GONE SHOPPING: A THEORY OF RATINGS INFLATION

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Abstract

Many blame the recent financial market turmoil on malfeasance of ratings agencies, who had incentives to bias their ratings. But these incentives had existed for decades. Why did the ratings bias issue only recently emerge? We model asset issuers who can shop for ratings – observe multiple ratings and disclose only a subset – before auctioning their assets. When assets are simple, agencies’ ratings are similar and the incentive to shop is low. When assets are sufficiently complex, ratings differ enough that an incentive to shop emerges. Thus an increase in the complexity of recently-issued securities could create a systematic bias in disclosed ratings. This is true even if each ratings agency discloses an unbiased estimate of the asset’s true quality. Increasing competition among agencies would not solve this problem. Switching to a buyer-initiated ratings system alleviates the bias, but could collapse the market for information.

The recent surge in defaults on subprime mortgages caught the markets off guard because the ratings on their associated financial securities were high. The disparity between the credit rating agencies’ assessment and the realized risk of mortgage-backed securities caused many to allege that the ratings agencies were issuing biased ratings to attract business from asset issuers. But the same institutional structure and the same incentives has existed for decades and for decades. If ratings had been biased for decades, why were investors surprised to learn this in the summer of 2008? What changed to inflate the bias in asset ratings?

We show that even if ratings agencies were each issuing an unbiased rating, the bias in disclosed ratings could have increased. In other words, if asset issuers shop around and choose the highest rating to disclose, then even though each rating was an unbiased draw, the rating that is ultimately announced in the maximum realized rating, which is a biased signal of the asset’s true quality. The incentive to engage in such ratings shopping increases dramatically if new, more complex financial securities are issued, which were harder to rate. Harder to rate securities generate noisier ratings. With more noise, ratings have more dispersion and the difference between the highest and lowest rating grows. If this difference were small, asset issuers would announce all ratings because more information reduces investor uncertainty and increases the price they are willing to pay for the asset. But if the difference between ratings is large, the benefit of reduced uncertainty is outweighed by having investors expect higher returns. With dispersed ratings, shopping results in a higher price
being paid for the issued securities. Thus asset complexity creates an incentive to shop, which prompts ratings inflation.

The link between asset complexity and ratings shopping feeds back on itself. Not only does complexity create an incentive to ratings shop, but once issuers know they will shop for ratings, they prefer to design securities that are more complex. Thus, ratings inflation can feed on itself.

Section 1. models an environment where an asset issuer can purchase and make public one or more signals about the payoff of an asset. We call these signals “ratings.” After choosing how many signals to observe and which ones to make public, the issuer holds a menu auction for his assets. After each asset buyer submits a list of price-quantity pairs, the asset issuer sets the highest market-clearing price for his asset and all buyers pay that price per share.

Section 2. presents our main results. It shows that if assets became more complex and harder to rate, this could explain the increase in ratings shopping and the subsequent increase in the bias of disclosed ratings. Furthermore, if an asset issuer can choose to make his asset more complex, then knowing he will shop for ratings can make even more asset complexity desirable.

Since the ratings bias in this model comes from how asset issuers use and manipulate ratings, and not from the ratings themselves, a natural question is whether prohibiting issuers from acquiring information – only allowing investors who purchase the assets pay for ratings – is a viable or desirable solution. Section 3. extends the model to allow any fraction of investors to observe an asset’s rating at a cost. A fraction of buyers in the model are subject to investible securities regulation, which only allows them to purchase assets whose ratings exceed a minimum threshold. These buyers have higher expected utility if they can acquire inflated ratings, because higher ratings expand their investment possibility set. However, conditional on buying the asset, every investor should use all the information in his information set. This keeps ratings bias from inflating asset prices without bound. The only price effect comes from the increase in the price of assets that were previously not investible and are now investible. Furthermore, because buyers cannot view a rating and credibly commit to pay for it afterwards, they cannot exert any pressure on ratings agencies to issue biased ratings. Thus, investor-initiated ratings are innocuous, compared to issuer-initiated ratings.

This does not mean that investor-initiated ratings have no downside. The problem is not with bias, but with market externalities that prevent the efficient provision of information. We show how investors can free-ride on others’ information and how, with an endogenous price for information, the ratings market can easily collapse for some assets. If buyers cannot overcome these problems to acquire some information for themselves, perhaps some – even biased – information provided by asset issuers is better than none at all.

Both for the case of issuer initiated, and investor initiated ratings we investigate whether, and to what extent the degree of bias of the information produced depends on the degree of competition of the rating agency industry, as well as on other institutions and rules such as the investible securities legislation, and the NRSRO stamp of approval. For the case of issuer initiated ratings, we show that having more rating agencies would exacerbate the problem of bias because there will be more ratings to choose from. For the investor initiated ratings model, increasing the number of agencies is inconsequential.
In an extension of the present work (sketched in Appendix B) we examine situations where rating agencies report biased ratings in an attempt to increase their business. There they face a trade-off between selling many ratings and not losing reputation in case market participant realize that their ratings are systematically biased in an upward way. In such situations the investible securities legislation reinforces incentives to produce biased ratings.

From a broader perspective our analysis and findings highlight the role that institutions, rules and market structure play in an industry that produces information. A central question in economics is what institutions achieve desirable economic outcomes when information is dispersed and private among members in the economy. The mechanism design literature tackles such problems. This paper goes a step deeper and looks at an industry that produces information. We examine how institutions, rules and market structure affect both the quality and the degree of dissemination of the information produced. As the recent crisis highlights, when information production goes wrong, large economic fluctuations can result.

1. An Auction Model of Asset Issuances with a Continuum of Traders

This is a static model. It consists of an asset issuer, who controls the supply of the asset. The demand is determined by the investors (or bidders). Each bidder submits a bidding function that specifies the maximum amount that the bidder is willing to pay for \( q \) units as a function of his information. These bid functions determine the aggregate demand. The auctioneer specifies a market clearing price \( p \) that equates aggregate demand and supply and each trader pays this price for each unit purchased (uniform price auction). The asset’s value is unknown to the market participants. Information about the value of the asset is produced and sold by the credit rating agencies. This information can be purchased either by the issuer or by the investors or by both. Below we investigate how each of these different arrangements about who purchases the information affects the quality and the amount of information available to market participants.

We now move on to provide more details about the assets, the asset issuer, the investors and the credit rating agencies.

**Assets** There are two assets, one offers riskless return \( r \), the risky asset pays \( u \), which is normally distributed \( u \sim N(\bar{u}, \sigma_u^2) \). Price of the riskless asset = 1 (numeraire), price of risky asset is \( p \) (endogenous).

**Asset Issuer** The issuer of the asset is endowed with \( x \) shares of the risky asset. When we investigate the scenario where investors buy the information, in order to ensure that prices are not perfectly revealing, we will modify the assumption that the supply is known and fixed. We will instead assume that the issuer of the asset is endowed with \( x + \bar{x} = \int q d\bar{q} \) shares of the asset. It is partly random: \( \bar{x} \sim N(0, \sigma_x) \). This randomness keeps investors from being able to free-ride on the information other investors know.
Investors There is a continuum of ex-ante identical investors have utility:

$$U^I = -e^{-\rho W}$$  \hspace{1cm} (1)

where $W$ is wealth at the end. Each agent is endowed with $m_0^i$ units of riskless asset, but can borrow and lend that asset freely at the riskless rate $r$. Investors’ end of period wealth is $W = m_i r + q u$, were $q$ is the number of shares the investor buys of the risky asset.

