Asset Returns with Earnings Management

Bo Sun

Federal Reserve Board

November 20, 2009
Motivation: earnings management

- Earnings management occurs when managers use judgment in financial reporting to intentionally alter the financial reports to
  - mislead shareholders
  - influence the contractual outcome that depends on the accounting numbers (Healy and Wahlen 1999).

- Playing games with their accounting, prompting the government to seize control of the companies.
  - Richard Shelby, U.S. Senator

- Earnings management is found to be widespread (Burgstahler and Dichev 1997, Loomis 1999, Grahama et al. 2005).
Research question

- Why is earnings management prevalent?
- How does earnings management affect asset return dynamics?
This paper

- Construct a principal-agent model that generates earnings management
- Embed the contract model into an asset-pricing model
- Analyze the behavior of asset prices and returns
Stylized financial facts

Stock returns:

- Volatility clustering: Diebold and Nerlove (1989); Shimokawa et al. (2007)
Stylized financial facts

Stock returns:

▶ Volatility clustering: Diebold and Nerlove (1989); Shimokawa et al. (2007)

▶ Asymmetric volatility: Black and Scholes (1973); French (1987) and Campbell (1992)
Stylized financial facts

Stock returns:

- Volatility clustering: Diebold and Nerlove (1989); Shimokawa et al. (2007)

- Asymmetric volatility: Black and Scholes (1973); French (1987) and Campbell (1992)

- Increasing idiosyncratic volatility: Campbell et al. (2001); Rajgopal and Venkatachalam (2007)
Model feature

- Investors contract with a newly-hired manager each period
- Asymmetric information about realized earnings and effort
- Actual earnings are autocorrelated
- Realized earnings are periodically revealed
- Investors pay monetary penalties if earnings management is detected
- Dividend is (reported earnings − wage − fines)
New features of the model

- Complete characterization of optimal contract with earnings management
- Bayesian updating by the investors
Presentation outline

- Model setup
  - Optimal contract
  - Asset prices
- Continuous earnings
- Calibration
- Robustness check
- Conclusion
Roadmap

- Model setup
  - Optimal contract
    - Asset prices
- Continuous earnings
- Calibration
- Robustness check
- Conclusion
Contract model

- Manager makes unobserved effort
- Earnings are stochastic, and manager may misreport earnings
- Basic tradeoff:
  - Strong pay-performance relationship: Reward manager’s effort
  - Weak pay-performance relationship:
    - Provide insurance
    - Disincentive to manipulate
Contract timeline

- Manager exerts effort $e \in \{L, H\}$
- Earnings realize $y \in \{l, h\}$ with $p_e = \Pr(y = h|e)$, $p_H > p_L$
- Manager privately receives earnings management opportunity with probability $x$
- Misreporting incurs cost $\phi(0) = 0$ and $\phi(h - l) = \psi > 0$
- Manager makes a report $R(y) \in \{\tilde{l}, \tilde{h}\}$
- Manager is compensated based on the report: $u_{R(y)}$
The principal’s problem

Minimize \{ \text{expected cost of inducing effort} \}

Subject to

- \((IC_e)\) manager maximizes his utility at \(e = H\) (desired effort)
- \((IC_R)\) manager maximizes his utility at \(R(y)\) (reporting choice)
- \((PC)\) manager’s utility \(\geq\) outside opportunity
Optimal contract results

$$u_h - u_l$$

$$45^\circ$$

honest
Optimal contract results

\[ u_\hat{h} - u_\hat{l} \]

\[ 45^\circ \]

honest
Optimal contract results

\[ u_{\tilde{h}} - u_{\tilde{l}} \]

\[ \psi \]

\[ \frac{c}{p_H - p_L} \]

misreport ← → honest
Optimal contract results

\[ u_{\tilde{h}} - u_{\tilde{l}} \]

\[
\psi \left( \frac{c - x(p_H - p_L)}{(1-x)(p_H - p_L)} \right)
\]

\[
\psi \left( \frac{c}{p_H - p_L} \right)
\]

misreport \( \leftarrow \) honest

honest

45°
Optimal contract results

\[ u_h - u_l \]

\[ c - x(p_H - p_L)\psi \]

\[ \frac{c}{p_H - p_L} \]

\[ 45^\circ \]

misreport ← → honest

honest
The Revelation Principle

RP requires: it is costless to establish communication channels.

