International Disaster Risk, Business Cycles, and Exchange Rates – Preliminary and Incomplete –

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Abstract

What accounts for the unprecedented decline in world trade during the crisis? What have been the consequences of shifting risk appetites for international capital flows? How have they differed across developed and developing economies? We answer these questions in an international real business cycle model with time-varying disaster risk. We interpret the recent crisis as an increase in the probability of disasters. It leads to a decrease of investment and a recession worldwide. In our model, capital pulls out of the riskier country, which experience the largest recession. Hence, the real exchange rate depreciates. Both stock markets tank, and short-term safe interest rates fall.

Keywords: business cycles, equity premium, time-varying risk premium, return predictability, disasters, rare events, jumps.


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1 Introduction

What accounts for the unprecedented decline in world trade during the crisis? What have been the consequences of shifting risk appetites for international capital flows? How have they differed across developed and developing economies? We answer these questions in an international real business cycle (IRBC) model with time-varying disaster risk. We interpret the recent crisis as an increase in the probability of disasters. It leads to a decrease of investment and to a recession worldwide. In our model, capital pulls out of the riskier country, which experience the largest recession. Hence, the real exchange rate depreciates. Both stock markets tank, and short-term safe interest rates fall.

In this paper, we explore whether a class of disaster-based models that postulate the existence of rare but large adverse aggregate shocks can account for some macroeconomic features of the recent crisis and address some of the remaining puzzles in international economics. Our strategy is to start from a model that replicates the basic stylized facts on equity, currency and risk-free bond prices, and then study its implications for international macroeconomic quantities. Disaster-based models offer such a starting point. Pioneered by ?, and ?, this class of models has received much attention recently in the macroeconomics and finance literature. These models can potentially reproduce the dynamics of equity and currency risk premia, provided that there is some time-variation in the quantity or price of disaster risk. Here, we do so by assuming that the disaster probability is time-varying as in ?, ?, and ?.

Our IRBC model differs from the standard models because of the presence of disaster risk. As is standard in the IRBC literature, there is in each country a representative firm operating a neoclassical production function with costs of adjusting the capital stock. The production function depends on the stock of capital, the amount of labor and productivity. We assume that the two productivity processes are perfectly correlated. Countries can trade but they incur some quadratic trade costs. Financial markets are complete. We consider three different shocks: a standard normally distributed shock to total factor productivity (TFP), a disaster shock that might reduce TFP and destroy different portions of the home and foreign capital stocks, and a change in the probability of a disaster which follows a a Markov chain. The last two shocks are novel in the IRBC framework.

We consider two setups. We start with a two-country IRBC model with standard expected utility preferences. We report standard business cycle statistics. As ? shows in a closed-economy setting, this model has counterfactual implications that can be overcome by switching to ? preferences. With such preferences, the risk-aversion coefficient is no longer equal to the inverse of

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the intertemporal elasticity of substitution. It is thus possible to have them both simultaneously above unity. This is is key to produce our results. In this case, a higher disaster probability leads to a lower investment, higher consumption and a real exchange rate depreciation (instead of a real exchange rate appreciation with power utility). Unfortunately, solving a two-country IRBC model with \( \pi \) preferences is a daunting task because these preferences imply that Pareto weights in the social planner maximization problem are not constant. To circumvent this difficulty, we focus on the case of a small open economy with \( \pi \) preferences. We solve for the optimal behavior of the large economy (the US) as if it were closed. In essence, we assume that international trade is negligible in this first step. In the second step, we consider the small open economy. There, consumers and firms then decide on consumption, investment, labor and output, and thus net exports, taking as given the world prices (including the risk prices) set by the large economy.

Beyond the current crisis, our paper relates to a large literature in international economics. This literature offers a long list of puzzles, some on quantities, some on prices. We rapidly review here these puzzles because, again, our strategy is to build a model that can account for these price puzzles, and then study its implications for quantities.

Looking at quantities, two puzzles stand out, broadly grouped under co-movement and sudden-stop quandaries. In the data, the cross-country correlation of consumption is lower than the cross-country correlation of gross domestic product (GDP). The cross-country correlation of investment and hours worked are positive. The correlation between net exports and GDP is negative. In the benchmark international RBC model of \( \pi \), the opposite is true. Emerging countries experience sudden stops: a drop in output, investment and consumption and an increase in net exports and current accounts. In emerging countries, the volatility of consumption is higher than the volatility of output. These facts are difficult to replicate in international RBC models.

Looking at prices, four stylized facts remain puzzling for the international RBC literature: the uncovered interest rate parity (UIP) puzzle, the low correlation between real exchange rates and relative consumption growth rates, the volatility of real exchange rates, and the high cross-country correlation of stock prices compared to the low cross-country correlation of dividends and output. According to the UIP condition, the expected change in exchange rates should be equal to the interest rate differential between foreign and domestic risk-free bonds. The UIP condition implies that a regression of exchange rate changes on interest rate differentials should produce a slope coefficient of 1. Instead, empirical work following \( \pi \) and \( \pi \) consistently reveals a slope coefficient that is smaller than 1 and very often negative. The international economics literature refers to these negative UIP slope coefficients as the UIP puzzle or forward premium anomaly. Most RBC models imply tiny departure from UIP. \( \pi \) show that a necessary condition for a model to replicate the empirical evidence on currency risk premia is the existence of a common component in

\[ \text{footnote} 2: \text{and } \pi \text{ obtain a positive correlation between savings and investment in models with perfect capital mobility and a negative correlation between net exports and output.} \]
stochastic discount factors across countries and some heterogeneity in the loadings on this common component. In this paper, countries suffer the same reduction in TFP should a disaster happen. This implies the existence of a common component in their pricing kernels. The countries differ, however, in how much capital they will lose should a disaster occur. This introduces a key source of heterogeneity across countries.

? note that in complete markets and with power utility, the change in the real exchange rate is equal to the relative consumption growth in two countries times the risk-aversion coefficient, thus implying a perfect correlation between the consumption growth and real exchange rate variations. Yet, in the data, ? find that the actual correlation between exchange rate changes and consumption growth rates is low and often negative. , , and ? confirm their findings. In our model with ? preferences, real exchange rates are negatively correlated to consumption growth rates.