Main Steps of the Analysis of the Investors’ Behavior:

We will now state the bid functions and verify that they form an equilibrium.

$$b(q, \text{info}) = \frac{E[u|\text{info}, \text{I pay } b \text{ per unit}] - q \rho Var[u|\text{info}, \text{I pay } b \text{ per unit}]}{r}$$

Note: the expectation of $u$ is taken conditional on the information that the trader has as well the info he infers from $b$ being the price paid per unit (this is because of the uniform price character of the auction. The price paid per unit is a exogenous from each bidder’s perspective because he is infinitesimal compared to the rest of the market, implying that the price he faces is determined completely by aggregate demand (determined by all other bidder’s bid functions) together with the aggregate supply.

Interpretations of the bidding functions: $b(q,\text{info})$: given what I know (\text{info}) I want to pay up to $b$ to acquire $q$ units. Now in the event that I end up paying $b$ this, together with the auction rules, will tell me something about the behavior of the other bidders, which also depends on their information. Given that there is a continuum of bidders the market clearing price is independent of my bidding behavior. It is then straightforward to verify that the above bidding function is a best response given everyone else’s bid function. This is because the bidding function $b(q,\text{info})$ can be viewed as an inverse demand function (where we replace $b$ (\$ paid) by $p$ :

$$q(p, \text{info}) = \frac{1}{\rho Var[u|\text{info}, \text{I pay } p \text{ per unit}]}(E[u|\text{info}, \text{I pay } p \text{ per unit}] - pr)$$  \hspace{1cm} (2)

which is exactly the demand function obtained via utility maximization for a given price $p$.

Deriving the demand function:

$$EU = -E[\exp(-\rho(uq_i + m_i r))]$$

$$= -E[\exp(-\rho(uq_i + r(m_0^i - pq_i)))]$$

$$= -E[\exp(-\rho q_i(u - pr)) \exp(-pr(m_0^i)))]$$

The second term is a constant. It won’t matter in the maximization. The first term is a log-normal variable at time 2. $pr$ is known and $u$ is normally distributed. Mean of a log normal is:

$$E[e^{-\rho q(u-pr)}|E[u], V[u]] = e^{-\rho q(E[u]-pr) + \frac{\rho^2 q^2}{2}V[u]}.$$
It can be easily verified that the objective is concave in \( q \) so that the first order condition describes the optimal portfolio

\[
-\rho(E[u] - pr) + 2\rho^2 V[u]q = 0.
\]

Rearranging,

\[
q = \frac{1}{\rho} V[u]^{-1}(E[u] - pr).
\] (3)

The budget constraint is then satisfied with purchases of the riskless asset. That pins down \( m_i \).

This first order condition holds irrespective of the information that each investor conditions on. In order to obtain the asset demand of an investor as a function of his information we have to condition \( E[u] \) and \( V[u] \) on the information that is available and can be inferred from equilibrium play.

**Ratings Agencies** There is a finite number of rating agencies operating. In reality there are essentially three major agencies: Moody’s, S&P and Fitch. Agencies produce ratings. The rating is a signal \( \theta \) that reveals some information about the asset payoff \( u \). A rating agency bases its rating primarily on publicly observable information, but has access to exclusive know-how, such as statistical tools, and industry experience, that translate this information into a relevant signal about the risky asset’s return.

We first assume that the rating \( \theta \) is an unbiased signal about the asset’s return, that is \( \theta \sim N(u, \sigma_\theta^2) \) and is produced at marginal cost \( \chi \). All rating agencies produce the same service (they all have the NRSRO stamp of approval) and compete in a Bertrand way. Given this market structure, the price per rating is \( \chi \).

2. **Shopping for Issuer-Initiated Ratings**

When fixed-income securities are issued, the issuer typically pays the ratings agency to publicly issue a rating that summarizes the probability that the asset issuer will default. Therefore, we explore the incentives to acquire and disclose ratings in a setting with issuer-initiated ratings before exploring the counter-factual possibility that buyers might initiate ratings in the following section.

Our objective is to explore why securities ratings might be biased and what institutional arrangements might correct this bias. To explore this question in a non-trivial way, we need to make assumptions to ensure that the bias is not irrelevant. In particular, sophisticated buyers should not be able to infer the expected bias, subtract that expected bias from the announced rating, and neutralize its effects through their actions. To prevent this, we assume that buyers are naive: They treat each disclosed rating as an unbiased signal about the asset’s true quality. We re-solve the model with sophisticated buyers in appendix B. The incentive for issuers to shop for ratings and make assets complex still exists, but it no longer affects the average price of assets.

The assumption that buyers believed ratings to be unbiased is consistent with our main argument that much of the bias was a relatively recent phenomena. If buyers get no information about
asset complexity, they would not have known that a change in asset characteristics had taken place. They would rationally infer ratings bias from the past history of ratings and asset outcomes. Since most assets in the past were simple assets, the bias would have been small. Thus, even rational buyers with incomplete information about asset characteristics might not have detected the ratings bias until it was too late.

Before the auction begins, the asset issuer can decide whether or not to acquire a rating and if yes, how many ratings he chooses to purchase. A rating can be acquired at a price $\chi$. Paying that cost entitles the issuer to freely distribute the signal to all investors. We assume that the asset issuer must pay for the rating before he can observe its value, but we show that allowing issuers to observe ratings first and then pay to publicize them does not overturn any of our main results. Issuers’ utility function is

$$U^S = px - \tau \chi$$  \hspace{1cm} (4)

where $\tau$ is the number of ratings bought by the issuer. The benefit the issuer gets from paying to get his asset rated is that the rating makes the asset less risky to the investors, which increases the price they are willing to pay for it.

**Assumption: Misinformed Investors** In this section, we maintain the assumption that investors are naive and do not condition on the fact that an unrated asset is possibly an asset that was rated, but got a bad rating, or that the rating disclosed may be the maximum rating of a set of observed ratings. In a static model, these investors are just not fully rational. In a repeated version of this model, this could be is equivalent to assuming that the complexity of assets increased and investors were not aware of this change. If investors thought assets were still simple and could be rated accurately, then they would rationally infer that little or no ratings shopping was taking place and they would not try to correct disclosed signals for any bias.

This assumption is perhaps a bit crude, but is necessary to have any meaningful discussion of ratings bias. If investors are sophisticated, ratings may or may not still be biased, but that bias will be inconsequential. It will not affect asset prices. We analyze the case of sophisticated buyers in appendix B.

### 2.1 Ratings demand with one ratings agency

Suppose that the issuer decides whether or not to buy a single rating ($\tau \in \{0, 1\}$). The first step to figuring out demand for ratings is to solve for investors’ bidding functions and determine the market price, which in turn determines the issuer’s payoff through (4).

When the issuer does not obtain a rating everybody remains uninformed. In this case, integrating over the asset demand for each investor given by (2) and equating that with the asset supply delivers the equilibrium price

$$p = \frac{1}{r} (\bar{u} - \rho(\sigma_u^2)x).$$ \hspace{1cm} (5)

Now when the issuer acquires a rating the price he faces depends on whether he sees the rating...
before purchasing it or not. We first assume that the issuer cannot condition the decision to acquire a rating or not on the particular realization of the rating.