RP is not applicable in this model because of limited message space.
Roadmap

- Model setup
  - Optimal contract
  - Asset prices
- Continuous earnings
- Calibration
- Robustness check
- Conclusion
Model timeline

← $t$ (1st period) →← $t + 1$ (2nd period) →

<table>
<thead>
<tr>
<th>Price $q_1(y_{t-1})$</th>
<th>Manager$_t$ makes a report $r_t$ and is paid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price $q_2(y_{t-1}, r_t)$</td>
<td>Manager$<em>{t+1}$ makes a report $r</em>{t+1}$ and is paid</td>
</tr>
<tr>
<td>$y_t$ and $y_{t+1}$ are revealed</td>
<td></td>
</tr>
<tr>
<td>Fines $F_1$ or $F_2$ are charged</td>
<td></td>
</tr>
</tbody>
</table>

Binary earnings: $\Pr[y_{t+1} = j | y_t = i] = \pi_{ij}, \forall i \in \{l, h\}, \forall j \in \{l, h\}$

Continuous earnings: $y_{t+1} = \rho y_t + k + \epsilon_{t+1}, \epsilon \sim N(0, \sigma^2)$
Price formulation

- Binary earnings (exposition)
  - Updating the beliefs
  - Components of the asset prices

- Continuous earnings (calibration)
  - Quantitative analysis
1st period: price

Given $y_{t-1}$, the asset price $q_1(y_{t-1})$ is

$$q_1(h) = \pi_{hh}[d_h + \beta q_2(h, \tilde{h})] + \pi_{hl}x[d_h + \beta q_2(h, \tilde{h})] + \pi_{hl}(1-x)[d_l + \beta q_2(h, \tilde{l})]$$

where $d_r = r - w_r$, $r \in \{\tilde{l}, \tilde{h}\}$
Given $r_t$ and $y_{t-1}$, investors update their belief about the true state at time $t$$$
abla_1 = \Pr(y_t = h | r_t = \tilde{h}, y_{t-1} = h)
= \frac{\pi_{hh}}{\pi_{hh} + \pi_{hl}}$$
2nd period: price

The price of the firm is

\[ q_2(h, \tilde{h}) = \]
\[ p_1 \{ \pi_{hh} [d_{\tilde{h}} + \beta q_1(h)] + \pi_{hl} x [d_{\tilde{h}} - F_1 + \beta q_1(l)] + \pi_{hl} (1 - x) [d_l + \beta q_1(l)] \} \]
\[ + (1 - p_1) \{ \pi_{lh} [d_{\tilde{h}} - F_1 + \beta q_1(h)] + \pi_{ll} x [d_{\tilde{h}} - F_2 + \beta q_1(l)] + \pi_{ll} (1 - x) [d_l - F_1 + \beta q_1(l)] \} \]

- an honest report in period 1
  - an honest high report in period 2
  - a false high report in period 2
  - an honest low report in period 2

- a false report in period 1
  - an honest high report in period 2
  - a false high report in period 2
  - an honest low report in period 2
2nd period: price

The price of the firm is

\[ q_2(h, \tilde{h}) = \]
\[ p_1 \{ \pi_{hh} [d_{\tilde{h}} + \beta q_1(h)] + \pi_{hl} [d_{\tilde{h}} - F_1 + \beta q_1(l)] + \pi_{ll} (1 - x) [d_l + \beta q_1(l)] \} \]
\[ + (1 - p_1) \{ \pi_{lh} [d_{\tilde{h}} - F_1 + \beta q_1(h)] + \pi_{ll} [d_{\tilde{h}} - F_2 + \beta q_1(l)] + \pi_{ll} (1 - x) [d_l - F_1 + \beta q_1(l)] \} \]

