Private and Public Liquidity, Intermediation, and Monetary Policy

Stephen Williamson
Washington University St. Louis
Richmond Fed
St. Louis Fed

December 2009
Questions

- What is the role of a central bank?
- What is a liquid asset, and how do privately-provided and publicly-provided liquid assets play in exchange?
- Do open market operations matter, and if so, why?
- How should we evaluate the current actions of our own central bank?
Plan for the Talk

- Construct a model with financial intermediaries that can issue tradeable deposits and currency.
- Intermediaries have delegated-monitoring and Diamond-Dybvig roles.
- Look at equilibria with and without government.
- With government, fiscal policy determines the path for total nominal debt, monetary policy determines its composition.
- Analyze effects of changes in fiscal and monetary policy.
- Consider an example related to current Fed policy.
Model

- Basic structure from Lagos-Wright (2005)
- Some results related to: Lagos (2008), Lagos-Rocheteau (2008), Lester-Postlewaite-Wright (2009)
- \( t = 0, 1, 2, \ldots, \) and two subperiods, day (centralized trading) and night (decentralized trading).
- Continuum of buyers with unit mass, each having preferences

\[
E_0 \sum_{t=0}^{\infty} \beta^t [ -H_t + u(x_t) ],
\]

- Define \( q^* \) by \( u'(q^*) = 1. \)
- Continuum of sellers with unit mass, each having preferences

\[
E_0 \sum_{t=0}^{\infty} \beta^t [ X_t - h_t ],
\]
Production: one unit labor supply produces one unit output for buyers in the day and for sellers in the night.

Each period, a continuum of entrepreneurs with mass $\alpha$ is born, and each lives until the day of the following period.

Each entrepreneur risk neutral, consumes in the next day.

Has access to an investment project - requires one unit consumption good as input, yields return $w$ the next day.

Distribution function $F(w)$; density function $f(w)$ strictly positive on $[0, \bar{w}]$, where $\bar{w} > 0$.

Costly state verification - verification cost $\gamma$ specific to the entrepreneur.

$G(\gamma)$ denotes the distribution of verification costs across entrepreneurs, with $\gamma \geq 0$. 
- Day: Everyone meets in the same place, and trade on a Walrasian market.

- Night: Each buyer matched at random with a seller - buyer makes a take-it-or-leave-it offer.

  \[ \rho = \text{fraction of nighttime bilateral meetings not monitored} \]

  \[ 1 - \rho = \text{fraction of monitored meetings at night.} \]

- Personal IOUs never accepted at night - buyer’s history not observable.

- Particular types of claims on financial intermediaries (see later) may be tradeable in monitored and non-monitored meetings.

- Buyer does not know type of meeting in the subsequent night when production, consumption decisions made in the day - learns this at the end of the day.
Financial Intermediation

- Rule out mixed strategy verification - implies efficient contractual arrangement is perfectly-diversified financial intermediaries that write debt contracts with depositors and entrepreneurs.
- Pay a certain gross rate of return per unit invested $r_t$ to holders of liabilities and lend to entrepreneur with verification cost $\gamma$ at gross loan rate $R_t(\gamma)$.
- Zero profits:

$$r_t = R_t(\gamma) - \gamma F[R_t(\gamma)] - \int_0^{R_t(\gamma)} F(w)dw$$

- Default premium for entrepreneur $\gamma$:

$$D_t(\gamma) = \gamma F[R_t(\gamma)] + \int_0^{R_t(\gamma)} F(w)dw$$
Expected Payoff to the Financial Intermediary

$\text{Gross Loan Rate } R$

$\text{Expected Payoff}$
Marginal borrower \((\gamma^*_t, R^*_t)\) solving

\[
1 - \gamma_t^* f(R^*_t) - F(R^*_t) = 0,
\]

\[
r_t = R^*_t - \gamma^*_t F(R^*_t) - \int_0^{R^*_t} F(w)dw
\]

Total loans:

\[
L_t = \alpha G(\gamma^*_t),
\]

or more simply,

\[
L_t = L(r_t).
\]
Loan Interest Rates

Gross Loan Rate $R$

Expected Payoff

$\gamma_1, \gamma_2, \gamma^*, \gamma_3$
Financial intermediaries issue two kinds of liabilities:

- circulating notes (cost = \( \sigma \) per unit of value in nighttime trades, to encode the message that this is a claim on a particular intermediary, and what it is a claim to). Could include more complicated costs associated with currency exchange - counterfeiter and thieves - but this is simple.
- deposits - can only be traded in monitored meetings in the night, when the technology is available to transfer the claim from the buyer to the seller (checking or debit card transaction).
No Government

- Banking during the day:
  - Buyer produces, makes deposit in the bank.
  - Bank lends to entrepreneurs.
  - Buyers learn whether they will be in monitored or non-monitored meetings in the night.
  - Buyers in non-monitored meetings withdraw circulating notes.
Deposit Contracts

