Impatience vs. Incentives

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July 2013
Motivation

- **Incentives**: Self-enforcing contracts between the principal and the agent play an important role because contract enforcement may either be too costly or even impossible (Levin, 2003)
  - expropriation risk / sovereign debt
  - worker relationships

- **Impatience**: Recent applied papers make assumption of relative impatience on the side of the agent (Acemoglu et al. (2008, 2010), Aguiar, Amador, Gopinath (2009), DeMarzo Fishman (2007), and many others)
  - sometimes motivated by underlying economic situation:
    - "Politicians are often argued to be more short-sighted than the agents, and the impact of political economy on intertemporal distortions is one of the questions motivating our analysis" (Acemoglu et al., 2008)
  - sometimes purely an auxiliary assumption
Differential Discounting and Dynamic Agency

- Two forces:
  - Differential discounting creates dynamic trading gains: impatient player should receive frontloaded rewards
  - Incentive provision: Agent payoffs should be backloaded

- Strong conflict (impatient agent, both forces matter) implies
  - Oscillation is a generic feature of optimal contracts
    - even in the absence of exogenous uncertainty
    - non-convergence to unique steady if oscillation is explosive
  - Efficiency of deviations from static optimum investment

- Weak conflict (patient agent, or one force dominates)
  - Convergence to steady state is ensured
  - Lehrer-Pauzner (1999) and Ray (2002) can be understood as different limits
Related Literature

- Incentive provision via backloading to agent (homogeneous discounting)

- Repeated games with *heterogeneous* discounting:
  - Lehrer Pauzner (1999) Dynamic trading gains should lead to frontloaded rewards (in the limit as both discount factors approach one)

- We study an environment where both incentives and impatience matter: Ray (2002) and Lehrer Pauzner (1999) can be understood as limiting cases.
  - If both forces are aligned, i.e., principal impatient, then backloading to the agent (see also Acemoglu et al., 2008)
  - Tension between *Incentives* (Ray 2002) and *Impatience* (Lehrer Pauzner, 1999) if agent is made more impatient
Outline

- Strong Impatience / Incentives conflict: transferable utility framework
  - Agent is strictly impatient
  - Neither impatience or incentives dominates
- Weak Impatience / Incentives conflict: general framework
Setup: Preferences

- Infinite horizon repeated interaction between principal $P$ and agent $A$
  - discrete time, perfect public information (as in Ray, 2002)
  - no exogenous uncertainty
- Transferable utility framework with $\delta_A < \delta_P$

$$U_{i,t} = \sum_{s=t}^{\infty} \delta_i^{s-t} u_{i,s} = u_i(e_t, m_t) + \delta_i U_{i,t+1} \quad \text{where}$$

$$u_A = r_A(e) + m$$

$$u_P = r_P(e) - m$$

- Optimum static action $e^*$ satisfies $r'_A(e) + r'_P(e) = 0$
IC and PC constraints

- Self-enforcing contracts represent subgame perfect equilibria
- Players’ participation constraints are exogenous

\[ U_{i,t} \geq O_i \]

- Only agent faces an incentive constraint:

\[
\delta_A (U_{A,t+1} - O_A) \geq d(e_t) - (1 - \theta) m_t
\]

Net Future Deviation Loss \[\geq\] Net Deviation Payoff today

with \( d' \geq 0 \) and \( 0 \leq \theta < 1 \) governs the insensitivity of the incentive constraint to monetary transfers
Example

**Expropriation:** In Thomas and Worrall (1994), the government (A) allows a multinational firm (P) to invest \( e \) in the country and generate returns \( Y(e) \). In exchange, the multinational firm pays the government taxes \( \tau \). The government can deviate by expropriating the return \( Y(e) \) but then forfeits tax income. Then:

\[
\begin{align*}
    r_A(e) & = 0 \\
    r_P(e) & = Y(e) - e \\
    m & = \tau \\
    D(e, \tau) & = Y(e) - \tau \\
    \theta & = 0
\end{align*}
\]

- Related papers: Acemoglu et al. (2008), Aguir et al. (2009) and Opp (2012)
Example

**Shirking:** In a simplified version of Thomas and Worrall (1988), a worker (A) exerts costly effort $c(e)$ to produce $e$ for the owner (P). In return, the worker receives an upfront wage $w$. The worker can deviate by shirking and stealing the wage. Then:

$$r_A(e) = -c(e)$$
$$r_P(e) = e$$
$$m = w$$
$$D(e, w) = c(e)$$
$$\theta = 1$$
Example