**Issuer purchases rating prior to observing it.** First, let’s assume that the issuer releases the rating irrespective of its realization:

If the issuer initiates a rating, the expected value of the asset is 
\[ E[u|\theta] = \left(\sigma_u^{-2}\bar{u} + \sigma_\theta^{-2}\theta\right)/\left(\sigma_u^{-2} + \sigma_\theta^{-2}\right) \]
and the conditional variance of the asset will be 
\[ V[u|\theta] = 1/\left(\sigma_u^{-2} + \sigma_\theta^{-2}\right). \]
Since we assumed that the issuer acquires the rating before he knows its outcome, we need to compute the expected price 
\[ \bar{p}_1 \equiv E[p] = \frac{1}{r}(\bar{u} - \rho V[u|\theta]x). \]
If he does not buy the rating, the price will be 
\[ \bar{p}_0 \equiv \frac{1}{r}(\bar{u} - \rho(\sigma_u^2)x). \]
Thus, the difference in issuer utility from buying information is 
\[ U_{\tau=1}^S - U_{\tau=0}^S = (\bar{p}_1 - \bar{p}_0)x - \chi \]
\[ = \frac{1}{r} \left( \frac{\sigma_u^{-2}\bar{u} + \sigma_\theta^{-2}\theta - \rho x}{\sigma_u^{-2} + \sigma_\theta^{-2}} \right) \rho x - \chi. \]

Next, consider the case where the issuer releases ratings selectively

Suppose now that the issuer still purchases the rating prior to observing it, but he can decide to release it or not conditional on it being sufficiently favorable, say above a threshold \( \tilde{\theta} \). Then for all \( \theta > \tilde{\theta} \) the realized price will be

\[ p = \frac{1}{r} \left( \frac{\sigma_u^{-2}\bar{u} + \sigma_\theta^{-2}\theta - \rho x}{\sigma_u^{-2} + \sigma_\theta^{-2}} \right) \]
and for all \( \theta < \tilde{\theta} \) the price will be given by (5). Then the expected price without rating remains (5). Whereas, the expected price with rating is

\[ \bar{p}_1 = E\left[\frac{1}{r} \left( \frac{\sigma_u^{-2}\bar{u} + \sigma_\theta^{-2}\theta - \rho x}{\sigma_u^{-2} + \sigma_\theta^{-2}} \right) | \theta > \tilde{\theta} \right] + E\left[\frac{1}{r}(\bar{u} - \rho(\sigma_u^2)x)|\theta < \tilde{\theta}\right]. \]

where \( \bar{p}_1 \) stands always for the expected price arising from purchasing a single rating

Then, the difference in the issuers’ utility from buying information is

\[ U_{\tau=1}^S - U_{\tau=0}^S = (\bar{p}_1 - \bar{p}_0)x - \chi \]

\[ = \left( \frac{E\left[\frac{1}{r} \left( \frac{\sigma_u^{-2}\bar{u} + \sigma_\theta^{-2}\theta - \rho x}{\sigma_u^{-2} + \sigma_\theta^{-2}} \right) | \theta > \tilde{\theta} \right]}{r} \right) - \chi. \]
This expression determines for each rating disclosing rule $\tilde{\theta}$ whether the issuer will find it profitable to buy information or not. At an equilibrium the issuer chooses $\tilde{\theta}$ to maximize $U_{\tau=1}^S - U_{\tau=0}^S$. This is done as follows. Notice that $\phi(\theta) \equiv \frac{x}{r} \frac{\sigma_u^{-2}(\theta - \tilde{u}) + \rho x \sigma_u^2 \sigma_{\theta}^{-2}}{\sigma_u^{-2} + \sigma_{\theta}^{-2}}$ is strictly increasing in $\theta$. From this observation, it is easy to see that $\tilde{\theta}$ is the smallest $\theta$ with the property that $\frac{x}{r} \frac{\sigma_u^{-2}(\theta - \tilde{u}) + \rho x \sigma_u^2 \sigma_{\theta}^{-2}}{\sigma_u^{-2} + \sigma_{\theta}^{-2}} - \chi \geq 0$, hence in an equilibrium

$$\tilde{\theta} = \min\{\theta \in \mathbb{R} : \frac{x}{r} \frac{\sigma_u^{-2}(\theta - \tilde{u}) + \rho x \sigma_u^2 \sigma_{\theta}^{-2}}{\sigma_u^{-2} + \sigma_{\theta}^{-2}} - \chi \geq 0\}. \quad (7)$$

**Purchase rating after observing it.** Suppose now that the issuer can also decide whether to purchase the rating or not depending on its realization. In this case the issuer purchases a price only if he is intending to reveal it. In particular, say that he decides to purchase the rating if $\theta > \hat{\theta}$. In this case the difference in the issuer’s payoff that results from purchasing information depends on the rating realization - since the issuer now can condition on this information when purchasing the rating - and it is given by

$$U_{\tau=1}^S - U_{\tau=0}^S = x \frac{\sigma_u^{-2}(\theta - \tilde{u}) + \rho x \sigma_u^2 \sigma_{\theta}^{-2}}{\sigma_u^{-2} + \sigma_{\theta}^{-2}} - \frac{x}{r} (\tilde{u} - \rho \sigma_u^{-2} x) - \chi.$$

As before, $\hat{\theta}$ in an equilibrium must be chosen optimally. The optimal cut-off of $\theta$ for deciding to disclose the rating in this case is given by

$$\hat{\theta} = \min\{\theta \in \mathbb{R} : \frac{x}{r} \frac{\sigma_u^{-2}(\theta - \tilde{u}) + \rho x \sigma_u^2 \sigma_{\theta}^{-2}}{\sigma_u^{-2} + \sigma_{\theta}^{-2}} - \chi \geq 0\},$$

which turns out to be the same rule as in the case where the issuer did not observe the rating.

**Conclusion:** In an environment where the issuer can purchase a single rating, whether the issuer observes the rating prior to or after purchasing it is inconsequential, as long as the issuer can disclose the rating only if he wishes to do so.

### 2.2 Ratings shopping with multiple ratings

Next consider a setting where there are $N$ ratings agencies. Issuers can purchase multiple ratings and after observing the rating, decide which information to reveal to investors. As we will see, issuers may selectively reveal signals, creating signal bias on average.

**Purchase Ratings Prior to Observing them** Here we assume that the issuer can purchase multiple ratings prior to observing them, however he can choose to release a subset of them if he wishes to do so.
Suppose that the issuer purchases two ratings, \( \theta_1 > \theta_2 \), both with variance \( \sigma^2_\theta \). Consider the choice of whether to reveal both ratings, or only one of them. If the issuer reveals both ratings, the posterior variance of the asset payoff will be lower for investors. The resulting asset price will be

\[
p_2 = \frac{1}{r} \frac{\sigma_u^{-2} \bar{u} + \sigma_\theta^{-2}(\theta_1 + \theta_2) - \rho x}{\sigma_u^{-2} + 2\sigma_\theta^{-2}}.
\]  

(8)

If the issuer only reveals the more favorable rating, the posterior variance of the asset payoff will be higher for investors, but the mean expected payoff will be higher as well. The resulting asset price will be

\[
p_1 = \frac{1}{r} \frac{\sigma_u^{-2} \bar{u} + \sigma_\theta^{-2}\theta_1 - \rho x}{\sigma_u^{-2} + \sigma_\theta^{-2}}.
\]  

(9)

Suppose that the issuer has paid for both ratings and the question is whether it is worth revealing both of them. Then he will choose to reveal only the highest rating if

\[
p_1 - p_2 > 0,
\]

which can be rewritten as:

\[
\sigma_\theta^{-2} \left( \frac{\sigma_u^{-2}(\bar{u} - \theta_2) + \sigma_\theta^{-2}(\theta_1 - \theta_2) - \rho x}{\sigma_u^{-2} + \sigma_\theta^{-2}} \right) > 0.
\]

Selectively revealing ratings will be optimal if \( p_1 - p_2 > 0 \). This will hold when \( \theta_1 \) and \( \theta_2 \) are sufficiently far apart, in particular we need that

\[
\sigma_u^{-2}(\bar{u} - \theta_2) + \sigma_\theta^{-2}(\theta_1 - \theta_2) - \rho x > 0.
\]

(10)

If condition (10) holds, then even though every rating agency is issuing honest, unbiased ratings, the ratings that are revealed will be systematically upward biased, because in this case the issuer chooses to reveal the highest of the two ratings. This is an undesirable situation because asset investors are systematically paying too high a price for assets, given their fundamental risk and return. If such bias is uncovered and revealed to investors, prices would fall precipitously.