- an honest report in period 1
  - an honest high report in period 2
  - a false high report in period 2
  - an honest low report in period 2
- a false report in period 1
  - an honest high report in period 2
  - a false high report in period 2
  - an honest low report in period 2
Quantitative model: $y \in \mathcal{R}$

Optimal contract

- Setup: analogous to the binary example
- Result: manager overstates earnings by an amount $a$ if $y_t$ is below a threshold level
\( x = 0 \)

\[ q(y) = \frac{\rho y}{(1 - \beta \rho)} + \frac{k}{(1 - \beta)(1 - \beta \rho)} \]

\[ q' = q(y') = q(\rho y + k + \epsilon) = \rho q + \frac{(1 - \rho)k}{(1 - \beta)(1 - \beta \rho)} + \frac{\rho}{(1 - \beta \rho)}(k + \epsilon) \]

- Without earnings management, prices follow an AR(1) process (no EGARCH effect).
Two effects of earnings management

- **Level effect**
  - lower price because of possible earnings management

- **Additional distortion**
  - when $r$ is greater than $y^* + a$, it is truthful
  - when $r$ is slightly smaller than or equal to $y^* + a$, it is likely to be overstated
  - when $r$ is much smaller than $y^* + a$, it is likely to be true
Numerical example: pricing functions

Figure: Pricing function with continuous earnings
Numerical example: pricing functions

Figure: Pricing function with continuous earnings
Numerical example: pricing functions

Figure: Pricing function with continuous earnings
Figure: Pricing function in period 2
Calibration strategy

- Incorporate aggregate and idiosyncratic productivity shocks:
  \[ y' = \rho y + k + \epsilon_a + \epsilon_i \]

- Simulate realizations of productivity shocks and earnings management opportunities for a large number of firms

- Gather return sequences together and match the aggregate targets

- Data: net income in quarterly industrial Compustat database; S&P 500 index returns in quarterly CRSP files
## Benchmark parameterization: restated data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.98</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Autoregressive parameter</td>
<td>0.77</td>
</tr>
<tr>
<td>$k$</td>
<td>Constant term</td>
<td>0.23</td>
</tr>
<tr>
<td>$a$</td>
<td>Amount of overstatement</td>
<td>0.07</td>
</tr>
<tr>
<td>$x$</td>
<td>Likelihood of earnings management</td>
<td>0.04</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Std. Dev of aggregate productivity shock</td>
<td>0.12</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>Std. Dev of idiosyncratic productivity shock</td>
<td>0.17</td>
</tr>
<tr>
<td>$F_1$</td>
<td>Monetary loss for one restatement</td>
<td>1.06</td>
</tr>
<tr>
<td>$F_2$</td>
<td>Monetary loss for two restatements</td>
<td>2.12</td>
</tr>
</tbody>
</table>
EGARCH(1,1) model

- ARCH (Autoregressive conditional heteroskedasticity)
- GARCH (Generalized ARCH)
- EGARCH (Exponential GARCH)

\[
\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]
\]

\[
s_t = C + \epsilon_t
\]

\[
\sigma_t^2 = E_{t-1}(\epsilon_t^2)
\]

- \(G\): volatility clustering
- \(L\): asymmetric volatility
Benchmark results

<table>
<thead>
<tr>
<th></th>
<th>Model data $x = 0$</th>
<th>Model data $x = 0.04$</th>
<th>S&amp;P 500 data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>0.0028</td>
<td>0.5260**</td>
<td>0.7365**</td>
</tr>
<tr>
<td>$A$</td>
<td>0.0116</td>
<td>0.0529**</td>
<td>0.3058**</td>
</tr>
<tr>
<td>$L$</td>
<td>0.0009</td>
<td>−0.0234**</td>
<td>−0.2557**</td>
</tr>
</tbody>
</table>