**Lending and Investment:** In a simplified version of Albuquerque Hopenhayn (2004), an entrepreneur \((A)\) borrows money \(l\) from a lender \((P)\) and invests in projects that generate \(F(l)\), where \(r\) is the riskless rate. In return, the lender receives a loan repayment \(R\). The entrepreneur can deviate by not repaying the loan. Then:

\[
\begin{align*}
    r_A(l) &= F(l) \\
    r_P(l) &= -(1 + r)l \\
    m &= -R, \\
    D(l, R) &= R \\
    \theta &= 0
\end{align*}
\]
Goal: Characterize all Pareto optimal infinite sequences of \( \{(e_t, m_t)\}_{t=0}^{\infty} \) subject to incentive compatibility and participation constraints

\[
\delta_A U_{A,t+1} \geq d(e_t) - (1 - \theta) m_t + \delta_A O_A \forall t \\
U_{A,t} \geq O_A \forall t \\
U_{P,t} \geq O_P \forall t
\]

Equilibrium payoffs within relationship:

\[
U_{P,t} = \sum_{s=t}^{\infty} \delta_{P}^{s-t} (r_P(e_s) - m_s) \\
U_{A,t} = \sum_{s=t}^{\infty} \delta_{A}^{s-t} (r_A(e_s) + m_s)
\]
Recursive Formulation

\[ U_P (U_A) = \max_{e,m,U_A^+} r_P (e) - m + \delta_P U_P (U_A^+) \quad \text{s.t.} \]

\begin{align*}
\text{#} & \quad \text{Constraint} & \quad \text{Lagrange} \\
1) & \quad r_A (e) + m + \delta_A U_A^+ = U_A & \lambda_{PK} \\
2) & \quad m (1 - \theta) - d (e) + \delta_A U_A^+ \geq \delta_A O_A & \lambda_{IC} \\
3) & \quad U_i^+ \geq O_i & \delta_i \lambda_{Oi}
\end{align*}

- Assume that \( O_A \) and \( O_P \) are sufficiently weak such that \( e^* \) can satisfy \( IC \) in the long run. Define \( m^* = r_P (e^*) - O_P \), then this implies

\[
m^*(1 - \theta) - d (e^*) + \delta_A \frac{r_A (e^*) + m^*}{1 - \delta_A} \geq \delta_A O_A
\]
Analysis

Problem

Preview

Agent value $U_A$
Principal value $U_P$

$V_F(U_A)$

Pareto frontier $V(U_A)$

Inefficient part

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Benchmark: Homogeneous Discounting

Assuming that $O_P$ is sufficiently low, we obtain

**Solution**

*Under homogeneous discounting, any efficient self-enforcing contract is backloaded and can be implemented as follows*

a) *In the first period* $e_1 \leq e^*$ and $m_1 \leq m^*$

b) *From the second period onwards, $e = e^*$ and $m = m^* = r_P(e^*) - O_P$*

- **Features:**
  - The stationary allocation features efficient investment.
  - There is a continuum of efficient stationary contracts
  - The slope of the Pareto frontier in the efficient region is 1
First-order Conditions and Envelope Condition

\[ e : r_P' (e) = -\lambda_{PK} r_A' (e) + \lambda_{IC} d' (e) \]  
\[ m : 1 = \lambda_{PK} + \lambda_{IC} (1 - \theta) \]  
\[ U_A^+ : -\frac{\delta_A}{\delta P} \frac{\lambda_{PK} + \lambda_{IC} + \lambda_{OA}}{1 + \lambda_{OP}} = U_P' (U_A^+) \]  
\[ EC : -\lambda_{PK} = U_P' (U_A) \]

Using Eq. 1 and 2, effort distortions can be linked to slope \( \lambda_{PK} \)

\[ \frac{r_P' (e) + r_A' (e)}{r_A' (e) + \frac{d'(e)}{1-\theta}} = 1 - \lambda_{PK} \]  

Hence: \( \lambda_{PK} = 1 \Rightarrow e = e^* \) and \( \lambda_{PK} < 1 \Rightarrow e < e^* \)
Steady State

Lemma

There exists a unique steady state with $\bar{e} < e^*$.

$$\bar{\lambda}_{PK} = \frac{\delta_A}{(1 - \theta) \delta_P + \theta \delta_A} < 1$$

$\Leftrightarrow$ The incentive constraint binds in the steady state

Proof.

