Aggregate Fluctuations and the Industry Structure of the US Economy*

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Abstract
Reallocation of inputs in production and substitution across them, are mechanisms through which the economy adjusts to changes in relative efficiency in production across sectors. When input productivity moves along relative prices, cost shares are constant and the input output structure of the economy does not change. I document that cost shares of intermediate inputs fluctuate on average 5% in the investment sector (equipment production) and 3% in the consumption sector. Furthermore, cost shares of intermediate inputs from the equipment (consumption) sector are countercyclical (procyclical) across the economy. These facts are used to discipline the behavior of a multisector RBC model with intermediate goods. I compare the predictions of the model against a comparable economy calibrated to the same steady state, under constant cost shares. Most of the volatility of output is explained by sector specific shocks. However, I find that neutral shocks become relatively more important in generating output volatility when cost shares are allowed to fluctuate as in the data. Additionally, the paper provides conditions for the existence of a balanced growth path in which all intermediate inputs are used in production, and the economy displays investment specific technological change. [JEL CODES: E23,E22,L6,L7]

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1 Introduction

The input output structure (a summary of the trade in intermediate inputs across sectors) is usually assumed constant in time. However, recent input output data reported at annual frequencies, suggests that the structure changes in time and that those changes are correlated with aggregate activity in the economy:

1. The average absolute change of intermediate inputs’ cost shares in the equipment production (consumption) sector is 5% (3%) annually.

2. Cost shares of intermediate inputs produced by the equipment (consumption) sector are countercyclical (procyclical).

3. Inputs’ costs shares in the equipment sector are more cyclical than those in the consumption sector.

Changes in the entries of the input-output matrix are a reflection of the pattern of reallocation in the economy in response to changes in relative prices.

In view of these stylized facts, the paper revisits an old question in business cycles theory: What is the role of sectoral and aggregate (neutral) shocks in determining output volatility? After the work of Greenwood et al. (1997) we have seen the development of a fruitful research agenda that studies the role of investment specific versus neutral shocks in aggregate fluctuations. The canonical model in the literature is homomorphic to a two sector economy where one sector produces capital and the other one consumption goods. In this paper, I augment that economy to allow for intermediate good linkages across sectors. Then, I compare two economies that coincide in the steady state industry structure, and the allocation of labor and capital, but where one of them replicates the patterns of cost share behavior observed in the data. In both economies, sector specific shocks account for most of the volatility of output (in line
with Foerster et al. (2008) and Atalay (2014)). However, the economy that abstracts away from changes in the input output structure predicts a more important role for sectoral shocks in determining output volatility. This is the case, because in the data there is not as much substitution towards the intermediate goods that are produced relatively more efficiently as predicted by a constant cost share economy.

In a one sector model, the share of intermediate goods in production has a role in determining the level of GDP, Jones (2011). But GDP growth depends only on aggregate productivity growth, measured as the change in output not explained by a change in primary inputs (labor and capital). In other words, intermediate inputs are irrelevant in determining aggregate fluctuations. In a multisector neoclassical model instead, the production possibility frontier is a weighted measure of the Solow residuals in each sector, Hulten (1978). The computed solow residuals depend not only on the allocation of primary factors but also on the allocation of intermediate goods across sectors. Hence, aggregate output fluctuations are determined by changes in the allocation of intermediate goods across sectors.

If we assume that production technologies are well approximated by a constant return to scale technology and markets are competitive, cost shares of inputs are equal to the elasticity of inputs in production, i.e.

\[ C_{sh}^{iJ} = \frac{p^i M^i}{p^J M^J} = \frac{\partial Y^J}{\partial M^i} \frac{M^i}{Y_J} = \varepsilon^{iJ} \]

where \( p \) index prices, \( Y \) gross output and \( M \) intermediate good consumption. In a frictionless economy, changes in relative prices reflect changes in relative productivity across sectors. When input productivity moves along relative prices one to one, cost shares are constant and the input output structure of the economy does not change. Substitution towards the now more productively produced input may generate output increases in the sector producing the intermediate good as well as in the one consuming it. Alternatively, cost shares can go up or down. If sectors switch away from other
intermediate inputs that are produced less productively to the more productively pro-
duced one, the cost share can go up and output boosts above the previously predicted
level. On the other hand, if there is no substitution, output per sector need not change
(except from the sector experiencing the productivity improvement), cost shares can
drop and aggregate output will move little. Hence, cost shares movements, jointly with
the behavior of relative prices bring us direct evidence of the pattern of reallocation.

Why do these patterns imply different roles for sector specific and neutral shocks?
To illustrate, let’s think of an economy with three sectors. Two sectors produce in-
termediate goods out of a linear technology in sectoral productivity. The third sector
combines these two inputs to generate the consumption good in the economy using a
Leontief technology.

\[ Y = \min \{ aM^1, M^2 \} \quad M^1 = A^1, \ M^2 = A^2 \]

where \( A^i \) is exogenous. The equilibrium price of output satisfies

\[ p = p^2 + \frac{1}{a}p^1 \]

Suppose that productivity improves in sector 2. Then \( \Delta A^2 > 0 \), and the cost share
for input 2 drops as \( \Delta p^2 < 0 \). The cost share of input 2 is just the relative price of
input to output. Total value added does not change because \( Y = aA^1 \), but aggregate
productivity improves as \( \Delta TFP = \frac{A^2}{aM^1} \Delta A^2 \). Hence, a purely sectoral shock has no
impact on aggregate productivity but a neutral shock (that raises both \( A^1 \) and \( A^2 \)) does.
Furthermore, should the economist analyzing the economy had imposed a constant cost
share structure, it would have predicted an increase in aggregate output after the shock.
Substitution towards the now more efficiently produced input would have induced an
increase in output of \( \Delta Y = Csh_{2Y} \Delta A^2 \).

While in the example the disparity in cost share behavior is fully characterized by
the underlying production function describing output in each sector, there are many
other mechanisms for which cost shares may change differently across sectors, even when operating the same technology. Input specificity is one of them. When looking at aggregate sectoral data, many goods are bundled together. Movements in cost shares may reflect the inability to easily switch across goods that are close together (belong to the same 3 digit NAICS code) but not the same. Another potential source of cost share fluctuations is the presence of inventories. If a firm has stock up enough intermediate inputs for production for a year of production, changes in relative prices anytime during the year will not be reflected in its input intake. In this paper, I am agnostic as of the sources of the movements in cost shares. They are generated by differences in production technologies alone. This allows me to assess the quantitative impact of changes in cost shares while keeping a structure that is very close to the canonical two sector model in the literature.

In the paper, conditions are provided for the existence of a balanced growth path in which all inputs are used in production, yet productivity growth rates in the equipment and consumption sector may differ. When the detrended economy is calibrated to predict the patterns of cost share movements observed in the data, the contribution of neutral shocks to output volatility increases. In other words, the variance decomposition of an economy calibrated to the same steady state in which constant cost shares are imposed (Cobb-Douglas technologies), indicates that neutral shocks contribute 7% less to aggregate output volatility than they do in a flexible cost share economy. Aggregate output impulse responses to persistent and fully temporary shocks depend on the underlying reallocation patterns embedded in the economy. The impact of sectoral shocks is magnified in the constant cost share economy versus the flexible one.

The rest of the paper is organized as follows. Section 2 described the related literature, Section 3 documents the main finding in the data. Section 4 describes the model and the characterization of the BGP. Section 4 presents the calibration and quantitative results. Section 5 concludes.
2 Literature Review

The literature on the role of sectoral shocks is extensive. The seminal work by Hulten (1978) paved the way for the study of the role of input output linkages in the transmission of sectoral shocks. While the authors find a substantial role for sectoral shocks in shaping aggregate fluctuations in output, much discussion has been triggered since on the plausibility of transmission of idiosyncratic shocks to the aggregate economy. At the heart of the arguments is whether law of large arguments apply to the units that we define as sectors.

Dupor (1999) shows that when the network that describes the input output structure is a balanced one, sectoral shocks indeed do not affect aggregates. We have learn much about the characteristics of the network structure since. Horvath (2000) shows that when the input output structure is sparse (as is the case in the data) sectoral shocks do not fade away in the aggregate. Alternatively, Carvalho and Gabaix (2013) show that when the role of sectors in the economy is unbalanced, in the sense that a few sectors account for most of the value added, the law of large numbers fails and sectoral shocks can have aggregate impact. Along the same line are the network results by Acemoglu et al. (2012) and Oberfield (2011). Hence, there is Nowadays consensus that sectoral shocks can be transmitted to the aggregate economy and have quantitative impact.