**Table**: Comparison of EGARCH(1,1) estimation results

$$\log \sigma^2_t = K + G \log \sigma^2_{t-1} + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$$.
Intuition behind EGARCH effect

Volatility is state-dependent and state is persistent

- Consequences of earnings management
  - low earnings lead to earnings restatements
  - earnings restatements enhance volatility

- Suspicion of earnings management
  - low reports lead to intensive suspicion
  - suspicion causes volatility

⇒ Volatility is persistent and exhibits asymmetry
Returns volatility

Table: Comparison of data volatility

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model $x = 0$</td>
<td>0.0134</td>
</tr>
<tr>
<td>Model $x = 0.04$</td>
<td>0.0714</td>
</tr>
<tr>
<td>Data</td>
<td>0.1106</td>
</tr>
</tbody>
</table>

$x \uparrow \Rightarrow$ more frequent earnings restatements $\Rightarrow$ volatility $\uparrow$
Robustness check

- Alternative calibration
- Stochastic monitoring
## Alternative Parameterization: unrestated data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.98</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Autoregressive parameter</td>
<td>0.82</td>
</tr>
<tr>
<td>$k$</td>
<td>Constant term</td>
<td>0.18</td>
</tr>
<tr>
<td>$a$</td>
<td>Amount of overstatement</td>
<td>0.03</td>
</tr>
<tr>
<td>$x$</td>
<td>Likelihood of earnings management</td>
<td>0.04</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Std.Dev of aggregate productivity shock</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>Std.Dev of idiosyncratic productivity shock</td>
<td>0.08</td>
</tr>
<tr>
<td>$F_1$</td>
<td>Monetary loss for one restatement</td>
<td>0.49</td>
</tr>
<tr>
<td>$F_2$</td>
<td>Monetary loss for two restatements</td>
<td>0.98</td>
</tr>
</tbody>
</table>

**Table:** Alternative Parameterization
Alternative calibration results: EGARCH effect

<table>
<thead>
<tr>
<th></th>
<th>Model data x=0</th>
<th>Model data x=0.04</th>
<th>S&amp;P 500 data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>0.0028</td>
<td>0.6584**</td>
<td>0.7365**</td>
</tr>
<tr>
<td>$A$</td>
<td>0.0116</td>
<td>0.0339**</td>
<td>0.3057**</td>
</tr>
<tr>
<td>$L$</td>
<td>0.0009</td>
<td>$-0.0231$**</td>
<td>$-0.2557$**</td>
</tr>
</tbody>
</table>

Table: Comparison of EGARCH(1,1) estimation results

$$
\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|e_{t-1}|/\sigma_{t-1} - E\{|e_{t-1}|/\sigma_{t-1}\}] + L[e_{t-1}/\sigma_{t-1}].
$$
Return moments

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model $x = 0$</td>
<td>0.0134</td>
</tr>
<tr>
<td>Model $x = 0.04$</td>
<td>0.0300</td>
</tr>
<tr>
<td>Data</td>
<td>0.1106</td>
</tr>
</tbody>
</table>

**Table:** Comparison of data volatility
Stochastic investigation

Model dynamics are *not* driven by the revelation cycles

- Investigation is conducted stochastically
- Prices are formulated in a recursive manner
- Results and intuition go through
Timeline

- Investigation takes place with probability $\lambda$. Investors incur financial costs associated with detected misstatement $\kappa n$

- Manager exerts effort $e \in \{L, H\}$

- Earnings realize $y \in \{l, h\}$ with $p_e = \Pr(y = h|e)$

- Manager privately receives earnings management opportunity with probability $x$