Note that condition (10) gives us information about whether the issuer will choose to display both versus one of the two ratings conditional on having purchased both of them. If (10) fails then the issuer chooses to display either one or no ratings at all. In particular, if \( \theta_1 > \tilde{\theta} \) where \( \tilde{\theta} \) satisfies (7), then the highest rating will be displayed, whereas otherwise no ratings will be displayed even though the issuer bought two of them.

Conditional on having purchased a single rating the disclosing rule is still characterized by \( \hat{\theta} \) that satisfies (7).

In order to complete the analysis for this we need to answer the following questions: When does the issuer buy two versus one versus no ratings?
The expected price from buying two ratings is given by:

\[
\bar{p}_2 \equiv E[p|\text{two ratings}] = E \left[ \frac{1}{r} \frac{\sigma_u^{-2} \bar{u} + \sigma_\theta^{-2} (\theta_1 + \theta_2) - \rho x}{\sigma_u^{-2} + 2\sigma_\theta^{-2}} | \theta_1 - \theta_2 \leq \hat{\Delta} \text{ and } \theta_1 \geq \hat{\theta} \right] \\
+ E \left[ \frac{1}{r} \frac{\sigma_u^{-2} \bar{u} + \sigma_\theta^{-2} \theta_1 - \rho x}{\sigma_u^{-2} + \sigma_\theta^{-2}} | \theta_1 - \theta_2 > \hat{\Delta} \text{ and } \theta_1 \geq \hat{\theta} \right] \\
+ E \left[ (\bar{u} - \rho (\sigma_u^2) x) | \theta_1 < \hat{\theta} \right]
\]

The expected price from buying a single rating is given by:

\[
\bar{p}_1 \equiv E[p|\text{one rating}] = E \left[ \frac{1}{r} \frac{\sigma_u^{-2} \bar{u} + \sigma_\theta^{-2} \theta - \rho x}{\sigma_u^{-2} + \sigma_\theta^{-2}} | \theta \geq \hat{\theta} \right] \\
+ E \left[ (\bar{u} - \rho (\sigma_u^2) x) | \theta < \hat{\theta} \right]
\]

Finally, as usual the expected price when no rating is purchased is given by \( \bar{p}_0 = \bar{u} - \rho (\sigma_u^2) x \).

With the help of the above we can calculate under which conditions the issuer will choose to buy two versus one versus no zero ratings. In particular the issuer will choose to buy two versus one rating iff

\[
(\bar{p}_2 - \bar{p}_1) x - \chi \geq 0
\]

The issuer will choose to buy one versus no ratings iff

\[
(\bar{p}_1 - \bar{p}_0) x - \chi \geq 0
\]

**Purchase Ratings After Observing them:**

Suppose now that the issuer can observe all available ratings before deciding how many to purchase and disclose. Again in the case where the issuer can first see a rating and then purchase it, the decision to buy a rating and to disclose it is the same, since there is no point in purchasing a rating if there is no intention to displaying it.

Under observability, the issuer will choose to buy at least one rating, if the highest of the two satisfies:

\[
U^S_{\tau=1} - U^S_{\tau=0} = \frac{x}{r} \left( \frac{\sigma_u^{-2} \bar{u} + \sigma_\theta^{-2} (\theta_1 - \bar{u}) - \rho x}{\sigma_u^{-2} + \sigma_\theta^{-2}} \right) - \frac{x}{r} (\bar{u} - \rho (\sigma_u^2) x) - \chi
\]

\[
= \frac{x}{r} \left( \frac{\sigma_\theta^{-2} (\theta_1 - \bar{u}) + \rho x \sigma_u^2 \sigma_\theta^{-2}}{\sigma_u^{-2} + \sigma_\theta^{-2}} \right) - \chi.
\]

The issuer will buy only the highest rating of the highest rating of the two iff

\[
(p_1 - p_2) x + \chi \geq 0
\]
that is if
\[
\frac{\sigma_u^{-2} \bar{u} + \sigma_\theta^{-2} \theta_1}{\sigma_u^{-2} + \sigma_\theta^{-2}} - \frac{\sigma_u^{-2} \bar{u} + \sigma_\theta^{-2} (\theta_1 + \theta_2)}{\sigma_u^{-2} + 2\sigma_\theta^{-2}} + \chi \geq 0
\]
which can be rewritten as a function of ratings variance being positive
\[
F(\sigma_\theta^2) \equiv \frac{\sigma_\theta^{-2}}{r} \left( \frac{\sigma_u^{-2} (\bar{u} - \theta_2) + \sigma_\theta^{-2} (\theta_1 - \theta_2) - \rho x}{(\sigma_u^{-2} + \sigma_\theta^{-2}) (\sigma_u^{-2} + 2\sigma_\theta^{-2})} \right) + \chi \geq 0. \quad (11)
\]
If \(F(\sigma_\theta^2) < 0\), he will buy both ratings.

2.3 The effect of greater asset complexity

Our main result, that greater asset complexity could have triggered ratings inflation, follows directly from the previous results. But, first, we need to define asset complexity.

**Definition 1** A more complex asset is one with more noise in its ratings: It has a higher \(\sigma_\theta^2\).

Consider a firm who chooses whether to structure a simple security or a complex security. Both securities have the same payoff distribution, but the distribution of the ratings is \(\theta_i \sim N(u, \sigma_\theta^2\theta_S)\) for the simple security and \(\theta_i \sim N(u, \sigma_\theta^2\theta_C)\) for the complex security, where \(\sigma_\theta\theta_S < \sigma_\theta\theta_C\). The firm chooses how to structure the security before it observes its ratings.

If the firm does not engage in ratings shopping, its expected asset price is the expectation of (8)
\[
E[p_2] = \frac{\sigma_u^{-2} \bar{u} + 2\sigma_\theta^{-2} \bar{u} - \rho x}{\sigma_u^{-2} + 2\sigma_\theta^{-2}}.
\]
\[
(12)
\]
In this case, making the security more complex lowers the precision \(\sigma_\theta^{-2}\), which unambiguously lowers the expected asset price. So, there is no incentive to make the asset complex.

If the firm does engage in ratings shopping, its expected asset price is the expectation of (9)
\[
E[p_1] = \frac{\sigma_u^{-2} \bar{u} + \sigma_\theta^{-2} E[\theta_1|\theta_1 > \theta_2] - \rho x}{\sigma_u^{-2} + \sigma_\theta^{-2}}.
\]
\[
(13)
\]
Now, making \(\theta_1\) and \(\theta_2\) higher variance has two effects. It lowers \(\sigma_\theta^{-2}\), which lowers the price, but it also increases \(E[\theta_1|\theta_1 > \theta_2]\).

**Result 2** The issuer of a more complex asset is more likely to shop for ratings: \(\partial F(\sigma_\theta^2)/\partial \sigma_\theta^2 > 0\).

The proof of this and all further results and lemmas is in appendix A.

When the issuer practices ratings shopping then instead of \(E[\theta_1] = \bar{u}\) the expected price is determined in part by \(E[\max\{\theta_1, \theta_2\}]\) which is greater than \(E[\theta_1]\). The more draws the issuer can observe before choosing a rating, i.e. the larger the number of rating agencies, the higher this bias will be.
2.4 Issuers who choose asset complexity

Recall that a firm shops for ratings when \( \theta_1 \) and \( \theta_2 \) are sufficiently far apart. If \( \theta_1 \) and \( \theta_2 \) are drawn from a higher-variance distribution, they will be further apart on average, and that makes ratings-shopping more likely. Thus, added security complexity feeds the incentive to shop for ratings. It also means that ratings-shopping creates the incentive to structure more complex securities.

