Occupational Mobility and Wage Dynamics Within and Between Firms

Francis Kramarz*   Fabien Postel-Vinay†   Jean-Marc Robin‡

January 2014 — INCOMPLETE

Abstract

Recent research has emphasized the key importance of the equilibrium allocation of heterogeneous workers into heterogeneous jobs or occupations as a determinant of economic efficiency. Most of the literature on this subject envisions worker (re-)allocation as occurring between employers. Yet, the data suggest that a very large amount of reallocation occurs within firms, in the form of internal promotions or de-motions. We construct a structural job search model with internal and external labor markets. Internal labor markets mediate occupational mobility and wage dynamics within firms, whereas the external labor market organizes any mobility involving an employer change, and related wage dynamics. The aim of this construction is to understand and quantify the role of within-firm reallocation in the assignment process of workers into jobs. The model is estimated on a large-scale matched employer-employee data set covering the entire French business sector.

* Crest-Insee. Address: CREST-INSEE, Timbre J301, 15 boulevard Gabriel Péri, 94245 Malakoff cedex, France. E-mail: francis.kramarz@ensae.fr.
† University College London and Sciences Po. Address: Department of Economics, University College London, Drayton House, 30 Gordon Street, London WC1H 0AX, UK. E-mail: f.postel-vinay@ucl.ac.uk. Postel-Vinay is also affiliated with CEPR (London), IZA (Bonn), and CIfM (London).
‡ Sciences Po and UCL. Address: Department of Economics, Sciences Po, 28 rue des Saints Pères, 75007 Paris, France. E-mail: jeanmarc.robin@sciences-po.fr.
1 Introduction

We construct a model with internal and external labor markets. Internal labor markets mediate occupational mobility and wage dynamics within firms, whereas the external labor market refers to occupational mobility with employer change, and related wage dynamics.\footnote{At this early stage, we use to ‘occupation’ to refer to a set of job characteristics, which may include industry (e.g., manufacturing, services, etc.) and/or the degree of managerial responsibility that goes with the job (e.g. unskilled worker, supervisor, manager, etc.).} The aim of this construction is to understand and quantify the forces at work in a labor market where, at one extreme, workers spend their entire careers with one single employer (as was the norm in France in the three decades following WWII), gradually moving up some hierarchical ladder within a single firm, and at the other extreme, workers change employers regularly to prevent their career from stalling. Intuitively, the first type of equilibrium is stable if there is little complementarity between worker skill and firm technology. Biased technical progress, however, increases complementarity at the same time as TFP, and is responsible for the observed shift in workers’ careers from a single “job-for-life” to a sequence of different employments.

Internal labor markets (ILMs) have been a focus of both institutional labor economics (see Doeringer and Piore, 1972; Osterman, 2011 for a recent review) and new personnel economics (Lazear, 2000). The approach and focus of those two strands of literature are quite different: the former emphasizes rules, regulations, and group practices, whereas the latter views the functioning of ILMs as the outcome of individual optimizing behavior.

Specifically, personnel economics “draws on four theoretical building blocks (Gibbons and Waldman, 1999): human capital theory, optimal job assignment models, incentive contracting, and tournaments. [...] Human capital theory addresses the acquisition of skill and, in the context of a firm, the determination and consequences of learning trajectories. Assignment models try to determine to what extent organizations can best match skills and job requirements. Incentive contracting concerns solutions to the well-known agent-principal problem whereas tournament models focus on the use of promotions to provide incentives and to
sort employees by quality.” (Osterman, 2011). By contrast, institutional labor economics holds the view that “organizations are characterized by groups with competing objectives and perspectives regarding the legitimate purposes of the enterprise. [...] The ILM outcomes we observe are the result of an internal political process that is made necessary by the need to resolve these conflicts.” (Osterman, 2011).2

In their research based on case studies of individual firms’ personnel records, Baker and Holmstrom (1995) find “a very distinct and stable hierarchy with very clearly identified career ladders”. They show that “employees advance along a few well-defined paths, usually one level at a time. There are few lateral moves and essentially no demotions”. These authors further find that the link between wages and job positions, a tenet of ILMs, is weak. There are systematic winners and losers, and they conclude that promotions and wage growth are related through a common variable: “the natural candidate for this common driver is ability”. Finally, they find that market forces do shape wages at entry but play a much weaker role inside the firm.

Our approach remains agnostic on the foundations of ILMs (rules vs optimization). We simply acknowledge their existence and examine their impact on worker and firm search and wage setting behavior. In addition, by contrast to most of the empirical personnel economics literature, we base our inference on a very large data set, covering the universe of all French firms and establishments over 10 employees.

2 Related Literature

Three recent contributions are worth mentioning. They are clearly in line with the personnel economics approach, although they follow from different traditions in labor economics. Papageorgiou (2010) presents a model of ILMs capturing differences between small and large

2In between those two academic traditions, Human Resource Management scholars have started combining economic considerations with organizational data (see among others, Pfeffer and Cohen, 1984 for an early investigation). They conclude that ILMs tend to prevail within organizations where both unions and personnel departments are present, even though unions seem in conflict with ILM arrangements.
firms in separation rates and wages. Switching between firms is costly, while switching within a firm is not. This affects the way matches are formed and dissolved. Bose and Lang (2011) connect ILMs and monitoring. Workers have different qualities and may be allocated to more or less difficult tasks, with or without monitoring. In equilibrium, workers with known intermediate qualities will be monitored. In this model, both steady increases in wages with seniority and large changes in wages in case of promotions are present, as observed in ILMs. Finally, Pastorino (2012) examines careers in firms by estimating a structural model of learning, job assignment, and human capital accumulation. She uses the same data from a single large firm as Baker, Gibbs, and Holmstrom (1994a, 1994b). The estimated model appears to replicate well several features of the data: transitions across levels of the hierarchy, the distribution of performance ratings, the distribution of wages and their evolution with tenure. In addition, the estimated parameters show that uncertainty at entry over managers’ ability is large and heterogeneous, and that learning takes time. Pastorino (2012) also estimates costs of moving managers across positions and other features of this particular ILM.

3 A Description of Job and Occupational Mobility in France

Our description of worker mobility within and between firms draws from two main data sources. The first one is the Déclarations Annuelles de Données Sociales (DADS), an exhaustive matched employee-employer data set based on employers’ payroll tax filings which allows us to directly measure individual mobility. The second one, the FIChier Unifié de Statistiques d’entreprises (FICUS) compiles various administrative data sources on other forms of employer tax filings. The FICUS files contain detailed accounting information on virtually all firms in the French economy. We use both data sets for the years 2006 and 2007. We describe these data sets in turn, together with the construction of our variables of interest.
3.1 Measuring Mobility

The DADS consists of the payroll tax filings of all private and semi-public French employers (i.e. it does not cover public sector employees, self-employed or unemployed individuals). Individuals $i$ are identified by their national insurance number. Each employer $j$ also has a unique identifier. A typical record for a person $i$/employer $j$/year $t$ consists of all wage payments (net and gross pay) in year $t$, the first and last date of employment of person $i$ at establishment $j$ in year $t$, the number of hours worked over the employment period, type of contract (open ended or fixed term), and four-digit occupation. DADS also conveys limited information about individual characteristics: sex, age, nationality, location/city of the individual’s residence.

Job Movers and Job Stayers. Based on that information, we distinguish between two types of workers: Job Stayers and Job Movers. Job stayers are defined as workers who are observed as working for an employer $j$ at some point in January 2006 and for a period of more than 6 months over the year 2006, and who were again working for that same employer $j$ at some point in December 2007 and for a period of more than 6 months over the year 2007.\(^3\) We have a total of 14,426,325 worker observations satisfying the first conditions in 2007, and 14,127,805 in 2006. Among those, 9,438,841 workers are job stayers, using our definition.

Job movers (from employer $j$ to employer $j'$) are defined as workers who are observed as working for an employer $j$ for a period of more than 6 months during 2006, and who were observed as working for a different employer $j'$ for a period of more than 6 months during

\(^3\)We thus focus on relatively stable workers. While those workers may be employed under either permanent or fixed-term contracts, the latter are typically much more unstable: a very large fraction of those fixed-term contracts last less than one month, and fixed-term contract holders often have multiple employers in any given year, even at the same time, and often go through periods of unemployment (which we cannot accurately measure). Even though fixed-term contracts account for a large fraction of new hires, their share in the stock of employment is quite low in those years (below 7%) because of their short duration. There are strong legal limitations to the renewal of fixed-term contracts, and a fraction (approximately a third) of those contracts get converted to permanent contracts when they end. Their mobility status is measured, both within firm and between firm, when the contract in their destination firm is also a CDI or a long CDD, the most likely case for workers with a CDI or a long CDD in the origin firm. \(\text{????}\)
2007. Among workers satisfying the above conditions, 922,865 are classified as job movers based on our definition.

**Skills and Occupations.** We group four-digit occupation codes into four broad categories, which we interpret as skill levels. Those are, by increasing level of skill: (1) unskilled blue-collar workers, clerical workers or sales agents; (2) skilled blue-collar workers, clerical workers or sales agents; (3) foremen, technicians, mid-level professionals and administrators; (4) engineers, professionals, and executives. These skill categories will be used to assess occupational mobility both within and between firms, which is the main focus of our analysis. A change of four-digit occupation code *within* one of those four broad skill categories will be regarded as a “horizontal” change, not reflecting a promotion or a demotion.

**Firm-Level Performance Measures.** From the firm-level FICUS data, we extract measures of total employment, sales, exports, value-added, operating profit, wage bill, payroll taxes, investment, and industry affiliation. Our data contain 2,661,856 firm-level observations for 2006, and 2,698,734 for 2007.

**The Analysis File.** To compute meaningful measures of within-firm mobility, we restrict attention to firms with at least 10 observations in the establishment in the first of the two years, 2006. Table 1 shows how the firm size, value-added per worker and total sales per workers are distributed across workers and across firms.