This paper is methodologically closely related to recent work by Foerster et al. (2008). Using a factors model as in Long and Plosser (1983), Foerster et al. (2008) show that the role of sectoral shocks in explaining aggregate volatility has increased (in relative terms) after the great moderation. Key to the econometric strategy of the paper is the assumption that the input output structure is stable. This paper departs fundamentally from it by allowing trade intensities in intermediate goods to change across time. Still, the model economy studied in this paper is tractable and displays a balanced growth path where cost shares remain constant. More recently, Atalay (2014)
has used intermediate input purchases to identify the relative importance of industry-specific shocks. In his framework he estimates an elasticity of substitution between value added and intermediate goods different than one. In this paper, I assume the elasticity is unitary so that the economy is consistent with a balance growth path, and a long term trend in investment specific technological change.

While the papers described earlier allow for a large degree of heterogeneity across sectors in the economy, this paper focuses on a two sector economy. After the work of Greenwood et al. (1997), the analysis of economies with neutral and investment specific shocks, is the preferred choice in the literature studying business cycles. I would like to keep the model as close to this benchmark economy as possible, so that the role of intermediate goods linkages can be uncovered. Among others, the model can be used to understand the role of investment specific and neutral shocks. The question regarding the relevance of each of these shocks for output volatility is not new. While research is extensive, some of the most recent developments are Schmitt-Grohe and Uribe (2012) and Justiniano et al. (2011). In all cases, the structure of intermediate goods trade across sectors is abstracted away. This paper contributes to the literature by disciplining the patterns of reallocation across sectors to match the movements observed in the data.

Finally, there is a surging literature studying the role of intermediate goods in aggregate factor productivity. Moro (2012) argues that the intensity of intermediate goods usage is key to explain the dynamic of aggregate TFP. In his paper, as the elasticity of gross output to intermediate input increases, aggregate productivity declines. Jones (2011) also relates the intensity of intermediate goods usage to aggregate productivity, this time to understand the large differences in income that we observe across countries. In that paper, distortions in the allocation of intermediate goods across technologies generates losses in both aggregate productivity and output. In either paper, the input output structure is constant.
3 Data

3.1 Input-Output structure

I study make-use tables from 1993 to 2008 as reported by the Bureau of Labor Statistics based on BEA data. The series are presented for 195 sectors, and values are chained weighted at 2005 US dollars. I aggregate sectors to build an Input Output matrix with 31 industries, which can be further classified as investment/equipment, consumption sector and agricultural and mining sector. The "investment" sector is comprised by equipment producing sectors which include Machinery, Electrical and optical equipment, Transport equipment, and Repair and installation of machinery and equipment. The consumption sector comprise all remaining sectors (except agricultural and mining), which full listing can be found in the appendix.

I focus on the disparities in cost share behavior of equipment and consumption sectors. I abstract from the behavior of agricultural and mining sectors for several reasons. First, the assumption of constant returns to scale in technology that I use later in the model economy is unlikely to hold in these sectors, where there are large fixed costs of operation. Second, price fluctuations in these commodities within the country may not be as closely tight to changes in relative productivity as it is the case for equipment production and consumption goods production.

The cost share of input i in sector j is defined as

\[ C_{sh}^{ij} = \frac{p_j M^{ij}}{p_j Y^j} \]

where \( Y^j \) denotes gross output in sector j, \( M^{ij} \) is the intermediate good i intake of sector j and \( p \) denotes prices. Hence, cost shares fluctuate whenever changes in relative prices are not fully translated into changes in input productivity \( \frac{Y^j}{M^{ij}} \).
I compute yearly absolute changes in cost shares for each of the inputs in each sector. Table (1) reports its average (unweighted) and standard deviation per sector across inputs of production.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equipment</td>
<td>4.9%</td>
<td>6.3%</td>
</tr>
<tr>
<td>Consumption</td>
<td>3.1%</td>
<td>4.5%</td>
</tr>
</tbody>
</table>

Table 1: Yearly average absolute change

Cost share changes in the equipment sectors are on average larger than for inputs in the consumption sectors. The standard deviation is also larger, with changes that can reach above 10% in a year within one standard deviation in the investment sector, and above 7% in the consumption sector. It is possible that such changes only take place for particular inputs in a given sector. To study variation across inputs I compute average absolute changes accordingly. Those are presented in table (2).

<table>
<thead>
<tr>
<th>Input/Sector</th>
<th>Equipment</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equipment</td>
<td>3.2%</td>
<td>3.1%</td>
</tr>
<tr>
<td>Consumption</td>
<td>4.7%</td>
<td>2.3%</td>
</tr>
</tbody>
</table>

Table 2: Yearly average absolute change

Notice that the diagonal terms in the matrix equalize input productivity changes. The table indicates that changes in cost shares are mostly influenced by changes in the off diagonal terms. Movements in relative prices, that are not fully compensated by changes in relative prices.

Finally, let me describe the composition of input intake as depicted in table (3)
Table 3: Share of Service and Manufacturing Inputs (intermediate goods from the Agriculture and Mining sector have been factored out)

As expected the share of equipment intermediate inputs across sectors is relatively small. Note that equipment sector accounts for about 7% of gross output and 4% of value added at current prices for all sectors under analysis.

The asymmetry in the industry structure, vis a vis the role of each sector in gross output is an important feature that the model economy should capture. Elasticities of substitution across inputs (hence, cost shares) depend on the industry structure as summarized by (3). However, the relevance of shocks in the aggregate depend on their contributions to gross output.

Before moving on into the modelling strategy, let me describe the cyclical patterns observed for the actual changes in cost shares. Table (4) shows that 2) cost shares in the equipment sector are more cyclical than their counterparts in the consumption sector; 3) cost shares of equipment goods across sectors are countercyclical.

Table 4: Correlation with Industrial Value Added

This last characteristic is important as it has been extensively documented that the price of equipment is countercyclical. Countercyclical cost shares indicate that input
productivity does not drop one to one with the drop in relative prices. Take for example the behavior of intermediate investment sector goods into the investment sector. The countercyclicality of its cost share implies that input productivity is increasing in an expansions. Hence, there is not enough substitution between consumption and equipment intermediate goods that would make the input productivity of equipment goods in investment drop. The mirror image of this behavior is that of cost shares of consumption goods in the equipment sector. When the relative price of equipment goods drop, intermediate consumption goods are not substituted away, which makes input productivity relatively lower to what a constant cost share environment would dictate. Similar arguments apply for the consumption sector.
4 Model

4.1 Environment

This is a discrete time, infinite horizon economy.

There are two final goods produced in the economy, equipment and consumption goods. Additionally, intermediate equipment goods are produced.

There are three production sectors in the economy. I assume there is a representative firm in each sector. All markets are competitive and the technologies are constant returns to scale. The diagram 5 displays the input output structure of the economy under analysis. A cross indicates a positive entry in the matrix. To distinguish between goods produced by the equipment sector, I call \( X_2 \) the sector producing intermediate equipment and \( X_1 \) the sector producing investment goods (capital).

<table>
<thead>
<tr>
<th>Sector</th>
<th>Intermediate Demand</th>
<th>Final Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E ( X_2 ) ( X_1 ) Y</td>
<td>Equipment Consumption</td>
</tr>
<tr>
<td>E</td>
<td>x ( X_2 ) x ( X_1 ) x</td>
<td>( x )</td>
</tr>
<tr>
<td>C</td>
<td>x x x</td>
<td>( x )</td>
</tr>
<tr>
<td>CAPITAL</td>
<td>x x x</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Input Output matrix of the economy

Total value added in this economy equalizes the sum across entries in the last two columns (GDP). Gross output per sector corresponds to the row sum across all columns. Total cost corresponds to the column sum per sector. The cost share is the ratio between a particular entry in the intermediate demand section and gross output. The cost shares in the model are constructed as in the data, separating out expenses in capital services.
There is a representative household with standard preferences over final consumption goods. She maximizes lifetime utility by choosing a consumption stream as well as purchases of investment goods.