Next, suppose that an asset issuer can choose to make their security more complex. A more complex security is one that is harder to rate. Thus, the signals the ratings agencies get have higher variance. Consider a firm who chooses whether to structure a simple security or a complex security. Both securities have the same payoff distribution, but the distribution of the ratings is \( \theta_i \sim N(u, \sigma_{\theta S}^2) \) for the simple security and \( \theta_i \sim N(u, \sigma_{\theta C}^2) \) for the complex security, where \( \sigma_{\theta S} < \sigma_{\theta C} \).

The firm chooses how to structure the security before it observes its ratings.

If the firm does not engage in ratings shopping, its expected asset price is the expectation of (8)

\[
E[p_2] = \frac{1}{r} \frac{\sigma_u^{-2} \bar{u} + 2\sigma_{\theta}^{-2} \bar{u} - \rho x}{\sigma_u^{-2} + 2\sigma_{\theta}^{-2}}.
\]

In this case, making the security more complex lowers the precision \( \sigma_{\theta}^{-2} \), which unambiguously lowers the expected asset price. So, there is no incentive to make the asset complex.

If the firm does engage in ratings shopping, its expected asset price is the expectation of (9)

\[
E[p_1] = \frac{1}{r} \frac{\sigma_u^{-2} \bar{u} + \sigma_{\theta}^{-2}E[\theta_1 | \theta_1 > \theta_2] - \rho x}{\sigma_u^{-2} + \sigma_{\theta}^{-2}}.
\]

Now, making \( \theta_1 \) and \( \theta_2 \) higher variance has two effects. It lowers \( \sigma_{\theta}^{-2} \), which lowers the price, but it also increases \( E[\theta_1 | \theta_1 > \theta_2] \).

Result 3 An asset issuer who will shop for ratings prefers to issue a more complex security if \( \partial E[p_1]/\partial \sigma_{\theta}^2 > 0 \).

When the issuer practices ratings shopping then instead of \( E[\theta_1] = \bar{u} \) the expected price is determined in part by \( E[\max\{\theta_1, \theta_2\}] \) which is greater than \( E[\theta_1] \). The more draws the issuer can observe before choosing a rating, i.e. the larger the number of rating agencies, the higher this bias will be.

Recall that a firm shops for ratings when \( \theta_1 \) and \( \theta_2 \) are sufficiently far apart. If \( \theta_1 \) and \( \theta_2 \) are drawn from a higher-variance distribution, they will be further apart on average, and that makes ratings-shopping more likely. Thus, added security complexity feeds the incentive to shop for ratings and ratings-shopping creates the incentive to structure more complex securities.

complexity \( \rightarrow \) rating-shopping

\( \leftarrow \)
2.5 Does more competition help?

Result 4 If the asset issuer will shop for ratings, the increasing the number of ratings agencies will increase the bias of the disclosed rating.

2.6 The effect of investible securities legislation

Another target for criticism in the ratings scandal has been the role that risk-management rules played. Many banks and pension funds are required to hold only investible securities. These are assets who earn sufficiently high ratings from one of the five nationally-recognized statistical ratings organizations (NRSRO’s). This rule puts an enormous amount of pressure on asset issuers to ensure their assets achieve this rating. Without it, the pool of potential buyers is considerably smaller and the asset’s prices will be considerably lower.

Just repealing the investible securities regulations will not solve the problem of ratings shopping. None of the analysis so far has relied on any such demand effect. However, it is likely that this regulation further encouraged ratings shopping by increasing the payoff for acquiring a high rating, for a given level of ratings uncertainty.

3. Investor-initiated ratings

One possible solution to the problem of ratings bias is to replace the system of issuer-initiated ratings with investor-initiated ratings. We show that even though some buyers, those subject to the investible securities regulations, would prefer biased ratings, they cannot shop for ratings or pressure ratings agencies to bias their ratings like asset issuers can. However, a buyer-initiated market for ratings may provide too little information or can collapse entirely.

3.1 A model of investor information acquisition

Since investors are ex-ante identical, either all investors buy information (\(\lambda = 1\)), all investors do not buy information (\(\lambda = 0\)) or all investors are indifferent. We use the indifference condition to solve for equilibria where \(\lambda \in (0, 1)\). In order to do so we need to calculate the expected utility of the informed and the uninformed investors. The first step to do so is to derive the demand for the informed and the uniformed investors.

Substituting the correct posterior mean and variance in (3) we get the asset demands of informed investors

\[
q^I = \frac{1}{\rho} \sigma^{-2}(\theta - pr)
\]

For uninformed investors, the asset demand is more complicated. The reason is that uninformed investors learn something about \(\theta\) from observing the asset price \(p\). At the same time, asset prices are influenced by uninformed investors’ demand for the asset. This is a fixed point problem. We need to solve for \(p\) and \(q^U\) jointly.
The price of the risky asset is determined by the market clearing condition
\[ \lambda q^I + (1 - \lambda)q^U = x + \bar{x}. \] (17)

**Lemma 5** The price of the risky asset is a linear function of the signal and the asset supply:
\[ p = A + B\theta + Cx. \]

Uninformed investors combine their prior belief that \( \bar{\theta} \sim N(\theta, \sigma_\theta^2) \) and their signal from the price \( (p - A)/B \sim N(\theta, (C/B)^2\sigma_x) \) to form their posterior belief: \( \theta \sim N(\hat{\mu}, \sigma_{f|p}) \) where the posterior variance is
\[ \sigma_{f|p} \equiv V ar[\theta|\hat{\theta}, p] = (\sigma_\theta^{-2} + ((C/B)^2\sigma_x)^{-1})^{-1} \] (18) and the posterior mean is
\[ \hat{\mu} = \sigma_{f|p}(\sigma_\theta^{-2}\bar{\theta} + ((C/B)^2\sigma_x)^{-1}(P - A)/B) \]

Therefore, the uninformed investors’ optimal portfolio is
\[ q^U = \frac{1}{\rho \sigma_{f|p}^{-2}}(\hat{\mu} - pr). \] (19)

Note that on average, informed agents demand more of the risky asset. On average, the conditional mean will be
\[ E\left[ \frac{\sigma_\theta^{-2}\bar{\theta} + (C^2\sigma_x)^{-1}(P - A)/B}{\sigma_\theta^{-2} + (C^2\sigma_x)^{-1}} \right] = \theta \]
but the conditional variance for informed investors is less \( \sigma_e^2 < \sigma_{f|p}^2 \). Since higher demand for the asset pushes up its price, the more investors observe the rating, the higher the asset price will be. Furthermore, the more precise the information that informed investors observe, the higher their expected demand for the asset and the higher its expected price.

Now that we have the demands for the risky asset both for the informed and the uninformed investors, we can move on to calculate their corresponding expected utilities.

The argument of the utility function is risk aversion times wealth: \( \rho W = \rho q(u - pr) = (E[u] - pr)'V[u]^{-1}(u - pr) \). This is a product of correlated normal variables. To take expected utility, we need to know the expectation of the exponential of this.

**Lemma 6** The ratio of informed investors’ expected utility to uninformed investors’ expected utility, before accounting for information cost, is
\[ \frac{E[U^I]}{E[U^U]} = \left( \frac{\sigma_e^2}{\sigma_e^2 + \sigma_{f|p}^2} \right)^{\frac{1}{2}}. \]
Note that this ratio is less than one because utility is negative. Informed investors’ utility is higher when it is less negative and therefore smaller in absolute value.