### 3.2 A Descriptive Analysis of Mobility

We split our 5,390,066 workers into job stayers and job movers, as explained above. We further divide job stayers and job movers into “promoted”, “demoted” and “stable” workers based on their occupation in both periods. Specifically, any movement up the occupation scale defined in Subsection 3.1 is considered a promotion, and any movement down that scale
<table>
<thead>
<tr>
<th></th>
<th>Across workers</th>
<th>Across firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Size (DADS)</td>
<td>Size (FICUS)</td>
</tr>
<tr>
<td>D1</td>
<td>25</td>
<td>24</td>
</tr>
<tr>
<td>Q1</td>
<td>57</td>
<td>54</td>
</tr>
<tr>
<td>Median</td>
<td>295</td>
<td>286</td>
</tr>
<tr>
<td>Q3</td>
<td>1965</td>
<td>1875</td>
</tr>
<tr>
<td>D9</td>
<td>15382</td>
<td>14053</td>
</tr>
<tr>
<td>Number of observations</td>
<td>5345412</td>
<td>5390066</td>
</tr>
</tbody>
</table>

Table 1: Sample Composition

<table>
<thead>
<tr>
<th></th>
<th>Out, down</th>
<th>Out, up</th>
<th>Out, same</th>
<th>In, down</th>
<th>In, up</th>
<th>In, same</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manager</td>
<td>7725</td>
<td>49869</td>
<td>41814</td>
<td>873953</td>
<td>973361</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.79%</td>
<td>5.12%</td>
<td>4.30%</td>
<td>89.79%</td>
<td>100.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technician, Admin</td>
<td>13175</td>
<td>11306</td>
<td>35288</td>
<td>552929</td>
<td>52713</td>
<td>1152124</td>
<td>1318535</td>
</tr>
<tr>
<td></td>
<td>1.00%</td>
<td>0.86%</td>
<td>2.68%</td>
<td>4.09%</td>
<td>4.00%</td>
<td>87.38%</td>
<td>100.00%</td>
</tr>
<tr>
<td>Skilled white</td>
<td>3172</td>
<td>13082</td>
<td>25450</td>
<td>7871</td>
<td>50294</td>
<td>709573</td>
<td>809442</td>
</tr>
<tr>
<td></td>
<td>0.39%</td>
<td>1.62%</td>
<td>3.14%</td>
<td>0.97%</td>
<td>6.21%</td>
<td>87.66%</td>
<td>100.00%</td>
</tr>
<tr>
<td>Skilled blue</td>
<td>11414</td>
<td>8572</td>
<td>61672</td>
<td>26104</td>
<td>42925</td>
<td>1456705</td>
<td>1607392</td>
</tr>
<tr>
<td></td>
<td>0.71%</td>
<td>0.53%</td>
<td>3.84%</td>
<td>1.62%</td>
<td>2.67%</td>
<td>90.63%</td>
<td>100.00%</td>
</tr>
<tr>
<td>Total skilled</td>
<td>14586</td>
<td>21654</td>
<td>87122</td>
<td>33975</td>
<td>93219</td>
<td>2166278</td>
<td>2416834</td>
</tr>
<tr>
<td></td>
<td>0.60%</td>
<td>0.90%</td>
<td>3.60%</td>
<td>1.41%</td>
<td>3.86%</td>
<td>89.63%</td>
<td>100.00%</td>
</tr>
<tr>
<td>Unskilled white</td>
<td>1896</td>
<td>5677</td>
<td>14113</td>
<td>135039</td>
<td>156725</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.21%</td>
<td>3.62%</td>
<td>9.00%</td>
<td>86.16%</td>
<td>100.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unskilled blue</td>
<td>25362</td>
<td>21438</td>
<td>54423</td>
<td>413248</td>
<td>514471</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.93%</td>
<td>4.17%</td>
<td>10.58%</td>
<td>80.32%</td>
<td>100.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total unskilled</td>
<td>27258</td>
<td>27115</td>
<td>68536</td>
<td>548287</td>
<td>671196</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.06%</td>
<td>4.04%</td>
<td>10.21%</td>
<td>81.69%</td>
<td>100.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>35486</td>
<td>60218</td>
<td>199394</td>
<td>129718</td>
<td>214468</td>
<td>4740642</td>
<td>5379926</td>
</tr>
<tr>
<td></td>
<td>0.66%</td>
<td>1.12%</td>
<td>3.71%</td>
<td>2.41%</td>
<td>3.99%</td>
<td>88.12%</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

Table 2: Job and occupational mobility - Gross flows
is a demotion. Workers staying on the same rung of that skill scale between 2006 and 2007 (while possibly changing four-digit occupation) are classified as stable.

Table 2 shows a basic description of occupational mobility. About 5.5% of workers are classified as job movers. Unsurprisingly, occupational stability is by far the most common occurrence for both job movers and job stayers, with demotion being the least common (almost half as frequent as promotions). About 88% of workers in the sample are stable job stayers.

Also unsurprisingly, the frequency of promotions decreases with the initial skill level, and is heterogeneous across occupation types. Roughly 10% of unskilled job stayers received a within-employer promotion over the two years considered. This proportion goes down to 6% among skilled white collar workers, and less than 3% for skilled blue collar workers. Finally, engineers, professionals, and managers (who can only be demoted or stable on our skill scale) are very stable within firms (90%) as well as between firms (5%).

Promotion, demotion, and mobility rates vary between firms. For instance, the within-firm probability of demotion has a 3% mean but most firms do not demote their workers: the 3rd quartile of the distribution is almost equal to 4% and the ninth decile to 10%. The equivalent numbers for the stability rate of job stayers vary between 82% for the first quartile and 97% for the third quartile of the distribution. As for within-firm promotions, the median is again zero, the third quartile is below 7% and the ninth decile 12.5%.

When workers change employers, stability or promotion are the most likely outcomes with values of Q3 or D9 of 5% and 10%, respectively. These numbers are not very different from those seen for job stayers. However, conditional on changing employers, promotion is much more likely and stability much less, at all levels of the distribution.

Interestingly, the very high (above 85%) fraction of stable job stayers conceals a lot of “horizontal” occupational mobility. The fraction of stable job stayers who change two-digit occupation is approximately 3% (less for the unskilled white collar but 6% for engineers, professionals, and managers). Furthermore, over 8% move between four-digit occupations.
Now focusing on stable job movers, the equivalent numbers are much higher: more than 20% change 2-digit occupation and over 60% change 4-digit occupation. Summing up, approximately 17% of workers change their 4-digit occupation every year. For workers staying in their firm, the equivalent number is just below 15%. Hence, even focusing on the most stable workers (workers employed under permanent, or long fixed-term contracts), occupational transitions both within and between firms are important.

In order to understand the role of firms in the mobility process, we characterize firms in the spirit of the “Heterogeneous Firms” literature (see Melitz, 2003 or Eaton, Kortum, and Kramarz, 2011, EKK hereafter) and summarize productivity with a measure of sales in France as in EKK. Then, we compute the various measures summarized above by percentile of the distribution of sales in France among our firms.\textsuperscript{4} Results are quite clear. Despite some mild evidence of a negative (positive) correlation between within-firm demotions (promotions) and sales, or of a positive correlation between between-firm stability and sales in the origin firm, the main message is that all such rates are, on first approximation, independent of size and productivity. To go a step further, we examine how various outcomes and characteristics of the origin and the (potential) destination firm simultaneously affect mobility and promotion/demotions using a multinomial logit.

The outcome can be internal demotion (except for unskilled workers), internal stability, internal promotion (except for engineers etc.), external demotion (id.), external stability, and external promotion (id.). A different regression is performed for each of our four broad occupation categories. The explanatory variables are a constant that captures the base tendency of each category to move (by type of move), the log-employment of the origin firm in 2006, the log-sales per employee of the origin firm in 2006, the proportion of unskilled workers in the origin firm in 2006, the proportion of engineers, technicians etc. in the origin firm in 2006, the difference between log-employment of the destination and of the origin firm (for mobile workers) both measured in the first year i.e. 2006 hence before mobility,

\textsuperscript{4}To assess robustness, we do the same with the centiles of the distribution of value-added per employee.
<table>
<thead>
<tr>
<th></th>
<th>Unskilled industrial and service worker</th>
<th>Skilled industrial and service worker</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log(sales/worker) ((t - 1))</td>
<td>-0.361</td>
<td>-1.110</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Change in log(sales/worker)</td>
<td>1.576</td>
<td>0.332</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Log(size) ((t - 1))</td>
<td>0.201</td>
<td>0.167</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Change in log(size)</td>
<td>-0.290</td>
<td>-0.280</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Share High Skilled ((t - 1))</td>
<td>-0.291</td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>Change in Share High Skilled</td>
<td>0.943</td>
<td>-2.189</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.053)</td>
</tr>
<tr>
<td>Share Low Skilled ((t - 1))</td>
<td>-3.940</td>
<td>4.643</td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>Change in Share Low Skilled</td>
<td>-8.565</td>
<td>2.508</td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
<td>(0.051)</td>
</tr>
<tr>
<td>Sample size</td>
<td>668,502</td>
<td>2,399,006</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Technician, foreman, assistant manager and sale worker</th>
<th>Administrative technical manager</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log(sales/worker) ((t - 1))</td>
<td>-0.357</td>
<td>-0.113</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Change in log(sales/worker)</td>
<td>0.692</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Log(size) ((t - 1))</td>
<td>0.030</td>
<td>-0.076</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Change in log(size)</td>
<td>-0.332</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Share High Skilled ((t - 1))</td>
<td>-2.335</td>
<td>-0.113</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>Change in Share High Skilled</td>
<td>-2.759</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>Share Low Skilled ((t - 1))</td>
<td>0.476</td>
<td>0.387</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>Change in Share Low Skilled</td>
<td>-2.154</td>
<td>-2.428</td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td>(0.065)</td>
</tr>
<tr>
<td>Sample size</td>
<td>1,334,854</td>
<td>972,918</td>
</tr>
</tbody>
</table>

Notes: (1) “In, same” (internal market - no change of occupation category) is the reference destination category. (2) This table contains the results of three different multinomial logit estimations with different firm characteristics. (3) All estimations contains intercepts and controls for age and gender.

Table 3: Job and occupational mobility - Firm effects
the difference between log-sales per employee of the destination and of the origin firm (for mobile workers) both measured in 2006, the difference between the proportions of unskilled workers of the destination and of the origin firm both measured in 2006, and the difference between the proportions of technicians, engineers et al. of the destination and of the origin firm both measured in 2006. Results are presented in Table 3.2a-d.

