House Prices and Time on the Market in Competitive Search

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Abstract

We build a competitive search model of the housing market capturing the illiquidity of housing assets, and also the fact that sellers who list lower prices get more visits and sell their property faster. We consider a symmetric environment where households suffer idiosyncratic shocks that affect how much they value their residence (e.g. they change jobs, have kids, divorce,...). If a shock occurs, a household may become mismatched and seek to move. Potential buyers take time to locate an appropriate unit, while owners of vacant units take time to locate potential buyers. In equilibrium, prices, sales and average time on the market are jointly determined as a result of the competitive search process. We characterize a steady state equilibrium in the short-run (when the housing stock is fixed), and in a long-run. In the short-run, as shocks become more frequent, the housing market "heats-up", so prices, sales, and liquidity increase. In the long run, an increase in the housing stock – due to a lagged response to the market heating up–decreases sales and prices, and increases the average time-the-market. These predictions are consistent with empirical evidence.

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1 Introduction

A household’s residence is usually its largest single asset. Household real net housing wealth is about half of the level of tangible financial assets held by households on average (according to the Board of Governors of the Federal Reserve System’s data for 2006). Residential real estate also represents a large share of both total wealth and GDP—33 and 11 percent, respectively, in the US in 2005 (see Merlo, Ortalo-Magne, and Rust 2007). There are some salient stylized facts of the housing sector that have been emphasized in the economics literature. In particular, housing prices are more volatile than GDP (displaying positive auto-correlation at high frequencies), and sales are even more volatile and they co-move with prices (see Sanchez-Marcos and Ríos-Rull 2007). Housing construction levels are also highly volatile and persistent over time (see Glaeser and Gyourko 2006), but construction reacts relatively slowly to changes in prices and vacancy rates (see Wheaton 1990).1

Despite the importance of the housing sector, our understanding of many aspects of this sector is, at best, incomplete. Attempts to model equilibrium determination of prices and sales in the housing market have proven difficult. One reason is that the housing market displays some key features that are hard to reconcile with the Walrasian market paradigm.

First and foremost, housing units are illiquid long-lived assets. They are costly to sell and buy, they can be partially financed and they are not movable.1 Incorporating these frictions to the Walrasian set-up help to match the observed price volatility (see, for instance, Nakajima 2005 or Nieuwerburgh and Weill 2006), but it is still hard to explain the positive co-movement of prices and sales (see, for instance, Sanchez-Marcos and Ríos-Rull 2007).

The illiquidity of houses, moreover, has an additional dimension. Anyone who has gone through the process of buying or selling a house knows it takes time and resources to buy and sell housing units. In particular, there are costs associated to acquiring relevant information in each potential transaction. For instance, buyers’ valuations typically depend not only on attributes of the units for sale which are listed together with the price by sellers or realtors, or verifiable at a negligible cost, say via a phone call or an internet search (e.g. size, number of bedrooms and bad rooms, neighborhood, proximity to public transportation, age, floor, ...).

1See Díaz and Luengo-Prado (2006) for a model of tenure choice with housing adjustment costs and collateralized borrowing constraints, for instance.
Buyers’ valuations also depend on idiosyncratic features of the unit that can only be verified by visiting and inspecting them. For instance, buyers may want to visit units for sale to check how well-kept the property is, whether there are sources of noise such as heavy traffic or loud neighbors, etc. In general, a typical buyer visits several units prior to purchasing one. Thus, time to sell is a key variable that we have to take into account to understand the joint movement of prices and sales.\(^2\)

This liquidity (e.g., time to sell) also seems to vary widely over time, and it co-moves with prices and sales. In “hot” real estate markets, price and sales are high and so is liquidity (e.g. properties sell fast and time-on-the-market is low), while in “cold” real estate markets prices, sales and liquidity are low, as reported by Krainer (2001). Thus, there is evidence that adjustments to changes in housing market conditions take place not only through prices and quantities, as in a Walrasian world, but also through the degree of liquidity; i.e., how easy/hard it is to sell a property. At the same time, there is also evidence that competitive forces are present in the housing market in developed economies. For instance, Merlo and Ortalo-Magne (2004) provide evidence that, by posting lower prices, sellers increase the number of visits and offers they get, and sell their property faster.

In this paper we build a competitive search equilibrium model of the housing market. Our model captures the aforementioned illiquidity of the housing market since, in order to trade, buyers and sellers must search for trading partners. We also assume that buyers’ valuations have an idiosyncratic match specific component that is realized when buyers visit and inspect the units for sale. Sometimes buyers will visit units that they do not like, and will then move on with their search. On the other hand, the competitive search equilibrium notion we adopt captures the fact that, by posting lower prices, sellers can increase the number of buyers who visit their property and hence the probability of selling the property in a given period.

Following Wheaton (1990), we consider a symmetric dynamic environment where households experience shocks throughout their life-time that affect how much they value their residence. We interpret these shocks are changes in a job or in the number of household

\(^2\)The interaction of financial constraints and search frictions are studied, for instance, by Genesove and Mayer (1997), and Stein (1995).
members (e.g. through marriage, birth or divorce) that render the households mismatched with their current residence. When a shock occurs a household who was formerly happily matched with its residence may become mismatched. Such a household may then seek to buy an appropriate unit and sell the current one. For instance, if a couple living in an apartment in period $t$ has a child in period $t+1$, the couple will become mismatched and may seek to buy a bigger unit and sell the apartment. We study both a short-run equilibrium where the housing stock is fixed, and a long-run equilibrium where the housing stock is endogenous.\(^3\)

The competitive search process can be described as follows.\(^4\) Each period market makers or intermediaries compete by creating submarkets. Buyers and sellers are free to choose in which submarket to participate. Market makers specify the attributes (type) of the units sold and their price in each submarket. Market makers may charge fees to those traders who choose to participate in the submarkets they create contingent on a transaction taking place. However, in equilibrium, competition among market makers drives their profits to zero, so the fees represent the cost of intermediation. Buyers and sellers who participate in a given submarket meet randomly according to an exogenously given matching function. Upon meeting a seller and visiting his property, the buyer learns her valuation. If she likes the unit, the buyer purchases the unit at the posted price. The key to the competitive search process is that buyers and sellers choose optimally among the existing submarkets in full knowledge of the price (and the units' listed attributes) and perfectly anticipating the probability with which they will trade. In equilibrium, competition by market makers implies that there is no gains from creating additional submarkets.

We characterize the steady state short-run equilibrium and derive comparative static results. We find that, as shocks become more frequent (because job changes or changes in the family structure are more frequent), the housing market “heats-up” so sales and liquidity increase. While in general the effect on prices is ambiguous, our simulations show that for

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\(^3\)For simplicity, we assume that all housing units are owned by the households and there is no rental housing market. We also assume that households want to consume at most one unit (the value they assign to additional units is zero).

\(^4\)Competitive search models are well-known in the labor literature. We follow the description in Moen (1997). An alternative (equivalent) description assumes that sellers compete by posting price offers, and buyers direct their search to the set of sellers posting the most attractive offers, as in Shimer (1996). These reduced form models build on earlier game-theoretic formulations by Montgomery (1991) and Peters (1991), with a large finite number of traders. See also Burdett, Shi, and Wright (2001).
parameter values that are consistent with the frequency range with which households move in the data, the effect on prices is positive. The mechanism is simple. Because more households become mismatched each period, the ratio of potential buyers over units for sale increases. This reduces the time it takes to sell a vacant unit, and increases total sales. Prices go up because, due to the competitive forces at play, sellers get a higher fraction of the total surplus and this surplus is higher (for reasonable parameter values). Symmetrically, if shocks become less frequent, the market cools down with sales and liquidity falling, and prices also falling (again for reasonable parameter values).