Set $\lambda_{OP} = \lambda_{OA} = 0$, then FOC (3) and envelope condition imply that slope follows oscillating first-order difference equation. In the steady state $\lambda_{PK} = \lambda^+_{PK} = \bar{\lambda}_{PK}$.

$$\lambda^+_{PK} = \frac{\delta_A}{\delta_P} \frac{1}{1 - \theta} - \frac{\delta_A}{\delta_P} \frac{\theta}{1 - \theta} \lambda_{PK}$$
Steady State Distortions

- Why does relative impatience of the agent cause long run distortions?
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- Tightened incentive constraint?
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- Recall $\bar{\lambda}_{PK} = \frac{\delta_A}{(1-\theta)\delta_P + \theta\delta_A}$ and consider logical thought experiment.
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  - Tightened incentive constraint?
  - Dynamic trading gains?

- Recall $\bar{\lambda}_{PK} = \frac{\delta_A}{(1-\theta)\delta_P + \theta \delta_A}$ and consider logical thought experiment
  - Start with $\delta_P = \delta_A \Rightarrow \bar{\lambda}_{PK} = 1$ and $\bar{e} = e^*$

Expropriation risk example:

$Y'(\bar{I}) = \delta_P \delta_A > 1$:
Marginal benefits from trading across time $\delta_P \delta_A$ equal marginal benefit from increasing stationary investment.
Steady State Distortions

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  - Tightened incentive constraint?
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  - Start with $\delta_p = \delta_A \Rightarrow \bar{\lambda}_{PK} = 1$ and $\bar{e} = e^*$
  - Increase $\delta_p$ above $\delta_A$
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★ original location still incentive compatible, BUT .....
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    - $\bar{\lambda}_{PK} < 1 \Rightarrow \bar{e} < e^*$
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    - $\bar{\lambda}_{PK} < 1 \Rightarrow \bar{e} < e^*$
  - Dynamic trading gains make it efficient to move away from $e^*$
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- Expropriation risk example: $Y' (\bar{I}) = \frac{\delta_P}{\delta_A} > 1$:
  Marginal benefits from trading across time $\frac{\delta_P}{\delta_A}$ equal marginal benefit from increasing stationary investment
Dynamics

- Steady state effort $\bar{e}$ captures the constrained efficient trade-off between incentives and impatience.
- If sustainable, it is clearly efficient to mandate $\bar{e}$ also along any non-stationary contract.
  $\Rightarrow$ linear frontier with slope $\bar{\lambda}_{PK}$

**Lemma**

Let $\theta \in (0, 1)$ and $\rho := \frac{1}{\delta_A} \frac{1-\theta}{\theta}$. Let $\{(\bar{m}, \bar{e})\}$ be the steady state with payoff $(\bar{U}_A, \bar{U}_P)$. For every $m \in \mathbb{R}$, the sequence $\{(\bar{m}^s + (-\rho)^t m, \bar{e})\}_{t=0}^{\infty}$ satisfies the IC-constraint at every date.

- Consider the relaxed problem without participation constraints, then these action sequences define the linear Pareto frontier.
Dynamics

- Continuation value evolves according to oscillating first-order difference equation (with $\rho = \frac{1}{\delta_A \frac{1-\theta}{\theta}}$)

$$U^+_A = d(e) + r_A(e)(1 - \theta) + \delta_A O_A \frac{\delta_A \theta}{\delta_A \theta} - \rho U_A$$
Dynamics

- Continuation value evolves according to oscillating first-order difference equation (with $\rho = \frac{1}{\delta_A} \frac{1-\theta}{\theta}$)

$$U_A^+ = \frac{d(\bar{e}) + r_A(\bar{e})(1-\theta) + \delta_A O_A}{\delta_A \theta} - \rho U_A$$

- Does this sequence of monetary transfers also satisfy participation constraints? Depends on $\rho$
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  - If $\rho < 1 \Rightarrow$ damped oscillation: Participation constraints do not (essentially) matter, convergence
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  - If $\rho = 1 \Rightarrow$ perpetual oscillation between two points, no convergence
  - If $\rho > 1 \Rightarrow$ explosive oscillation: arbitrarily low participation constraints will be violated

⇒ effort cannot be held constant
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  - If $\rho > 1 \Rightarrow$ explosive oscillation: arbitrarily low participation constraints will be violated $\Rightarrow$ effort cannot be held constant
Dynamics Case 1:

- If $\rho < 1$, i.e., $\theta > \frac{1}{1 + \delta_A}$, the value function is linear (with slope $\bar{\lambda}_{PK}$)
  - Participation constraints will not matter in the neighborhood of the steady state
  - Except for corner case $\theta = 1$, i.e., $\rho = 0$, monetary transfers damped oscillate

- Tension between impatience and incentives
  - $U_{A,t} > \bar{U}_A \Rightarrow$ Impatience dominates $\Rightarrow$ high $m_t$, low $U_{A,t+1}$
  - $U_{A,t} < \bar{U}_A \Rightarrow$ Incentives dominates $\Rightarrow$ low $m_t$, high $U_{A,t+1}$
Dynamics Case 1

**Case 1**: $\rho < 1 \iff \theta > (1 + \delta_A)^{-1}$

- Static Payoffs
- Pareto Frontier
- $\tilde{U}$ Steady State

**Graph**:
- **Actions**: $e^*$
- **Monetary Transfers**
- **Time**

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If $\rho > 1$, i.e., $\theta < \frac{1}{1+\delta_A}$, Pareto Frontier no longer linear.