First, and as seen in the raw statistics, internal stability (i.e. no mobility or change of skill category), is the most frequent outcome both conditionally as well as unconditionally for all skill groups and types of mobility except for the unskilled in the case of external promotion. This reflects the transitory nature of unskilled jobs for a fraction of the workforce who start out in such jobs and then move up to a skill category reflecting their education (not measured here).

Second, external mobility is always associated with low productivity of the origin firm, as measured by log-sales per worker. And, conditional on the origin firm’s productivity, external mobility is also always associated with the destination firm being more productive than the origin firm (both measured in 2006). This is a first element of empirical sorting due to selective mobility of the workforce. This fact is consistent with Postel-Vinay and Robin (2003)’s model.

Demotions and promotions are not extremely frequent, be they internal or external. Internal stability is the most likely outcome and for immobile workers, demotions or promotions mostly take place in smaller firms. This is potentially a reflection of stricter rules that must be followed in large firms (under bargaining collective agreements which often prevail there). However, internal promotions seem to be affected by the skill structure of the firm in conjunction with the worker’s skill. For instance, unskilled workers employed in firms with a large fraction of highly skilled workers are more likely to be internally promoted; the effect is very strong and much larger than the effect for internal stability. Conversely, internal demotion of engineers, professionals, and managers – a low probability event – is more likely in firms with many low-skilled workers. In addition, the skill structure of the
destination firm, in comparison with that of the origin firm, has a strong impact on the resulting external mobility. For instance, unskilled workers who move to a destination firm with a larger fraction of unskilled workers than that of the origin firm have essentially no chance of getting promoted in the year following their move.

4 An Equilibrium Model of Job Assignment and Occupational Mobility Within and Between Firms

We consider an economy populated by fixed continua of workers and firms, all infinitely lived, risk-neutral, and discounting the future at rate $r$. Time is discrete and the labor market is in steady state.

4.1 Firms, Workers and Jobs

Workers are characterized by a pair of scalars $(x, k) \in [\underline{x}, \overline{x}] \times \{1, \ldots, K\}$. The scalar $x$ is a time-invariant measure of the worker’s permanent potential ability, which is unobservable to the econometrician but observed by agents in the model. The (improper) density of type-$x$ workers is $\ell(x)$, with $\int \ell(x) \, dx = L$. The integer $k$ indicates the worker’s current occupation (one of a fixed number $K$ of different occupations observed in the market) which is time-varying, and observed by the econometrician as well as agents in the model. Worker can change occupations over time: a worker currently in occupation $k$ may be offered a job of type $k'$. Such mobility entails a conversion (or re-training) cost, and we allow for occupational mobility in both directions (up and down). As such, occupation is also a measure of human capital, which evolves with a worker’s labor market experience.

Workers face a continuum of firms indexed by $y \in [\underline{y}, \overline{y}]$, a general technology index, unobservable to the econometrician but observed by agents. The (proper) density of firm technologies is $\gamma(y)$, with $\int \gamma(y) \, dy = 1$. Each firm employs a continuum of workers of all types $x$, distributed over (up to) $K$ discrete occupations. The density of type-$x$ workers in
occupation \( k \) at any firm of type \( y \) is denoted by \( h_k(x|y) \), with \( \sum_{k=1}^{K} \int h_k(x|y) \, dx = n(y) \) being total firm size.

We further denote the density of type-\( x \) unemployed workers with skills \( k \) as \( u_k(x) \). The latter are defined as workers whose last occupation prior to job loss was \( k \). We thus assume that workers keep the skills relevant to their occupation during unemployment spells (for example, a skilled worker who is promoted to supervisor keeps his supervising skills after a layoff), and talk about “unemployed workers with skills \( k \)”. Finally, the total density of type-\( x \) unemployed workers is \( u(x) \equiv \sum_{k=1}^{K} u_k(x) = \ell(x) - \sum_{k=1}^{K} \int h_k(x|y) \gamma(y) \, dy \).

We introduce a hierarchy among matches as follows. A type-\((k, x, y)\) firm-worker match produces a constant output flow of \( p_k(x, y) \), which we assume increasing in \( k \) for all \((x, y)\). Occupation is thus an unambiguous indicator of a worker’s “quality” (or human capital). We make no further assumption about the monotonicity of \( p_k(x, y) \) w.r.t. \((x, y)\), to leave scope for possibly strong complementarities between worker and firm types. It may well be, for example, that certain technologies require a minimum worker type. Conversely, high-\( x \) workers may be over-qualified for certain technologies, and produce low output in firms with those technologies.

### 4.2 Within-Period Timing of Events

The reallocation of workers across jobs is mediated by both internal (within-firm) and external (between firms) labor markets. Specifically, workers may apply to the job vacancies posted by their current employer, as well as to vacancies posted by competing employers. Workers face frictions in both markets. Frictions on the internal labor market are modeled similarly, but interpreted slightly differently than the ones affecting the external labor market. Frictions in the labor market at large are generally thought of as capturing a broad range of information imperfections about various forms of underlying worker and job heterogeneity that hinder the reallocation of workers across jobs. While there may also be an element of that in the frictions affecting internal labor markets, here we think of those frictions partly
as capturing the limited ability that a firm has to promote (or, more generally, to reassign) its workers. In the background is the idea (not modeled) that a firm’s technology requires the proportions of clerical workers, supervisors, managers, etc. to remain within certain bounds, and that, in order for someone to be promoted from supervisor to, say, manager, a manager’s position must become vacant. Modeling those technological constraints explicitly would be highly intractable, and we therefore take the formal shortcut of a matching function to capture them.\(^5\)

Events take place in the following order within a period:

1. With probability \(\delta\), employed workers receive notice that their post will be shut down within the period.

2. Firms decide on the number \(v_k(y)\) of vacant jobs to open in each occupation \(k\).

3. All employees of any type-\(y\) firm, *including those that have been given notice at stage 1*, receive an opportunity to interview with their own firm for an alternate job in some occupation \(k\) with probability \(\lambda_k^I(y)\) (specified below). Conducting interviews and processing applications takes some time, so that the workers and vacancies involved in internal-market interviews cannot participate in any other recruitment activities this period. Interviews may or may not be successful, according to rules specified below.

4. Firms post the share of their initial stock of vacancies that are not currently involved in internal mobility interviews (stage 3) on the external market. Meanwhile, employees who are not currently busy interviewing on the internal market (stage 3) search for alternate jobs on the external market, where they interview for a random vacant job in occupation \(k\) in a firm of type \(y\) with probability \(s_1 \lambda_k^E(y)\) (specified below). Likewise,

\(^5\)Chiappori, Salanié and Valentin (1999), Papageorgiou (2013), and Pastorino (2013), among others, analyze a combination of symmetric learning about worker ability and exogenous human capital accumulation as an alternative driver of changes in optimal job assignment within firms. While our model shares some features with those studies, its focus is less on the foundations of internal labor markets, and more on the equilibrium consequences of their existence in the presence of frictions (as in Papageorgiou, 2013) and persistent worker heterogeneity that is partially portable between occupations (as in Pastorino, 2013).
unemployed workers search in the external market and meet an occupation-\( k \) vacancy in a type-\( y \) firm with probability \( s_0 \lambda_k^E(y) \).

5. Production and payments take place at the end of the period.

6. The interviewing round finishes on both markets, and hiring takes place. Employees who received notice of dismissal at stage 1, and who either did not get a chance to interview on either market, or did interview but had an unsuccessful interview, become unemployed.

Our timing assumptions bring about some comments. First, they do not allow workers to search simultaneously on both the internal and external markets. This is to avert the possibility of a worker receiving two different alternative job offers (one on the internal, and one on the external market) in the same period. We would then have to compare both offers and have the worker choose the better one, which would add complexity without adding much substance. What’s more, we do not allow workers who are selected for an internal promotion interview to pass up on that interview and try to go on the external market instead. This is a restriction, as in some (arguably rare, given the low value of \( \lambda_k^E \)) cases, it may be in a worker’s interest to do so. Second, they also rule out simultaneous search on the vacancy side, as vacancies committed to the internal market in stage 2 cannot be advertised on the external market. This ensures that the employer’s outside option (the value of a vacancy) is the same on both markets. Again, assuming otherwise would complicate the analysis without adding much substance. Finally, a perhaps more substantive feature of our timing assumptions is that firms first try to fill their job vacancies through internal promotions, and then advertise the vacancies they have left on the external market.\(^6\) We describe those two steps of the hiring process in turn.

\(^6\)As we show below, this feature is formally equivalent to assuming that firms commit a fixed, exogenous share of their vacancies to the internal market.
4.3 Meetings

**Internal labor markets.** The opening of $v_k(y)$ vacant jobs in occupation $k$ gives a type-$y$ firm the capacity to conduct $q^I v_k(y)$ “promotion interviews” on the internal market, with $q^I < 1$ a fixed, exogenous scalar. We make the simplifying assumptions that interviewees are drawn at random from the firm’s current workforce, including the fraction $\delta$ of workers who just got notice that their current post was terminated. The probability of the typical employee in that firm being interviewed for an internal job change to occupation $k$ is therefore

$$\lambda^I_k(y) = q^I v_k(y) \frac{v_k(y)}{n(y) v(y)},$$

where $n(y) = \sum_k \int h_k(x|y) \, dx$ is total firm size and $v(y) = \sum_k v_k(y)$ is the overall number of job openings in the firm, in that period. We further introduce the notation $\lambda^I(y) \equiv \sum_{k=1}^K \lambda^I_k(y) = q^I v(y)/n(y)$ for the unconditional chance of being interviewed for any occupation.

Note that our postulated internal-market meeting technology is linear in the number of vacancies: from the firm’s perspective, the probability of interviewing a candidate for any given job in the internal labor market is $q^I$, independent of the total number of job openings. This rules out congestion externalities between vacancies, and amounts to assuming that firms are able to some extent to coordinate their workers’ promotion applications. This assumption is not essential, however we view it as a reasonable description of a firm’s internal market.