The volatility of the real exchange rates is also puzzling from the perspective of standard IRBC. From a theoretical international trade perspective, actual real exchange rates appear often very volatile. From an asset pricing perspective, ? show that they are too smooth. In order to fit the equity premium, we know that the variance of the stochastic discount factor has to be high ( and ?). We also know that the correlation among consumption growth shocks across countries is low. Power utility thus implies a low correlation of stochastic discount factors. When financial markets are complete, changes in real exchange rates are equal to the ratio of the foreign to domestic stochastic discount factors. As a result, real exchange rates built from asset pricing kernels are much more volatile than the observed ones. To make matters worse, models can usually not reproduce with the same set of parameters both the pre-and post-Bretton Woods exchange rate volatilities because they start from shocks that are not that different across time periods; ? show that real consumption growth shocks have similar volatilities in both sub-periods. In our model, real exchange rates are determined on international good markets. But shocks to the disaster probability increase their volatility.

Finally, the cross-country correlation of stock prices is much greater than the correlation of fundamentals (for example, dividends). ? address this puzzle in a ?’s long run risk model but they need to assume that the time-varying means of consumption growth rates are perfectly correlated across countries. Our paper offers a different interpretation. Here, the cross-country correlation of stock prices is much greater than the correlation of fundamentals because asset prices are driven by the disaster probability which is common across countries. Yet countries differ in terms of fundamentals, in normal times as in times of disaster.

The rest of the paper is organized as follows: section I outlines the two-country model with standard preferences. Section II presents the small economy case with ? preferences. Section III reports the simulation results. Section IV concludes. An appendix reports the simulation algorithm.
2 A two-country IRBC model with disaster risk

In this section, we set up a two-country DSGE model. We follow closely the benchmark of the literature (e.g. ?) and depart from it only through our consideration of disaster risk. This allows us to consider how a time-varying risk of disaster affects the implications of this model for business cycle quantities, the current account, and the real exchange rate.

Throughout this section, the two countries (home and foreign) are symmetric, except where noted. We denote the foreign country with a superscript *. 

2.1 Preferences

In each country, there is a representative agent with standard expected utility preferences:

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t) \]

where

\[ u(C, 1 - N) = \frac{C^{\nu(1-\gamma)}(1 - N)^{(1-\nu)(1-\gamma)}}{1 - \gamma}. \]

We use preferences consistent with a balanced growth path, because our model will feature permanent shocks, which are somewhat easier to analyze.

2.2 Technology

In each country, there is a representative firm operating a neoclassical production function:

\[ Y_t = F(K_t, z_t N_t), \]
\[ Y_t^* = F(K_t^*, z_t^* N_t^*), \]

where \( K_t \) is capital (at the beginning of period \( t \)), \( N_t \) is labor, \( Y_t \) is output, and \( z_t \) is productivity. We will use the standard Cobb-Douglas formulation, \( F(K, zN) = K^\alpha (zN)^{1-\alpha} \).

We will make the following assumption, which simplifies the analysis substantially:

\[ z_t = z_t^*. \]

Hence, the two countries share exactly the same total factor productivity. It is easy to extend this to allow for a constant difference between the two countries, i.e. \( z_t = z_t^* \lambda \) for some fixed \( \lambda > 0 \).

As is standard in the IRBC literature, we assume that there are costs of adjusting the capital stock (otherwise, capital flows instantaneously to the highest possible return). Capital accumula-
tion takes the form:

\[ \tilde{K}_t = (1 - \delta)K_t + \phi \left( \frac{I_t}{K_t} \right) K_t, \]

where \( \tilde{K}_t \) is the capital at the end of period \( t \). The reason for this timing will be apparent in the next paragraph. The foreign country has the same adjustment cost technology.

### 2.3 Shocks

Our economy will be hit by three different shocks. First, we use a standard normally distributed shock to TFP, and we assume that TFP follows a unit root process. Second, with some probability each period, a disaster will occur. A disaster will (i) reduce TFP by \( b_z \% \), (ii) destroy \( b_k \% \) of the home economy’s capital stock, and (iii) destroy \( b^*_k \% \) of the foreign economy capital stock. Third, the probability of a disaster at time \( t + 1 \) is time-varying: more precisely, at the beginning of period \( t \), the probability of a disaster in the next period is drawn, according to a Markov chain with transition matrix \( Q \). Typically, one can think of the probability as following an AR(1), with the proviso that it must stay between 0 and 1.

The law of motion for productivity is:

\[ \log z_{t+1} = \log z_t + \mu + \sigma \varepsilon_{t+1} + x_{t+1} \log(1 - b_z), \]

where \( x_{t+1} = 1 \) with probability \( p_t \), and 0 with probability \( 1 - p_t \); and

\[ K_{t+1} = (1 - b_k x_{t+1}) \tilde{K}_t, \]

\[ K^*_{t+1} = (1 - b^*_k x_{t+1}) \tilde{K}^*_t, \]

Note that the two countries enjoy the same TFP at any point in time, and hence in particular they will suffer the same reduction in TFP should a disaster happen. The countries differ, however, in how much capital they will lose should a disaster occur. This makes one country riskier than the other. In our application, home will be the less risky country, and we assume it is a developed market economy (e.g. the U.S.). The foreign economy will be an emerging market. We think of this simple assumption as capturing the fact that countries have different risk, perhaps due to different industry compositions or different financial structures.

The assumption that a disaster reduces the capital stock requires some explanation. This could be due to a war which physically destroys capital, but there are alternative interpretations. For instance, \( b_k \) could reflect expropriation of capital holders (if the capital is taken away and then not used as effectively), or it could be a “technological revolution” that makes a large share of the capital worthless. It could also be that even though physical capital is not literally destroyed, some
intangible capital (such as matches between firms, employees, and customers) is lost. Finally, one can imagine a situation where the demand for some goods falls sharply, rendering worthless the factories which produce them.\footnote{In a large downturn, the demand for some luxury goods such as boats, private airplanes, etc. would likely fall sharply. If this situation were to last, the boats-producing factories would never operate at capacity.}

Note that we have made the assumption that the disaster is the same event, i.e. we are considering only worldwide disasters and not country-specific disasters. An interesting extension is to consider idiosyncratic disasters. Finally, in a richer setup, the risk parameters $b_k$ and $b_k^*$ may be changing over time.