- Manager makes a report and is paid

- Asset price realizes based on the report
Price is a function of

- current (hidden) state
  - $r$: the current earnings report
  - $\gamma$: $\Pr(y_t = h|r^t)$, $r^t$: reporting history
Price is a function of

- current (hidden) state
  - $r$: the current earnings report
  - $\gamma$: $\Pr(y_t = h| r^t)$, $r^t$: reporting history

- previous (hidden) history
  - $N$: the number of consecutive high reports until the previous period since the last low report or the last investigation, whichever is more recent

![Diagram](image-url)

- low report
- $N$
- previous investigation
- current period
- $r$ and $\gamma$
Price is a function of

- current (hidden) state
  - $r$: the current earnings report
  - $\gamma$: $\Pr(y_t = h | r^t)$, $r^t$: reporting history

- previous (hidden) history
  - $N$: the number of consecutive high reports until the previous period since the last low report or the last investigation, whichever is more recent
  - $\bar{y}$: the true earnings before the consecutive $N$ high reports starts
Price is a function of

- current (hidden) state
  - $r$: the current earnings report
  - $\gamma$: $\Pr(y_t = h | r^t)$, $r^t$: reporting history

- previous (hidden) history
  - $N$: the number of consecutive high reports until the previous period since the last low report or the last investigation, whichever is more recent
  - $\bar{y}$: the true earnings before the consecutive $N$ high reports starts
  - $Z$: the expected number of periods involving earnings management since the last investigation until the most recent low report
Price for a high report

\[ P(\gamma, Z, N, \tilde{h}, \tilde{y}) = \tilde{h} + \beta \left[ (1 - \lambda) W_n^h + \lambda W_i^h \right]. \]
Price for a high report

\[ P(\gamma, Z, N, \tilde{h}, \tilde{y}) = \tilde{h} + \beta \left[ (1 - \lambda) W_n^h + \lambda W_i^h \right]. \]

If the investigation does not take place in the beginning of the next period,

\[ W_n^h = \mu P(\gamma', Z, N + 1, \tilde{h}, \tilde{y}) + (1 - \mu) P(0, Z, N + 1, \tilde{l}, \tilde{y}). \]
Price for a high report

\[ P(\gamma, Z, N, \tilde{h}, \tilde{y}) = \tilde{h} + \beta \left[ (1 - \lambda) W_n^h + \lambda W_i^h \right]. \]

If the investigation does not take place in the beginning of the next period,

\[ W_n^h = \mu P(\gamma', Z, N + 1, \tilde{h}, \tilde{y}) + (1 - \mu) P(0, Z, N + 1, \tilde{l}, \tilde{y}). \]

If the investigation takes place in the next period:

\[ W_i^h = -\kappa [Z + f(N + 1; \tilde{y})] \]
\[ + \gamma \left[ \xi_1 P \left( \frac{\pi_{hh}}{\xi_1}, 0, 0, \tilde{h}, h \right) + (1 - \xi_1) P(0, 0, 0, \tilde{i}, h) \right] \]
\[ + (1 - \gamma) \left[ \xi_2 P \left( \frac{\pi_{ih}}{\xi_2}, 0, 0, \tilde{h}, l \right) + (1 - \xi_2) P(0, 0, 0, \tilde{i}, l) \right]. \]
Numerical example: volatility clustering & asymmetric volatility

<table>
<thead>
<tr>
<th>$x=0$</th>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>-5.0000</td>
<td>12.8300</td>
<td>-0.3897</td>
</tr>
<tr>
<td>$G$</td>
<td>0.0576</td>
<td>0.0880</td>
<td>0.6552</td>
</tr>
<tr>
<td>$A$</td>
<td>0.0033</td>
<td>0.0119</td>
<td>0.2838</td>
</tr>
<tr>
<td>$L$</td>
<td>0.0041</td>
<td>0.0066</td>
<td>0.6195</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x=0.1$</th>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>-2.0291</td>
<td>0.2979</td>
<td>-6.8092</td>
</tr>
<tr>
<td>$G$</td>
<td>0.7441</td>
<td>0.0376</td>
<td>19.7951</td>
</tr>
<tr>
<td>$A$</td>
<td>0.1068</td>
<td>0.0207</td>
<td>5.1616</td>
</tr>
<tr>
<td>$L$</td>
<td>-0.0841</td>
<td>0.0197</td>
<td>-4.2789</td>
</tr>
</tbody>
</table>