In equilibrium, either the expected net benefit of information \((E[U^I] - E[U^U])\) must equal the expected utility cost \(-e^{\rho c} U^I\), or there must be a corner solution \(\lambda = \{0, 1\}\). Thus, the condition for an interior equilibrium is

\[
\left( \frac{\sigma_e^2 + \sigma_f^2}{\sigma_e^2} \right)^{1/2} = e^{\rho c},
\]

(20)

### 3.2 Buyer’s Incentive to Obtain Biased Ratings

Suppose that some subset of buyers can only buy assets that are “investment grade.” That means the asset’s rating surpasses a threshold \((\theta > \bar{\theta})\). This group of buyers then achieves higher expected utility if they obtain an upward-biased rating.

Let \(\Phi\) be the cumulative probability density function for the unbiased rating \(\theta\). Suppose that instead of \(\theta\), the ratings agency, with the knowledge of the asset buyer, issues a rating \(\theta + \epsilon\). Then with probability \(\Phi(\bar{\theta} - \epsilon)\), the investor gets a rating that is too low, cannot invest, and gets no income from the risky security. With probability \(1 - \Phi(\bar{\theta} - \epsilon)\), the investor can invest and chooses the optimal portfolio in equation (16). This investor’s expected utility is

\[
U^{\text{bias}} = -\exp \left[ -\rho r (m_0 - c) \right] \int_{\theta - \epsilon}^{\infty} E \left[ \exp \left[ \frac{(\theta - pr)^2}{\sigma_e^2} \right] \right] d\theta.
\]

(21)

Since the bias \(\epsilon\) enters only in the bounds of integration and expands them, and since the integrand is strictly positive, signal bias unambiguously increases expected utility.

### 3.3 Why Buyers Cannot Bias Ratings or Inflate Asset Prices

If the buyers shop for ratings, it is in order to find a ratings agency that gives the asset an investible grade rating. But once the buyer find that the asset is in his feasible investment set, he should use all available information to determine the optimal bidding strategy for the asset. Since buyers use all available information in forming their asset demands, prices are not affected by their ratings shopping.

Another key difference between asset buyers and asset issuers when they purchase information is that buyers want to know the information and issuers potentially want to publicize the information. While asset issuers who are shown a rating still have an incentive to pay for the ratings agency to publicize the rating and put their reputation behind it, asset buyers have no such incentive. They do not want other buyers to know this information. A buyer who is given a preview of the information they buy should never rationally pay for the information.

Because asset buyers cannot see a rating before the buy it, the ratings agency has no incentive to make the rating more positive. The buyer is just as likely to buy a positive rating as a negative one.
3.4 Downsides to Buyer-Initiated Ratings

While buyers cannot induce ratings bias, they can create two other problems due to information market externalities: information leakage and market collapse due to demand complementarity. Since information requires a fixed cost to discover and is free (or at least quite cheap) to replicate, efficiency dictates that a discovered piece of information should be distributed to every asset buyer so that all buyers benefit from lower asset payoff risk. Yet, when buyers have to pay for ratings themselves, either no buyers or too few buyers may end up being informed.

Information Leakage One reason that asset buyers may decide not to buy information is that they can partially free-ride on the information others observe. The price of the asset will depend on what informed investors know. While the uninformed investors cannot literally observe the price before they bid, they can condition on the price when they submit their menu of bids. When deciding on the quantity they will associate with each realized price, the buyer asks himself, “If that is the realized price, what would it tell me about what the informed buyers have learned?” In this way, uninformed buyers can use information contained in prices to free ride on what others have learned.

An increase in the number of informed investors \( \lambda \) reduces the posterior uncertainty of uninformed investors, conditional on the price level \( \sigma_{f|p} \). Recall that the noise in the asset price about the signal \( \theta \) is \( (C/B)^2 \sigma_x \). Having more informed investors (higher \( \lambda \)) reduces \( (C/B)^2 \) (equations 23 and 24 in appendix). This makes prices more informative (reduces \( \sigma_{f|p} \) in equation 18) and reduces the benefit of acquiring information (lemma 6). This is a form of strategic substitutability that makes it unlikely that asset buyers will ever all choose to be informed.

Complementarity in Information Demand and Market Collapse The basic model exhibits strategic substitutability. But endogenizing the price of ratings can introduce complementarity that also creates problems. The market for information can collapse. This happens when no buyers buy signal because no other buyers are buying signals. Such a collapse can arise in situations where the seller would be willing to provide information to all buyers for free.

Instead of assuming that signals are provided at a fixed cost, we consider a profit-maximizing information-production sector. The sector has three crucial features: First, information can be produced with a fixed-cost technology. A signal \( \theta \) can be discovered by any agent for a fixed cost \( \chi \). This can be interpreted as the cost of hiring an analyst to interview people and find primary sources of information. Once discovered, the information can be replicated at zero marginal cost and sold to other investors at a price \( c \). Second, reselling purchased information is forbidden. The realistic counterpart to this assumption is intellectual property law that prohibits copying a publication and re-distributing it for profit. Third, there is free entry. Any agent can discover information at any time, even after other information producers have announced their prices \( c \). That information markets are competitive is crucial. The exact market structure is not. Veldkamp (2006) analyzes Cournot and monopolistic competition markets for information. All three markets
produce information prices that decrease in demand.

**Lemma 7** The equilibrium price for information $c(\lambda)$ is decreasing in the quantity of information sold $\lambda$. Specifically, $c(\lambda) = \chi/\lambda$.

**Figure 1:** The costs and benefits of information, as the fraction of informed buyers varies.

**Reconciling substitutability and complementarity** Figure 1 plots the costs and benefits of information as the fraction of the investors that buy the signal increases. Points where the cost and benefit line cross are potentially equilibria. With an exogenous cost, there is one crossing point and a unique equilibrium. With the endogenous price for information that comes from the information market, there are three possibilities: $\lambda = 0$ is an equilibrium because at that point, the costs exceed the benefit of information, and the two crossing points in the figure. However, the lower crossing point is not a stable equilibrium. If one additional investor buys information, the price of information will fall, making it optimal for others to buy as well. Thus, if we restrict attention to stable equilibria, the two possible outcomes are no demand for information ($\lambda = 0$), or a thick information market (right crossing point).

4. Conclusions

Many markets supply information or certification services: academic testing services, appraisals, job head-hunters, ... Our paper raises a more general question about all of these markets. How does the market structure affect the quality of the information produced and how might the nature of the evaluated products change to game the ratings system? This makes the paper related to a strand of the applied micro literature that includes Bar-Isaac, Caruana, and Cuñat (2008).
A Technical Appendix: Proofs

1.1 Proof of Proposition 2.3

The linear price result is analogous to that in Grossman and Stiglitz (1980). We guess that prices take the linear form $p = A + B\theta + Cx$. Determine what this price implies for risky asset demands, substitute those demand functions into the market clearing conditions and match coefficients to verify the hypothesis. Substituting asset demands (16) and (19) in to the market clearing condition (17), yields

$$\lambda \sigma^2_x (\theta - \mu - pr) + (1 - \lambda) \frac{\hat{\sigma}^2_{\theta|p}}{\sigma^2_{\theta|p} + \sigma^2_e} \left[ \left( \frac{B}{C} \right)^2 \frac{\sigma^2_x}{B} - A \right] = \rho(x + x)$$

where $\hat{\sigma}^2_{\theta|p} = (B/C)^2 \sigma^2_x + \sigma^2_{\theta} - 1$ is the variance of the price signal $(p - A)/B$, conditional on the true state $\theta$.