### 2.4 Resource constraint

As usual, we assume that the labor market is national, but the capital market is international. Financial markets are assumed to be complete. However, we assume that there are some costs to trading goods between the two countries. The world equilibrium resource constraint for goods is:

$$C_t + C_t^* + I_t + I_t^* = Y_t + Y_t^* - \kappa \left( \frac{N X_t}{Y_t} \right) Y_t,$$

where $N X_t = Y_t - C_t - I_t$ are net exports of the home country, and $\kappa(.)$ captures trade costs. $\kappa$ is assumed to satisfy the following properties:

\begin{align*}
\{ & \kappa(x) \geq 0, \\
& \kappa'(x) > 0 \text{ for } x > 0 \text{ and } \kappa'(x) < 0 \text{ for } x < 0, \\
& \kappa \text{ is convex.} \}
\end{align*}

An example is: $\kappa(x) = \kappa x^\xi \frac{x}{x^2}$, for $\xi \geq 1$. A trading firm which employs good to ship goods can justify this trade cost function.

### 2.5 Solution of the model

Since markets are complete, the first welfare theorem holds and the market equilibrium can be computed as the solution to a social planner problem. The sequence problem is, denoting $\lambda_1$ and $\lambda_2$ the Pareto weights:

$$\max_{\{C_{1,t}, C_{2,t}, I_{1,t}, I_{2,t}, N_{1,t}, N_{2,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \lambda_1 u(C_{1,t}, N_{1,t}) + \lambda_2 u(C_{2,t}, N_{2,t}) \right)$$
\[s.t.:\]
\[
K_{1,t+1} = \left( (1 - \delta)K_{1,t} + \phi \left( \frac{I_{1,t}}{K_{1,t}} \right) K_{1,t} \right) (1 - x_{t+1}b_{k,1})
\]
\[
K_{2,t+1} = \left( (1 - \delta)K_{2,t} + \phi \left( \frac{I_{2,t}}{K_{2,t}} \right) K_{2,t} \right) (1 - x_{t+1}b_{k,2})
\]

\[x_{t+1} = 1 \text{ if disaster (w/prob } p_t), 0 \text{ otherwise}\]

\[p_{t+1}|p_t \text{ Markov chain with transition } Q\]

and the resource constraint is

\[C_{1,t} + C_{2,t} + I_{1,t} + I_{2,t} = Y_{1,t} + Y_{2,t} - \kappa \left( \frac{Y_{1,t} - I_{1,t} - C_{1,t}}{Y_{1,t}} \right) Y_{1,t}.\]

We can write the first-order conditions of this problem. Yet, due to the highly non-linear nature of the solution, we cannot linearize the system of first-order conditions as is commonly done in the literature. In contrast, we provide a full non-linear solution, which we obtain by studying the problem recursively.

### 2.5.1 Bellman Equation

The Bellman equation corresponding to the planner problem is:

\[
W(K_1, K_2, z, p) = \max_{C_1, C_2, N_1, N_2, I_1, I_2} \left\{ \lambda_1^1 C_1^{(1-\gamma)(1-N_1)(1-\gamma)(1-\gamma)} + \lambda_2^2 C_2^{(1-\gamma)(1-N_2)(1-\gamma)(1-\gamma)} + \beta E[W(K_1', K_2', z', p')] \right\}
\]

\[s.t.:\]
\[
K_1' = \left( (1 - \delta)K_{1,t} + \phi \left( \frac{I_{1,t}}{K_{1,t}} \right) \right) (1 - x'b_{k,1})
\]
\[
K_2' = \left( (1 - \delta)K_{2,t} + \phi \left( \frac{I_{2,t}}{K_{2,t}} \right) \right) (1 - x'b_{k,2})
\]
\[
\log z' = \log z + \mu + \sigma z' + x' \log(1 - b_z)
\]
\[
C_1 + C_2 + I_1 + I_2 = Y_1 + Y_2 - \kappa \left( \frac{N X}{Y_1} \right) Y_1.
\]

We now use a standard trick to simplify the problem, by rescaling variables by the TFP shock. Define \( k_i = K_i/z \), and similarly \( y_i = Y_i/z = k_i^{\alpha} N_i^{1-\alpha} \), \( c_i = C_i/z \). We have then the following law of
motions for the two rescaled capital stocks:

\[
k'_1 = \frac{1 - \delta + \phi \left( \frac{I}{K_1} \right)}{e^{\mu + \sigma \varepsilon}(1 - x'b_z)} k_1(1 - x'b_{k,1})
\]

\[
k'_2 = \frac{1 - \delta + \phi \left( \frac{I}{K_2} \right)}{e^{\mu + \sigma \varepsilon}(1 - x'b_z)} k_2(1 - x'b_{k,2})
\]

We can now guess and verify that the value function takes the specific form:

\[
W(K_1, K_2, z, p) = \frac{z^{v(1-\gamma)}}{1-\gamma} g(k_1, k_2, p),
\]

where \( g \) satisfies the following equation:

\[
g(k_1, k_2, p) = \max_{c_1, c_2, N_1, N_2, i_1, i_2} \left\{ \lambda_1 c_1^{v(1-\gamma)}(1 - N_1)\frac{(1-v)(1-\gamma)}{1-\gamma} + \lambda_2 c_2^{v(1-\gamma)}(1 - N_2)\frac{(1-v)(1-\gamma)}{1-\gamma} + \beta E_{p', x', \varepsilon'} e^{v(1-\gamma)(\mu + \sigma \varepsilon)}(1 - b_{x})x'v(1-\gamma)g(k'_1, k'_2, p') \right\}
\]

s.t.

\[
k'_1 = \frac{1 - \delta + \phi \left( \frac{I}{K_1} \right)}{e^{\mu + \sigma \varepsilon}(1 - x'b_z)} k_1(1 - x'b_{k,1})
\]

\[
k'_2 = \frac{1 - \delta + \phi \left( \frac{I}{K_2} \right)}{e^{\mu + \sigma \varepsilon}(1 - x'b_z)} k_2(1 - x'b_{k,2})
\]

\[
c_1 + c_2 + i_1 + i_2 = y_1 + y_2 - \kappa \left( \frac{y_1 - c_1 - i_1}{y_1} \right) y_1.
\]

To prove this, it is enough to check that the guess works, i.e.