We also characterize a long-run equilibrium where the housing stock is endogenous. We find that an increase in the housing stock—say due to a lagged reaction to the market heating up—in the long-run increases the average time-the-market, and decreases sales and prices (for reasonable conditions on the parameters). We are currently investigating the transitional dynamics to ascertain the magnitude and correlation of changes in prices and sales at high frequencies when the market heats up, and the magnitude and correlation of the lagged response in construction.

Our benchmark model abstracts away from several important issues, including financial frictions, rental housing markets, and population growth (e.g. immigration). In future work, we plan to embed our model of the housing market in general equilibrium to study these issues in future work. We view the model in this paper as a simple benchmark that, while leaving out several important factors that affect the housing market, is a step forward in that it can account for the co-movement of prices, liquidity and sales we see in the data. It is remarkable to us that in the model this co-movement arises solely from the interaction of search and matching frictions with competitive forces.

2 Related literature

We are not the first ones to suggest that search models are a natural benchmark to study the housing market. Search-theoretic models of the housing markets have been proposed by Wheaton (1990), Yavaş (1992), Williams (1995), Arnold (1999), Krainer (2001), Albrecht, Anderson, Smith, and Vroman (2007), Yui and Zhang (2007), among others. The key differ-
ence with our work is that, in all existing search models the terms of trade are determined by bargaining, after buyers and sellers meet. The competitive search construct assumes that prices are posted ex ante, prior to the search process, and that buyers can direct their search to sellers posting lower prices. Hence, sellers can affect the arrival rate of buyers and sell their property faster by posting low prices. As we have already noted, this feature of the model is consistent with empirical evidence, and is not present in bargaining models.

Krainer (2001) is the closest to our paper. In his model, households’ valuations for their residence are subject to idiosyncratic shocks that may render households mismatched, but also to aggregate shocks. He finds that, as long as there is enough persistence in the aggregate shock, prices, sales and liquidity are higher in the high state (when all valuations are higher) than in the low state in a short-run equilibrium. A different mechanism operates in our model (where there are no aggregate shocks) by which more frequent idiosyncratic shocks also heat up the market. This mechanism arises from competitive forces and it is not present in Krainer’s model (where more frequent shocks increase prices but decrease liquidity and sales).

Yui and Zhang’s (2007) model is also closely related to ours. Their model is halfway between the bargaining and the competitive search model. On the one hand, the terms of trade are determined by ex post bargaining. On the other hand, there are different submarkets where sellers can choose to trade ex ante. Buyers cannot move across submarkets (as in our model), so they cannot directly target submarkets where lower prices are posted. However, sellers who trade in submarkets where prices are lower sell their property faster in equilibrium. The reason is that in equilibrium sellers are indifferent between existing submarkets, so time on the market is lower in submarkets with lower prices. The authors also assume free entry of sellers (thus making endogenous the number of units for sale), so their’s is a long-run equilibrium model. They find a positive co-movement between in housing prices, sales and the rate at which units are sold when buyers differ in their waiting costs and their search intensity is endogenous. Without endogenous search intensity, the movement in liquidity is counterfactual (when the market “heats up”, say because waiting costs are higher, prices and sales go up but the rate at which units are sold goes down). Our model delivers the co-movement in housing prices, sales and time on the market also in the short
run (so it is consistent with high-frequency data), and with ex ante symmetric buyers and sellers and no choice of search intensity.

Another important difference with the existing search literature (including Yui and Zhang 2007) has to do with the efficiency properties of the equilibrium allocation. It is well-known that with ex-post bargaining, the equilibrium allocation is typically constrained inefficient. The reason is that the share of the surplus is exogenous and does not reflect the traders' contribution to the matching process. The competitive search process endogeneizes the share of the surplus, and in equilibrium this share reflects the traders' contribution to the matching process. Hence, with competitive search, the equilibrium allocation is constrained efficient (see Moen 1997). This difference is important for policy analysis.

As reported by Merlo and Ortalo-Magne (2004), while sellers list their prices to affect the number of visit and offers they get, often times they revise their listed prices downward. Listing price revisions take place more often the longer properties have been on the market without receiving any offer. Sometimes bargaining further reduces the sale price (e.g. in their sample properties sell at about 96 percent of their current listed price). Some times negotiations between buyers and sellers break down (e.g. 1/3 of buyers whose first offer is turned down walk away in their sample). Our focus is on the co-movement of aggregate variables in the housing market and our model is too simple to capture the details of the strategic bilateral interaction between buyers and sellers (see, however, Merlo, Ortalo-Magne, and Rust 2007). We make a strong commitment assumption that all transactions take place at the listed price each period (though this price may change from one period to the next). Under this assumption, every match with positive joint surplus results in trade. In any case, Merlo and Ortalo-Magne (2004) report that more than 3/4 of the units in their sample sell without any revision of their initial listed price, in the vast majority of cases a property is sold to the first potential buyer who makes an offer. In this sense, our model is fairly as representative of a substantial fraction of the total trading volume.
3 The model economy

3.1 The environment

There is a continuum of infinitely-lived households who derive utility from the services of housing units. There are two types of households, \( i = 1, 2 \), and two types of (indivisible) housing units, \( h = 1, 2 \). We assume that type-1 households prefer type-1 units, and type-2 households prefer type-2 units. Suppose, for example, that the type of a unit represents its location and the type of a household the location of the household’s job. Most households prefer to live and work in the same location rather than to commute. Alternatively, the type of a unit may represent its size and the type of the household the number of household members. Larger households typically prefer to live in larger units. For simplicity, we assume that each household wants to consume the services of a single unit each period.

We say that a type-\( i \) household is matched if it consumes the services of a type-\( i \) unit, and is mismatched if it consumes the services of a unit of the other type (\( h \neq i \)). Matching frictions arise in this environment because household types change over time due to exogenous factors (e.g. because the location of the household’s job or the number of household members change over time).\(^5\) As a result of these changes, a formerly matched household may become mismatched. Such a household will then seek to move to a different unit.

In our benchmark model, all units are owned by the households and there is no rental market for housing. We assume that, in period 0, the measure of households of each type is equal and normalized to one. The stock \( H \) of housing units of each type is also equal, and is exogenously fixed in the short run. For simplicity, we assume that \( 1 < H < 2 \), and that in period 0 households own either one or two units. The proportion of households with two units is then \( H - 1 \). We also assume that households who own two units in period 0 own both a type-1 and a type-2 unit. Initially, the flow (per period) value of a type-\( i \) household assigns to the services provided by a type-\( h \) unit is \( \bar{v} \) if \( h = i \), and \( v \) if \( h \neq i \). Households

\(^5\)Matching frictions could also arise from positive income shocks that make households want to upgrade their residence more often (or negative shocks that make them revert this upgrading process). In our symmetric environment there no houses that are uniformly better than others, however. It's just the different households have different valuations for different types of units. However, the model could be extended in this direction.
owning two units assign zero value to their second unit. Utility is transferable, so implicit is the existence of a divisible good (a “money good”) which yields constant marginal utility to all households.

