implied } x \text{ and let}
\]
\[
\bar{\kappa} \equiv -\int x(\theta, \rho)dF^\rho
\]
\[
\bar{\alpha} \equiv \int \tilde{H}(-x(\theta, \rho))dF^\rho
\]
Then \( x^\rho \equiv x(\cdot, \rho) \) must solve
\[\max_{x^\rho} \mathcal{V}_D^\rho \left( c_1^\rho \right) \equiv \int u(c_1(\theta, \rho) + x(\theta, \rho))dG^\rho \quad \text{s.t. } \int x(\theta, \rho)dF^\rho \leq -\bar{\kappa}
\]
\[
\int \tilde{H}(-x(\theta, \rho))dF^\rho \leq \bar{\alpha}.
\]
Hence, the deviation program (40) has the same structure, for each given \( \rho \), as (DP) in Section 4, except for the fact that the integrals in the objective and the constraints involve different weights \( G \) and \( F \). To extend the comparative statics result from Lemma 4, we therefore require a generalized version of our technical Lemma 2, which is as follows:
\[\text{Lemma 11. Consider the same setting as in Lemma 2 but with the modified program}
\]
\[
W \equiv \sup_{\rho} \int \int w(a, t(\theta))dPdG
\]
\[\text{subject to}
\]
\[
\int \int y_x(a)dPdF \leq 0 \text{ for all } x \in X.
\]
Assume that \( w \) is continuous and has strictly increasing differences in \( a \) and \( t(\theta) \), \( y_x \) is continuous for all \( x \in X \), and \( t \) is right-continuous. Suppose that either
(i) \( w(a,t) \) is non-decreasing in \( a \) and \( g(\theta) / f(\theta) \) is non-decreasing in \( \theta \), or  
(ii) \( w(a,t) \) is non-increasing in \( a \) and \( g(\theta) / f(\theta) \) is non-increasing in \( \theta \).  
Then for any solutions \( P' \) and \( P'' \), if \( t(\theta') < t(\theta'') \) then \( a(\theta') \leq a(\theta'') \) for all \( a(\theta') \in \text{supp} \ P'(\theta') \) and \( a(\theta'') \in \text{supp} \ P''(\theta'') \). Moreover, if \( W < \infty \), there exists a deterministic solution in which \( a \) is non-decreasing in \( t \).

**Proof.** The proof follows the same variational argument as the proof of Lemma 2, where the perturbed randomized schedule \( \hat{P} \) is defined as before. As before, \( \hat{P} \) satisfies the constraints. The difference appears when comparing the value of the objective under \( \hat{P} \) and \( P \), which becomes

\[
\int \int w(a,t(\theta))d\hat{P}dG - \int \int w(a,t(\theta))dPdG = \varepsilon \int_{\Theta'} \int w(a,t(\theta))\tilde{\gamma}(\theta)dP(\phi(\theta)) - \int w(a,t(\theta))\tilde{\gamma}(\phi(\theta))d\hat{P}(\theta) \frac{g(\theta)}{f(\theta)}d\theta \\
+ \varepsilon \int_{\Theta'} \int w(a,t(\theta))\tilde{\gamma}(\theta)d\hat{P}(\phi^{-1}(\theta)) - \int w(a,t(\theta))\tilde{\gamma}(\phi^{-1}(\theta))dP(\theta) \frac{g(\theta)}{f(\theta)}d\theta \\
= \varepsilon \int_{\Theta'} \int \left[ \int w(a,t(\theta))\frac{g(\theta)}{f(\theta)} - \int w(a,t(\theta))\frac{g(\phi(\theta))}{f(\phi(\theta))} \right] \tilde{\gamma}(\theta)dP(\phi(\theta))d\theta.
\]

Note that

\[
w(a,t(\theta)) \frac{g(\theta)}{f(\theta)} - w(a,t(\phi(\theta))) \frac{g(\phi(\theta))}{f(\phi(\theta))} = \left[ w(a,t(\theta)) - w(a,t(\phi(\theta))) \right] \frac{g(\theta)}{f(\theta)} + w(a,t(\phi(\theta))) \frac{g(\phi(\theta))}{f(\phi(\theta))} - w(a,t(\phi(\theta))) \frac{g(\phi(\theta))}{f(\phi(\theta))}.
\]

For each \( \theta \in \Theta' \), \( t(\theta) < t(\phi(\theta)) \), and because \( w \) has increasing differences, the first term in the second line is non-increasing in \( a \) as before. Moreover, the second term is also non-increasing in \( a \) if either condition (i) or (ii) in the lemma is satisfied. This ensures that the entire expression in the first line is decreasing in \( a \) for all \( \theta \in \Theta' \), so the rest of the proof of Lemma 2 goes through.