4.2 Representative Household

The representative household maximizes lifetime utility subject to its budget constraint. Her income stems from the rental of capital to the firms in the economy and from claims to the profits of those firms. Note that capital is specific to a sector and hence capital cannot be instantaneously reallocated from one sector to another. The non-negativity constraint in investment goods should hold for each capital type.

$$\max_{c_t, i_t, x_t} \sum_{t=0}^{\infty} \beta^t U(c_t)$$

subject to

$$p^y_t c_t + p^x_t x_1 \leq r_t (k_{x1} + k_{x2} + k_y) + \sum_{j=y, x1, x2} \pi^j_t$$

$$k^x_{x1} - k_{x1}(1 - \delta) = i_x$$ \quad \text{(}\kappa_x\text{)}

$$k^x_{x2} - k_{x2}(1 - \delta) = i_x$$ \quad \text{(}\kappa_x\text{)}

$$k^y_{y} - k_y(1 - \delta) = i_y$$ \quad \text{(}\kappa_y\text{)}

$$i_y + i_{x1} + i_{x2} = x_1 \quad \text{and} \quad i_j \geq 0 \text{ for } j = y, x_1, x_2$$

where $\beta$ is the discount factor; $p^j$ indexes prices for alternative goods $j = y, x_1$, i.e. final consumption and investment goods, respectively; the capital stock is $k^j_t$ with rental rate $r_t$ and the profits of the firms in each sector are $\pi^j_t$. 

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4.3 Consumption Goods Sector

The representative firm in the consumption sector maximizes profits each period. It has available a constant returns technology that uses intermediate goods \( M \) and capital goods. Once the productivity of all sectors is realized, it chooses its input purchases. The problem of the firm reads

\[
\max_{M_t^{yy}, M_t^{xy}, k_t^y} p_t^y Y_t^y - p_t^y M_t^{yy} - p_t^{xy} M_t^{xy} - r_t k_t^y
\]

subject to

\[
Y_t = \exp(A_t) \left( k_t^y \right)^{\alpha_{y1}} M_t^{yy}_{t} + \left( 1 - \alpha_{y2} \right) M_t^{xy}_{t} \frac{\alpha_{my}}{\rho_y}
\]

where \( A_t^{y} \) is a Hicks Neutral productivity shock. For simplicity we have assumed that shocks to the productivity of the consumption good sector correspond to aggregate shocks\(^1\). The first one corresponds to a sector specific shock, and the second one to an aggregate one. The intermediate good purchases from sector \( j \) are \( M_t^{jy} \); \( k_t^y \) is the stock of capital used in production, \( \alpha_{y1} \) is the share of the intermediate goods in gross output; \( (1 - \rho_y)^{-1} \in (-\infty, 1) \) is the elasticity of substitution across intermediate goods in the equipment sector; \( \alpha_{my} \) is the share of intermediates in value added; and \( \alpha_{y2} \) corresponds to the share of consumption intermediate inputs in the production of consumption goods when \( \rho_y = 0 \) (Cobb-Douglas technology).

\(^1\)Shocks particular to this sector can be incorporated. However, we expect the predictions of that economy to be analogous to this one. The current setup correspond in which any change in idiosyncratic productivity in this sector is reflected in changes in the relative productivity of the other two sectors in the economy. Quantitatively, the modeling strategy may make a difference in the variance decomposition exercise, so robustness checks will be run.
4.4 Equipment Sectors

4.4.1 Investment Goods

The representative firm in the equipment investment sector maximizes profits by choosing intermediate good purchases of both consumption and intermediate equipment goods. Its problem reads

$$\max_{M_i^{xx}, M_i^{yx}} p_t^{x1} X_i^1 - p_t^{x2} M_i^{xx1} - p_t^{y} M_i^{yx1}$$

subject to

$$X_i^1 = \exp(A_t^g) \exp(A_t^{x1}) (h_t^{x1})^{\alpha_{x1}} (\alpha_{x2} (M_t^{xx1})^{\rho_x} + (1 - \alpha_{x2}) (M_t^{yx1})^{\rho_x})^{\alpha_{mx}}$$

where $A_t^{x1}$ is a Hicks Neutral sectoral productivity shock, $M_i^{ix}$ are intermediate good $i$ purchases in sector $X_1$; $(1 - \rho_x)^{-1} \in (-\infty, 1)$ is the elasticity of substitution across intermediate goods in the equipment sector; $\alpha_{mx}$ is the share of intermediates in value added; and $\alpha_{x2}$ corresponds to the share of intermediate equipment inputs in the production of investment goods when $\rho_x = 0$.

4.4.2 Intermediate Goods

The representative firm in this sector maximizes profits by choosing capital and intermediate goods from the consumption sector. Its problem reads

$$\max_{M_t^{yx2}, k_t^y} p_t^{x2} X_i^2 - p_t^{y} M_t^{yx2} - r_t k_t^{x2}$$

subject to

$$X_i^2 = \exp(A_t^g) \exp(A_t^{x2}) (h_t^{x2})^{\zeta} (M_t^{yx2})^{\alpha_{mx2}}$$
where $A_t^{x2}$ is a Hicks Neutral productivity shock, $M_t^{yx2}$ is the purchase of intermediate consumption goods; $k_t^{x2}$ is the stock of capital used in production; and $\zeta$ corresponds to the share of capital in the production of intermediate equipment goods and $\alpha_{m,x2}$ is the share of intermediates in value added.

### 4.5 Productivity

Each sector takes the realization of the productivity process as given. Productivity has two elements, a deterministic trend and a noise term. Let $A_t \equiv \{A_t^0, A_t^{r1}, A_t^{r2}\}$ be the current realization of the shocks in the economy. The dynamic of $A$ is described as

$$A_t = \Gamma(A_{t-1})$$

$$A_t = (1 + \gamma_t)A_0 + \Lambda_t$$

where $\gamma_t$ is a vector collecting the time trends and $\Lambda_t$ is the noise in the process, $E(\Lambda_t) = 0$.

The noise term has in turn two elements. One that is purely temporary and I call $\epsilon_t$ and a persistent component $z_t$ with persistence $\theta$ and innovation $\eta_t$. In other words, the noise structure is

$$\Lambda_t = z_t + \epsilon_t$$

$$z_t = \theta z_{t-1} + \eta_t$$

$$\Lambda_t = \theta \Lambda_{t-1} - \theta \epsilon_{t-1} + \eta_t + \epsilon_t$$

$\epsilon_t \sim N(0, \Sigma^\epsilon)$ and $\eta_t \sim N(0, \Sigma^\eta)$. The variance covariance matrix of the shocks are $\Sigma^\epsilon$ and $\Sigma^\eta$ independent from each other.
5 Equilibrium

Before defining the equilibrium let me introduce some additional notation. Let \( p_t \equiv \{p_t^y, p_t^{x1}, p_t^{x2}\} \) be the vector of prices in the economy, \( M_t^y \equiv \{M_t^{yx1}, M_t^{yx2}, M_t^{yy}\} \) be the vector of intermediate consumption goods, \( M_t^x \equiv \{M_t^{xx1}, M_t^{xx2}\} \) be the vector of intermediate equipment goods.