Each period \( t \) matched households change type with probability \( \alpha \in (0, 1) \), and this implies a change in the household’s preferences for housing. Shocks are independent across households. When a shock occurs a matched household owning a single unit will become mismatched. Such a household will then seek to buy an appropriate unit and sell the current one. On the other hand, households owning two units are always matched even if their type changes. When a shock occurs, these households simply move to their vacant unit and put the other unit up for sale.\(^9\)

Housing units are traded bilaterally in a decentralized market where households seeking to buy or sell a unit meet potential trading partners at random. In Section 3.2 we describe in detail the random meeting process. We also describe how the terms of trade are determined by a competitive search process (as in Moen’s 1997 and Shimer’s 1996 models of the labor market). As we shall see, units of different types will be traded in different markets.

In the market for type-\( i \) units, the flow (per period) value a type-\( i \) household assigns to a type-\( i \) unit visited at random is a random variable \( v \). This random variable measures the quality of a match between a type-\( i \) household and a type-\( i \) unit, and is realized when the household visits the unit. For simplicity, we assume that \( v \) can take two values \( \{ \bar{v}, g \} \) with respective probabilities \( q \) and \( 1 - q \), for each \( i = 1, 2 \). Intuitively, \( v \) captures the fact that household valuations depend not only on the type but also on idiosyncratic features of the unit. Typically these features can only be verified by visiting an inspecting it. For instance, someone who has trouble sleeping at night prefers to live in a quiet place. Such a person may want to visit units for sale to check for possible sources of noise such as heavy traffic or loud neighbors. The parameter \( q \in (0, 1) \) captures the extent of the matching frictions. The lower the value of \( q \), the larger the number of units households need to visit until they find

\(^9\)For simplicity, we assume that only matched households change type. This means that the only way for a mismatched household to become matched again is by purchasing an appropriate unit. We could alternatively assume that any household can change type. Then a mismatched household can become matched if it fails to purchase an appropriate unit but it experiences another type change. Our qualitative results would not change under this alternative assumption.
one they like (because they are more “picky”).\footnote{We have assumed for simplicity that initially all matched households like their unit; i.e., their value in period 0 is \( \tilde{v} \).}

Given the above, a household can be in one of three states each period \( t \): matched with one unit (not seeking to trade), mismatched with one unit (seeking to buy an appropriate unit), and matched with two units (seeking to sell the unit it does not value).\footnote{We assume that the utility of being homeless is \(-\infty\), so no household will sell a unit unless they have another unit to live in.} Denote the measure of type-\( i \) households in each of these states by \( n_{it}, b_{it} \) and \( s_{it} \), respectively. Denote the total measure of type-\( i \) households in period \( t \) by \( \gamma_{it} \). Then,

\[
\gamma_{it} = n_{it} + b_{it} + s_{it}, \quad i = 1, 2, \quad \text{with} \quad \gamma_{1t} + \gamma_{2t} = 2. \tag{1}
\]

By assumption, \( \gamma_{10} = \gamma_{20} = 1 \). Note that each period \( t \) some of the type-\( i \) units are owned by type-\( i \) households who are matched with one or two units. The rest are owned by type-\( j \) households who are either mismatched with one unit or matched with two units. Hence,

\[
H = n_{it} + s_{it} + b_{jt} + s_{jt}, \quad i = 1, 2. \tag{3}
\]

From these, only the units owned by type-\( j \) households with two units are vacant (the rest are occupied). Therefore, the measure of type-\( i \) units potentially for sale is \( s_{jt} \).

In the housing market, buying and selling housing units takes time because of search frictions as we shall see. Let \( \pi_{it}^b \in [0, 1] \) denote the probability that a mismatched type-\( i \) household purchases a type-\( i \) unit in period \( t \). Similarly, let \( \pi_{it}^s \in [0, 1] \) denote the probability that a vacant type-\( i \) unit is sold in period \( t \). These probabilities will be endogenously in equilibrium, but for the moment we take them as given. The number of type-\( i \) units purchased in period \( t \) is equal to the number of type-\( i \) units sold so\footnote{We assume that the Law of Large Numbers holds.}

\[
\pi_{it}^b b_{it} = \pi_{it}^s s_{jt}. \tag{4}
\]
composition of the population in period $t$, $\{n_{it}, b_{it}, s_{it}\}_{i=1,2}$, the measure of type-$i$ households who do not seek to trade in period $t+1$ is
\[ n_{i,t+1} = (1 - \alpha)[n_{it} + \pi_{it}^s s_{it}]. \] (5)

These are the type-$i$ households who were matched with one unit at the end of period $t$ and do not change type at the start of $t+1$. These households either did not seek to trade in period $t$ (e.g. families living in a large unit), or they were sellers who sold their $j$-unit that period (e.g. families living in a large unit who sold their vacant small unit). The measure type-$i$ buyers in period $t+1$ is
\[ b_{i,t+1} = (1 - \pi_{it}^b) b_{it} + \alpha[n_{jt} + \pi_{it}^s s_{jt}]. \] (6)

This includes the type-$i$ buyers who did not trade in period $t$, and continue to search for a unit in $t+1$. It also includes the type-$j$ households who were matched with one unit at the end of period $t$ and become mismatched at the start of $t+1$ (e.g. families living in a large unit in period $t$ who went through divorce in $t+1$). Finally, the measure of type-$i$ sellers in period $t+1$ is
\[ s_{i,t+1} = (1 - \alpha)(1 - \pi_{jt}^s)s_{it} + \pi_{it}^b b_{it} + \alpha(1 - \pi_{it}^s)s_{jt}. \] (7)

This includes the type-$i$ sellers who did not trade in period $t$ and (provided their type does not change) continue to search for a buyer in $t+1$. It also includes the type-$i$ buyers who bought an $i$-unit in period $t$ and seek to sell their $j$-unit in $t+1$ (e.g. families who bought a large unit and now seek to sell their vacant small unit). Finally, it includes the type-$j$ sellers who did not trade period $t$ and become of type $i$ in $t+1$. In period $t+1$ these households simply move to their $i$-unit and put up for sale their $j$-unit.

Equations (1) and (5)-(7) imply that the measure of households of each type evolves according to the following differential equation
\[ \gamma_{i,t+1} = \gamma_{i,t} - \alpha(n_{it} + s_{it}) + \alpha(n_{jt} + s_{jt}), \quad i = 1, 2, \] (8)
since we have assumed that only matched households change type.

### 3.2 Competitive Search

In this section, we study how housing prices and average time on the market are endogenously determined in competitive search. To ease notation we drop all \( t \) subscripts in most of the expressions below (since the competitive search process is the same every period),

Each period the housing market consists of a collection of submarkets created by competing market makers or intermediaries. Market makers specify the type \( i \) of the units sold in each submarket and a common price \( z \) for these units. While the type of a unit can be perfectly advertised at no cost, the units’ idiosyncratic features can only be verified when buyers visit and inspect them.

Buyers and sellers are free to choose in which submarket to participate. The market will be segmented since buyers and sellers of type-\( i \) units will only participate in submarkets for type-\( i \) units. Market makers may charge fees to those traders who choose to participate in the submarkets they create contingent on a transaction taking place. However, in equilibrium, competition among market makers drives their profits to zero, so the fees measure the cost of intermediation (assumed zero, for simplicity).\(^{10}\) In a competitive search equilibrium, there must be no gains from creating additional submarkets.

Buyers and sellers who choose to participate in a given submarket meet bilaterally and at random. For simplicity, we assume that each trader experiences at most one match each period. Let \( b \) and \( s \) be the measures of buyers and sellers in the submarket. A standard matching function \( \mathcal{M}(b, s) \) determines the total numbers of matches as a function of \( b \) and \( s \). As usual, \( \mathcal{M} : R_+^2 \to R_+ \) is continuously differentiable, increasing in both arguments, strictly concave, and homogeneous of degree one. Also, the total number of matches cannot exceed the number of traders in the short side of the submarket, so \( \mathcal{M}(b, s) \leq \min\{b, s\} \). In particular, \( \mathcal{M}(0, s) = \mathcal{M}(b, 0) = 0 \).