- Equilibrium contract: Choose \((x_t, y_{t+1}^n, y_{t+1}^d)\) to maximize expected utility of the depositor, subject to zero-profit constraint

\[
(x_t - \rho \sigma \beta y_{t+1}^n) r_t = \rho y_{t+1}^n + (1 - \rho) y_{t+1}^d
\]

- Arbitrage implies \(r_t \leq \frac{1}{\beta}\) in equilibrium. Then, solution is characterized by

\[
u'(\beta y_{t+1}^n) = \frac{1 + \sigma r_t \beta}{r_t \beta}, \quad u'(\beta y_{t+1}^d) = \frac{1}{r_t \beta}, \quad \text{if } r_t < \frac{1}{\beta},
\]

\[
y_{t+1}^d \geq \frac{q^*}{\beta}, \quad \text{if } r_t = \frac{1}{\beta},
\]

Williamson ()

Private and Public Liquidity, Intermediation, and Monetary Policy

December 2009 11 / 24
Equilibrium with Sufficient Liquidity

- Stationary equilibrium with $r_t = r = \frac{1}{\beta}$ for all $t$.
- $y^d = \frac{q^*}{\beta}$ (monitored trades are efficient)
- For trading with circulating notes

$$u'(\beta \tilde{y}^n) = 1 + \sigma,$$

so $\beta \tilde{y}^n < q^*$, and as $\sigma$ increases, consumption by buyers in nonmonitored trades falls.

- Loans to entrepreneurs $= L \left( \frac{1}{\beta} \right)$.
- Sufficient liquidity iff

$$L \left( \frac{1}{\beta} \right) \geq \beta \rho \tilde{y}^n + (1 - \rho) q^*,$$

which requires a sufficient quantity of investment projects with sufficiently high returns and sufficiently low verification costs.
Insufficient Liquidity

- This case arises when
  \[ L \left( \frac{1}{\beta} \right) < \beta \rho \tilde{y}^n + (1 - \rho) q^*, \]
  which implies \( r < \frac{1}{\beta} \).
- Inefficient exchange in all nighttime meetings.
Equilibrium with Government

- Government can
  - tax buyers lump sum during the day.
  - issue currency, subject to the same costs as was the case for the private sector.
  - ban circulating private notes, at no cost.

- Government budget constraints:

\[
\phi_t [M_t - M_{t-1}] + \phi_t B_t = \sigma q_t + \phi_t z_t B_{t-1} + \tau_t, \text{ for } t = 1, 2, \ldots
\]

\[
\phi_0 M_0 + \phi_0 B_0 = \sigma q_0 + \tau_0.
\]

Williamson (Private and Public Liquidity, Intermediation, and Monetary Policy, December 2009)
Assume government bans private circulating note issue.

Bank now acquires a portfolio of currency, loans to entrepreneurs, and bonds (which don’t need to be intermediated).

\[ r_t = \frac{\phi_{t+1} z_{t+1}}{\phi_t}. \]

\[ \frac{\phi_{t+1}}{\phi_t} \leq r_t \leq \frac{1}{\beta}. \]
Government Policy

- Interest on bonds and costs of maintaining the stock of currency financed by lump-sum taxes.
- Total nominal debt, $M_t + B_t$, grows at gross rate $\mu$.
- \[ M_t = \delta(M_t + B_t), \text{ for all } t. \]
- Fiscal policy sets $\mu$, monetary policy sets $\delta$. 
• $q^*$ is traded in monitored exchanges, $r = \frac{1}{\beta}$, $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$.

•

$$\frac{\beta}{\mu} u' \left( \frac{\beta}{\mu} \hat{m} \right) = 1,$$

• Sufficient liquidity if and only if

$$\frac{1 - \delta}{\delta} \geq \frac{q^*(1 - \rho) - L(\frac{1}{\beta})}{\rho \hat{m}}.$$

• Bonds are the key liquid asset.
Equilibrium with Government - Insufficient Liquidity

- \( r < \frac{1}{\beta} \)
- First-order conditions, balance sheet constraint for the intermediary:
  \[
  \beta ru'(\beta rd) = 1.
  \]
  \[
  \frac{\beta}{\mu} u' \left( \frac{\beta}{\mu} m \right) = 1.
  \]
  \[
  d = \frac{L(r)}{1 - \rho} + \frac{(1 - \delta) \rho m}{\delta(1 - \rho)}
  \]
- Sufficient condition (weaker than before) for existence and uniqueness:
  \[
  - \frac{cu''(c)}{u'(c)} \leq 1, \text{ for } c > 0.
  \]
- Insufficient liquidity iff
  \[
  \frac{1 - \delta}{\delta} < \frac{q^*(1 - \rho) - L(\frac{1}{\beta})}{\rho m}.
  \]
Demand and Supply of Liquidity - Insufficient Liquidity Case

\[ (1 - \rho) \omega(r) \]

\[ L(r) + \frac{[(1 - \delta)\rho m]}{\delta} \]
Policy Experiments

- $\mu$ increases: $m$ decreases (so real quantity of bonds falls as well), $r$ decreases, $L(r)$ increases, $d$ decreases.