Table: EGARCH(1,1) estimation results

$$\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E[|\epsilon_{t-1}|/\sigma_{t-1}]] + L[\epsilon_{t-1}/\sigma_{t-1}]$$
Increased volatility

<table>
<thead>
<tr>
<th>$x$</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0134</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0193</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0201</td>
</tr>
</tbody>
</table>

Table: Volatility of the model returns
Conclusions

- Earnings management causes price distortion
- Earnings management may lead to a range of financial anomalies
  - Volatility clustering
  - Asymmetric volatility
  - Increasing individual volatility
Motivation: market response upon restatements

Returns

Days before and after restatements (announcement is made at day 0)

Parameter values in the numerical example (binary earnings)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>50</td>
</tr>
<tr>
<td>$l$</td>
<td>0</td>
</tr>
<tr>
<td>$\pi_{hh}$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\pi_{ll}$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.95</td>
</tr>
<tr>
<td>$F_1$</td>
<td>$1.2(h - l)/\beta$</td>
</tr>
<tr>
<td>$F_2$</td>
<td>$2F_1$</td>
</tr>
</tbody>
</table>

Table: Parameter values in the numerical example with binary earnings
Updating beliefs in the second period in the binary example

\[ p_1 = \frac{\Pr(y_t = h | r_t = \tilde{h}, y_{t-1} = h)}{\Pr(r_t = \tilde{h} | y_{t-1} = h)} \]
\[ = \frac{\Pr(y_t = h, r_t = \tilde{h} | y_{t-1} = h)}{\Pr(r_t = \tilde{h} | y_{t-1} = h)} \]
\[ = \frac{\Pr(r_t = \tilde{h} | y_t = h) \Pr(y_t = h | y_{t-1} = h)}{\Pr(r_t = \tilde{h} | y_{t-1} = h)} \]
\[ = \frac{\pi_{hh}}{\pi_{hh} + \pi_{hl}X}, \]

If the previously revealed outcome is low, the posterior belief of the current high report being honest is

\[ p_2 = \frac{\Pr(y_t = h | r_t = \tilde{h}, y_{t-1} = l)}{\Pr(r_t = \tilde{h} | y_{t-1} = l)} \]
\[ = \frac{\Pr(y_t = h, r_t = \tilde{h} | y_{t-1} = l)}{\Pr(r_t = \tilde{h} | y_{t-1} = l)} \]
\[ = \frac{\Pr(r_t = \tilde{h} | y_t = h) \Pr(y_t = h | y_{t-1} = l)}{\Pr(r_t = \tilde{h} | y_{t-1} = l)} \]
\[ = \frac{\pi_{lh}}{\pi_{lh} + \pi_{ll}X}. \]
Updating beliefs in the second period with continuous earnings

$p$ is the posterior belief about the likelihood of having an honest report in period 1, that is, $p = \Pr[y' = r|y]$. Following Bayes’ Rule,

$$p = \begin{cases} 
1 & \text{if } r \in [y^* + a, \infty), \\
\frac{f(r - k - \rho y)}{f(r - k - \rho y) + xf(r - a - k - \rho y)} & \text{if } r \in (y^*, y^* + a), \\
\frac{(1-x)f(r - k - \rho y)}{(1-x)f(r - k - \rho y) + xf(r - a - k - \rho y)} & \text{if } r \in (-\infty, y^*].
\end{cases}$$
Asset prices in the first period in the binary example