The probabilities with which buyers and sellers meet potential trading partners depend on the ratio of buyers over sellers in the submarket where they participate: \( \theta = b/s \). Specifically,

\(^{10}\)See, however, Remark 2 below.
by the Law of Large Numbers, the probability that a seller meets a buyer is

$$m^s(\theta) = \frac{\mathcal{M}(b, s)}{s} = \mathcal{M}(\theta, 1),$$  \hspace{1cm} (9)

where \( m^s : R_+ \to [0, 1] \) is continuously differentiable, strictly increasing and strictly concave, with \( m^s(0) = 0 \) and \( \lim_{\theta \to \infty} m^s(\theta) = 1 \). Intuitively, the higher the value of \( \theta \) the easier it is for sellers to meet buyers. As this ratio goes to infinity (zero) the probability that a seller meets a buyer goes to one (zero). Similarly, the probability that a buyer meets a seller is

$$m^b(\theta) = \frac{\mathcal{M}(b, s)}{b} = \mathcal{M}(1, \theta^{-1}),$$  \hspace{1cm} (10)

where \( m^b : R_+ \to [0, 1] \) a continuously differentiable, strictly decreasing, with \( \lim_{\theta \to 0} m^b(\theta) = 1 \) and \( \lim_{\theta \to \infty} m^b(\theta) = 0 \). In this case, the higher the value of \( \theta \) the harder it is for buyers to meet sellers. As this ratio goes to zero (infinity) the probability that a buyer meets a seller goes to one (zero). A key parameter in the determination of the competitive search equilibrium is the elasticity of \( m^b(\theta) \):

$$\eta(\theta) = \frac{-m^b'(\theta)\theta}{m^b(\theta)} \in [0, 1],$$  \hspace{1cm} (11)

which is assumed to be non-decreasing. It is also useful to write \( m^s(\theta) = m^b(\theta)\theta \).

Given the above, the probabilities with which buyers and sellers trade in the submarket where they participate are:

$$\pi^b(\theta) = m^b(\theta)q,$$  \hspace{1cm} (12)

$$\pi^s(\theta) = m^b(\theta)\theta q.$$  \hspace{1cm} (13)

This is so because buyers locate a vacant unit with probability \( m^b(\theta) \) and like this unit with probability \( q \). Similarly, sellers are contacted by a buyer with probability \( m^b(\theta)\theta \) and the buyer likes their unit with probability \( q \).

A submarket is fully characterized by a triple \((i, z, \theta)\), where \( i \) is the type of unit sold, \( z \) is the price of a unit, and \( \theta \) is the ratio of buyers over sellers in the submarket. Denote the set of submarkets that are created by market makers in period \( t \) by \( \Omega_t \subset \{1, 2\} \times R \times R_+ \).
(i.e., the attract positive masses of both buyers and sellers).\textsuperscript{11} It is useful to extend the set \( \Omega_t \) to incorporate an artificial submarket \( \omega_0 \) representing the no-trade option. A choice of \( \omega_0 \) by a buyer (seller) is then associated to \( \pi^b = 0 \) (\( \pi^s = 0 \) respectively). In what follows, we assume that submarkets for both types of units are created every period (they will be in equilibrium, as we shall see, as long as the bilateral trade surplus is positive).

Buyers and sellers of types 1 and 2 choose optimally among these submarkets in full knowledge of the price and perfectly anticipating the expected time on the market (the probability with which they will trade). Denote the values of buyers, sellers, and "non-traders" of type-\( i \) by \( W^{ib} \), \( W^{is} \) and \( W^{im} \), respectively. To ease notation, the subscript \( t \) is omitted; i.e., below \( W^{ib} \) stands for \( W^{ib}_t \) and \( W^{ib}_{t+1} \) for \( W^{ib}_{t+1} \). Denote the discount factor by \( \beta \in (0, 1) \).

The Bellman equation of a type-\( i \) buyer is

\[
W^{ib} = \max_{(i, z, \theta) \in \Omega_t \cup \omega_0} \left\{ \pi^b(\theta) [\bar{v} - z + \beta((1 - \alpha)W^{is}_{t+1} + \alpha W^{js}_{t+1})] + (1 - \pi^b(\theta))[v + \beta W^{ib}_{t+1}] \right\},
\]

where \( j \neq i \). Buyers choose optimally among existing submarkets each period. If they choose to participate in submarket \( (i, z, \theta) \in \Omega_t \), buyers purchase a unit with probability \( \pi^b(\theta) \). In this event, they get utility \( \bar{v} \), pay \( z \) dollars and become sellers in the following period (possibly of type \( j \)). With complementary probability buyers do not trade, in which case they get utility \( v \) from their current unit and continue to be buyers in the following period. Remember that buyers can always choose not to trade, in which case they always get utility \( v \) from their current unit and continue to be buyers in the period \( t + 1 \). The Bellman equation of a type-\( i \) seller is

\[
W^{is} = \max_{(j, z, \theta) \in \Omega_t \cup \omega_0} \left\{ \bar{v} + \pi^s(\theta) [z + \beta((1 - \alpha)W^{ib}_{t+1} + \alpha W^{jb}_{t+1})] + \beta(1 - \pi^s(\theta))[(1 - \alpha)W^{is}_{t+1} + \alpha W^{js}_{t+1}] \right\}.
\]

Sellers always get utility \( \bar{v} \) for the unit they occupy, whether or not they trade. Remember that type-\( i \) sellers only participate in submarkets for type-\( j \) units. If they who choose to

\textsuperscript{11}Suppose there is a negligible but positive cost of creating a submarket. Then only submarkets which attract both buyers and sellers will be created in equilibrium.
participate in submarket \((j, z, \theta) \in \Omega_t\), they sell their \(j\)-unit with probability \(\pi^*(\theta)\). In this event, they receive \(z\) dollars and, as long as their type does not change, they exit the housing market the following period. If their type does change, they become mismatched and are buyers in the following period. With complementary probability sellers do not trade, and they continue to be sellers the following period (possibly of type \(j\)). Sellers can always choose not to trade, in which case they continue to be sellers in period \(t + 1\). Finally, the Bellman equation of a non-trader of type-\(i\) is

\[
W^{in} = \bar{v} + \beta[1 - (1 - \alpha)W^{in}_{t+1} + \alpha W^{jb}_{t+1}]
\]  

(16)

These households get flow utility \(\bar{v}\) for their unit and do not participate in the housing market in period \(t\). However, with probability \(\alpha\) they become mismatched in period \(t + 1\).