Two first-order difference equations simultaneously determine dynamics (until PC constraints bind)

$$U^+_A = f(e(\lambda_{PK})) - \rho U_A \quad \text{and} \quad \lambda^+_{PK} = \frac{\delta_A}{\delta_P} \frac{1}{1-\theta} - \frac{1}{\delta_P \rho} \lambda_{PK}$$

$\Rightarrow$ Comovement of effort $e(\lambda_{PK})$ and monetary transfers implied by:

$$\frac{r'_P(e) + r'_A(e)}{r'_A(e) + \frac{d'(e)}{1-\theta}} = 1 - \lambda_{PK}$$

Distinguish two subcases for $\rho > 1$

- If $1 < \rho < \frac{1}{\delta_P}$ both difference equations are explosive
- If $\rho > \frac{1}{\delta_P}$, the slope $\lambda_{PK}$, i.e., effort, converges to the steady state
Dynamics Case 2:

- If $1 < \rho < \frac{1}{\delta_P}$, i.e., $\frac{\delta_P}{\delta_P + \delta_A} < \theta < \frac{1}{1 + \delta_A}$, continuation values and slopes explosively oscillate until participation constraints are hit.

$$U_A^+ = k(\lambda_{PK}) - \rho U_A \text{ and } \lambda_{PK}^+ = \kappa - \frac{1}{\delta_P \rho} \lambda_{PK}$$

$\Rightarrow$ Arbitrarily weak participation constraints bind in the long run.

- In the long run, oscillation between two unique allocations.
Analysis
Dynamics

Dynamics Case 2

Case 2: $1 < \rho < \delta_P^{-1} \iff \delta_P(\delta_P + \delta_A)^{-1} < \theta < (1 + \delta_A)^{-1}$
Dynamics Case 3

- If $\rho > \frac{1}{\delta_P}$, i.e., $\theta < \frac{\delta_P}{\delta_P + \delta_A}$, slopes damped oscillate

$$\lambda_{PK,t+1} = \kappa - \frac{1}{\delta_P \rho} \lambda_{PK,t} \text{ and } U_{A,t+1} = k (\lambda_{PK,t}) - \rho U_{A,t}$$

(6)

- Convergence to steady state.
  - Speed of convergence inversely related to $\rho$.
  - Convergence is monotone when $\theta = 0$.

- Subtle role of participation constraints:
  - Participation constraints are never reached in the optimal contract
  - But, if participation constraints were removed $\Rightarrow$ explosive oscillation
Dynamics Case 3

**Case 3:** $\rho > \delta_P^{-1} \iff \theta < \delta_P (\delta_P + \delta_A)^{-1}$

- Static Payoffs
- Pareto Frontier
- $\bar{U}$ Steady State
- $U^*$
- $U_{\text{max}}$

**Graph:***
- Y-axis: $U_P(1 - \delta_P)$
- X-axis: $U_A(1 - \delta_A)$

**Legend:**
- *: Monetary Transfers
- X: Effort

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Summary of Strong Impatience / Incentives Conflict

- Optimality of oscillation contracts when $\theta \in (0, 1)$
  - Incentives dominates when $U_{A,t} < \bar{U}_A$
  - Impatience dominates when $U_{A,t} > \bar{U}_A$
- Comovement of effort and transfers when $\theta < \frac{1}{1+\delta_A}$, otherwise just oscillation in transfers
- Convergence to distortionary steady state unless $\theta \in \left(\frac{\delta_P}{\delta_P+\delta_A}, \frac{1}{1+\delta_A}\right)$
- Steady state trades off dynamic trading gains with effort distortions
Weak Impatience / Incentives Conflict

- Convex, compact action space $\Omega$ with continuous utility function $u_i$
- Continuous net deviation function $D : \Omega \rightarrow \mathbb{R}$, so that self-enforcing contract is defined by:

$$
\delta_A \left( U_{A,t+1} - O_A \right) \geq D \forall t
$$

$$
U_i, t \geq O_i \forall t
$$

- $D$ is semi-convex over $(\Omega, u_A, u_P)$ if for any $a_1, a_2 \in \Omega$ and $\lambda \in [0, 1]$ there exists an $a_{\lambda}$ such that

$$
u_i (a_{\lambda}) \geq \lambda u_i (a_1) + (1 - \lambda) u_i (a_2) \text{ for } i = A, P \text{ and }$$

$$D (a_{\lambda}) \leq \lambda D (a_1) + (1 - \lambda) D (a_2) .$$

- Semi-Convexity implies that Pareto frontier $V (U_A)$ is weakly concave with compact domain.
General Results

- Lemma 6: There exists at least one stationary $V$-contract. It is unique if $D$ is strictly semi-convex.