Given the revelation of the true outcome in the end of the last cycle \( y_{t-1} \), the investors form the price of the firm’s asset \( q_1(y_{t-1}) \) as follows.

\[
q_1(h) = \pi_{hh}[d_{\tilde{h}} + \beta q_2(h, \tilde{h})] + \pi_{hl}x[d_{\tilde{h}} + \beta q_2(h, \tilde{h})] + \pi_{hl}(1-x)[d_{l} + \beta q_2(h, \tilde{h})],
\]

and

\[
q_1(l) = \pi_{lh}[d_{\tilde{h}} + \beta q_2(l, \tilde{h})] + \pi_{ll}x[d_{\tilde{h}} + \beta q_2(l, \tilde{h})] + \pi_{ll}(1-x)[d_{l} + \beta q_2(l, \tilde{h})],
\]
Asset prices in the second period in the binary example

The price of the firm is given by the discounted expected future net dividends (reported earnings net of the labor wage and the expected penalty.)

\[ q_2(l, \tilde{l}) = q_2(h, \tilde{l}) = \]
\[ \pi_{lh} [d_{\tilde{h}} + \beta q_1 (h)] + \pi_{llx} [d_{\tilde{h}} - F_1 + \beta q_1 (l)] + \pi_{ll} (1 - x) [d_{\tilde{h}} + \beta q_1 (l)] . \]

\[ q_2(h, \tilde{h}) = \]
\[ p_1 \{ \pi_{hh} [d_{\tilde{h}} + \beta q_1 (h)] + \pi_{hll} [d_{\tilde{h}} - F_1 + \beta q_1 (l)] + \pi_{hl} (1 - x) [d_{\tilde{l}} + \beta q_1 (l)] \} \]
\[ + (1 - p_1) \{ \pi_{lh} [d_{\tilde{h}} - F_1 + \beta q_1 (h)] + \pi_{llx} [d_{\tilde{h}} - F_2 + \beta q_1 (l)] + \pi_{ll} (1 - x) [d_{\tilde{l}} - F_1 + \beta q_1 (l)] \} \]

\[ q_2(l, \tilde{h}) = \]
\[ p_2 \{ \pi_{hh} [d_{\tilde{h}} + \beta q_1 (h)] + \pi_{hll} [d_{\tilde{h}} - F_1 + \beta q_1 (l)] + \pi_{hl} (1 - x) [d_{\tilde{l}} + \beta q_1 (l)] \} \]
\[ + (1 - p_2) \{ \pi_{lh} [d_{\tilde{h}} - F_1 + \beta q_1 (h)] + \pi_{llx} [d_{\tilde{h}} - F_2 + \beta q_1 (l)] + \pi_{ll} (1 - x) [d_{\tilde{l}} - F_1 + \beta q_1 (l)] \} \]
Asset prices in the first period with continuous earnings

Given the revelation of the true outcome in the end of the last cycle $y$, the investors form the price of the firm’s asset $q_1(y)$ as follows.

$$q_1(y) = \Pr[y' \geq y^* | y] E \left[ (\rho y + k + \epsilon) + \beta q_2(y, \rho y + k + \epsilon) | y' \geq y^* \right]$$
$$+ \Pr[y' < y^* | y] x E \left[ (\rho y + k + \epsilon + a) + \beta q_2(y, \rho y + k + \epsilon + a) | y' < y^* \right]$$
$$+ \Pr[y' < y^* | y] (1 - x) E \left[ (\rho y + k + \epsilon) + \beta q_2(y, \rho y + k + \epsilon) | y' < y^* \right].$$
Asset prices in the second period with continuous earnings

\[ q_2(y, r) = p\Omega + (1 - p)\bar{\Omega}, \]