It is useful to write the Bellman equations (14)-(16) as

\[
W^{ib} = v + \beta W^{ib}_{t+1} + \max_{(i, z, \theta) \in \Omega_t \cup \Omega_0} S^{ib}(i, \theta, z),
\]

(17)

\[
W^{is} = \bar{v} + \beta[1 - (1 - \alpha)W^{is}_{t+1} + \alpha W^{jb}_{t+1}] + \max_{(j, z, \theta) \in \Omega_t \cup \Omega_0} S^{is}(j, \theta, z),
\]

(18)

\[
W^{in} = \bar{v} + \beta[1 - (1 - \alpha)W^{in}_{t+1} + \alpha W^{jb}_{t+1}].
\]

(19)

The first and second terms in (17)-(19) are the flow utility and the continuation value of the households in each state when they do not trade. The terms \(S^{ib}(i, z, \theta)\) and \(S^{is}(j, z, \theta)\) in (17) and (18) are the expected trade surpluses for buyers and sellers of type-\(i\) depending on the submarket where they choose to participate in period \(t\); i.e.,

\[
S^{ib}(i, z, \theta) = \pi^b(\theta)[\bar{v} - v - z + \beta((1 - \alpha)W^{is}_{t+1} + \alpha W^{jb}_{t+1} - W^{ib}_{t+1})],
\]

(20)

\[
S^{is}(j, z, \theta) = \pi^s(\theta)[z + \beta((1 - \alpha)W^{in}_{t+1} + \alpha W^{jb}_{t+1} - (1 - \alpha)W^{is}_{t+1} - \alpha W^{jb}_{t+1})].
\]

(21)

When the buyer trades, she gets an increase \(\bar{v} - v\) in current (flow) utility from housing services net of a total payment of \(z\) dollars. Her continuation utility also changes since she becomes a seller the following period (possibly of type \(j\)). Similarly, when the seller trades, he receives \(z\) dollars. His continuation utility also changes since, at the start of \(t + 1\), he exists the housing market with probability \(1 - \alpha\) and becomes mismatched with complementary
probability. A choice of $\omega_n$ is associated to no trade; i.e., to $S^{ib} = 0$ and $S^{is} = 0$ respectively.

Equations (17) and (18) say that buyers and sellers choose to participate in the submarket that yields highest expected trade surplus. The expressions of the expected trade surpluses in equations (20) and (21) highlight the fact that traders care not only about prices but also about the expected time on the market. In particular, buyers prefer lower prices and lower expected time on the market (so $S^{ib}$ is decreasing in $z$ and $\theta$). Similarly, sellers prefer higher prices and lower expected time on the market (so $S^{is}$ is increasing $z$ and $\theta$). This trade-off between prices and expected time on the market is key in the determination of as competitive search equilibrium.\(^{12}\)

It is easy to characterize the equilibrium outcome given the expected trade surpluses in (20) and (21). First, since all type-$i$ buyers are ex ante symmetric and are free to choose among all active submarkets, they must get the same expected surplus $S^{ib}$ in equilibrium. The same is true for sellers. Also, these expected surpluses are positive: $S^{ib}, S^{is} > 0$. Otherwise, agents would not trade. On the other hand, it cannot be not possible to create additional submarkets that would attract both buyers and sellers of type-$i$ units and yield strictly higher surplus to either of them. That is, there is no $(i, z', \theta') \in \{1, 2\} \times R \times R_+$ yielding $S^{ib}(i, z', \theta') \geq \tilde{S}^{ib}$ to buyers and $S^{is}(i, z', \theta') \geq \tilde{S}^{is}$ to sellers, with strict inequality for one (or both) of them.\(^{13}\) Formally, for each submarket $(i, z, \theta) \in \Omega_i$, $(z, \theta)$ must solve

$$
\tilde{S}^{is} = \max_{(z, \theta) \in H \times R_+} S^{is}(i, z, \theta) \text{ s.t. } \tilde{S}^{ib} = S^{ib}(i, z, \theta), \quad j \neq i. \quad (22)
$$

It is easy to check that the solution of this convex program is unique and interior for given $S^{ib}, S^{is} > 0$.\(^{14}\) This means that all type-$i$ units are traded at the same price and with identical probability. Equivalently, only two submarkets are created in equilibrium, one for type 1 units and another one for type 2 units. Buyers of type 1 and sellers of type 2 trade in the first submarket, while buyers of type 2 and sellers of type 1 trade in the second submarket.

The submarket where type $i$ units are traded is then characterized by the following

---

\(^{12}\)Traders also care about how their continuation values change when they trade, but these changes does not depend on where they choose to trade.

\(^{13}\)Remember that type-$i$ units are sold by type-$j$ households with two units ("type-$j$ sellers").

\(^{14}\)Substituting $z$ as a function of $\theta$ from the constraint into the objective function yields an strictly concave function (because $m^s(\theta)$ is strictly concave). Also, because $\tilde{S}^{is}, \tilde{S}^{ib} > 0$, the unique solution must satisfy $0 < \theta < \infty$. 

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tangency condition:
\[
\frac{\partial S^{b,i}(z_0, \theta)}{\partial z} = \frac{\partial S^{s,i}(z_0, \theta)}{\partial \theta}.
\]

Differentiating (20) and (21), substituting into (23), and using equations (12) and (13), we obtain
\[
\frac{\bar{v} - \bar{u} - z_i + \beta((1 - \alpha) W^{i*}_{i+1} + \alpha W^{j*}_{i+1} - W^{j*}_{i+1})}{z_i + \beta((1 - \alpha) W^{j*}_{i+1} + \alpha W^{i*}_{i+1} - (1 - \alpha) W^{j*}_{i+1} - \alpha W^{i*}_{i+1})} = \frac{1 - \eta(\theta_i)}{\eta(\theta_i)},
\]
where \( \theta_i \) is the ratio of buyers and sellers in the submarket, and \( \eta \) is the elasticity of \( m^t \) in equation (11). The numerator and denominator of the left-hand side of (24) are the ex post surpluses of buyers and sellers. Hence, the sum of these two terms gives the total bilateral trade surplus. Equation (24) thus says that, in a competitive search equilibrium, buyers receive a share \( 1 - \eta(\theta_i) \) of the total bilateral trade surplus, while sellers receive a share \( \eta(\theta_i) \). The interpretation of this condition is well-known (see Moen 1997). For a given \( \theta \), the elasticity \( \eta(\theta) \) measures the contribution of sellers to the matching process, while \( 1 - \eta(\theta) \) measures the contribution of buyers to this process. Condition (24) says that, in a competitive search equilibrium, buyers and sellers receive a share to the total bilateral trade surplus which is equal to their respective contribution to the matching process. In other words, a competitive search equilibrium endogenously generates the sharing rule in Hosios (1990), thus attaining a constrained efficient allocation.

To complete the competitive search equilibrium characterization, note that the equilibrium ratio of buyers over sellers in period \( t \) is
\[
g_{it} = \frac{b_{it}}{s_{it}},
\]
since all type-\( i \) buyers and all type-\( j \) sellers participate in this submarket. Since \( \eta(\theta) \) is non-increasing in \( \theta \), when the number of type-\( i \) buyers increases relative to the number of type-\( j \) sellers, sellers (buyers) get a higher (lower) fraction of the surplus and vice versa.