\[
\frac{z^{v(1-\gamma)}}{1-\gamma} g(k_1, k_2, p) = \max_{c_1, c_2, N_1, N_2, i_1, i_2} \left\{ \lambda_1 c_1^{v(1-\gamma)}(1 - N_1)\frac{(1-v)(1-\gamma)}{1-\gamma} + \lambda_2 c_2^{v(1-\gamma)}(1 - N_2)\frac{(1-v)(1-\gamma)}{1-\gamma} + \beta E_{p', x', \varepsilon'} e^{v(1-\gamma)(\mu + \sigma \varepsilon)}(1 - b_{x})x'v(1-\gamma)g(k'_1, k'_2, p') \right\}
\]

s.t.

\[
k'_1 = \frac{1 - \delta + \phi \left( \frac{I}{K_1} \right)}{e^{\mu + \sigma \varepsilon}(1 - x'b_z)} k_1(1 - x'b_{k,1})
\]

\[
k'_2 = \frac{1 - \delta + \phi \left( \frac{I}{K_2} \right)}{e^{\mu + \sigma \varepsilon}(1 - x'b_z)} k_2(1 - x'b_{k,2})
\]

\[
C_1 + C_2 + I_1 + I_2 = \frac{Y_1 + Y_2}{z} - \kappa \left( \frac{NX}{Y_1} \right) \frac{Y_1}{z}
\]

\[\text{Note that we multiplied by } 1 - \gamma. \text{ Hence if } \gamma > 1, \text{ the max needs to be transformed into a min.}\]
Note that $k_j'$ depends on $k_j, i_j, \varepsilon'$ and $x'$. And $i_j$ itself depends on $k_j$ and $p$.

## 2.6 Calibration

At this stage, we wish to simply illustrate the mechanism in the model. We pick most parameters based on the previous literature. For now, we abstract from adjustment costs and set the adjustment cost function to $\phi(x) = x$.

[Table 1 about here.]

We consider two possible trade cost function $\kappa$ : either $\kappa(x) = \frac{\kappa}{2}x^2$, or $\kappa(x) = \kappa x^\eta$ more generally, for $\eta$ even (this coefficient must be positive to accommodate for negative values of $x$).

We start with a two-state process for the disaster probability: it can either take a low value $p$ or high value $\overline{p}$. The transition matrix is:

\[
Q = \begin{pmatrix}
\rho & 1 - \rho \\
1 - \rho & \rho
\end{pmatrix}.
\]

## 2.7 Results

We first consider impulse response functions to highlight the mechanisms of the model. Figures 1 and 2 report impulse response functions to TFP shocks. Figures 3 and 4 report impulse response functions to shocks on the disaster probability. Finally, figures 5 and 6 focus on the impact of disasters.

We then turn to business cycle statistics. In Table 2 we report standard deviations (as a fraction of the output volatility) and cross-correlations of macro-variables in the model (Consumption $C$, investment $I$ and hours worked $N$). These moments are obtained from a simulation with 20,000 periods, excluding disasters. The model is simulated at quarterly frequency using the parameters in Table 1.

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

[Figure 5 about here.]

[Figure 6 about here.]

[Table 2 about here.]
3 Small open economy with complete markets

We turn now to the case of a small open economy (SOE) with preferences. We suppose that there is a large economy and a small one. We solve the large economy as if it were closed (i.e. as in ?). This gives a stochastic discount factor (SDF) denoted $M$. Then, the small open economy, taking these prices (notably $M$) as given, makes its optimal consumption and investment decisions.

3.1 The large, safe economy (the U.S.)

The large, safe economy is as in ?. The representative household has preferences:

$$V_t = \left( u(C_t, N_t)^{1-\gamma} + \beta E_t \left( V_{t+1}^{1-\theta} \right)^{\frac{1}{1-\theta}} \right)^{\frac{1}{1-\gamma}}$$

with

$$u(C, N) = C^\nu (1 - N)^{1-\nu}.$$  

There is a representative firm, which produces output using a standard Cobb-Douglas production function:

$$Y_t = K_t^\alpha (z_t N_t)^{1-\alpha},$$

where $z_t$ is total factor productivity (TFP). The firm accumulates capital subject to adjustment costs:

$$K_{t+1} = (1 - \delta) K_t + \phi \left( \frac{I_t}{K_t} \right) K_t, \text{ if } x_{t+1} = 0,$$

$$= \left( (1 - \delta) K_t + \phi \left( \frac{I_t}{K_t} \right) K_t \right) (1 - b_k), \text{ if } x_{t+1} = 1,$$

where $\phi$ is an increasing and concave function, which curvature captures adjustment costs, and $x_{t+1}$ is 1 if there is a disaster at time $t+1$ (with probability $p_t$) and 0 otherwise (probability $1 - p_t$).

The resource constraint is

$$C_t + I_t \leq Y_t,$$

i.e. trade is negligible for this economy. Total factor productivity is affected by the “normal shocks” $\varepsilon_t$ as well as the disasters. A disaster reduces TFP by a permanent amount $b_{t fp}$:

$$\log z_{t+1} = \log z_t + \mu + \sigma \varepsilon_{t+1} + x_{t+1} \log(1 - b_{t fp})$$

where $\mu$ is the drift of TFP, and $\sigma$ is the standard deviation of “normal shocks”. Last, $p_t$ follows a stationary Markov process with transition function $Q$.

This model has three states: capital $K$, technology $z$ and probability of disaster $p$; two in-
dependent controls: consumption $C$ and hours worked $N$; and three shocks: the realization of disaster $x' \in \{0,1\}$, the draw of the new probability of disaster $p'$, and the “normal shock” $\varepsilon'$. The first welfare theorem holds, hence the competitive equilibrium is equivalent to a social planner problem, which is easier to solve. Denote $V(K, z, p)$ the value function, and define $W(K, z, p) = V(K, z, p)^{1-\gamma}$. The social planning problem can be formulated as:

$$W(K, z, p) = \max_{C,I,N} \left\{ \left( C^\gamma (1 - N)^{(1-\nu)} \right)^{1-\gamma} + \beta \left( E_{p',z',x'} W(K', z', p') \right)^{\frac{1-\theta}{1-\gamma}} \right\},$$

s.t.:

$$C + I \leq z^{1-\alpha} K^{\alpha} N^{1-\alpha},$$

$$K' = \left( (1 - \delta) K + \phi \left( \frac{I}{K} \right) K \right) (1 - x'b_k),$$

$$\log z' = \log z + \mu + \sigma \varepsilon' + x' \log(1 - b_{tp}).$$

We can write $W(K, z, p) = z^{\nu(1-\gamma)} g(k, p)$, where $k = K/z$, and $g$ satisfies the associated Bellman equation:

$$g(k, p) = \max_{c,i,N} \left\{ c^{\nu(1-\gamma)} (1 - N)^{(1-\nu)(1-\gamma)} + \beta e^{\nu(1-\gamma)} \left( E_{p',z',x'} e^{\sigma \varepsilon' (1-\theta)} (1 - x' + x'(1 - b_{tp}))^{\nu(1-\theta)} \right) g(k', p')^{\frac{1-\theta}{1-\gamma}} \right\},$$

s.t.:

$$c = k^\alpha N^{1-\alpha} - i,$$

$$k' = \frac{(1 - x'b_k) \left( (1 - \delta) K + \phi \left( \frac{I}{K} \right) K \right)}{e^{\mu + \sigma \varepsilon'} (1 - x'b_{tp}).}$$