Definition 1 A competitive equilibrium is an allocation of consumption, investment and capital \( \{c_t, \{i_t^j, k_{t+1}^j\}_{j=y,x_1,x_2}\}_{t=0}^\infty \), as well as intermediate good consumption \( \{M_t^y, M_t^x\}_{t=0}^\infty \), such that given a system of prices, \( \{p(A_t), r(A_t)\}_{t=0}^\infty \), the exogenous dynamic for sectoral productivity \( A_{t+1} = \Gamma(A_t) \) and the initial stock of capital \( k_0^j \),

1. The representative household maximizes utility
2. The representative firm in each sector maximizes profits
3. Markets clear:
   (a) \( c_t + M_t^{yy} + M_t^{yx1} + M_t^{yx2} = Y_t \)
   (b) \( i_t^y + i_t^{x1} + i_t^{x2} = X_t^1 \)
   (c) \( M_t^{xx1} + M_t^{xy} = X_t^2 \)

I now describe how the production possibility frontier changes in this multisector economy. As in Hulten (1978), the PPF is a weighted average of Solow residuals of different sectors in the economy. Let \( \tilde{T}_t \) be the log deviation of aggregate efficiency in period \( t \), then

\[
\tilde{T}_t = \frac{p_t^1 Y_t^j}{\sum_j p_t^1 (Y_t^j \gamma_j)} \tilde{Z}_t^j
\]

(1)

Fluctuations in the solow residual in each sector are characterized by

\[
\tilde{Z}_t^j = \tilde{Y}_t^j - \sum_{i=1}^S C s h^{ij} \tilde{M}_t^{ij} - C s h^{kj} \tilde{K}_t^j
\]

(2)
where $M$ are intermediate inputs and $K$ is capital. The residual is the change in aggregate output not explained by changes in the input of production\(^2\). By definition, movements in intermediate input intake per sector are characterized by

$$\tilde{M}_{it}^{ij} = \tilde{C}_{sh_{it}}^{ij} + \tilde{p}_{it}^{ij} - \tilde{p}_{it}^{i} + \tilde{Y}_{it}^{j}$$

Hence, whether equilibrium prices adjust so that the actual movement in intermediate purchases is the same in the constant and flexible cost share economy should be assessed in the context of a general equilibrium model. A priori, there is no reason to believe this will be the case. Hence, productivity residuals will differ across economy, and through them aggregate productivity changes.

6 Balanced Growth Path

**Theorem 1** A Balanced Growth Path (BGP) where all intermediate goods are used in production exists iff technology is Cobb Douglas in capital and intermediates and either 1) the elasticity of substitution across intermediate goods, equals unity (Cobb-Douglas technology); or 2) there are no linkages through intermediate goods between the equipment and consumption sector; 3) productivity growth in the consumption sector and intermediate equipment sector are proportional by a factor $(g^{x})^{\alpha_{y1}-\zeta}(g^{y})^{\alpha_{my}-\alpha_{mx2}}$, where $g^{x}$ is the growth rate of gross output in the investment sector and $g^{y}$ is the one in the consumption sector.

The first result is analogous to that in Ngai and Pissarides (2007) in an economy with structural change. The second result is well known as it reduces the economy to one

\(^2\)Alternatively, the residual can be expressed as

$$\bar{Z}_{it}^{j} = \sum_{i} \left| \tilde{z}_{it}^{ij} \right| \tilde{A}_{it}^{i}$$
like the one in Greenwood et al. (1997). The third one allows me to study the economy that has been described in the previous section. One with non-trivial heterogeneous productivity processes across different sectors while allowing for fluctuations in cost shares. It is worth mentioning that while cost shares are allowed to change, in equilibrium they will be constant along the BGP. In the third case, productivity growth in the intermediate equipment and consumption sector are allowed to differ iff the shares of capital and intermediates in value added are different across sectors. If productivity growth rates are proportional, then output in the intermediate equipment sector and in the output sector grow at the same rate.

**Corollary 1** For a given productivity growth rate in the investment sector, it is possible to find a set of parameters (share of capital and intermediates) such that productivity growth in the intermediate equipment sector equals the one in the investment sector and the BGP is preserved. Productivity gains should satisfy

\[
\gamma^x = \gamma^x_2 = \left(\gamma^y\right)^{1+\psi_{x2}(\alpha_{y1} - \zeta)+\psi_{y2}(\alpha_{my} - \alpha_{mx2})}
\]

Hence, if the previous relationship is satisfied between productivity gains in the investment and the consumption sector, the economy resembles an economy with two sectors in which the only shocks are a neutral and an investment specific one.

It is worth describing equilibrium growth rates for the case that will be analyzed in the rest of the paper (3). Growth rates of gross output along the BGP are convex combinations of the productivity growth in the investment and consumption sector.

\[
g^x = (\gamma^x)^{\psi_{x1}} (\gamma^y)^{\psi_{x2}}
\]

\[
g^y = (\gamma^x)^{\psi_{y1}} (\gamma^y)^{\psi_{y2}} = g^x_2
\]
\[ \psi_{x1} = \frac{1-a_{my}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mz}a_{y1}} \quad \text{and} \quad \psi_{x2} = \frac{a_{mx}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mz}a_{y1}}. \]

Also, \[ \psi_{y1} = \frac{a_{y3}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mz}a_{y1}} \]

and \[ \psi_{y2} = \frac{1-a_{x1}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mz}a_{y1}}. \]

In an economy with constant returns to scale and no labor, so that \( a_{y1} + a_{my} = 1 \) and \( a_{x1} + a_{mx} = 1 \), the growth rates of output are identical across sectors. In an economy where \( a_{y1} + a_{my} < 1 \) and \( a_{x1} + a_{mx} = 1 \), the consumption sector gross output will grow slower than the investment sector. If instead, \( a_{y1} + a_{my} = 1 \) and \( a_{x1} + a_{mx} < 1 \), the consumption sector will grow faster than the investment equipment sector.

In other words, whenever the share of intermediates in production in the equipment (consumption) sector is relatively small, the consumption sector grows faster (slower) than the investment equipment sector.

### 7 Quantitative Exercises

The quantitative strategy is as follows. First the model is detrended, using the characterization of the GDP from the previous section. Second, I calibrate the model to match the steady state behavior of the industry structure (i.e. the share of intermediate inputs in each sector, as well as value added in gross output) and standard features of RBC models. To match the cyclical behavior of cost shares observed in the data, I calibrate the variance covariance matrix of the shock structure.

Third, I calibrate a comparable economy with Cobb-Douglas technology to generate the same steady state of the baseline economy.

With these two economies I run alternative experiments. First, I compute impulse responses for identical shocks to test the propagation properties of each economy. Second, I simulate each economy and compute a variance decomposition of the generated path for output and aggregate TFP, for neutral and investment specific shocks.

I have used data from the Capital Flow Table of 1997 to compute investment levels across sectors. Capital stocks for the same year across sectors were obtained from the EUKLEMS database. Nominal shares of intermediate inputs were obtained from annual
Input Output tables at chained dollars of 2005 as reported by BLS. The relative price of equipment to consumption good was obtained as averages of quarterly data as reported in DiCecio (2009), computed following Cummins and Violante (2002) methodology.

For these exercises, I assume that the share of intermediates in value added is the residual after deducting the share of capital. Under such specification, growth rates of gross output are the same across all sectors.

### 7.1 Calibration

The model is calibrated to annual frequencies mainly because the data on intermediate good cost shares is available at that frequency. Table 13 describes the set of parameters that were set independently of the model conditions. The persistence of the permanent component of productivity was set to 0.98, equal across shocks. The discount factor was set consistent with an annual interest rate of 2%, and capital depreciation was set to 5% per year (as in Cooley and Prescott, 1995). I also need to calibrate the growth rate along the BGP. The trends are obtained as gross output weighted average sector growth rates by KLEMS. These are computed 1978 to 2010\(^3\).

Table 6 describes the set of parameters chosen to generate a steady state relationship indicated by the model.

From the optimality conditions for capital (\(x^*\) corresponds to the steady state value of variable \(x\)) we obtain,

\[
\frac{1 + g^x - \beta(1 - \delta) k_{x1}^*}{\beta} X_1^* = \alpha_{x1}
\]

Hence, I need either a measure of capital output ratio in the investment sector, or a measure of capital capital services in gross output, \(\alpha_{x1}\). I use the latter. The feasibility condition that dictates that gross output in sector \(X_1\) corresponds to total investment

\(^3\)If the average is computed over a time frame comparable to the input output data, the growth rate in the equipment sector raises 1% and in the consumption sector raises 0.2%. Productivity growth for these period is 5.99% in the equipment sector and 1.5% in the consumption sector (value added measures of TFP).
in the economy as reported in the Flow of Funds. Capital services are obtained from KLEMS data.