**Remark 1.** While we have assumed a general matching function, the urn-ball matching process is specially suitable for the competitive search environment. Peters (1991) constructs
a price-posting game that generates this matching process endogenously when the number of
traders is large.\textsuperscript{15} In this case,

\begin{align}
m^s(\theta_{it}) &= 1 - \exp\{-\theta_{it}\}, \quad (26) \\
m^b(\theta_{it}) &= \frac{1 - \exp\{-\theta_{it}\}}{\theta_{it}}, \quad \text{and} \\
\eta(\theta) &= \frac{\exp\{\theta\} - (1 + \theta)}{\exp\{\theta\} - 1} \quad (27) \\
\eta'(\theta) &= 0, \lim_{\theta \to 0} \eta(\theta) = 0, \text{ and } \lim_{\theta \to \infty} \eta(\theta) = 1.\textsuperscript{16}
\end{align}

so \eta'(\theta) > 0, \lim_{\theta \to 0} \eta(\theta) = 0, \text{ and } \lim_{\theta \to \infty} \eta(\theta) = 1.\textsuperscript{16}

Remark 2. Our benchmark model assumes that the cost of intermediation is negligible. Suppose this cost was positive, \( F > 0 \), but not too high (so it does not exceed the bilateral trade surplus). The difference is that in this case, the price paid by type-i buyers is different from the price received by sellers of type-i units: \( z^b_i = z^s_i + F \). Hence, the equilibrium will be characterized as above except that (24) is replaced by

\begin{align}
\frac{\bar{v} - v - z^s_i - F + \beta((1 - \alpha)W^s_{i+1} + \alpha W^b_{i+1} - W^s_{i+1})}{z^s_i + \beta((1 - \alpha)W^b_{i+1} + \alpha W^s_{i+1} - (1 - \alpha)W^s_{i+1} - \alpha W^s_{i+1})} &= \frac{1 - \eta(\theta_i)}{\eta(\theta_i)} \quad (30)
\end{align}

where \( z^s_i \) is the price received by sellers of type-i units. If, in addition to intermediation costs, there are taxes \( T \), then \( F \) should be replaced by \( F + T \) in the above equation.

4 Equilibrium

We are now ready to define a competitive search equilibrium for the housing market.

\textsuperscript{15}In Peters (1991), sellers post and commit to ex ante price offers to attract buyers to match with them. Buyers then observe all the posted offers and simultaneously select a seller as a potential trading partner (their selection strategy being described as a probability measure on the set of sellers posting offers). A symmetric equilibrium is characterized where all buyers and all sellers play identical strategies. The matching function (26) describes how buyers’ selection strategies respond to unilateral price deviations by sellers in the symmetric equilibrium.

\textsuperscript{16}Also of interest is the limiting situation where search frictions vanish:

\begin{align}
m^s(\theta_{it}) &= \min\{\theta_{it}, 1\} \quad (29)
\end{align}

In this case, the short side of the market is always served, while the large side of the market is rationed. If \( \theta_{it} > 1 \), there are more buyers than vacant units, so \( m^s(\theta_{it}) = 1 \) and \( m^b(\theta_{it}) = \min\{1, \theta_{it}^{-1}\} < 1 \); otherwise \( m^s(\theta_{it}) = \min\{\theta_{it}, 1\} < 1 \) and \( m^b(\theta_{it}) = 1 \). While the matching probabilities are not differentiable in this case, one can use similar arguments to the ones in this Section to characterize an equilibrium in the limiting frictionless case.
Definition 1. A competitive search equilibrium is a set

$$\{\{\gamma_{tt}, n_{tt}, b_{tt}, s_{tt}, \zeta_{tt}, \theta_{tt}, \pi_{tt}^s, \pi_{tt}^b, W_{i}^{tb}, W_{i}^{is}, W_{i}^{in}, S_{i}^{tb}, S_{i}^{is}\}_{i=1,2}\}_{t=t}$$

that satisfies the system of equations (1)-(3), (5)-(7), (time and type dependent versions of) (11)-(13), (17)-(21), (24) and (25), given an initial composition of the population $\{\gamma_{t0}, n_{t0}, b_{t0}, s_{t0}\}_{i=1,2}$ satisfying (1)-(3).\(^1\)

Since the environment is symmetric, in a steady state, the equilibrium is symmetric. In particular, the mass of each type is constant and equal to one. Dropping the $i$ and $j$ indexes from the above system of equations, it is direct to characterize the steady state by the following system of equations

\begin{align*}
1 & = n + b + s, \quad (31) \\
H & = n + b + 2s, \quad (32) \\
b & = \theta s, \quad (33) \\
\alpha n & = (1 - \alpha) m^b(\theta) q s, \quad (34) \\
(1 - \beta) W^b & = \bar{v} + m^b(\theta) q[\bar{v} - \bar{v} - z + \beta(W^s - W^b)], \quad (35) \\
(1 - \beta) W^s & = \bar{v} + m^b(\theta) q[z + \beta((1 - \alpha) W^n + \alpha W^b - W^s)], \quad (36) \\
(1 - \beta) W^n & = \bar{v} - \beta \alpha (W^n - W^b), \quad (37) \\
\frac{\bar{v} - \bar{v} - z + \beta(W^s - W^b)}{z + \beta((1 - \alpha) W^n + \alpha W^b - W^s)} & = \frac{1 - \eta(\theta)}{\eta(\theta)}, \quad (38) \\
\eta(\theta) & = \frac{-m^b(\theta) \theta}{m^b(\theta)}. \quad (39)
\end{align*}

Equation (31) is the adding-up condition for the population of each type. Equation (32) describes the ownership distribution for each type of unit among households in different states. Equation (33) gives the (common) ratio $\theta$ of buyers over sellers for each type of unit. Equation (34) ensures that the number of no-traders who become mismatched is equal to the number of households who become no traders each period (so the flows in and out of this state are equal). Households who become no-traders of a given type are sellers of that type.

\(^1\)It is easy to check that, if (2) holds at $t = 0$, it also holds at any $t \geq 1$ by (1) and (5)-(7). Similarly, if (3) holds at $t = 0$, it also holds for all $t \geq 1$ by (1),(2), (5)-(6),(12)-(13), and (25). So equations (2) and (3) are redundant.
who successfully sold their vacant unit and do not change type. Equation (35) describes the flow value of a buyer as the sum of the flow value from housing services when mismatched and the expected trade surplus in a steady state. Equation (36) is a similar expression for the seller. Equation (37) describes the flow value of a no-trader as the sum of the flow value from housing services when matched and the discounted expected loss of becoming mismatched in the following period. The numerator and denominator of the left-hand side of (38) are the ex post surpluses of buyers and sellers in a steady state. Equation (38) thus says that the buyers’ (sellers’) ex post surplus is a fraction $1 - \eta(\theta)$ ($\eta(\theta)$ respectively) of the total bilateral trade surplus. Finally, equation (39) says that this fraction is equal to the elasticity of $m^b$ evaluated at the equilibrium ratio $\theta$.

Equations (31)-(33) yield $s, b$ and $n$ as a function of $\theta$:

$$s = H - 1, \quad b = \theta(H - 1), \quad n = 2 - H - \theta(H - 1). \quad (40)$$

From (34), $\theta$ is then given by:

$$m^b(\theta)\eta q = \frac{1}{1 - \alpha} \left( \frac{2 - H}{H - 1} - \theta \right). \quad (43)$$

Finally, it is direct to determine $z, W^b, W^s$, and $W^n$ as a function of $\theta$ using the remaining equations:

$$\frac{z}{\bar{v} - \underline{v}} = 1 + \left( \frac{1}{1 - \beta} \right) \frac{(1 + \beta \theta \eta(\theta) - 1)}{1 - \beta (1 - \alpha)[1 - m^b(\theta)q(1 - \eta(\theta))]}, \quad (44)$$

$$(1 - \beta)W^n = \bar{v} - \frac{\beta \alpha (\bar{v} - \underline{v})}{1 - m^b(\theta)q(1 - \eta(\theta))} - \beta (1 - \alpha), \quad (45)$$

$$(1 - \beta)W^b = \frac{1 - \beta (1 - \alpha)}{m^b(\theta)q(1 - \eta(\theta)) + \beta (1 - \alpha)}, \quad (46)$$

$$(1 - \beta)W^s = \bar{v} + \frac{\beta \alpha (\bar{v} - \underline{v})}{1 - m^b(\theta)\eta q(1 - \eta(\theta))} + \beta (1 - \alpha) \frac{(1 - \eta(\theta))}{\theta^2}, \quad (47)$$

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Remark 3: Housing prices can also be expressed as

\[ z = \bar{v} - v + \frac{1}{1 - \beta}(1 + \beta \theta)\eta(\theta) - 1\] \(G\).  