Collecting price terms reveals that the equilibrium price formula is linear in $x$ and $\theta$

$$A = -\psi \left[ \rho \bar{x} + (1 - \lambda) \frac{\hat{\sigma}^2_{\theta|p}}{\sigma^2_{\theta|p} + \sigma^2_e} \left( \frac{AB \sigma^2_x}{C^2} - \sigma^2_{\theta} \mu \right) \right]$$

(22)

$$B = \psi \lambda \sigma^2_x$$

(23)

$$C = -\psi \rho$$

(24)

where $\psi = [\lambda \sigma^2_x + (1 - \lambda) \frac{\hat{\sigma}^2_{\theta|p}}{\sigma^2_{\theta|p} + \sigma^2_e} \lambda \sigma^2_x \sigma^2_{\theta} \rho]^2$.

1.5 Proof of Lemma 6

To compute the value of information, we proceed in three steps, using the law of iterated expectations:

$$E[e^{-\rho W}] = E[E[e^{-\rho W}|E[u], p]|p]$$

Expected Utility for Uninformed Investor We’re just going to take the first expectation, over $u$. Now, investment risk is $\sigma^2_x + \sigma^2_{f|p}$. Let $\hat{\mu} \equiv E[u|pr]$. Use mean of a log-normal - exp of mean - 1/2 times coefficient squared, times variance of $u$.

$$E[u^U|pr] = -\exp \left[ \frac{(\mu - pr)^2}{2 \sigma^2_x + \sigma^2_{f|p}} \right]$$

$$E[u^U|pr] = -\exp \left[ -\frac{1}{2} \left( \frac{(\hat{\mu} - pr)^2}{2 \sigma^2_x + \sigma^2_{f|p}} \right) \right]$$

There is a second expectation over $pr$ that we haven’t taken. It turns out we won’t need to. So, we leave this line of argument here for now.
Expected Utility for Informed Investor  For informed investors, following the same steps yields:

$$E[e^{-\frac{1}{2}(\theta - pr)\sigma_{e}^{-2}(\theta - pr)}]$$

where \(\theta\) replaced \(\hat{\theta}\) as the conditional mean and \(\sigma_{e}^{2}\) is now the conditional variance. Next step: take expectation over \(\theta\), but not \(pr\).

Now, this is a moment-generating-function of a quadratic normal (called a Wishart). General formula for multivariate quadratic forms: If \(z \sim N(0, \Sigma)\),

$$E[e^{z'Fz+Gz+H}] = |I - 2\Sigma F|^{-1/2} \exp\left[\frac{1}{2}G'(I - 2\Sigma F)^{-1}\Sigma G + H\right]$$

We need this more general form because \(\theta - pr\) is not mean-zero, conditional on \(pr\). It has mean \(\hat{\theta} - pr\).

$$\rho W^I = \rho q' (u - pr) = (\theta - pr)\sigma_{e}^{-2}(u - pr)$$

The \(\theta - pr\) in the informed investor’s expected utility, is a r.v, conditional on \(pr\). It’s mean is \(\hat{\theta} - pr\) and its variance is \(\text{var}[\theta|pr] = \sigma_{f|pr}\).

The mean-zero random variable in the moment-generating function formula is \(\theta - \hat{\theta}\).

$$F = -\frac{1}{2} \sigma_{e}^{-2}$$

$$G' = -(\hat{\theta} - pr)\sigma_{e}^{-2}$$

$$H = -\frac{1}{2}(\hat{\theta} - pr)^2 \sigma_{e}^{-2}$$

$$\Sigma = \sigma_{f|pr}$$

Apply formula:

$$E[U^I|pr] = -|I - 2\sigma_{f|pr}(\frac{1}{2})\sigma_{e}^{-2}|^{-1/2} \exp\left[\frac{1}{2}(\mu - pr)^2 \sigma_{e}^{-2}(I + \frac{1}{2}\sigma_{f|pr}\sigma_{e}^{-2})^{-1}\sigma_{f|pr} - \frac{1}{2}(\hat{\theta} - pr)^2 \sigma_{e}^{-2}\right]$$

Note that if we multiply numerator and denominator by \(\sigma_{e}^{2}\), \((I + 2\sigma_{f|pr}(\frac{1}{2})\sigma_{e}^{-2}) = \frac{\sigma_{e}^{2}}{\sigma_{e}^{2} + \sigma_{f|pr}}\),

$$= -\left(\frac{\sigma_{e}^{2}}{\sigma_{e}^{2} + \sigma_{f|pr}}\right)^{1/2} \exp\left[\frac{1}{2}(\mu - pr)^2 \sigma_{e}^{-2}(\frac{\sigma_{f|pr}}{\sigma_{e}^{2} + \sigma_{f|pr}} - \frac{\sigma_{e}^{2}}{\sigma_{e}^{2} + \sigma_{f|pr}})\right]$$

Cancelling \(\sigma_{e}^{2}\)\(\sigma_{e}^{-2}\) and rewriting 1,

$$= -\left(\frac{\sigma_{e}^{2}}{\sigma_{e}^{2} + \sigma_{f|pr}}\right)^{1/2} \exp\left[\frac{1}{2}(\mu - pr)^2 \sigma_{e}^{-2}\left(\frac{\sigma_{f|pr}}{\sigma_{e}^{2} + \sigma_{f|pr}} - \frac{\sigma_{e}^{2}}{\sigma_{e}^{2} + \sigma_{f|pr}}\right)\right]$$

collecting terms in the numerator and setting \(\sigma_{f|pr} - \sigma_{f|pr} = 0\),

$$= -\left(\frac{\sigma_{e}^{2}}{\sigma_{e}^{2} + \sigma_{f|pr}}\right)^{1/2} \exp[-\frac{1}{2}(\mu - pr)^2 \sigma_{e}^{-2}\left(\frac{-\sigma_{e}^{2}}{\sigma_{e}^{2} + \sigma_{f|pr}}\right)]$$

note that \(\sigma_{e}^{-2} \times (-\sigma_{e}^{2}) = -1\).

$$= \left(\frac{\sigma_{e}^{2}}{\sigma_{e}^{2} + \sigma_{f|pr}}\right)^{1/2} \exp[-\frac{1}{2}(\mu - pr)^2 \sigma_{f|pr}]$$

$$E[U^I|pr] = \left(\frac{\sigma_{e}^{2}}{\sigma_{e}^{2} + \sigma_{f|pr}}\right)^{1/2} E[U^I|pr]$$

The ratio of variances is a known quantity when information is acquired. All agents can infer the equilibrium strategies of other agents and deduce how much information will be revealed through the price level. Since the two expected utilities are related by a known constant, their unconditional expectations \(E[U^I]\) and \(E[U^U]\) must be proportional as well.
1.6 Proof of proposition 7: Information price is \( \chi/\lambda \)

Let \( d_{it} = 1 \) if agent \( i \) decides to discover information in period \( t \) and \( d_{it} = 0 \) otherwise. Let per capita demand for information with price \( c_{it} \), given all other posted prices \( c_{-it} \), be \( I(\cdot, \cdot) \). Then the objective of the information producer is to maximize profit:

\[
\max_{d_{it}, c_{it}} d_{it}(c_{it}I(c_{it}, c_{-it}) - \chi).
\]

Suppose the equilibrium information price was above average cost \( c > \chi/\lambda \). Then, an alternate supplier could enter the market with a slightly lower price, and make a profit. If a supplier set price below marginal cost, they would make a loss. This strategy would be dominated by no information provision. If there are two or more suppliers, then either price is above marginal cost, which can’t be an equilibrium by the first argument, or both firms price at (or below) marginal cost, split the market, and make a loss, which is dominated by exit.