- $\delta$ increases: $r$ decreases, $L(r)$ increases, $m$ (and price level path) unchanged, consumption in non-monitored trades unchanged, consumption in monitored trades falls.

Money not neutral - a one-time open market purchase reduces the real quantity of public liquid assets, the real interest rate falls, and there is an increase in private lending.

Williamson () Private and Public Liquidity, Intermediation, Intermediation December 2009 19 / 24
Increase in $\mu$ or $\delta$ - Insufficient Liquidity

$$\text{Liquidity}$$
Optimal Policy

- Planner chooses $q$, $s$ to maximize
  \[ W = \rho [u(q) - q] + (1 - \rho) [u(s) - s] - \sigma \rho q. \]

- Solution: $q = \hat{q}$ and $s = q^*$ where $\hat{q}$ solves
  \[ u'(\hat{q}) = 1 + \sigma \]

- Supported as an equilibrium by setting $\mu$, $\delta$ according to
  \[ \mu = \beta (1 + \sigma) \]

  \[ \frac{1 - \delta}{\delta} \geq \frac{q^*(1 - \rho) - L \left( \frac{1}{\beta} \right)}{\rho (1 + \sigma) \hat{q}} \]

- Nominal interest rate at the optimum is $\sigma$.

- At the optimum, the inflation tax finances the entire cost of maintaining the currency stock.
Some Key Results

- Monetary policy is nonneutral only because the central bank is doing something suboptimal.

- If $L\left(\frac{1}{\beta}\right) \geq \rho \hat{q} + (1 - \rho)q^*$, then there is no role for the government - optimal policy supports the same allocation as with note-issuing private intermediaries.

- If $L\left(\frac{1}{\beta}\right) < \rho \hat{q} + (1 - \rho)q^*$, then there is a role for government in supplying liquid outside assets. Two ways to achieve efficiency:
  - Appropriate growth in total nominal debt with appropriate monetary policy, and prohibition on private note issue.
  - Supply sufficient interest-bearing debt (real bonds) and permit private intermediaries to issue circulating notes.
Take fiscal policy as given: $\mu = \hat{\mu}$, $t = 0, 1, 2, ...$

Fix future monetary policy: $\delta = \hat{\delta}$, $t = 1, 2, 3, ...$

Consider effects of changes in $\delta_0$ (open market purchases at $t = 0$ which are undone at $t = 1$) when the nominal interest rate is positive at $t = 0$:

- Sufficient liquidity: Increase in $\delta_0$ increases price level at $t = 0$, there is more trade in non-monitored meetings, and $r_0$ unchanged.
- Insufficient liquidity: Increase in $\delta_0$ increases price level at $t = 0$, there is more trade in non-monitored meetings and less trade in monitored meetings, $r_0$ falls, lending increases.

always exists a $\tilde{\delta}_0$ sufficiently large that:

- if $\delta_0 \geq \tilde{\delta}_0$, then the nominal interest rate at $t = 0$ is zero.
- if $\delta_0 \geq \tilde{\delta}_0$ and $\delta_0$ increases, this has no effect on any quantities or prices - *liquidity trap*, with extra cash injection held as reserves until $t = 1$. 
Central Bank Lending

Suppose central bank wants to increase lending - frustrated by the liquidity trap, it attempts direct lending to the private sector.

Central bank lending on same terms as lending by the private sector:

- irrelevant.
- displaces an equal quantity of private lending.
- large increase in the money supply at $t = 0$, retired at $t = 1$ with returns on central bank portfolio.

Central bank lending on better terms - gross rate of return $r_0$ on loans implies losses on central bank portfolio.

- lowers nominal interest rates on loans.
- increases lending.
- central bank has to make up for the losses on central bank portfolio - taxation results in redistribution, printing money unwinds the zero nominal interest rate policy in period 0.
Conclusions

- Model with financial intermediation, where deposit claims and currency can be used in exchange.
- Model can be used to analyze monetary policy.
- Role for the government can arise if there is not enough capacity in the private economy to produce sufficient liquid assets, or assets that can be made liquid through the intermediation process.
- Monetary policy may be nonneutral - a one-time open market purchase reduces the real interest rate and increases lending.
- Nonneutralities arise only when the central bank is behaving suboptimally.
- Can achieve optimality with the appropriate supply of liquid government bonds and private note issue.