(48)

where \(G\) represents the total trade bilateral surplus in a steady state. In turn, \(G\) is determined by the sum of the (ex post) trade surplus of the buyer and the seller (given by the numerator and the denominator in the left-hand side of (38) respectively):

\[ G = \bar{v} - v + \beta(1 - \alpha)(W_n - W_b). \]

(49)

The total bilateral surplus reflects the instantaneous gain from a match, \(\bar{v} - v\), as well as the gain in terms of the expected change in the discounted continuation utilities of buyers and sellers, \(\beta(1 - \alpha)(W_n - W_b)\). This gain is realized when the agents trade so the buyer becomes a seller, and the seller’s type does not change in \(t + 1\) so he becomes a no-trader. It is direct to check that this gain is always positive, and it is given by

\[ W_n - W_b = \frac{(\bar{v} - v)[1 - m^b(\theta)q(1 - \eta(\theta))]}{1 - \beta(1 - \alpha)[1 - m^b(\theta)q(1 - \eta(\theta))]} > 0. \]

(50)

It is also direct to check that both \(W_n - W_b\) and \(G\) increase with \(\theta\) (other things equal). This is intuitive. When \(\theta\) is higher, it takes more time to buy a unit. This increases the loss from experiencing a shock that renders the agent mismatched (measured by \(W_n - W_b\)), and hence the gains from bilateral trade. It is also easy to check that \(W^n\) increases with \(\theta\). Intuitively, the value of being a seller increases when \(\theta\) increases because expected sales time decreases and the seller’s trade surplus is higher (since the total surplus \(G\) is higher and sellers get a higher fraction of \(G\)). On the other hand, both \(W^n\) and \(W^b\) fall with \(\theta\). The value of being a buyer decreases because it takes longer on average to purchase a unit and (even though the total bilateral surplus is higher) the buyers’ share of the surplus is lower. The value of non-traders also falls because, in the event they become mismatched in the future, their value will be lower. Note however that the effect on prices is ambiguous. In particular, equation (48) implies that \(z\) always increases with \(\theta\) if \(\theta\) is sufficiently high (so \((1 + \beta \theta)\eta(\theta) > 1\) but the effect is ambiguous if \(\theta\) is low (so \((1 + \beta \theta)\eta(\theta) < 1\)). Remember, for instance, that for the exponential matching function in \(\lim_{\theta \to 0} \eta(\theta) = 0\), and \(\lim_{\theta \to \infty} \eta(\theta) = 1\).
4.1 Steady-State Comparative Statics

The following comparative statics results follow directly from the previous characterization of a steady state. In this characterization, (43) is a key equation which captures how the number \( \theta \) of buyers per seller in the housing market changes due to changes in the housing stock \( H \), the probability \( \alpha \) that a matched household changes type, and the extent of matching frictions as measured by \( q \).

An increase in the rate at which household’s change type (e.g. more frequent changes in job location or in the family structure) decreases average time on the market and increases total sales. While in general the effect on prices is ambiguous, our simulations show that, when one restricts to ranges of \( \alpha \) that are consistent with the frequency range with which households move in the data, the effect on prices is also positive.

**Proposition 1.** Changes in the rate \( \alpha \) at which matched households’s change type have the following effects on a steady-state equilibrium:

\[
\frac{\partial \theta}{\partial \alpha} > 0, \quad \frac{\partial \pi^s}{\partial \alpha} > 0, \quad \frac{\partial \pi^b}{\partial \alpha} < 0, \quad \frac{\partial b}{\partial \alpha} > 0, \quad \frac{\partial s}{\partial \alpha} = 0, \quad \frac{\partial n}{\partial \alpha} < 0, \quad \text{and} \quad \frac{\partial (\pi^s s)}{\partial \alpha} > 0.
\]

The effect on \( z \) and \( G \) is ambiguous.

When \( \alpha \) increases, households are more likely to become mismatched each period. This increases the ratio \( \theta \) of buyers over sellers in the housing market. As a result, the time it takes to sell a vacant unit decreases, while the time it takes to buy one increases. The number of units for sale \( s \) does not change because it depends only in the housing stock. But since the probability of a sale increases, so do sales. In the new steady state, more households are mismatched and fewer are matched with one unit (\( b \) increases and \( n \) decreases). The effect on the total surplus and on prices is ambiguous for the following reason. On the one hand, the gain in terms of the change in the discounted continuation utilities of buyers and sellers, \( W_n - W_b \), is higher (equation (50)). On the other hand, it is now less likely that this gain is realized, because once they trade sellers are more likely to become mismatched (\( 1 - \alpha \) is lower). If the first effect dominates the second, the total bilateral surplus increases, and so do prices provided \( \eta(\theta)(1 + \theta) > 1 \). This is the case for reasonable parameter values.
A decrease in the extent of matching frictions (e.g. a higher \( q \)) decreases the time it takes to sell a unit and increases total sales. The total bilateral surplus decreases, and so do prices (provided \( \eta(\theta)(1 + \theta) > 1 \)).

**Proposition 2.** Changes in the probability \( q \) that a mismatched household likes a unit visited at random have the following effects on a steady-state equilibrium:

\[
\begin{align*}
\frac{\partial \theta}{\partial q} &< 0; \quad \frac{\partial \pi^s}{\partial q} > 0; \quad \frac{\partial \pi^b}{\partial q} > 0; \quad \frac{\partial b}{\partial q} < 0; \quad \frac{\partial s}{\partial q} = 0; \quad \frac{\partial n}{\partial q} > 0; \quad \frac{\partial (\pi^s s)}{\partial q} > 0; \\
\frac{\partial G}{\partial q} &< 0; \quad \frac{\partial z}{\partial q} < 0, \text{ if } (1 + \beta \theta) \eta(\theta) > 1 \text{ (otherwise the effect is ambiguous)}. \end{align*}
\]

When \( q \) increases, it becomes easier for buyers to find a unit they like. Such a change could reflect for instance an improvement in the intermediation technology, or the access to more information about the units for sale through new channels like the internet. When \( q \) is higher, buyers are more likely to visit units which are more likely to buy. This means that fewer households remain in a mismatched state. Since the number \( s \) of units for sale does not change, the number of households matched with one unit increases. Also, \( \theta \) decreases, and so do prices provided \( (1 + \beta \theta) \eta(\theta) > 1 \). While the probability that a potential buyer likes the house is higher, sellers are now less likely to find a buyer. It is easy to check that the first effect dominates and average time on the market decreases. Hence, more meetings result in trade, and more units are sold.

The higher the housing stock the lower the total bilateral surplus, and the longer the average time on the market. Prices are also lower as long as \( (1 + \beta \theta) \eta(\theta) > 1 \).