**External labor market.** As the external market opens, a type-$y$ firm is left with a number $(1 - q^I)v_k(y)$ of vacancies in occupation $k$ not currently processing internal-market applications, which it advertises on the external market.\(^7\) On the supply side, a measure $u_k(x)$ of

---

\(^7\) Note that our assumption of a linear matching technology on the internal market implies that firms are *de facto* committing a fixed share $q^I$ of their vacancies to internal reassignment. This share would become endogenous (a function of firm size and total vacancies posted) if we assumed that the internal market was affected by congestion externalities.
type-$x$ workers with current skills $k$ are unemployed and searching in the external market, while a measure $[1 - \lambda^I(y)] h_k(x|y)$ of such workers are employed at firms of type $y$ and available for external-market interviews, as they are not busy interviewing on the internal market. Aggregate worker search effort is given by

\[
S^E = s_0 \sum_{k=1}^K \int w_k(x) \, dx + s_1 \sum_{k=1}^K \iiint [1 - \lambda^I(y)] h_k(x|y) \gamma(y) \, dx \, dy,
\]

where $s_0, s_1 > 0$ are the exogenous search intensity of unemployed and employed workers, assumed to be independent of occupation or type. Encounters between job seekers and vacant jobs on the external market are also governed by a standard random matching process. The aggregate flow of such meetings is $M^E = m^E [S^E, (1 - q^I) V]$, where $V = \sum_{k=1}^K \int v_k(y) \gamma(y) \, dy$ denotes the total number of job vacancies opened by firms at the onset of the period. The function $m^E$ has all the properties of a standard matching function. The instantaneous probability of a worker contacting a vacancy of any type is $s_0 M^E / S^E$ if that worker is unemployed, and $s_1 M^E / S^E$ if she is employed. Next, the probability of an unemployed job seeker meeting a vacancy in a specific occupation $k$ and firm type $y$ is simply

\[
\frac{s_0 M^E v_k(y) \gamma(y)}{S^E V} \equiv s_0 \lambda^E_k(y).
\]

It is $s_1 \lambda^E_k(y)$ for employed job seekers.

We further introduce the notation $q^E = M^E / [(1 - q^I) V]$ for the unconditional probability of a vacancy in any occupation $k$ and firm $y$ contacting any job applicant. Note that this is, in general, a decreasing function of the aggregate volume of vacancies left in the external labor market $(1 - q^I) V$, as the external market is affected by the congestion externality inherent to conventional matching functions.
4.4 Rent Sharing and Wage Setting

Let $W_k(w, x, y)$ denote the present value of a wage $w$ for a type-$x$ worker in occupation $k$ at a firm of type $y$. Let $U(x)$ denote the value of unemployment, assumed independent of the worker’s occupation $k$ for reasons discussed later in Subsection 4.6. Let $P_k(x, y)$ denote the expected present value of all future output generated by a worker with her current and future employers.$^8$ Anticipating on the firms’ optimal job creation behavior, we assume that the (marginal) value of all vacancy types is zero (more below). Firms pay a flow cost to post job adverts (see below), but job creation entails no other investment that the firm would retain if a negotiation fails.

External wage competition. Wages are determined by Bertrand competition between employers, as in Postel-Vinay and Robin (2002). Let us first consider the case of an unemployed worker of type $x$ and skills $k$ who is currently unemployed or has received notice of layoff without internal reassignment. If she meets a vacancy of type $(k', y')$, she is hired if total potential match value net of the value of unemployment, $P_{k'}(x, y') - U(x)$, exceeds the conversion cost, which we specify as $c^E(k, k') + z \geq 0$, where $z$ is a non-negative match-specific shock drawn from a distribution $F$. Note that, absent job creation costs, the value of a vacancy is zero for the employer. In this case, the worker is hired at a wage $w$ which solves

$$W_{k'}(w, x, y') = U(x).$$

In other words, the worker receives her reservation value (implemented by her reservation wage), and thus makes zero net gain, while the firm appropriates the whole match surplus net of the conversion cost: $P_{k'}(x, y') - U(x) - c^E(k, k') - z$.

Next consider a type-$x$ employed worker in a match of type $(k, y)$, paid a wage $w$, who has not received notice that her post would be shut down at the end of the period. Suppose

$^8$The assumption that a firm’s output is the sum of individual match output over all matches within the firm ensures that $P_k(x, y)$ only depends on the occupation and type of a single worker, and not on the entire distribution of worker and occupation types within the firm.
this employee meets a vacancy of type \((k', y')\) such that the difference in match present values 
\(P_{k'}(x, y') - P_k(x, y)\) is greater than the re-training cost \(c^E(k, k') + z\), the worker ends up in the higher-surplus match with a wage \(w'\) which solves 

\[ \mathcal{W}_{k'}(w', x, y') = P_k(x, y). \]

The poacher thus receives the supplement in match value minus the conversion cost, \(P_{k'}(x, y') - P_k(x, y) - c^E(k, k') - z\). Under that rule, the worker always makes a positive net gain from the move, as her previous employment value was necessarily no greater than the total value of her previous type-\((k, y)\) job, \(P_k(x, y)\), otherwise her previous employer would not have found it profitable to employ her. Both parties agree to the move iff \(P_{k'}(x, y') - P_k(x, y) \geq c^E(k, k') + z\).

If \(P_k(x, y) > P_{k'}(x, y') - c^E(k, k') - z > \mathcal{W}_k(w, x, y)\), the worker does not move as the type-\((k', y')\) job cannot profitably outbid the initial type-\((k, y)\) job. Yet, in order to retain the worker, the incumbent employer must match the best offer made by the poacher, namely \(P_{k'}(x, y') - c^E(k, k') - z\). As a result, the worker is offered a wage increase to \(w'\), defined by 

\[ \mathcal{W}_k(w', x, y) = P_{k'}(x, y') - c^E(k, k') - z. \]

If \(P_{k'}(x, y') - c^E(k, k') - z \leq \mathcal{W}_k(w, x, y)\) then the total value of a potential match with the poaching firm is not enough to even compensate the worker for the value she receives from her current contract. In this case, the worker cannot benefit from using the outside offer to prompt a wage renegotiation, so she simply discards the offer and carries on with her current contract.

**Internal wage mobility.** We finally turn to the description of internal wage mobility. Two distinct situations may arise. The first is one in which an employed worker initially earning a wage \(w\) in a job of type \((k, y)\), and who has not received notice that her position was shut down, is offered an alternate position with occupation \(k'\) in her own firm (i.e. meets a vacancy of type \((k', y)\) on her own firm’s internal market). If the worker accepts it, we refer to this case as a *promotion*, as the firm-worker match can only gain from this situation.
Indeed, from the joint perspective of the worker and her employer, it is efficient to go ahead with the promotion if it affords a net gain in total match value, i.e. if $P_{k'}(x, y) - P_k(x, y)$ exceeds the conversion cost $c^I(k, k') + z$. Yet the worker will only accept the promotion if offered a value $W' \geq W_k(w, x, y)$, and the employer will find the promotion profitable if

$$P_{k'}(x, y) - W' - c^I(k, k') - z \geq P_k(x, y) - W_k(w, x, y).$$

That is, the employer’s profit from promoting the worker to occupation $k'$, net of the training cost $c^I(k, k') + z$, must be greater than the current profit $P_k(x, y) - W_k(w, x, y)$ from keeping the worker where she is. If implementing the promotion is jointly efficient, then there will exist an interval of worker values,

$$W' \in [W_k(w, x, y), W_k(w, x, y) + P_{k'}(x, y) - P_k(x, y) - c^I(k, k') - z],$$

that are acceptable to both the worker and the employer. The key assumption we make here is that the employer and the worker pick a value in this interval, according to some rule guaranteeing that all privately efficient promotions are implemented. In essence, we are assuming that the institutional arrangements on firms’ internal labor markets are sufficiently well-functioning to guarantee constrained-efficient within-firm labor reallocation. This criterion is fulfilled by a simple fixed-share surplus-splitting rule setting the worker’s promotion value to

$$W' = W_k(w, x, y) + \beta \left[ P_{k'}(x, y) - P_k(x, y) - c^I(k, k') - z \right],$$

for some $\beta \in (0, 1)$. Under that rule, the worker ends up being paid the wage $w'$ such that $W_{k'}(w', x, y) = W'$. We adopt this rule for its simplicity and parametric parsimony.

---

9Observe in passing that we have added indexes $E$ and $I$ to the re-training cost, to allow for different training costs following a change of occupations coming along with a change of firms, and a change of occupations within a firm. The latter may for instance be less costly than the former if there is a firm-specific component to human capital which is common to all occupations in a firm, but non-portable across firms, such as “corporate culture”.

20
The second type of internal mobility occurring in the model involves workers initially in occupation \( k \) who did receive a job termination notice at the beginning of the period (with probability \( \delta \)), and were subsequently drawn to interview for a job in occupation \( k' \) on the internal market (with probability \( \lambda_{k'}^I(y) \)). By contrast to the previous case, we refer to this situation as a demotion. We apply the same wage-setting rule as for promotions, except that in this case, the outside options in the bargain are the value of unemployment, \( U(x) \), for the worker, and the value of a marginal vacancy, that is 0, for the employer. Both parties will therefore agree to go ahead with the internal reassignment if \( \mathcal{P}_k'(x, y) - c^I(k, k') - z \geq U(x) \), in which case the worker comes out of the process with a new value

\[
W' = U(x) + \beta \left[ \mathcal{P}_k'(x, y) - U(x) - c^I(k, k') - z \right].
\]

Conversely, if \( \mathcal{P}_k'(x, y) - c^I(k, k') - z < U(x) \), the worker and the firm agree to pass up on the reassignment opportunity: the worker then becomes unemployed, and the firm is left with a vacant post to advertise on the external market.