- Proposition 2: If the only steady state is $V_R$, the right most payoff of $V$, then every $V$-contract converges monotonically to the steady state (backloading to the agent). A mirror result holds if $V_R$ is replaced with $V_L$ (frontloading to the agent).
  $\Rightarrow$ Ray and Lehrer Pauzner comparison in a more general setting...

- Proposition 3: If $\delta_A \geq \delta_P$, then every $V$-payoff can be supported by a convergent contract.
  $\Rightarrow$ Discontinuity when moving from $\delta_A = \delta_P$ to $\delta_A = \delta_P - \varepsilon$. 

Corollary

**Ray Limit** *(Incentives dominates, $\bar{V} = V_R$):* In the transferable utility model, there exists some threshold level $\tilde{O}_A$ such that for $O_A > \tilde{O}_A$ any efficient self-enforcing contract is backloaded.

Corollary

**Lehrer-Pauzner Limit** *(Impatience dominates, $\bar{V} = V_L$):* In the transferable utility model, fix $r = \frac{1-\delta_P}{1-\delta_A}$ and consider the limit as $\delta_P$ approaches 1. Any self-efficient contract provides frontloaded payoffs to the agent and features efficient investment $e^*$. 

Ray vs. Lehrer-Pauzner

Ray Limit

Agent Payoff $U_A(1 - \delta_A)$

Principal Payoff $U_P(1 - \delta_P)$

Lehrer-Pauzner Limit

Agent Payoff $U_A(1 - \delta_A)$

Principal Payoff $U_P(1 - \delta_P)$

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Positive (negative) Implications

- Efficient contracts try to tap “dynamic trading gains” as well as static relationship gains (investment).

- When the conflict “impatience vs. incentives” is strong:
  - Oscillation is a natural feature of efficient contracts
  - Subtle role of participation constraints

- When the conflict “impatience vs. incentives” is weak, efficient contracts are always well-behaved.

- How to interpret our results?
  - Is impatience motivated by the underlying economic environment?
    ⇒ positive prediction about real-life phenomena
  - Or just an auxiliary assumption?
    ⇒ negative result: strong side-effects
Example

**Lending and Investment (2):** In Fuchs/Green a NGO/multinational company supplies a quantity $q$ of a product to a seller in a third-world country. The cost per good to the NGO is $c$. The seller obtains net revenue $r(q)$ (net of effort cost of selling). In return for the goods, the seller pays the NGO an amount $T$ after he has received the revenue. If he deviates, he will simply not pay $T$. Since the good is socially desirable (externalities), the NGO internalizes an additional value of $a < c$ per unit sold.

$$r_A(q) = r(q)$$
$$r_P(q) = (c - a) q$$
$$m = - T,$$
$$D(q, T) = T$$
$$\theta = 0$$
Fuchs/ Green solution

- Make agent $\varepsilon$ impatient and set $O_A = 0$.
  - unique steady state quantity $\bar{q}$ characterized by $r'(\bar{q}) = c - a$ (assuming $r(q)$ well-behaved)
  - incentive constraint binds in the steady state. That is $\bar{T}$ solves
    \[
    \delta_A \left( \frac{r(\bar{q}) - \bar{T}}{1 - \delta_A} \right) = \bar{T}
    \]
  - since $\theta = 0$ monotone convergence to steady state (that is: unless constraints on $T$ binds in the first period, steady state is reached in the second period).
Comparative statics of deviation payoff

Comparative Statics of Productivity $u_{min}$

Graph showing the comparative statics of productivity with different values of $\rho$: $\rho = 0$, $\rho = 0.25$, $\rho = \tilde{\rho} = 0.51$, $\rho = \tilde{\rho} = 0.59$. The graph plots the firm value $U_P$ against the government value $U_A$. The lines represent different values of $\rho$. The axes range from 0 to 1.4 for $U_A$ and -0.2 to 1.2 for $U_P$. The graph includes points labeled $v_A$, $\hat{U}$, and $U_{max}$.