The SDF is:

$$M_{t,t+1}^* = \beta \left( \frac{C_{t+1}^*}{C_t^*} \right)^{u(1-\gamma) - 1} \left( \frac{1 - N_{t+1}^*}{1 - N_t^*} \right)^{(1-\nu)(1-\gamma)} \left( \frac{V_{t+1}^*}{E_t \left( V_{t+1}^{1-\theta} \right)^{\frac{1}{1-\theta}}} \right)^{\gamma - \theta}. $$

### 3.2 The small open economy

It has the same structure, but the disaster size $b_k$ is different. We assume that the disaster is worldwide.

The household preferences are the same as in the U.S.:

$$V_t = \left( u(C_t, N_t)^{1-\gamma} + \beta E_t \left( V_{t+1}^{1-\theta} \right)^{\frac{1}{1-\theta}} \right)^{\frac{1}{1-\gamma}}$$
The foreign country also shares the same production function:

\[ Y_t = K_t^\alpha (z_t N_t)^{1-\alpha}, \]

and same TFP:

\[ \log z_{t+1} = \log z_t + \mu + \sigma \varepsilon_{t+1} + x_{t+1} \log (1 - b_{tfp}). \]

The firm accumulates capital subject to adjustment costs:

\[
K_{t+1} = (1 - \delta) K_t + \phi \left( \frac{I_t}{K_t} \right) K_t, \text{ if } x_{t+1} = 0, \\
= \left( (1 - \delta) K_t + \phi \left( \frac{I_t}{K_t} \right) K_t \right) (1 - b_k^*), \text{ if } x_{t+1} = 1,
\]

where \( \phi \) is an increasing and concave function, which curvature captures adjustment costs, and \( x_{t+1} \) is 1 if there is a disaster at time \( t+1 \) (with probability \( p_t \)) and 0 otherwise (probability \( 1 - p_t \)). The crucial difference across countries is that \( b_k^* \) need not be the same as \( b_k \).

The country can trade in perfect state-contingent markets, but trade costs are incurred as resources are moved from one country to the other. One way to set up the problem is to think of choosing state-contingent claims to maximize the utility of the household. We will not do this here.

Let \( N X_t = Y_t - C_t - I_t \) be the net exports of the SOE. The real exchange rate satisfies

\[
\frac{1}{e_t} = 1 + \kappa' \left( \frac{N X_t}{Y_t} \right),
\]

with 1 US good = \( e_t \) SOE goods.

We know the SDF \( M_{t,t+1}^* \) of the foreign economy. Given complete markets, the SDF at home satisfies

\[
M_{t,t+1} = M_{t,t+1}^* \frac{e_{t+1}}{e_t}.
\]

The problem of the SOE can now be split into the household problem and the firm value maximization problem:

Let \( M_{0,t} = M_{0,1} \times M_{1,2} \times \ldots \times M_{t-1,t} \) be the SDF for a \( t \)-period cash flow. Taking \( \{M_{0,t}\} \) and
\(\{w_t\}\) as given, the firm solves
\[
\max_{\{N_t, I_t\}} \sum_{t=0}^{\infty} M_{0,t} (F(K_t, z_t N_t) - w_t N_t - I_t)
\]
\[s.t.: K_{t+1} = \left(1 - \delta + \phi \left(\frac{I_t}{K_t}\right)\right) K_t (1 - x_{t+1} b_k^p)
\]

We can rewrite this problem in recursive form. Note that the US state variable \(k^*\) will enter this problem since it affects prices.

Hence,
\[
J(K, z, p, k^*) = \max_{N, I} \{F(K, zN) - wN - I + E_{z', p', x'} (M(J(K', z', p', k^*))}\}
\]
\[K' = \left(1 - \delta + \phi \left(\frac{I}{K}\right)\right) K (1 - x'b_k^e)
\]

\[
J(K, z, p, k^*) = \max_{N, I} \{F(K, zN) - wN - I + E_{z', p', x'} (M^*(k^*, p, \varepsilon', x') \frac{\varepsilon'}{e}.J(K', z', p', k^*))\}
\]
\[K' = \left(1 - \delta + \phi \left(\frac{I}{K}\right)\right) K (1 - x'b_k^e)
\]

leading to the FOC wrt \(I\):
\[
1 = E_{z', p', x'} \left(M^*(k^*, p, \varepsilon', x') \frac{\varepsilon'}{e}.J_1(K', z', p', k^*) (1 - x'b_k^e) \phi' \left(\frac{I}{K}\right)\right)
\]
\[= E_{z', p', x'} \left(M^*(k^*, p, \varepsilon', x') \frac{\varepsilon'}{e}.J_1(K', z', p', k^*) (1 - x'b_k^e)\right) \times \phi' \left(\frac{I}{K}\right)
\]

and to the envelope condition:
\[
J_1(K, z, p, k^*) = F_1(K, zN) + E_{z', p', x'} \left(M^*(k^*, p, \varepsilon', x') \frac{\varepsilon'}{e}.J_1(K', z', p', k^*) (1 - x'b_k^e)\right) \left(1 - \delta + \phi \left(\frac{I}{K}\right) - \frac{I}{K} \phi' \left(\frac{I}{K}\right)\right)
\]
\[= F_1(K, zN) + E_{z', p', x'} \left(M^*(k^*, p, \varepsilon', x') \frac{\varepsilon'}{e}.J_1(K', z', p', k^*) (1 - x'b_k^e)\right) \times \left(1 - \delta + \phi \left(\frac{I}{K}\right) - \frac{I}{K} \phi' \left(\frac{I}{K}\right)\right)
\]
\[= F_1(K, zN) + \frac{1}{\phi' \left(\frac{I}{K}\right)} \times \left(1 - \delta + \phi \left(\frac{I}{K}\right) - \frac{I}{K} \phi' \left(\frac{I}{K}\right)\right)
\]
\[= F_1(K, zN) - I + \frac{1 - \delta + \phi \left(\frac{I}{K}\right)}{\phi' \left(\frac{I}{K}\right)}
\]
Hence, substituting back, we get the "standard" investment Euler equation:

\[
\frac{1}{\phi' \left( \frac{I'}{K'} \right)} = E_{\varepsilon', \zeta', x'} \left( M^* (k^*, p, \varepsilon', x') \frac{\varepsilon'}{e} \cdot (1 - x'b_k^*) \left( F_1 (K', z'N') - I' + \frac{1 - \delta + \phi \left( \frac{I'}{K'} \right)}{\phi' \left( \frac{I'}{K'} \right)} \right) \right).
\]

The other firm FOC is simply

\[ F_2 (K_t, z_t N_t) = w_t \]

As for households, we have because of complete markets (equalize the MU of 1 unit of good in the US: you can use it to ship to the SOE, or keep it in the US)

\[
\frac{\partial V_t}{\partial C_t} = \mu e_t \frac{\partial V^*_t}{\partial C^*_t},
\]

for some real \( \mu \). (Taking the ratio of this condition at \( t + 1 \) and \( t \) yields \( M_{t,t+1} = \frac{e_{t+1}}{e_t} M^*_{t,t+1} \). \( \mu \) is a Pareto weight.

Rewriting the utility recursion as

\[ V_t^{1-\gamma} = \left( C_t^u (1 - N_t)^{1-v} \right)^{1-\gamma} + \beta E_t \left( V_{t+1}^{1-\theta} \right)^{\frac{1-\gamma}{1-\theta}}, \]

note that

\[ (1 - \gamma) \frac{\partial V_{t-\gamma}}{\partial C_t} = (1 - \gamma) v C_t^{u(1-\gamma)-1} (1 - N_t)^{(1-v)(1-\gamma)} \]

hence

\[
\frac{\partial V_t}{\partial C_t} = \frac{v \left( C_t^u (1 - N_t)^{1-v} \right)^{(1-\gamma)}}{C_t} V_t^{\gamma}.
\]

Collecting the results, we obtain in equilibrium that:

\[
u \left( \frac{C_t^u (1 - N_t)^{1-v}}{C_t} \right)^{(1-\gamma)} V_t^{\gamma} = \mu \left( 1 + \kappa' \left( \frac{N X_t}{Y_t} \right) \right) \frac{\partial V^*_t}{\partial C^*_t} \] (3.1)

and as usual

\[
u_2(C, N) = \frac{u_2(C, N)}{u_1(C, N)} = F_2 (K_t, z_t N_t) = w_t \] (3.2)

and

\[ V_t^{1-\gamma} = \left( C_t^u (1 - N_t)^{1-v} \right)^{1-\gamma} + \beta E_t \left( V_{t+1}^{1-\theta} \right)^{\frac{1-\gamma}{1-\theta}} \] (3.3)

and from the firm problem:

\[
\frac{1}{\phi' \left( \frac{I'}{K'} \right)} = E_{\varepsilon', \zeta', x'} \left( M^* (k^*, p, \varepsilon', x') \frac{\varepsilon'}{e} \cdot (1 - x'b_k^*) \left( F_1 (K', z'N') - I' + \frac{1 - \delta + \phi \left( \frac{I'}{K'} \right)}{\phi' \left( \frac{I'}{K'} \right)} \right) \right) \] (3.4)
We have 4 equations in 4 unknowns: $C_t, I_t, V_t, K_t$. (The constant $\mu$ determines the wealth of the SOE, which is not pinned down. We can set it = 1.) To solve this, first we rescale these equations to make them stationary, then we solve for the policy functions which solve this system of equation. They will depend on the state variables of the country, $k, p$ as well as the foreign economy $k^*$ since $k^*$ affects the SDF.

More precisely: $i = i(k, p, k^*)$ satisfies

$$\frac{1}{\phi'(\frac{x}{k})} = E_{\varepsilon', p', x'} \left( M^*(k^*, p, p', \varepsilon', x') \frac{1 + \kappa'(\frac{N}{X})}{1 + \kappa'(\frac{X}{Y})} \cdot (1 - x' b^*_k) \left( \alpha k'^{\alpha - 1} N^{1 - \alpha} - \frac{i'}{k'} + \frac{1 - \delta + \phi(\frac{\mu}{k'})}{\phi'(\frac{\mu}{k'})} \right) \right)$$

$$= \sum_{\varepsilon', p', x'} \pi(\varepsilon')Q(p, p')pr(x') M^*(k^*, p, p', \varepsilon', x') \frac{1 + \kappa'(\frac{N x}{y}(k^', p^', k^*))}{1 + \kappa'(\frac{N x}{y}(k^, p, k^*))} \cdot (1 - x' b^*_k) \left( \alpha k'^{\alpha - 1} N(k^', p^', k^*)^{1 - \gamma} \right)$$

$$\phi(x) = a_0 + a_1 \frac{x^{1 - \eta}}{1 - \eta}$$

$$\phi'(x) = a_1 x^{-\eta}$$

with

$$M^*(k^*, p, \varepsilon', p', x') = \beta \left( \frac{C^*_{t+1}}{C^*_t} \right)^{(1-\gamma)\gamma^\gamma} \left( \frac{1 - N^*_{t+1}}{1 - N^*_t} \right)^{1 - (1 - \gamma)\gamma^\gamma} \left( \frac{V^*_{t+1}}{E_t (V^*_{t+1})^{1 - \gamma}} \right)^{\gamma^\gamma}$$

i.e.

$$M^*(k^*, p, \varepsilon', p', x') = \beta \left( \frac{z'}{z} \right)^{(1 - \gamma)\gamma^\gamma} \left( \frac{c^*(k^*, p')}{c(k^*, p)} \right)^{v(1 - \gamma)} \times \ldots$$

$$\ldots \left( \frac{1 - N(k^*, p')}{1 - N(k^*, p)} \right)^{v(1 - \gamma)} \left( \frac{g(k^*, p')^{1 - \gamma}}{E_{\varepsilon', p', x'} \left( \frac{E_{\varepsilon', p', x'}}{g(k^*, p')^{1 - \gamma}} \right)^{1 - \gamma}} \right)^{\gamma^\gamma}.$$