### 4.5 Job Creation

Firms post vacancies of all types so as to maximize expected profit. Let \( c_k(v) \) denote the cost of posting \( v \) adverts for jobs in occupation \( k \). After posting \( v_k(y) \) job adverts in occupation \( k \), a type-\( y \) firms expects to conduct \( q^I v_k(y) \) interviews on the internal market, and post the remaining \( (1 - q^I) v_k(y) \) vacancies on the external market. The expected flow value to the firms of type \( y \) of opening \( v_k(y) \) vacancies adverts in occupation \( k \) is

\[
q^I v_k(y) J_k^I(y) + q^E (1 - q^I) v_k(y) J_k^E(y),
\]

where contact probabilities \( q^E \) and \( q^I \) were defined in Section 4.2, and where \( J_k^E(y) \) and \( J_k^I(y) \) are the expected net values to the firm of filling the vacancy with a worker met on the external and internal labor market, respectively.
The former value is the sum of two components, \( J_{k}^{E}(y) = J_{k}^{E,0}(y) + J_{k}^{E,1}(y) \), where \( J_{k}^{E,0}(y) \) is the expected return to hiring an unemployed or laid-off worker:

\[
J_{k}^{E,0}(y) = \sum_{k'=1}^{K} \int \frac{s_{0}u_{k'}(x') + s_{1}h_{k'}(x',y')\gamma(y')}{S_{E}} \, dy' \times \mathbb{E} \max \{ P_{k}(x',y) - U(x') - c^{E}(k',k) - z, 0 \} \, dx',
\]

and \( J_{k}^{E,1}(y) \) is the return to poaching an employee from another firm:

\[
J_{k}^{E,1}(y) = \sum_{k'=1}^{K} \int \frac{(1 - \delta)h_{k'}(x',y')\gamma(y')}{S_{E}} \times \mathbb{E} \max \{ P_{k}(x',y) - P_{k'}(x',y') - c^{E}(k',k) - z, 0 \} \, dx' \, dy'.
\]

Under the fixed-share surplus-splitting rule described in Paragraph 4.4, the expected value of an interview on the internal market is also the sum of two components, \( J_{k}^{I}(y) = J_{k}^{I,+}(y) + J_{k}^{I,-}(y) \), where \( J_{k}^{I,+}(y) \) reflects promotions:

\[
J_{k}^{I,+}(y) = (1 - \beta) \sum_{k'=1}^{K} \int \frac{(1 - \delta)h_{k'}(x',y)}{n(y)} \mathbb{E} \max \{ P_{k}(x',y) - P_{k'}(x',y) - c^{I}(k',k) - z, 0 \} \, dx',
\]

and \( J_{k}^{I,-}(y) \) reflects demotions:

\[
J_{k}^{I,-}(y) = (1 - \beta) \sum_{k'=1}^{H} \int \frac{\delta h_{k'}(x',y)}{n(y)} \mathbb{E} \max \{ P_{k}(x',y) - U(x') - c^{I}(k',k) - z, 0 \} \, dx'.
\]

The optimal number of posted vacancies \( v_{k}(y) \) equates the cost of the marginal vacancy to its expected value:

\[
c'_{k}[v_{k}(y)] = q^{I} J_{k}^{I}(y) + q^{E}(1 - q^{I}) J_{k}^{E}(y).
\]
4.6 Unemployment, Job and Match Values

We now derive the Bellman equations determining $U(x), P_k(x, y)$ and $W_k(w, x, y)$.

The value of unemployment. Consider an unemployed worker of type $x$ and skills $k$. During unemployment, she earns a flow income of $b(x)$, which we allow to depend on the worker’s fixed type $x$, but not on her skills. This worker expects to be offered a job of type $(k', y')$ at any point in time with probability $\lambda^E_k(y')F \left( P_{k'}(x, y') - U(x) - c^E(k, k') \right)$. Now, whether she takes the job or not, the continuation value is the value of unemployment, $U(x)$. Hence

$$rU(x) = b(x). \quad (7)$$

We should acknowledge at this juncture that our assumption that $b(x)$ is independent of $k$, combined with the assumption that employers extract the whole surplus by making workers take-it-or-leave-it offers, is key in ensuring that the value of unemployment, $U(x)$, is independent of the worker’s last job occupation, $k$. While this independence is not a particularly appealing feature of the model, it affords great simplifications in the rules governing the acceptance of job offers, described in 4.4 above.\(^{10}\) We opt for simplicity over realism in this particular instance.

Match value. Consider a match $(x, y, k)$. The output flow of this match is $p_k(x, y)$. With probability $\delta$, the match is dissolved at the beginning of the period. However, with probability $\delta \lambda^I_k(y)$, the worker is offered a chance to be reassigned to a new occupation $k'$ in the same firm. This reassignment possibility is implemented if the associated joint continuation value for the firm-worker collective, $\mathcal{P}_{k'}(x, y) - c^I(k, k') - z$, is larger than the collective outside value, i.e. $U(x)$ as the value of a vacancy is equal to zero. In case a

\(^{10}\)If $U$ was a function of $k$, workers would change reservation values each time they change occupations, and we would have to check that any offer made to a worker involving a change from occupation $k$ to $k'$ is greater than max $\{U_k(x), U_{k'}(x)\}$, so that no worker is ever tempted to quit into unemployment right after a potential new employer has paid for the upgrade of her skills from $k$ to $k'$. This would be feasible, but it would make the derivation of equilibrium worker distributions more cumbersome.
reassignment opportunity fails to arise, or if one arises but turns out to be unprofitable, the worker becomes either unemployed or finds an alternate match on the external market, leaving the firm with a vacant job, and receiving the value of unemployment. The joint continuation value is therefore \( U(x) \) in both cases. Next, with probability \((1 - \delta)\lambda_k(y)\), the worker is offered a promotion to occupation \( k' \) within the firm, which is implemented if the associated continuation gain, \( P_{k'}(x, y) - P_k(x, y) - c^I(k, k') - z \), is positive. Finally, the worker may also be poached by a vacancy of type \((k', y')\). In this case, the worker moves and pockets a value of \( P_k(x, y) \), while the firm is left with a vacant job worth 0, so that the net continuation gain for the firm-worker collective is 0. Summing up, match surplus solves:

\[
(1 + r)P_k(x, y) = p_k(x, y) + \delta \sum_{k'=1}^{K} \lambda_{k'}(y) \mathbb{E} \max \{ P_{k'}(x, y) - c^I(k, k') - z, U(x) \} \\
+ (1 - \delta) \sum_{k'=1}^{K} \lambda_{k'}(y) \mathbb{E} \max \{ P_{k'}(x, y) - c^I(k, k') - z, P_k(x, y) \} \\
+ \delta (1 - \nu^I(y))U(x) + (1 - \delta)(1 - \nu^I(y))P_k(x, y).
\]  

The match value function \( P(x, y) = (P_1(x, y), \ldots, P_K(x, y)) \) is therefore the fixed point of the following contraction operator (for given job arrival rates):

\[
\mathcal{P}_k(x, y) = U(x) + \frac{1}{r + \delta} \left( p_k(x, y) - rU(x) \\
+ \delta \sum_{k'=1}^{K} \lambda_{k'}(y) \mathbb{E} \max \{ P_{k'}(x, y) - U(x) - c^I(k, k') - z, 0 \} \\
(1 - \delta) \sum_{k'=1}^{K} \lambda_{k'}(y) \mathbb{E} \max \{ P_{k'}(x, y) - P_k(x, y) - c^I(k, k') - z, 0 \} \right).
\]

**Present value of a wage contract.** Applying the wage setting rules laid out in Subsection 4.4, we have that

\[
r \mathcal{W}_k(w, x, y) = w + (1) + (2) + (3) + (4),
\]
where the 4 components of the continuation value reflect the expected capital gains associated
with the various random events (job offers, job destruction) that the worker faces, and are
as follows:

- Poaching:

\[
(1) = (1 - \delta) \left[ 1 - \lambda^I(y) \right] s_1 \sum_{k' = 1}^{K} \int \lambda_k^E(y') \ldots \\
\times \mathbb{E} \max \left\{ \min \left\{ P_k(x, y), P_{k'}(x, y') - c^E(k, k') - z \right\} - W_k(w, x, y), 0 \right\} dy';
\]

- Promotion:

\[
(2) = (1 - \delta) \sum_{k' = 1}^{K} \lambda_{k'}^I(y) \beta \mathbb{E} \max \left\{ P_{k'}(x, y) - P_k(x, y) - c^I(k, k') - z, 0 \right\};
\]

- Demotion:

\[
(3) = \delta \sum_{k' = 1}^{K} \lambda_{k'}^I(y) \left[ \beta \mathbb{E} \max \left\{ P_{k'}(x, y) - \mathcal{U}(x) - c^I(k, k') - z, 0 \right\} + \mathcal{U}(x) - W_k(w, x, y) \right];
\]

- Layoff:

\[
(4) = \delta \left[ 1 - \lambda^I(y) \right] \left[ \mathcal{U}(x) - W_k(w, x, y) \right].
\]
This Bellman equation also defines the wage \( w \) as an increasing function of the contract value \( \mathcal{W} \) given \((x, y, k)\). Rearranging, for any \( \mathcal{U}(x) \leq \mathcal{W} \leq \mathcal{P}_k(x, y) \):

\[
w_k(\mathcal{W}, x, y) = (r + \delta)\mathcal{W} - \delta \mathcal{U}(x)
\]

\[
- (1 - \delta) \left[ 1 - \lambda^I(y) \right] s_1 \sum_{k'=1}^K \lambda^{E}_k(y') \left( \mathbb{E} \max \{ \mathcal{P}_{k'}(x, y') - \mathcal{W} - c^E(k, k') - z, 0 \} 
\right.
\]

\[
- \mathbb{E} \max \{ \mathcal{P}_{k'}(x, y') - \mathcal{P}_k(x, y) - c^E(k, k') - z, 0 \} \right) dy'
\]

\[- \beta(1 - \delta) \sum_{k'=1}^K \lambda^{I}_k(y) \mathbb{E} \max \{ \mathcal{P}_{k'}(x, y) - \mathcal{P}_k(x, y) - c^I(k, k') - z, 0 \}
\]

\[- \beta \delta \sum_{k'=1}^K \lambda^{I}_k(y) \mathbb{E} \max \{ \mathcal{P}_{k'}(x, y) - \mathcal{U}(x) - c^I(k, k') - z, 0 \} \}.
\]

(10)

Note in passing that, in some situations, a worker can receive a wage strictly greater than her productivity, \( p_k(x, y) \).11 The reason is that the worker is rewarded by the employer for being an input into the internal market matching process: total match surplus will increase when the worker gets promoted (or demoted, thus averting the complete loss of the match value) at some point in the future, and our surplus-sharing rule ensures that the employer will receive a share \((1 - \beta)\) of that surplus gain. This is factored into today’s wage.