Applying Lemma 11(ii) to the modified deviation program (40) with \( a = -x^p(\theta), t(\theta) = c_1^p(\theta) \) and \( w(a,t) = u(c_1 - x) \) immediately implies that, for any given \( \rho \), \( x^p(\theta) \) is non-decreasing in \( c^p(\theta) \) when \( g(\theta, \rho) / f(\theta, \rho) \) is non-decreasing in \( \theta \), which is the desired result.

We can also reproduce Lemma 6 as follows:

**Lemma 12.** In any deterministic solution, for almost all non-pooled \( \theta \),

\[
\frac{\tau_k(\theta, \rho)}{1 - \tau_k(\theta, \rho)} = R \eta \left[ u'(c_1(\theta, \rho) + x(\theta, \rho)) - u'(c_1(\theta, \rho)) \right] g(\theta, \rho) / f(\theta, \rho).
\]

**Proof.** Observe first that, for each given \( \rho \), \( c_1^p \) must solve

\[
\min_{c_1} \int \left( u^{-1}(U(\theta, \rho) - \beta u(c_1(\theta, \rho))) + \frac{1}{R} c_1(\theta, \rho) \right) dF^p \text{ s.t. } \int u(c_1(\theta, \rho))dG^p \geq \nabla_D^p(c_1^p),
\]

where \( \nabla_D^p(c_1^p) \) is defined in (40). This is the same as (18) except that the constraint involves the weights \( G \) rather than \( F \). Accounting for this when following the same steps as in the proof of Lemma 6 yields the result.
Since Lemma 10 has shown that $x(\theta, \rho)$ is decreasing in $c_1(\theta, \rho)$ for any given $\rho$, Lemma 12 immediately yields the following analogue of Proposition 4:

**Proposition 9.** Suppose $g(\theta, \rho)/f(\theta, \rho)$ is non-increasing in $\theta$ for each $\rho$. In a deterministic solution, for each $\rho$, there exists a threshold level of period 1 consumption $c_1^{\rho^*}$ such for almost all non-pooled agents, capital is subsidized for agents with $c_1(\theta, \rho) < c_1^{\rho^*}$ and taxed for those with $c_1(\theta, \rho) > c_1^{\rho^*}$.

**Proof.** Analogous to the proofs of Propositions 4 and 6, now using Lemmas 10 and 12. □

By inspection of the formula in (42), Proposition 5 also goes through if $c_1(\theta, \rho)$ is increasing and $g(\theta, \rho)/f(\theta, \rho)$ is decreasing in $\theta$ given $\rho$. In words, the marginal capital tax is increasing in $\theta$ in this case for non-pooled agents, as it was in Section 4.

The one major way in which the results in this appendix are weaker than those in Section 4 is that we cannot conclude that $c_1(\theta, \rho)$ is non-decreasing in $\theta$, even when $g(\theta, \rho)/f(\theta, \rho)$ is decreasing in $\theta$. The reason is that our generalized monotonicity lemma, Lemma 11, is still not general enough to deliver comparative statics with respect to $c_1(\theta, \rho)$. This is because Lemma 11 requires that $w(a, t)$ is monotone in $a$. Yet, the objective in (43), while supermodular, is not monotone in $c_1$. The monotonicity requirement on the objective can be dropped only when the relative weights $g(\theta, \rho)/f(\theta, \rho)$ are constant in $\theta$. Thus, the full set of comparative statics from Section 4 can be guaranteed only when the government is utilitarian within each group $\rho$.\(^{27}\)

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\(^{27}\)Of course, the conditions on $g(\theta, \rho)/f(\theta, \rho)$ in Lemma 11 are only sufficient, so it is certainly possible that our full set of results goes through even when the government is not utilitarian.