where

\[
\Omega \equiv \Pr[y'' \geq y^* | y' = r] E \left[ (\rho r + k + \epsilon) + \beta q_1 (\rho r + k + \epsilon) | y'' \geq y^* \right] \\
+ \Pr[y'' < y^* | y' = r] x E \left[ (\rho r + k + \epsilon + a) - F_1 + \beta q_1 (\rho r + k + \epsilon) | y'' < y^* \right] \\
+ \Pr[y'' < y^* | y' = r] (1 - x) E \left[ (\rho r + k + \epsilon) + \beta q_1 (\rho r + k + \epsilon) | y'' < y^* \right],
\]

and

\[
\bar{\Omega} \equiv \Pr[y'' \geq y^* | y' = r - a] E \left[ (\rho (r - a) + k + \epsilon) - F_1 + \beta q_1 (\rho (r - a) + k + \epsilon) | y'' \geq y^* \right] \\
+ \Pr[y'' < y^* | y' = r - a] x E \left[ (\rho (r - a) + k + \epsilon + a) - F_2 + \beta q_1 (\rho (r - a) + k + \epsilon) | y'' < y^* \right] \\
+ \Pr[y'' < y^* | y' = r - a] (1 - x) E \left[ (\rho (r - a) + k + \epsilon) - F_1 + \beta q_1 (\rho (r - a) + k + \epsilon) | y'' < y^* \right].
\]
Parameter values in the numerical example with stochastic monitoring

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>20</td>
</tr>
<tr>
<td>$l$</td>
<td>10</td>
</tr>
<tr>
<td>$\pi_{hh}$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\pi_{ll}$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.98</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>15</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Posterior probabilities (high report)

\[
\gamma' = \begin{cases} 
\gamma \pi_{hh} + (1 - \gamma) \pi_{lh} & r = h \text{ at } t + 1, \\
0, & r = l \text{ at } t + 1.
\end{cases}
\]

\[
\mu = \gamma \pi_{hh} + \gamma (1 - \pi_{hh}) x + (1 - \gamma) \pi_{lh} + (1 - \gamma) (1 - \pi_{lh}) x
\]

\[
\xi_1 = \pi_{hh} + (1 - \pi_{hh}) x
\]

\[
\xi_2 = \pi_{lh} + (1 - \pi_{lh}) x
\]
Price for low report

\[
P(0, Z, N, \tilde{l}, \tilde{y}) = \tilde{l} + \beta \left[ (1 - \lambda) W_n^l + \lambda W_i^l \right].
\]

If the investigation does not take place in the next period,

\[
W_n^l = \zeta P \left( \frac{\pi_{lh}}{\zeta}, Z + f(N; \bar{y}), 0, \tilde{h}, l \right) + (1 - \zeta) P(0, Z + f(N; \bar{y}), 0, \tilde{l}, l)
\]

where \( \zeta \) denotes the conditional probability that the manager makes a high report in the next period:

\[
\zeta = \pi_{lh} + (1 - \pi_{lh}) x
\]

If the investigation takes place in the next period:

\[
W_i^l = -\kappa [Z + f(N; \bar{y})]
\]

\[
+ \zeta P \left( \frac{\pi_{lh}}{\zeta}, 0, 0, \tilde{h}, l \right)
\]

\[
+ (1 - \zeta) P(0, 0, 0, \tilde{l}, l)
\]
## Data moments

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std.Dev</th>
<th>Autocorr</th>
<th>Std.Dev of avg. earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings</td>
<td>0.06</td>
<td>0.20</td>
<td>0.77</td>
<td>0.12</td>
</tr>
</tbody>
</table>

**Table:** Moments of semi-annual scaled earnings: Compustat Restated earnings

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std.Dev</th>
<th>Autocorr</th>
<th>Std.Dev of avg. earnings</th>
<th>Avg. of Std.Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reports</td>
<td>0.10</td>
<td>0.22</td>
<td>0.82</td>
<td>0.03</td>
<td>0.15</td>
</tr>
</tbody>
</table>

**Table:** Moments of semi-annual scaled reports: Compustat Unrestated earnings