Note that $k^* = f(k^*, p, x', \varepsilon')$ and that $p$ is the same for both countries.

$$\frac{z'}{z} = e^{\mu + \sigma x' + x' \log(1 - b)}$$

and

$$\frac{NX}{\gamma} = \frac{y - c - i}{y}$$

The second equation in this system of four equations is:
\[ V_t^{1-\gamma} = \left( C_t^v (1 - N_t)^{1-v} \right)^{1-\gamma} + \beta E_t \left( V_{t+1}^{1-\theta} \right)^{\frac{1}{1-\theta}}. \]

Now define \( v = V^{1-\sigma}/z^{v(1-\gamma)} = v(k, p, k^*) \). This leads to:

\[
v(k, p, k^*) = (c(k, p, k^*)^v (1 - N(k, p, k^*))^{1-v})^{1-\gamma} + \beta e^\mu(1-\gamma) E_{x', y', \epsilon'} \left( e^{\sigma \epsilon' + x' \log(1-b_\epsilon)} \right)^{(1-\theta)} v(k', p', k^{*})^{1-\theta} \]  \tag{3.5}

The other equations are, with \( c = C/z \):

\[
v \frac{z^{v(1-\gamma)}}{zc(k, p, k^*)} (c(k, p, k^*)^v (1 - N(k, p, k^*))^{1-v})^{1-\gamma} V_t^\gamma = \mu \left( 1 + \kappa' \left( \frac{nx(k, p, k^*)}{y(k, p, k^*)} \right) \right) \frac{\partial V_t^{*}}{\partial C_t^*}
\]

we can plug in \( \frac{\partial V_t^{*}}{\partial C_t^*} \), which depends on \( z, k^*, \) and \( p \).

More precisely,

\[
v \frac{z^{v(1-\gamma)}}{zc(k, p, k^*)} (c(k, p, k^*)^v (1 - N(k, p, k^*))^{1-v})^{1-\gamma} V_t^\gamma = \mu \left( 1 + \kappa' \left( \frac{nx(k, p, k^*)}{y(k, p, k^*)} \right) \right) \frac{\partial V_t^{*}}{\partial C_t^*} \frac{z^{v(1-\gamma)}}{zc(k, p, k^*)} (c(k, p, k^*)^v (1 - N(k, p, k^*))^{1-v})^{1-\gamma} v(k, p, k^*)^\gamma \]

\[
c(k, p, k^*)^{v(1-\gamma)-1} (1 - N(k, p, k^*))^{(1-v)(1-\gamma)} v(k, p, k^*)^\gamma = \mu \left( 1 + \kappa' \left( \frac{nx(k, p, k^*)}{y(k, p, k^*)} \right) \right) c_t^{v(1-\gamma)-1} (1 - N_t^{*(1-v)}(1-\gamma) v) \]

The last equilibrium condition is:

\[
\frac{u_2(C, N)}{u_1(C, N)} = F_2(K_t, z_t N_t) \rightarrow 1 - \frac{c(k, p, k^*)}{v (1 - N(k, p, k^*))} = (1 - \alpha) k^\alpha N(k, p, k^*)^{-\alpha} \]  \tag{3.7}

Our solution method is as follows. We have a system of 4 equations in 4 functions \( c(k, p, k^*) \), \( i(k, p, k^*) \), \( N(k, p, k^*) \), and \( v(k, p, k^*) \). We start with a guess for these functions, e.g. the foreign country solution. Then, we update the guess using the equations above. We iterate until convergence. There is no need for optimization, as these are not Bellman equations.

\[ ^5 \text{Note that } y = k^\alpha N^{1-\alpha} \rightarrow nx = y - c - i, e(k, p, k^*) = 1 + \kappa' \left( \frac{nx}{y} \right). \]
4 Conclusion

To be done.
Table 1: Calibration Parameters - This table presents the parameters of the model.

<table>
<thead>
<tr>
<th>Parameter Values</th>
</tr>
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<tbody>
<tr>
<td>$\gamma$</td>
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<td>$\beta$</td>
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<tr>
<td>$b_z$</td>
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<tr>
<td>$b_{k,1}$</td>
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<tr>
<td>$b_{k,2}$</td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>$\mu$</td>
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<td>$\sigma$</td>
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<tr>
<td>$\lambda_1$</td>
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Table 2: Business Cycle Moments

<table>
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<tr>
<th></th>
<th>$\frac{\sigma(\Delta \log C)}{\sigma(\Delta \log Y)}$</th>
<th>$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$</th>
<th>$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$</th>
<th>$\rho_{\Delta \log C, \Delta \log Y}$</th>
<th>$\rho_{\Delta \log I, \Delta \log Y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.57</td>
<td>2.68</td>
<td>0.92</td>
<td>0.45</td>
<td>0.68</td>
</tr>
<tr>
<td>Model</td>
<td>1.20</td>
<td>3.96</td>
<td>0.50</td>
<td>0.83</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Notes: This table reports standard deviations (as a fraction of the output volatility) and cross-correlations of macro-variables in the model (Consumption $C$, investment $I$ and hours worked $N$). These moments are obtained from a simulation with 20,000 periods, excluding disasters. The model is simulated at quarterly frequency using the parameters in Table 1.
Figure 1: Impulse response to TFP shocks (I/II)

Each panel corresponds to the impulse response functions to a shock on total factor productivity. We consider the impact of such a shock on the growth rate of productivity, the disaster probability, the disaster realization, the capital, the ratio of net exports to GDP, and the real exchange rate.
Figure 2: Impulse response to TFP shocks (II/II)

Each panel corresponds to the impulse response functions to a shock on total factor productivity. We consider the impact of such a shock on investment, consumption, employment, output, the return on a risk-free bond and the return on domestic equity.
Figure 3: Impulse response to shocks on the disaster probability (I/II)

Each panel corresponds to the impulse response functions to a shock on the disaster probability. We consider the impact of such a shock on investment, consumption, employment, output, the return on a risk-free bond and the return on domestic equity.
Figure 4: Impulse response to shocks on the disaster probability (II/II)