It follows from this analysis that the relevant state variable for any worker state is the quadruple \((\mathcal{W}, x, y, k)\). The worker type \( x \) is fixed over time, and the triple \((\mathcal{W}, y, k)\) evolves according to the following rules:

- A type-\( x \) unemployed worker with skills \( k \) getting an opportunity to form match of type \((k', y')\) takes that opportunity if \( \mathcal{P}_{k'}(x, y') - \mathcal{U}(x) \geq c^E(k, k') + z \). In this case, her state changes from \((\mathcal{U}(x), x, 0, k)\) to \((\mathcal{U}(x), x, y', k')\) and her new wage is \( w_{k'}(\mathcal{U}(x), x, y') \).12

- A type-\( x \) worker, employed in a match of type \((k, y)\) with value \( \mathcal{W} \), and not having received a layoff notice, who gets an opportunity to move to a match of type \((k', y')\),

---

11 This can be seen by applying (10) at the maximum value \( \mathcal{W} = \mathcal{P}_k(x, y) \) and comparing it with (9).

12 By convention, we assign the value \( y = 0 \) to the state of unemployment.
takes that opportunity if $P_{k'}(x, y') \geq P_k(x, y) + c^E(k, k') + z$. Her state then changes from $(W, x, y, k)$ to $(P_k(x, y), x, y', k')$, and her new wage is $w_{k'}(P_k(x, y), x, y')$.

- A type-$x$ worker, employed in a match of type $(k, y)$ with value $W$, who has received a layoff notice, and who gets an opportunity to move to a match of type $(k', y')$, takes that opportunity if $P_{k'}(x, y') \geq U(x) + c^E(k, k') + z$. Her state then changes from $(W, x, y, k)$ to $(U(x), x, y', k')$, and her new wage is $w_{k'}(U(x), x, y')$.

- A type-$x$ worker employed in a match of type $(k, y)$ with value $W$ getting an opportunity to move to a match of type $(k', y')$ uses that opportunity to renegotiate her contract while staying with her incumbent employer if $P_k(x, y) > P_{k'}(x, y') - c^E(k, k') - z > W$. Her state then changes from $(W, x, y, k)$ to $(P_{k'}(x, y') - c^E(k, k') - z, x, y, k)$, and her new wage is $w_k(P_{k'}(x, y') - c^E(k, k') - z, x, y)$.

- If an opportunity arises for a worker type-$x$ employed worker in a match of type $(k, y)$ with value $W$ to interview for a job in occupation $k'$ in her own firm, and if that employee has not received prior notice that her post would be shut down in the period, the interview is successful if $P_{k'}(x, y) > P_k(x, y) + c^I(k, k') + z$. The worker’s state then changes from $(W, x, y, k)$ to $(W', x, y, k')$, with $W' = W + \beta[P_{k'}(x, y) - P_k(x, y) - c^I(k, k') - z]$, and her new wage is $w_{k'}(W', x, y, k')$.

- Finally, if an opportunity arises for a worker type-$x$ worker currently in a match of type $(k, y)$ with value $W$ who has received a layoff notice to be reassigned to occupation $k'$ in her own firm, that opportunity is pursued if $P_{k'}(x, y) > U(x) + c^I(k, k') + z$. The worker’s state then changes from $(W, x, y, k)$ to $(W', x, y, k')$, with $W' = U(x) + \beta[P_{k'}(x, y) - U(x) - c^I(k, k') - z]$, and her new wage is $w_{k'}(W', x, y, k')$.

## 4.7 Worker Flows and Equilibrium Employment Distributions

Having established the job offer acceptance rules, we now turn to their implications in terms of worker and job turnover, which determine the equilibrium distribution of workers across
firm and occupation types.

4.7.1 Steady-state Unemployment

The flow of type-$x$ unemployed job seekers with skills $k$ into vacancies of type $k'$ at firms of type $y$ is $u_k(x)s_0 \lambda^E_k(y)F \left[ \mathcal{P}_{k'}(x, y) - \mathcal{U}(x) - c^E(k, k') \right]$, as $s_0 \lambda^E_k(y)$ is the probability of meeting a vacancy of type $(k, y)$ and $F \left[ \mathcal{P}_{k'}(x, y) - \mathcal{U}(x) - c^E(k, k') \right]$ is the probability of the match being profitable. The aggregate unemployment exit rate for type-$x$ workers with skills $k$ is therefore $s_0 \sum_{k'=1}^K \lambda^E_k(y)F \left[ \mathcal{P}_{k'}(x, y) - \mathcal{U}(x) - c^E(k, k') \right]$. Conversely, the flow into unemployment of type-$x$ workers with skills $k$ out of type-$y$ firms is composed of those among the $h_k(x|y)\gamma(y)$ workers who received a layoff notice (probability $\delta$), and had either no interview on either market or had an unsuccessful one. The combined probability of those events is

$$
\sum_{k'=1}^K \lambda^I_{k'}(y) \left( 1 - F \left[ \mathcal{P}_{k'}(x, y) - \mathcal{U}(x) - c^I(k, k') \right] \right) \\
+ (1 - \lambda^I(y)) \left( 1 - s_1 \sum_{k'=1}^K \int \lambda^E_{k'}(y')F \left[ \mathcal{P}_{k'}(x, y') - \mathcal{U}(x) - c^E(k, k') \right] \ dy' \right)
$$

Integrating over all employer types, we obtain the aggregate inflow of type-$x$ workers with skills $k$. Aggregate unemployment inflow and outflows must be equal in steady state, so that:

$$
\delta \int h_k(x|y)\gamma(y) \left\{ \sum_{k'=1}^K \lambda^I_{k'}(y) \left( 1 - F \left[ \mathcal{P}_{k'}(x, y) - \mathcal{U}(x) - c^I(k, k') \right] \right) \\
+ (1 - \lambda^I(y)) \left( 1 - s_1 \sum_{k'=1}^K \int \lambda^E_{k'}(y')F \left[ \mathcal{P}_{k'}(x, y') - \mathcal{U}(x) - c^E(k, k') \right] \ dy' \right) \right\} \ dy \\
= u_k(x) \sum_{k'=1}^K s_0 \lambda^E_{k'}(y)F \left[ \mathcal{P}_{k'}(x, y) - \mathcal{U}(x) - c^E(k, k') \right] \ dy,
$$

is the flow-balance equation that determines $u_k(x)$ in steady state.
4.7.2 Steady-State Employment

The flow-balance equation defining the steady-state equilibrium distribution of match types equates the flows into and out of the stock \( h_k(x|y)\gamma(y) \) of matches \((k, x, y)\). These flows have the following components.

**Outflow by layoff.** This is the flow of workers in the stock \( h_k(x|y)\gamma(y) \) who are hit by a shock \( \delta \). They may be reassigned to another job in the internal or the external market:

\[
(1) = h_k(x|y)\gamma(y) \times \delta.
\]

Note that a reassignment to a different task classified in the same occupation category \((k' = k)\) does not add to the net outflow. However, we shall count these reassignments also in the inflow.

**Outflow by internal promotion.** These are the workers whose job is not shut down, and who are offered a reassignment to a different occupation in the same firm, which they may pass up on and keep their current job:

\[
(2) = h_k(x|y)\gamma(y) \times (1 - \delta) \sum_{k'=1}^{K} \lambda^{I}_{k'}(y)F \left[ \mathcal{P}_{k'}(x, y) - \mathcal{P}_k(x, y) - c^I(k, k') \right].
\]

**Outflow by external mobility.** These workers did not get notice of termination and were poached by a competing employer:

\[
(3) = h_k(x|y)\gamma(y) \times (1 - \delta) \left[ 1 - \lambda^I(y) \right] s_1 \sum_{k'=1}^{K} \int \lambda_{k'}^{E}(y')F \left[ \mathcal{P}_{k'}(x, y') - \mathcal{P}_k(x, y) - c^E(k, k') \right] dy'.
\]

**Inflow from unemployment.** There are two types of workers whose outside option is unemployment: workers in the stock of unemployment at the beginning of the period, and workers under layoff notice who were not reassigned to another task in their firm. Together
they generate the following inflow:

\[
(4) = \lambda_k^E (y) \sum_{k'=1}^{K} \left( s_0 u_{k'}(x) + s_1 \delta \int \left[ 1 - \lambda^I (y') \right] h_{k'}(x|y') \gamma(y') \, dy' \right) \\
\times F \left[ \mathcal{P}_k(x, y) - \mathcal{U}(x) - c^E(k', k) \right].
\]

**Inflow by poaching.** This is simply the inflow counterpart of (3):

\[
(5) = s_1 \lambda_k^E (y) \sum_{k'=1}^{K} \int \left( 1 - \delta \right) \left[ 1 - \lambda^I (y') \right] h_{k'}(x|y') \gamma(y') F \left[ \mathcal{P}_k(x, y) - \mathcal{P}_{k'}(x, y') - c^E(k', k) \right] \, dy'.
\]

**Inflow by internal reassignment.** There are two components to this flow: workers may be reassigned to another occupation following a job termination, or they may be promoted to a higher occupation within the firm:

\[
(6) = \lambda_k^I (y) \sum_{k'=1}^{K} h_{k'}(x, y) \gamma(y) \left\{ \delta F \left[ \mathcal{P}_k(x, y) - \mathcal{U}(x) - c^I(k', k) \right] \\
+ (1 - \delta) F \left[ \mathcal{P}_k(x, y) - \mathcal{P}_{k'}(x, y) - c^I(k', k) \right] \right\}.
\]

**Steady-state.** At the steady-state equilibrium, inflows must balance outflows:

\[
(1) + (2) + (3) = (4) + (5) + (6).
\]

Although this equation seems cumbersome at first, it is linear in \( h_k(x|y) \), and therefore amenable to fast numerical solving.