Following a similar strategy we can calibrate the share of capital in the consumption sector as

\[
\frac{1 + g^x - \beta(1 - \delta) \lambda_x^* h_y^*}{\beta \lambda_y^* Y^*} = \alpha_{y1}
\]

The shares of intermediate goes in sectoral output do not have an analytical solution in terms of the data. In particular we are left to calibrate, two elasticities of substitution \((\frac{1}{1 - \rho_x}, \frac{1}{1 - \rho_y})\) and the shares of input in production. We can use data on gross output, intermediate expenses and the price of investment goods (to deflate intermediate expenses in the last term of the LHS) to evaluate the next two equations

\[
(1 - \alpha_{y1}) \alpha_{y2} \left( \frac{\lambda_y^* Y^*}{\lambda_y^* M_{yy}^*} \right) \frac{1}{\alpha_{y2} + (1 - \alpha_{y2}) \left( \frac{M_{yy}^*}{M_{yy}^*} \right)^{\rho_y}} = 1
\]

\[
(1 - \alpha_{y1}) \alpha_{y2} \left( \frac{\lambda_y^* Y^*}{\lambda_y^* M_{yy}^*} \right) \frac{1}{\alpha_{y2} + (1 - \alpha_{y2}) \left( \frac{M_{yy}^*}{M_{yy}^*} \right)^{\rho_y}} \left[ \frac{(1 - \alpha_{y1}) \alpha_{y2} - \frac{1}{\rho_y}}{\alpha_{y2} + (1 - \alpha_{y2}) \left( \frac{M_{yy}^*}{M_{yy}^*} \right)^{\rho_y} + \rho_y} - 1 \right] = 0
\]

These are necessary and sufficient conditions for an interior solution in intermediate inputs. These two equations form a system of two equations in two unknowns that can be solved for rho and alpha whenever relative prices are available.\(^4\)

For the shares of intermediate inputs in sector \(X_1\), as well as the elasticity of substitution across inputs, a similar argument yields two equations to be solved in two unknowns.

Finally, I choose \(\zeta\) to assure that the steady state of the model is well defined, i.e. there is a set of non-negative prices that solve the allocation. Equilibrium prices in steady state are obtained from the feasibility constraints in the three sectors.

\(^4\)I am currently running an alternative calibration where both the elasticity of substitution and the shares of intermediates are calibrated jointly matching the steady state behavior of the economy.
The full list of parameters is described in table 6. The first column reports the calibrated shares. Consistent with the literature (Hornstein and Praschnik (1997) and Huffman and Wynne (1999)), the share of capital in the consumption sector is slightly higher than the one in the investment sector. The share of capital in the intermediate equipment sector however is calibrated roughly three times that in the consumption sector. In both sectors intermediate inputs are identified as complements in production, with larger degree of complementarity in the consumption sector.

7.1.1 Industry Structure

The predicted steady state share of intermediate inputs per sector in the model is reported in table 7. Using the calibration strategy presented in the previous section, the model predicts accurately the share of intermediate inputs in the consumption sector. However, it underpredicts the share of intermediate consumption goods in the equipment sector by 17%. Still, it predicts a larger share of consumption intermediate inputs than equipment intermediates in the production of other equipment.

Overall, the model is able to predict the disparate share of consumption input versus equipment intermediate input across all sectors in the economy. Matching such disparity is important in assessing the relevance of shocks to each sector because the elasticity of cost shares to changes in relative prices depend on the initial input cost share.

7.1.2 Productivity shocks

An important element of the parametrization are the features of the productivity shocks. The pattern of reallocation observed in the data depends not only on the available technologies for production but also on the persistence and the relative size of different shocks. The variance covariance structure of shocks is set to mimic the cyclical behavior of cost shares in the economy as well as to match the aggregate volatility of output in the economy. The calibrated parameters are reported in the table 8.
The shocks structure identified indicate smaller volatility for aggregate shocks, and for the permanent component of the equipment intermediate sector vis a vis the equipment investment sector. Transitory shocks have a slightly higher volatility in the equipment sector than their permanent shock counterpart. Finally, the permanent component of the shocks in the equipment sector where assumed positively correlated (0.9).

**Cyclicality of cost shares**  Under this parametrization the predicted cyclical behavior of cost shares and aggregate volatility of output is presented in table 9.

The model generates correlations that are a lower bound to the correlations between cost shares and aggregate output observed in the data. Correlations increase when the size of the shocks, in particular the neutral shock, increases. However, higher volatility of the neutral shocks implies a much larger volatility of output than observed in the data. The model is unable to predict the disparity in correlations across sectors. In the model economy inputs in the equipment sector as slightly more cyclical but negligible if compared to the orders of magnitude observed in the data. The model however, generates the correct cyclical patterns for inputs from the consumption and equipment sectors.

8 **Results**

8.1 **Aggregate and Sectoral Shocks in a flexible cost share economy**

Table 10 presents the variance decomposition across shocks for aggregate value added, aggregate productivity and consumption. Shocks to the equipment sector explain 80% of the volatility of output. Roughly 60% of those movements are accounted by the volatility of the transitory component of the equipment intermediate sector, and the remaining to the persistent component of the innovations in the investment equipment
sector. The relevance of shock to the production of intermediates highlights the importance of modeling the input output structure and dynamic. More than 10% of the volatility of output is explained by aggregate shocks.

As of the volatility of total factor productivity, most of it is explained by transitory shocks. Recall that in a multisector economy with intermediate inputs, aggregate PPT shifts with changes in the relative intensity with which inputs are used in production across different sectors, and the relevance of each sector for gross output. Shocks to the equipment sector account for 70% of the volatility of aggregate productivity.

Finally, the volatility of consumption in the model is also mostly explained by transitory shocks. About one third of it stems from aggregate shocks and the remaining stems from shocks to the production of investment equipment goods.

8.2 Constant versus Flexible Cost Shares

As pointed out in the simple example in the introduction, the direction of the change in cost share will indicate whether output and productivity is over or under estimated if the model is calibrated assuming constant cost shares. To compare a constant and a flexible cost share economy, I recalibrate the previous economy, to one which input structure in steady state can be generated by a constant cost share economy. Again, I choose $\zeta$ to assure that the steady state of the model is well defined. Table 11 reports the parametrization of the model when Cobb Douglas technologies are assumed. The share of capital are identical across specifications. The elasticities of substitution across intermediate inputs are set to 1 ($\rho = 0$), so that the technology is Cobb Douglas in all inputs. The share of equipment intermediates in the capital producing sector is lower than in the Cobb Douglas technology. Also, the share of consumption intermediates in consumption production sector is set 0.04 higher than under the CES technology.

Under the calibrated economies, the Cobb-Douglas technology generates the same intermediate input intake in the consumption sector than the CES economy (See table
12). As in the Baseline calibration, both models predict relatively well the input composition in the consumption sector, but fail to account for all the disparity in intermediate input intake in the equipment sector. The industry structure cannot be matched up exactly because some parameters are restricted to equalize those under the CES structure (capital shares for example), and the parametrization has to be consistent with the existence of non-negative prices for all goods produced in the economy (see the Steady State Section in Appendix (B)).

**Variance Decomposition** Table 13 displays the contribution to the variance of GDP, aggregate productivity and aggregate consumption from neutral and sectoral specific shocks.

As I have pointed out in the baseline economy, shocks to the production of intermediate equipment goods contribute substantially to GDP volatility. Shocks to the capital producing sector have less impact because they do not affect the capital stock in the current period directly. The production of intermediate equipment does. Also, the contribution of shocks to intermediate equipment producing sectors in aggregate productivity is explained by the complementarity in production inputs. The calibration dictates that inputs are complementary in both sectors, hence when a shock hits the production of intermediate equipment goods, output in the consumption sector boost generating additional intermediate consumption goods, which feeds back into the economy.

If compared to the constant cost share economy, the latter assign almost 80% of the volatility of output to shocks in the investment equipment sector. Half stems from purely transitory shocks and half from persistent shocks. The constant cost share economy, assigns 26% of the volatility of output to persistent shocks in the investment equipment sector, and no role for transitory shocks. Furthermore, the constant cost share economy indicates that the persistent component of aggregate shocks has no contribution to output volatility, while the flexible cost share economy, indicates they
contribute 10% of the movements in output.

The relative contribution of shocks to the volatility of aggregate consumption is almost identical.

This is not the case, for aggregate productivity. Both models imply that transitory shocks explain the large bulk of the volatility of productivity. However, the relative importance of each shock varies across economies. While the flexible cost share economy indicates that 1/5 of productivity volatility is explained by shocks to the production of intermediate equipment, the constant cost share economy assigns a negligible role. On the flip side, the constant cost share economy dictates that aggregate shocks contribute 20% more to the volatility of productivity than the flexible cost share economy does.