**Proposition 3.** Changes in the housing stock \( H \) have the following effects on a steady-state equilibrium:

\[
\begin{align*}
\frac{\partial \theta}{\partial H} &< 0; \quad \frac{\partial \pi^s}{\partial H} < 0; \quad \frac{\partial \pi^b}{\partial H} > 0; \quad \frac{\partial s}{\partial H} = 1; \quad \frac{\partial b}{\partial H} < 0; \\
\frac{\partial n}{\partial H} &> 0, \quad \frac{\partial (\pi^s s)}{\partial H} > 0 \text{ if } (1 + \theta) \eta(\theta) > 1, \quad \frac{\partial n}{\partial H} < 0, \quad \frac{\partial (\pi^s s)}{\partial H} < 0 \text{ if } (1 + \theta) \eta(\theta) < 1; \\
\frac{\partial G}{\partial H} &< 0; \text{ and } \frac{\partial z}{\partial H} < 0 \text{ if } (1 + \beta \theta) \eta(\theta) > 1 \text{ (otherwise the effect is ambiguous)}. \end{align*}
\]

As new vacant units are added, the ratio \( \theta \) of buyers over sellers decreases (from (43)).
This increases the time it takes to sell a vacant unit, and decreases the time it takes to buy one ($\pi^s$ falls and $\pi^b$ increases). The total bilateral surplus decreases because the loss from being mismatched is now lower (equation (50)). Prices $z$ also fall provided $(1 + \beta \theta)\eta(\theta) > 1$. The number $s$ of households matched with two units increases one-to-one with $H$. The number of mismatched household $b$ falls. The number $n$ of matched households with one unit may increase (if $\eta(\theta)(1 + \theta) > 1$) or fall. The same is true about total sales. The reason is that while the number of units for sale increases, it is now harder to sell a unit.

4.2 A quantitative exercise

Here we conduct several quantitative exercises to check the ability of our model to account for the observed co-movements of the price, sales and time on the market.

We assume that the model period is a week. The discount factor $\beta$, in annual terms is set equal to 0.96. The search friction $q$ is set equal to 0.2. That is, a buyer likes one in 5 houses visited. The mobility parameter $\alpha$ is equal to 0.0032. This implies that a household becomes mismatched every 6 years, on average.

In our first exercise we study the steady state effects of changes in the housing stock. This is shown in Figure 1. Notice that the housing stock ranges from 1.01 to 1.06. That is, the number of vacancies goes from 1 percent of the housing stock of each type of houses to 6 percent. The equilibrium price has a non monotonic behavior. This is so because, for small values of the housing stock, there are few vacancies. In this case, the probability that a buyer finds a suitable match is so low that the market is not very active. As the housing stock increases, the probability of finding a suitable place increases and, therefore, sales increase and the price increase.

Next, we turn to examine the effect of mobility at the steady state. In Figure 2 we have plotted the steady state for different values of $\alpha$. The lowest value of $\alpha$ implies that households are hit by the mobility shock every 8 years, on average. The largest value of $\alpha$ implies that households become mismatched every 4 years. The housing stock is 1.04, that is, the number of vacancies is 4 percent. Notice that as $\alpha$ increases the price increases, sales increase and time on the market (the time needed to sell a house) decreases. That is,
the larger the degree of mobility, the “hotter” the market and the lower its liquidity. The point is that a larger \( \alpha \) implies a larger the number of buyers. This increases the price, the volume of sales and lowers the time needed to sell a house. Next, in Figure 3 we show the same exercise for a lower level of vacancies, just 2 percent. Notice that, in this case, the behavior of the price is not monotonic with \( \alpha \). This is so because if the number of vacancies is low, sellers today have a very large probability of becoming mismatched next period, which reduces their trade surplus and therefore, their willingness to sell the house.

Finally, we assess the effect of the idiosyncratic friction \( q \). Remember that \( q \) measures the probability that a buyer likes the house offered by the seller. Figure 4 shows the steady state for different values of \( q \), assuming a number of vacancies equal to 4 percent of the stock of houses. As expected, the price declines with \( q \). This is so because the larger \( q \), the fewer households are mismatched at any given period. Since it is easy for mismatched households to find a suitable house, the number of sales increases. In a way, a larger \( q \) implies that the supply of houses suitable for any given buyer increases. Hence, the price must decreases as well as time on the market whereas sales increase. Figure 5 shows the steady state when the number of vacancies is equal to 2 percent. In this case, again the price is not monotonic with \( q \) for very small values of \( q \). Notice that if \( q = 0 \) the market is inactive. As \( q \) increases, buyers find that there is another house that give them higher utility, thus, they are willing to pay a positive price for it. As \( q \) increases, the effect that there are many suitable houses implies that the price must decline again. This interaction does not appear if the stock of houses is large enough, as in the previous case.

4.3 Dynamics

To be written.

5 Endogenous Housing Stock

In this section, we characterize a long-run equilibrium in the housing market by making endogenous the housing stock \( H \). The description of equilibrium is almost identical to that
in Section 3.2 except that now we assume that each period \( t \) households who do not trade can build new units at a cost.

Specifically, we assume that each non-trader of type \( i \) can build a unit of type \( h \neq i \) by paying a fixed cost \( \kappa > 0 \). To allow for possible decreasing returns in construction (e.g., fixed land), \( \kappa \) is assumed to be convex function of the housing stock \( H \). If they build a unit, these households will become sellers in period \( t + 1 \). Hence, the analysis is identical to that in Section 3.2, except that the Bellman equation of a non-traders in (19) now becomes

\[
W^{in} = \bar{v} + \max\{\beta[(1 - \alpha)W^{in}_{t+1} + \alpha W^{jh}_{t+1}], \beta[(1 - \alpha)W^{is}_{t+1} + \alpha W^{js}_{t+1}] - \kappa\}. \tag{51}
\]

since non-traders optimally choose whether or not to build a new unit. The net gain from building a unit is the gain in terms of the change in the discounted continuation value of the household:

\[
\beta[(1 - \alpha)W^{is}_{t+1} + \alpha W^{js}_{t+1}] - \beta[(1 - \alpha)W^{in}_{t+1} + \alpha W^{jh}_{t+1}]. \tag{52}
\]

The cost from building a unit is \( \kappa \). Moreover, in a long-run equilibrium, the housing stock must be such that the marginal benefit from building a new unit is zero so

\[
\beta[(1 - \alpha)W^{is}_{t+1} + \alpha W^{js}_{t+1}] - \beta[(1 - \alpha)W^{in}_{t+1} + \alpha W^{jh}_{t+1}] = \kappa. \tag{53}
\]

Substituting (53) into (51) yields again (19).

Therefore, a competitive search equilibrium is characterized as in Definition 1, adding \( \{H_t\} \) to the array of equilibrium variables and (53) to the system of equations.

Similarly, a steady state is characterized adding \( H \) as an additional variable and equation

\[
\beta W^s - \beta[(1 - \alpha)W^n + \alpha W^0] = \kappa \tag{54}
\]

to the system in (40)-(47). Substituting (45)-(47) into (54) then yields are relation between \( \theta \) and \( H \). This equation together with (43) determines the steady state values of \( \theta \) and \( H \) in the long run.

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Figure 1: Comparative statics for different values of the housing stock. $\beta = 0.96$ in annual terms, $q = 0.2$, and $\alpha = 0.0032$. 
Figure 2: Comparative statics for different values of the mobility parameter. $\beta = 0.96$ in annual terms, $q = 0.2$, and vacancies are 4 percent. The range of $\alpha$ implies that the household is hit by a mobility shock from every 4 years to 8 years.
Figure 3: Comparative statics for different values of the mobility parameter. $\beta = 0.96$ in annual terms, $q = 0.2$, and vacancies are 2 percent. The range of $\alpha$ implies that the household is hit by a mobility shock from every 4 years to 8 years.
Figure 4: Comparative statics for different values of the friction $q$. $\beta = 0.96$ in annual terms, $\alpha = 0.0032$, and vacancies are 4 percent.
Figure 5: Comparative statics for different values of the friction $q$. $\beta = 0.96$ in annual terms, $\alpha = 0.0032$, and vacancies are 2 percent.
References