### 4.7.3 Within-firm flows and the distribution of worker values

A similar flow-balance equation can be obtained for the conditional distribution of contract values, \( g_k(\mathcal{W}|x, y) \), where \( g_k(\mathcal{W}|x, y) \) denotes the measure of type-\( x \) workers with value \( \mathcal{W} \) in type-(\( k, y \)) jobs. It is derived in Appendix A.
4.8 Equilibrium Definition and Simulation

We are now in a position to define a steady-state equilibrium of our model, and implement that definition for simulation and estimation.

**Definition 1.** A **steady-state equilibrium** is a tuple \( \{ P_k(x, y), w_k(W, x, y), v_k(y), u_k(x), h_k(x|y), g_k(W|x, y) \} \), where the match value function \( P_k(x, y) \) is a fixed point of the operator defined in (9), the wage function \( w_k(W, x, y) \) is defined in (10), the vacancy function \( v_k(y) \) is defined in (6), and the distribution of the worker state \( \{ u_k(x), g_k(W|x, y) h_k(x|y)\gamma(y) \} \) is jointly defined by equations (11), (12), and (13).

Based on this definition, we simulate the model’s equilibrium using the procedure described in Appendix B.

5 Application

5.1 Estimation Procedure

We estimate the model by Indirect Inference (a.k.a. Simulated Minimum Distance) in two steps. In a first step, we estimate a version of the model where the contact rates \( \lambda^I_k(y) \) and \( \lambda^E_k(y) \) are exogenous and specified as flexible functions of \( y \). This identifies the joint distribution of worker types, firm types and occupations in employment \( [h_k(x|y)] \), the distribution of worker types and occupations in unemployment \( [u_k(x)] \), the distribution of worker values \( [g_k(W|x, y)] \), the parameters governing match surpluses \( [p_k(x, y), b(x)] \) and the mobility costs \( c^I \) and \( c^E \), and the firm/occupation-specific contact rates \( [\lambda^I_k(y) \text{ and } \lambda^E_k(y)] \). In a second step, we estimate the vacancy functions \( v_k(y) \) and vacancy cost functions \( c_k(v) \) that rationalize those firm/occupation-specific contact rates given all the other parameters estimated in the first step, based on the firms’ optimal vacancy-posting condition (6).

Note that only the first step involves the direct matching of empirical moments. Indeed, our procedure is little else than a particular way of parameterizing the model: we choose to
specify $\lambda^I_k(y)$ and $\lambda^E_k(y)$ as flexible functions of $y$, then determine the vacancy cost functions that rationalize those contact rates given the model, rather than choosing a functional form for the vacancy cost function and using the implied contact rates in the estimation. The advantage of this procedure is that it obviates the need to solve for the equilibrium distribution of vacancies at each call of the moment-generating function during estimation, which greatly improves computation speed and stability.\footnote{The requirement to solve for the equilibrium $v_k(y)$ given model parameters turns the simulation algorithm into a high-dimensional, time-consuming fixed-point problem, as described in Appendix B, whereas solving for the model’s equilibrium for given $\lambda^I_k(y)$ and $\lambda^E_k(y)$ can be done directly and takes very little time.}

### 5.2 Empirical Specification and Targeted Moments

We use the parametric forms listed in Table 4. All specifications are largely straightforward, except maybe for the occupation/firm-specific contact rates $\lambda^X_k(y)$, which are constructed as fractions of the corresponding firm-level contact rate $\lambda^X(y)$ using a multinomial logit-like specification to ensure that $\sum_k \lambda^X_k(y) = \lambda^X(y)$. We should further point out that, as per our initial assumption, $p_k(x, y)$ is increasing in $k$ for all $(x, y)$, but is not necessarily monotonic in $x$ or $y$ and allows for a potentially strong level of complementarity between $x$ and $y$ (which would translate as a large, negative coefficient $a_{p,3}$).

Further note that we do not specify the distribution of firm types among firms, $\gamma(y)$, as the only objects that we can identify are the firm type sampling weights, $v_k(y)\gamma(y)$. We only normalize the support of $y$ to $[y_{\text{min}}, y_{\text{max}}] = [1, 2]$.

The parameterization in Table 4 has 38 parameters, in addition to which we also estimate $s_1$ (the relative search intensity of employed workers - $s_0$ being normalized to one) and $\delta$ (the job destruction rate). This leaves us with 40 scalar parameters to estimate. We fix the discount rate to the monthly equivalent of 10 percent p.a., and the bargaining power of workers on internal labor markets to $\beta = 0.2$. We further specify the distribution of conversion cost shocks $F(\cdot)$ as log-normal with mean (of the underlying normal distribution) $-1$ and variance 0.1. Finally, we set the total number of occupations to $K = 4$, reflecting
Distribution of worker types:

\[ \ell(x) = \frac{(1 - \alpha x) \cdot x^{-\alpha x}}{2^{1-\alpha x} - 1}, \quad x \in [1, 2] \]

Production function:

\[ p_k(x, y) = a^{(k)}_{p,0} \cdot \left[ 1 + a_{p,1}x + a_{p,2}y + a_{p,3} (y - x)^2 \right] \]

Unemployment income:

\[ b(x) = a_{b,0} + a_{b,1}x + a_{b,2}x^2 \]

Firm-level contact rates (\( X = I \) or \( E \)):

\[ \lambda^X(y) = (1 + \exp \left( a^X_{\lambda,0} + a^X_{\lambda,1}y + a^X_{\lambda,2}y^2 \right))^{-1} \]

Occupation-level contact rates (\( X = I \) or \( E \)):

\[ \lambda^X_{k}(y) = \frac{\exp(\sum_{k'} a^{(X, k')}_{\lambda,0} + a^{(X, k')}_{\lambda,1}y)}{\sum_{k'} \exp(\sum_{k''} a^{(X, k'')}_{\lambda,0} + a^{(X, k'')}_{\lambda,1}y)} \cdot \lambda^X(y) \]

with \( a^{(X, K)}_{\lambda,0} = a^{(X, K)}_{\lambda,1} = 0 \).

Conversion costs (\( X = I \) or \( E \)):

\[ c^X(k, k') = a_{c,up}^{X,k} \cdot \max(k' - k, 0)^2 + a_{c,down}^{X,k} \cdot \min(k' - k, 0)^2 \]

---

Table 4: Model Parameterization
the broad occupation types identified in the data (see Section 3.1).

Using this parameterization, we simulate panels of workers over 12 months, and take the observations from months 1 and 12 as model counterparts for the two observations we have in the data for years 2006 and 2007 (see Section 3.1). We then estimate our 33 parameters by matching the following moments: (i) employment share of each occupation (3 moments), (ii) mean frequency of mobility of each type (in-up, in-down, out-up, out-same, out-down), for each initial occupation (5 moments times four initial occupations equals 20 moments), (iii) coefficients of a regression of wage growth on a constant, the type of mobility, the shares of high-skill (occupation-K) and low-skill (occupation-1) workers in the firm of origin and the destination firm, for each initial occupation (10 moments for each of the four occupations), (iv) coefficients of a regression of wage growth on a constant, the type of mobility, log-output per worker in the firm of origin and the destination firm,\(^{14}\) for each initial occupation (8 moments for each of the four occupations), (v) variance of wage growth, for each initial occupation (4 moments), (vi) mean and variance of log output per worker, for each occupation (8 moments), (vii) for each initial occupation, coefficients of five linear probability models explaining the probability of each of the five types of transition as a function of a constant and the shares of high- and low-skill workers in the origin and destination firms (5 coefficients times five types of transition times four initial occupations equals 100 additional moments). The distance between the empirical and model-based moments is minimized using a combination of direct (pattern) search and gradient-based optimization algorithms.

\(^{14}\)Constructed from the model as \(\sum_k \int p_k(x,y)h_k(x|y) \, dx / \sum_k \int h_k(x|y) \, dx\).
A Worker Value Distributions

In light of the updating rules for wages (see Subsection 4.4), the steady-state distribution of the worker state \((W, x, y, k)\) solves the following flow-balance equation,

\[
(1) = (2) + (3) + (4) + (5) + (6),
\]

where the different components of this steady-state flow-balance equation are defined below, using \(g_k(W|x, y)\) to denote the measure of type-\(x\) workers with value \(W\) in type-(\(k, y\)) jobs.

- Outflow of worker stock \(g_k(W|x, y) h_k(x|y) \gamma(y)\):

\[
(1) = g_k(W|x, y) h_k(x|y) \gamma(y) \times \left\{ \delta + (1-\delta) \sum_{k'=1}^{K} \lambda_{k'}^I(y) F \left[ P_{k'}(x, y) - P_{k}(x, y) - c'(k, k') \right] 
+ s_1 (1-\delta) \left[ 1 - \lambda^I(y) \right] \sum_{k'=1}^{K} \lambda_{k'}^E(y') F \left[ P_{k'}(x, y') - c^E(k, k') - W \right] \right\}.
\]

- Inflow from unemployment and after layoff:

\[
(2) = \lambda_{k}^E(y) F \left[ P_k(x, y) - \mathcal{U}(x) - c^E(k', k) \right] \times \sum_{k'=1}^{K} \left( s_0 u_{k'}(x) \times \mathbf{1}\{W = \mathcal{U}(x)\} \right)
+ s_1 \int \delta \left[ 1 - \lambda^I(y') \right] h_{k'}(x|y') \gamma(y') \mathbf{1}\{P_{k'}(x, y') = \mathcal{U}(x)\} \left\{ W \right\} \lambda_{k'}^E(y') \mathbf{1}\{P_{k'}(x, y') = W\} dy'.
\]

where \(\mathbf{1}\{\text{cond}\}\) is equal to 1 if the condition in the brackets is true, and is 0 otherwise.