**Impulse Responses** This section presents the predicted responses of key macroeconomic variables to shocks to productivity in alternative sectors. The focus is on a comparative analysis of responses in the flexible versus the constant cost share economy.

I study the behavior of aggregate output, TFP, aggregate consumption, gross output in the consumption sector, investment, and the relative price of new capital goods versus consumption goods. Figures 13 to 13 depict the responses of this variables to transitory neutral shocks, investment specific shocks, and shocks to the equipment intermediates sector. Figures 13 to 13 display the responses to shocks in the persistent component of productivity for the same three alternatives.

The qualitative impact of transitory shocks is the same across economies except in response to a shock in the investment equipment sector. While in the flexible cost share economy GDP increases on impact, in the constant cost share economy, aggregate GDP drops below steady state levels on impact. Quantitatively, the economy with constant cost shares underestimates the reaction of aggregate value added to a neutral shock upon impact. When the shock originates in the intermediate equipment sector, the reallocation of factors induced by the change in relative productivity generates additionally higher predicted value added in the long run if compared to a flexible
cost share economy. Long run consumption also raises above its counterpart in the benchmark economy. Relative prices are countercyclical in both cases but the drop in the price of investment goods is one third larger in the constant cost share economy. When the shock originates in the investment sector, both aggregate value added and investment in the constant cost share economy raise above the level predicted in the benchmark economy.

When the shocks are persistent the picture is disparate across variables and shocks. In all cases, gains in productivity are predicted larger under the flexible cost share economy. The same is true for investment, value added, gross output, and consumption are predicted higher in the constant cost share economy. When the shock is neutral, the trends in variable are higher in the flexible cost share economy than in the constant cost share one. When the shock originates in the investment sector the predicted growth rates are also higher under the flexible cost share economy. Relative prices presents a non-monotonic dynamic mimic across both economies that has to do with the reallocation of factors and aggregate output shares across sectors. When the shock originates in the equipment intermediates sector, similar dynamics are observed for the macro variables. However, the behavior of relative prices is disparate. while the constant cost share economy predicts a large drop in relative prices, the flexible cost shares economy predicts an increase in relative prices to slowly converge to the same steady state.

9 Conclusion

This paper studies the effect of fluctuations in the cost shares of intermediate inputs for the volatility of output. The model economy is calibrated to match the industry structure of the USA economy and the cyclical cost share behavior documented in the data. When tested against a comparable constant cost share economy, neutral shocks account for 7% more of aggregate output volatility. Responses of aggregate output and
productivity to sectoral shocks are magnified when patterns of reallocation of factors are consistent with constant cost shares.

The disparities in cost share behavior across sectors may provide identifying restrictions for the nature of shocks to the economy. The results stems not only from the disparate contribution of sectors to value added, but also from degree of substitutability in inputs of production. Additional empirical analysis on the latter might be a promising avenue for further work. Furthermore, the disparities in the predictions of the dynamic of key aggregate variables may provide additional identification restrictions for the nature of shocks in the economy.

The paper illustrate the quantitative implications of disciplining the model economy to generate the pattern of reallocation observed in the data. It is still an open questions which are the mechanisms that generate those patterns. Are they consistent with factor specificity? do inventories play a role? do these patterns change when credit conditions change? Analysis of the input output structure dynamic for more disaggregated sector can shed light to some of these questions.
References


## Appendix (A)

<table>
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<tr>
<th>Equipment Sectors</th>
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11 Appendix (B)

11.1 Balance Growth Path

Theorem 2 A BGP where all intermediate goods are used in production exists iff technology is Cobb Douglas in capital and intermediates and either 1) the elasticity of substitution across intermediate goods, equals unity (Cobb-Douglas technology); or 2) there are no linkages through intermediate goods between the investment and consumption sector; 3) productivity growth in the consumption sector and intermediate equipment sector are proportional by a factor \( (g^x)^{\alpha_{y1} - \zeta} (g^y)^{\alpha_{my} - \alpha_{mx2}} \), where \( g^x \) is the growth rate of output in the investment sector and \( g^y \) is the one in the consumption sector.

1) and 2) are special cases of 3), hence I prove the latter first.

Proof. Suppose that productivity in the investment durable sector grows at rate \( \gamma^x \) and productivity in the consumption and intermediate equipment sector grows at rate \( \gamma^y \) and \( \gamma^{x2} \) respectively. Let \( g^j \) be the growth rate of output in sector \( j = y, x, x_2 \).

The feasibility restrictions in the economy imply

\[
\begin{align*}
g^y &= g^{My} = g^{My} = g^c \\
g^{x2} &= g^{Mx} = g^{My} \\
g^x &= g^{ij} = g^{kj} \text{ for any } j = y, x, x_2
\end{align*}
\]

Given production technologies, for intermediate goods from the consumption and equipment sector to be used in production along the BGP, the growth rate of output in the consumption and intermediate equipment should equalize. Such feature stems
from the optimality conditions of the firms in intermediate inputs.

\[
\alpha_{my} \alpha_{y2} \left( \frac{Y}{M_{yy}} \right) \frac{1}{\alpha_{y2} + (1 - \alpha_{y2}) \left( \frac{M_{xy}}{M_{yy}} \right) p_{y}} = 1
\]  \hspace{1cm} (3)

Unless the growth rates of input intake from the equipment and consumption sector are the same, the optimality condition would not be satisfied along the BGP.

\[ g^{x2} = g^{y} \]

From the production technology in the consumption sector and intermediate equipment sector we obtain

\[ g^{y} = \gamma^{y} (g^{x})^{\alpha_{y1}} (g^{y})^{\alpha_{my}} \]

\[ g^{x2} = \gamma^{x2} (g^{x})^{\xi} (g^{y})^{\alpha_{mx2}} \]

Hence,

\[ \gamma^{x2} = \gamma^{y} (g^{x})^{\alpha_{y1} - \zeta} (g^{y})^{\alpha_{my} - \alpha_{mx2}} \]

which depends on the relative capital intensity of the consumption sector and intermediate equipment sector.

Finally, from the investment sector technology, we have

\[ g^{x} = \gamma^{x} (g^{x})^{\alpha_{z1}} (g^{y})^{1 - \alpha_{z1}} \]

Combining this equation with the one describing growth rates in the consumption sector, we obtain the BGP of the economy, i.e.

\[ g^{x} = (\gamma^{x})^{\psi_{z1}} (\gamma^{y})^{\psi_{x2}} \]

\[ g^{y} = (\gamma^{x})^{\psi_{y1}} (\gamma^{y})^{\psi_{y2}} \]
\( \psi_{x1} = \frac{1-\alpha_{my}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}} \) and \( \psi_{x2} = \frac{\alpha_{mx}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}} \). Also, \( \psi_{y1} = \frac{a_{y2}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}} \) and \( \psi_{y2} = \frac{1-a_{x1}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}} \).

To show number 1), note that the problem that was pointed out in 3 is not present anymore, for \( \rho = 0 \). Hence, the condition \( g^{x2} = g^{y} \) need not hold. The algebra gets more cumbersome but it is possible to show that the BGP will solve

\[
\begin{align*}
g^{x} &= \gamma^{x} (g^{x})^{\alpha_{x1}} ((g^{x2})^{\alpha_{x2}} (g^{y})^{1-\alpha_{x2}})^{\alpha_{mx}} \\
g^{y} &= \gamma^{y} (g^{x})^{\alpha_{y1}} ((g^{x2})^{1-\alpha_{y2}} (g^{y})^{\alpha_{y2}})^{\alpha_{my}} \\
g^{x2} &= \gamma^{x2} (g^{x})^{\xi} (g^{x2})^{\alpha_{mx2}}
\end{align*}
\]

Number 2) is analogous to number 3) but now the system of equations to be solved is

\[
\begin{align*}
g^{x} &= \gamma^{x} (g^{x})^{\alpha_{x1}} ((g^{x2})^{\alpha_{x2}})^{\alpha_{mx}} \\
g^{y} &= \gamma^{y} (g^{x})^{\alpha_{y1}} ((g^{y})^{\alpha_{y2}})^{\alpha_{my}} \\
g^{x2} &= \gamma^{x2} (g^{x})^{\xi} (g^{x2})^{\alpha_{mx2}}
\end{align*}
\]
12 Appendix (C)