B Technical Appendix: Results with Sophisticated Buyers

2.1 Results with a Single Ratings Agency

Suppose that the issuer can purchase a single rating (as we have been assuming in this section). Then the assets that appear with a rating will have a rating about some threshold - depending on the rule that the issuer is following regarding purchasing/ disclosing the asset. For the rated assets, the naivete of investors does not matter. It matters though for the unrated assets, because now the population of unrated assets now includes assets that got a bad rating leading the issuer to choose either not to purchase or not to disclose the rating. If the largest fraction of investors need to buy rated assets with a rating above a threshold (investible securities legislation), whether they are naive or not is irrelevant for this version of the model where we assume that the rating agencies do not lie about the rating. In other words, when there is a single rating and the buyers are naive, the assets that are falsely perceived are the unrated ones. Hence the naivety will not matter in the extreme case where all investors need to buy assets with good enough ratings. (this assumes that rating agencies behave in a straightforward way).

Now assume that there is no investible securities legislation. Then investors can buy the asset even if is unrated. In this case, if buyers are naive, the unrated assets will be perceived to be better than they actually are, thus reducing the incentive of issuers to obtain a rating compared to the case buyers were not naive.

Comparisons between the Naive and Sophisticated Buyers Case: Irrespective of whether buyers are naive or not, rated assets are rated in an unbiased way in this model where we assume that rating agencies report the rating truthfully. In the section that follows we show that this stops being the case when the issuers can practice “rating-shopping”.

2.2 Results with Ratings Shopping

What happens in the ratings-shopping section when investors anticipate this behavior? Let us go through a heuristic logic first. If buyers anticipate ratings-shopping, once they see an asset with a single rating they can immediately infer that the second (and third...) rating is below the ration they observe. Given this the price of the asset where only one rating is revealed is given by:

\[
p_2^{(1)} = \frac{1}{r} \frac{\sigma_u^{-2} \bar{u} + \sigma_\theta^{-2} (\theta_1 + E(\theta_2 | \theta_1 > \theta_2)) - \rho \bar{x}}{\sigma_u^{-2} + \sigma_\theta^{-2} + \sigma_{\theta \theta}^{(1)} \bar{\theta}_1 < \theta_1}.
\]
The price when both ratings are revealed is still
\[ p_2 = \frac{1}{2} \left( \frac{\sigma_u \theta_1 + \sigma_\theta \theta_2}{\sigma_u + 2\sigma_\theta} \right) \] and the issuer will choose to reveal only the highest rating if \( p_2^{(1)} - p_2 > 0 \). This will be true when the distance between \( \theta_1 - \theta_2 \) is large enough. But given such behavior of the issuer, the price of an asset with a single rating will be
\[ p_2^{(2)} = \frac{1}{2} \left( \frac{\sigma_u \theta_1 + \sigma_\theta \theta_2}{\sigma_u + 2\sigma_\theta} \right) - \rho x, \]
but then we get a new condition for when the second rating will be hidden: \( p_2^{(2)} - p_2 > 0 \). An equilibrium will be then characterized by a \( \Delta^* \) such that
\[ p_2^{(E)} - p_2 = 0 \text{ when } \theta_1 - \theta_2 = \Delta^*, \]
where
\[ p_2^{(E)} = \frac{1}{2} \left( \frac{\sigma_u \theta_1 + \sigma_\theta \theta_2}{\sigma_u + 2\sigma_\theta} \right) - \rho x, \]
but then we get a new condition for when the second rating will be hidden: \( p_2^{(2)} - p_2 > 0 \). An equilibrium will be then characterized by a \( \Delta^* \) such that
\[ p_2^{(E)} - p_2 = 0 \text{ when } \theta_1 - \theta_2 = \Delta^*, \]
where
\[ p_2^{(E)} = \frac{1}{2} \left( \frac{\sigma_u \theta_1 + \sigma_\theta \theta_2}{\sigma_u + 2\sigma_\theta} \right) - \rho x. \]

Conjecture: If \( \theta \) are drawn from a bounded set, then issuers always prefer to reveal all ratings. Reason: an asset with a single rating is immediately perceived to be an asset that got a really bad rating - hence revealing the second rating cannot hurt more.

If this is conjecture is true then in a world with sophisticated buyers, and where rating agencies behave straightforwardly, then rating shopping does not matter.

### C  Technical Appendix: Ratings Agencies’ Incentive to Bias Ratings

Here we will be investigating the rating agencies incentives to produce more or less biased information for different market arrangements.

We return to a model without an issuer having a complexity choice, but add a choice on the part of the ratings agency about whether to issue biased ratings.

The agency’s utility depends on their profits and a reputation cost that is a quadratic function of the distance between their forecast and the true asset payoff. Recall that we use \( \chi \) to denote the price of a rating.

Suppose that once a rating is ordered, it is paid for. Then the rating agencies payoff is given by:
\[ U_r = \chi - c(\text{asset complexity}) - \alpha (\theta_i - u)^2. \]
Suppose that the investor of a rating can observe the rating before purchasing it. Then, in this case the profit of the rating agency depends on the probability that the asset issuer purchases their rating \( \pi(\theta_i) \) and the ratings price \( \chi \)
\[ U^r = \pi(\theta_i) \chi - \alpha (\theta_i - u)^2. \]

When the investor of a rating (so far we have looked at issuers buying ratings) buys the highest of all ratings, than
\[ \pi_i(\tilde{\theta}_i) = \max_{j \in I} \tilde{\theta}_j. \]

There are two equivalent ways to model “rating inflation.” One way is to assume that rating agencies draw a rating \( \theta_i \) from an unbiased distribution, but they report a different rating \( \tilde{\theta}_i = r(\theta_i) \). The other is to model rating agencies who draw from biased distributions. We use the first approach.
Assuming that all rating agencies use the same technology to produce ratings (draw ratings from the same distribution), and that all \( I \) rating agencies use the same monotonic (strictly increasing in \( \theta_i \)) reporting strategy, then
\[
\pi(\theta_i) = F^{I-1}(r^{-1}(\tilde{\theta}_i)),
\]
and the payoff of rating agency is given by
\[
U^r = F^{I-1}(r^{-1}(\tilde{\theta}_i))\chi - \alpha(\tilde{\theta}_i - u)^2.
\]

Heuristic derivation of the equilibrium reporting strategy:
First let \( G \) denote the distribution of the highest order statistic, namely \( G = F^{I-1} \).
We also use \( g \) to denote its density. Then \( U^r \) can be rewritten as:
\[
U^r = G(r^{-1}(\tilde{\theta}_i))\chi - \alpha(\tilde{\theta}_i - u)^2.
\]
Maximizing this expression with respect to \( \tilde{\theta}_i \) we get the following first order condition:
\[
\frac{g(r^{-1}(\tilde{\theta}_i))\chi}{r'(r^{-1}(\tilde{\theta}_i))} - 2\alpha(\tilde{\theta}_i - u) = 0
\]
Now recalling that \( \tilde{\theta}_i = r(\theta_i) \) the above first order condition can be rewritten as:
\[
\frac{g(\theta_i)\chi}{r'(\theta_i)} - 2\alpha(r(\theta_i) - u) = 0
\]
which in turn can be rewritten as:
\[
g(\theta_i)\chi = r'(\theta_i) \cdot 2\alpha(r(\theta_i) - u).
\]
This is a first order differential equation. To make things more transparent for its solution, let \( t = \theta_i \) and \( r = y \), then it can be rewritten as
\[
g(t)\chi = y'(t) \cdot 2\alpha(y(t) - u),
\]
or
\[
\frac{dy(t)}{dt} = \frac{g(t)\chi}{2\alpha(y(t) - u)}
\]
\[
\frac{dy(t)}{dt} \cdot 2\alpha(y(t) - u) = g(t)\chi \cdot dt
\]
integrating the LHS with respect to \( y \) and the RHS with respect to \( t \) we get that:
\[
\int_y 2\alpha(y(t) - u)dy(t) = \int_t g(t)\chi \cdot dt + c
\]
where \( c \) is a constant. The solution to this differential equation characterizes the optimal bias of the ratings agencies.