Each panel corresponds to the impulse response functions to a shock on the disaster probability. We consider the impact of such a shock on investment, consumption, employment, output, the return on a risk-free bond and the return on domestic equity.
Figure 5: Impulse response to disaster shocks (I/II)

Each panel corresponds to the impulse response functions to a disaster shock. We consider the impact of such a shock on the growth rate of productivity, the disaster probability, the disaster realization, the capital, the ratio of net exports to GDP, and the real exchange rate.
Figure 6: Impulse response to disaster shocks (II/II)

Each panel corresponds to the impulse response functions to a disaster shock. We consider the impact of such a shock on the growth rate of productivity, the disaster probability, the disaster realization, the capital, the ratio of net exports to GDP, and the real exchange rate.
A Algorithm to solve the model

(1) Preliminary: define grid for \( k_1 \) and grid for \( k_2 \). Use a large grid around the steady-state. (Find \( k^\ast \). then grid = linspace(.1*kstar,1.2*kstar,100) for instance). Discretize the normal shock \( \varepsilon \), with probabilities \( \pi(\varepsilon) \), e.g. use 5 pts from -2 to +2 with weights proportional to the standard normal pdf. (Make sure it has mean 0, variance 1.).

Create a grid for \( i_1 \) and a grid for \( i_2 \), from 0 to 2 times the steady-state investment.

(2) Note that the labor allocation in each country satisfies:

\[
\frac{u_2(C, N)}{u_1(C, N)} = F_2(K, zN) \rightarrow \frac{1 - \nu}{\nu} \frac{c}{1 - N} = (1 - \alpha) \left( \frac{k}{N} \right)^\alpha \\
\rightarrow c = c(k, N)
\]

First let’s consider the static problem, given \( k_1, k_2 \) and given desired \( i_1, i_2 \):

\[
H(k_1, k_2, i_1, i_2) = \max_{N_1, N_2, c_1, c_2} \{ \lambda_1 c_1^{v(1-\gamma)}(1 - N_1)^{(1-\nu)(1-\gamma)} + \lambda_2 c_2^{v(1-\gamma)}(1 - N_2)^{(1-\nu)(1-\gamma)} \}
\]

s.t. : \( c_1 + c_2 + i_1 + i_2 = k_1^\alpha N_1^{1-\alpha} + k_2^\alpha N_2^{1-\alpha} - \kappa \left( \frac{k_1^\alpha N_1^{1-\alpha} - c_1 - i_1}{k_1^\alpha N_1^{1-\alpha}} \right) k_1^\alpha N_1^{1-\alpha} \)

Substitute the expression above for \( c(k, N) \) in this program. Then we have a maximization problem in 2 variables, subject to one constraint. Solve it for any value of \( k_1, k_2, i_1, i_2 \) in the grid. (Use a standard solver such as ‘fsolve’, and put as initial guess the steady-state.)

Store the solution

\[
N_1 = N_1^\ast(k_1, k_2, i_1, i_2)
\]

and

\[
N_2 = N_2^\ast(k_1, k_2, i_1, i_2).
\]

as well as \( c(k, N) \) and the value \( H(k_1, k_2, i_1, i_2) \).

(3) Now we can look at the Bellman equation, which has now discrete state space and discrete action space:
\[ g(k_1, k_2, p) = \max_{i_1, i_2} \left\{ \begin{array}{c} \frac{H(k_1, k_2, i_1, i_2)}{\sum_{\varepsilon'} \sum_{p'} Q(p, p')\pi(\varepsilon')e^{(1-\gamma)(\mu+\sigma\varepsilon')}((1-p)g(k'_1, k'_2, p') + p(1-b_z)e^{(1-\gamma)}g(k''_1, k''_2, p') \right\} 
\]

where

\[ k'_1 = \frac{(1 - \delta + \phi(k'_1))}{e^{\mu+\sigma\varepsilon'}} k_1 \quad \text{and} \quad k'_1 = k'_1 \frac{1 - b_{k,1}}{1 - b_z} \]

\[ k'_2 = \frac{(1 - \delta + \phi(k'_2))}{e^{\mu+\sigma\varepsilon'}} k_2 \quad \text{and} \quad k'_2 = k'_2 \frac{1 - b_{k,2}}{1 - b_z} \]

Solve this Bellman equation by VFI. (Note: first write the code with VFI. Then it will probably be useful to do policy function iteration instead, since it seems to be significantly faster in this case.) Note that you will need to interpolate \( g \) to compute \( g(k'_1, k'_2, p') \) outside the grid. Use interp2 with spline or linear interpolation.

(4) We know have the value function and the policy function \( i^*_1 = i_1(k_1, k_2, p) \) and \( i^*_2 = i_2(k_1, k_2, p) \). Hence we can deduce the policy functions \( N_1 = N_1(k_1, k_2, p) = N_1^*(k_1, k_2, i_1(k_1, k_2, p), i_2(k_1, k_2, p)) \), and similarly for \( N_2, c_1 \) and \( c_2 \).

(5) Compute the exchange rate function:

\[ e(k_1, k_2, p) = 1 + \kappa' \left( \frac{NX}{Y_1} \right) = 1 + \kappa' \left( \frac{k_1^\alpha N_1^{1-\alpha} - c_1 - i_1}{k_1^\alpha N_1^{1-\alpha}} \right) \]

with 1 home good = \( e_t \) foreign goods (1 = home, 2 = foreign) /

(6) Simulate a time series for the model: draw a path for \( \{p_t\} \), for \( x_t \), and for \( \varepsilon_t \). Plot the path for \( c_{1,t}, i_{1,t}, N_{1,t}, k_{1,t}, NX_t \) etc.

(7) Using (6), feed in a special path: no shock to \( x \), no shock to \( \varepsilon \), and a one-time switch in \( p \). This gives something like an “impulse response function”.

(8) Compute the risk-free rate in each country: the SDF is

\[ M_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{\nu(1-\gamma)-1} \left( \frac{1 - N_{t+1}}{1 - N_t} \right)^{(1-\nu)(1-\gamma)} \]

and the RFR is

\[ R_f(k_1, k_2, p) = E_t M_{t,t+1}, \]

and similarly in the other country.

Check that in this case we have \( \frac{\varepsilon'_{t+1}}{\varepsilon_t} = \frac{M_{t+1}}{M_t} \rightarrow \) this must be true at the optimum given the way the solution is constructed!