12.1 Optimality and Steady State

Feasibility dictates

\[
\begin{align*}
    k'_{x1} - k_{x1} (1 - \delta) &= i_{x1} \quad (\kappa_x) \\
    k'_{x2} - k_{x2} (1 - \delta) &= i_{x2} \quad (\kappa_x) \\
    k'_{y} - k_{y} (1 - \delta) &= i_{y} \quad (\kappa_y) \\
    i_{y} + i_{x1} + i_{x2} &= X_1 \quad (\lambda_x) \\
    M_{xx} + M_{xy} &= X_2 \quad (\lambda_x) \\
    C + M_{yy} + M_{yx} &= Y \quad (\lambda_y) \\
    M_{yx} &= M_{yx1} + M_{yx2} 
\end{align*}
\]

The corresponding optimality conditions are

\[
\begin{align*}
    \lambda_x &= \beta \lambda'_{x} \left[ \alpha_{x1} \left( \frac{X_{1}'}{k'_{x1}} \right) + (1 - \delta) \right] \\
    \lambda_x &= \beta \lambda_{x} \left[ \frac{X_{2}'}{X_{x}} \xi_{x} \left( \frac{X_{2}'}{k'_{x2}} \right) + (1 - \delta) \right] \\
    \lambda_x &= \beta \lambda_{y} \left[ \frac{Y'}{X_{y}} \alpha_{y1} \left( \frac{Y'}{k'_{y}} \right) + (1 - \delta) \right] \\
    (1 - \alpha_{y1}) \alpha_{y2} \left( \frac{Y}{M_{yy}} \right) &\frac{(M_{yy})^{\rho_y}}{\alpha_{y2} (M_{yy})^{\rho_y} + (1 - \alpha_{y2}) (M_{xy})^{\rho_y}} = 1 \quad (M_{yy}) \\
    \lambda_{y} (1 - \alpha_{y1}) (1 - \alpha_{y2}) \left( \frac{Y}{M_{xy}} \right) &\frac{(M_{xy})^{\rho_y}}{\alpha_{y2} (M_{yy})^{\rho_y} + (1 - \alpha_{y2}) (M_{xy})^{\rho_y}} = \lambda_{x2} \quad (M_{xy}) 
\end{align*}
\]
\[
\lambda_x (1 - \alpha_{x1}) \alpha_{x2} \left( \frac{X_1}{M_{xx}} \right) \frac{(M_{xx})^{\rho_x}}{\alpha_{x2} (M_{xx})^{\rho_x} + (1 - \alpha_{x2}) (M_{yx1})^{\rho_x}} = \lambda_{x2} \quad (M_{xx})
\]
\[
\lambda_x (1 - \alpha_{x1}) (1 - \alpha_{x2}) \left( \frac{X_1}{M_{yx1}} \right) \frac{(M_{yx1})^{\rho_x}}{\alpha_{x2} (M_{xx})^{\rho_x} + (1 - \alpha_{x2}) (M_{yx1})^{\rho_x}} = \lambda_y \quad (M_{yx})
\]
\[
\lambda_{x2} (1 - \zeta_x) \left( \frac{X_2}{M_{yx2}} \right) = \lambda_y \quad (M_{yx2})
\]

This is a standard convex economy. Hence, the equilibrium exits and its unique. Also the welfare theorems hold.

### 12.2 Steady State

From the production function, we obtain

\[
\frac{X_1}{M_{xx}} = \left( \frac{k_{x1}}{M_{xx}} \right)^{\alpha_{x1}} \left( \alpha_{x2} + (1 - \alpha_{x2}) \left( \frac{M_{yx1}}{M_{xx}} \right)^{\rho_x} \right)^{1 - \alpha_{x1}}
\]

Using the optimality condition in intermediate goods and capital we can rewrite the equation as

\[
\frac{\lambda_{x2}}{\lambda_x (1 - \alpha_{x1}) \alpha_{x2}} = \left( \frac{\alpha_{x1}}{1 - \alpha_{x1}} \right) \frac{1 - \beta (1 - \delta)}{\lambda_x} \left[ \frac{1}{\alpha_{x2} + (1 - \alpha_{x2}) \left( \frac{1 - \alpha_{x2}}{\alpha_{x2} \lambda_y} \right)^{\rho_x} \left( 1 - \alpha_{y1} \right)^{1 - \rho_x}} \right]
\]

which defines the equilibrium relative prices of investment goods versus consumption goods.

Using the production function in the final good sector we can solve for \( \frac{\lambda_{x2}}{\lambda_y} \) as

\[
\frac{Y}{M_{yy}} = \left( \frac{k_y}{M_{yy}} \right)^{\alpha_{y1}} \left( \alpha_{y2} + (1 - \alpha_{y2}) \left( \frac{M_{xy}}{M_{yy}} \right)^{\rho_y} \right)^{1 - \alpha_{y1}}
\]

Following the same procedure as before, we can express this equation as a function of the relative price of investment and final goods.
\[
\frac{1}{(1 - \alpha_{y1}) \alpha_{y2}} = \left( \frac{\alpha_{y1}}{(1 - \alpha_{y1}) \alpha_{y2}} \frac{\beta}{1 - \beta(1 - \delta)} \frac{\lambda_x}{\lambda_y} \right)^{\alpha_{y1}} \left[ \alpha_{y2} + (1 - \alpha_{y2}) \left( \frac{1 - \alpha_{y2}}{\alpha_{y2}} \frac{\lambda_y}{\lambda_{x2}} \right)^{\rho_y - \rho_y} \right]^{(1 - \alpha_{y1}) \alpha_{y2}} \tag{5}
\]

Finally, the production technology in the third sector dictates
\[
\frac{X_2}{k_{x2}} = \left( \frac{M_{y, x2}}{k_{x2}} \right)^{(1-\varsigma_x)}
\]
\[
\left( \frac{1 - \beta (1 - \delta)}{\beta} \frac{\lambda_x}{\lambda_{x2}} \frac{1}{\varsigma_x} \right) = \left( \frac{1 - \beta (1 - \delta)}{\beta} \frac{\lambda_y}{\lambda_{y2}} \frac{1}{\varsigma_x} \right)^{(1-\varsigma_x)} \tag{6}
\]

Hence, equations (4), (5), (6) define a system of three equations and three unknowns.

Given the calibrated parameters I impose conditions on the share of value added in the third sector so that the system is exactly determined.

From the feasibility condition in intermediate goods of the investment sector we obtain
\[
\frac{M_{x, x}}{k_{x2}} + \frac{M_{x, y}}{k_{x2}} = \frac{X_2}{k_{x2}}
\]
\[
\frac{\lambda_x}{\lambda_{x2}} \frac{(1 - \alpha_{x1}) \alpha_{x2}}{\alpha_{x2} + (1 - \alpha_{x2}) \left( \frac{1-\alpha_{x2}}{\alpha_{x2}} \frac{\lambda_y}{\lambda_y} \right)^{\rho_y - \rho_y}} \frac{X_1}{X_2} + \frac{\lambda_y}{\lambda_{y2}} \frac{(1 - \alpha_{y1}) (1 - \alpha_{y2})}{\alpha_{y2} \left( \frac{\alpha_{y2}}{1-\alpha_{y2}} \frac{\lambda_y}{\lambda_y} \right)^{\rho_y - \rho_y}} \frac{Y}{X_2} = 1
\]

which determines the ratio of gross output in the production of equipment, as well as the ratio of consumption good production to intermediate investment goods, as a function of parameters and equilibrium prices.

If we now turn to the feasibility condition in the final production investment sector, we have
\[
\delta (1 + \frac{k_y}{k_x} + \frac{k_{x2}}{k_x}) = \frac{X_1}{k_x}
\]
\[ \delta(1 + \frac{\lambda_y Y}{\lambda_x X_2} \alpha_{y1} X_2 + \left( \frac{\lambda_{x2}}{\lambda_x} \right) \frac{\zeta_x}{\alpha_{x1}} \frac{X_2}{X_1}) = \frac{1 - \beta(1 - \delta)}{\beta} \frac{1}{\alpha_{x1}} \]

If we put both feasibility conditions together we obtain a system of two equations in two unknowns, i.e. the ratios of gross output across sectors.