- Inflow by poaching:

\[
(3) = s_1 \lambda_{k}^E(y) \sum_{k'=1}^{K} (1-\delta) \left[ 1 - \lambda^I(y') \right] h_{k'}(x|y') \gamma(y')
\times F \left[ P_k(x, y) - P_{k'}(x, y') - c^E(k', k) \right] \mathbf{1}\{P_{k'}(x, y') = W\} \lambda_{k'}^E(y') \mathbf{1}\{P_{k'}(x, y') = W\} dy'.
\]
• Inflow by wage renegotiation to counter outside offers:

\[
(4) = G_k \left( W^\prime | x, y \right) h_k(x|y) \gamma(y) \\
\times (1 - \delta)(1 - \lambda^I(y)) s_1 \sum_{k'=1}^K \lambda_{k'}^E(y') F' \left[ P_{k'}(x, y') - c^E(k, k') - W \right] dy'.
\]

• Inflow by internal promotion:

\[
(5) = (1 - \delta) \lambda^I(y) \sum_{k'=1}^K g_{k'} (W - \beta \left[ P_k(x, y) - P_{k'}(x, y) - c^I(k', k) - z \right] | x, y) \\
\times h_{k'}(x, y) \gamma(y) 1 \left\{ P_k(x, y) - P_{k'}(x, y) - c^I(k', k) - z \geq 0 \right\} dF(z),
\]

where the condition \( P_k(x, y) - P_{k'}(x, y) - c^I(k', k) - z \geq 0 \) is about \( k' \) and \( z \). It is equivalent to

\[
P_k(x, y) - P_{k'}(x, y) - c^I(k', k) \geq 0 \text{ and } 0 \leq z \leq P_k(x, y) - P_{k'}(x, y) - c^I(k', k).
\]

• Inflow by internal demotion:

\[
(6) = \delta \lambda^I(y) \sum_{k'=1}^K \frac{1}{\beta} f \left( P_k(x, y) - U(x) - c^I(k', k) - \frac{W - U(x)}{\beta} \right) h_{k'}(x, y) \gamma(y),
\]

where \( f = F' \).

Integration w.r.t \( W \) yields, after rearranging, the flow-balance equation for the integrated stock \( G_k(W|x, y) h_k(x|y) \gamma(y) \) of workers of type \( x \) with value less than \( W \) in firms of type \( y \):

\[
(1) = (2) + (3) + (4) + (5),
\]

(14)
• Outflow of worker stock $G_k(W|x, y) h_k(x|y) \gamma(y)$:

$$(1) = G_k(W|x, y) h_k(x|y) \gamma(y) \left\{ \delta + (1 - \delta) \sum_{k'=1}^{K} \lambda_k'(y) F \left[ P_{k'}(x, y) - P_k(x, y) - c^I(k', k) \right] + s_1(1 - \delta) \left[ 1 - \lambda^I(y) \right] \sum_{k'=1}^{K} \lambda_k'(y) F \left[ P_{k'}(x, y) - c^E(k', k) - W \right] dy' \right\}. $$

• Inflow from unemployment or layoff:

$$(2) = s_0 \lambda_k^E(y) F \left[ P_k(x, y) - U(x) - c^E(k', k) \right] \times \sum_{k'=1}^{K} \left( s_0 u_{k'}(x) + s_1 \int \delta \left[ 1 - \lambda^I(y') \right] h_{k'}(x|y') \gamma(y') dy' \right). $$

• Inflow by poaching:

$$(3) = s_1 \lambda_k^E(y) \sum_{k'=1}^{K} \int (1 - \delta) \left[ 1 - \lambda^I(y') \right] h_{k'}(x|y') \gamma(y') \times F \left[ P_k(x, y) - P_{k'}(x, y) - c^E(k', k) \right] \mathbf{1} \{ U(x) \leq P_{k'}(x, y') \leq W \} \{ U(x) \leq P_{k'}(x, y') \leq W \} dy'. $$

• Inflow by internal promotion:

$$(4) = (1 - \delta) \lambda_k^I(y) \sum_{k'=1}^{K} \int G_{k'} \left( \mathcal{W} - \beta \left[ P_k(x, y) - P_{k'}(x, y) - c^I(k', k) - z \right] \right) |x, y \rangle \times h_{k'}(x|y') \gamma(y') \mathbf{1} \{ P_k(x, y) - P_{k'}(x, y) - c^I(k', k) - z \geq 0 \} dF(z). $$

• Inflow by internal demotion:

$$(5) = \delta \lambda_k^I(y) \sum_{k'=1}^{K} \left[ F \left( P_k(x, y) - U(x) - c^I(k', k) \right) - F \left( P_k(x, y) - U(x) - c^I(k', k) - \frac{W - U(x)}{\beta} \right) \right] h_{k'}(x|y) \gamma(y). $$
This flow-balance equation, when applied at the maximum worker surplus, $\mathcal{W} = \mathcal{P}_k(x, y)$, yields (12).

**B The Simulation Algorithm**

We simulate the model’s equilibrium using an iterative procedure that can be outlined as follows. In each outer-iteration, for a given guess of the vacancy distribution $v_k(y)$, we iterate the surplus operator (9) until convergence to update $\mathcal{P}_k(x, y)$, then solve the linear steady-state flow equations (11) and (12) for $u_k(x)$ and $h_k(x|y)$. Finally, we update $v_k(y)$ by solving the nonlinear optimality condition for vacancies, (6). After convergence, we compute the steady-state distribution of worker values, $g_k(\mathcal{W}|x, y)$, by solving (13).\(^\text{15}\) All functions and integrals are approximated using Chebyshev nodes and Clenshaw-Curtis quadrature.

**C Additional Tables**

\(^\text{15}\)More precisely, it is easier to solve for the conditional cdf $G_k(\mathcal{W}|x, y)$, then obtain the corresponding conditional density by numerical differentiation. See the details of the expression of $G_k(\mathcal{W}|x, y)$ in Appendix A.
### Table 5: Job and occupational mobility - Demographic effects

<table>
<thead>
<tr>
<th>Category</th>
<th>Out, down</th>
<th>Out, up</th>
<th>Out, same</th>
<th>In, down</th>
<th>In, up</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unskilled industrial and service worker</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>-0.076</td>
<td>-0.053</td>
<td></td>
<td>-0.026</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>0.399</td>
<td>0.233</td>
<td></td>
<td>0.256</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.015)</td>
<td></td>
<td>(0.009)</td>
<td></td>
</tr>
<tr>
<td><strong>Skilled industrial and service worker</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>-0.096</td>
<td>-0.102</td>
<td>-0.070</td>
<td>-0.018</td>
<td>-0.027</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Male</td>
<td>0.664</td>
<td>-0.022</td>
<td>0.371</td>
<td>0.104</td>
<td>-0.226</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.015)</td>
<td>(0.009)</td>
<td>(0.012)</td>
<td>(0.007)</td>
</tr>
<tr>
<td><strong>Technician, foreman, assistant manager and sale worker</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>-0.107</td>
<td>-0.084</td>
<td>-0.083</td>
<td>-0.028</td>
<td>-0.016</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Male</td>
<td>-0.144</td>
<td>0.253</td>
<td>0.168</td>
<td>-0.249</td>
<td>0.089</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.021)</td>
<td>(0.012)</td>
<td>(0.009)</td>
<td>(0.010)</td>
</tr>
<tr>
<td><strong>Administrative and technical manager</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>-0.093</td>
<td>-0.075</td>
<td>-0.032</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>-0.218</td>
<td>0.192</td>
<td>-0.431</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.011)</td>
<td>(0.011)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: (1) “In, same” (internal market - no change of occupation category) is the reference destination category. (2) This table contains the results of three different multinomial logit estimations with different firm characteristics. All estimations contains intercepts and controls for firm effects.
Table 6: Log-Wage Regressions - Industrial and service workers
<table>
<thead>
<tr>
<th></th>
<th>Middle level</th>
<th>Top level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.0852</td>
<td>0.0872</td>
</tr>
<tr>
<td></td>
<td>(0.0009)</td>
<td>(0.0012)</td>
</tr>
<tr>
<td>Log(sales/worker) (t – 1)</td>
<td>0.0011</td>
<td>0.0029</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>Change in Log(sales/worker)</td>
<td>0.0289</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td></td>
</tr>
<tr>
<td>Log(size) (t – 1)</td>
<td>-0.0020</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td>Change in log(size)</td>
<td>0.0005</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td>Share High Skilled (t – 1)</td>
<td>0.0043</td>
<td>0.0173</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0005)</td>
</tr>
<tr>
<td>Change in Share High Skilled</td>
<td>0.0184</td>
<td>-0.0289</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0012)</td>
</tr>
<tr>
<td>Share Low Skilled (t – 1)</td>
<td>-0.0134</td>
<td>0.0447</td>
</tr>
<tr>
<td></td>
<td>(0.0008)</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>Change in Share Low Skilled</td>
<td>-0.0366</td>
<td>-0.0779</td>
</tr>
<tr>
<td></td>
<td>(0.0009)</td>
<td>(0.0013)</td>
</tr>
<tr>
<td>Age</td>
<td>-0.0013</td>
<td>-0.0013</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Male</td>
<td>0.0063</td>
<td>0.0060</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>External Demotion</td>
<td>-0.0645</td>
<td>-0.1153</td>
</tr>
<tr>
<td></td>
<td>(0.0012)</td>
<td>(0.0019)</td>
</tr>
<tr>
<td>External Promotion</td>
<td>0.0777</td>
<td>0.0800</td>
</tr>
<tr>
<td></td>
<td>(0.0013)</td>
<td>(0.0013)</td>
</tr>
<tr>
<td>External Stability</td>
<td>-0.0083</td>
<td>-0.0161</td>
</tr>
<tr>
<td></td>
<td>(0.0008)</td>
<td>(0.0008)</td>
</tr>
<tr>
<td>Internal Demotion</td>
<td>-0.0877</td>
<td>-0.2067</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.0008)</td>
</tr>
<tr>
<td>Internal Promotion</td>
<td>0.0523</td>
<td>0.0520</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>Internal Stability</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>R-square</td>
<td>0.0387</td>
<td>0.0756</td>
</tr>
<tr>
<td></td>
<td>0.0348</td>
<td>0.0674</td>
</tr>
<tr>
<td>Sample Size</td>
<td>1,313,925</td>
<td>962,781</td>
</tr>
<tr>
<td></td>
<td>1,315,913</td>
<td>965,858</td>
</tr>
<tr>
<td></td>
<td>1,334,854</td>
<td>972,918</td>
</tr>
</tbody>
</table>

Table 7: Log-Wage Regressions - Engineers, professionals and executives