To pin down the levels of the variables use the feasibility constraint in the consumption good sector.

\[ \frac{U^{-1}(\lambda_y^*)}{M_{yy}} + 1 + \frac{M_{yx}}{M_{yy}} = \frac{Y}{M_{yy}} \]

where \( \frac{Y}{M_{yy}} = \frac{\alpha_{y2} + (1 - \alpha_{y2})(\frac{M_{yy}}{M_{yy}})^{\rho_y}}{(1 - \alpha_{y1})\alpha_{y2}}. \) As we have shown before, \( \frac{M_{yy}}{M_{yy}} \) is a function of the prices in the economy. In other words, \( M_{yy} \) solves

\[ \frac{U^{-1}(\lambda_y^*)}{M_{yy}} + 1 + \frac{\alpha_{y2} + (1 - \alpha_{y2})\left(\frac{M_{yy}}{M_{yy}}\right)^{\rho_y}}{(1 - \alpha_{y1})\alpha_{y2}} = \frac{X_2}{Y} \left[ 1 - \frac{\lambda_x}{\lambda_y} \right] \frac{X_1}{\alpha_{x1}} \frac{\lambda_x}{\lambda_y} \frac{1 - \alpha_{x1}}{1 - \alpha_{x2}} \frac{Y}{\alpha_{x2}} \left( \frac{M_{xx}}{M_{yy}} \right)^{\rho_x} + (1 - \alpha_{x2}) \]

\[ = \frac{\alpha_{y2} + (1 - \alpha_{y2})\left(\frac{M_{yy}}{M_{yy}}\right)^{\rho_y}}{(1 - \alpha_{y1})\alpha_{y2}} \]

where \( \frac{M_{yy}}{M_{yy}} = \left( \frac{\lambda_y}{\lambda_{x2}} \frac{1 - \alpha_{y2}}{\alpha_{y2}} \right)^{1 - \rho_y} \) and \( \frac{M_{yx}}{M_{yy}} = \left( \frac{\alpha_{x2}}{1 - \alpha_{x2}} \frac{\lambda_y}{\lambda_{x2}} \right)^{1 - \rho_x}. \)

Once \( M_{yy}^* \) is determined \( M_{yx}^* \) is too, as well as \( Y^* \), \( k_y^* \), \( M_{xx}^* \) from the optimality conditions. \( k_x^* \) is determined using \( M_{xx}^* \) and then \( K^* \) can be computed. \( X^* \) is solved by the equilibrium ratio \( \frac{X^*}{Y^*} \) and \( Y^* \).
## Appendix (D)

### Parameter Definition Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{x_1,x_2,g}$</td>
<td>Persistence Permanent Shock</td>
<td>0.98</td>
</tr>
<tr>
<td>$g^x$</td>
<td>Gross Output Growth Rate, Equipment</td>
<td>3.15%</td>
</tr>
<tr>
<td>$g^y$</td>
<td>Gross Output Growth Rate, Consumption</td>
<td>1.50%</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount Factor</td>
<td>0.98</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital Depreciation</td>
<td>0.05</td>
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### Parameter Model Equation (Steady State) Value

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model Equation (Steady State)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_x$</td>
<td>Nec. and Suf. conditions for optimality</td>
<td>-0.625</td>
</tr>
<tr>
<td>$\alpha_{x_2}$</td>
<td>in intermediate inputs, Equipment</td>
<td>0.588</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>Nec. and Suf. conditions for optimality</td>
<td>-0.789</td>
</tr>
<tr>
<td>$\alpha_{y_2}$</td>
<td>in intermediate inputs, Consumption</td>
<td>0.8</td>
</tr>
<tr>
<td>$\alpha_{x_1}$</td>
<td>optimality in capital, Equipment</td>
<td>0.182</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>consistency in prices</td>
<td>0.984</td>
</tr>
<tr>
<td>$\alpha_{y_1}$</td>
<td>optimality in capital, Consumption</td>
<td>0.210</td>
</tr>
</tbody>
</table>

Table 6: Technology parameters

<table>
<thead>
<tr>
<th>Input/Sector</th>
<th>Equipment</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Equipment</td>
<td>15%</td>
<td>32%</td>
</tr>
<tr>
<td>Consumption</td>
<td>85%</td>
<td>68%</td>
</tr>
</tbody>
</table>

Table 7: Share of Equipment and Manufacturing Intermediate Inputs
<table>
<thead>
<tr>
<th>Definition</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Permanent Shocks</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_\eta^g$</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\eta^{x1}$</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\eta^{x2}$</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>$\text{Corr}(\eta^{x2}, \eta^{x1})$</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td><strong>Transitory Shocks</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\sigma_\varepsilon^g)$</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>$(\sigma_\varepsilon^{x1})$</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>$(\sigma_\varepsilon^{x2})$</td>
<td>0.085</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Structural Parameters, Permanent-Transitory Model

<table>
<thead>
<tr>
<th>TARGETS</th>
<th>DATA</th>
<th>MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP hp-filtered(6.25)</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td>Equipment in Equipment</td>
<td>-0.55</td>
<td>-0.21</td>
</tr>
<tr>
<td>Consumption in Equipment</td>
<td>0.5</td>
<td>0.21</td>
</tr>
<tr>
<td>Equipment in Consumption</td>
<td>-0.21</td>
<td>-0.20</td>
</tr>
<tr>
<td>Consumption in Consumption</td>
<td>0.29</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 9: Correlation with GDP

<table>
<thead>
<tr>
<th></th>
<th>Transitory, $\varepsilon$</th>
<th>Persistent, $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A^g$</td>
<td>$A^{x1}$</td>
</tr>
<tr>
<td>Aggregate Output</td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>Total Factor Productivity</td>
<td>0.30</td>
<td>0.47</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.31</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Table 10: Variance Decomposition, Baseline
Table 11: Parametrization, Baseline CES vs. Cobb Douglas technology

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>DEFINITION</th>
<th>CES</th>
<th>COBB-Douglas</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_x$</td>
<td>Elasticity of substitution, Equipment</td>
<td>-0.625</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha_{x2}$</td>
<td>Share of equipment intermediates, Equipment</td>
<td>0.76</td>
<td>0.42</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>Elasticity of substitution, Consumption</td>
<td>-0.789</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha_{y2}$</td>
<td>Share of consumption intermediates, Consumption</td>
<td>0.81</td>
<td>0.85</td>
</tr>
<tr>
<td>$\alpha_{x1}$</td>
<td>Share of capital, Equipment</td>
<td>0.182</td>
<td>0.182</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Share of capital, Equipment intermediates</td>
<td>0.881</td>
<td>0.995</td>
</tr>
<tr>
<td>$\alpha_{y1}$</td>
<td>Share of capital, Consumption</td>
<td>0.210</td>
<td>0.210</td>
</tr>
</tbody>
</table>

Table 12: Cost shares across inputs and sectors

<table>
<thead>
<tr>
<th>COST SHARES</th>
<th>DATA</th>
<th>CES</th>
<th>COBB-Douglas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equipment in Equipment</td>
<td>0.15</td>
<td>0.43</td>
<td>0.42</td>
</tr>
<tr>
<td>Consumption in Equipment</td>
<td>0.85</td>
<td>0.57</td>
<td>0.58</td>
</tr>
<tr>
<td>Equipment in Consumption</td>
<td>0.12</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td>Consumption in Consumption</td>
<td>0.88</td>
<td>0.85</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Table 13: Variance Decomposition
Intermediate Investment Shock, Persistent
Constant Cost Shares, Red. Flexible Cost Shares, Blue *

Student Version of MATLAB
Equipment Intermediates Sector Shock, Persistent
Constant Cost Shares, Red. Flexible Cost Shares, Blue *

GDP

Gross Final Output

TFP

Consumption

Investment

Relative Prices

Student Version of MATLAB
Equipment Investment Sector Shock, Transitory
Constant Cost Shares, Red. Flexible Cost Shares, Blue

Student Version of MATLAB
Equipment Intermediates Sector Shock, Transitory
Constant Cost Shares, Red. Flexible Cost Shares, Blue∗

GDP

Gross Final Output

TFP

Consumption

Investment

Relative Prices

Student Version of MATLAB

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